A Heat Transfer Model for Industrial Food Processes

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ABSTRACT

The industrial pasteurization process of jars is investigated theoretically and experimentally. The proposed methodology is a simplified Finite Elemant Analysis (FEA) model for industrial pasteurization that is based on a conduction-only approximation. An FEA software package (Autodesk Algor Simulation) is used to carry out the computations. The time-temperature profiles produced by the FEA software are compared to a simplified conduction approximation and experimental data. The results of the simulation show that the FEA simulation can be used for the range of the samples tested and their properties. A valuable application of this model for optimization is demonstrated for the process and packaging parameters. For the food industry, there is a potentially tremendous value in having an effective methodology to predict the heat transfer behavior for thermally processed foods.

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1 INTRODUCTION

Through recent years, the cost of energy has steadily increased. This economic and environmental growth makes energy efficiency and optimization important factors to consider when designing new manufacturing facilities or processes. Currently, in food processing involving heat transfer, the cost of electric energy and fuels can comprise the majority of production cost. In order to control, predict and optimize these types of processes, it is necessary to know the thermal properties of the food used: specific heat, thermal conductivity and thermal diffusivity. With these properties, optimization of the heat transfer process can be explored through changes in geometry, input parameters, and boundary conditions. However, before optimization of the process can be explored, a proven methodology for predicting and analyzing their time-temperature profiles must be established.

Currently in food research there are two methodologies for exploring optimization in thermal processes. Experimental trials can be performed for changes in process conditions and product modifications. In this case, optimization is achieved through trial and error, which consumes production time and resources. Complex theoretical models for mixed conduction and convection are another method used. Although these models are accurate, they typically include lengthy algorithms and include many variables.

In this research, a simplified method for analysis of thermally treated foods is developed. Although convection and conduction may both occur during the heating and cooling process, a simplified, purely conductive model with convective boundary conditions is proposed. The simplified methodology provides flexibility for design conditions, allowing a wide range of data to be interpolated. This will reduce the need for experimentation, complex calculations and prohibitively expensive software.

1.1 History of Pasteurization

Canning is a method of preserving food. The food is processed and sealed in an airtight container. This provides a typical shelf life of one to five years by preventing microorganisms from entering and proliferating inside. The process was first discovered by Nicolas Appert in 1810 (Wiley, 1994), as a way to prolong the use packaged food for the French military. He noticed that if food was cooked inside a jar, it did not spoil unless the seals leaked, leading him to develop a method of sealing food in glass jars.

Many methods have been developed since the discovery of food preservation in the 1800s. Prevention of food spoilage has become a driving motivator in food industries, because of the increase in demand for food quality. To prevent the food from being spoiled before or during storage, quite a number of methods are used: pasteurization, boiling, refrigeration, freezing, drying, vacuum treatment, antimicrobial agents, acid, base or other microbe-challenging environments. Microorganisms such as bacteria, protozoa, molds and yeasts can be reduced at elevated temperatures. Therefore, one of the most dependable methods of food preservation is heat treatment, which reduces the number of viable pathogens so they are unlikely to cause disease (Kumar, Bhattacharya, and Blaylock, 1990).

An effective method for the production of extended shelf life food products is heat treatment in a tunnel pasteurizer. Pasteurization is a process of heating a food, usually liquid, to an elevated temperature (at least 72 °C) for a minimum of 15 seconds (Grant, Ball, and Rowe, 1998) and then cooling it immediately. In a tunnel pasteurizer, the heat needed to reduce the microorganisms that cause spoilage is applied by spraying the containers with water. Thepasteurizer is divided into several heating zones followed by cooling zones, each of different temperatures. Fig 1-1 shows the design modeled for the present work. After the precooking and packaging of the food or beverage, the containers are passed through the heating and cooling zones on a conveyor belt.

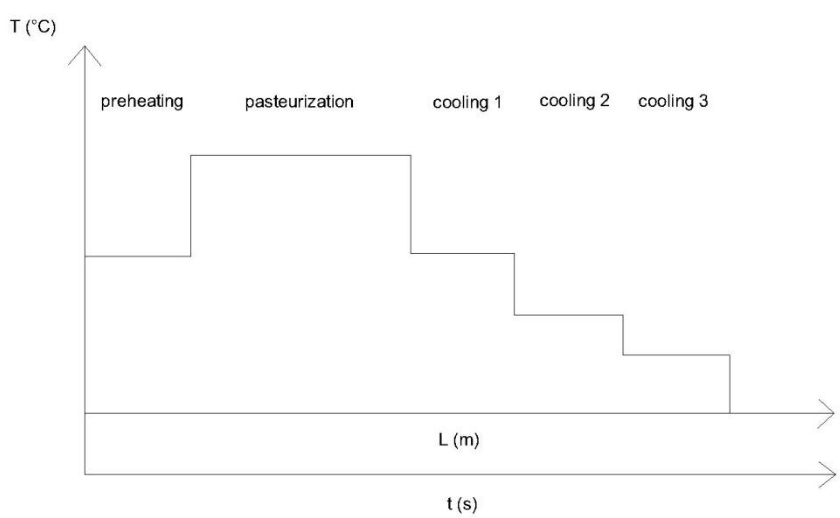

Fig 1-1: Simplified scheme of industrial pasteurizer

Common practice within the food industry is to optimize these processes by means of experimental methods using trial and error methods. The use of the tunnel pasteurizer for the procurement of data for new products is very expensive. The conditions for temperature and speed needed in each zone are typically selected with the aid of basic heat transfer models for the entire container. Process planning through numerical modeling and optimization offers a way to reduce the cost of these experiments, increasing the efficiency and improving the design of the equipment.

1.2 Process Planning

The growth of international markets has created a higher level of competition that demands rapidly reconfigurable manufacturing systems and changes in production standards. In today's global manufacturing market it has become essential to develop strategies to increase the integration, production and distribution of processes. Manufacturing process planning determines how a product will be manufactured. It is a process of selecting and sequencing manufacturing processes, so that certain results can be achieved. Goals may include improvements in cost, processing time, new product design, quality, and productivity.

Specialized software programs exist for various types of design including: three-dimensional modeling, engineering, electronics, and roadways. Within the traditional manufacturing industries (fabrication, metalwork, etc.), process planning and the need for quick turnover are facilitated by the use of Computer-Aided Design (CAD) and Computer Aided Manufacturing (CAM). CAD software can be used in tandem with

Computer-Aided engineering (CAE) software, specifically Algor's Autodesk finite element analysis (FEA), for detailed structural or thermal analysis of three-dimensional models.

CAD software provides the user with input-tools for the purpose of streamlining design processes, drafting, documentation, and manufacturing processes. It is used in the engineering process from conceptual design and dimensions of products, through strength, dynamical and thermal analysis of assemblies to definition of manufacturing methods of products. CAD software has become an important technology within the scope of computer- aided technologies. Associated with using software are benefits such as lower product development costs and a greatly shortened design cycle. CAD is a strategic tool used by engineers. It enables designers to develop work on screen, save time on drawings, analyze components, and print out and save the data for later use.

CAD is used together with other tools as an integrated module or stand-alone product. CAE is one of these modules that can be used in collaboration with CAD software. It can assist in the development of new products and processes by applying computational capability to the CAD models. With the improvement of graphics displays, engineering workstations, and graphics standards, CAE has become an essential tool for solving complex engineering problems and for representing those solutions with the assistance of interactive computer graphics. A wide variety of CAE tools are currently available and span most engineering disciplines. In current engineering practice it is not uncommon to find that the solution of a single engineering problem may require the application of several CAE programs.

Finite element analysis (FEA), sometimes called finite element method (FEM), is one application of CAE that can be helpful when developing new products and processes. It is a numerical technique for finding approximate solutions of partial differential equations (PDE), particularly in complex, three-dimensional objects. It originated from the need for solving complex elasticity and structural analysis problems in civil and aeronautical engineering (Huebner, Dewhirst, Smith, and Byrom, 2001). Today it can be used to solve complex transient and static heat transfer algorithms. Although the approaches taken by each of these methods are different, each shares a critical characteristic. They both utilize mesh discretization of a continuous domain into a set of discrete sub-domains, usually called elements. These elements define the shape and properties of the object to be analyzed. The design is then subjected to virtual forces, temperatures, or other inputs for which the resulting behavior of the material might be observed.

FEA can be used in new product design or remediation of an existing product. A proposed design is able to be verified by a company, ensuring it meets the client's specifications prior to manufacturing or construction. Modifying the parameters of an existing product or structure is allows them to be qualified for new service conditions. FEA may be used to help determine the design modifications necessary to meet the new conditions. In order to solve the complex algorithms of a three-dimensional model, a series of points called nodes are used which make a grid called a mesh (Fig 1-2). The structure will react to certain physical inputs according to the mesh, which is programmed to contain the material and structural properties of the model. As a result a web of vectors, creates small, discrete regions called elements. These elements are what carries the material properties of the object.

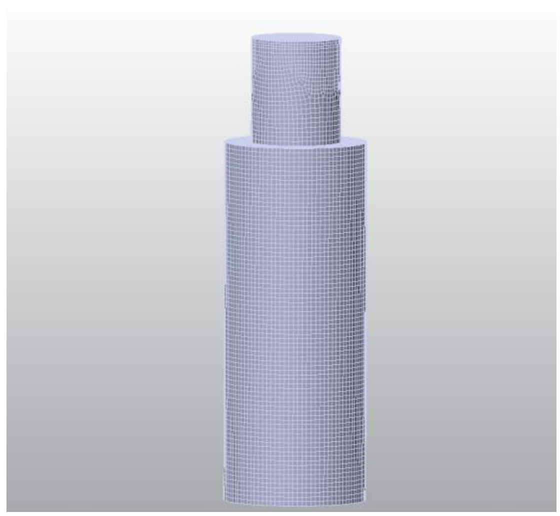

Fig 1-2: FEA mesh of jar

A wide range of objective functions are available for minimization or maximization:

- Mass, volume, temperature
- Strain energy, stress
- Force, displacement, velocity, acceleration

There are multiple loading conditions which may be applied to a system:

- Point, pressure, thermal, gravity, and centrifugal static loads
- Thermal loads from solution of heat transfer analysis
- Enforced displacements

- Heat flux and convection

- Point, pressure and gravity dynamic loads

1.3 Computer-aided Process Planning

Computer-aided process planning is a link between design and manufacturing. Commonly used by manufacturing engineers, CAPP can be used to develop a product manufacturing plan based on projected variables such as cost, lead times, equipment availability, product volumes, potential material substitution routings and testing requirements.

Prior to CAPP, manufacturers attempted to overcome the problems of manual process planning by classifying parts into families and developing standardized processes plans for parts families. When a new part or modification was introduced, the family process plan would be retrieved, marked-up and redone. Although this may have improved productivity, it did not consider the differences between parts in a family or evaluate the production process for improvements. Therefore, the quality of the planning process was not improved. As the design process is supported by many computer-aided tools, CAPP has evolved to simplify and improve process planning and achieve more effective use of manufacturing resources.

There are substantial benefits that have been demonstrated from the use of CAPP. In a detailed survey of twenty-two large and small companies using generative-type CAPP systems, the following estimated cost savings were achieved (Crow, 1992):

- 58% reduction in process planning effort
- 10%saving in direct labor
- 4%saving in material
- 10%saving in scrap
- 12%saving in tooling
- 6%reduction in work-in-progress

In addition, there are intangible benefits as follows:

- Reduced process planning and production lead-time; faster response to engineering changes
- Greater process plan consistency; access to up-to-date information in a central database
- Improved cost estimating procedures and fewer calculation errors
- More complete and detailed process plans
- Improved production scheduling and capacity utilization

1.3.1 Applications of Computer-Aided Process Planning

Since the first CAPP system was developed in the early 1960's, there has been rapid growth in the development of various CAPP systems. This growth spans from industrial prototypes to modification of existing products and processes. CAPP has evolved to simplify and improve process planning and achieve more effective use of manufacturing resources. Currently there are two general approaches to CAPP, generative and variant. Both are associated with specific planning techniques. The variant approach involves retrieving an existing plan for a similar part and making the necessary modifications to the plan for the new part. It follows the principle that similar parts require similar plans. The generative approach involves generation of new process plans by means of decision logics and process knowledge. It utilizes decision logic, formulae, manufacturing rules, and geometry based data to determine processes required to convert raw materials into finished products (Ahmad, Haque, and Hasin, 2001).

1.3.2 Current Research in Process Planning

In the last twenty years significant research work has been developed in different areas in CAPP. Ahmad, Haque and Hasin have researched and categorized the types of these works by part and geometric modeling techniques. CAPP systems have been introduced to improve the design of various prismatic and cylindrical parts (Jaekoo; Kyoung; Kim, Kim, Pariente, and Wang; Younis and Wahab, 1997), fabrication of sheet metal (Abo-Rayia), assembly systems (Arieh and Kramer, 1994; Arnold, 1996; Wang, 1989), and foundry (Ajmal). Feature based and solid model based techniques included a solid model based automated process planning system for integrating Cad and CAM systems (Nasser, El-Gayar, Zahran, Parsaei, and Leep, 1996), a feature based approach for cylindrical surface machining (Kim, Kim, Pariente, and Wang), and a feature based process planning for sculptured pocket machining (Jaekoo).

In traditional manufacturing CAPP is typically used for new product design and implementation. With it, the routine clerical tasks of the manufacturing engineers are reduced, more time may be spent on productive tasks. With the aid of computers it becomes easier to generate production routings that are rational and consistent without depending on the experience or the judgment of the individuals involved. Tools such as CAD, CAM and FEA can also be used outside the scope of new product design. Computer-aided engineering can be used to improve process efficiency and explore new methods of engineering through research. CAE software allows the user to simulate real life production scenarios without costly and time consuming experimental trials. This allows researchers to explore the effects changes in process parameters, boundary conditions, geometry, process time, etc., theoretically before investing capital in design changes.

CAPP, specifically CAD and FEA, has not been widely used in the food industry. One major reason is that the food industry is often not recognized as “manufacturing” in the traditional sense. Those in the industry sometimes resist that designation, and those outside the industry do not generally think of food when they think of manufacturing. Principles, practices, and even personnel of each industry segment seldom cross the borders between traditional manufacturing and the food industry.

Also, since food quality is usually the target of concern, the industry tends to lean towards manufacturing techniques that have been proven for their particular domain of products. It has not been typical for food manufacturers to explore new production methodologies or process efficiency in an attempt to maintain quality standards. However, with the growing awareness of energy consumption more industries are focusing on lean and green manufacturing. Exploring process efficiency has become more prevalent than ever. With the increasing knowledge of food quality, FEA and CAD software can be used to analyze energy efficiency in food processes.

Introducing CAPP to the food industry will not only improve product and process efficiency and quality, but can reduce the labor time required for process improvements. For processes including heat treatment of food products, the heat transfer can be optimized through computer simulation. Some of the potential benefits from applying CAD and FEA to food manufacturing include:

- Reduced process planning
- Reduced process time
- Improved heat transfer efficiency
- Saving in labor

- Reduced production lead-time
- Improved production scheduling and capacity utilization
- Improved ability to introduce new products and manufacturing technology

1.4 Problem Statement

The objective of this research is to develop a predictive heat transfer model for thermal sterilization and pasteurization of food products in glass containers. Such a model would enhance the capabilities of process planning, reduce the need for iterative trial and error methodologies, and increase the level of integration of computer-aided design and computer-aided engineering software. This model should provide significant adaptive capability for the various design parameters needed to meet the quality standards of cooling and heat treatment of food manufacturing. Furthermore, it is the objective of this research to obtain the stated results by method of transient heat conduction and determine under which conditions the model applies. Such a model is preferable to a convection model because of the large cost differential for FEA software that is capable of the Computational Fluid Dynamics (CFD) necessary to address convection. A finite element method and mathematical model will be implemented as an alternative to a costly and time intensive trial period of new products needing heat treatment or cooling.

1.5 Purpose of Research

This research will be focused upon the development of a heating model for pasteurization of substantially liquid food products in 12-32oz glass containers. A CAPP system architecture will be used utilizing CAE, specifically FEA software, to analyze thermal loading conditions and process times for pasteurization of a variety of foods. It is the goal of this research to develop a model that encompasses the high-variety and high volume product mix that is needed for a sterilization process that includes a wide range of foods with different thermal properties. The process has many features of the domain which, for any given product, may need to be controlled or improved for process optimization. These distinctive features include:

1. 1. Processes that are time dependent. Production is always running on a schedule and process inputs may need to be manipulated in order to meet deadlines.

2. 2. Multiple pasteurization sections that have independent temperatures. Each section of the pasteurizer is capable of producing a given range of temperatures. These chambers may have time or temperature constraints due to energy availability.

3. 3. The large number or parts involved for multiple packaging requirements. For any given product there can be multiple container shapes including multiple volumes and surface areas.

4. 4. High variation in processes at the same station for a given container. While the sequence of operations are similar, the processes within the operation may vary significantly. For example the pasteurizer may be used for both thermal sterilization or cooling of product for any given geometry.

5. 5. Variation in equipment or layout between facilities. Manufacturing facilities that require thermal processing may vary in process procedures. The content and geometry of each container will influence which process or methods that will be included.

6. 6. Advances in technology. The industry is growing and becoming more dynamic as new processes are developed.

In summary, the purpose of this research is to advance the integration of computer aided simulation for thermal treatment of the industrial food process and develop a model that can adequately predict the time-temperature profiles of a variety of foods through a heat conduction approximation. It is important that the model has the flexibility and adaptability necessary to simulate both the pasteurization and/or cooling scenarios that are present in food manufacturing facilities. Existing computational fluid dynamic (CFD) analysis for various bottled liquids and mathematical models for heat conduction of canned foods have been studied as discussed in section 2.4 (Ghani, Farid, Chen and Richards, 1999; Varma and Kannan, 2006; Kumar, Bhattacharya and Blaylock, 1990). The contribution of this research is to develop an architecture for the pasteurization and cooling process which minimizes the complexity of fluid mechanics and presents an alternate solution to tedious trial and error methodologies used in new product development that waste energy and consume manufacturing time. The model-based approach is a viable solution to process planning in any manufacturing environment and provides the basis for adaptable analysis within that type of environment.

1.6 Methods of Research

The methodologies in the presented work that will provide significant evidence of function while supporting the objectives stated above are:

- A calorimetric approach to determine the basic thermal properties of the foods tested, such as specific heat and mass density (Perez-Martin, Gallardo, Banga, and Casares, 1989).
- CAD software, SolidWorks, for the geometrical make-up and foundation for the creation, exchange, visualization, animation, interrogation and annotation of digital models of the containers and product used for experiments.
- Finite element analysis software, Autodesk's Algor Simulation, for the prediction and analysis of transient heat transfer with the capability of reproducing experimental and typical manufacturing results (Comini, Cortella and Saro, 1994)

- A testing facility designed to represent a complete section of a tunnel pasteurizer with identical operating conditions to an industrial process and special emphasis on the reproducibility of process conditions and automation of experimental procedures (Horn, Franke, and Blakemore, 1997).

