DEFLECTION OF A SYMMETRICAL, PARABOLIC, FIXED-END ARCH

by

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ABSTRACT

DEFLECTION OF A SYMMETRICAL, PARABOLIC,

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The purpose of this thesis was to solve the complexity of the analysis of symmetrical, parabolic fixed-end arches by a numerical analysis method. The conditions of loading were vertical concentrated, vertical uniformly distributed and horizontal uniformly distributed.

The analysis for the given types of loading was accomplished in three steps:

1) Computer programs were written that would numerically analyze an experimental arch. The programs were developed to compute the three redundants (horizontal reaction, vertical reaction and clockwise fixed-end moment) at the right fixed end; along with outputs for the vertical and horizontal deflection for any point on the span of the arch.

2) The results from the computer analysis of the experimental arch are validated by several techniques, to verify results.

3) The experimental arch was analyzed for various loads and rise-to-span ratios and the data were applied to the construction of a set of design charts.

The design charts show:

1) The vertical deflection for any point on any symmetrical, parabolic, fixed-end arch with different types of loading;

2) the horizontal deflection for the same data;

3) the horizontal reaction;

4) the vertical reaction;

5) the fixed-end moment.

With the design charts constructed, results were obtained applicable for any rise-to-span ratio, length of the span, and intensity of load.

To

My Parents

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S.P.

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CHAPTER I

INTRODUCTION

The structural analysis of arch members is much more complex than that of the straight members. For example it can be seen that, by the principle of virtual work, the deflection equation is given by

$$\Delta = \int_0^{L_A} \frac{Mm}{EI}\, ds$$

where Δ represents the deflection of the desired point in the direction of the dummy unit load;

M represents the bending moment in the member caused by the real loads;

m represents the bending moment in the member caused by the dummy unit load;

E represents the Modulus of elasticity of the member;

I represents the moment of inertia of the member;

ds represents the increment of the member along the actual length;

L_A represents the actual (axial) length of the member.

For straight members $ds = dx$ where $0 \leqslant x \leqslant L$, and

where dx is the increment of the member along the straight length (usually in horizontal),

x is the distance of any point in the member from the left fixed end support, and

L is the span length of the member.

But for arch members $ds = \sqrt{dx^2 + dy^2}$

$$= \sqrt{1 + (\frac{dy}{dx})^2} \; dx$$

for $0 \leq x \leq L$ where y is the ordinate distance, and has the geometric relation to the absissa x.

The equation for the member with curved axes becomes

$$\Delta = \int_0^L \frac{Mm}{EI} \sqrt{1 + (\frac{dy}{dx})^2} \; dx$$

To simplify matters, however, it is often assumed that the moment of inertia of the cross-section of the curved member varies directly as the secant of the angle β, where β is the angle between the horizontal and a tangent drawn from a point on the axis of curved member, that is, (see Figure 1.1)

$$I = I_c \text{ Sec } \beta \text{ ,}$$

where I_c = moment of inertia of the cross section of the curved member at the crown.

If that is the case, then

$$\Delta = \int_0^{L_A} \frac{Mm \; ds}{EI_c \sec \beta} \; .$$

Since ds in the curved axis member is ds = Sec β dx (See Fig. 1.1), hence

$$\Delta = \int_0^L \frac{Mm \; dx}{EI_c} \; .$$

Then, the integration is further simplified. However, this assumption holds and gives the accurate results only when the curve is very flat, and the greater the curvature becomes, the less accurate the result is. Therefore, some other methods must be introduced; ones which will not involve a complicated integration, and which will give accurate results.

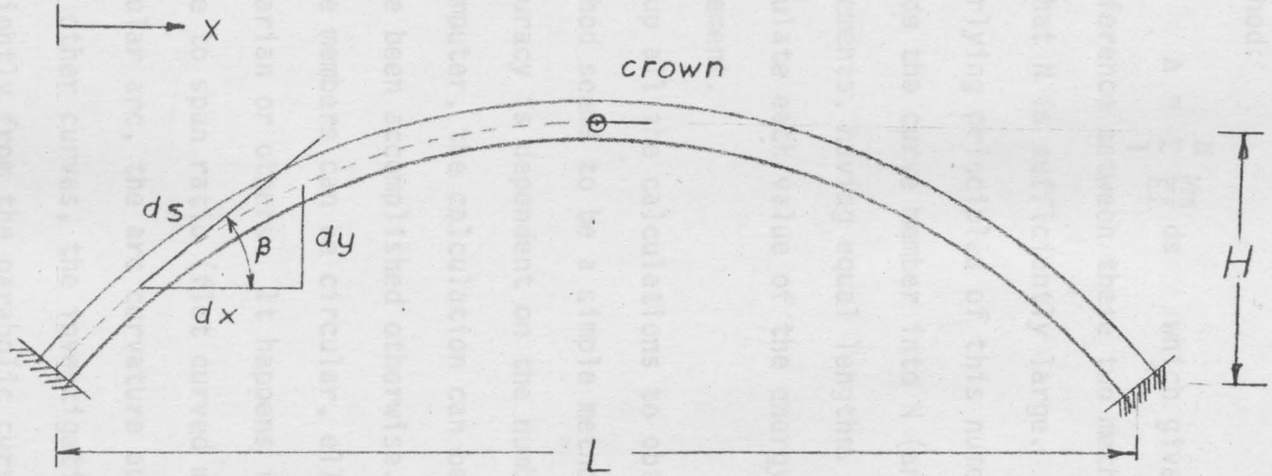


Fig. 1.1 The arch and parameters

Mathematically, the problem can be handled by changing the following integration:

$$\Delta = \int_0^{L_A} \frac{Mm}{EI}\, ds$$

which yields the exact results to the numerical method:

$$\Delta = \sum_1^N \frac{Mm}{EI}\, ds$$

which gives only approximate results. The difference between these two methods, however, is negligible, provided that N is sufficiently large.

The underlying principles of this numerical method are:

1) Divide the curve member into N (any arbitrary number) increments, having equal lengths.

2) Calculate each value of the energy function for each increment.

3) Sum up all the calculations to obtain the final result.

This method seems to be a simple method, but unfortunately, the degree of accuracy is dependent on the number of increments. With the aid of the computer, the calculation can be made at a greater speed than it could have been accomplished otherwise.

The curve members can be circular, elliptical, parabolic, transformed catenarian or others. It happens that in a segmental member of low rise to span ratio (flat curved member) with the axis defined by a circular arc, the arc curvature approaches that of a parabola. As for other curves, the investigations also indicate results, deviating only slightly from the parabolic curve. That means that the solutions derived for flat arched members with parabolic axes are applicable to flat arched members with various curvatures axes and provide sufficiently accurate results in the range of a low rise to span ratio.

Hence, the basis of this thesis involved the development of a computer program that would numerically analyze a symmetrical, parabolic, fixed end arch subjected to a vertical concentrated load, a vertical uniformly distributed load over any portion of the span, a horizontal uniformly distributed load over any half-left portion of the span and would print out the horizontal and vertical deflections for any point along the arch axis.

Finally the influence diagrams,or so called design charts, were constructed for vertical and horizontal deflection at any point on the parabolic fixed end arch due to various types of loading conditions.

CHAPTER II

THE COMPUTER PROGRAM

A representative arch will be introduced and analyzed, the arch being a constant 100 feet in length; the concentrated load 1000 pounds and the uniformly distributed load 100 pounds per foot; the rises being 25, 30, 40, and 50 feet respectively.

The assumptions made for the analysis are:

1) E and I are constant
2) The members are within proportional limit, that is, Hooke's Law holds (stress is proportional to strain)
3) The arch geometry is symmetrical
4) All deflections are arch center line deflections
5) The effect from the axial deformation is negligible. This assumption is valid only for arches with rise-to-span ratios greater than 0.2. For smaller ratios, i.e., flat arches, axial effects can be significant.

With the use of Fortran IV computer language, programs were developed to iterate the increments. The program consists of a main program and ten subprograms. The main program will read and write the following:

1) Specification of arch (span, rise)
2) Loading conditions (concentrated, uniforms, horizontal, vertical, span and intensity of load)
3) Positions and numbers of points where deflections are to be calculated.

4) Print out data in number 1, 2, and 3

5) Call the subprograms from storages

6) Print out the desired data and deflection

Since fixed-end arches are statically indeterminate to the third degree, it is required to solve the determinants for three simultaneous equations in order to determine the unknown reactions at the right end of the fixed-end arch.

The following subprograms are used to solve for the reactions at both fixed ends:

ACTVC (due to vertical concentrated loading)

ACTHC (due to horizontal concentrated loading)

ACTVU (due to vertical uniform loading)

ACTHU (due to horizontal uniform loading)

The following subprograms are used to solve for vertical deflections:

VDFVC (due to the vertical concentrated load; other subprogram needed is ACTVC)

VDFVU (due to the vertical uniformed load; other subprograms needed are ACTVU and ACTVC)

VDFHU (due to the horizontal uniformed load; other subprograms needed are ACTHU and ACTVC)

The following subprograms are used to solve for horizontal deflection:

HDFVC (due to the vertical concentrated load; other subprograms needed are ACTVC and ACTHC)

HDFVU (due to the vertical uniformed load; other subprograms needed are ACTVU and ACTHC)

HDFHU (due to the horizontal uniformed load; other subprograms needed are ACTHU and ACTHC)

A basic flow chart of the program along with computer print-outs of each subprogram are shown in Figures 2.1 through 2.14 inclusive. Figures 2.15 through 2.19 show typical input and output data.

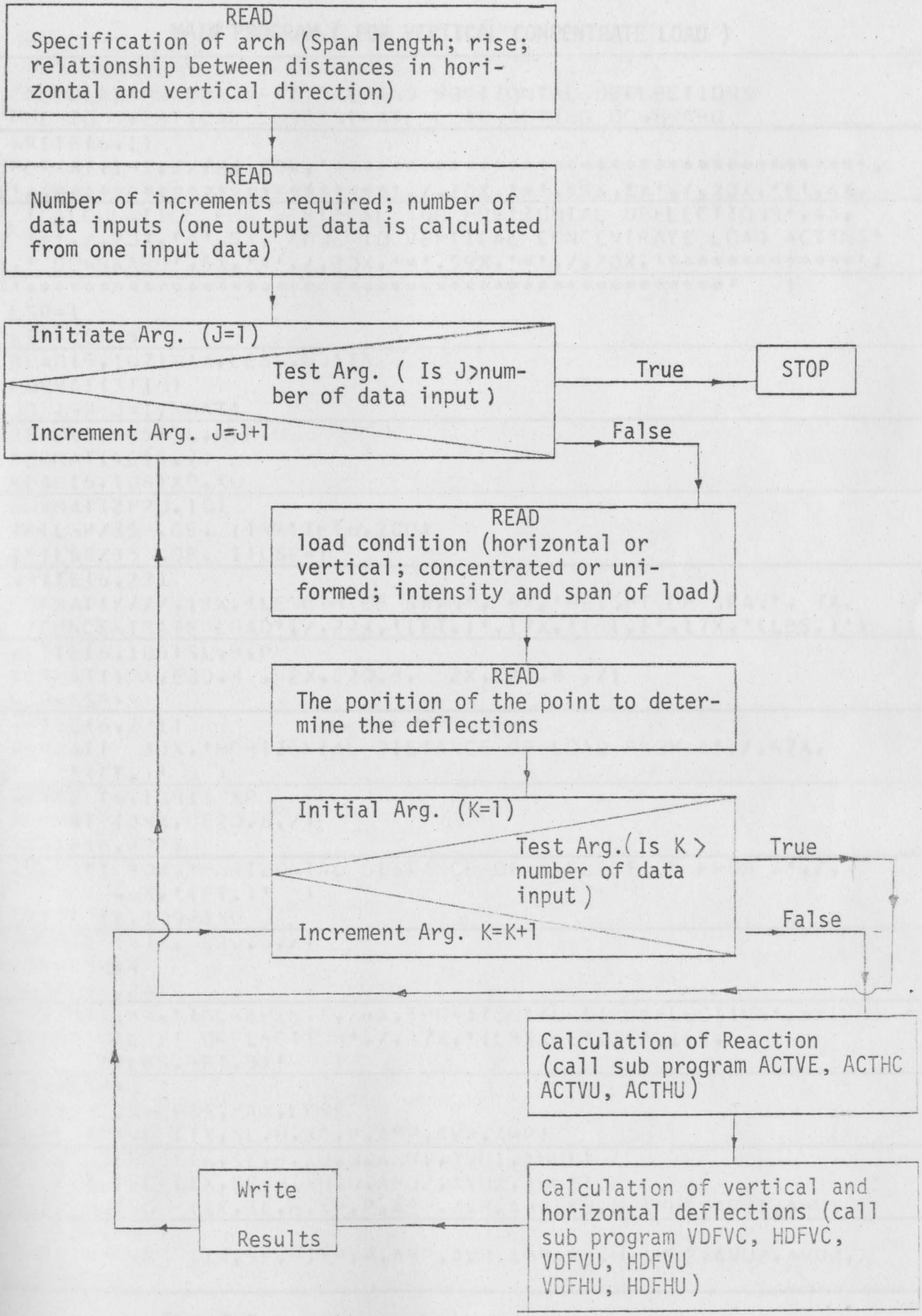


Fig. 2.1 General flow chart of computer program

```
C
C            MAIN PROGRAM ( FOR VERTICAL CONCENTRATE LOAD )
C
C
C     CALCULATION FOR VERTICAL AND HORIZONTAL DEFLECTIONS
C     DUE TO  VERTICAL CONCENTRATE LOAD ACTING DOWNWARD
      WRITE(6,1)
    1 FORMAT(1H1,/////,20X,'***************************************',
     1'*************************',/,20X,'*',59X,'*',/,20X,'*',4X,
     2 'CALCULATION FOR VERTICAL AND HORIZONTAL DEFLECTIONS',4X,
     3 '*',/,20X,'*',5X, 'DUE TO VERTICAL CONCENTRATE LOAD ACTING'
     4,' DOWNWARD',6X,'*',/,20X,'*',59X,'*',/,20X,'************',
     5'*****************************************************'     )
      LSP=1
      LSP=LSP+5
      READ(5,107)MAX,LESS,NDATA
  107 FORMAT(3I10)
      DO 199 J=1,NDATA
      READ(5,105)SL,H,P,U
  105 FORMAT(4E15.7)
      READ(5,108)XP,XU
  108 FORMAT(2F20.10)
      IF(LSP/35 .GE. 1)WRITE(6,200)
      IF(LSP/35 .GE. 1)LSP=1
      WRITE(6,22)
   22 FORMAT(////,18X,'LENGTH OF SPAN', 8X,'HEIGHT OF SPAN', 7X,
     1 'CONCENTRATE LOAD',/,22X,'(FT.)',17X,'(FT.)',17X,'(LBS.)')
      WRITE(6,106)SL,H,P
  106 FORMAT(12X,E20.8 , 2X,E20.8,  2X,E20.8 ,/)
      LSP=LSP+8
      WRITE(6,231)
  231 FORMAT(  30X,'HORIZONTAL DISTANCE OF LOAD FROM A',/,47X,
     1     '(FT.)'   )
      WRITE (6,1091) XP
 1091 FORMAT (38X, E20.8,/)
      WRITE(6,232)
  232 FORMAT( 30X,'HORIZONTAL DISTANCE OF DEFLECTION FROM A',/,
     1        46X,'(FT.)'   )
      WRITE (6,1092)XU
 1092 FORMAT (37X, E20.8,/)
      LSP=LSP+4
      WRITE(6,24)
   24 FORMAT(18X,'INCREMENT', 4X,'HORIZONTAL EI DEFLECTION',5X,
     1'VERTICAL EI DEFLECTION',/,37X,'(LBS.-FT.3)',18X,
     2      '(LBS.-FT.3)'    )
      LSP=LSP+3
      DO 198 IX=LESS,MAX,LESS
      CALL ACTVC (IX,SL,H,XP,P,AMP,AVP,AHP)
      CALL ACTHC (IX,SL,H,XU,U,AMU1,AVU1,AHU1)
      CALL ACTVC (IX,SL,H,XU,U,AMU2,AVU2,AHU2)
      CALL HDFVC  (IX,SL,H,XP,P,AMP,AVP,AHP,XU,U,AMU1,AVU1,AHU1,
     1   HDF,YU )
      CALL VDFVC  (IX,SL,H,XP,P,AMP,AVP,AHP,XU,U,AMU2,AVU2,AHU2,
     1   VDF )
C
C
C
C
```

Fig. 2.2 Computer print-out of main program(A)

```
C
C
C
  198   WRITE(6,110)IX,    HDF,VDF
  110   FORMAT(18X,I6,     7X,          E20.10, 9X,E20.10)
        LSP=LSP+MAX/LESS
  199   CONTINUE
  200   FORMAT(1H1,/// )
        STOP
        END
```

Fig. 2.2 (continue)

```
C
C                MAIN PROGRAM ( FOR VERTICAL UNIFORM LOAD )
C
C
C     CALCULATION FOR VERTICAL AND HORIZONTAL DEFLECTIONS
C     DUE TO  VERTICAL  UNIFORM LOAD ACTING DOWNWARD
      WRITE(6,1)
1     FORMAT(1H1,/////,20X,'***************************************',
     1'*************************',/,20X,'*',59X,'*',/,20X,'*',4X,
     2 'CALCULATION FOR VERTICAL AND HORIZONTAL DEFLECTIONS',4X,
     3 '*',/,20X,'*',7X,    'DUE TO VERTICAL UNIFORM LOAD ACTING ',
     4 'DOWNWARD',8X, '*',/,20X,'*',59X,'*',/,20X,'*************',
     5'***************************************************'     )
      LSP=1
      LSP=LSP+3
      READ(5,107)MAX,LESS,NDATA
107   FORMAT(3I10)
      DO 199 J=1,NDATA
      READ(5,105)SL,H,W,U
105   FORMAT(4E15.7)
      READ(5,108)XW1,XW2,XU
108   FORMAT(3E20.10)
      IF(LSP/32 .GE. 1)WRITE(6,200)
      IF(LSP/32 .GE. 1)LSP=1
      WRITE(6,22)
22    FORMAT(////,18X,'LENGTH OF SPAN', 8X,'HEIGHT OF SPAN',10X,
     1     'UNIFORM LOAD',/,22X,'(FT.)',17X,'(FT.)',14X,'(LBS. '
     2, 'PER FT.)'   )
      WRITE(6,106)SL,H,W
106   FORMAT(12X,E20.8 , 2X,E20.8,  3X,E20.8 ,/)
      LSP=LSP+6
      WRITE(6,231)
231   FORMAT(  32X,    'HORIZONTAL SPAN OF LOAD FROM A',/,35X,
     1 '(FT.)', 15X, '(FT.)'     )
      WRITE (6,1091) XW1,XW2
1091  FORMAT (25X, E20.8,      E20.8,/)
      WRITE(6,232)
232   FORMAT( 27X,'HORIZONTAL DISTANCE OF DEFLECTION FROM A',/,
     1       46X,'(FT.)'  )
      WRITE (6,1092)XU
1092  FORMAT (36X, E20.8,/)
      LSP=LSP+4
      WRITE(6,24)
24    FORMAT(18X,'INCREMENT', 4X,'HORIZONTAL EI DEFLECTION',5X,
     1'VERTICAL EI DEFLECTION',/,37X,'(LBS.-FT.3)',18X,
     2      '(LBS.-FT.3)'    )
      LSP=LSP+3
      DO 198 IX=LESS,MAX,LESS
         CALL ACTVU  (IX, SL,H,XW1,XW2,W,AMP,AVP,AHP)
      CALL ACTVC (IX,SL,H,XU,U,AMU2,AVU2,AHU2)
      CALL ACTHC (IX,SL,H,XU,U,AMU1,AVU1,AHU1)
      CALL VDFVU (IX,SL,H,XW1,XW2,W,AMP,AVP,AHP,XU,U,AMU2,AVU2,
     1 AHU2,VDF)
C
C
C
C
C
```

Fig. 2.3 Computer print-out of main program(B)

```
C
C
C
      CALL HDFVU (IX,SL,H,XW1,XW2,W,AMP,AVP,AHP,XU,U,AMU1,AVU1,
     1 AHU1,HDF,YU)
 198  WRITE(6,110)IX, HDF,VDF
 110  FORMAT(18X,I6,    7X,          E20.10, 9X,E20.10)
      LSP=LSP+MAX/LESS
 199  CONTINUE
 200  FORMAT(1H1,///    )
      STOP
      END
```

Fig. 2.3 (continue)

```
C
C              MAIN PROGRAM ( FOR HORIZONTAL UNIFORM LOAD )
C
C
C     CALCULATION FOR VERTICAL AND HORIZONTAL DEFLECTIONS
C     DUE TO HORIZONTAL UNIFORM LOAD ACTING RIGHTWARD
      WRITE(6,1)
    1 FORMAT(1H1,/////,20X,'****************************************',
     1'**************************',/,20X,'*',59X,'*',/,20X,'*',4X,
     2 'CALCULATION FOR VERTICAL AND HORIZONTAL DEFLECTIONS',4X,
     3 '*',/,20X,'*',6X, 'DUE TO HORIZONTAL UNIFORM LOAD ACTING ',
     4 'RIGHTWARD',6X,'*',/,20X,'*',59X,'*',/,20X,'*************',
     5'**********************************************'    )
      LSP=1
      LSP=LSP+6
      READ(5,107)MAX,LESS,NDATA
  107 FORMAT(3I10)
      DO 199 J=1,NDATA
      IF(LSP/32 .GE. 1)WRITE(6,200)
      IF(LSP/32 .GE. 1)LSP=1
      READ(5,105)SL,H,W,U
  105 FORMAT(4E15.7)
      READ(5,108)XW1,XW2,XU
  108 FORMAT(3F20.10)
      WRITE(6,22)
   22 FORMAT(////,18X,'LENGTH OF SPAN', 8X,'HEIGHT OF SPAN',10X,
     1       'UNIFORM LOAD',/,22X,'(FT.)',17X,'(FT.)',14X,'(LBS. '
     2, 'PER FT.)'   )
      LSP=LSP+3
      WRITE(6,106)SL,H,W
  106 FORMAT(12X,E20.8 , 2X,E20.8,  3X,E20.8 ,/)
      WRITE(6,231)
  231 FORMAT(  33X,        'VERTICAL SPAN OF LOAD FROM A',/,35X,
     1   '(FT.)',15X,'(FT.)'     )
      WRITE (6,1091) XW1,XW2
 1091 FORMAT (25X, E20.8,       F20.8,/)
      WRITE(6,232)
  232 FORMAT( 27X,'HORIZONTAL DISTANCE OF DEFLECTION FROM A',/,
     1        46X,'(FT.)'   )
      WRITE (6,1092)XU
 1092 FORMAT (36X, E20.8,/)
      LSP=LSP+4
      WRITE(6,24)
   24 FORMAT(18X,'INCREMENT', 4X,'HORIZONTAL EI DEFLECTION',5X,
     1'VERTICAL EI DEFLECTION',/,37X,'(LBS.-FT.3)',18X,
     2       '(LBS.-FT.3)'    )
      LSP=LSP+3
      DO 198 IX=LESS,MAX,LESS
      CALL ACTHU  (IX,SL,H,XW1,XW2,W,AMP,AVP,AHP)
      CALL ACTVC  (IX,SL,H,XU,U,AMU2,AVU2,AHU2)
      CALL ACTHC  (IX,SL,H,XU,U,AMU1,AVU1,AHU1)
      CALL HDFHU  (IX,SL,H,XW1,XW2,W,AMP,AVP,AHP,XU,U,AMU1,AVU1,
     1  AHU1,HDF,YU)
C
C
C
C
```

Fig. 2.4 Computer print-out of main program(C)

```
C
C
C
      CALL VDFHU  (IX,SL,H,XW1,XW2,W,AMP,AVP,AHP,XU,U,AMU2,AVU2,
     1 AHU2,VDF)
  108 WRITE(6,110)IX,    HDF,VDF
  110 FORMAT(18X,I6,   7X,          E20.10, 9X,E20.10)
      LSP=LSP+MAX/LESS
  199 CONTINUE
  200 FORMAT(1H1,////       )
      STOP
      END
```

Fig. 2.4 (continue)

```
C
C                          SUBPROGRAM  ACTVC
C
      SUBROUTINE  ACTVC (K,SL,H,XP,P,AM,AV,AH)
C     **SUBPROGRAM TO FIND REACTIONS**
C     *** DUE TO VERTICAL CONCENTRATE LOAD ***
C     THIS PROGRAM IS USED FOR FINDING THE VALUES OF M, V AND H
C      AT A AND B FOR A FIXED END PARABOLIC ARCH.
C     (REMOVE ALL REACTIONS AND CHANGE TO FREE END AT B)
      Z=K
      DELX=SL/Z
      X=DELX/2.
C     INITIALIZE VALUES FOR ALL FUNCTION-INCREMENTS
      FN1=0.0
      FN2=0.0
      FN3=0.0
      FN4=0.0
      FN5=0.0
      FN6=0.0
      FN7=0.0
      FN8=0.0
      FN9=0.0
C     THE FOLLOWING STATEMENTS ARE USED FOR CALCULATING SUMMATIONS
C       OF ALL FUNCTION-INCREMENTS
      DO 97 I=1,K
      E=4.0*H/SL**2
      F=SL-2.0*X
      G=(E*F)**2
      DS=(SQRT(1.0+G))*DELX
      Y=E*(SL*X-X**2)
      IF(X .GE. 0. .AND. X .LT. XP) C1=0.00
      IF(X .GE. XP .AND. X .LE. SL) C1=1.00
      PM=P*(X-XP)*(1.0-C1)
      FUNCT1=DS*PM
      FUNCT2=DS*PM*(X-SL)
      FUNCT3=DS*PM*Y
      FUNCT4=DS
      FUNCT5=DS*(X-SL)
      FUNCT6=DS*Y
      FUNCT7=DS*((X-SL)**2)
      FUNCT8=DS*Y*(X-SL)
      FUNCT9=DS*(Y**2)
      FN1=FN1+FUNCT1
      FN2=FN2+FUNCT2
      FN3=FN3+FUNCT3
      FN4=FN4+FUNCT4
      FN5=FN5+FUNCT5
      FN6=FN6+FUNCT6
      FN7=FN7+FUNCT7
      FN8=FN8+FUNCT8
      FN9=FN9+FUNCT9
      X=X+DELX
   97 CONTINUE
```