The primary methods used in the work presented for an improved process planning system and design of a predictive heat transfer model are calorimetry, solid modeling, finite element analysis, and a pasteurizing testing facility. CAD and CAE software (Solidworks and Autodesk Algor Simulation) provide the flexibility needed for simulation of the various geometric shapes and properties needed for the scope of this research. The testing facility can produce a realistic basis for which the software can be compared. Each of these methods can be introduced by the parameters found using a simple calorimeter, scale and volume of the product being used.

1.7 Expected Results of Research

The results of the research shall be to demonstrate a finite element analysis and mathematical model for thermal pasteurization and cooling for a range of food products and their glass containers. It is expected that the analysis methodology will significantly supplement the process planning system for industrial food processing with results which:

- resemble the physical characteristics of the range of products used within their given geometry,

- accurately analyze the time-temperature profiles of the samples through the use of three-dimensional modeling and FEA software,
- correspond to proven methodologies of solving transient heat conduction of cylindrical and rectangular coordinates, and
- reflect “real time” production scenarios that may range from several production hours to minutes depending upon the values assigned.

2 LITERATURE SURVEY

2.1 Introduction

As a result of energy prices increasing, industries have turned their attention to developing methods for reducing energy consumption. Energy efficiency and optimization have become important factors to consider when designing and modifying manufacturing facilities or processes. In the food industry, processes involving heat transfer can comprise of the majority of manufacturing costs. There have been many approaches to understanding efficiency in thermally treated foods researched. This literature survey aims to address the areas that have been studied and identify how our understanding of the process can be improved.

In order to control, predict and optimize thermal processes, it is necessary to know certain properties of the food used: specific heat, thermal conductivity and thermal diffusivity. Perez-Martin, Gallardo, Banga and Casares (1989) determined the thermal conductivity values of dried and raw albacore muscle via a thermal conductivity probe. A microcalorimetric method and differential scanning calorimetry were used to obtain the specific heat values of the dried and raw muscle at several temperatures. Thermal diffusivity was calculated during the heat treatment of cylindrical cans filled with edible parts of precooked albacore. The experimental temperature curves were adjusted to those obtained from an analytical solution of heat transfer equations where cylindrical geometry and the heat transfer through conduction were assumed.

Simulation of the heating and cooling processes in the food products can be carried out through the use of numerical modeling. Numerical modeling allows improvements in quality, safety, and shelf life to be explored in a cost reducing and time saving manner. Broad experimental conditions can be simulated to produce information about the processes within a short time. Process experiments are often restricted to special conditions due to time and cost limitations. It is, however, necessary to use this experimental approach for the validation of numerical models. With the aid of predictive models, processes can be analyzed for better understanding of complex mechanisms and evaluating the process for safety and quality while designing and optimizing food processes and systems.

Methodologies for process planning and for modeling the processes used to produce thermally treated food have evolved over the past thirty years. The use of numerical methods to describe the heating and cooling processes in the food industry has produced many different models. Most analytical heat and mass transfer models involve lengthy and complex equations. Typically these problems can only be solved by hand for simple cases. Numerical methods are a useful tool for estimating heat transfer under the realistic complex conditions that include time dependent boundary conditions, variation in initial temperature and irregular shaped bodies (Puri and Anantheswaran, 1993).

These models have been developed with the use of finite difference, finite element and finite volume methods (Wang and Sun, 2003). Review of the literature reveals that finite element and finite difference methods in heating and cooling processes of foods have been developed mostly in the late 1990's and early twenty-first century (Schmalko, Morawicki and Ramallo, 1997; Gowda, Narismham, and Murthy, 1997; Fasina and Fleming, 2001; Akterian, 1996; Avila, Manso and Silva, 1996; Wang and Sun, 2003; Zhao, Kolbe and Craven, 1998; Comini, Cortella and Saro, 1994; Nicolai and Baerdemaeker, 1996; Varga and Oliveira, 2000; Lin, Anantheswaran and Puri, 1992). Methodologies for analyzing and predicting pasteurization in the industrial processes have been typically modeled for purely liquid foods. In recent years, the finite volume method has been the main component of computational fluid dynamics (CFD) software. The use of CFD software to simulate thermal processes has increased due to its ability to analyze complex flow behavior of foods (section 2.4).

2.2 Finite Difference Method

The transport equations involved in governing the mechanism of heating and cooling processes are a differential type, because variables such as temperature and moisture depend on time and position. In mathematics, the finite difference method is a numerical method for approximating the solutions to differential equations using finite difference equations to approximate derivatives. The solutions to differential equations are found by replacing derivative expressions with approximately equivalent difference quotients. It is simple to formulate a set of discretized equations from the transport differential equations in a differential manner (Chandra and Singh, 1994). The surface temperature predictions by the FD method are less satisfactory with foods of irregular shapes due to geometric simplification. Therefore, the FD method is widely used for simple geometries such as cylinders, spheres and plates. This section summarizes some of the key work that has been published on the use of FD for modeling of thermally processed foods.

Schmalko, Morawicki and Ramallo (1997) simultaneously determined the specific heat capacity and thermal conductivity of twigs of yerba mate (Ilex paraguariensis Saint Hilaire) using the finite-difference method. Transient heating data and the FD method were used in determining its specific heat capacity and steady-state data for thermal conductivity. It was concluded that both properties could be determined with mean error of 9.87%. Error was found to increase as a function of moisture content.

Ansari (1999) used the FD method to analyze problems involving pure heat transfer and simultaneous heat and mass transfer from the surface of solids exposed to a cooling environment. His worked was aimed at finding an optimized finite difference scheme for regular shaped bodies such as an infinite slab, infinite cylinder and sphere. A range of their characteristic dimensions (thickness or radius) was analyzed with all possible options of dimensionless time increments to find the best convergence and truncation error criteria. Forced-air precooling of spherical foods has also been studied using the FD method (Gowda, Narismham, and Murthy, 1997). Their numerical study was performed using similar varying process parameters over a wide range. The results included the variation of moist air properties along the height of the packaging and the effect of each parameter on the process time. The range of parameters for optimal precooling was identified and correlations for the process time based on the product center and mass-averaged temperatures in terms of the process parameters.

Fasina and Fleming (2001) developed a FD model for the heat transfer characteristics of cucumbers during blanching (rapid water heating). A two-dimensional, cylindrical coordinate, heat diffusion equation was used to simulate the heat transfer characteristics during the process. They were solved by explicit form. Thermal conductivity, specific heat and density of the cucumbers were determined to solve the differential heat transfer equations. It was determined that the two-dimensional diffusion equation could be used to predict center and surface temperatures with an error less than 4.5°C. This method has also been analyzed for an on-line control process of sausages (Akterian, 1996). He integrated a generalized heat conduction equation for thermal processing of solid foodstuffs with an irregular shape. Computed functions of sensitivity related to three kinds of cooked sausages were given.

Avila, Manso and Silva (1996) modeled the thermal sterilization of foods inside packs with two divisions using the FD method. A three dimensional mathematical model, using a single step standard explicit finite differences method with non-capacitance surface nodes was developed to describe the heat transfer into packs with two rectangular divisions. The sterility value was calculated as a function of position in order to identify the location of the least lethality point. The product heating rate and the surface heat transfer coefficients proved to be the most important variable affecting the position of this point. This point has also been identified for cylindrical containers

(Silvia and Korczak, 1994). For uniform surface heat transfer coefficient the least lethality point is always located at the axis of symmetry with the smallest dimension. Several other authors have published mathematical models to describe the transfer into rectangular packages (Tucker, 1991; Manson, Zahradnik and Stumbo, 1970; Tucker and Clark, 1990; Tucker and Holdsworth, 1991).

Research has shown that the finite difference method has been a valuable tool for analyzing the effects of heat and mass transfer during thermal treatment of foods. Thermal properties, such as specific heat and thermal conductivity, can be solved by approximating solutions to differential equations. Problems involving pure heat transfer and simultaneous heat and mass transfer from surfaces of solids exposed to heating and cooling environments have been analyzed as well. Its use is typically limited to geometries of regular shape, including: infinite cylinders, spheres, and infinite plates. Optimization of simple geometries has been explored using a finite difference scheme.

2.3 Finite Element Method

In a continuum problem of any dimension, the field variable (pressure, temperature, displacement or stress) possesses an infinite number of values because it is a function of each generic point in the solution region. Therefore, the problem has an infinite number of unknowns. The finite element discretization process reduces the problem to one of a finite number of unknowns by dividing the solution region into elements and expressing the unknown field variable in terms of assumed approximating functions within each element (Huebner, Dewhirst, Smith, and Byrom, 2001). The finite element method may perform better than the finite difference method for irregular geometries, complex boundary conditions and heterogeneous materials. When discretizing the large domain into small elements, element equations are assembled and solved (Wang and Sun, 2003). The relationship between heat transfer rates and temperatures of an element are given by the temperature matrix (Bocioaga, 1996):

$$[\ K\ ]element\ \{\ t\}element = \{\ f\ \} \tag{1}$$

The temperature in the interior of the finite element is written as:

$$t = \sum_{k=1}^{n} t_k v_k = v_k \quad t_k \tag{2}$$

Here t_k, where k = 1 through n are the temperature values in the nodes. The functions v_k are polynomial functions and belong to the Solbolev space function. Because they are forming a base in the space these functions are linear independent. These functions are forming a base because any temperature from the finite element can be written like a linear combination with them. The FE discretization of the governing differential equations uses interpolating polynomials to describe the variation of the analyzed variables within an element. Although this discretization is different when comparing the FE method to the FD method, it is usual to use the FD method for the time progression in a transient problem (Rao, 1989; Stasa, 1985). The FE method has been successfully used in various analysis regarding thermal sterilization and the heating and cooling of different foods.

Wang and Sun (2003) modeled conventional cooling processes of cooked meat using the finite element method. A two-dimensional transient heat transfer problem for cooling by means of slow air, air blast and water immersion were solved for the cooling of ellipsoid shaped precooked meats. Variations in thermal properties of the food and in operating conditions during the cooling process were included in the model. The authors developed a program to solve the finite element model and compared it with experimental results. The maximum deviation between the predicted and experimental core temperatures for all cooling processes was within 2.9°C. Computational simulation has also been used to analyze the chilling and freezing of albacore tuna (Zhao, Kolbe and Craven, 1998). A commercial finite element computer program package, PDEase, was used to simulate an infinite elliptical cylinder, where nonuniform boundary conditions present complications in computer simulation. Temperature-dependent thermal properties and time-dependent ambient temperature conditions were used in the developed equations that defined the elliptical boundary. Measured time-temperature profiles were compared with the results from the finite element program. The deviation was found to be between 3 to 6%. Similar approaches using finite element analysis of coupled conduction and convection in refrigerated transport have been researched (Comini, Cortella and Saro, 1994).

Nicolai and Baerdemaeker (1996) developed a new method to compute the sensitivity of the time-temperature profile inside conduction heated foods with respect to the surface heat transfer coefficient. Their method was based on the finite element formulation of the heat conduction equation. Sensitivity charts relating the dimensionless center temperature (θ) and Biot number (Bi) were presented for plate, cylindrical and spherical shapes. It was concluded that in cases with low surface heat transfer coefficients small deviations in the surface heat transfer coefficient resulted in large deviations in the core temperature of the food.

Varga and Oliveira (2000) determined the external heat transfer coefficient in steam retort processing by applying finite element and using the actual retort temperature profile as boundary conditions. It was determined that reliable results for the external heat transfer coefficient at any time could be obtained from the average heat transfer coefficient of the course of time. The heat transfer coefficient increased sharply in the first few minutes and then remained relatively constant. It was determined that constant heat transfer coefficients during heating and cooling were sufficient to describe temperature profiles inside conduction heated products during retort.

Lin, Anantheswaran and Puri (1995) studied the thermal effects of microwave heating solid foods with rectangular and cylindrical geometries using finite element analysis. A two-dimensional commercial finite element software, TWODEPEP, was used for the model. Absorbed microwave power density for locations in the tested material was derived as a function of dielectric properties and the geometry of the material. Sodium alginate gel was used as the testing material for validation of the simulated model. The temperature predictions by the software agreed with the experimental measurement in slab-shaped models. However, in cylindrical samples, the experimental data and finite element predicted values differed at the central region. They suggested a three-dimensional finite element model may be necessary to take into account more complicated sample geometries and manage the complicated field distribution for processes within the oven.

The finite element method is useful when considering objects of irregular geometry, complex boundary conditions and heterogeneous materials. Commercial finite element computer program packages can be used to model irregular shaped geometries that would otherwise require complex algorithms and differential equations. Heat transfer coefficients and temperature profiles have been produced by several authors for solid foods of cylindrical and rectangular geometries. With the knowledge of how food reacts to thermal process, sensitivity analysis can be performed on the process parameters.

2.4 Computational Fluid Dynamics

Computational fluid dynamics (CFD) is a simulation tool for the solution of fluid flow and heat transfer problems. Recently, CFD, derived from the finite volume (FV) method, has been the method of choice when simulating thermal processes of foods for analyzing complex flow behavior (Scott and Richardson, 1998; Sun 2002). In CFD calculations, the continuity equation, Navier-Stokes transport equations, and the first law of thermodynamics are numerically solved to give predictions of velocity, pressure, temperature and other fluid flow characteristics (Sun, 2002). The FV method was derived from the finite difference method. The domain is divided into discrete control volumes, where the transport equations are integrated over a control volume to yield a discretized equation at its nodal points (Vertseeg and Malalaskera, 1995). Although CFD has been applied to aerospace, automotive and nuclear industries since its arrival, only recently has it been applied to the food processing industry to the rapid development in computer and commercial software packages. A review of CFD in the food industry has been done by been researched by multiple authors (Scott and Richardson, 1997; Xia and Sun, 2002).

Ghani, Farid, Chen and Richards (1999) simulated liquid food during sterilization by the governing equations for continuity, momentum and energy conservation for an axisymmetiric case using commercial CFD software packages. Liquid models enclosed in cylindrical cans and comprised of sodium carboxy-methyl cellulos (CMC) and water were used for analysis. These liquid models were assumed to have constant thermal properties except for viscosity and density. The research showed that natural convection forced the slowest heating zone towards the bottom of the can as expected. The results of the simulation show a recirculating flow inside the can consisting of liquid rising near the wall, radial flow, and uniform core flow downwards near the axis. For CMC the coldest region covers the whole cross sectional area of the can at early stages of heating, then migrates towards the bottom of the can. Similar work was done by Datta and Teixeira (1988) also used a numerical model to predict transient flow patterns and temperature profiles during natural convection heat transfer of a liquid in a uniformly heated cylindrical can. The research concluded that the slowest heating location in the fluid was a donut-shaped region close to the bottom of the can about one-tenth the can height.

Varma and Kannan (2006) studied the natural convective heating of canned food in both conical and cylindrical containers. They explored the enhancement of natural convecting heat transfer through modifications to container geometry and its orientation. A conical geometry, of equal volume and height of a cylinder, was one such geometry considered. CMC liquid was used as the test food material and its laminar flow behavior investigated using CFD software. The movement of the slowest heating zone temperature was tracked and compared for the cone and cylinder of different orientation. In the case of the cone pointing downwards the heat transfer rate at the bottom was found to be inhibited by convection. The results of the study showed that the cone pointing up achieved an improvement of 15% over that of the cylinder and 23% over that of the cone pointing downward. These results confirmed that the efficiency of heat transfer of thin viscous foods can be improved by geometry modifications and without agitation or rotation.

Kumar, Bhattacharya and Blaylock (1990) developed a numerical simulation of natural convection heating of canned thick viscous liquid food products in a metal can sitting in an upright position and heated from side wall only. The liquid proved to have a temperature dependent viscosity but constant specific heat and thermal conductivity. Equations of mass, motion and energy conservation for axisymmetric cases were solved and compared to the temperature profiles of pure conduction contour plots. Results verified that slowest heating zone to be near the bottom of the can. The slowest heating zone heated up at a slower rate than one would predict by assuming conduction heating.

In summary, the finite volume method has been applied to thermally processed liquid foods using computational fluid dynamic software under various conditions. Liquid food is typically thermally treated in cylindrical containers. Several authors have explored the reaction of the food during this processes using CFD models. The effects of geometry shape and orientation have been compared during convection heating. The slowest heating zone during these processes have been determined in simple conical and cylindrical geometries. By applying the methods researched, optimization through changes in geometry and boundary conditions can be explored for a wide range of process conditions.

2.5 Pasteurization Models

There has been a limited amount of research done on the prediction of three dimensional temperature distributions of cylindrical geometries during tunnel pasteurization. As shown in the previous sections, multiple models have been developed for thermal processing of canned foods without motion, which is not the case in a pasteurizer. During pasteurization, jars are transported through different temperature sections. Due to the complex nature of fluid dynamics within the jar, the evaluation of the temperature profiles within the jar can prove to be a difficult task. An analytical solution for two extreme theoretical cases, perfectly mixed content and heat conduction through non-mixed food in the can, have been developed (Merson, Singh, and Carroad, 1978; Lenz and Lund, 1977). However, in many pasteurization scenarios with dynamic operational conditions and varied initial and boundary conditions can make it impossible to solve the problem analytically.

Plazl, Lakner and Koloini (2004) modeled the temperature distribution in canned tomato based dip during industrial pasteurization. The model follows the pasteurization process through six zones of industrial pasteurization with different operational conditions. The thermal diffusivity and specific heat of the tomato based dip was experimentally determined. A finite difference method was used to solve a three-dimensional heat conduction equation with variable boundary and initial conditions. With slight modifications, it may be used to design, optimize or control the industrial pasteurization process for various canned food products.

Dilay, Vargas, Amico and Ordonez (2006) introduced a general computational model for the simulation and optimization of beer pasteurization tunnels. A combination of the proposed simplified physical model with an adopted finite volume scheme were verified by a direct comparison with actual temperature experimental data. This data was measured with a mobile temperature recorder traveling within such a tunnel at a brewing company. The experimentally validated model was used in an optimization study to determine the optimal geometry for minimum energy consumption by the tunnel. The methods used incorporated a converged mesh through the experimental validation of numerical results which combined numerical accuracy with a low computational time. This resulted in a model that is useful for simulation, design and optimization of pasteurization tunnels.

The mathematical simulation of pasteurization using transient heat conduction is another way to investigate the time-temperature profiles of canned foods. Bhowmik and Shin (1991) proposed a mathematical model to evaluate the cylindrical cans undergoing transient heat conduction using convective boundary conditions. The model included convective heat transfer coefficients for heating and cooling media, thermal diffusivities of plastic can wall and the canned food, and contact conductance between the plastic wall and the canned food. The predicted slowest heating zone of the can was calculated through partial differential equations of heat conduction and compared to experimental results obtained by the use of thermocouple wiring. Temperatures estimated by the model at the coldest point in a can agreed closely with those determined experimentally during thermal processing. It was determined that thermal diffusivity of the can wall and heat transfer coefficients of heating the heating and cooling media considerably influenced the sterilizing values the food.