Fig. 2.5 Computer print-out of subprogram ACTVC

```
C
C
      A=FN1*(FN7*FN9-FN8**2)-FN5*(FN2*FN9-FN3*FN8)
     1 +   FN6*(FN2*FN8-FN3*FN7       )
      B=FN4*(FN2*FN9-FN3*FN8)-FN1*(FN5*FN9-FN6*FN8)
     1 +   FN6*(FN5*FN3-FN6*FN2       )
      C=FN4*(FN7*FN3-FN8*FN2)-FN5*(FN5*FN3-FN6*FN2)
     1 + FN1*(FN5*FN8-FN6*FN7  )
      D=FN4*(FN7*FN9-FN8**2)-FN5*(FN5*FN9-FN6*FN8)
     1 +   FN6*(FN5*FN8-FN6*FN7   )
  900 FORMAT(40X,4E15.8)
      IF(D .EQ. 0.) WRITE(6,900)A,B,C,D
      BM=A/D
      BV=B/D
      BH=C/D
      AH= BH
      AV=P-BV
      AM=BV*SL-BM-P*XP
      RETURN
      END
```

Fig. 2.5 (continue)

```
C
C                          SUBPROGRAM  ACTHC
C
      SUBROUTINE  ACTHC (K,SL,H,XP,P,AM,AV,AH)
C     **SUBPROGRAM TO FIND REACTIONS**
C     ***  DUE TO HORIZONTAL CONCENTRATE LOAD  ***
C     THIS PROGRAM IS USED FOR FINDING THE VALUES OF M, V AND H
C     AT A AND B FOR A FIXED END PARABOLIC ARCH.
C     (REMOVE ALL REACTIONS AND CHANGE TO FREE END AT B)
      Z=K
      DELX=SL/Z
      X=DELX/2.
C     INITIALIZE VALUES FOR ALL FUNCTION_INCREMENTS
      FN1=0.0
      FN2=0.0
      FN3=0.0
      FN4=0.0
      FN5=0.0
      FN6=0.0
      FN7=0.0
      FN8=0.0
      FN9=0.0
C     THE FOLLOWING STATEMENTS ARE USED FOR CALCULATING SUMMATIONS
C       OF ALL FUNCTION-INCREMENTS
      DO 97 I=1,K
      E=4.0*H/SL**2
      F=SL-2.0*X
      G=(E*F)**2
      DS=(SQRT(1.0+G))*DELX
      Y=E*(SL*X-X**2)
      YP=E*(SL*XP-XP**2)
      IF(X .GE. 0. .AND. X .LE. XP)PM=P*(Y-YP)
      IF(X .GT. XP .AND. X .LE. SL)PM=0.0
      FUNCT1=DS*PM
      FUNCT2=DS*PM*(X-SL)
      FUNCT3=DS*PM*Y
      FUNCT4=DS
      FUNCT5=DS*(X-SL)
      FUNCT6=DS*Y
      FUNCT7=DS*((X-SL)**2)
      FUNCT8=DS*Y*(X-SL)
      FUNCT9=DS*(Y**2)
      FN1=FN1+FUNCT1
      FN2=FN2+FUNCT2
      FN3=FN3+FUNCT3
      FN4=FN4+FUNCT4
      FN5=FN5+FUNCT5
      FN6=FN6+FUNCT6
      FN7=FN7+FUNCT7
      FN8=FN8+FUNCT8
      FN9=FN9+FUNCT9
      X=X+DELX
   97 CONTINUE
```

Fig. 2.6 Computer print-out of subprogram ACTHC

```
C
C
      A=FN1*(FN7*FN9-FN8**2)-FN5*(FN2*FN9-FN3*FN8)
     1   +  FN6*(FN2*FN8-FN3*FN7    )
      B=FN4*(FN2*FN9-FN3*FN8)-FN1*(FN5*FN9-FN6*FN8)
     1   +  FN5*(FN5*FN3-FN6*FN2    )
      C=FN4*(FN7*FN3-FN8*FN2)-FN5*(FN5*FN3-FN6*FN2)
     1   +  FN1*(FN5*FN8-FN6*FN7    )
      D=FN4*(FN7*FN9-FN8**2)-FN5*(FN5*FN9-FN6*FN8)
     1   +  FN6*(FN5*FN8-FN6*FN7    )
      BM=A/D
      BV=B/D
      BH=C/D
      AH=BH-P
      AV=-BV
      AM=BV*SL-BM-P*YP
      RETURN
      END
```

Fig. 2.6 (continue)

```
C
C                          SUBPROGRAM  ACTVU
C
C
      SUBROUTINE ACTVU  (K,SL,H,XW1,XW2,W,AM,AV,AH)
C     **SUBPROGRAM TO FIND REACTIONS**
C     ***  DUE TO VERTICAL UNIFORM LOAD   ***
C     THIS PROGRAM IS USED FOR FINDING THE VALUES OF M, V AND H
C       AT A AND B FOR A FIXED END PARABOLIC ARCH.
C     (REMOVE ALL REACTIONS AND CHANGE TO FREE END AT B)
      Z=K
      DELX=SL/Z
      X=DELX/2.
C     INITIALIZE VALUES FOR ALL FUNCTION_INCREMENTS
      FN1=0.0
      FN2=0.0
      FN3=0.0
      FN4=0.0
      FN5=0.0
      FN6=0.0
      FN7=0.0
      FN8=0.0
      FN9=0.0
C     THE FOLLOWING STATEMENTS ARE USED FOR CALCULATING
C       SUMMATIONS OF ALL FUNCTION-INCREMENTS
      DO 97 I=1,K
      E=4.0*H/SL**2
      F=SL-2.0*X
      G=(E*F)**2
      DS=(SQRT(1.0+G))*DELX
      Y=E*(SL*X-X**2)
      IF(X .GE. 0. .AND. X .LE. XW1)PM=(XW2-XW1)*W*X-((XW2**2)
     1  - (XW1**2))*W/2.0
      IF(X .GT. XW1 .AND. X .LE. XW2)PM=XW2*X*W-((XW2**2)
     1  + (X**2))*W/2.0
      IF(X .GT. XW2 .AND. X .LE.  SL)PM=0.0
      FUNCT1=DS*PM
      FUNCT2=DS*PM*(X-SL)
      FUNCT3=DS*PM*Y
      FUNCT4=DS
      FUNCT5=DS*(X-SL)
      FUNCT6=DS*Y
      FUNCT7=DS*((X-SL)**2)
      FUNCT8=DS*Y*(X-SL)
      FUNCT9=DS*(Y**2)
      FN1=FN1+FUNCT1
      FN2=FN2+FUNCT2
      FN3=FN3+FUNCT3
      FN4=FN4+FUNCT4
      FN5=FN5+FUNCT5
      FN6=FN6+FUNCT6
      FN7=FN7+FUNCT7
      FN8=FN8+FUNCT8
```

Fig. 2.7 Computer print-out of subprogram ACTVU

```
      C
      C
      C
         FN9=FN9+FUNCT9
         X=X+DELX
   97    CONTINUE
         A=FN1*(FN7*FN9-FN8**2)-FN5*(FN2*FN9-FN3*FN8)
        1  +   FN6*(FN2*FN8-FN3*FN7        )
         B=FN4*(FN2*FN9-FN3*FN8)-FN1*(FN5*FN9-FN6*FN8)
        1  +   FN6*(FN5*FN3-FN6*FN2      )
         C=FN4*(FN7*FN3-FN8*FN2)-FN5*(FN5*FN3-FN6*FN2)
        1  +   FN1*(FN5*FN8-FN6*FN7     )
         D=FN4*(FN7*FN9-FN8**2)-FN5*(FN5*FN9-FN6*FN8)
        1  +   FN6*(FN5*FN8-FN6*FN7     )
         BM=A/D
         BV=B/D
         BH=C/D
         AH= BH
         AV=W*(XW2-XW1)-BV
         AM=BV*SL-BM-W*((XW2**2)-(XW1**2))/2.0
         RETURN
         END
```

Fig. 2.7 (continue)

```
C
C                         SUBPROGRAM  ACTHU
C
      SUBROUTINE  ACTHU  (K,SL,H,XW1,XW2,W,AM,AV,AH)
C     **SUBPROGRAM TO FIND REACTIONS**
C     ***  DUE TO HORIZONTAL UNIFORM LOAD   ***
C     THIS PROGRAM IS USED FOR FINDING THE VALUES OF M, V AND H
C      AT A AND B FOR A FIXED END PARABOLIC ARCH.
C     (REMOVE ALL REACTIONS AND CHANGE TO FREE END AT B)
      Z=K
      DELX=SL/Z
      X=DELX/2.
C     INITIALIZE VALUES FOR ALL FUNCTION_INCREMENTS
      FN1=0.0
      FN2=0.0
      FN3=0.0
      FN4=0.0
      FN5=0.0
      FN6=0.0
      FN7=0.0
      FN8=0.0
      FN9=0.0
C     THE FOLLOWING STATEMENTS ARE USED FOR CALCULATING SUMMATIONS
C       OF ALL FUNCTION-INCREMENTS
      DO 97 I=1,K
      E=4.0*H/SL**2
      F=SL-2.0*X
      G=(E*F)**2
      DS=(SQRT(1.0+G))*DELX
      Y=E*(SL*X-X**2)
      YW1=E*(SL*XW1-XW1**2)
      YW2=E*(SL*XW2-XW2**2)
      IF(X .GE. 0. .AND. X .LE. XW1)PM=(YW2-YW1)*W*Y
     1  - ((YW2**2)-(YW1**2))*W/2.0
      IF(X .GT. XW1 .AND. X .LE. XW2)PM=YW2*Y*W-((YW2**2)
     1    + (Y**2))*W/2.0
      IF(X .GT. XW2 .AND. X .LE.  SL)PM=0.0
      FUNCT2=DS*PM*(X-SL)
      FUNCT3=DS*PM*Y
      FUNCT4=DS
      FUNCT5=DS*(X-SL)
      FUNCT6=DS*Y
      FUNCT7=DS*((X-SL)**2)
      FUNCT8=DS*Y*(X-SL)
      FUNCT9=DS*(Y**2)
      FN2=FN2+FUNCT2
      FN3=FN3+FUNCT3
      FN4=FN4+FUNCT4
      FN5=FN5+FUNCT5
      FN6=FN6+FUNCT6
      FN7=FN7+FUNCT7
      FN8=FN8+FUNCT8
      FN9=FN9+FUNCT9
      X=X+DELX
```

Fig. 2.8 Computer print-out of subprogram ACTHU

```
      C
      C
      C
   97 CONTINUE
      A=FN1*(FN7*FN9-FN8**2)-FN5*(FN2*FN9-FN3*FN8)
     1    +   FN6*(FN2*FN8-FN3*FN7    )
      B=FN4*(FN2*FN9-FN3*FN8)-FN1*(+N5*FN9-FN6*FN8)
     1  +   FN6*(FN5*FN3-FN6*FN2     )
      C=FN4*(FN7*FN3-FN8*FN2)-FN5*(FN5*FN3-FNo*FN2)
     1  +   FN1*(FN5*FN8-FN6*FN7    )
      D=FN4*(FN7*FN9-FN8**2)-FN5*(FN5*FN9-FN6*FN8)
     1  +   FN6*(FN5*FN8-FN6*FN7    )
      BM=A/D
      BV=B/D
      BH=C/D
      AM=BV*SL-BM-W*((YW2**2)-(YW1**2))/2.0
      AH= BH-W*((YW2-YW1))
      AV= -BV
      RETURN
      END
```

Fig. 2.8 (continue)

```
C
C                          SUBPROGRAM  VDFVC
C
      SUBROUTINE VDFVC (K,SL,H,XP,P,AMP,AVP,AHP,XU,U,AMU,AVU,AHU,
     1  VDF )
C     **SUBPROGRAM TO FIND  VERTICAL  DEFLECTIONS**
C     ***DUE TO VERTICAL CONCENTRATE LOAD***
      Z=K
      DELX=SL/Z
      X=DELX/2.
C     INITIALIZE VALUE FOR VERTICAL EI DEFLECTIONS
      VDF=0.0
C     THE FOLLOWING STATEMENTS ARE USED FOR CALCULATING
C     VERTICAL EI DEFLECTIONS
      DO 197 I=1,K
      F=4.0*H/SL**2
      F=SL-2.0*X
      G=(E*F)**2
      DS=(SQRT(1.0+G))*DELX
      Y=F*(SL*X-X**2)
      IF(X .GE. 0. .AND. X .LE. XP)C2=0.0
      IF(X .GT. XP .AND. X .LE. SL)C2=1.0
      PM=AMP+AVP*X-AHP*Y-C2*P*(X-XP)
      IF(X .GE. 0. .AND. X .LE. XU)C3=0.0
      IF(X .GT. XU .AND. X .LE. SL)C3=1.0
      UM=AMU+AVU*X-AHU*Y-C3*U*(X-XU)
      DF=PM*UM*DS
      VDF=VDF+DF
      X=X+DELX
  197 CONTINUE
      RETURN
      END
```

Fig. 2.9 Computer print-out of subprogram VDFVC

```
C
C                              SUBPROGRAM  HDFVC
C
      SUBROUTINE HDFVC (K,SL,H,XP,P,AMP,AVP,AHP,XU,U,AMU,AVU,AHU,
     1  HDF,YU )
C     **SUBPROGRAM TO FIND HORIZONTAL DEFLECTIONS**
C     ***DUE TO VERTICAL CONCENTRATE LOAD***
      Z=K
      DELX=SL/Z
      X=DELX/2.
C     INITIALIZE VALUE FOR HORIZONTAL DEFLECTIONS
      HDF=0.0
C     THE FOLLOWING STATEMENTS ARE USED FOR CALCULATING
C     HORIZONTAL EI DEFLECTIONS
      DO 197 I=1,K
      E=4.0*H/SL**2
      F=SL-2.0*X
      G=(F*E)**2
      DS=(SQRT(1.0+G))*DELX
      Y=E*(SL*X-X**2)
      YU=E*(SL*XU-XU**2)
      IF(X .GE. 0. .AND. X .LE. XP)G1=0.0
      IF(X .GT. XP .AND. X .LE. SL)G1=1.0
      PM=AMP+AVP*X-AHP*Y-G1*(X-XP)*P
      IF(X .GE. 0. .AND. X .LE. XU)G2=0.0
      IF(X .GT. XU .AND. X .LE. SL)G2=1.0
      UM=AMU+AVU*X-AHU*Y-G2*U*(Y-YU)
      DF=PM*UM*DS
      HDF=HDF+DF
      X=X+DELX
  197 CONTINUE
      RETURN
      END
```

Fig. 2.10 Computer print-out of subprogram HDFVC

```
C
C                          SUBPROGRAM VDFVU
C
      SUBROUTINE VDFVU (K,SL,H,XW1,XW2,W,AM,AV,AH,XU,U,AMU,AVU,
     1  AHU,VDF )
C     **SUBPROGRAM SOLVE FOR  VERTICAL  EI DEFLECTIONS***
C     ***DUE TO  VERTICAL  UNIFORM LOAD***
      Z=K
      DELX=SL/Z
      X=DELX/2.
C     INITIALIZE VALUE FOR VERTICAL EI DEFLECTIONS
      VDF=0.0
C     THE FOLLOWING STATEMENTS ARE USED FOR CALCULATING
C     VERTICAL EI DEFLECTIONS
      DO 197 I=1,K
      E=4.0*H/SL**2
      F=SL-2.0*X
      G=(E*F)**2
      DS=(SQRT(1.0+G))*DELX
      Y=E*(SL*X-X**2)
      IF(X .GE. 0. .AND. X .LE. XW1)PM=AM+AV*X-AH*Y
      IF(X .GT. XW1 .AND. X .LE. XW2)PM=AM+AV*X-AH*Y
     1 -((X-XW1)**2)*W/2.0
      IF(X .GT. XW2 .AND. X .LE. SL )PM=AM+AV*X-AH*Y
     1  -(2.0*X-XW2-XW1)*(XW2-XW1)*W/2.0
      IF(X .GE. 0. .AND. X .LE. XU)C3=0.0
      IF(X .GT. XU .AND. X .LE. SL)C3=1.0
      UM=AMU+AVU*X-AHU*Y-C3*U*(X-XU)
      DF=PM*UM*DS
      VDF=VDF+DF
      X=X+DELX
  197 CONTINUE
      RETURN
      END
```

Fig. 2.11 Computer print-out of subprogram VDFVU

```
C
C                          SUBPROGRAM  HDFVU
C
      SUBROUTINE HDFVU (K,SL,H,XW1,XW2,W,AM,AV,AH,XU,U,AMU,AVU,
     1  AHU,HDF,YU )
C     **SUBPROGRAM TO FIND HORIZONTAL DEFLECTIONS**
C     ***DUE TO VERTICAL   UNIFORM   LOAD***
      Z=K
      DELX=SL/Z
      X=DELX/2.
C     INITIALIZE VALUE FOR HORIZONTAL EI DEFLECTIONS
      HDF=0.0
C     THE FOLLOWING STATEMENTS ARE USED FOR CALCULATING
C     HORIZONTAL EI DEFLECTIONS
      DO 197 I=1,K
      E=4.0*H/SL**2
      F=SL-2.0*X
      G=(E*F)**2
      DS=(SQRT(1.0+G))*DELX
      Y=E*(SL*X-X**2)
      YU=E*(SL*XU-XU**2)
      IF(X .GE. 0. .AND. X .LE. XW1)PM=AM+AV*X-AH*Y
      IF(X .GT. XW1 .AND. X .LE. XW2)PM=AM+AV*X-AH*Y
     1 -  ((X-XW1)**2)*W/2.0
      IF(X .GT. XW2 .AND. X .LE. SL )PM=AM+AV*X-AH*Y
     1 -  (2.0*X-XW2-XW1)*(XW2-XW1)*W/2.0
      IF(X .GE. 0. .AND. X .LE. XU)C3=0.0
      IF(X .GT. XU .AND. X .LE. SL)C3=1.0
      UM=AMU+AVU*X-AHU*Y-C3*U*(Y-YU)
      DF=PM*UM*DS
      HDF=HDF+DF
      X=X+DELX
  197 CONTINUE
      RETURN
      END
```

Fig. 2.12 Computer print-out of subprogram HDFVU

```
C
C                              SUBPROGRAM  VDFHU
C
      SUBROUTINE VDFHU (K,SL,H,XW1,XW2,W,AMP,AVP,AHP,XU,U,AMU
     1 ,AVU,AHU,VDF)
C     **SUBPROGRAM TO FIND  VERTICAL   DEFLECTIONS**
C     ***DUE TO HORIZONTAL   UNIFORM   LOAD***
      Z=K
      DELX=SL/Z
      X=DELX/2.
C     INITIALIZE VALUE FOR VERTICAL EI DEFLECTIONS
      VDF=0.0
C     THE FOLLOWING STATEMENTS ARE USED FOR CALCULATING
C     VERTICAL EI DEFLECTIONS
      DO 197 I=1,K
      E=4.0*H/SL**2
      F=SL-2.0*X
      G=(E*F)**2
      DS=(SQRT(1.0+G))*DELX
      Y=E*(SL*X-X**2)
      YW1=E*(SL*XW1-XW1**2)
      YW2=E*(SL*XW2-XW2**2)
      IF(X .GE. 0. .AND. X .LE. XW1)PM=AMP+AVP*X-AHP*Y
      IF(X .GT. XW1 .AND. X .LE. XW2)PM=AMP+AVP*X-AHP*Y
     1 - ((Y-YW1)**2)*W/2.0
      IF(X .GT. XW2 .AND. X .LE. SL) PM=AMP+AVP*X-AHP*Y
     1  -(YW2-YW1)*(Y*2.0-YW1-YW2)*W/2.0
      IF(X .GE. 0. .AND. X .LE. XU)C3=0.0
      IF(X .GT. XU .AND. X .LE. SL)C3=1.0
      UM=AMU+AVU*X-AHU*Y-C3*U*(X-XU)
      DF=PM*UM*DS
      VDF=VDF+DF
      X=X+DELX
  197 CONTINUE
      RETURN
      END
```

Fig. 2.13 Computer print-out of subprogram VDFHU

```
C
C                              SUBPROGRAM  HDFHU
C
      SUBROUTINE HDFHU (K,SL,H,XW1,XW2,W,AMP,AVP,AHP,XU,U,AMU,
     1 AVU,AHU,HDF,YU)
C     **SUBPROGRAM TO FIND HORIZONTAL DEFLECTIONS**
C     ***DUE TO HORIZONTAL   UNIFORM   LOAD***
      Z=K
      DELX=SL/Z
      X=DELX/2.
C     INITIALIZE VALUE FOR HORIZONTAL DEFLECTION
      HDF=0.0
C     THE FOLLOWING STATEMENTS ARE USED FOR CALCULATING
C     HORIZONTAL EI DEFLECTION
      DO 197 I=1,K
      E=4.0*H/SL**2
      F=SL-2.0*X
      G=(E*F)**2
      DS=(SQRT(1.0+G))*DELX
      Y=E*(SL*X-X**2)
      YU=E*(SL*XU-XU**2)
      YW1=E*(SL*XW1-XW1**2)
      YW2=E*(SL*XW2-XW2**2)
      IF(X .GE. 0. .AND. X .LE. XW1)PM=AMP+AVP*X-AHP*Y
      IF(X .GT. XW1 .AND. X .LE. XW2)PM=AMP+AVP*X-AHP*Y
     1 -  ((Y-YW1)**2)*W/2.0
      IF(X .GT. XW2 .AND. X .LE. SL) PM=AMP+AVP*X-AHP*Y
     1 - (YW2-YW1)*(Y*2.0-YW1-YW2)*W/2.0
      IF(X .GE. 0. .AND. X .LE. XU)C3=0.0
      IF(X .GT. XU .AND. X .LE. SL)C3=1.0
      UM=AMU+AVU*X-AHU*Y-C3*U*(Y-YU)
      DF=PM*UM*DS
      HDF=HDF+DF
      X=X+DELX
  197 CONTINUE
      RETURN
      END
```

Fig. 2.14 Computer print-out of subprogram HDFHU

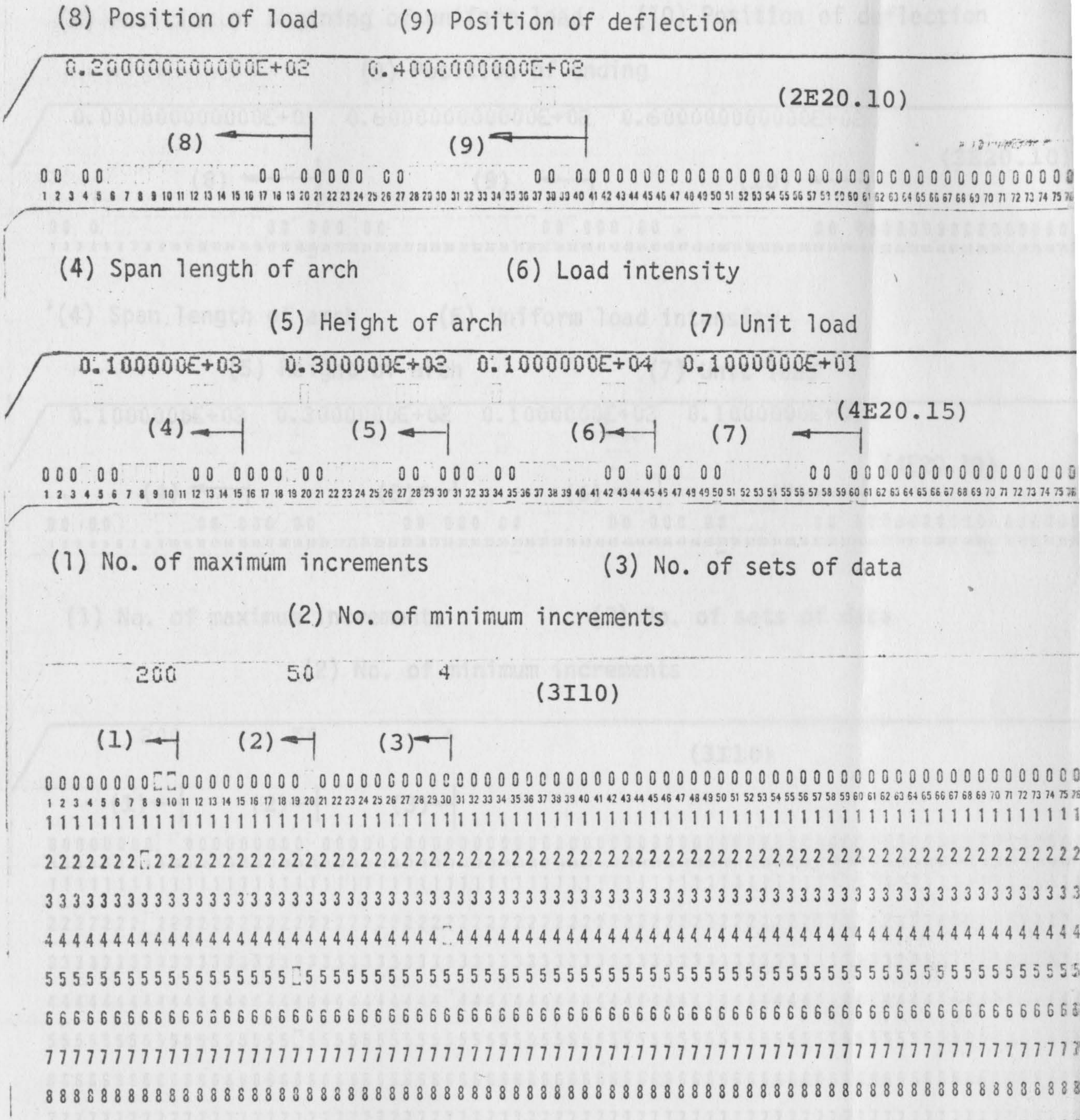


Fig. 2.15 In-put data for concentrate load

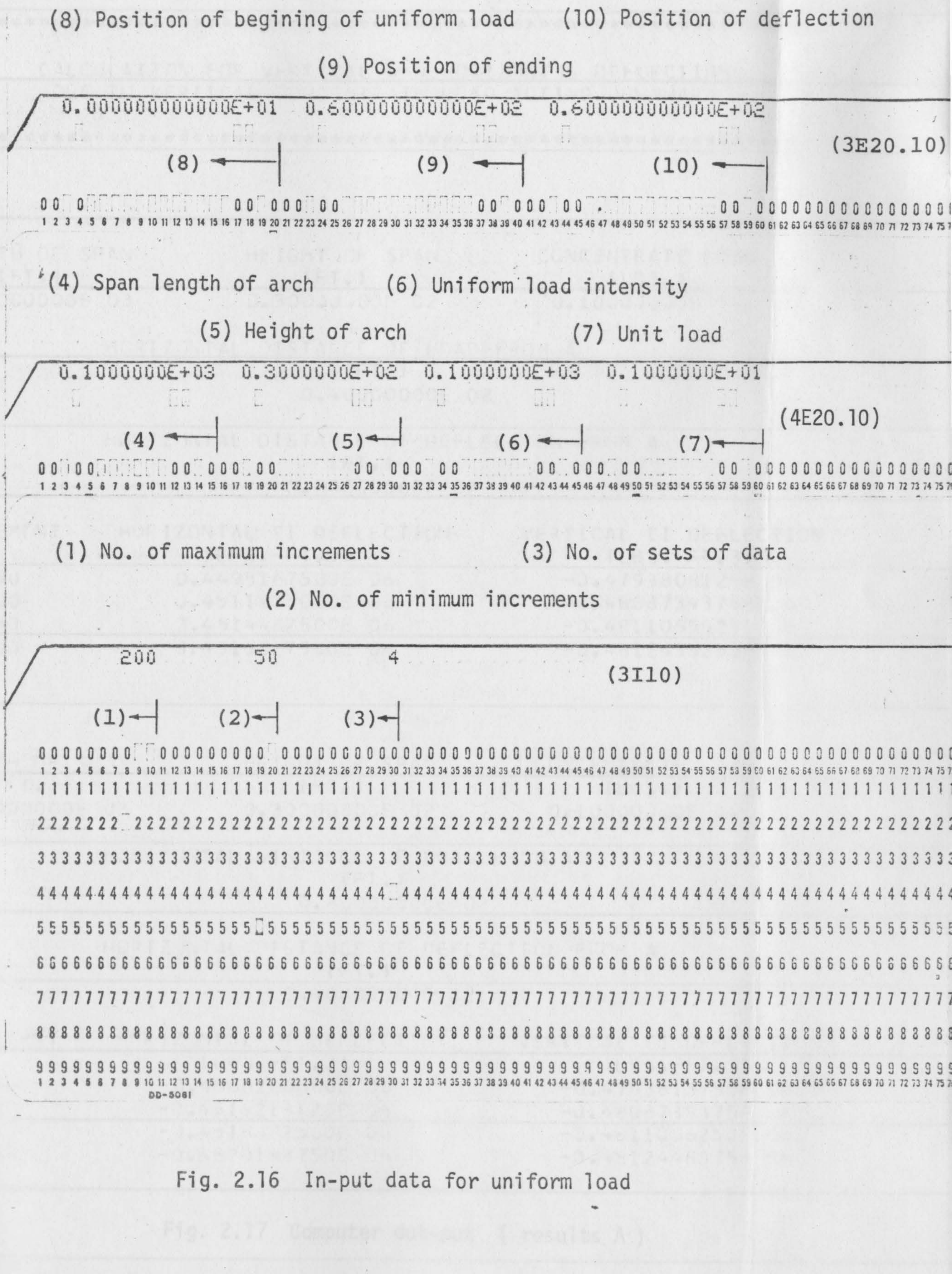


Fig. 2.16 In-put data for uniform load

```
**************************************************************
*                                                            *
*    CALCULATION FOR VERTICAL AND HORIZONTAL DEFLECTIONS     *
*     DUE TO VERTICAL CONCENTRATE LOAD ACTING DOWNWARD       *
*                                                            *
**************************************************************


LENGTH OF SPAN          HEIGHT OF SPAN          CONCENTRATE LOAD
    (FT.)                   (FT.)                    (LBS.)
0.10000000E 03          0.30000000E 02           0.10000000E 04

            HORIZONTAL DISTANCE OF LOAD FROM A
                          (FT.)
                       0.40000000E 02

            HORIZONTAL DISTANCE OF DEFLECTION FROM A
                          (FT.)
                       0.80000000E 02

INCREMENT      HORIZONTAL EI DEFLECTION       VERTICAL EI DEFLECTION
                     (LBS.-FT.3)                   (LBS.-FT.3)
   50             0.4495167500E 06              -0.4793808125E 06
  100             0.4511875000E 06              -0.4808734375E 06
  150             0.4514462500E 06              -0.4811055625E 06
  200             0.4516027500E 06              -0.4812438250E 06


LENGTH OF SPAN          HEIGHT OF SPAN          CONCENTRATE LOAD
    (FT.)                   (FT.)                    (LBS.)
0.10000000E 03          0.30000000E 02           0.10000000E 04

            HORIZONTAL DISTANCE OF LOAD FROM A
                          (FT.)
                       0.80000000E 02

            HORIZONTAL DISTANCE OF DEFLECTION FROM A
                          (FT.)
                       0.40000000E 02

INCREMENT      HORIZONTAL EI DEFLECTION       VERTICAL EI DEFLECTION
                     (LBS.-FT.3)                   (LBS.-FT.3)
   50            -0.4500260000E 06              -0.4793813750E 06
  100            -0.4516213125E 06              -0.4808739375E 06
  150            -0.4518802500E 06              -0.4811056250E 06
  200            -0.4520188750E 06              -0.4812446875E 06
```

Fig. 2.17 Computer out-put (results A)

LENGTH OF SPAN (FT.)	HEIGHT OF SPAN (FT.)	CONCENTRATE LOAD (LBS.)
0.50000000E 02	0.15000000E 02	0.30000000E 04

HORIZONTAL DISTANCE OF LOAD FROM A
(FT.)
0.40000000E 02

HORIZONTAL DISTANCE OF DEFLECTION FROM A
(FT.)
0.20000000E 02

INCREMENT	HORIZONTAL EI DEFLECTION (LBS.-FT.3)	VERTICAL EI DEFLECTION (LBS.-FT.3)
50	-0.1687594375E 06	-0.1797679375E 06
100	-0.1693576875E 06	-0.1803271875E 06
150	-0.1694556250E 06	-0.1804176250E 06
200	-0.1695057500E 06	-0.1804666250E 06

LENGTH OF SPAN (FT.)	HEIGHT OF SPAN (FT.)	CONCENTRATE LOAD (LBS.)
0.50000000E 02	0.15000000E 02	0.30000000E 04

HORIZONTAL DISTANCE OF LOAD FROM A
(FT.)
0.20000000E 02

HORIZONTAL DISTANCE OF DEFLECTION FROM A
(FT.)
0.40000000E 02

INCREMENT	HORIZONTAL EI DEFLECTION (LBS.-FT.3)	VERTICAL EI DEFLECTION (LBS.-FT.3)
50	0.1685690000E 06	-0.1797679375E 06
100	0.1691948750E 06	-0.1803271250E 06
150	0.1692958125E 06	-0.1804173750E 06
200	0.1693510000E 06	-0.1804663750E 06

Fig. 2.17 (continue)

```
****************************************************************
*                                                              *
*   CALCULATION FOR VERTICAL AND HORIZONTAL DEFLECTIONS        *
*   DUE TO VERTICAL UNIFORM LOAD ACTING DOWNWARD               *
*                                                              *
****************************************************************

LENGTH OF SPAN          HEIGHT OF SPAN              UNIFORM LOAD
    (FT.)                   (FT.)                  (LBS. PER FT.)
0.10000000E 03          0.30000000E 02             0.10000000E 03

              HORIZONTAL SPAN OF LOAD FROM A
                  (FT.)                  (FT.)
              0.0                 0.40000000E 02

          HORIZONTAL DISTANCE OF DEFLECTION FROM A
                             (FT.)
                        0.40000000E 02

INCREMENT     HORIZONTAL EI DEFLECTION       VERTICAL EI DEFLECTION
                    (LBS.-FT.3)                   (LBS.-FT.3)
   50             0.1429023000E 07             0.9847812500E 06
  100             0.1433513000E 07             0.9866920000E 06
  150             0.1434275000E 07             0.9869683125E 06
  200             0.1434613000E 07             0.9871689375E 06


LENGTH OF SPAN          HEIGHT OF SPAN              UNIFORM LOAD
    (FT.)                   (FT.)                  (LBS. PER FT.)
0.10000000E 03          0.30000000E 02             0.10000000E 03

              HORIZONTAL SPAN OF LOAD FROM A
                  (FT.)                  (FT.)
              0.0                 0.60000000E 02

          HORIZONTAL DISTANCE OF DEFLECTION FROM A
                             (FT.)
                        0.60000000E 02

INCREMENT     HORIZONTAL EI DEFLECTION       VERTICAL EI DEFLECTION
                    (LBS.-FT.3)                   (LBS.-FT.3)
   50             0.1429026000E 07            -0.9847810625E 06
  100             0.1433514000E 07            -0.9866907500E 06
  150             0.1434235000E 07            -0.9870512500E 06
  200             0.1434623000E 07            -0.9871643750E 06
```

Fig. 2.1 8 Computer out-put (results B)

```
LENGTH OF SPAN        HEIGHT OF SPAN          UNIFORM LOAD
    (FT.)                 (FT.)               (LBS. PER FT.)
0.50000000E 02        0.15000000E 02          0.20000000E 03

              HORIZONTAL SPAN OF LOAD FROM A
                   (FT.)              (FT.)
           0.0                0.20000000E 02

        HORIZONTAL DISTANCE OF DEFLECTION FROM A
                          (FT.)
                     0.20000000E 02

INCREMENT     HORIZONTAL EI DEFLECTION      VERTICAL EI DEFLECTION
                   (LBS.-FT.3)                   (LBS.-FT.3)
    50         0.1786287500E 06              0.1230981250E 06
   100         0.1791897500E 06              0.1233368125E 06
   150         0.1792861250E 06              0.1233726875E 06
   200         0.1793276875E 06              0.1233956875E 06


LENGTH OF SPAN        HEIGHT OF SPAN          UNIFORM LOAD
    (FT.)                 (FT.)               (LBS. PER FT.)
0.50000000E 02        0.15000000E 02          0.20000000E 03

              HORIZONTAL SPAN OF LOAD FROM A
                   (FT.)              (FT.)
           0.0                0.30000000E 02

        HORIZONTAL DISTANCE OF DEFLECTION FROM A
                          (FT.)
                     0.30000000E 02

INCREMENT     HORIZONTAL EI DEFLECTION      VERTICAL EI DEFLECTION
                   (LBS.-FT.3)                   (LBS.-FT.3)
    50         0.1786286250E 06             -0.1230980625E 06
   100         0.1791897500E 06             -0.1233358750E 06
   150         0.1792816875E 06             -0.1233795625E 06
   200         0.1793277500E 06             -0.1233928750E 06
```

Fig. 2.18 (continue)

```
***************************************************************
*                                                             *
*   CALCULATION FOR VERTICAL AND HORIZONTAL DEFLECTIONS       *
*      DUE TO HORIZONTAL UNIFORM LOAD ACTING RIGHTWARD        *
*                                                             *
***************************************************************

LENGTH OF SPAN         HEIGHT OF SPAN            UNIFORM LOAD
    (FT.)                  (FT.)               (LBS. PER FT.)
0.10000000E 03         0.30000000E 02           0.10000000E 03

            VERTICAL SPAN OF LOAD FROM A
                (FT.)              (FT.)
         0.0                  0.60000000E 01

         HORIZONTAL DISTANCE OF DEFLECTION FROM A
                          (FT.)
                     0.40000000E 02

INCREMENT    HORIZONTAL EI DEFLECTION      VERTICAL EI DEFLECTION
                  (LBS.-FT.3)                   (LBS.-FT.3)
   50          0.2102060547E 05              0.1600891602E 04
  100          0.2178113281E 05              0.1530780762E 04
  150          0.2191772266E 05              0.1509249756E 04
  200          0.2196790234E 05              0.1503244629E 04


LENGTH OF SPAN         HEIGHT OF SPAN            UNIFORM LOAD
    (FT.)                  (FT.)               (LBS. PER FT.)
0.10000000E 03         0.30000000E 02           0.10000000E 03

            VERTICAL SPAN OF LOAD FROM A
                (FT.)              (FT.)
         0.0                  0.60000000E 01

         HORIZONTAL DISTANCE OF DEFLECTION FROM A
                          (FT.)
                     0.60000000E 02

INCREMENT    HORIZONTAL EI DEFLECTION      VERTICAL EI DEFLECTION
                  (LBS.-FT.3)                   (LBS.-FT.3)
   50          0.1996794922E 05             -0.2331940234E 05
  100          0.2069448438E 05             -0.2419616797E 05
  150          0.2082568359E 05             -0.2437362500E 05
  200          0.2087414063E 05             -0.2443322656E 05
```

Fig. 2.19 Computer out-put (results C)

LENGTH OF SPAN (FT.)	HEIGHT OF SPAN (FT.)	UNIFORM LOAD (LBS. PER FT.)
0.50000000E 02	0.15000000E 02	0.10000000E 03

VERTICAL SPAN OF LOAD (FT.)	FROM A (FT.)
0.0	0.30000000E 01

HORIZONTAL DISTANCE OF DEFLECTION FROM A (FT.)

0.20000000E 02

INCREMENT	HORIZONTAL EI DEFLECTION (LBS.-FT.3)	VERTICAL EI DEFLECTION (LBS.-FT.3)
50	0.1313749268E 04	0.1001010132E 03
100	0.1361384033E 04	0.9618304443E 02
150	0.1370334961E 04	0.9570225525E 02
200	0.1373259766E 04	0.9531489563E 02

LENGTH OF SPAN (FT.)	HEIGHT OF SPAN (FT.)	UNIFORM LOAD (LBS. PER FT.)
0.50000000E 02	0.15000000E 02	0.10000000E 03

VERTICAL SPAN OF LOAD (FT.)	FROM A (FT.)
0.0	0.30000000E 01

HORIZONTAL DISTANCE OF DEFLECTION FROM A (FT.)

0.30000000E 02

INCREMENT	HORIZONTAL EI DEFLECTION (LBS.-FT.3)	VERTICAL EI DEFLECTION (LBS.-FT.3)
50	0.1247907227E 04	-0.1456986084E 04
100	0.1293355225E 04	-0.1510839111E 04
150	0.1301937500E 04	-0.1520516113E 04
200	0.1304708008E 04	-0.1524161133E 04

Fig. 2.19 (continue)

CHAPTER III

PRESENTATION OF RESULTS

The various computer program results were used to construct design charts which show the horizontal and vertical deflections for any point on a symmetrical, parabolic, fixed-end arch with three types of loading. Since the arch used in the analysis was 100 feet in length, the rise proportional to the span length, and the intensity of the loads were 1000 pounds for the concentrated load and 100 pounds per foot for the uniform distributed load, there must be some conversion factors associated with the values calculated. That is:

For concentrated load

$$EI\Delta\left(\frac{1000}{P}\right)\left(\frac{100}{L}\right)^3 = X$$

and for uniformly distributed load

$$EI\Delta\left(\frac{100}{W}\right)\left(\frac{100}{L}\right)^4 = X$$

where

Δ = the value of the deflection in feet

P = any concentrated loading in pounds

W = any uniformly distributed loading in pounds

L = span length of any arch in feet

I = moment inertia of the arch in $ft.^4$

E = Modulus of Elasticity of the arch in $lbs./ft.^2$

X = the value read from the charts.

The following will be a brief explanation of the charts:

The horizontal axis will show horizontal distance of any point on the arch in tenth of the span length (L) having the left fixed-end as the initial.

The vertical axis will show the value of either the expression

$$EI\Delta\left(\frac{1000}{P}\right)\left(\frac{100}{L}\right)^3 \quad \text{or} \quad EI\Delta\left(\frac{100}{W}\right)\left(\frac{100}{L}\right)^4$$

Sign Convention

For vertical deflection

positive sign indicates the downward movement of the point

negative sign indicates the upward movement of the point

For horizontal deflection

positive sign indicates the rightward movement of the point

negative sign indicates the leftward movement of the point

The design curves, reaction curves and fixed-end moment curves are shown in Figure 3.1 to 3.56 inclusive.

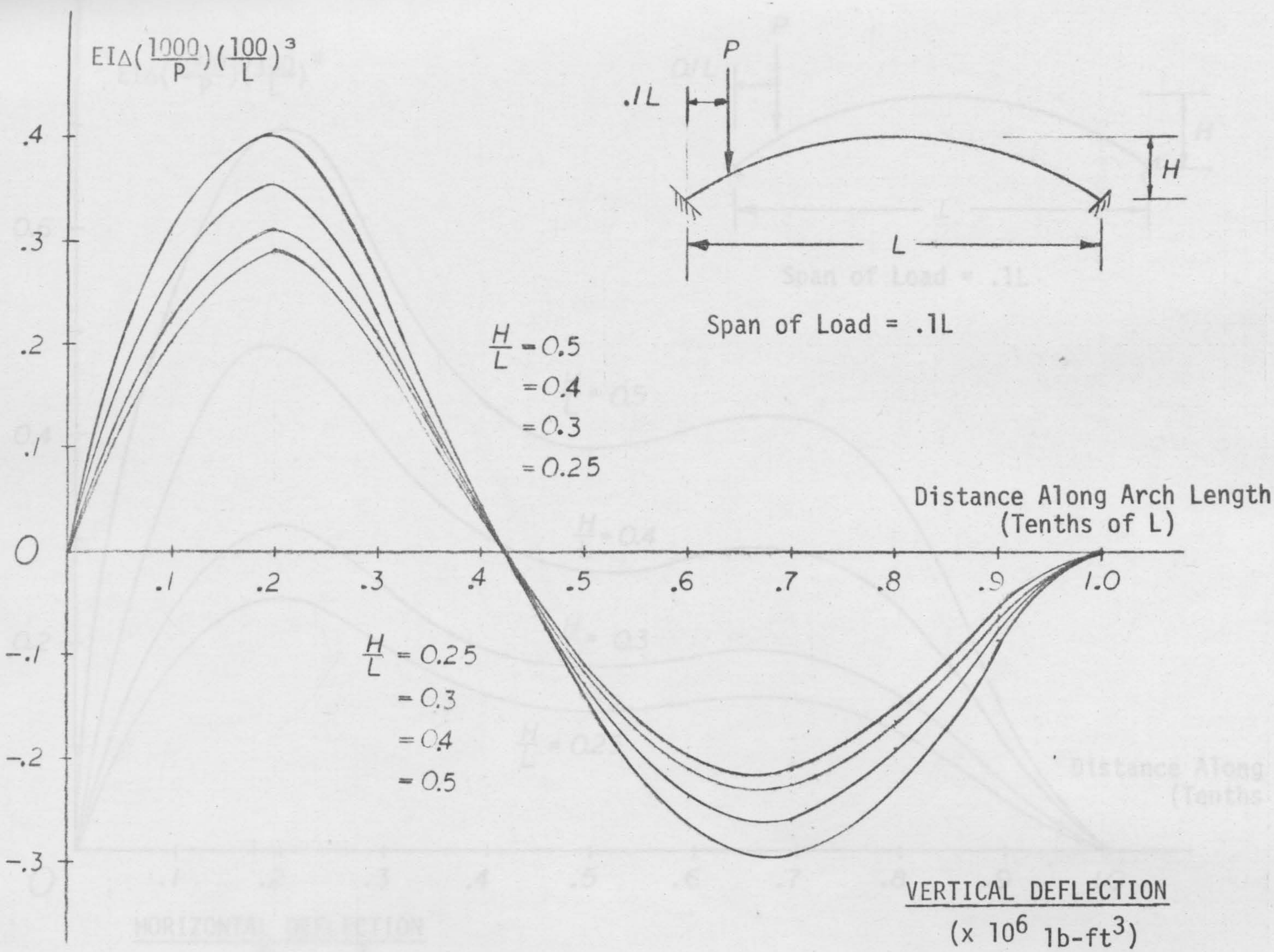


Fig. 3.1 Design chart for the vertical deflection for any point on the span of an arch with the position of load at .1 the span of the arch.

*Note: Positive values on design charts indicate downward deflection.

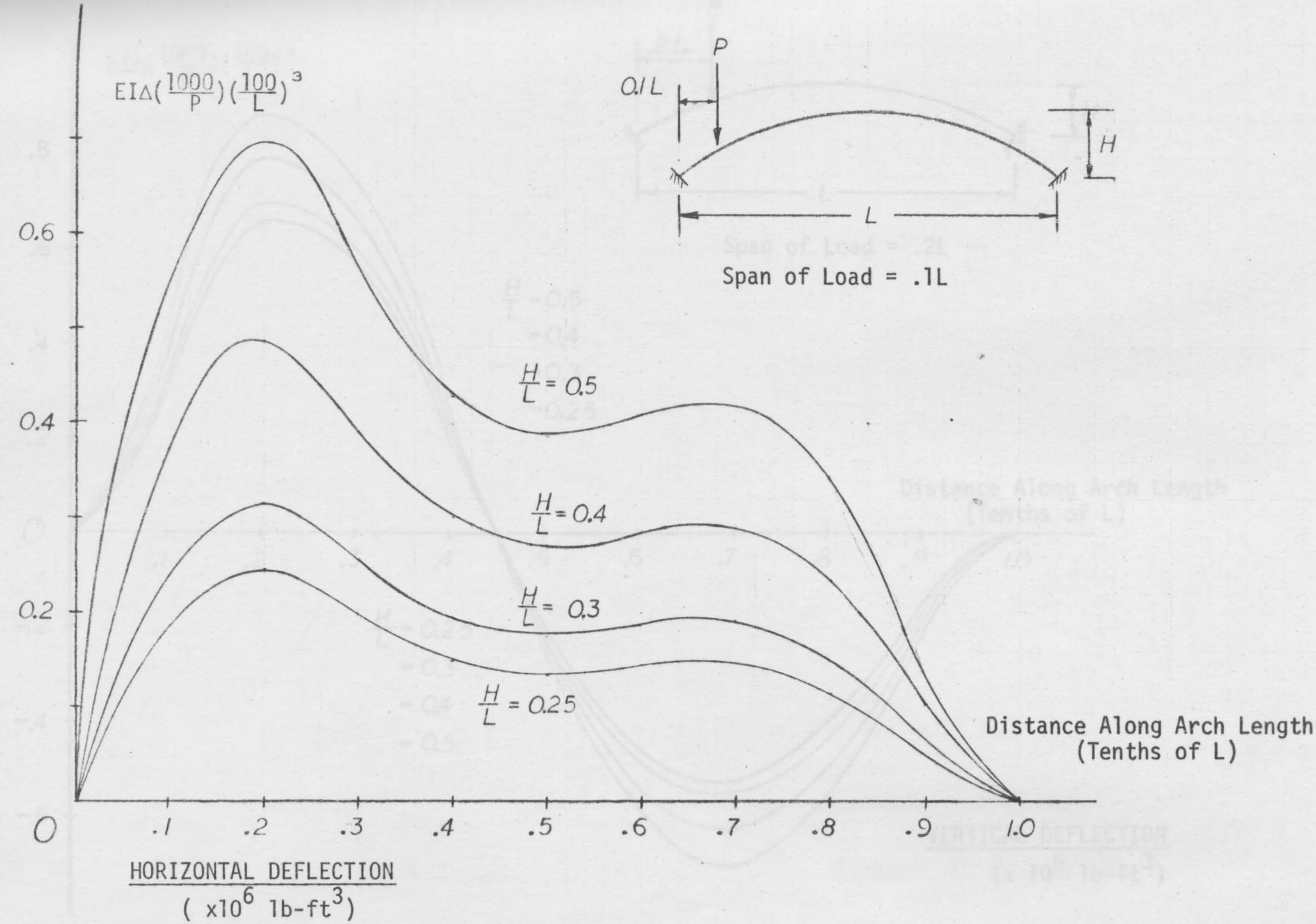


Fig. 3.2 Design chart for the horizontal deflection for any point on the span of an arch with the position of load at .1 the span of the arch.
*Note: Positive values on design charts indicate rightward deflection

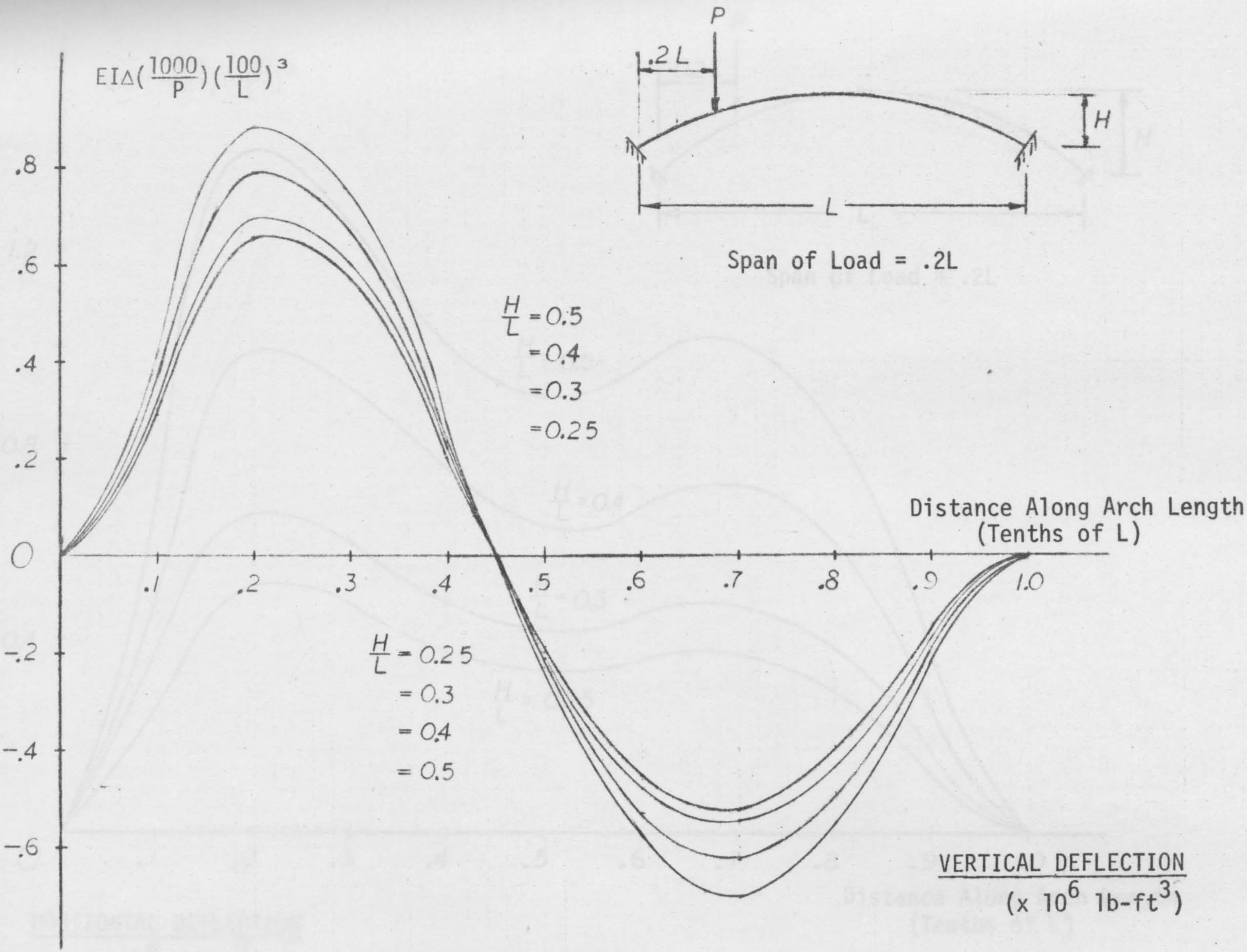


Fig. 3.3 Design chart for the vertical deflection for any point on the span of an arch with the position of load at .2 the span of the arch.