When designing a tunnel pasteurization plant, cost, time, energy and productivity are usually the basis for developing the system. It is not frequent that the convective transport of microorganisms and staling effects are considered during the design process. Recently, there have been an increase in demand for product quality and shelf life and these parameters need to be considered when designing processes. Horn, Franke, Blakemore and Stannek (1997) developed a model for the unsteady convective heat transfer inside a bottle, taking into account the influence of the convective flow on pasteurization and staling effects. Pasteurizing units were used to describe the effects achieved through one minute of heating at 60°C and was developed by Del Vecchio (1951) during the study of beer spoilage organisms. The simulations results of Horn, Franke, Blakemore and Stannek indicated that the model gave a good representation of experimental results. However, simulation studies show that the procedure for determining pasteurization units can overestimate the actually effect considerably if the reference point is not chosen accurately with reference to the bottle size and shape. The study suggested that a similar unit can be defined for the pasteurization induced staling effect.

2.6 Food Quality

In recent years, much attention has been shifted to maximizing nutrient quality during the sterilization of canned foods. Computer simulation has made optimizing the quality of thermally processed foods possible since kinetics of microorganisms and quality factors are well understood and can be described with mathematical models (Ball and Olson, 1957; Stumbo, 1973). These foods are subject to reduction in microorganism concentration under these heating and cooling conditions. Most studies have been based on the optimization of some functional form for sterilization temperature profiles. Saguy and Karel (1979) applied Pontryagin's Minimum Principle (Pontryagin, Boltyanskii, Gamkrelidze, and Mishchenko, 1962) to a lumped model for the for nutrient and microorganism concentration, while retaining the distributed nature of the thermal conduction process. Their research indicated that the time-temperature profile of the food would be required to optimize nutrient retention.

Nadkarni and Hatton (1995) derived optimal control policies for maximizing nutrient retention for a given reduction in microorganism concentration within a can during sterilization. Their mathematical model for optimal control of retort temperatures during the sterilization process assumed that heat transfer inside the cans occurs primarily by heat conduction. Their research showed that the heating and cooling rates should be as rapid as is permitted by process constraints and that control of retort temperature is the optimal control strategy to adopt. There should be only one heating and cooling cycle during the sterilization process and not a series of heating and cooling steps as indicated by other research (Teixeira, Zinsmeister, and Zahradnik, 1975; Saguy and Karel, 1979).

Kseibat, Mittal and Bair (2004) developed an artificial neural network (ANN) for reliably predicting the process temperature and process time for minimum quality degradation during thermal processing of canned foods. Can geometry, initial temperature, thermal diffusivity and a sensitivity indicator of microorganism and quality were used to predict the process variables. The trained ANN can provide optimum values of temperature, time and quality degradation which is important in conduction-heated foods, where large temperature gradients exit due to slow heating behaviors. By selecting the desired quality degradation, the food quality can also be maintained during food sterilization in a canning process.

Food quality and taste are among the main concerns of any industrial food processor. Research has determined that various design conditions can have an effect on the quality of food. Using the mathematical methods presented (finite difference method, finite element method, and finite volume method), sensitivity analysis has been performed on nutrient retention and food quality. It was determined that to maximize nutrient retention, heating and cooling rates should be as rapid as is permitted by process constraints and that control of process temperatures is the optimal control strategy to adopt.

2.7 Analytical Mathematical Modeling

To predict cylindrical dynamic temperature distributions in foods, solving of a heat conduction equation is necessary. The cylindrical geometry of the jar dictates the axis-symmetrical heat transport functions that are used. Preliminary calculations of the layer of air in the head space indicate that the heat transport through conduction can be neglected in comparison with the conduction heat transport through the food product. Also, the heat transfer resistance between the surface of the jar and the circumferential fluid can be neglected (Hanzawa, Wang, Suzuki and Sakai, 1998).

The time-temperature profile of a conduction heated food in cylindrical containers is normally characterized by Fourier's law and is expressed by the following (Bhowmik and Shin, 1991).

For the container wall:

$$\frac{1}{\alpha_w}\frac{\delta T_w}{\delta t} = (\frac{\delta^2 T_w}{dr^2} + \frac{1}{r}\frac{\delta T_w}{\delta t} + \frac{\delta^2 T_w}{\delta y^2}) \tag{3}$$

For the food:

$$\frac{1}{\alpha_f}\frac{\delta T_f}{\delta t} = (\frac{\delta^2 T_f}{dr^2} + \frac{1}{r}\frac{\delta T_f}{\delta t} + \frac{\delta^2 T_f}{\delta y^2}) \tag{4}$$

Where T = temperature in the jar as a function of location and time, t; T_w=$T_f = T_i$ at t=0; $\frac{\delta T_w}{\delta t}$ = outward normal gradient of temperature; r is radial distance and y is axial distance; α_w is the thermal diffusivity and can be expressed as:

$$\alpha = \frac{k}{\rho c_{p_f}} \tag{5}$$

Thermal conductivity, specific heat and heat transfer coefficients are considered in the boundary conditions. The above differential equation can be summarized into one analytical solution for transient heat transfer of a cylinder (Cengel, 2007):

$$\theta = \sum_{n=1}^{\infty} \frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} e^{-\lambda_n^2 \tau} J_0(\frac{\lambda_n r}{r_0}) \tag{6}$$

$\lambda_n{}'s$ are the roots of:

$$\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = Bi \tag{7}$$

The dimensionless temperature:

$$\theta\ r,\tau\ = \frac{T\ r,t\ - T_\infty}{T_i - T_\infty} \tag{8}$$

The dimensionless time, Fourier number:

$$\tau = \frac{\alpha t}{{r_0}^2} \tag{9}$$

The dimensionless heat transfer coefficient, Biot number:

$$\mathrm{Bi} = \frac{\mathrm{hr_0}}{\mathrm{k}} \tag{10}$$

The analytical solutions derived in the above equations for one-dimensional transient conduction in a cylinder involves infinite series and implicit equations which are difficult to evaluate. These analytical solutions can be simplified and presented in tabular or graphical forms using simple relations. One-term approximation is one such tabular form developed for transient heat conduction. The dimensionless temperature, $\theta\ r,\tau$, can be determined for any given point along the radius of the cylinder:

$$\theta\ r,\tau\ = A_1 e^{-\lambda_1^2 \tau} J_0(\frac{\lambda_1 r}{r_0}) \tag{11}$$

The constants A_1 and λ_1 are functions of the Biot number located in Table 2-2. The function J_0 is the zeroth-order Bessel function of the first kind, and can be determined from Table 2-1. If you consider the center of a cylinder where cos(0)= J_0(0)=1 from the table mentioned and the limit of (sin(x))/x is also 1, these relations can be simplified at the center of the cylinder. Therefore, the time-temperature profile of the center of a transient conduction for heated food in a cylindrical glass jar can be solved using this method:

$$\frac{T\ x,t\ - T_\infty}{T_i - T_\infty} = A_1 e^{-{\lambda_1}^2 \tau} \tag{12}$$

The terms in the series of solutions converge rapidly with increasing time, and for τ >.2, keeping the first term and neglecting all the remaining terms in the series results in an error under 2 percent. We are usually interested in the solution for times with τ>.2, and thus it is very convenient to express the solution using one-term approximation.

The system is assumed to have an infinite supply of heat, therefore assuming a Biot number of infinity. The coefficients A_1 and λ_1 used in the one-term approximation solution can be interpolated by the Biot number shown in Table 2-1.

Table 2-1: The zeroth- and first-order Bessel function of the first kind (Cengel, 2007)

H	(η)	(η)
0.0	1.0000	0.0000

0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.9463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	-0.0968	-0.4708
2.8	-0.1850	-0.4097
3.0	-0.2601	-0.3391
3.2	-0.3202	-0.2613

Table 2-2: One-term approximation constants (Cengel, 2007)

	Plane Wall		*Cylinder*		*Sphere*	
Bi	**Λ**	**A**	**Λ**	**A**	**Λ**	**A**
0.01	0.0998	1.0002	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.4450	1.0060

0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40	1.5325	1.2723	2.3455	1.5993	2.0632	1.9942
50	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

The energy required to heat or cool the product to a desired temperature is represented by the following heat transfer equation:

$$Q = c_{p_f} m_f \Delta T \tag{13}$$

The energy required to heat or cool the product can be used to determine costs associated with the process. In order to determine Joules (J) required for the process the specific heat and mass of the product needs to be determined. By using the first law of thermodynamics and water of known mass and specific heat the following relationship was created:

$$c_{p_w} m_w \Delta T = c_{p_f} m_f \Delta T \tag{14}$$

3 RESEARCH PLAN AND METHODOLOGY

3.1 Introduction

For industry, there is potentially tremendous value in having an effective methodology to predict the heat transfer behavior for thermally processed foods. The methodology should accurately model a wide range of foods and be able to account for changes in process parameters and packaging. It should also be cost effective to implement. Many of the most common methodologies developed to-date fall short of these objectives in several key ways:

- Industry's experimental trial-and-error approach does not facilitate cost-effective exploration of a broader solution set of packaging and process parameters.
- Analytical mathematical models can only be applied for very simplified heat transfer characteristics and basic packaging geometries.
- Comprehensive FEA and CFD models that incorporate both conduction and convection mechanisms are expensive, complex, and may lack robustness in terms of their correlation to real physical behaviors.

The objectives of this research are to:

1. Propose a simplified FEA modeling methodology for thermal food processing that is based on a conduction-only approximation.

2. Determine the range of product characteristics over which the model can be validated against experimental results and analytical predictions.

3. Demonstrate the value use of this model for optimization of process and packaging parameters.

To achieve these research objectives, the following steps were performed:

1. Determination of thermodynamic and heat transfer characteristics for representative food samples.

2. Analytical modeling of representative food samples in simple packaging.

3. Development and application of FEA model of representative samples in simple packaging.

4. Model validation and revision.

5. Demonstration of the model capability for cost optimization.

3.2 Determination of Thermodynamic and Heat Transfer Characteristics

The goal of this research is to develop a heat transfer model that provides a real-time, flexible, and transparent architecture which supports the objectives defined in Chapter 1. In order to accurately simulate "real time" production during pasteurization and cooling, a wide range of foods were sampled for testing and verification. The characteristics of the products selected should encompass the variety of production pasteurization is capable of handling. Therefore, sample foods were chosen of varying viscosity, including ones of homogenous, aggregate and inhomogeneous properties.

To analytically and computationally represent temperature changes by heat conduction, certain thermodynamic and heat transfer characteristics need to be found for the samples tested. For the simplified cylindrical heat transfer model described in section 2.7, thermal conductivity, specific heat and mass density are required to determine the temperature at any given time during transient heat conduction. The FEA software simulates transient heat transfer purely by conduction, so these properties are required for computational analysis as well. Variation of properties between batches, lots, different brands of the same food, and foods with similar characteristics were compared to determine consistency of similar foods.

Experimental verification of the thermal properties discovered is determined by a testing facility designed to represent the operating conditions of a pasteurizing and cooling machine (Fig 3-1). The testing facility was designed to represent a complete section of a tunnel pasteurizer. It included identical operating conditions to the industrial process and special emphasis on the reproducibility of process conditions and automation of experimental procedures. Unlike the tunnel pasteurizer, the facility is comprised of a single chamber controlled by a programming logic controller (PLC), which monitors and controls a single water bath and spray system. The system is capable of monitoring two jars simultaneously. Iron constantan thermocouple wiring is used to monitor and display the internal temperatures of the two jars and water bath (Fig 3-2). The water bath temperature, sample internal temperature and volumetric flow rate of the water can be recorded from a digital display (Fig 3-3).

Fig 3-1: Pasteurization testing facility

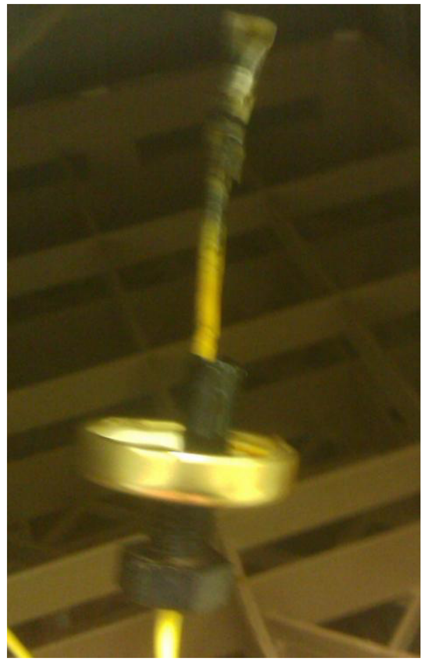

Fig 3-2: Thermocouple wiring used for temperature probe

Fig 3-3: Pasteurization facility's digital displays

To determine the specific heat of the sample a calorimetric approach was taken. The essential strategy in calorimetry is to use temperature change and heat flow to determine an unknown heat capacity. In this experiment, all substances have the same final temperature (T_{final}), but different initial temperatures (T_{iw} and T_{if}). An electronic balance was used to weigh the empty, dry calorimeter (Fig 3-4). Approximately 1200mL of water were placed into the calorimeter and weighed again, recording the mass (m_w). The initial water temperature was recorded using thermocouple wiring (T_{iw}). The specific heat capacity of water is known (4.184 J/°C).

Fig 3-4: Calorimeter used for preliminary calculations

The product of unknown specific heat was heated to an elevated temperature using the testing facility described above. Once the product reached the desired temperature (96°C), the sample was stirred until uniform temperature was reached and recorded (T_{if}). The heated sample was then placed into the calorimeter and the top sealed. The mixture was continuously stirred until the maximum temperature (T_{final}) was reached and recorded. Once all the necessary temperatures were taken, the calorimeter was weighed for the final time. The mass of the food (m_f) analyzed was determined by subtracting the mass of the water (m_w) and the calorimeter (Perez-Martin, Gallardo, Banga and Casares, 1989). The values procured above were used to solve the energy balance formula below, which was derived in section 2.7. The heat capacity of the sampled product was determined by Equation 12 (Tables 3 and 4).

In food manufacturing, many ingredients are combined and cooked together, each with varying thermodynamic properties. Some foods have an aggregate-like characteristic, where multiple solid foods are suspended in a liquid medium. To simplify the number of variables required to solve a transient heat conduction equation, a single specific heat and mass density value was assigned to each product tested. This may not give a true representation of the heat transfer occurring, however, it avoids the complexity of considering multiple thermal properties that could lead to a less reliable model. The significance of this simplification will be determined when the FEA simulation is compared to the experimental results.

To determine thermal conductivity of the food substance, the dynamic temperature profiles at the center of the jar were measured for the variety of foods ranging from homogeneous, inhomogeneous, aggregate and varying viscosity. The testing facility designed to represent a complete section of a tunnel pasteurizer was used to produce the time-temperature profile of the samples used. The facilitiy'sidentical operating conditions allow accurate temperature distributions to be reproduced without the use of a production scale pasteurizing machine. A 16 oz glass jar of 8cm diameter and 13.73 cm height was used. This jar, filled with product at an ambient state, was inserted into the pasteurizing machine designed for research and development. The sample was then heated by the spray system of water, maintained by a PLC at 98°C. A temperature probe inserted into the product was used to record the dynamic temperature at the center of the product. Approximately 15 temperature readings were recorded during the process, and a temperature curve fit to the data.

Using Autodesk Algor Simulation, a Finite Element Analysis (FEA) software, the process was simulated using different thermal conductivity values until the difference between the experimental and simulated temperature curves were negligible. The software finds approximate solutions of partial differential equations as well as integral equations by rendering the partial differential equation into an approximating system of ordinary differential equations, which are numerically integrated. The detailed development and application of the FEA model can be found in section 3.4.

Similar simplifications were made for determining the heat transfer and thermodynamic characteristics. A single thermal conductivity value was assumed for the entire product. In some cases, convection may occur in the liquid medium and conduction for individual food particles inside the glass container. It was assumed that the overall heat transfer was by conduction only. By assuming purely heat conduction, temperature approximations can be used as opposed to complex differential analysis. This allows cost and sensitivity analysis to be performed on boundary conditions and geometry changes without repeating complex analytical solutions. The significance of these assumptions can be determined when comparing the analytical and simulated results with experimental data.

3.3 Analytical Model of Food Samples

This research proposes an analysis of heat transfer by conduction only, which can be an acceptable model for many food applications. Whereas conduction occurs through a static medium, convection is a more efficient method of heat transfer because it adds the element of motion. However, when a mixture of solid and liquid foods is heated in a rigid container, there is little room for the motion of natural convection. This makes the heat transfer process behave more like a conduction model, even though both may occur. In the present work, a thermal conductivity value was assigned to the samples tested based on that assumption of pure conduction. Therefore, the model assumes that the product mixture is a single component during the analytical solution.

In the analytical model developed for the work presented, a simplified conduction model is used for the verification of FEA simulations. For infinitely long cylinders, one-dimensional heat transfer is used to determine the temperature at any point along the radius. A one-term approximation approach can be used to find the temperature at the center of the cylinder at any given time (Equation 10).

Where $T(x,t)$ is the temperature at the center of the jar, T_∞ is the temperature of the heating medium, T_i is the initial product temperature, A_1 and λ_1 can be found in Table 1 and Table 2, and solutions were only considered for τ>.2. The glass thickness was neglected due to its high thermal conductivity value in comparison to the food samples tested, so the internal height and diameter of the jar were used.

One-term approximation and one-dimensional transient conduction can only be applied to heat transfer of simple geometries such as a plane wall, cylinder or sphere. When analyzing more complex containers, a two- or three-dimensional analytical solution may be required. In these cases, a simplified approximation cannot be used, and often require the solution of complex differential equations. To accurately solve these equations would require lengthy calculations or the use of expensive software, making it very difficult to expand this analysis to irregular shaped containers.

There is an advantage to using a conduction model instead of convection. A conduction model is less complex because of the reduced number of variables involved to solve the differential equations. When considering convection, additional variables are needed to solve the corresponding differential equations. Since these variables for the sampled products are all unknown, their potential for error is greater. The analytical solution obtained for one-dimensional transient heat conduction involves infinite series and implicit equations, which can be difficult to evaluate as well. One-term approximation simplifies the analytical solutions and presents them in tabular from using simple relations. The disadvantage to using a one-term approximation solution is it can only be applied to simple geometries: plates, cylinders and spheres. Therefore, it is more practical to use simple analytical techniques to verify FEA simulation software that is capable of analyzing more complex shapes.

3.4 Development and Application of FEA Model

For simulation of conductive heat transfer a Finite Element Analysis can be used to define a complex system of points called nodes which make a grid called a mesh. This mesh is programmed to contain the material and structural properties which define how the structure will react to certain loading conditions. The finite element method performs better than the finite difference method for irregular geometries, complex boundary conditions and heterogeneous materials. Computational Fluid Dynamics (CFD) is a simulation tool for the solution of fluid flow and heat transfer problems. It is used for the complex analysis of convective heat transfer and is derived from the finite volume method. It is the method of choice when simulating thermal processes of foods for analyzing complex flow behavior. During pasteurization of mixtures of solid and liquid foods in solid containers, there is limited if any fluid flow occurring. By selecting a conduction model for simulation, relative thermodynamic and heat transfer properties can be determined for the liquid-food and a simplified analytical approach can be used to verify the results. If a CFD model were to be used, additional variables would need to be considered in the analytical and computational analysis which could decrease the reliability of results.