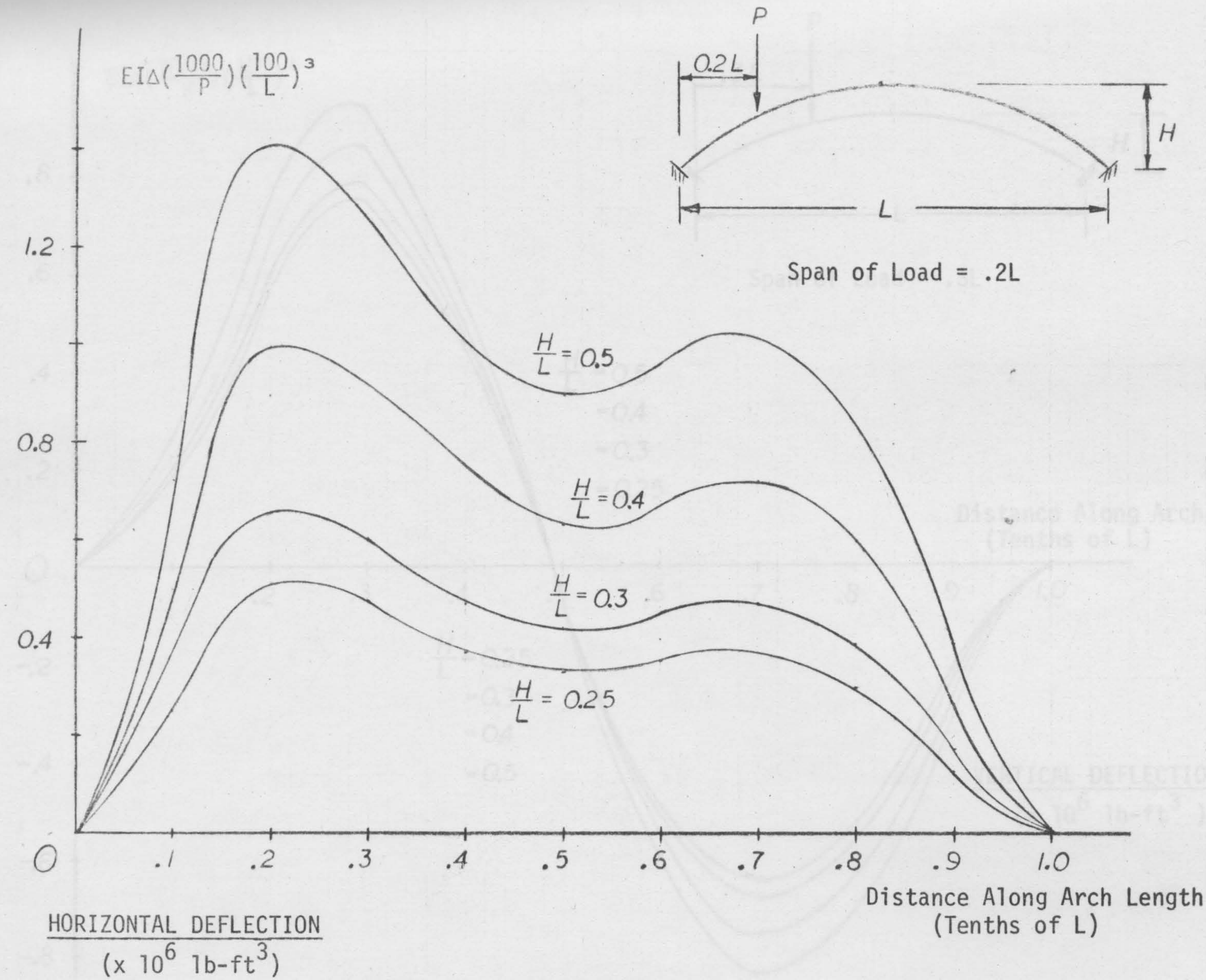


Fig. 3.4 Design chart for the horizontal deflection for any point on the span of an arch with the position of load at .2 the span of the arch.

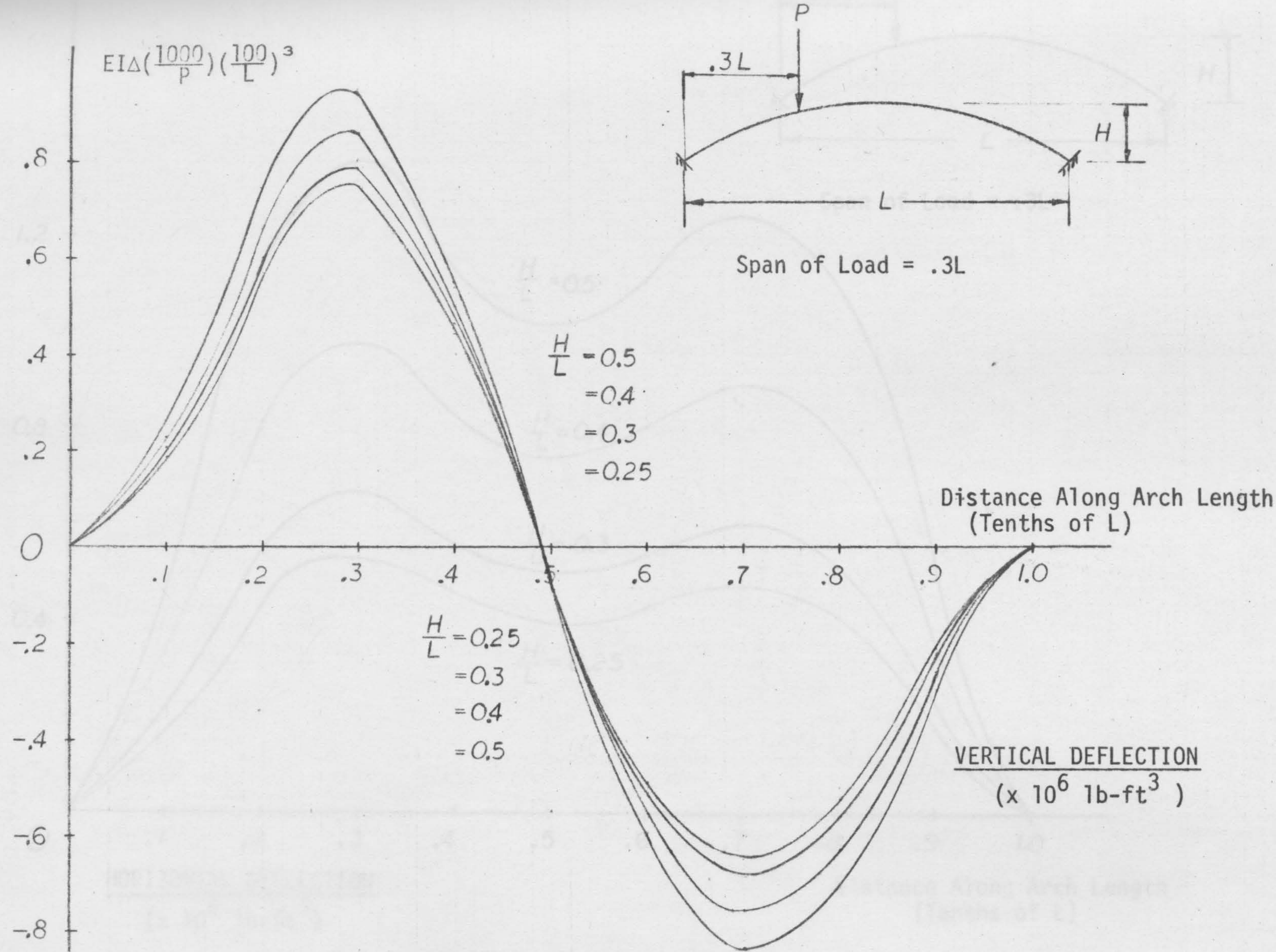


Fig. 3.5 Design chart for the vertical deflection for any point on the span of an arch with the position of load at .3 the span of the arch.

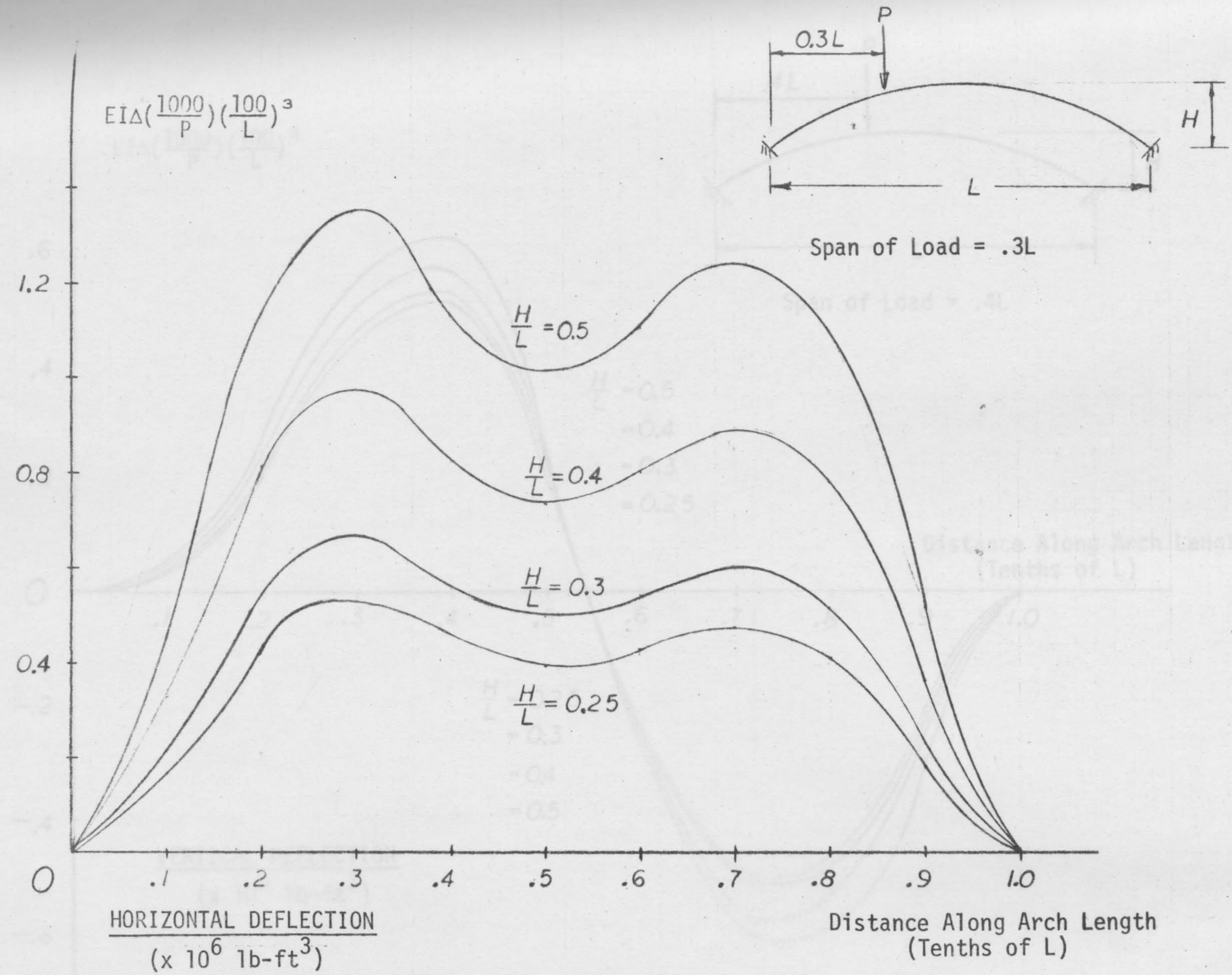


Fig. 3.6 Design chart for the horizontal deflection for any point on the span of an arch with the position of load at .3 the span of the arch.

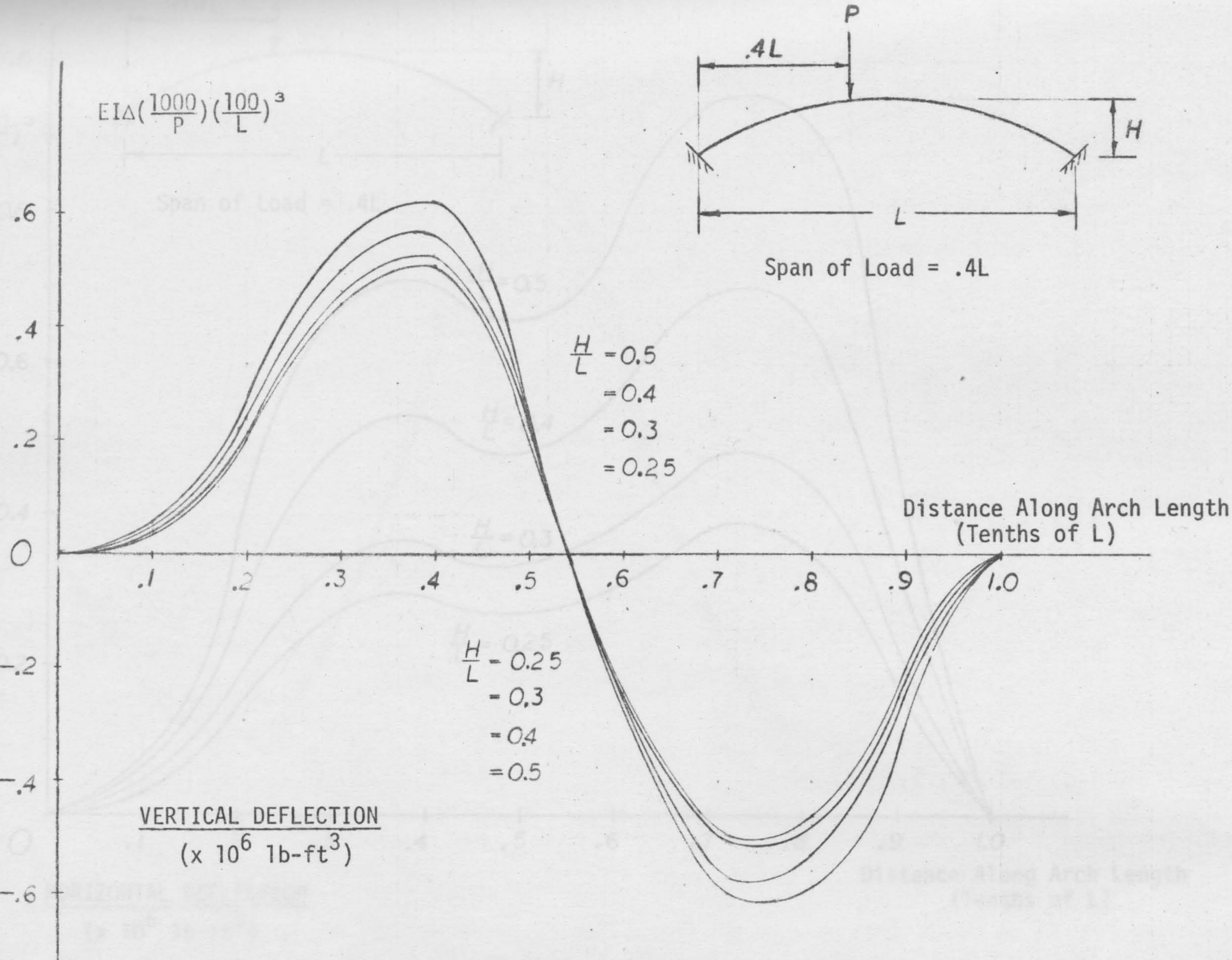


Fig. 3.7 Design chart for the vertical deflection for any point on the span of an arch with the position of load at .4 the span of the arch.

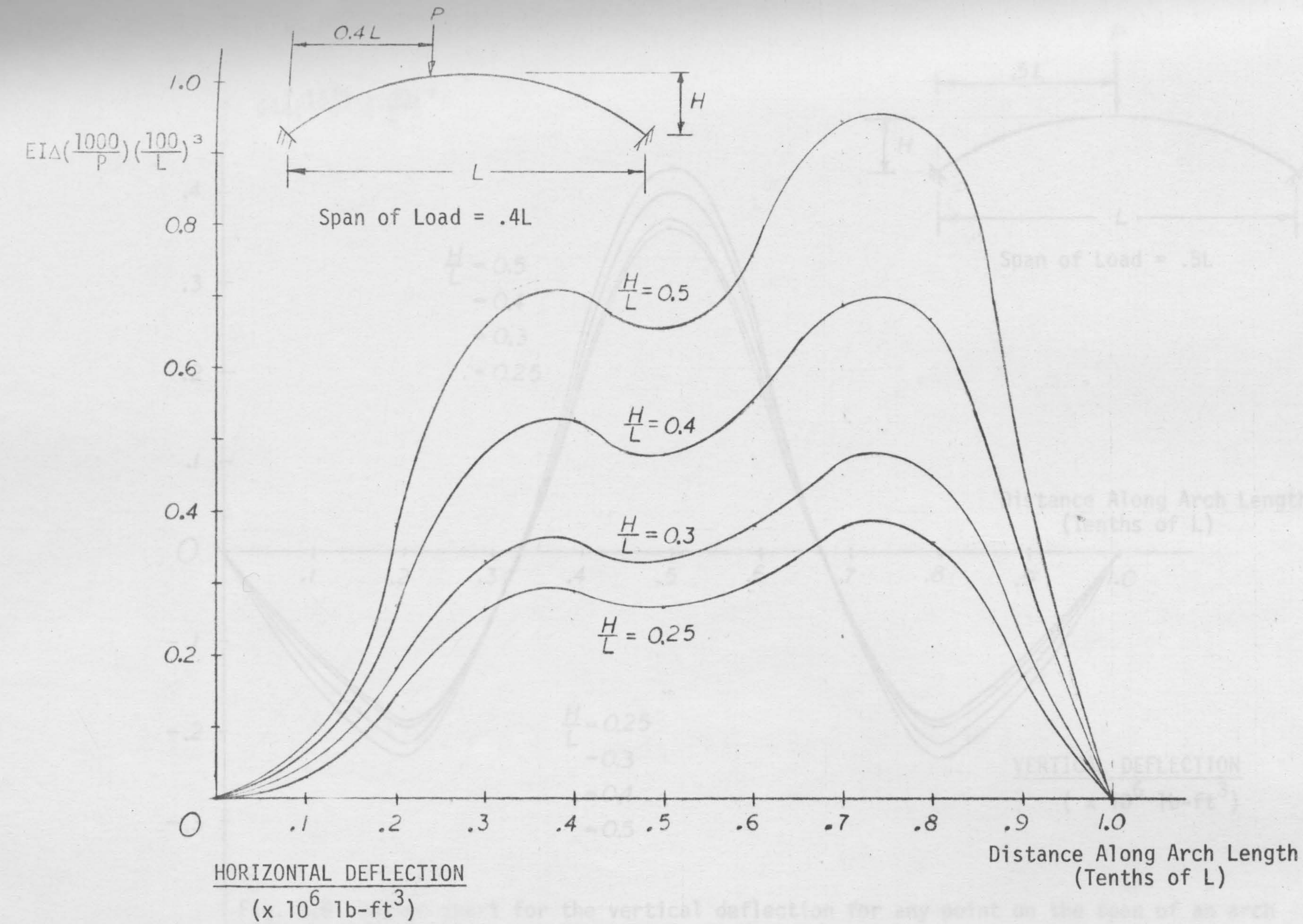


Fig. 3.8 Design chart for the horizontal deflection for any point on the span of an arch with the position of load at .4 the span of the arch.

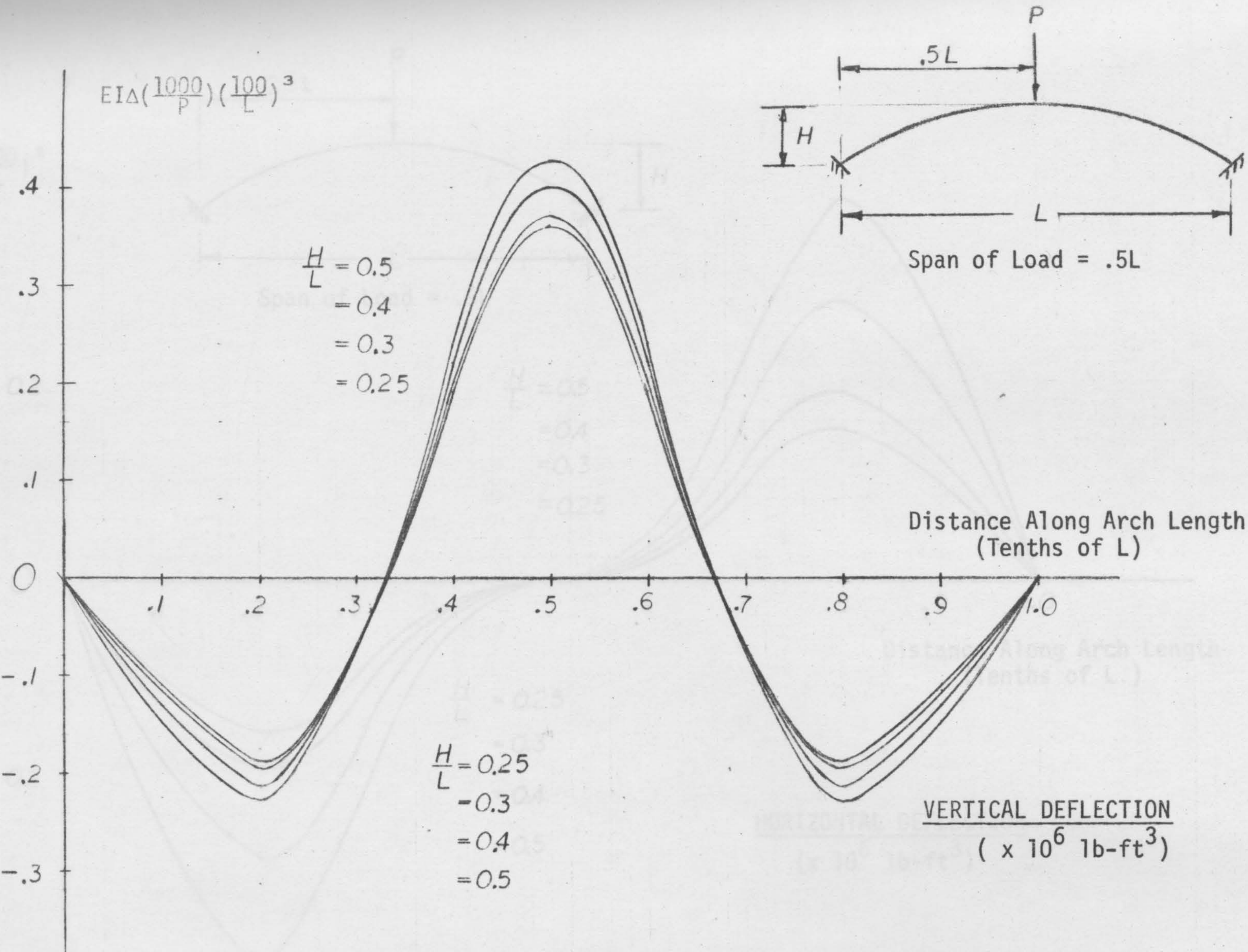


Fig. 3.9 Design chart for the vertical deflection for any point on the span of an arch with the position of load at .5 the span of the arch.

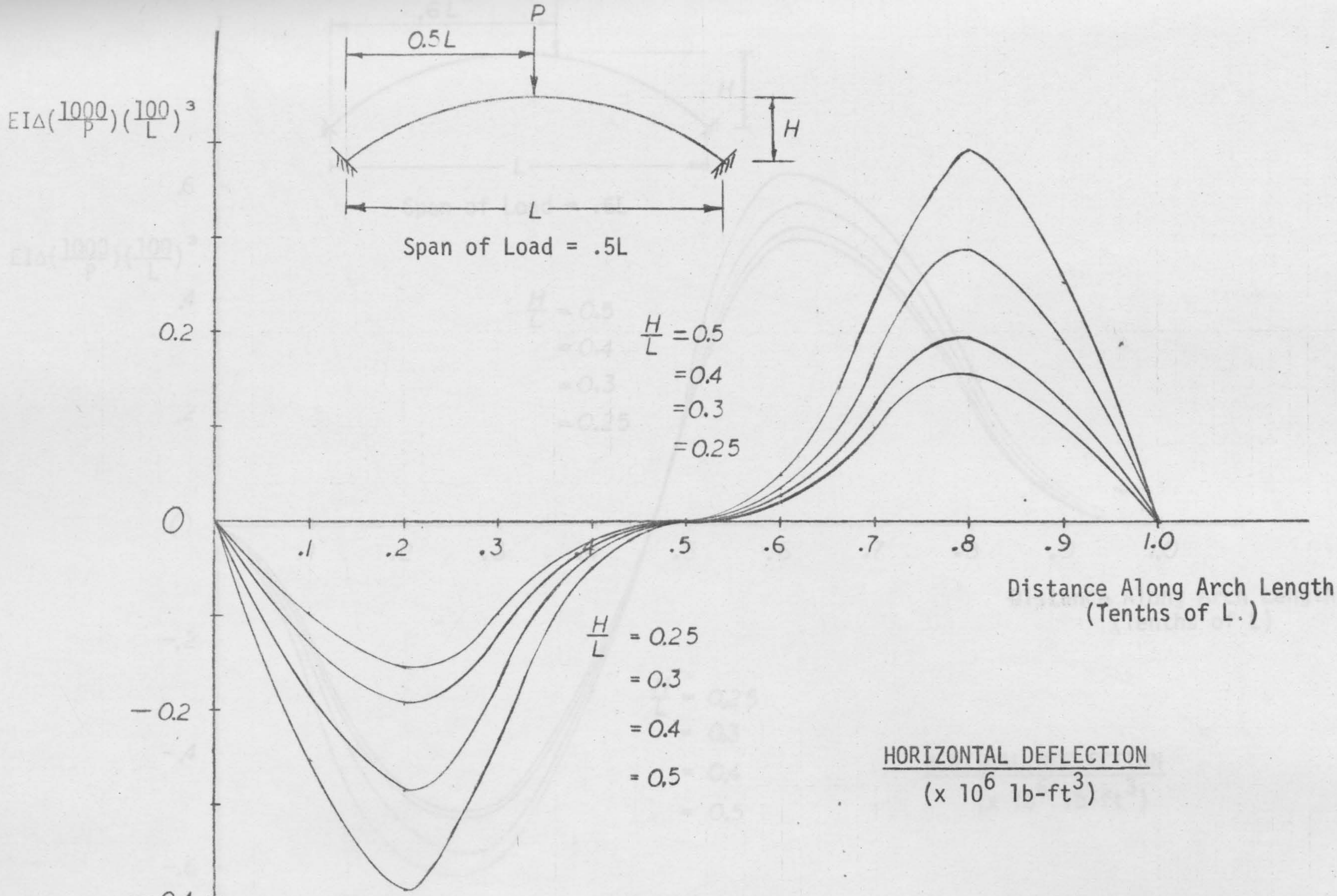


Fig. 3.10 Design chart for the horizontal deflection for any point on the span of an arch with the position of load at .5 the span of the arch.

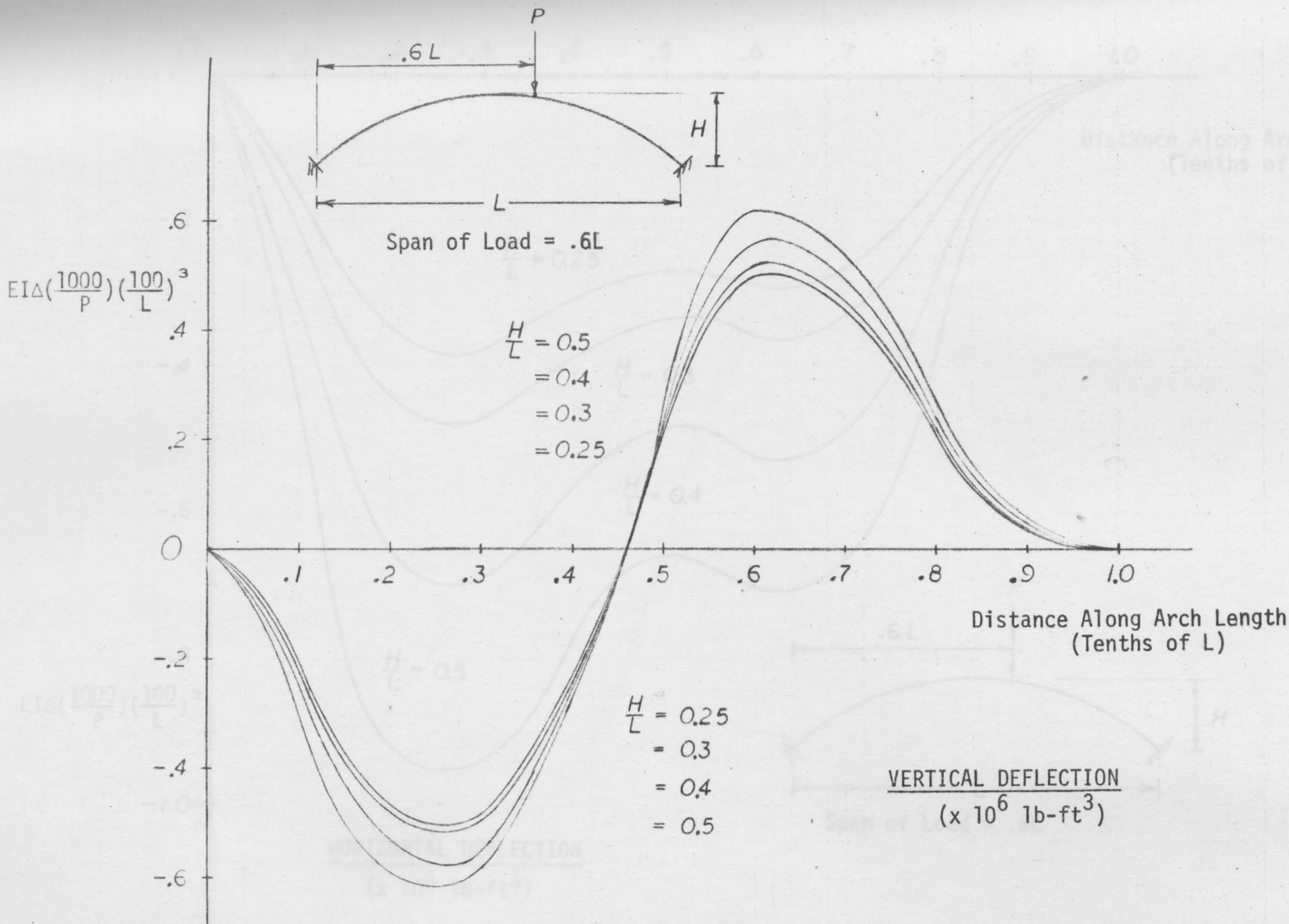


Fig. 3.11 Design chart for the vertical deflection for any point on the span of an arch with the position of load at .6 the span of the arch.

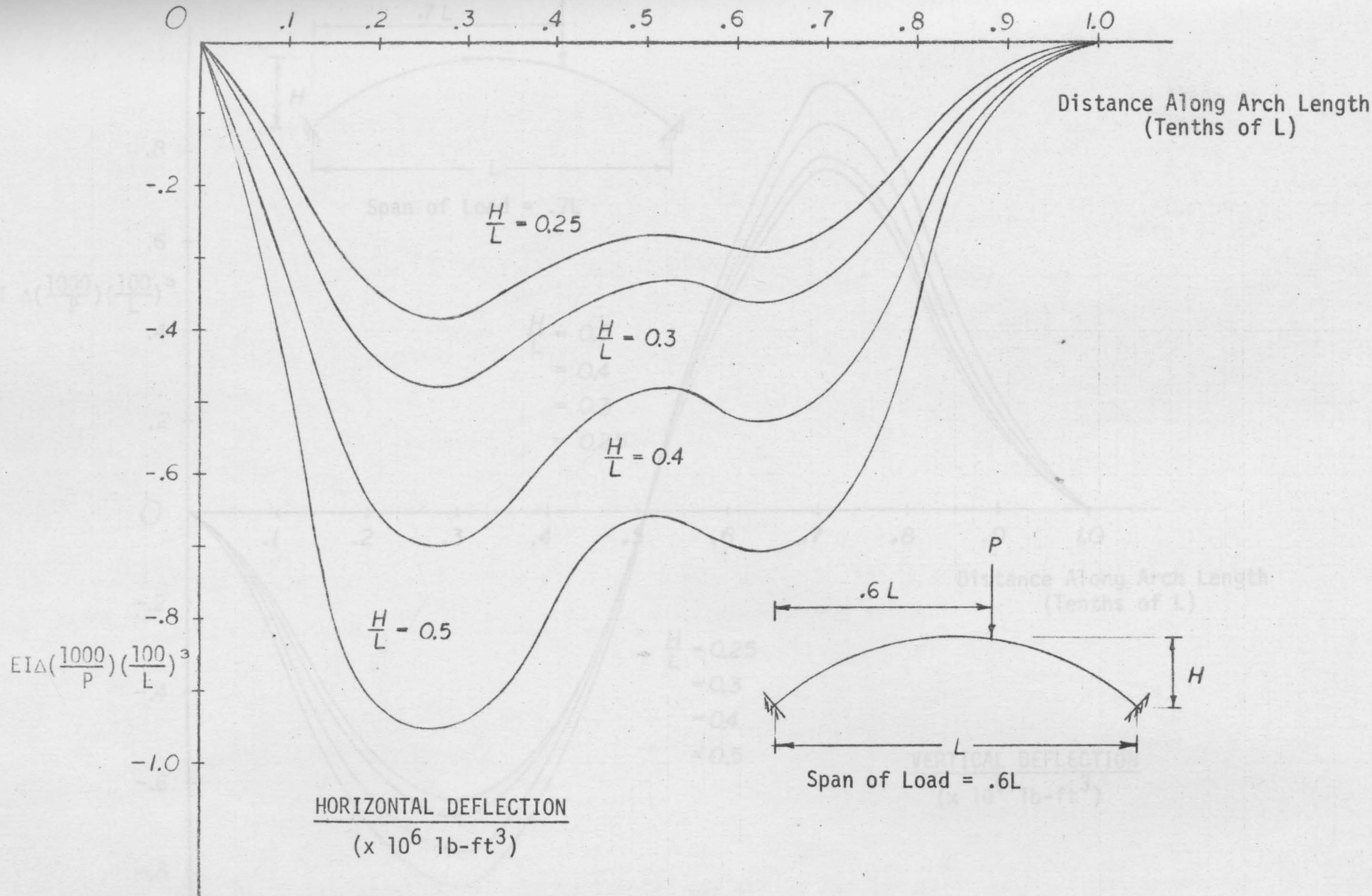


Fig. 3.12 Design chart for the horizontal deflection for any point on the span of an arch with the position of load at .6 the span of the arch.

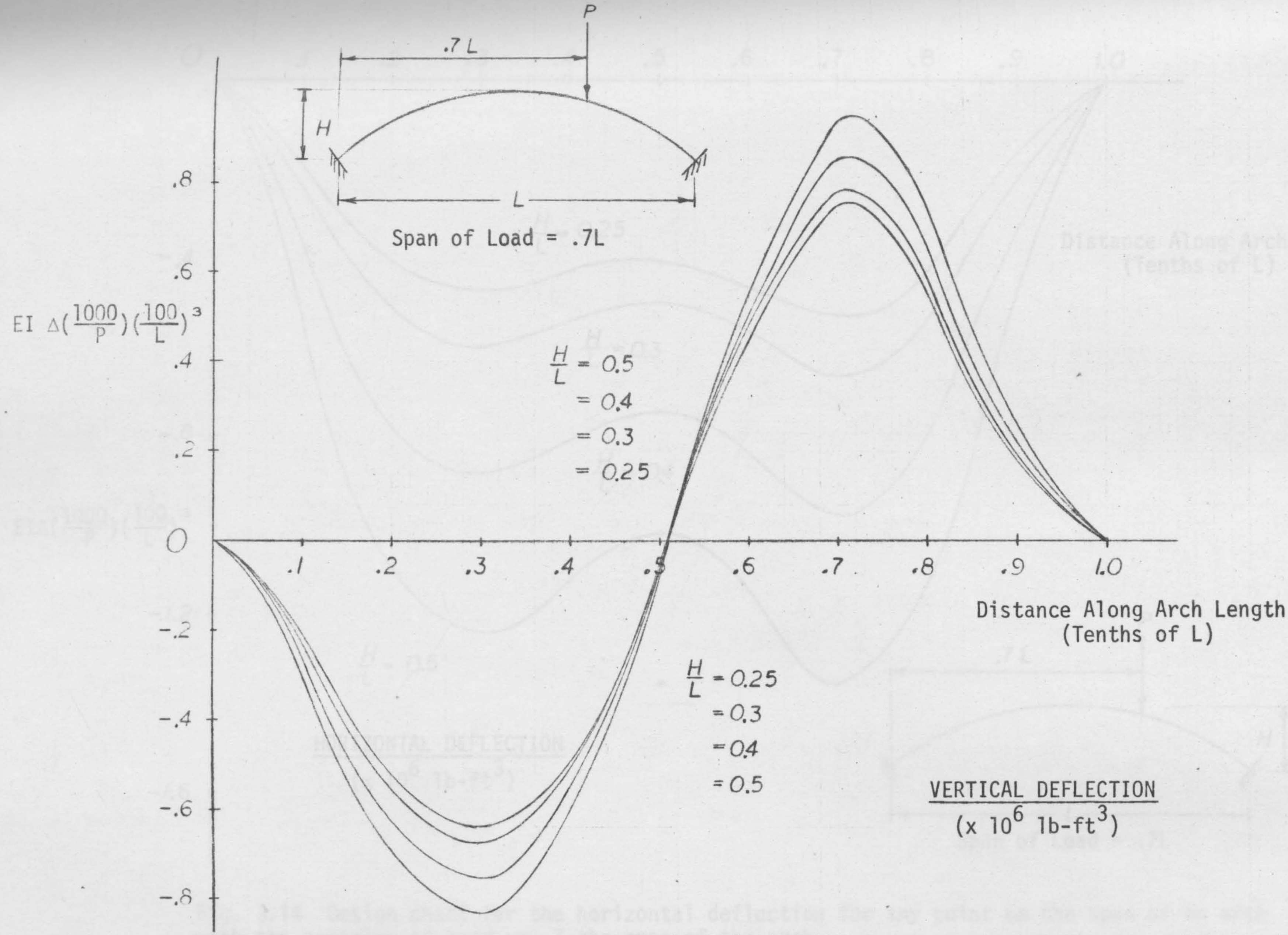


Fig. 3.13 Design chart for the vertical deflection for any point on the span of an arch with the position of load at .7 the span of the arch.

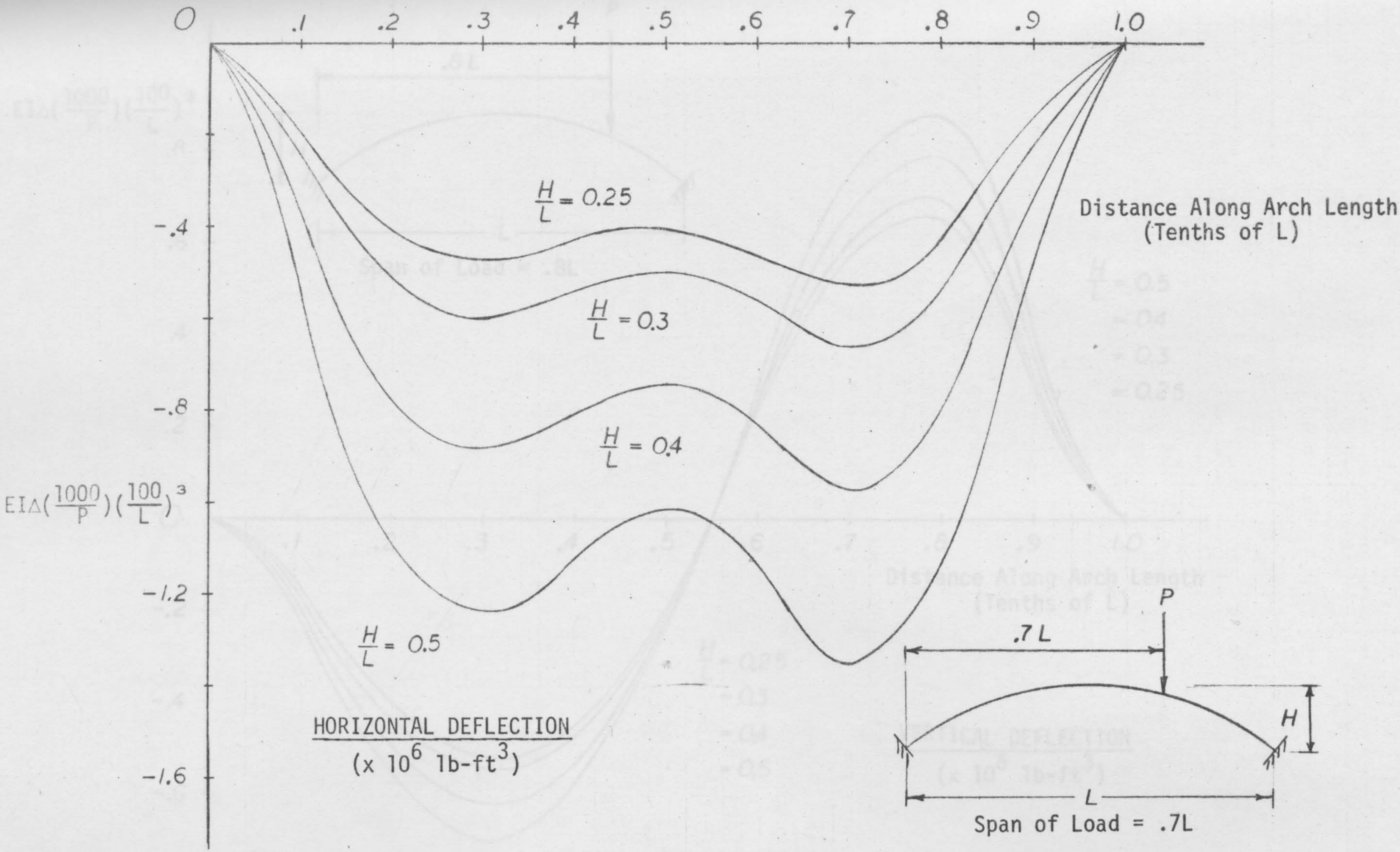


Fig. 3.14 Design chart for the horizontal deflection for any point on the span of an arch with the position of load at .7 the span of the arch.

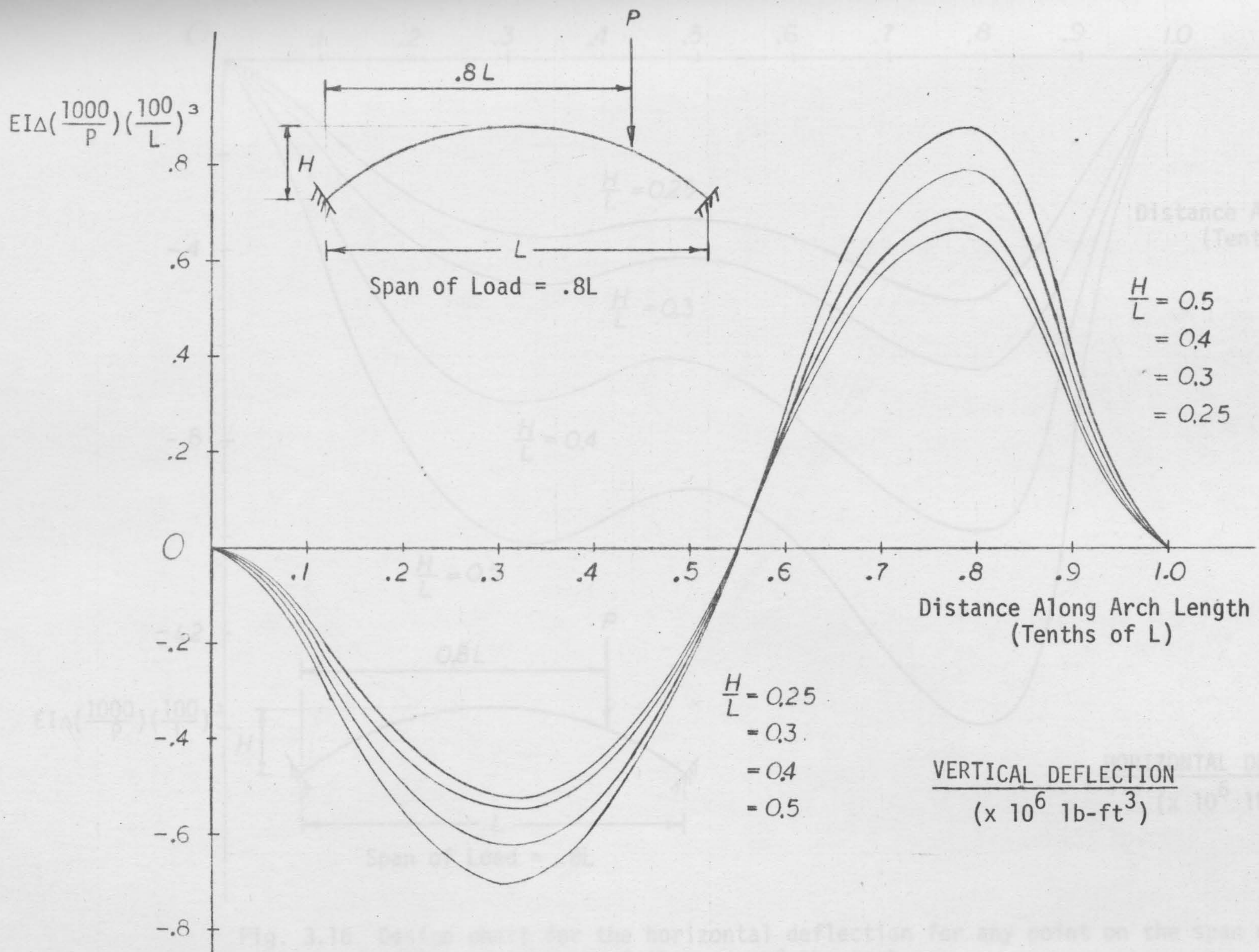


Fig. 3.15 Design chart for the vertical deflection for any point on the span of an arch with the position of load at .8 the span of the arch.

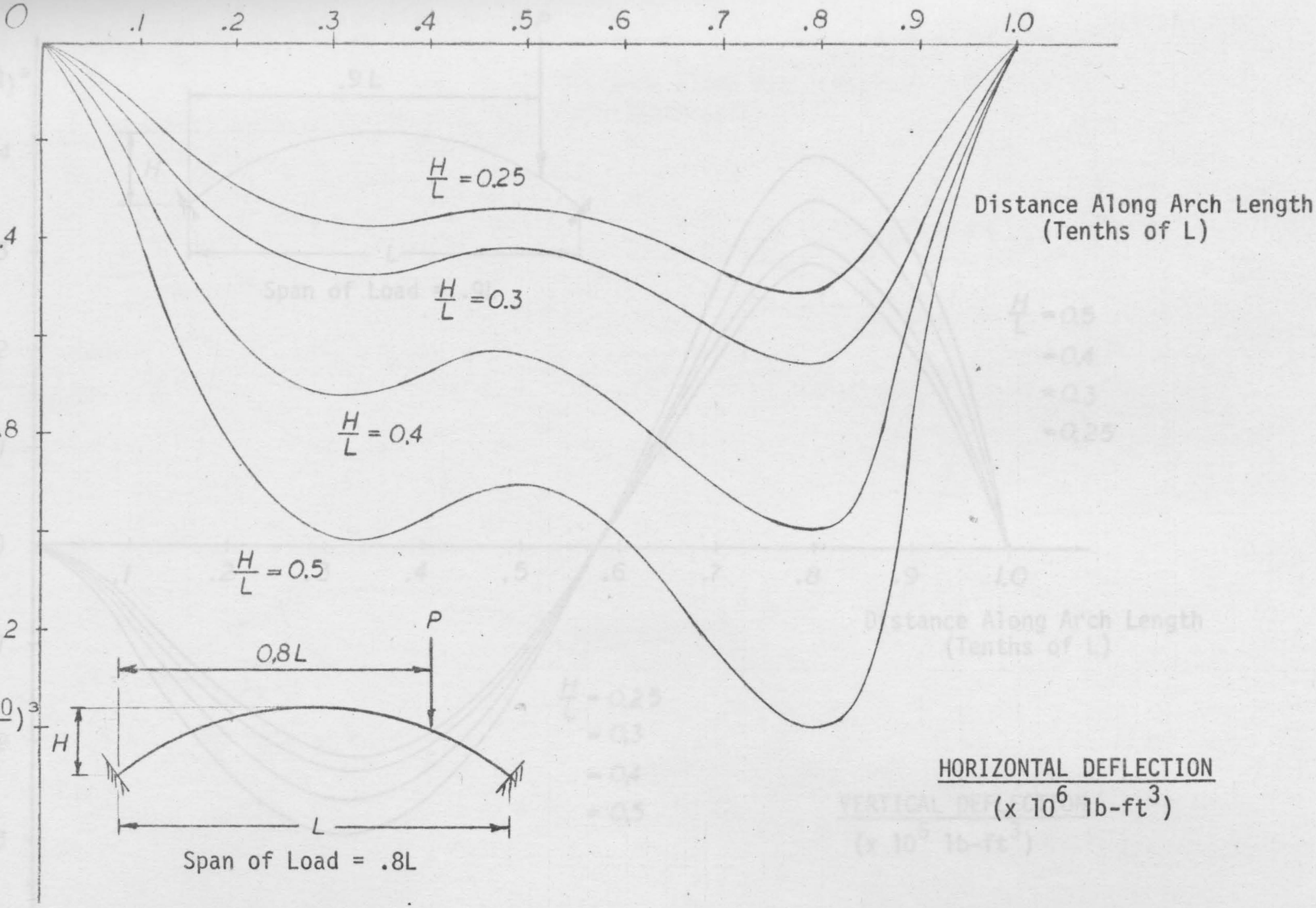


Fig. 3.16 Design chart for the horizontal deflection for any point on the span of an arch with the position of load at .8 the span of the arch.