CAD software (SolidWorks) was used to develop a three-dimensional model for FEA simulation. A 16 oz glass jar of 8 cm diameter and 13.73 cm height was used. Individual three-dimensional parts were created for the jar, lid, air pocket and food and assembled. The jar thickness used was .003175 m, and air pocket between the lid and food was .0087 m. The FEA software used can only visually display surface temperatures for individual parts, so the food was divided into two hemi-cylindrical shapes so the center temperature profiles could be viewed during simulation.

The three-dimensional model created by SolidWorks was uploaded to Autodesk Algor Simulation. A mesh was created for the solid model and a default nodal temperature set at 23 °C to represent each model starting at ambient state. Mesh convergence based on temperature was performed at the center of the modulus to determine the optimal mesh size. Consecutive analysis was performed with finer mesh sizes and the temperatures were considered converged when consecutive center temperatures differed by less than 2%. The geometry of the jar and food product was subdivided by 5 parts and 19107 solid elements. A brick element, a basic 3D element, was used for the building block of the model and allowed for the creation of stiffness matrices and force matrices in terms of a global coordinate system (Budynas, 1999). Each solid element allows three degrees of freedom at each of its nodes. These degrees of freedom represent the heat transfer that can exist at a node in the x-, y-, and z-directions, respectively.

In all cases, the geometry was that of a finite cylinder with negligible surface heat resistance at the sample surfaces contacting the heating medium (Hanzawa, Wang, Suzuki, and Sakai, 1998). It was assumed that the thermal properties were constant in the temperature range studied and the heat transfer was only by conduction (Bhowmik and Shin, 1991). The analytical solution of heat transfer with appropriate boundary conditions was used.

To solve these differential equations, thermal diffusivities of the glass wall and heating medium, heat capacities, thermal conductivities, and mass densities of all parts involved in the tree-dimensional model are required. Aluminum Alloy 2014-T6 was selected as the material used for the jar lid and Victrex PEEKr 150G Easy Flow for the glass container. Values of air at ambient state were used for the air pocket between the food sample and lid. Although these may not be the materials used during the experiment, their values were assumed to be arbitrary since the container and lid transfer heat at a much higher rate than the products being tested. The values of the parts involved can be found in Table 3 and the products tested in Table 4 and Table 5.

For the FEA, a transient heat transfer model was selected to analyze the process. The three-dimensional model of the food product was divided into two hemi-cylindrical shapes for an easier analysis and visual representation of the heat transfer phenomena. An applied surface temperature of 98 °C was used on all surfaces to represent the convective boundary conditions of the heating process. A stiffness value of one million was used to represent the assumed infinite supply of heat. In a transient heat transfer analysis, an applied temperature can be used to control the temperature of a node throughout the analysis. The magnitude of the applied temperature at any given time is calculated as (Autodesk Algor Simulation Help):

$$T_{ref} = \frac{current\ loading\ function\ \ * (scale\ factor)}{(stiffness)} + Nodal\ temperature \qquad (15)$$

Where T_{ref} is the temperature of the ambient source/sink, the loading function is the factor defined for the load curve, the scale factor is the scale value defined on the applied temperature dialog, and the stiffness is the stiffness defined on the applied temperature dialog.

Constant mass density and specific heat values of each product were used that were found from previous experiments (section 3.2). Varying thermal conductivity values were used in the software until the time-temperature profile matched that experimental values. The series was concluded when the consecutive terms were different by less than 5%.

Table 3-1: Input values for parts used in finite element analysis

Product	**Specific Heat (J/kg C)**	**Thermal Conductivity (J/ s m C)**	**Mass Density (kg/m^3)**
Aluminum Alloy 2014-T6 (lid)	917.4175	192.1632	2789.2796
Victrex PEEKr 150G, Easy Flow glass	2160	0.25	1300

It is expected that the FEA model follows the same temperature curve of the experimental data for each scenario. The thermal conductivity values of each product were adjusted through multiple FEA simulations until they reflected the data acquired from the testing facility. The solution of the analytical model will justify whether the methods used to determine the thermodynamic and heat transfer characteristics were accurate.

3.5 Model Validation and Revision

The thermal properties obtained by the experimental results and FEA were verified analytically using the equations formulated in section 2.7 of the present work. Temperature curves were created for the analytical, FEA and experimental data and compared on a single graph to check for inconsistencies. The analytical equations assumed that the thickness of the glass jar and lid could be neglected due to their thermal conductivity being considerably larger than that of the product being tested. The FEA simulation was considered valid if the values from the experimental results and mathematical model were consistent.

A wide range of samples of different thermal properties were considered for the conduction model. The intention of selecting such a variety of samples was to determine which characteristics of foods that can be analyzed by a pure conduction. The comparison between the FEA, analytical model and experimental data demonstrate the range that the simplified conduction model is capable of predicting. Liquid foods with characteristics similar to Brand A and Brand B sports drinks have typically been modeled by CFD software using the finite volume method to predict convective heat transfer (section 2.4). The results of this research will show how accurately an equivalent conduction model can predict the time temperature profiles of these types of products as well.

Once the FEA model accurately represented the experimental data that was taken from the testing facility, the capability of predicting time temperature profiles for containers of different geometry was explored. The FEA analysis was run for a jar of different geometry with a radius of .0301625 m and a height of .193675 m before the experimental data was collected. Brand A tomato bruschetta properties were used for the FEA simulation that were solved from previous experiments. Identical boundary conditions were used as well (ambient product temperature of 23 °C and an infinitely stiff surface applied temperature of 98 °C). The time-temperature profile at the center of the jar from the analysis was graphically represented. The Brand A tomato bruschetta was then experimentally tested, using these process conditions, and the time-temperature profile at the center of the jar was recorded using thermocouple wiring from the testing facility. The results were represented graphically and compared to the FEA analysis.

3.6 Demonstration of Model Capability for Cost Optimization

Once a proven methodology for predicting and analyzing temperature distributions during pasteurization has been developed, optimization of the process can be explored. Costs associated with container size, energy consumed, process time and shipping can be simulated using changes in geometry, boundary conditions, and operating parameters. Using tabular and graphical data constructed from the mathematical and FEA models, design conditions can be compared and process improvements made.

To represent the optimization capabilities of the model researched, cost efficiency was explored through changes in geometry of a few products sampled in the work presented. The samples selected included the entire range of thermal conductivity values investigated and are as follows: Brand A Corn, Brand A Baked Beans, and Brand A Tomato Bruschetta. An FEA analysis was performed on the three products with a constant volume of 16oz and varying surface areas. Time-temperature profiles were constructed from the FEA software, which were used to determine cost advantages and disadvantages associated the geometric change. The costs associated with running the pasteurizing machine that were determined were: container material, natural gas consumption to heat water, electricity required to run water pumps and shipping.

The cost of the glass jar material was based on one 16oz glass jar costing $0.90. Based on this price a volumetric cost of glass was calculated to be $0.008598 $/cm^3$. The volume of material for each container size was determined and the corresponding costs calculated. The costs associated with natural gas consumption were based on the pasteurizing machine consuming 1.4 million BTU/hr during production. The time required to heat each sample to 85°C was determined by the FEA analysis and used as a basis to calculate production time. The cost of natural gas delivered was estimated (Dominion gas company) to be $9.80 per ccf (approximately $9.80 mmBTU) delivered. Using the production time, pasteurizer energy consumption, a combined efficiency of

50%, and the cost of natural gas, a price can be determined for gas used during the process. It was assumed that the process was continuous, so an operational cost per jar was calculated based on the volume of product the pasteurizer can hold at any given time. To calculate the product volume, it was determined how many jars could fit side-by-side in a pasteurizer of 12.192 m long (40 ft) and 2.4384 m wide (8 ft). It was also assumed that each jar's orientation was upright, regardless of the jar size. To determine the amount of energy required to heat just one jar to temperature, a simple heat transfer equation can be used with the newly found m_f and ΔT (Equation 11).

A 40 hp pump is utilized during pasteurization. Costs associated with the pump can be determined by simple conversions of HP to KW (1.431 kw = 1 hp) and a kwh rate of $0.075 (Ohio Edison). An operational cost per jar was calculated from the total product volume the pasteurizer can hold at any given time.

Shipping costs are another factor that should be considered when optimizing the process. Jar geometry not only affects the process time and cost associated with manufacturing, but has an impact on how much product can be transported in one shipment (i.e. the lower the packing volume and weight of the product, the more that can be shipped at one time). The packing volume was determined the rectangular volume that encompasses the cylindrical jar. A maximum volume (32.58 m^3) and maximum weight (15890 kg) was used for each shipment. It was calculated for each size container and product whether a shipment would reach maximum volume or weight first and how much product could fit in one shipment. Using a shipping rate of $5.47

per kilometer and an arbitrary distance of 100 km, a rate was associated with the cost of shipping per container for each scenario analyzed.

4 RESULTS AND DISCUSSION

4.1 Determination of Thermodynamic and Heat Transfer Characteristics

Time-temperature profiles of a jar during thermal processing were found in the model developed in this study. Detailed knowledge of food characteristics are required to accurately predict temperature profiles. Using a calorimetric approach described in section 3.2, the specific heat (c_p) and mass density (ρ) of each sample was calculated. Each sample was tested five times and their average values can be found in Table 4-1 and Table 4-2. Table 4-1 includes foods of high viscosity and thermal conductivity levels and Table 4-2 of low viscosity and thermal conductivity values. Victrex PEEKr 150G, Easy Flow glass was used as the material for the cylindrical jar container and Aluminum Alloy 2014-T6 used for the lid that seals the container. These materials' thermal properties can be found in Table 3-1 of this research.

Table 4-1: Input parameters to the Finite Element Analysis Software

Product	Specific Heat (J/kg C)	Thermal Conductivity (J/ s m C)	Mass Density (kg/m^3)
Brand A Sports Drink	3835.151	0.031	4.095
Brand B Sports Drink	3853.632	0.033	4.159
Brand A Corn	3065.496	0.03	4.347
Brand B Corn	3935.003	0.028	3.990
Brand A Peas	3785.893	0.03	3.748
Brand B Peas	3481.213	0.025	4.027

Table 4-2: Additional Input parameters to the Finite Element Analysis Software

Product	Specific Heat (J/kg C)	Thermal Conductivity (J/ s m C)	Mass Density (kg/m^3)
Brand A Baked Beans	4336.202	0.005	4.409
Brand B Baked Beans	3892.369	0.0046	4.603
Brand A Pizza Sauce	3919.427	0.0045	4.159
Brand A Tomato Bruschetta	3697.594	0.00425	4.222

4.2 Development and Application of FEA Model

The solution of transient multi-dimensional heat conduction of a cylinder is explicitly expressed by the finite element analysis software package (Autodesk Algor Simulation). Heat treatment of the food samples and cylindrical containers at ambient state, using an applied temperature of 98°C, were simulated by the software. For easy comparison, the corresponding temperatures as a function of time solved by Finite Element Analysis, the mathematical model described in Chapter 3, and experimental values are superimposed on one graph (Fig 4-1-Fig 4-10). During the heating period, for the products with high viscosity and thermal conductivity values, estimated temperatures by the model given shows little difference with experimental values. It is evident that the heating of the liquid foods is mainly due to conduction and that the heat transfer simulated by FEA software is accurate.

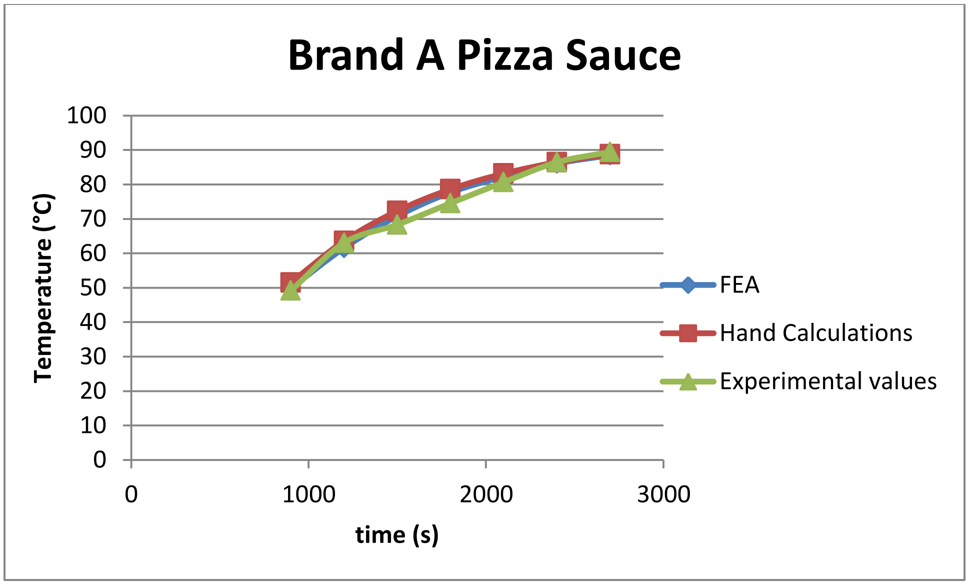

Fig 4-1: Brand A Pizza Sauce time-temperature profile

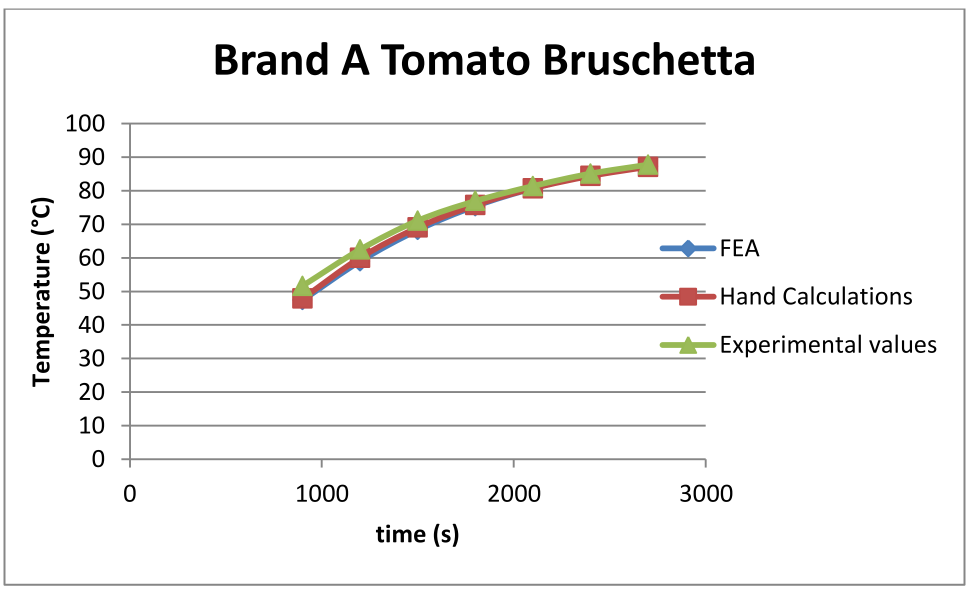

Fig 4-2: Brand A Tomato Bruschetta time-temperature profile

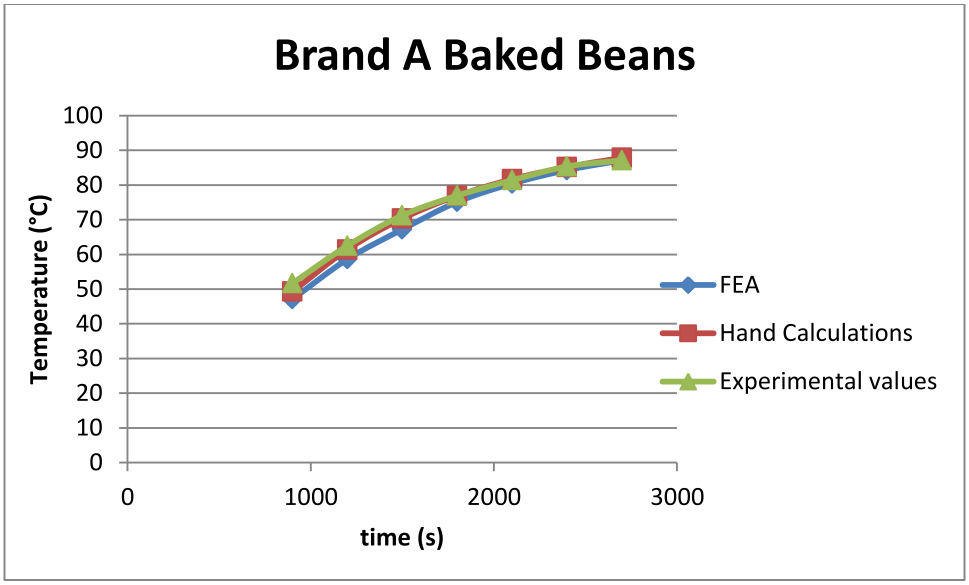

Fig 4-3: Brand A Baked Beans time-temperature profile

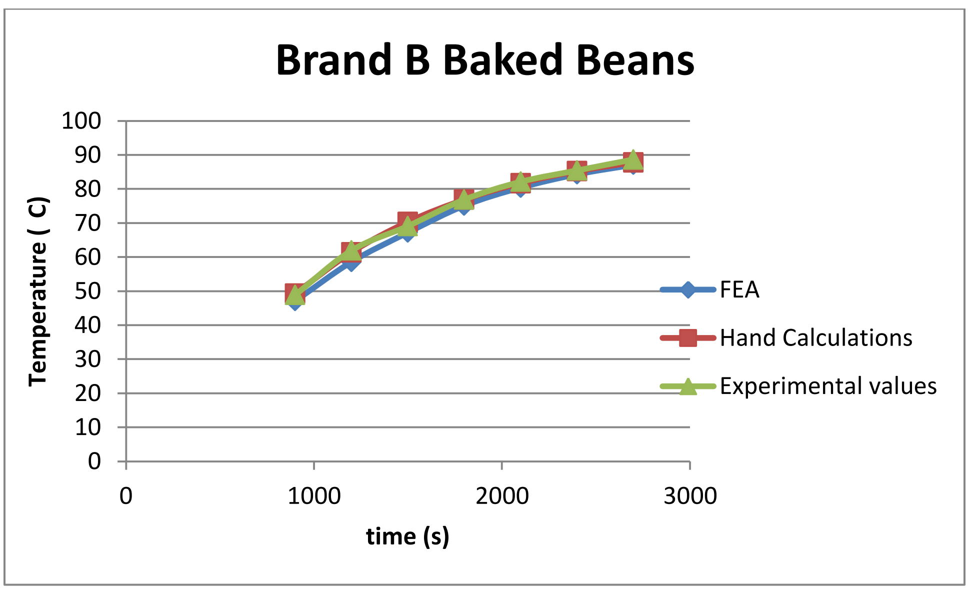

Fig 4-4: Brand B Baked Beans time-temperature profile

The transient temperature profiles shown of products with high viscosity and thermal conductivity were analyzed and compared as well. The temperatures determined from the mathematical model derived in Chapter 3 deviated considerably from the experimental and computational results (Fig 4-5-Fig 4-10). During the heating period, the mathematical model predicted the product's temperature to rise at a higher rate than experimental and simulated results. It was initially unclear why the mathematical model differed from the finite element analysis software. Each model is strictly defined as a transient heat conduction analysis with identical boundary conditions. Therefore, it would seem to be appropriate that each model would share similar results. The finite element analysis results were investigated to determine the reason for the large deviation in results.