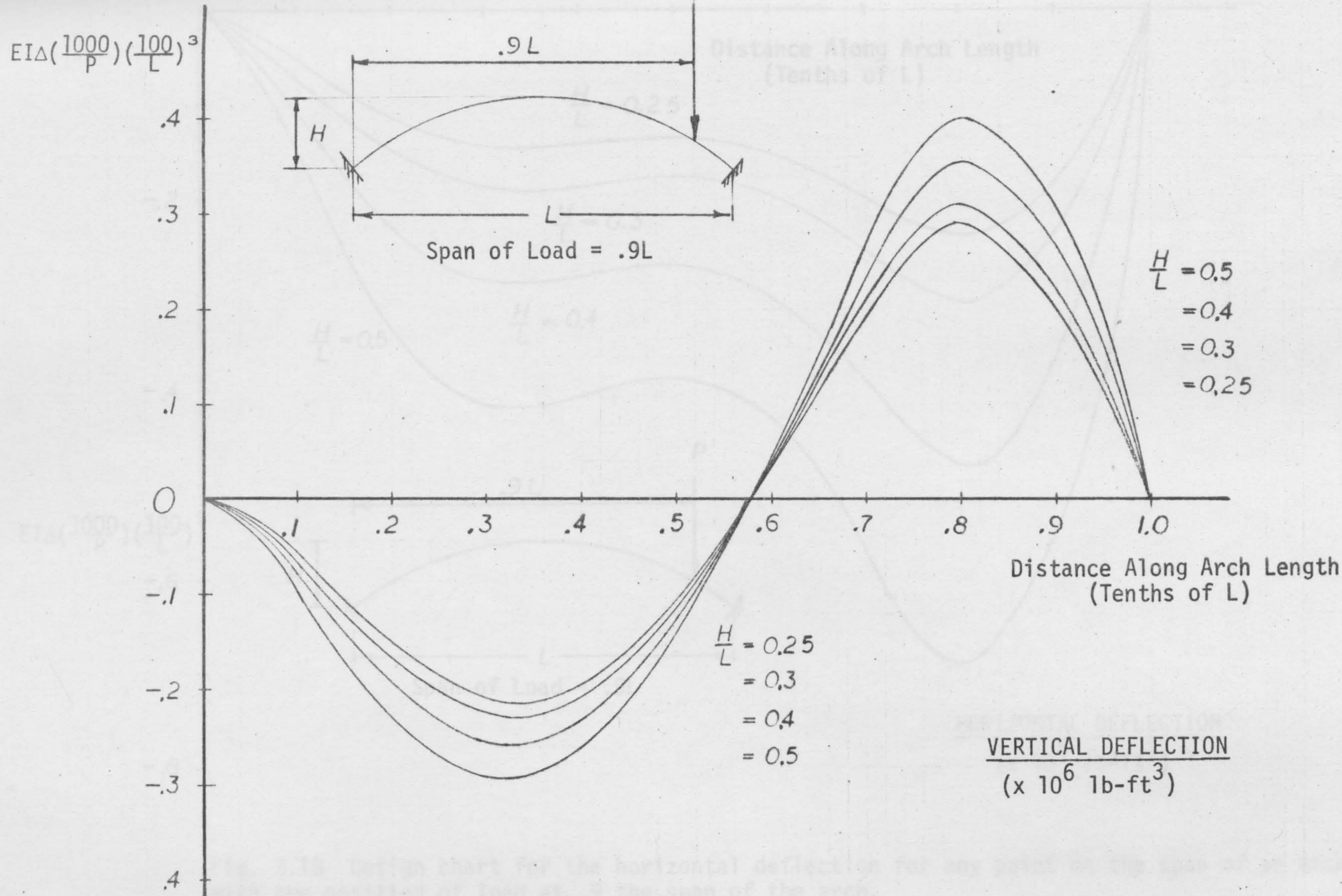


Fig. 3.17 Design chart for the vertical deflection for any point on the span of an arch with the position of load at .9 the span of the arch.

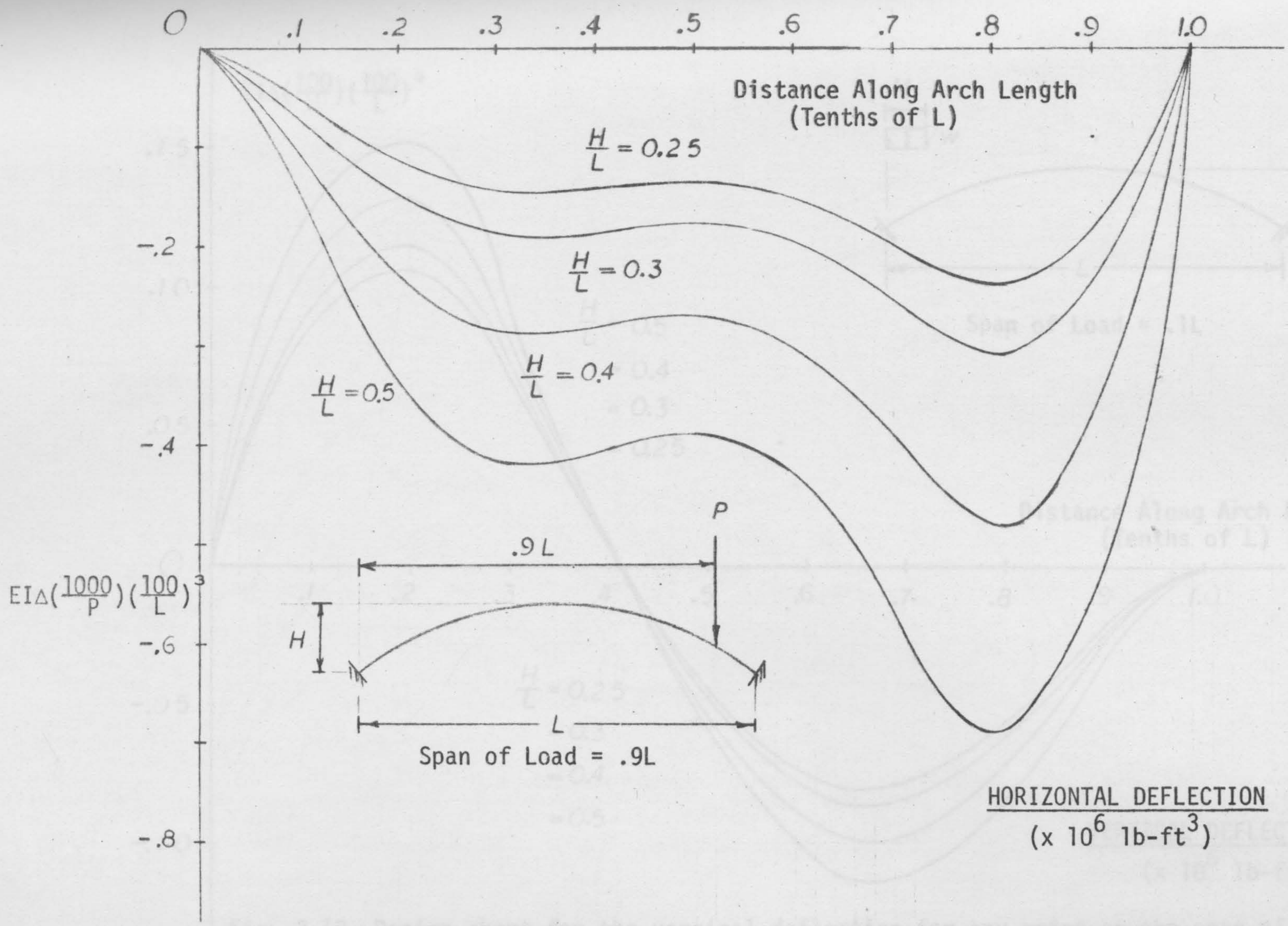


Fig. 3.18 Design chart for the horizontal deflection for any point on the span of an arch with the position of load at .9 the span of the arch.

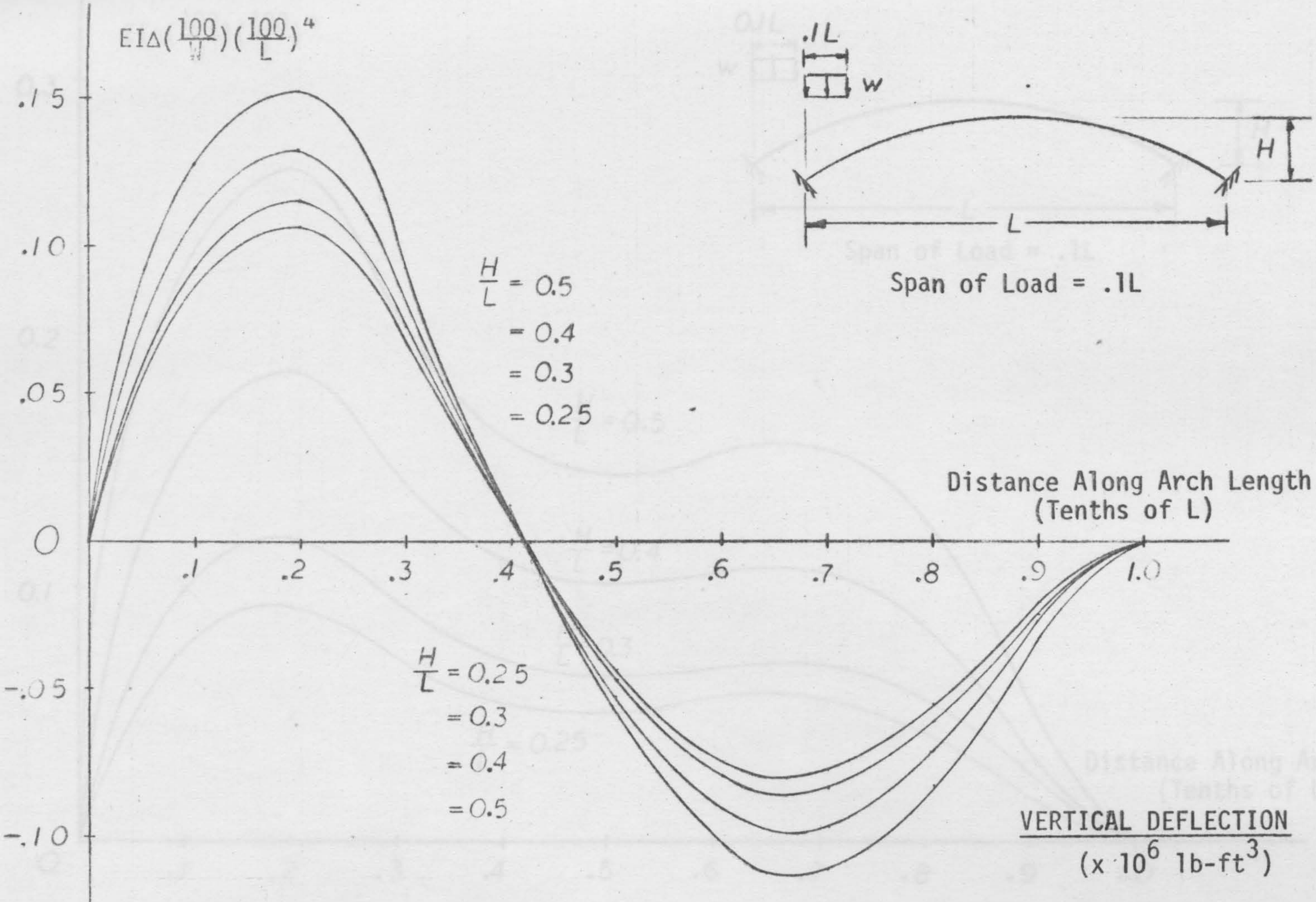


Fig. 3.19 Design chart for the vertical deflection for any point on the span of an arch with a span of load equal to .1 the span of the arch.

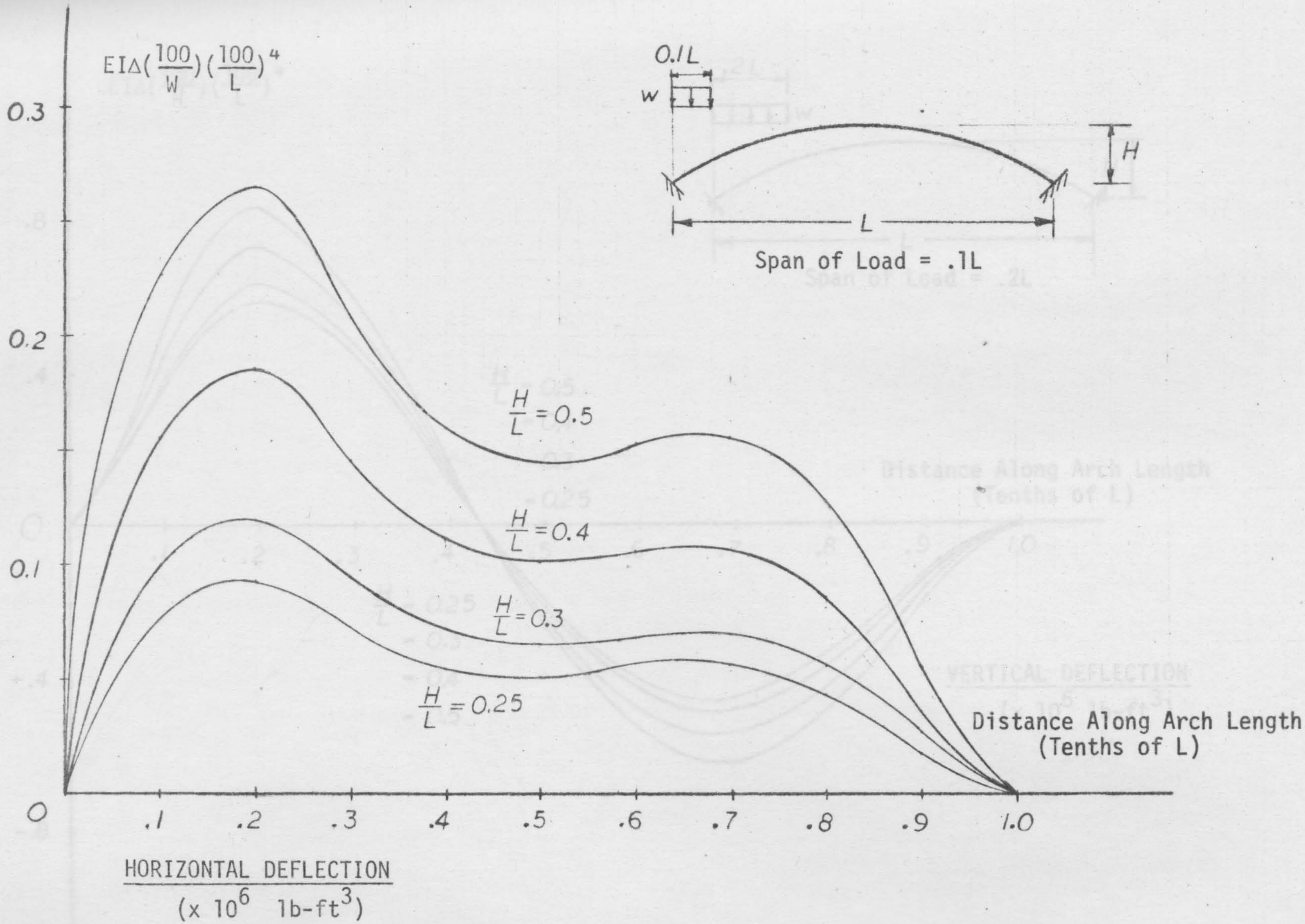


Fig. 3.20 Design chart for the horizontal deflection for any point on the span of an arch with a span of load equal to .1 the span of the arch.

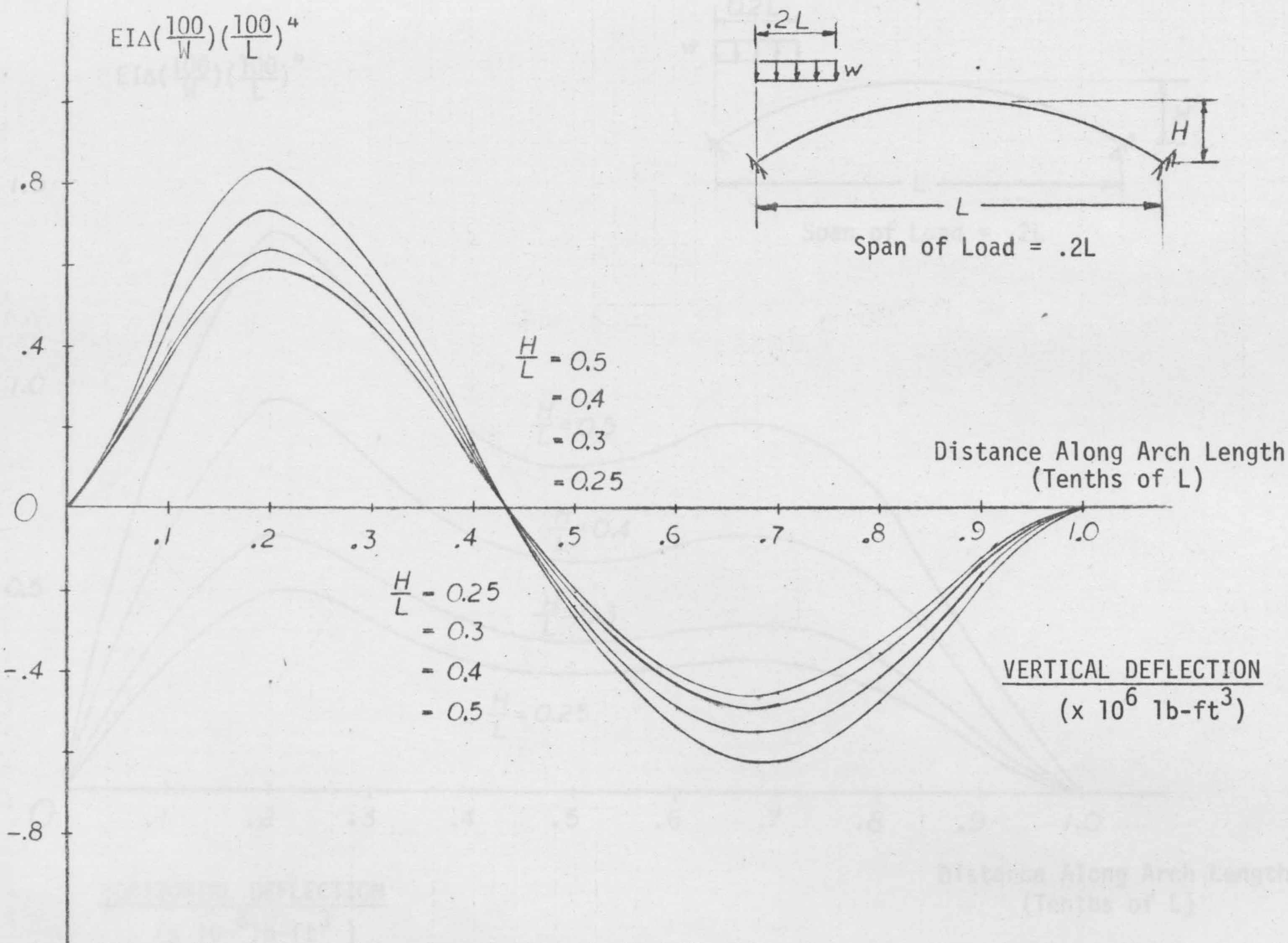


Fig. 3.21 Design chart for the vertical deflection for any point on the span of an arch with a span of load equal to .2 the span of the arch.

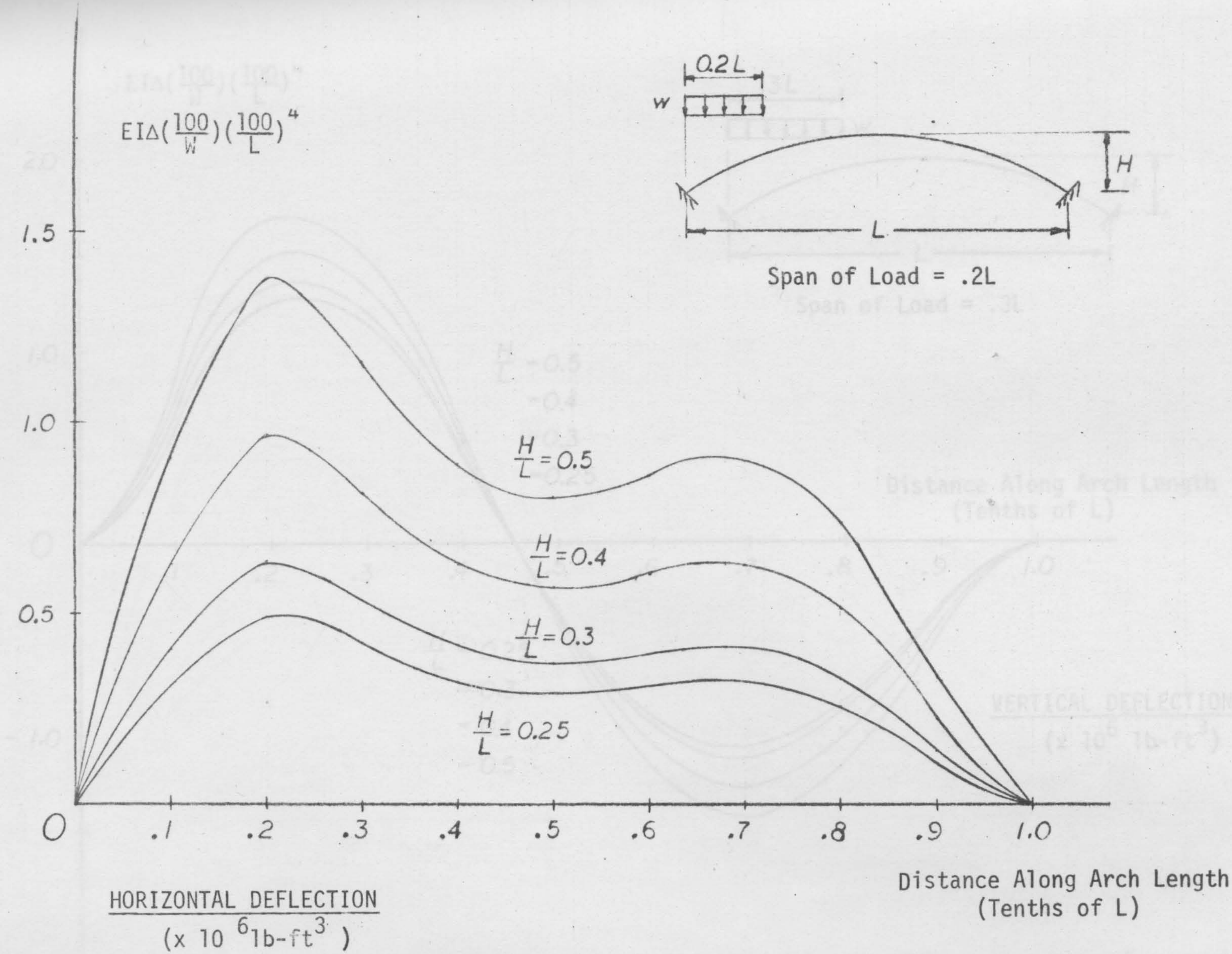


Fig. 3.22 Design chart for the horizontal deflection for any point on the span of an arch with a span of load equal to .2 the span of the arch.

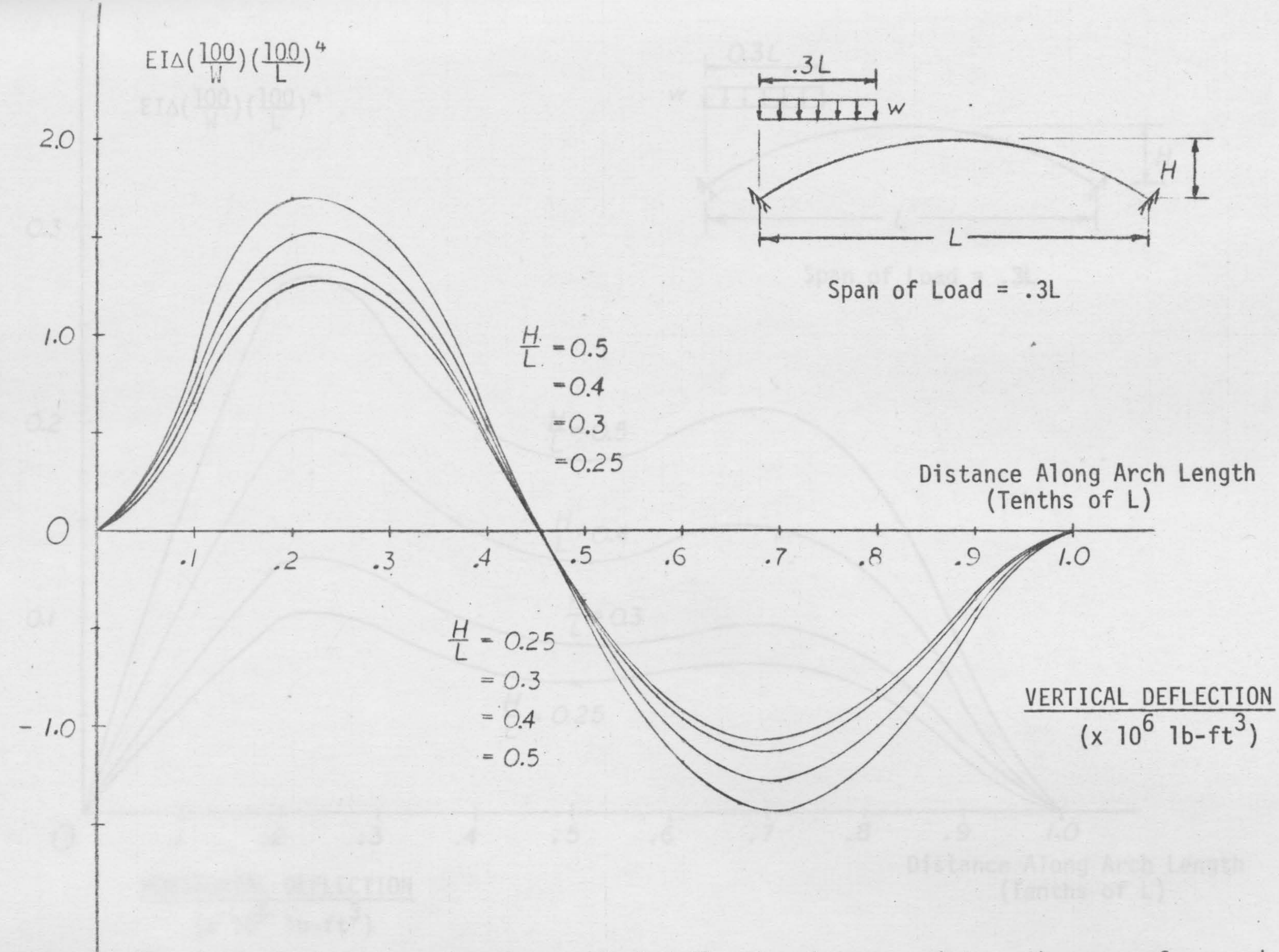


Fig. 3.23 Design chart for the vertical deflection for any point on the span of an arch with a span of load equal to .3 the span of the arch.