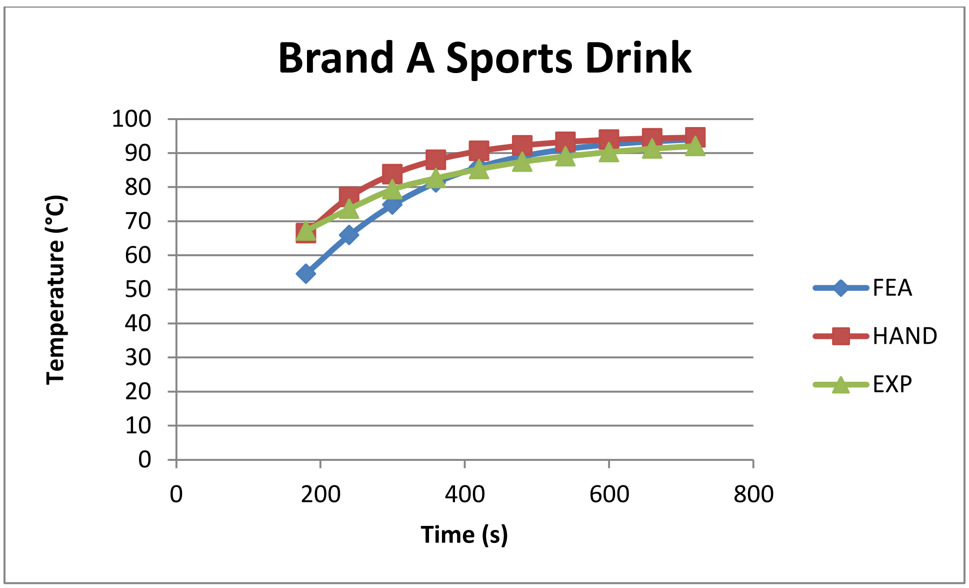

Fig 4-5: Brand A Sports Drink time-temperature profile

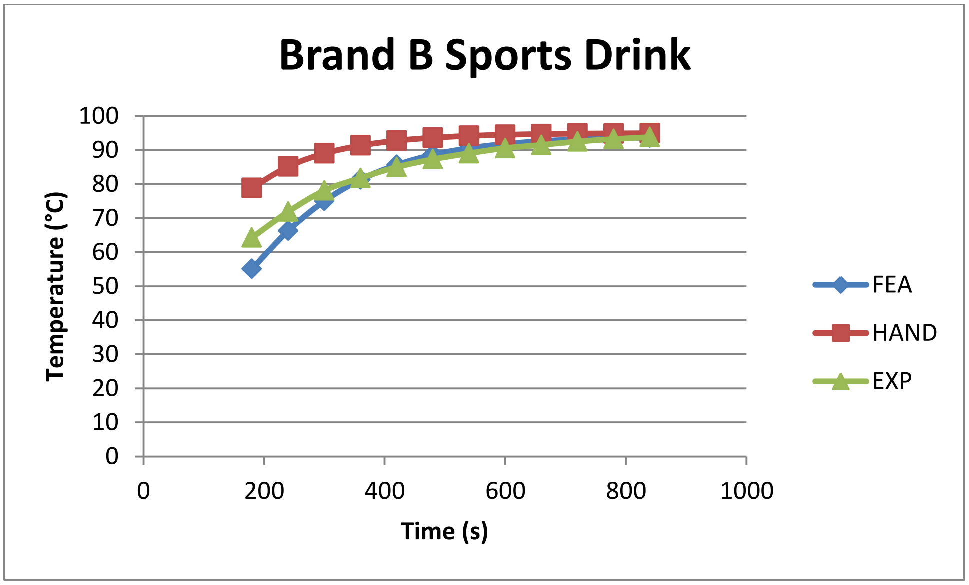

Fig 4-6: Brand B Sports Drink time-temperature profile

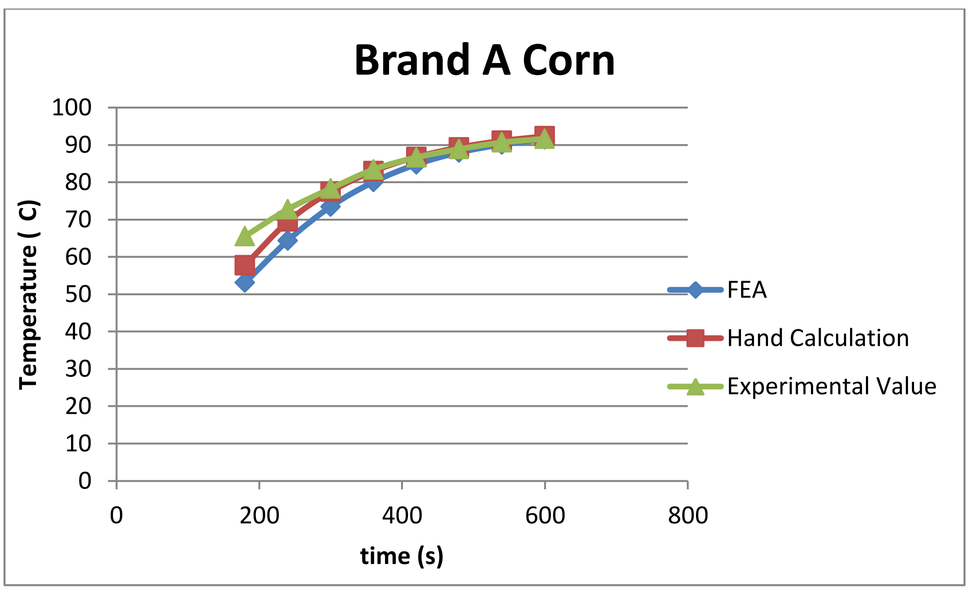

Fig 4-7: Brand A Corn time-temperature profile

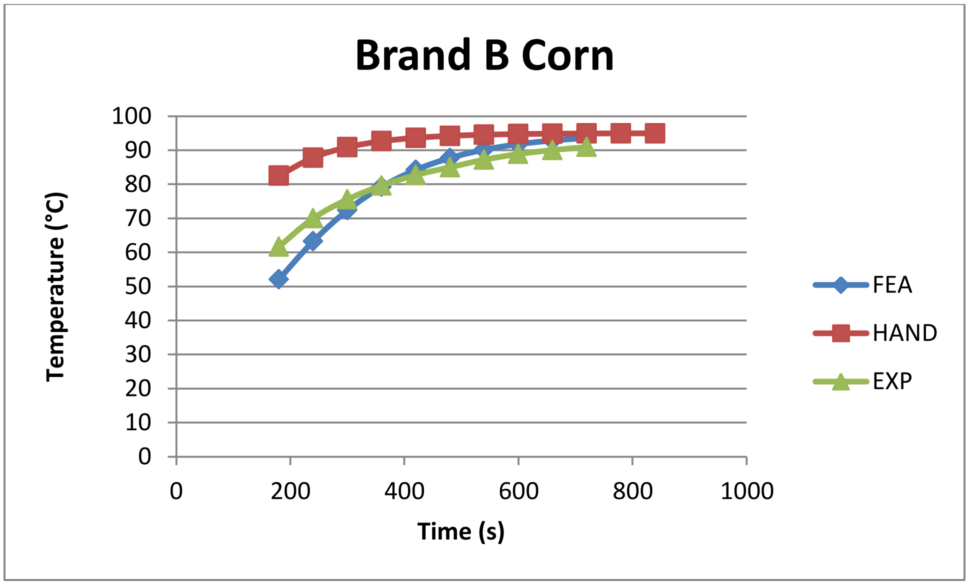

Fig 4-8: Brand B Corn time-temperature profile

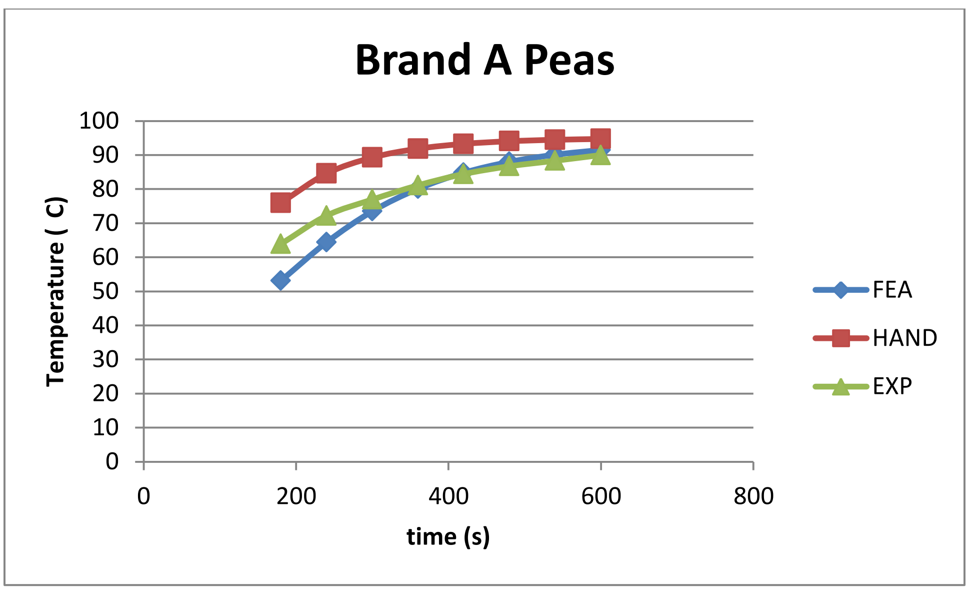

Fig 4-9: Brand A Peas time-temperature profile

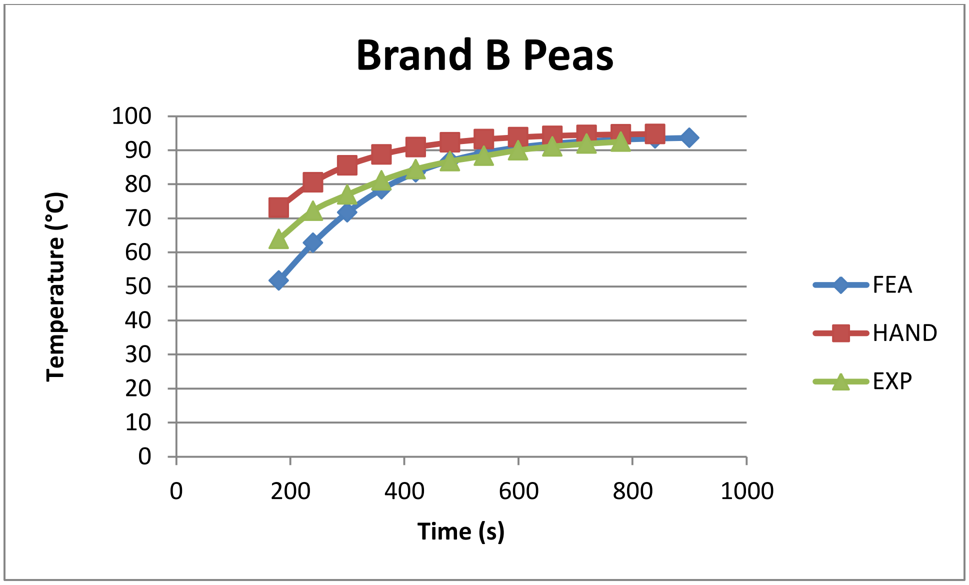

Fig 4-10: Brand B Peas time-temperature profile

4.3 Model Validation and Revision

For products of high thermal conductivity values, inconsistency was found between the mathematical model and FEA simulation (section 4.2). Therefore, incorrect assumptions about the heat transfer process were considered. It was originally assumed that the glass thickness could be neglected for all products because of its much higher thermal conductivity value. In order to determine whether this assumption was correct, the mathematical model and FEA simulation needed to be compared more closely.

An FEA model was created to simulate a comparison between the mathematical and experimental models. If the assumption were true, the FEA simulation of the mathematical (neglecting glass thickness) and experimental model (including glass thickness) would be consistent. A solid three-dimensional model was assembled for the mathematical model, neglecting the glass and lid thickness. The three-dimensional mathematical model was simulated in the FEA software using identical thermal properties, mesh sizes, elements, boundary conditions and cycle time to their original counterpart. The time-temperature profiles produced by the simulation were then compared to the model including the glass container and shown in Fig 4-11-Fig 4-20.

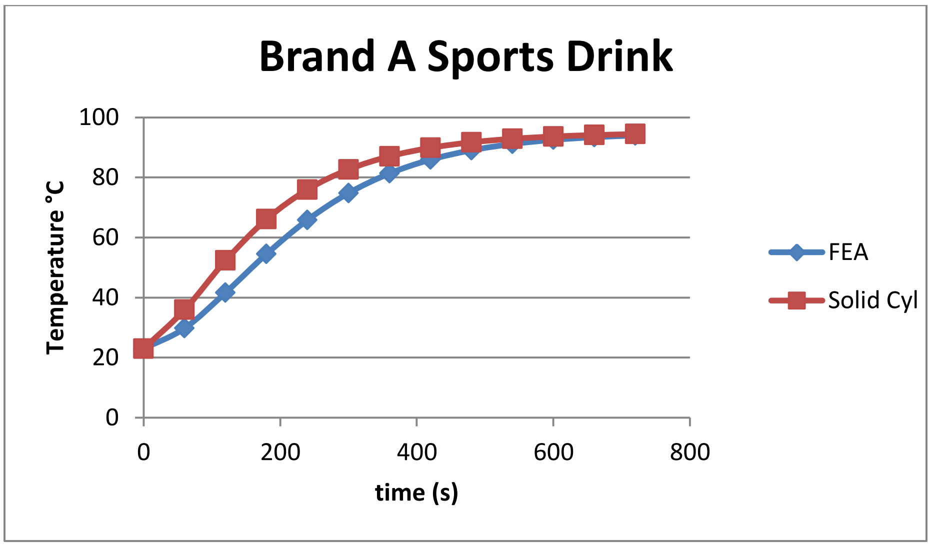

Fig 4-11: Brand A Sports Drink FEA, solid cylinder vs. three-dimensional model comparison

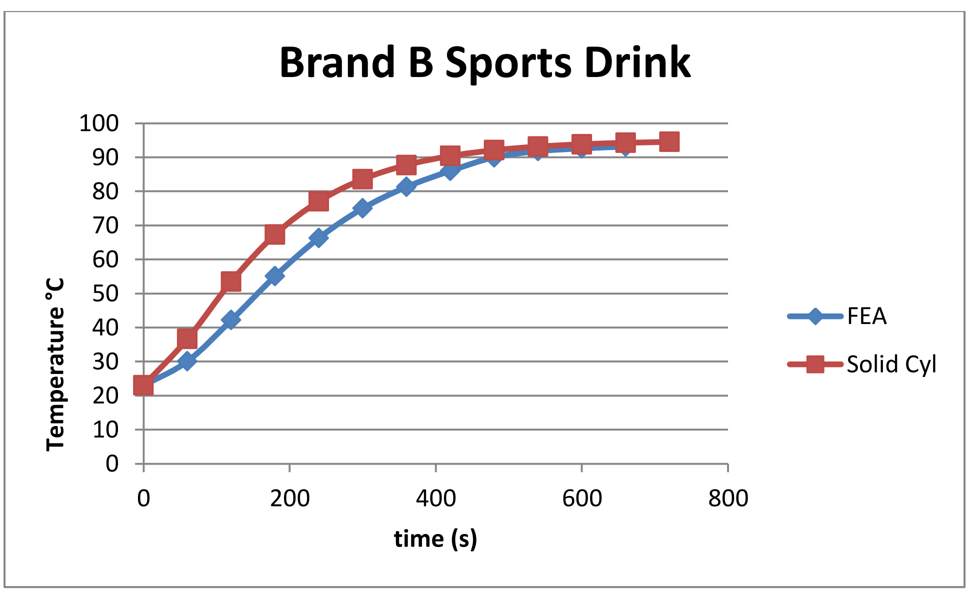

Fig 4-12: Brand B Sports Drink FEA, solid cylinder vs. three-dimensional model comparison

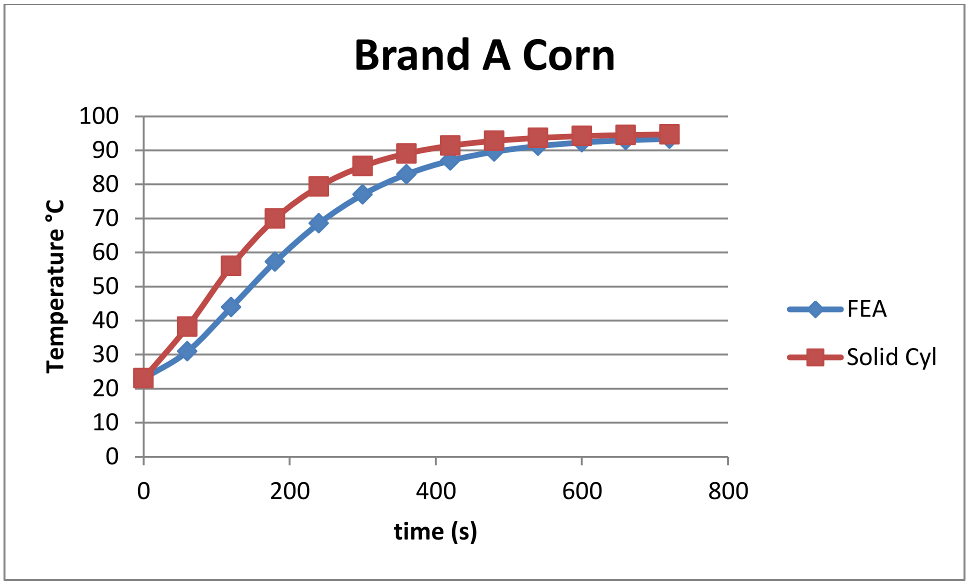

Fig 4-13: Brand A Corn FEA, solid cylinder vs. three-dimensional model comparison