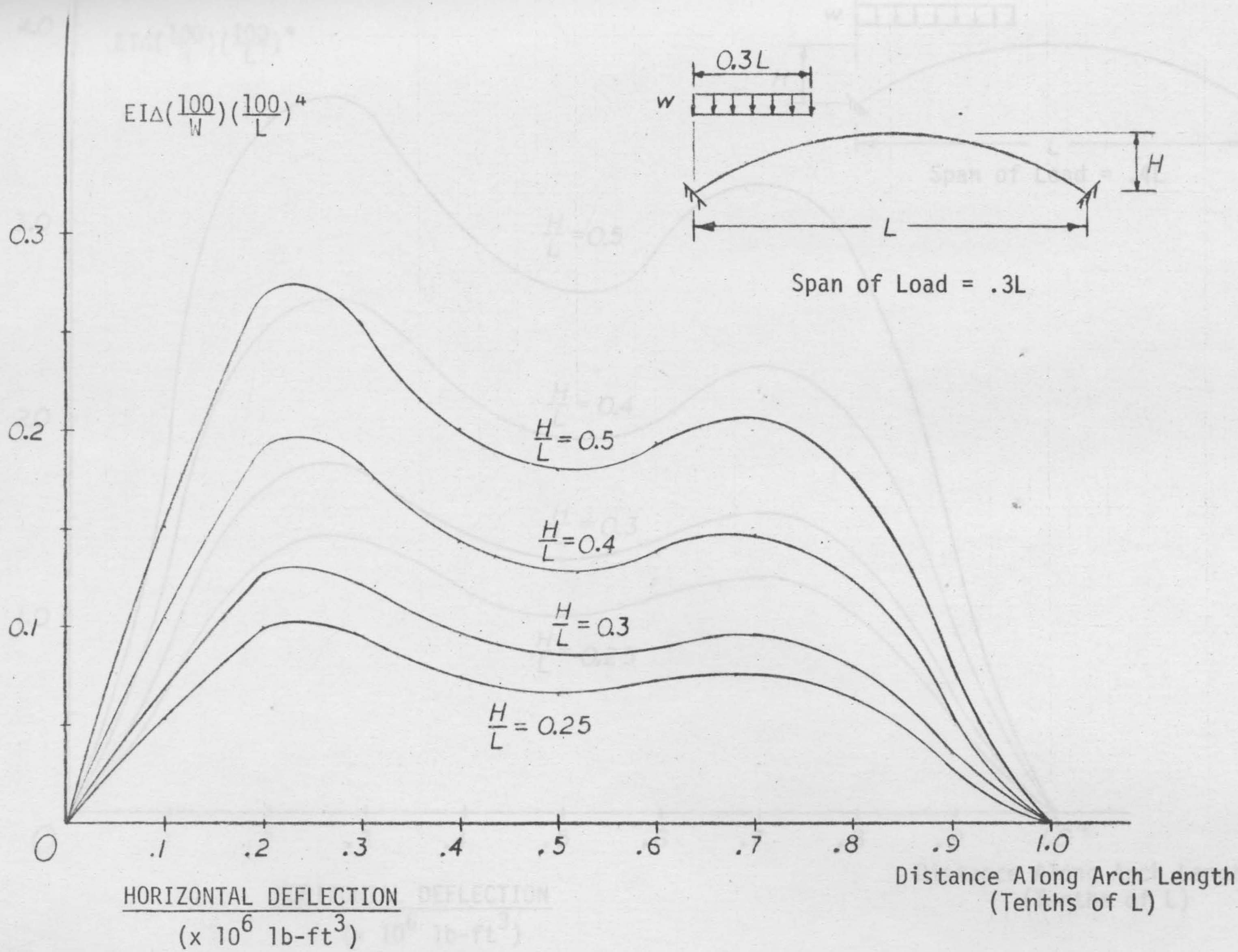


Fig. 3.24 Design chart for the horizontal deflection for any point on the span of an arch with a span of load equal to .3 the span of the arch.

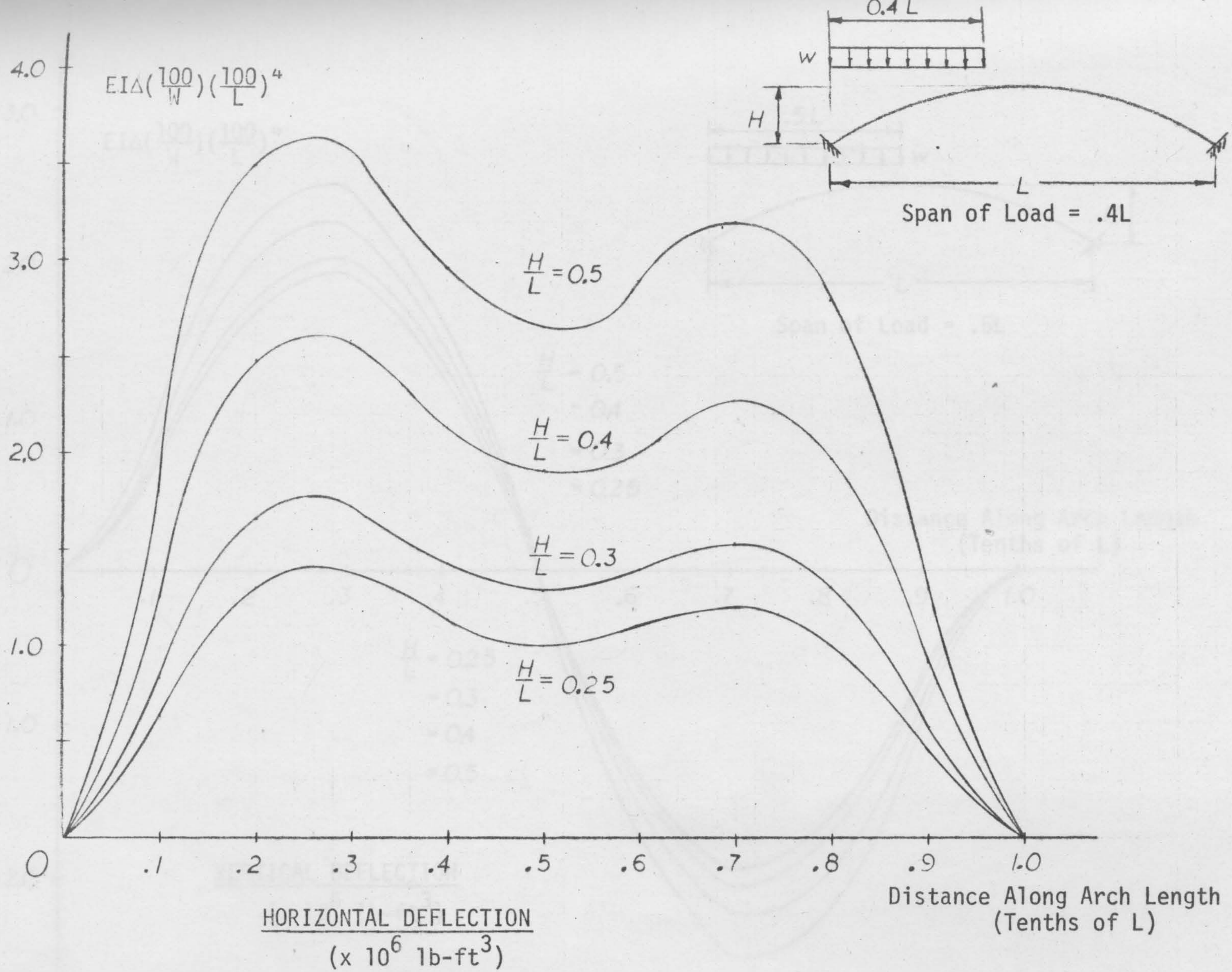


Fig. 3.26 Design chart for the horizontal deflection for any point on the span of an arch with a span of load equal to .4 the span of the arch.

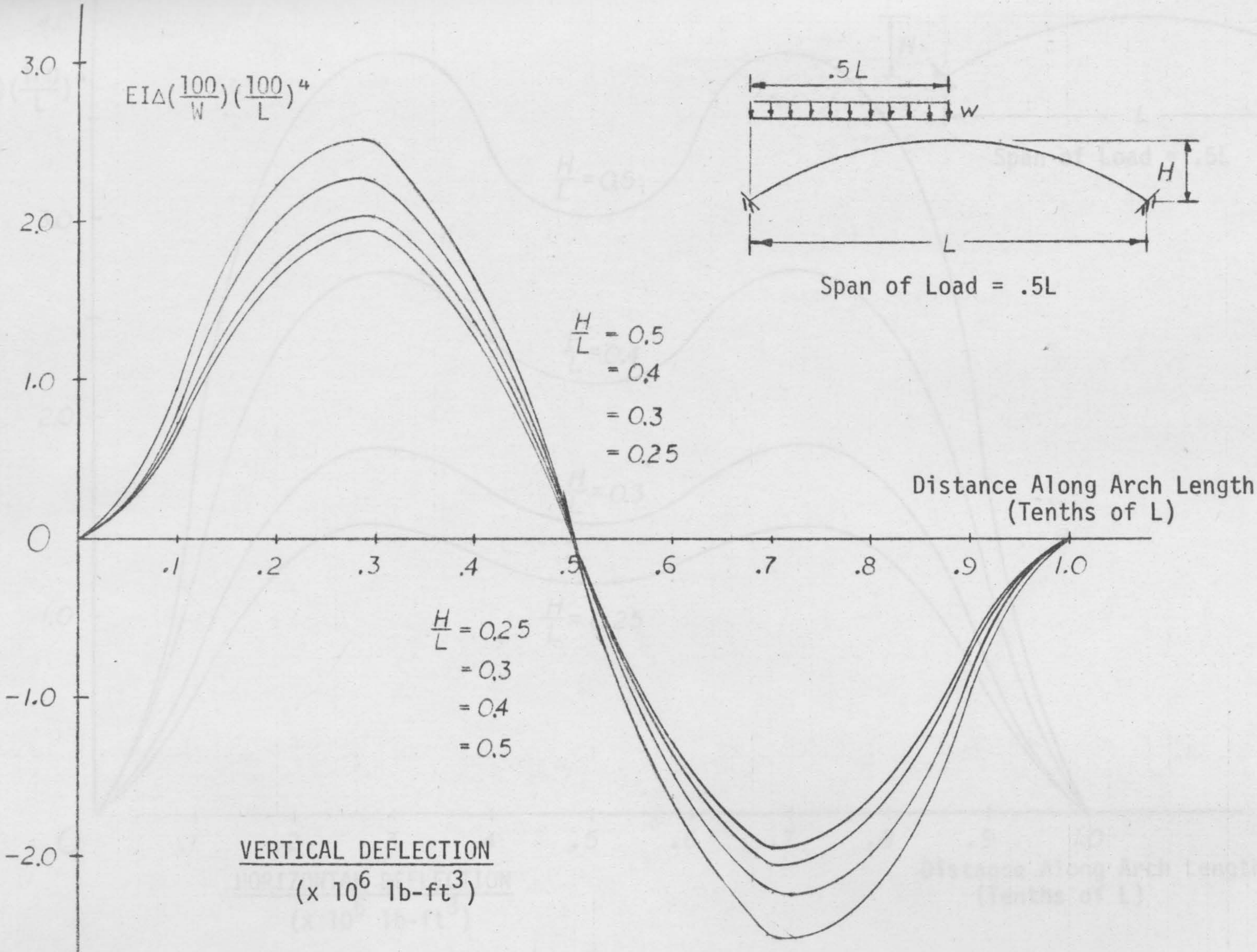


Fig. 3.27 Design chart for the vertical deflection for any point on the span of an arch with a span of load equal to .5 the span of the arch.

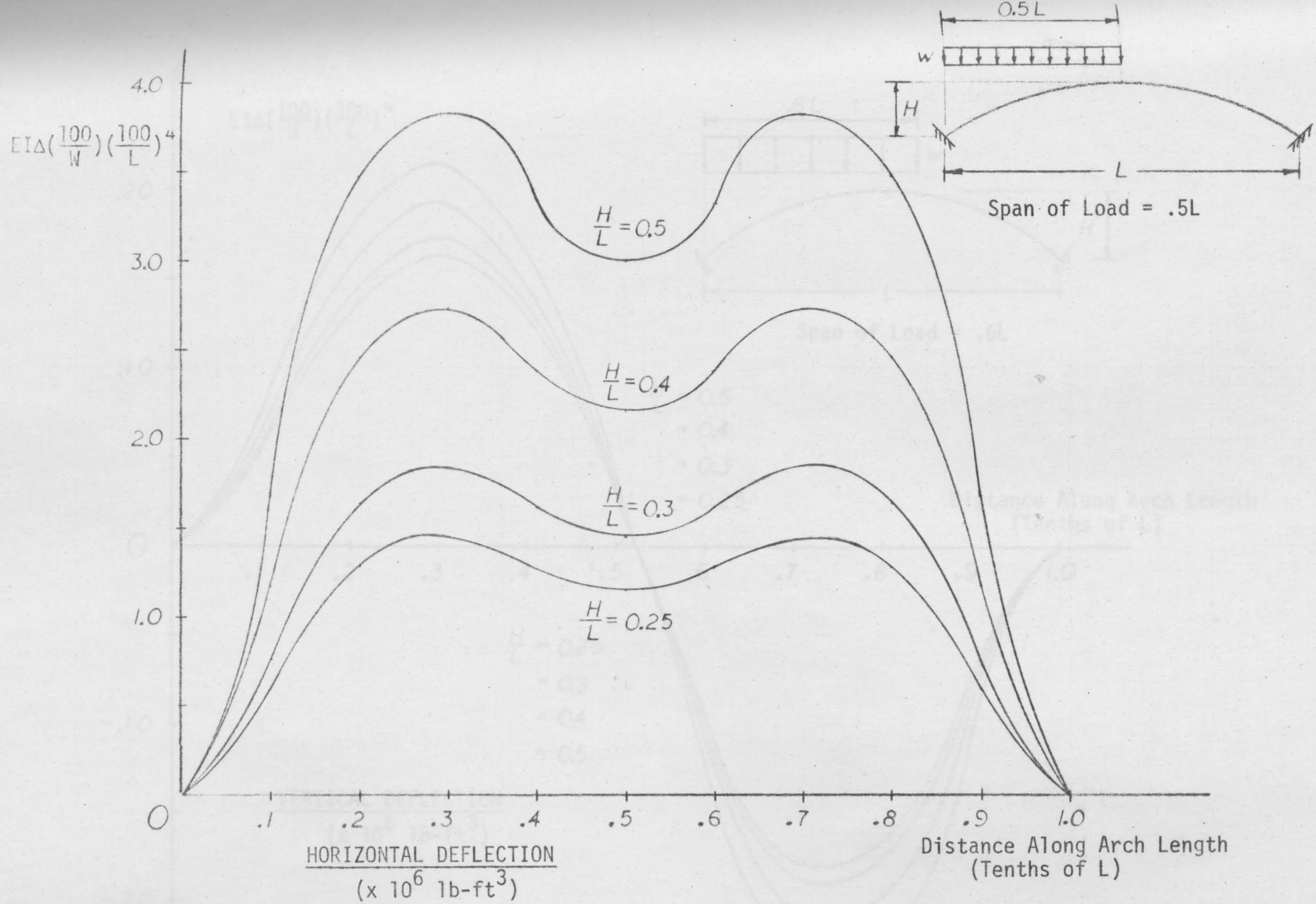


Fig. 3.28 Design chart for the horizontal deflection for any point on the span of an arch with a span of load equal to .5 the span of the arch.

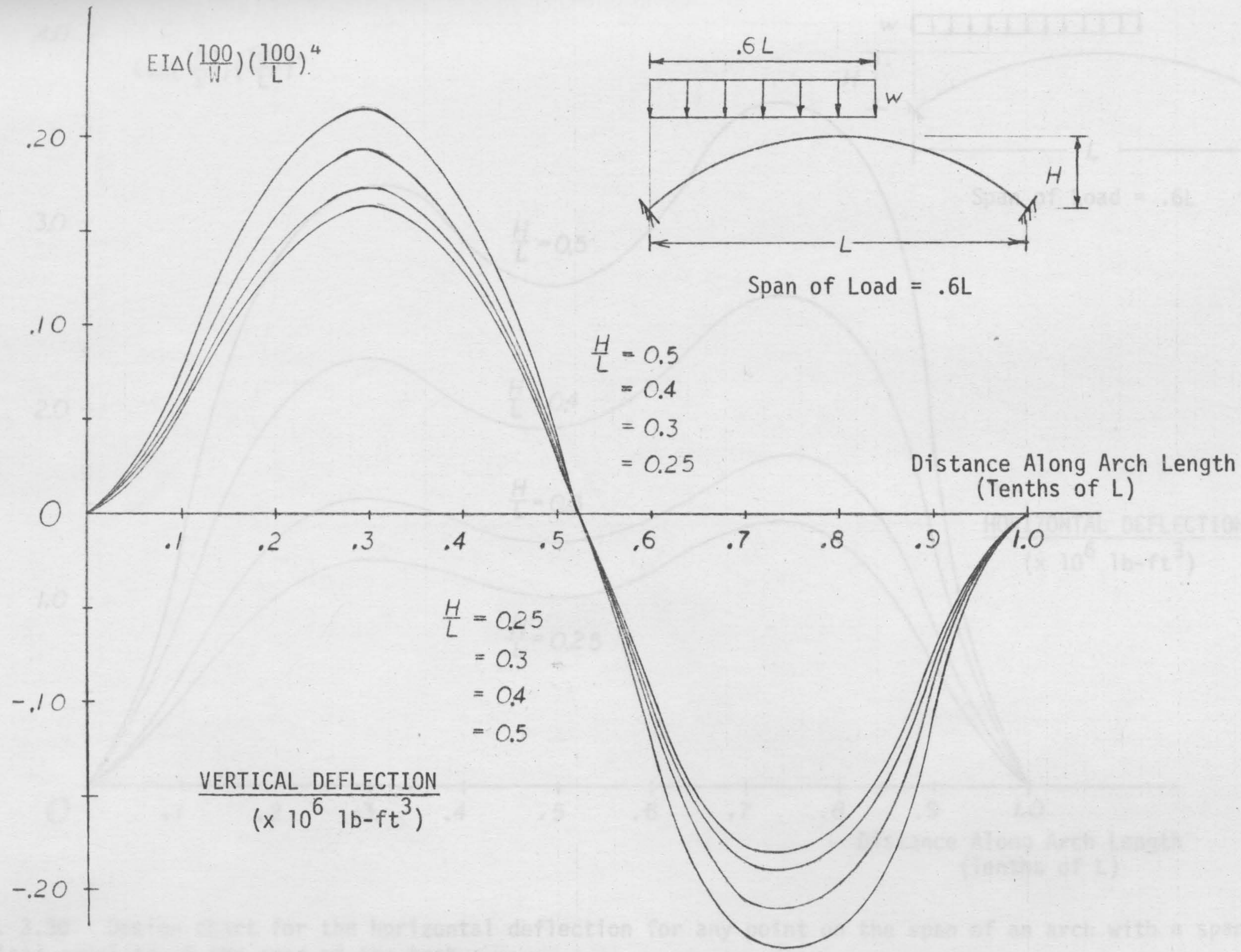


Fig. 3.29 Design chart for the vertical deflection for any point on the span of an arch with a span of load equal to .6 the span of the arch.

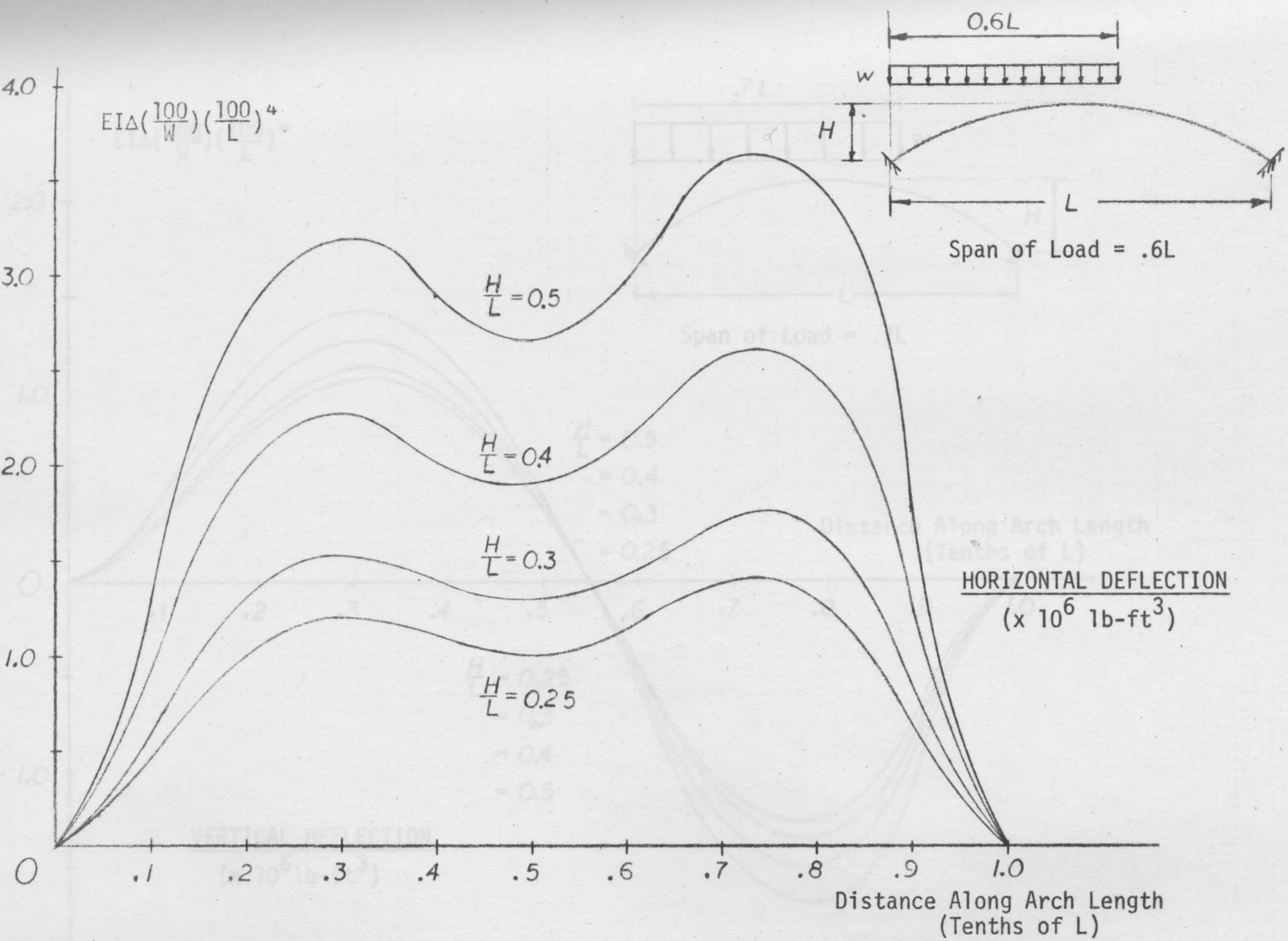


Fig. 3.30 Design chart for the horizontal deflection for any point on the span of an arch with a span of load equal to .6 the span of the arch.

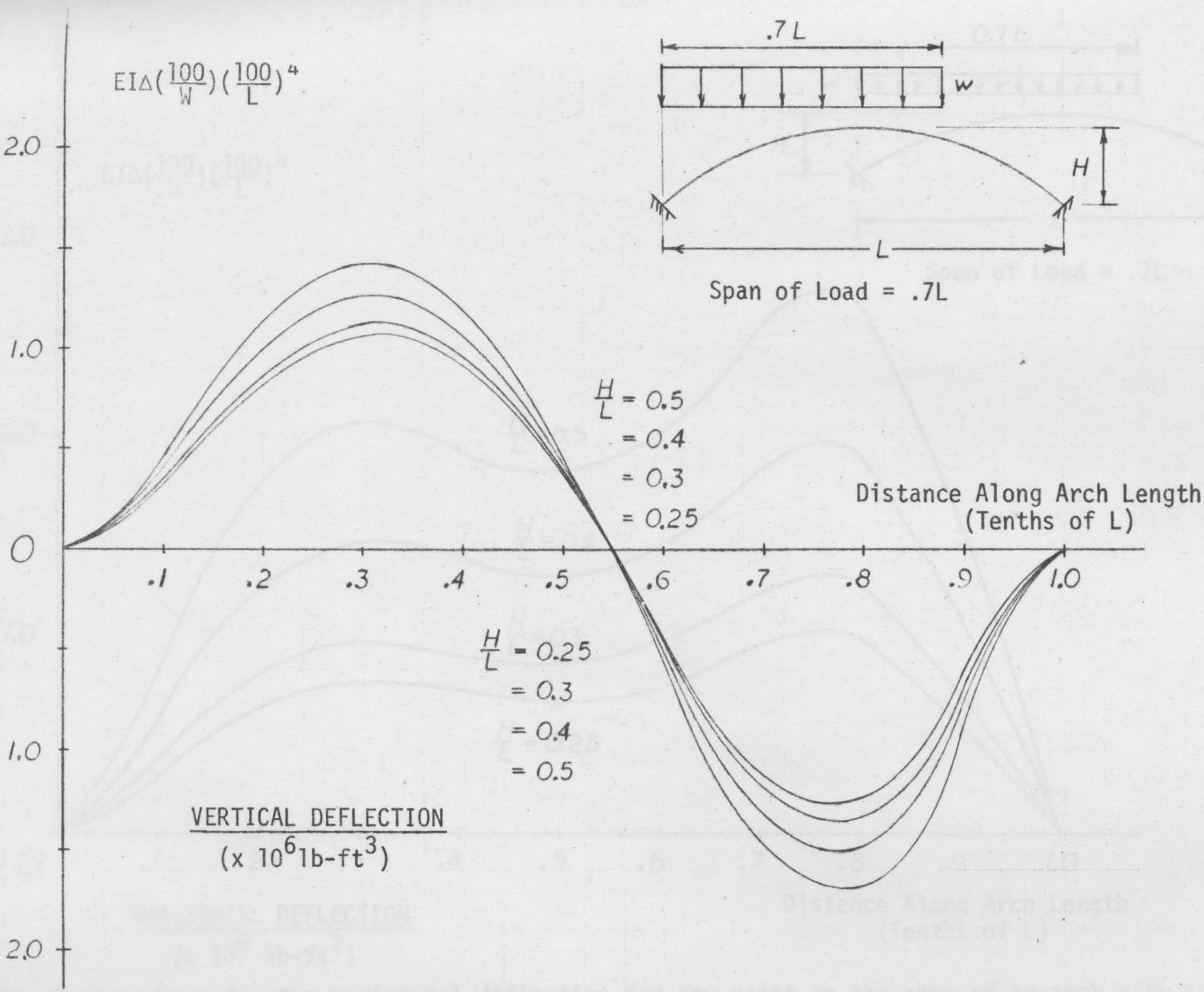


Fig. 3.31 Design chart for the vertical deflection for any point on the span of an arch with a span of load equal to .7 the span of the arch.