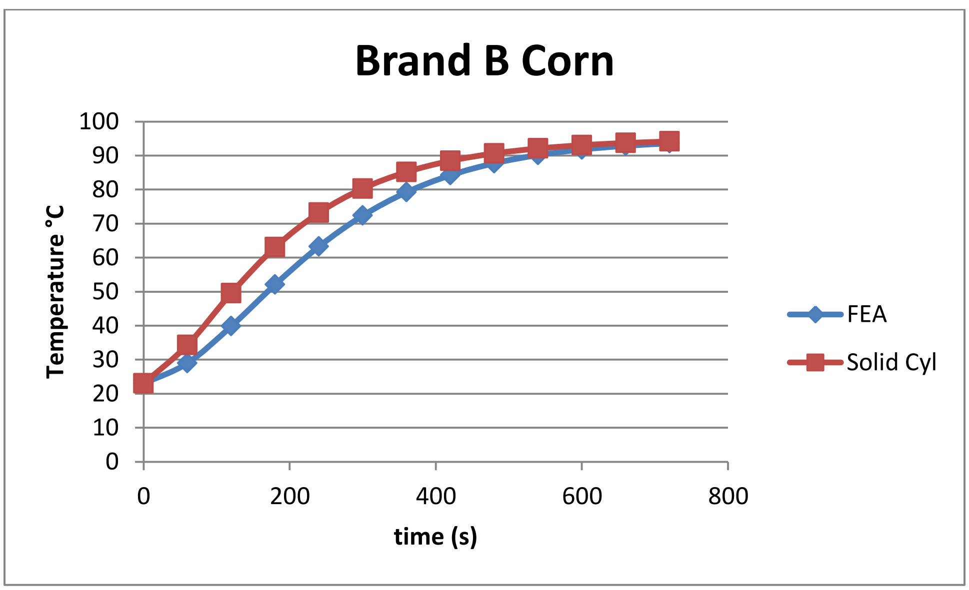

Fig 4-14: Brand B Corn FEA, solid cylinder vs. three-dimensional model comparison

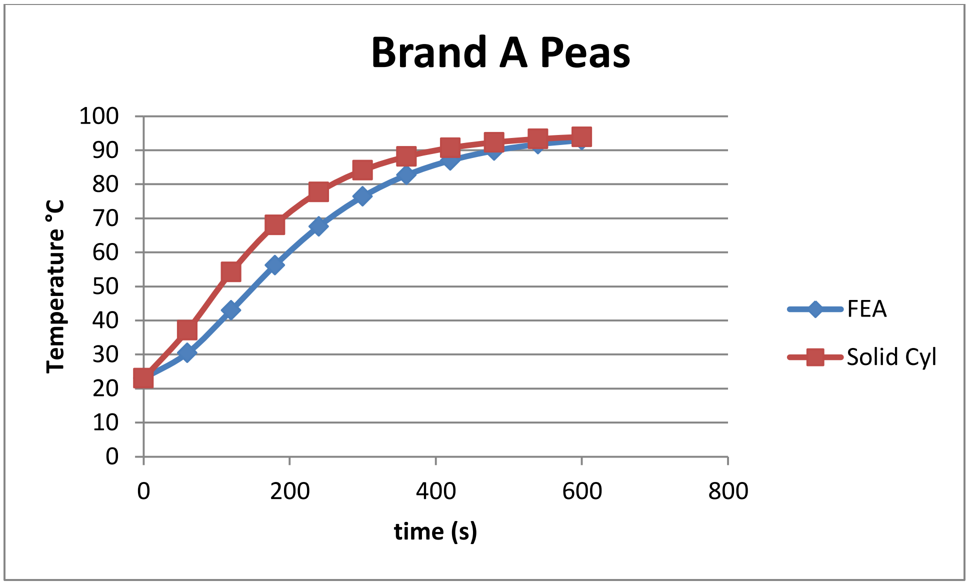

Fig 4-15: Brand A Peas FEA, solid cylinder vs. three-dimensional model comparison

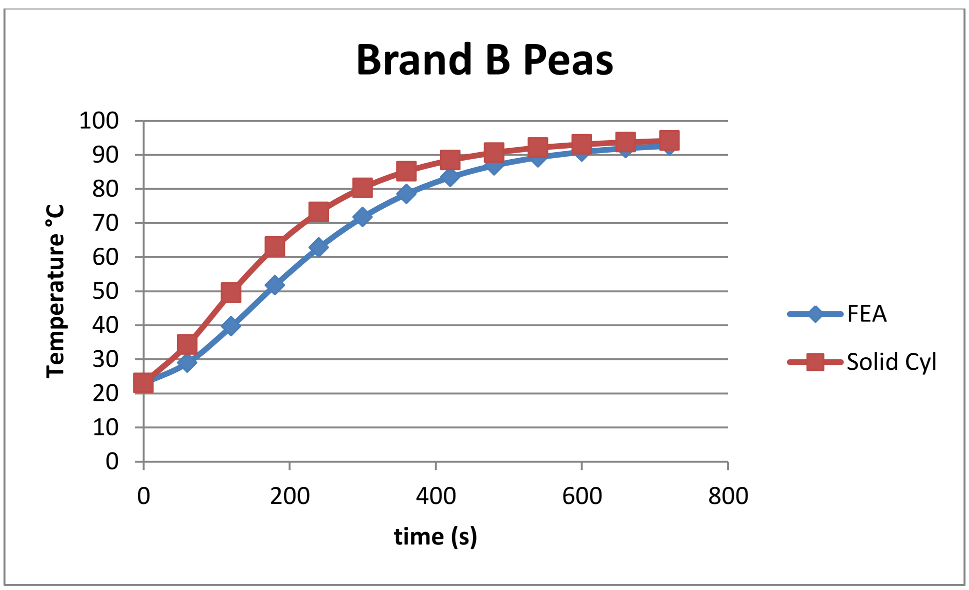

Fig 4-16: Brand B Peas FEA, solid cylinder vs. three-dimensional model comparison

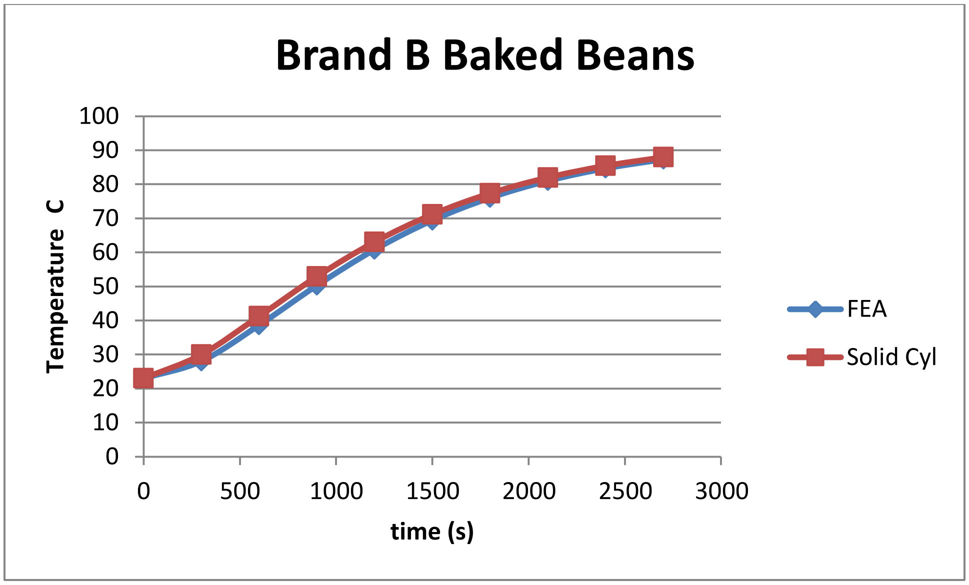

Fig 4-17: Brand B Baked Beans FEA, solid cylinder vs. three-dimensional model comparison

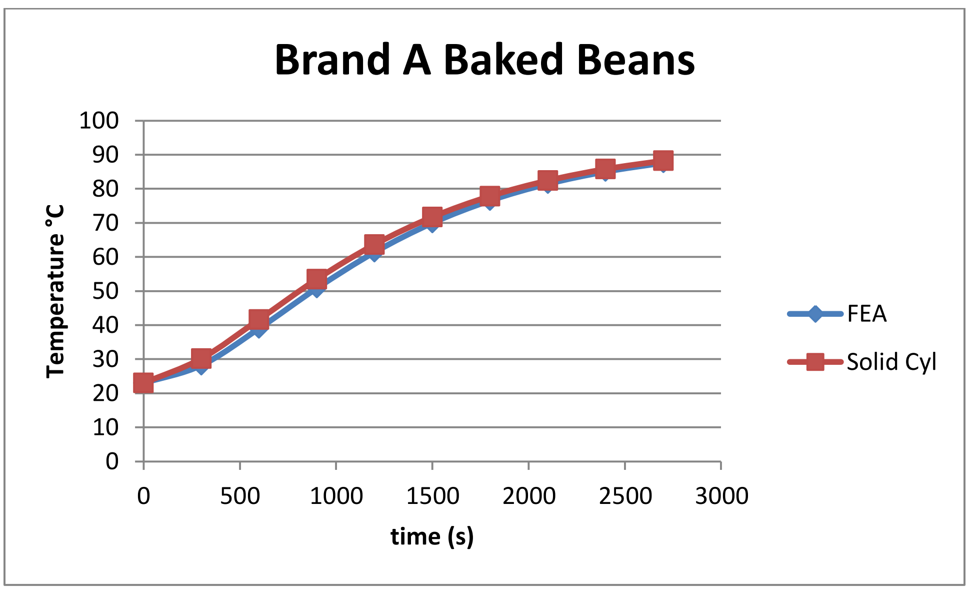

Fig 4-18: Brand A Baked Beans FEA, solid cylinder vs. three-dimensional model comparison

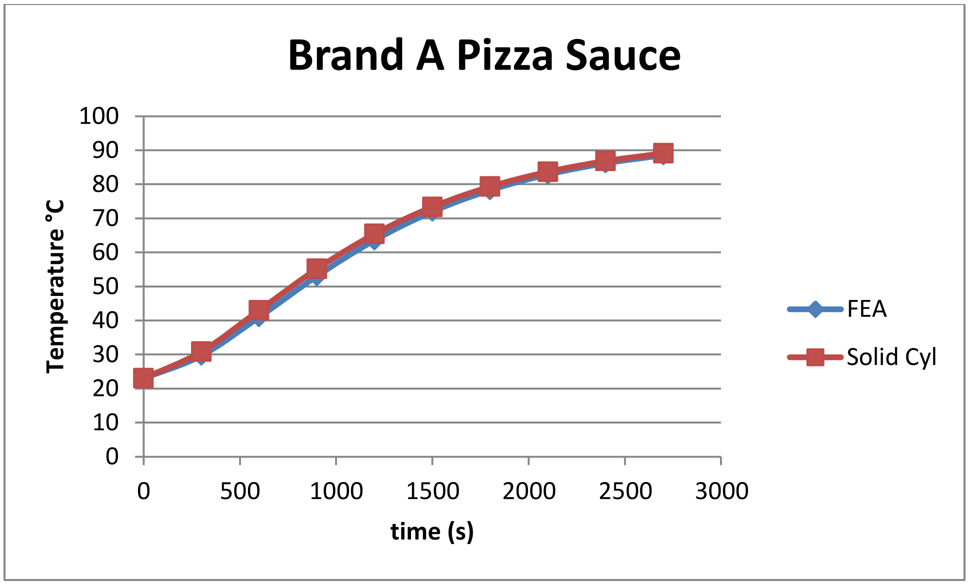

Fig 4-19: Brand A Pizza Sauce FEA, solid cylinder vs. three-dimensional model comparison

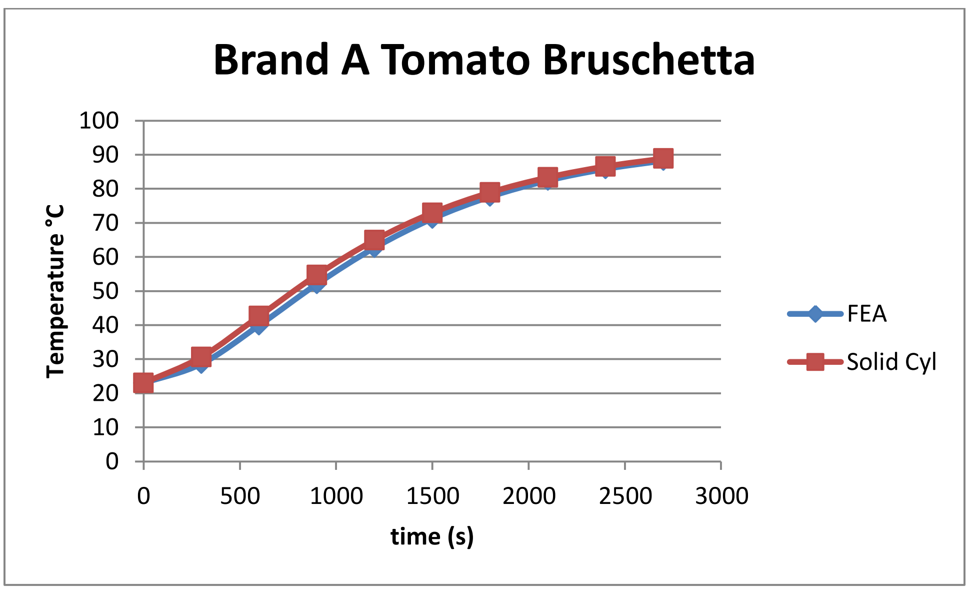

Fig 4-20: Brand A Tomato Bruschetta FEA, solid cylinder vs. three-dimensional model comparison

The time-temperature profiles above represent a strong indication that some of the assumptions initially made in the mathematical model are incorrect. A solid uniform cylinder described in the mathematical calculations and a full three-dimensional realistic model including the glass jar and lid were compared. In Fig 4-5-Fig 4-10 a deviation similar to the mathematical and FEA model (Fig 4-11-Fig 4-16) is shown. These products share common thermal properties, including a higher thermal conductivity with values ranging from 0.025 to 0.033 $\frac{J}{s*m*°\mathrm{C}}$. This deviation in temperatures does not exist when comparing the models that were consistent in mathematical, experimental and simulated results (Figures 14-17). The thermal conductivity of these products range from 0.00425 to 0.005 $\frac{J}{s*m*°\mathrm{C}}$, much lower than the previous examples.

The difference in temperature profiles is the result of the change in thermal conductivity between the two groups of products tested and not an error in the finite element analysis simulation. It was determined that the error in the analytical solution is the assumption that the transient heat conduction through the glass wall could be neglected due to its high thermal conductivity. This is not applicable when the thermal conductivity values of the food reach a certain threshold. However, in these cases of high thermal conductivity values, the FEA simulation is still a valuable method for predicting and analyzing the heat transfer phenomena. There is sufficient evidence in the research presented that the use of CAD and CAE software, specifically SolidWorks and Autodesk's Algor Finite Element Analysis software, can be an efficient and successful means for the optimization and prediction of transient heat conduction during industrial pasteurization.

The FEA model was tested for its predictability by using a container of different geometry. Brand A Tomato Bruschetta was simulated by the software using the dimensions and parameters described in section 3.5. After the FEA predicted the time-temperature profile of the food in the new container, an experimental trial was run to compare the results (Fig 4-21)

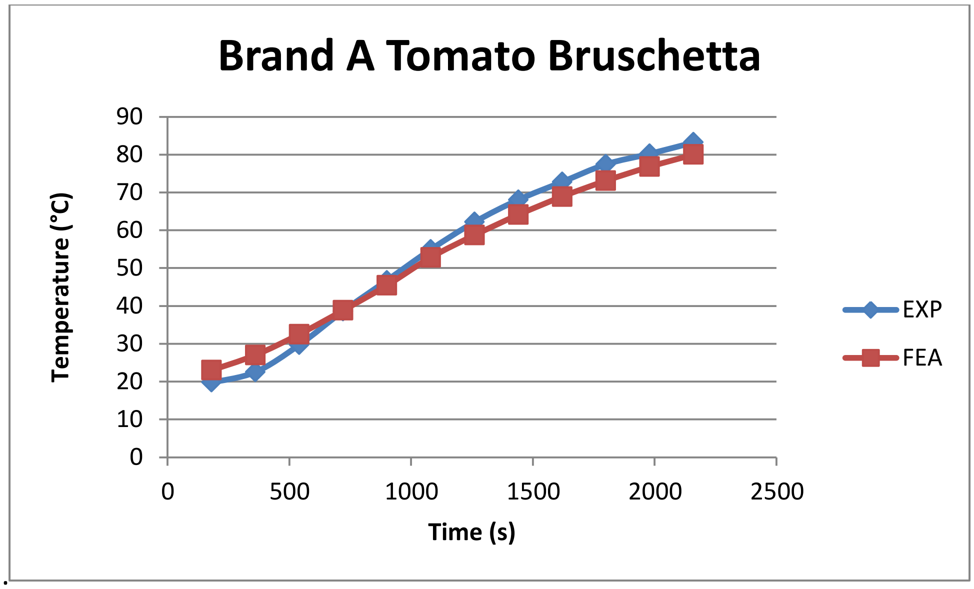

Fig 4-21: Brand A Tomato Bruschetta prediction model

Using FEA to predict time-temperature profiles can be beneficial for any food manufacturer. With a working model, optimization of process parameters can be explored without the cost of expensive trial and error methods. Container geometry, boundary conditions, and process conditions can be manipulated to optimize process time, energy consumption and even associated shipping costs. New products can now be designed for cost efficiency theoretically before implementation. A demonstration of the optimization capability of a prediction model is discussed in the next section.

4.4 Demonstration of Model Capability for Cost Optimization

With a proven methodology to analyze and predict time-temperature profiles of thermally treated foods in cylindrical containers, optimization can be explored through changes in geometry and process conditions. In the research presented, an efficiency model was created to demonstrate the costs associated with changes in geometry. In this particular example, a constant volume was used with varying surface area. The dimensions of the jar sizes simulated can be found in Table 4-3. Three samples of food were selected for analysis that included the range of thermal properties explored in the current research. The samples selected include: Brand A Tomato Bruschetta, Brand A Baked Beans, and Brand A Corn. Their thermal properties can be found in Table 4-1 and Table 4-2.

Table 4-3: Geometric dimensions of jar sizes used in efficiency model

	Radius (m)	Height (m)	Volume (m^3)	Surface Area (m^2)	Packing Volume (m^3)
Jar 1	0.0127	1.143	0.000579	0.092220761	0.0580644
Jar 2	0.0254	0.28575	0.000579	0.049657333	0.0290322
Jar 3	0.0381	0.127	0.000579	0.039523183	0.0193548
Jar 4	0.0508	0.071438	0.000579	0.039016635	0.014516202
Jar 5	0.0635	0.04572	0.000579	0.043576843	0.01161288
Jar 6	0.0762	0.03175	0.000579	0.051684163	0.0096774

Using the FEA software, time-temperature profiles were created of each product, initially at an ambient state and heated by a liquid medium of 98°C. By specifying a target final temperature (85 °C), the process time can be determined from the temperature graphs created (Fig 4-22-Fig 4-24). Process time is a direct reflection of the surface-area-to-volume ratio for each container, and is shown in Table 4-4. The process time can be used to calculate electricity, heating, shipping and material costs associated with each container's geometry.

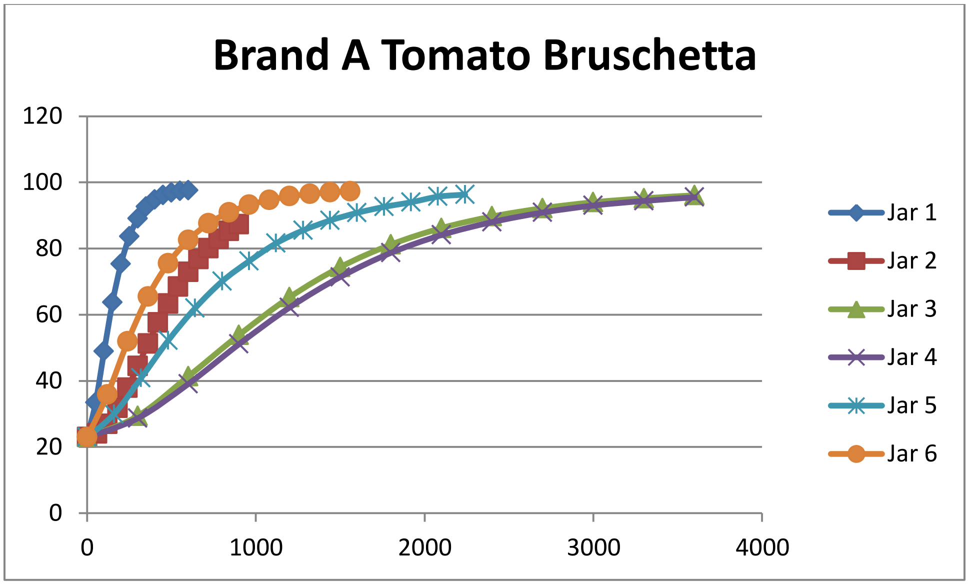

Fig 4-22: Center time-temperature profile of Brand A Tomato Brushchetta with varying surface area

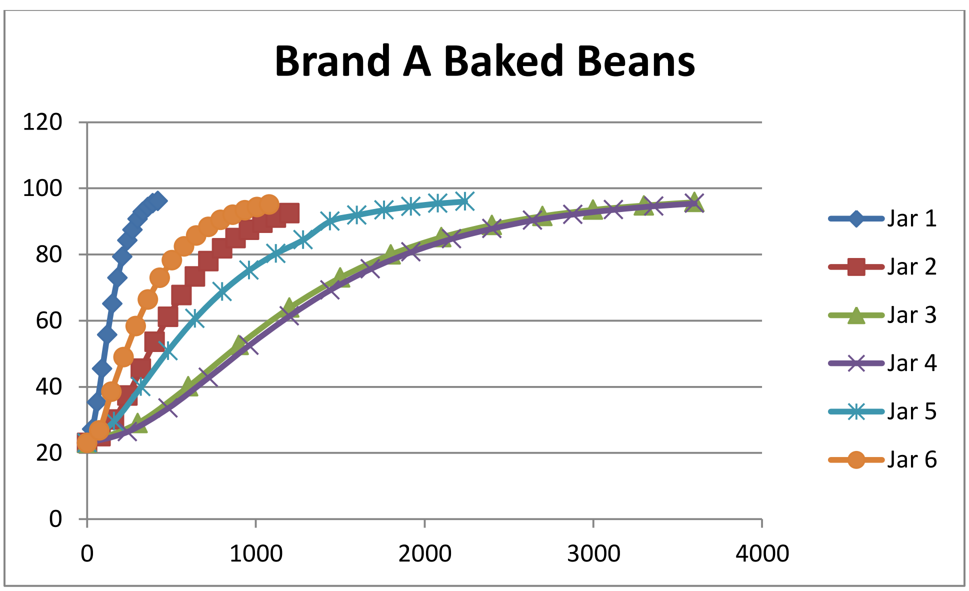

Fig 4-23: Center time-temperature profile of Brand A Baked Beans with varying surface area

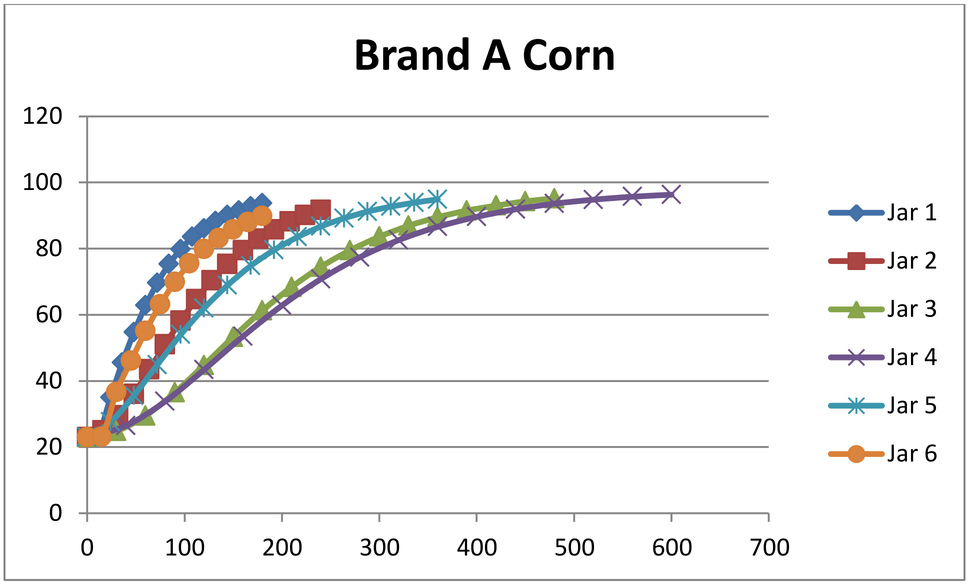

Fig 4-24: Center time-temperature profile of Brand A Corn with varying surface area

Table 4-4: Time to reach 85°C as a function of surface-area-to-volume ratio

Surface Area-to-Volume ratio	Brand A Beans Process Time (s)	Brand A Tomato Bruschetta Process Time (s)	Brand A Corn Process Time (s)
159.23010	248	263	115
85.73928	880	837	187
68.24147	2100	2028	313
67.36638	2183	2169	344
75.24059	1297	1260	227
89.23885	633	658	145

There are many factors that contribute to manufacturing costs. Prices of global variables such as natural gas, electricity, shipping, and materials may change frequently. The methods behind selecting the prices of these variables are described in section 3.6. Although the rates estimated may not be constant throughout the manufacturing industry, they provide an accurate representation of how the model can be used to calculate process efficiency. Specific data from individual companies or local rates can be used as a replacement for the values assigned to the model used in the current research. Tables Table 4-5, Table 4-6, andTable 4-7 show the manufacturing cost of producing one jar of each product by pasteurization.

Table 4-5: Manufacturing costs of Brand A Baked Beans

	Jar Material Cost	BTU Consumption Cost/Jar	Water Pump Energy Cost/Jar	Shipping Cost/Jar	Total Cost/Jar	% Cost Savings
Jar 1	$2.54	$0.000042	$0.000003	$0.97	$3.52	-155.8%
Jar 2	$1.41	$0.000594	$0.000047	$0.49	$1.89	-37.8%
Jar 3	$1.05	$0.003190	$0.000255	$0.32	$1.38	0.0%
Jar 4	$0.93	$0.005895	$0.000471	$0.24	$1.18	14.4%
Jar 5	$0.93	$0.005531	$0.000442	$0.19	$1.13	17.9%
Jar 6	$1.00	$0.003846	$0.000307	$0.16	$1.17	14.8%

Table 4-6: Manufacturing costs of Brand A Tomato Bruschetta

	Jar Material Cost	BTU Consumption Cost/Jar	Water Pump Energy Cost/Jar	Shipping Cost/Jar	Total Cost/Jar	% Cost Savings
Jar 1	$2.54	$0.000044	$0.000004	$0.97	$3.52	-155.8%
Jar 2	$1.41	$0.000565	$0.000045	$0.49	$1.89	-37.8%
Jar 3	$1.05	$0.003081	$0.000246	$0.32	$1.38	0.0%
Jar 4	$0.93	$0.005858	$0.000468	$0.24	$1.18	14.4%
Jar 5	$0.93	$0.005373	$0.000429	$0.19	$1.13	17.9%
Jar 6	$1.00	$0.003998	$0.000320	$0.16	$1.17	14.8%

Table 4-7: Manufacturing costs of Brand A Kennel Corn

	Jar Material Cost	BTU Consumption Cost/Jar	Water Pump Energy Cost/Jar	Shipping Cost/Jar	Total Cost/Jar	% Cost Savings
Jar 1	$2.54	$0.000019	$0.000002	$0.97	$3.52	-156.4%
Jar 2	$1.41	$0.000126	$0.000010	$0.49	$1.89	-38.0%
Jar 3	$1.05	$0.000475	$0.000038	$0.32	$1.37	0.0%
Jar 4	$0.93	$0.000929	$0.000074	$0.24	$1.17	14.6%
Jar 5	$0.93	$0.000968	$0.000077	$0.19	$1.12	18.1%
Jar 6	$1.00	$0.000881	$0.000070	$0.16	$1.17	14.9%

The jar material cost was based on the price of material estimated at \$0.008598 $/cm^3$. The cost of BTU consumption was calculated from the pasteurizing machine consuming 1.5 million BTUs for every hour of operation and a combined efficiency of 50%. The cost pumping water was calculated from the electrical consumption of a 40 hp pump and an estimated combined efficiency of 70%. It was determined whether a truck load of product would reach maximum capacity by weight or volume for each jar size and product. For every scenario determined the truck would reach maximum volume first, so the transportation cost of a jar was calculated based on the maximum product allowed per truck and an estimated cost for shipping 100 km.

Using the data from this efficiency model, optimal process conditions can determined based on a cost per product produced. Although having a high surface-area-to-volume ratio would have the greatest potential to reduce process time, it could have negative effects on the cost of shipping and material. To determine the container size that maximizes cost reduction, efficiency curves can be fit to each cost-based criteria and an optimal process condition can be selected. This methodology for process improvements can be applied to modifications in boundary and process conditions as well. By changing boundary conditions and constraints to the FEA model, virtually any design scenario can be simulated for pasteurization and the corresponding process times can be found.

5 CONCLUSIONS AND FUTURE WORK

The industrial pasteurization process of jars is investigated theoretically and experimentally. The mathematical model for the time prediction of temperature distributions in the glass jar considering the variableness in thermal properties between the products tested is developed. Transient temperature distribution evolving during conduction of a variety of foods in a cylindrical jar have been simulated by solving the governing equations for continuity, momentum and energy conservation using finite element method of solution. A FEA software package (Autodesk's Algor Simulation) was used to carry out the computations. The conduction model is tested and improves the understanding of the industrial pasteurization process of food in jars.

The results of the simulation show that the FEA simulation can be used for the range of the samples tested and their properties. The mathematical model takes a more simplistic approach to the thermal process than related articles and is only valid when evaluating foods of lower thermal conductivity values, where the transient heat transfer through the glass wall can be neglected. This research can be a valuable tool in the experimental design for heat transfer optimization in pasteurization. With slight modifications, it can be used in the design and prediction of various food products.

By using the FEA model for prediction of time-temperature profiles, process time required for the product to reach any given temperature could be calculated. This is a valuable tool for cost optimization. BTU consumption, electric energy consumption, process time, and shipping costs were calculated based on a set of global variables defined in the work presented. Although the values used may not be appropriate for every manufacturer, each industry can apply their own cost model based on local or in-house prices.

For the food industry, there is a potentially tremendous value in having an effective methodology to predict the heat transfer behavior for thermally processed foods. The proposed methodology is a simplified FEA model for thermal processing that is based on a conduction-only approximation, which makes it cost effective to implement. The FEA software could predict time-temperature profiles with a maximum deviation of 9.59 °C for high thermal conductivity values, and 4.2 °C for low. When using the simulation tool to model when products reach above the required 72 °C for sterilization, the model has a maximum deviation of 3.1 °C for high thermal conductivity values, and 1.1 °C for low. A valuable use of this model for optimization has been demonstrated for the process and packaging parameters. By knowing the container geometry, process time, and energy consumption of the pasteurization and cooling machine, the associated costs for production can be estimated. This allows cost optimization to be explored theoretically without time and money wasted on trial and error methodologies. This can prove to be especially beneficial during new product design and process modification. Under the conditions described for the demonstrative cost optimization model presented, a maximum savings of 18.1% was found from changes in jar geometry.

Although the method introduced has proven to be a sufficient way of accurately predicting heat transfer during industrial pasteurization, for what range of thermodynamic and heat transfer properties it may be applied. In order to determine the range of applicability, a larger variety of foods should be compared by the model. If possible, a substitution to food should be used for experiments so that the thermal properties can be manipulated to values of the researcher's choosing. This would allow virtually any range of heat transfer and thermodynamic characteristics to be explored.

To continue the current research, the next step would be to perform a sensitivity analysis on the effects the heat transfer characteristics have on the accuracy of the model. In the current research, the thermal conductivity values affected the way the model behaved the most. This should be the first process parameter to be analyzed by a sensitivity analysis. Cost efficiency can also be explored through improvements to the pasteurization process itself. Heat transfer rates change as products undergoing conduction approach equilibrium. By changing the temperature of the heating medium, heat transfer rates can be manipulated during the process. A sensitivity analysis can also be performed on the effects of changing these process parameters.

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NOMENCLATURE

Bi	Biot number
c_{p_f}	specific heat of food (J/kg*K)
c_{p_w}	specific heat of water (J/kg*K)
h	heat transfer coefficient (W/ m^2K)
J_0	0^{th} order Bessel number
J_1	1^{st} order Bessel number
K	thermal conductivity (J/ s*m*°C)
m_f	mass of food (kg)
m_w	mass of water (kg)
Q	heat gain or loss (J)
r	radius (m)
r_0	radius to outer wall (m)
T_f	temperature of food (°C)
T_{final}	final temperature (°C)
T_{if}	initial food temperature (°C)
T_{iw}	initial water temperature (°C)
T_w	water temperature (°C)
T_∞	temperature of heating medium (°C)
y	height (m)
α	thermal diffusivity (m^2/s)
α_f	thermal diffusivity of food (m^2/s)
α_w	thermal diffusivity of water (m^2/s)
θ	dimensionless temperature
λ_n	one-term approximation constant

ρ mass density (kg/m^3)

τ dimensionless time, Fourier number

APPENDIX A

Table 5-1:Brand A Pizza Sauce analytical results

Time (s)	τ	θ	T0
9.00E+02	0.168732	0.603824	51.52464
1200	0.224976	0.436165	63.5961
1500	0.28122	0.315059	72.31577
1.80E+03	0.337464	0.227579	78.61433
2100	0.393708	0.164389	83.16401
2400	0.449952	0.118744	86.45041
2700	0.506196	0.085773	88.82431

Table 5-2: Brand A Baked Beans analytical results

Time (s)	τ	θ	T0
900	0.160003	0.635086	49.27379
1200	0.213338	0.466531	61.40977
1500	0.266672	0.342711	70.3248
1800	0.320007	0.251754	76.87373
2100	0.373341	0.184937	81.68454
2400	0.426676	0.135854	85.21854
2700	0.48001	0.099797	87.81459

Table 5-3: Brand B Baked Beans analytical results

Time (s)	τ	θ	T0
900	0.155361	0.652369	48.02946
1200	0.207148	0.483535	60.18552
1500	0.258935	0.358395	69.19556
1800	0.310721	0.265642	75.87379
2100	0.362508	0.196893	80.82368
2400	0.414295	0.145937	84.49254
2700	0.466082	0.108168	87.21189

Table 5-4: Brand A Tomato Bruschetta analytical resutls

Time (s)	τ	θ	T0
900	0.154786	0.65454	47.87309
1200	0.206381	0.485682	60.03089
1500	0.257977	0.360386	69.05222
1800	0.309572	0.267413	75.74623
2100	0.361167	0.198426	80.71332
2400	0.412763	0.147236	84.399
2700	0.464358	0.109252	87.13384

Table 5-5: Brand A Corn analytical results

Time (s)	τ	θ	T0
60	0.065195	1.098877	15.88087
120	0.13039	0.753717	40.73236
180	0.195585	0.516973	57.77795
240	0.26078	0.354591	69.46948
300	0.325975	0.243213	77.48867
360	0.39117	0.166819	82.98902
420	0.456365	0.114421	86.76169
480	0.52156	0.078481	89.34936
540	0.586755	0.05383	91.12424
600	0.65195	0.036922	92.34163
660	0.717145	0.025325	93.17663
720	0.78234	0.01737	93.74935
780	0.847535	0.011914	94.14218
840	0.91273	0.008172	94.41163
900	0.977925	0.005605	94.59644

Table 5-6: Brand B Corn analytical results

Time (s)	τ	θ	T0
60	0.096093	0.919065	28.82735
120	0.192187	0.527233	57.03924
180	0.28828	0.302454	73.22334
240	0.384374	0.173506	82.50755
300	0.480467	0.099534	87.83355
360	0.57656	0.057099	90.88888
420	0.672654	0.032756	92.6416
480	0.768747	0.018791	93.64708
540	0.864841	0.010779	94.22388
600	0.960934	0.006184	94.55477
660	1.057027	0.003547	94.74459
720	1.153121	0.002035	94.85348
780	1.249214	0.001167	94.91595
840	1.345308	0.00067	94.95178
900	1.441401	0.000384	94.97234

Table 5-7: Brand A Peas analytical results

Time (s)	τ	θ	T0
60	0.089033	0.957365	26.06973
120	0.178067	0.572091	53.80942
180	0.2671	0.341864	70.3858
240	0.356134	0.204287	80.29132
300	0.445167	0.122076	86.21055
360	0.534201	0.072949	89.7477
420	0.623234	0.043592	91.86139
480	0.712267	0.026049	93.12447
540	0.801301	0.015566	93.87924
600	0.890334	0.009302	94.33027
660	0.979368	0.005558	94.59979
720	1.068401	0.003322	94.76085
780	1.157435	0.001985	94.85709
840	1.246468	0.001186	94.9146
900	1.335501	0.000709	94.94897

Table 5-8: Brand B Peas analytical results

Time (s)	τ	θ	T0
60	0.071938	1.05685	18.90682
120	0.143876	0.697167	44.80397
180	0.215814	0.459897	61.88742
240	0.287752	0.303378	73.15678
300	0.359691	0.200128	80.59079
360	0.431629	0.132017	85.49474
420	0.503567	0.087087	88.72971
480	0.575505	0.057449	90.86371
540	0.647443	0.037897	92.27143
600	0.719381	0.024999	93.20006
660	0.791319	0.016491	93.81264
720	0.863257	0.010879	94.21674
780	0.935196	0.007176	94.48331
840	1.007134	0.004734	94.65916
900	1.079072	0.003123	94.77516

Table 5-9: Brand A Peas preliminary results

	1	2	3	4	5	6
Mass of Product (g)	377.2	374.5	369.5	377.1	363.1	394
Product Temp (F)	196.5	200	200	198	200	199
Product Temp (C)	91.38889	93.33333	93.33333	92.22222	93.33333	92.77778
Mass of Water	1779.4	1772	1747.3	1752.6	1769.7	1760.2
Water Temp (F)	72.8	68.4	74.9	76.9	77.5	83.5
Water Temp (C)	22.66667	20.22222	23.83333	24.94444	25.27778	28.61111
Final Temp (F)	92.1	89.5	94.7	97	96.9	103.2
Final Temp (C)	33.38889	31.94444	34.83333	36.11111	36.05556	39.55556
Specific Heat	0.872085	0.903508	0.88918	0.924913	0.9171	0.918685
mass desnity	3.760771	3.733852	3.684001	3.759774	3.620191	3.928271

Table 5-10: Brand B Peas preliminary results

	1	2	3	4	5	6
Mass of Product (g)	411.8	401.7	393.3	407	409.4	400
Product Temp (F)	191	191	194.1	192	193.7	191.6
Product Temp (C)	88.33333	88.33333	90.05556	88.88889	89.83333	88.66667
Mass of Water	1587.4	1684.3	1675.4	1702.2	1691	1690.5
Water Temp (F)	85.4	93	95.7	98.3	92	86
Water Temp (C)	29.66667	33.88889	35.38889	36.83333	33.33333	30
Final Temp (F)	104.4	109.3	112	113.5	108.7	103.5
Final Temp (C)	40.22222	42.94444	44.44444	45.27778	42.61111	39.72222
Specific Heat	0.845738	0.836533	0.845744	0.809823	0.811509	0.839493
mass desnity	4.105741	4.005042	3.921292	4.057884	4.081813	3.988093

Table 5-11: Brand A Sports Drink preliminary results

	1	2	3	4	5
Mass of Product (g)	415	414.6	416.6	400	407.4
Product Temp (F)	196.3	199.6	199.8	206.3	201.8
Product Temp (C)	91.27777778	93.111111	93.22222	96.83333	94.33333
Mass of Water	1616.2	1552.2	1591.8	1647.4	1602.6
Water Temp (F)	77.3	77	78.7	72	98.6
Water Temp (C)	25.16666667	25	25.94444	22.22222	37
Final Temp (F)	99.5	101.4	102.6	96.1	118.1
Final Temp (C)	37.5	38.555556	39.22222	35.61111	47.83333
Specific Heat Cal/g C	0.893150453	0.9302437	0.939509	0.900688	0.916459
mass density (kg/ m^3)	4.13764623	4.1336581	4.153599	3.988093	4.061872

Table 5-12: Brand B Sports Drink preliminary results

	1	2	3	4	5	6
Mass of Product (g)	411.2	411.3	422	413.9	429.5	415
Product Temp (F)	200.2	202	197.7	198.9	200.6	199.8
Product Temp (C)	93.444444	94.44444	92.05556	92.72222	93.66667	93.22222
Mass of Water	1660.2	1630.4	1794	1687	1709	1718.7
Water Temp (F)	79	85.8	74.2	80.9	85.5	80.1
Water Temp (C)	26.111111	29.88889	23.44444	27.16667	29.72222	26.72222
Final Temp (F)	101.1	107.9	96.9	102.4	106.6	102.1
Final Temp (C)	38.388889	42.16667	36.05556	39.11111	41.44444	38.94444
Specific Heat Cal/g C	0.9003802	0.930975	0.95736	0.908094	0.893169	0.932567
mass density (kg/ m^3)	4.0997593	4.100756	4.207438	4.126679	4.282215	4.137646

Table 5-13: Brand A Corn preliminary results

	1	2	3	4	1	2	3	4
Mass of Product (g)	431.5	434.6	440	431	438	437.3	438.6	436.6
Product Temp (F)	201.7	200	200.4	200	158	148	148.7	149.5
Product Temp (C)	94.27778	93.33333	93.55556	93.33333	70	64.44444	64.83333	65.27778
Mass of Water	1716	2599	2612	2599	2623	2629	2632	2656
Water Temp (F)	91.1	76.9	76.2	74	61	58.8	61.8	58
Water Temp (C)	32.83333	24.94444	24.55556	23.33333	16.11111	14.88889	16.55556	14.44444
Final Temp (F)	109.1	90.5	90.4	88.8	70.6	67.6	71	68
Final Temp (C)	42.83333	32.5	32.44444	31.55556	21.44444	19.77778	21.66667	20
Specific Heat	0.773033	0.742748	0.766331	0.802576	0.657785	0.658018	0.710533	0.746426
mass density	4.302155	4.333063	4.386902	4.29717	4.366962	4.359982	4.372944	4.353003

Table 5-14: Brand B Corn preliminary results

	1	2	3	4	5	6
Mass of Product (g)	439.6	389.3	388.7	395.3	399	389
Product Temp (F)	201	200	200.5	199	199.5	200
Product Temp (C)	93.88889	93.33333	93.61111	92.77778	93.05556	93.33333
Mass of Water	1614.8	1779.3	1781.1	1803.1	1883.4	1780.6
Water Temp (F)	79.3	67.1	82.3	74.5	73	77.5
Water Temp (C)	26.27778	19.5	27.94444	23.61111	22.77778	25.27778
Final Temp (F)	103.2	89.6	104	95.6	94	97.8
Final Temp (C)	39.55556	32	40	35.33333	34.44444	36.55556
Specific Heat	0.897677	0.93149	1.030401	0.930797	0.939586	0.909205
mass density	4.382914	3.881411	3.875429	3.941233	3.978123	3.87842

Table 5-15: Brand A Baked Beans preliminary results

	1	2	3	4	5	6
Mass of Product (g)	445	440	442.5	440.2	430	455.7
Product Temp (F)	193	150	195.3	130	199	141
Product Temp (C)	89.44444	65.55556	90.72222	54.44444	92.77778	60.55556
Mass of Water	1669.5	1740.32	1747.6	1766	1735.4	1773.2
Water Temp (F)	84.5	79.6	82.3	72	80	68.1
Water Temp (C)	29.16667	26.44444	27.94444	22.22222	26.66667	20.05556
Final Temp (F)	104.7	93.7	102.4	87.1	102	85.1
Final Temp (C)	40.38889	34.27778	39.11111	30.61111	38.88889	29.5
Specific Heat	0.858256	0.990575	0.854494	1.412083	0.915339	1.183357
mass density	4.436753	4.386902	4.411828	4.388896	4.2872	4.543435

Table 5-16: Brand B Baked Beans preliminary results

	1	2	3	4	5	6	7	8	9
Mass of Product (g)	451	466.4	475	469	450	467	474	468	435
Product Temp (F)	193.5	135	201	145	197.5	138	166	199	152.6
Product Temp (C)	89.72222	57.22222	93.88889	62.77778	91.94444	58.88889	74.44444	92.77778	67
Mass of Water	1773.4	1695.41	1675.2	1683.9	1682.8	1678.5	1716	1763	1795.6
Water Temp (F)	74.8	75.7	78.9	84.3	79.3	63	69	59.7	60.5
Water Temp (C)	23.77778	24.27778	26.05556	29.05556	26.27778	17.22222	20.55556	15.38889	15.83333
Final Temp (F)	95.9	90	99.5	98.1	98.5	79.5	87.6	85.8	80.2
Final Temp (C)	35.5	32.22222	37.5	36.72222	36.94444	26.38889	30.88889	29.88889	26.77778
Specific Heat	0.850086	1.155154	0.715771	1.056452	0.725247	1.013754	0.858887	0.868561	1.123176
mass density	4.496575	4.650116	4.73586	4.676039	4.486604	4.656098	4.72589	4.666069	4.337051

Table 5-17: Brand A Pizza Sauce preliminary results

	1	2	3	4	5	6	7	8	9	10	11
Mass of Product (g)	426	420	435	430	404	405	419	425	405	413	407
Product Temp (F)	136.2	195.8	167	166	200	165	196.5	155	193	163.3	201.6
Product Temp (C)	57.88889	91	75	74.44444	93.33333	73.88889	91.38889	68.33333	89.44444	72.94444	94.22222
Mass of Water	1793.4	1758.4	1767	1663.8	1668.3	1664.8	1680.8	1687.4	1619	1674.6	1731.2
Water Temp (F)	59.3	73.3	75.2	75	80.4	78.3	76.5	76.2	80	64.3	70.5
Water Temp (C)	15.16667	22.94444	24	23.88889	26.88889	25.72222	24.72222	24.55556	26.66667	17.94444	21.38889
Final Temp (F)	80.1	95.8	92.1	92.7	100.3	93.6	97.4	90	98.7	82.5	92.2
Final Temp (C)	26.72222	35.44444	33.38889	33.72222	37.94444	34.22222	36.33333	32.22222	37.05556	28.05556	33.44444
Specific Heat	1.560875	0.942	0.916542	0.934334	0.824234	0.880847	0.846008	0.842936	0.792724	0.913316	0.843714
mass density	4.247319	4.187497	4.337051	4.2872	4.027974	4.037944	4.177527	4.237349	4.037944	4.117706	4.057884

Table 5-18: Brand A Tomato Bruschetta preliminary results

	1	2	3	4	5	6	7	8	9	10
Mass of Product (g)	417.5	417	425.3	434	397	423	449	434	438	399.5
Product Temp (F)	152.2	197.5	159	151	195	189.5	160.1	157.9	196.6	197.9
Product Temp (C)	66.77778	91.94444	70.55556	66.11111	90.55556	87.5	71.16667	69.94444	91.44444	92.16667
Mass of Water	1685.5	1648	1673.6	1660	1793.6	1776.5	1750.3	1737	1749	1789
Water Temp (F)	64.4	64.4	77.3	75.2	63.7	75	68	67	69	68.5
Water Temp (C)	18	18	25.16667	24	17.61111	23.88889	20	19.44444	20.55556	20.27778
Final Temp (F)	79.8	88.5	91.3	89.8	85.2	94.3	86.2	83.2	92.5	90.3
Final Temp (C)	26.55556	31.38889	32.94444	32.11111	29.55556	34.61111	30.11111	28.44444	33.61111	32.38889
Specific Heat	0.858726	0.873799	0.813759	0.912473	0.884649	0.851423	0.960048	0.86797	0.901432	0.907273
mass density	4.162572	4.157587	4.24034	4.327081	3.958182	4.217408	4.476634	4.327081	4.366962	3.983108

APPENDIX B

Table 5-19: Cost efficiency preliminary calculations

	Jar 1	Jar 2	Jar 3	Jar 4	Jar 5	Jar 6
Giant Eagle Beans (s)	248	880	2100	2183	1297	633
Tomato Bruschetta (s)	263	837	2028	2169	1260	658
Delmonte Corn (s)	115	187	313	344	227	145
outer radius (m)	0.0127	0.0254	0.0381	0.0508	0.0635	0.0762
inner radius	0.009525	0.022225	0.034925	0.047625	0.060325	0.073025
outer height (m)	1.143	0.28575	0.127	0.071438	0.04572	0.03175
inner hieght	1.139825	0.282575	0.123825	0.068263	0.042545	0.028575
outer volume (m^3)	0.000579167	0.000579167	0.000579167	0.000579	0.000579	0.000579
outer volume (cm^3)	579.1666486	579.1666486	579.1666486	579.1707	579.1666	579.1666
inner Volume m^3	0.000324876	0.000438498	0.000474494	0.000486	0.000486	0.000479
inner Volume cm^3	324.876292	438.4975268	474.4943428	486.4131	486.3994	478.7174
volume glass (m^3)	0.00025429	0.000140669	0.000104672	9.28E-05	9.28E-05	0.0001
volume glass (cm^3)	254.2903567	140.6691218	104.6723058	92.75765	92.76721	100.4492
Total Surface Area (m^2)	0.092220761	0.049657333	0.039523183	0.039017	0.043577	0.051684
Total Surface Area (cm^2)	922.207612	496.5733295	395.2318337	390.1664	435.7684	516.8416
Packaging Volume (m^3)	0.0580644	0.0290322	0.0193548	0.014516	0.011613	0.009677
Packaging Volume (cm^3)	58064.4	29032.2	19354.8	14516.2	11612.88	9677.4

Table 5-20: Jar size 1 preliminary results

Jar 1	Brand A Baked Beans	Brand A Tomato Bruschetta	Brand A Corn
Mass Density (kg/m^3)	4.409	4.222	4.347
volume (m^3)	0.000324876	0.000324876	0.000324876
mass	0.00143238	0.001371628	0.001412237
Specific Heat (J/kg C)	4336.202	3697.594	3065.496
Delta T	62	62	62
Energy (J)	385.087404	314.446787	268.4108721

Table 5-21: Jar size 2 preliminary results

Jar 2	Brand A Baked Beans	Brand A Tomato Bruschetta	Brand A Corn
Mass Density (kg/m^3)	4.409	4.222	4.347
volume (m^3)	0.000438498	0.000438498	0.000438498
mass	0.001933336	0.001851337	0.001906149
Specific Heat (J/kg C)	4336.202	3697.594	3065.496
Delta T	62	62	62
Energy (J)	519.766688	424.4204389	362.2840647

Table 5-22: Jar size 3 preliminary results

Jar 3	Brand A Baked Beans	Brand A Tomato Bruschetta	Brand A Corn
Mass Density (kg/m^3)	4.409	4.222	4.347
volume (m^3)	0.000474494	0.000474494	0.000474494
mass	0.002092046	0.002003315	0.002062627
Specific Heat (J/kg C)	4336.202	3697.594	3065.496
Delta T	62	62	62
Energy (J)	562.4349921	459.261649	392.0244213

Table 5-23: Jar size 4 preliminary results

Jar 4	Brand A Baked Beans	Brand A Tomato Bruschetta	Brand A Corn
Mass Density (kg/m^3)	4.409	4.222	4.347
volume (m^3)	0.000486413	0.000324876	0.000324876
mass	0.002144595	0.001371628	0.001412237
Specific Heat (J/kg C)	4336.202	3697.594	3065.496
Delta T	62	62	62
Energy (J)	576.5626623	314.446787	268.4108721

Table 5-24: Jar size 5 preliminary results

Jar 5	Brand A Baked Beans	Brand A Tomato Bruschetta	Brand A Corn
Mass Density (kg/m^3)	4.409	4.222	4.347
volume (m^3)	0.000486399	0.000486399	0.000486399
mass	0.002144535	0.002053578	0.002114378
Specific Heat (J/kg C)	4336.202	3697.594	3065.496
Delta T	62	62	62
Energy (J)	576.5465206	470.784552	401.8603381

Table 5-25: Jar size 6 preliminary results

Jar 6	Brand A Baked Beans	Brand A Tomato Bruschetta	Brand A Corn
Mass Density (kg/m^3)	4.409	4.222	4.347
volume (m^3)	0.000478717	0.000486399	0.000486399
mass	0.002110665	0.002053578	0.002114378
Specific Heat (J/kg C)	4336.202	3697.594	3065.496
Delta T	62	62	62
Energy (J)	567.4407708	470.784552	401.8603381

Table 5-26: Brand A Baked Beans cost analysis

Brand A Baked Beans	Jar 1	Jar 2	Jar 3	Jar 4	Jar 5	Jar 6
glass volume	254.2904	140.6691	104.6723	92.75765	92.76721	100.4492
Jar Cost	2.186455	1.20951	0.9	0.797555	0.797637	0.863689
food volume (m^3)	0.000426	0.000292	0.000394	0.000436	0.000434	0.000426
food weight (kg)	0.001877	0.001289	0.001738	0.00192	0.001914	0.001876
jar volume (m^3)	0.000254	0.000141	0.000105	9.28E-05	9.28E-05	0.0001
jar weight (kg)	0.330577	0.18287	0.136074	0.120585	0.120597	0.130584
Transport Cost/kg	0.3657	0.202574	0.151593	0.134756	0.134763	0.145706
Packaging volume (m^3)	0.058064	0.029032	0.019355	0.014516	0.011613	0.009677
Transport Cost/m^3	3.000188	1.500094	1.000063	0.750052	0.600038	0.500031
Time (s)	248	880	2100	2183	1297	633
BTU Cost	1.928889	6.844444	16.33333	16.97889	10.08778	4.923333
Pump Cost	0.154173	0.547067	1.3055	1.357098	0.806302	0.393515
Volume limit	561.098	1122.196	1683.294	2244.376	2805.49	3366.588
mass limit	47795.95	86284.32	115302.2	129708.9	129702.2	119960.6
Shipping Cost/jar	0.974874	0.487437	0.324958	0.24372	0.194975	0.162479

Table 5-27: Brand A Tomato Bruschetta cost analysis

Brand A Tomato Bruschetta	Jar 1	Jar 2	Jar 3	Jar 4	Jar 5	Jar 6
glass volume	254.2904	140.6691	104.6723	92.75765	92.76721	100.4492
Jar Cost	2.186455	1.20951	0.9	0.797555	0.797637	0.863689
food volume (m^3)	0.000426	0.000292	0.000394	0.000436	0.000434	0.000426
food weight (kg)	0.001798	0.001234	0.001664	0.001839	0.001833	0.001797
jar volume (m^3)	0.000254	0.000141	0.000105	9.28E-05	9.28E-05	0.0001
jar weight (kg)	0.330577	0.18287	0.136074	0.120585	0.120597	0.130584
Transport Cost/kg	0.365613	0.202514	0.151512	0.134666	0.134673	0.145619
Packaging volume (m^3)	0.058064	0.029032	0.019355	0.014516	0.011613	0.009677
Transport Cost/m^3	3.000188	1.500094	1.000063	0.750052	0.600038	0.500031
Time	263	837	2028	2169	1260	658
BTU Cost	2.045556	6.51	15.77333	16.87	9.8	5.117778
Pump Cost	0.163498	0.520335	1.26074	1.348395	0.7833	0.409057
Volume limit	561.098	1122.196	1683.294	2244.376	2805.49	3366.588
mass limit	47807.4	86309.94	115363.9	129795.2	129788.2	120032.7
Shipping Cost/jar	0.974874	0.487437	0.324958	0.24372	0.194975	0.162479

Table 5-28: Brand A Corn cost analysis

Brand A Corn	Jar 1	Jar 2	Jar 3	Jar 4	Jar 5	Jar 6
glass volume	254.2904	140.6691	104.6723	92.75765	92.76721	100.4492
Jar Cost	2.186455	1.20951	0.9	0.797555	0.797637	0.863689
food volume (m^3)	0.000426	0.000292	0.000394	0.000436	0.000434	0.000426
food weight (kg)	0.001851	0.001271	0.001713	0.001893	0.001887	0.00185
jar volume (m^3)	0.000254	0.000141	0.000105	9.28E-05	9.28E-05	0.0001
jar weight (kg)	0.330577	0.18287	0.136074	0.120585	0.120597	0.130584
Transport Cost/kg	0.365671	0.202555	0.151566	0.134726	0.134733	0.145677
Packaging volume (m^3)	0.058064	0.029032	0.019355	0.014516	0.011613	0.009677
Transport Cost/m^3	3.000188	1.500094	1.000063	0.750052	0.600038	0.500031
Time	115	187	313	344	227	145
BTU Cost	0.894444	1.454444	2.434444	2.675556	1.765556	1.127778
Pump Cost	0.071492	0.116252	0.194582	0.213853	0.141118	0.090142
Volume limit	561.098	1122.196	1683.294	2244.376	2805.49	3366.588
mass limit	47799.75	86292.81	115322.6	129737.5	129730.7	119984.5
Shipping Cost/jar	0.974874	0.487437	0.324958	0.24372	0.194975	0.162479