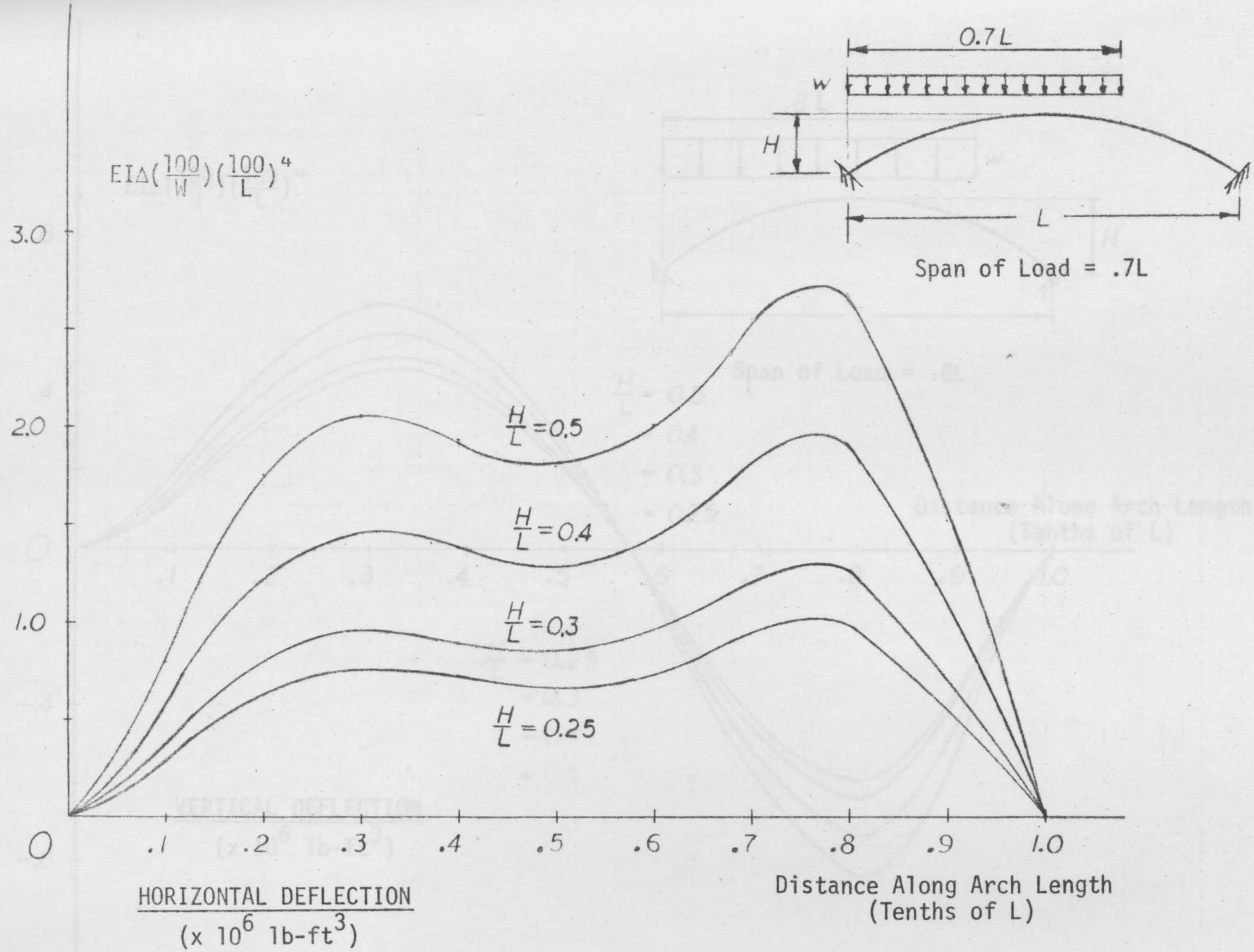


Fig. 3.32 Design chart for the horizontal deflection for any point on the span of an arch with a span of load equal to .7 the span of the arch.

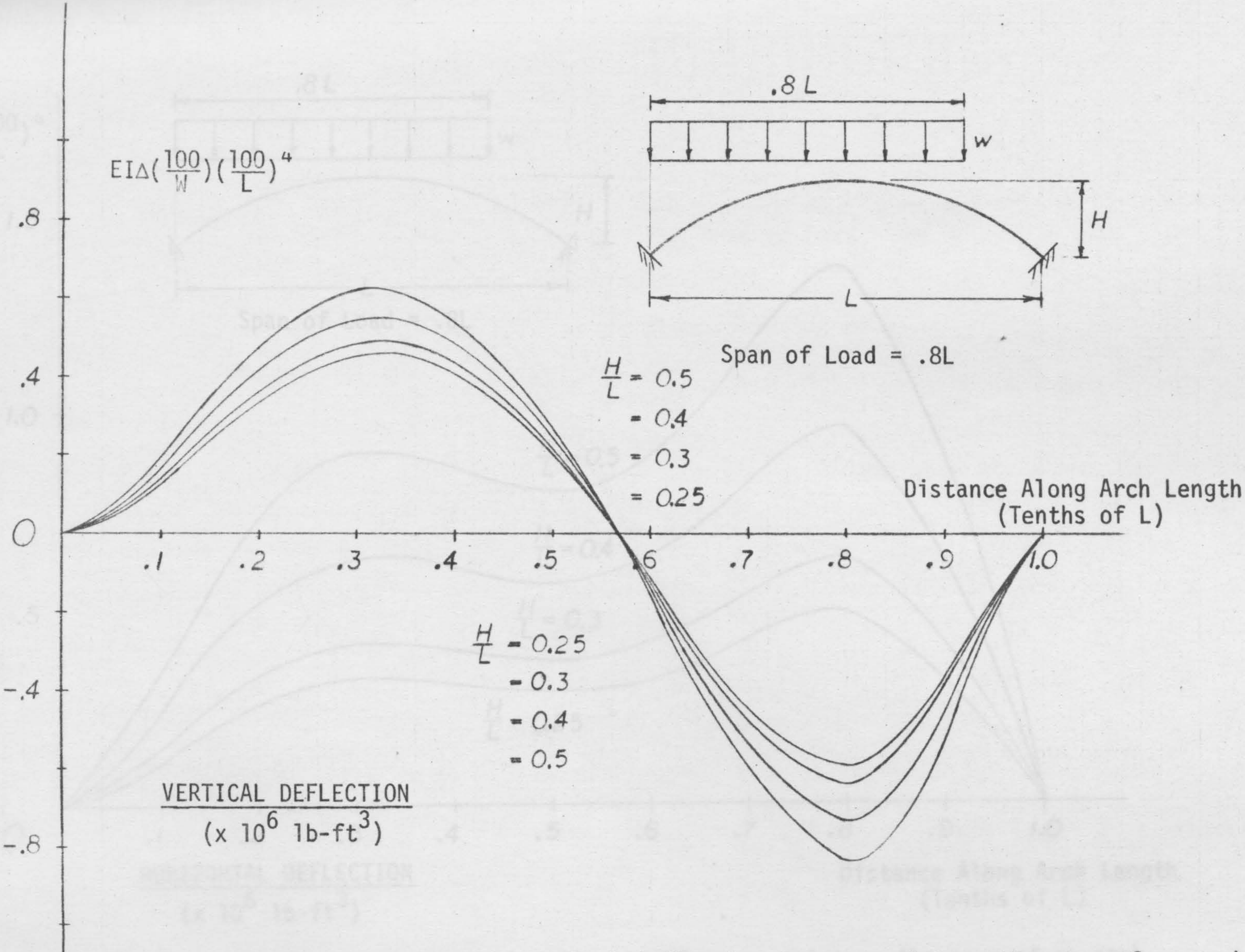


Fig. 3.33 Design chart for the vertical deflection for any point on the span of an arch with a span of load equal to .8 the span of the arch.

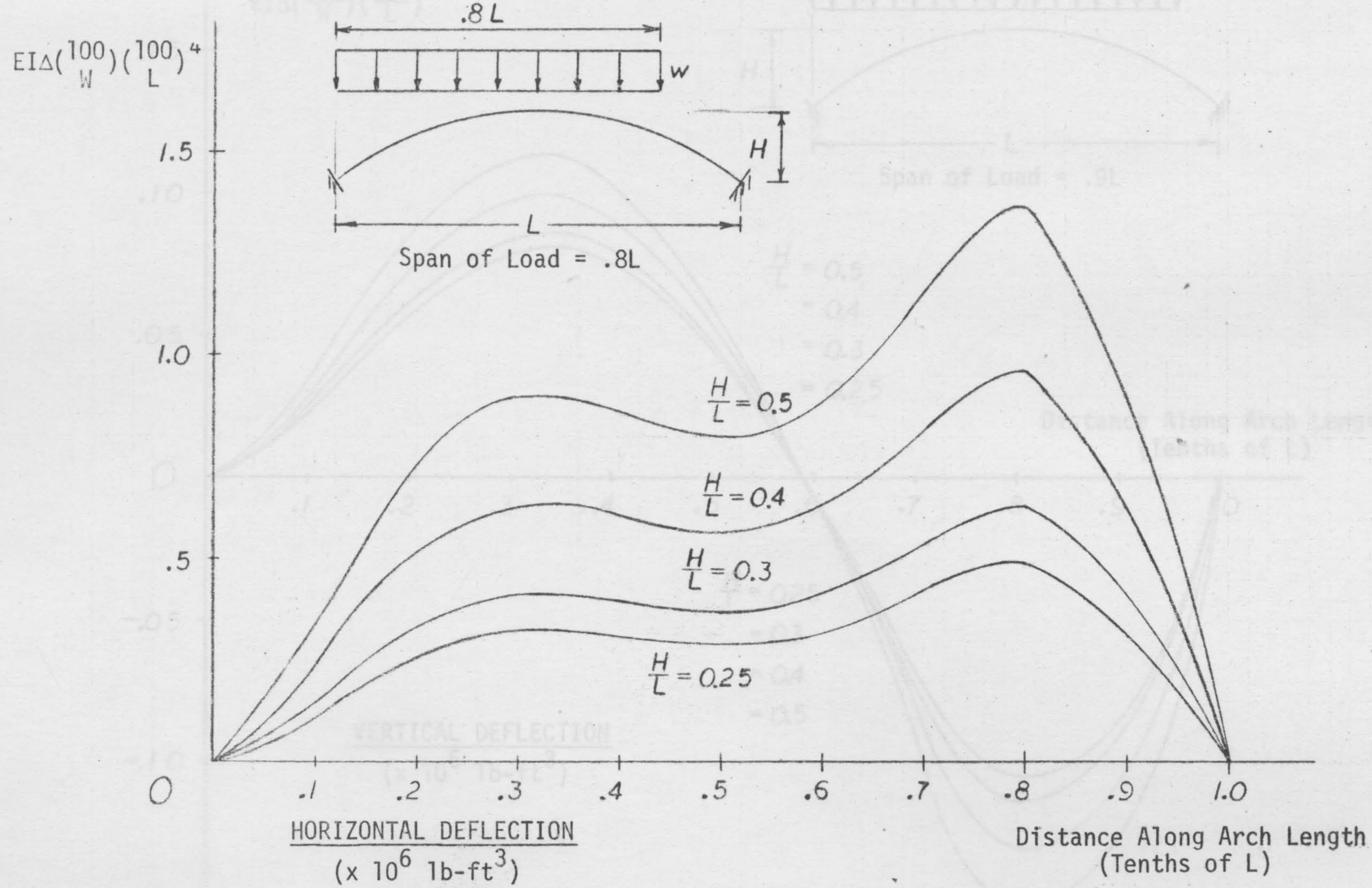


Fig. 3.34 Design chart for the horizontal deflection for any point on the span of an arch with a span of load equal to .8 the span of the arch.

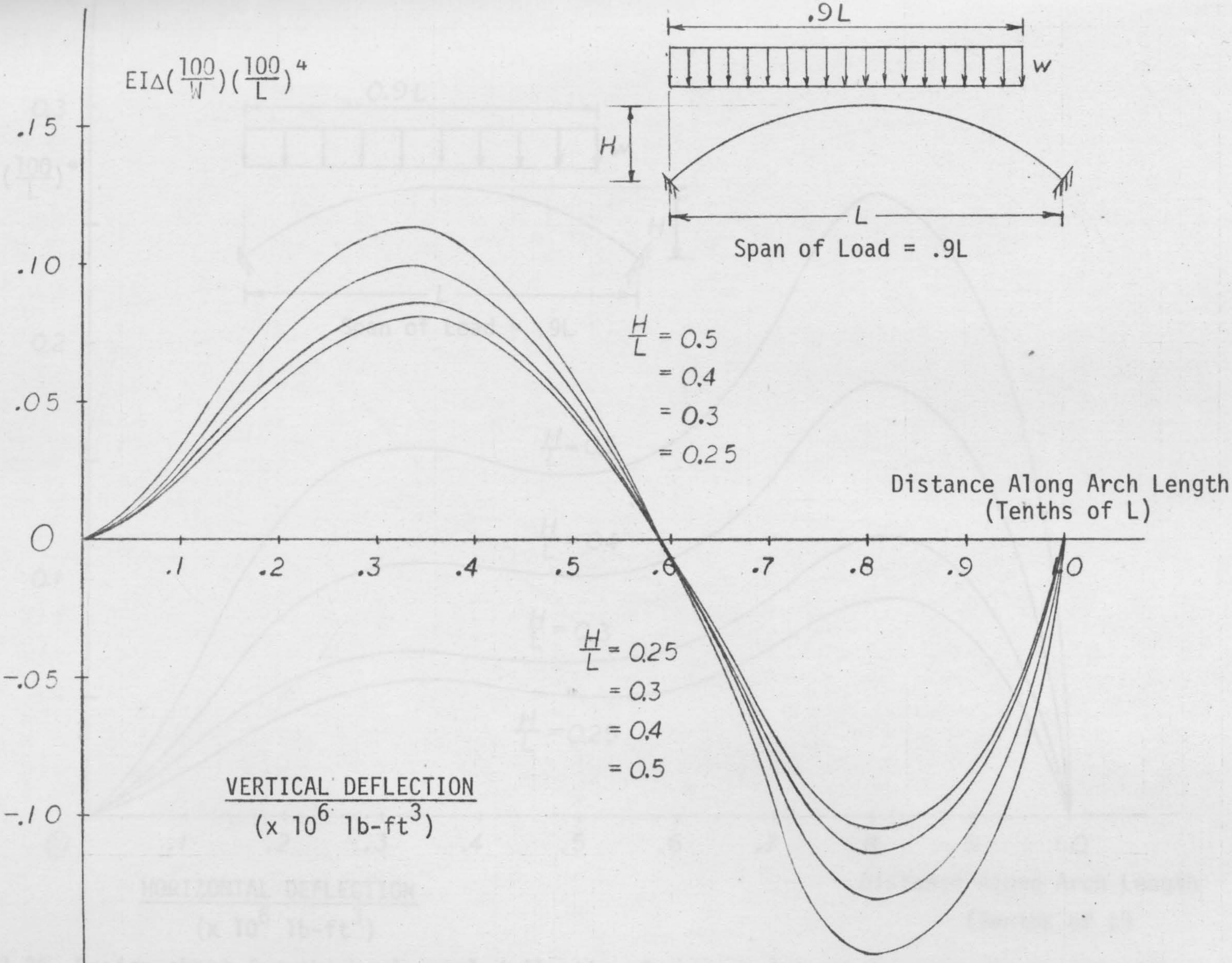


Fig. 3.35 Design chart for the vertical deflection for any point on the span of an arch with a span of load equal to .9 the span of the arch.

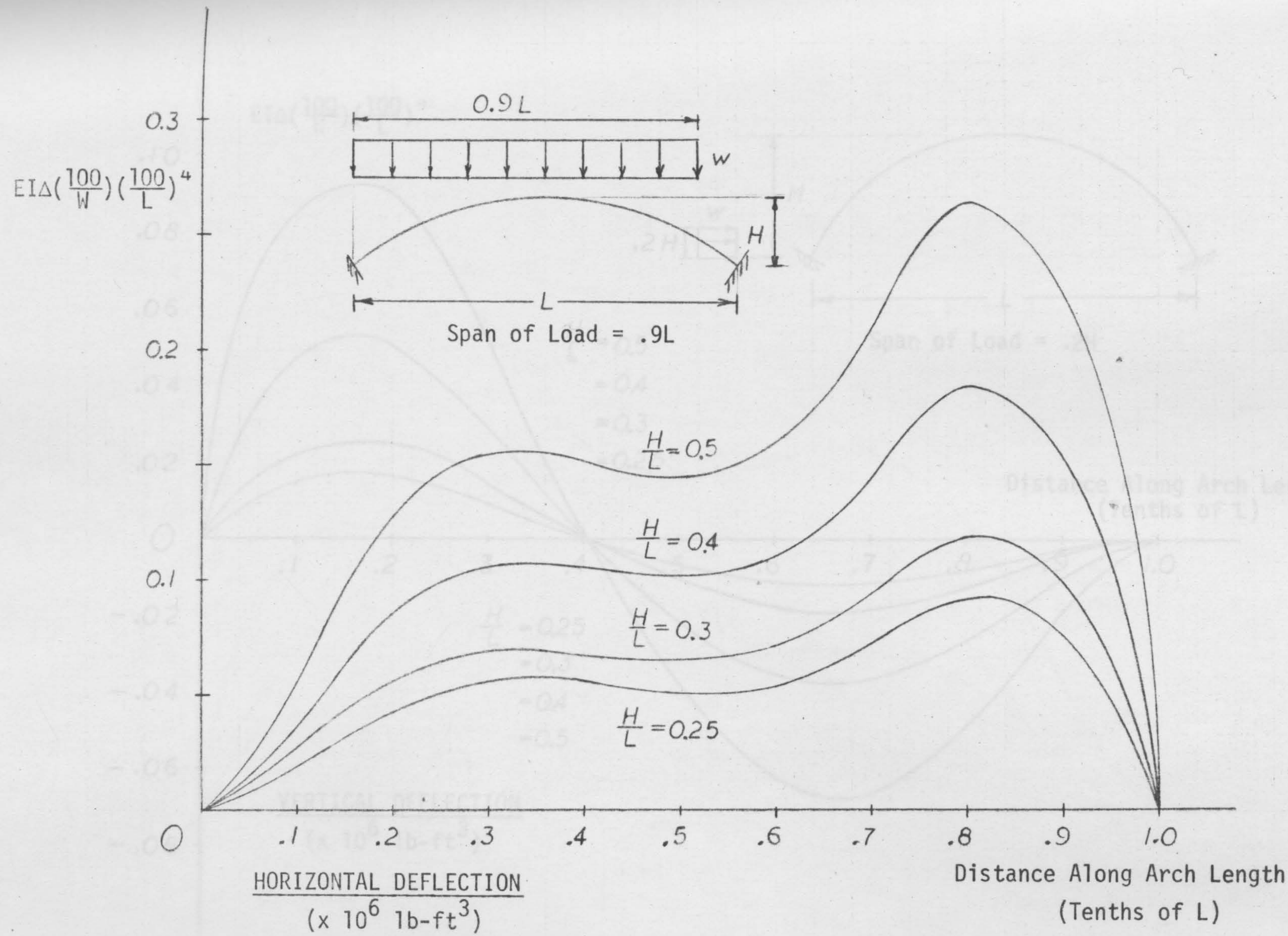


Fig. 3.36 Design chart for the horizontal deflection for any point on the span of an arch with a span of load equal to .9 the span of the arch.

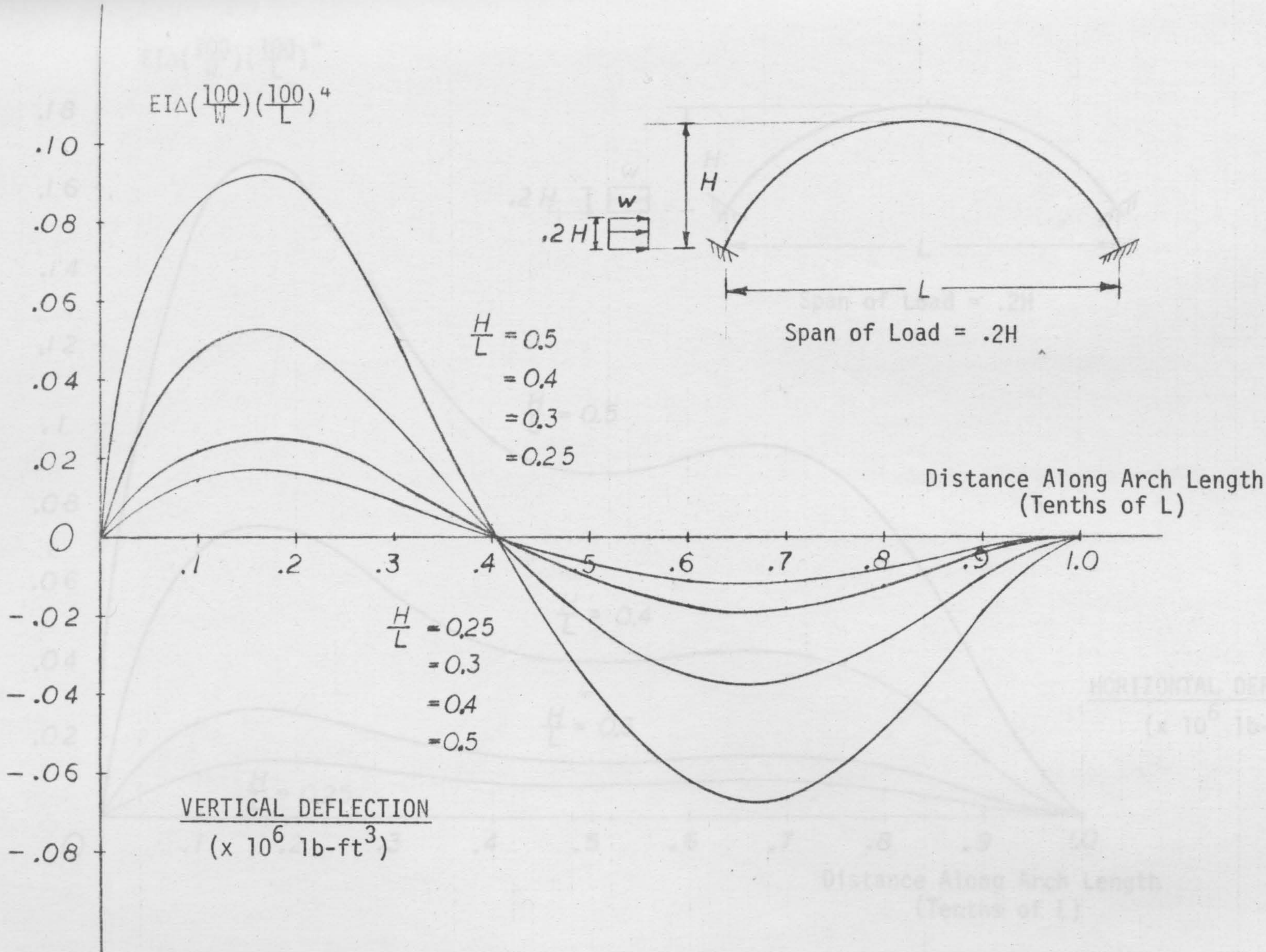


Fig. 3.37 Design chart for the vertical deflection for any point on the span of an arch with a span of load equal to .2 the rise of the arch.

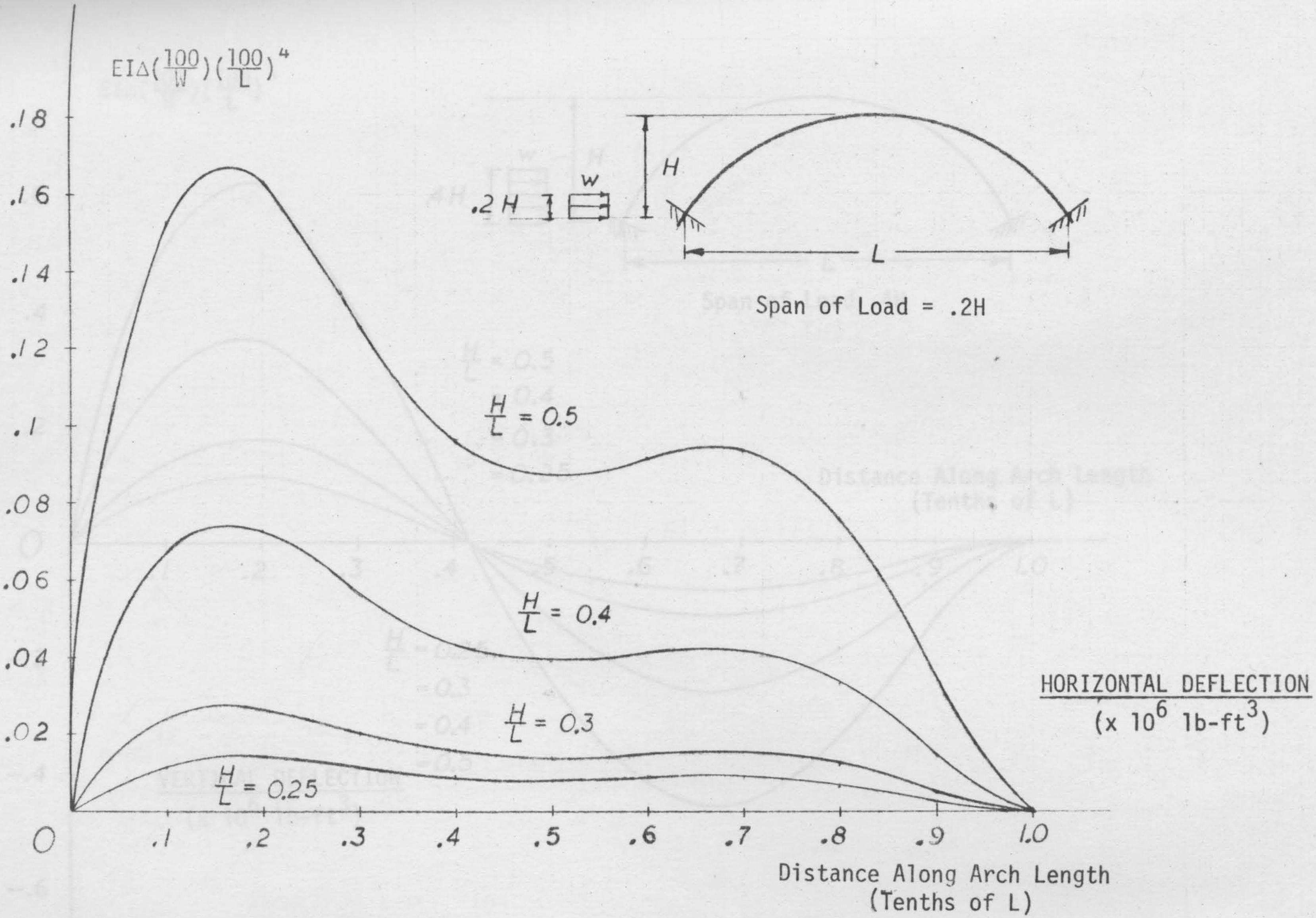


Fig. 3.38 Design chart for the horizontal deflection for any point on the span of an arch with a span of load equal .2 the rise of the arch.

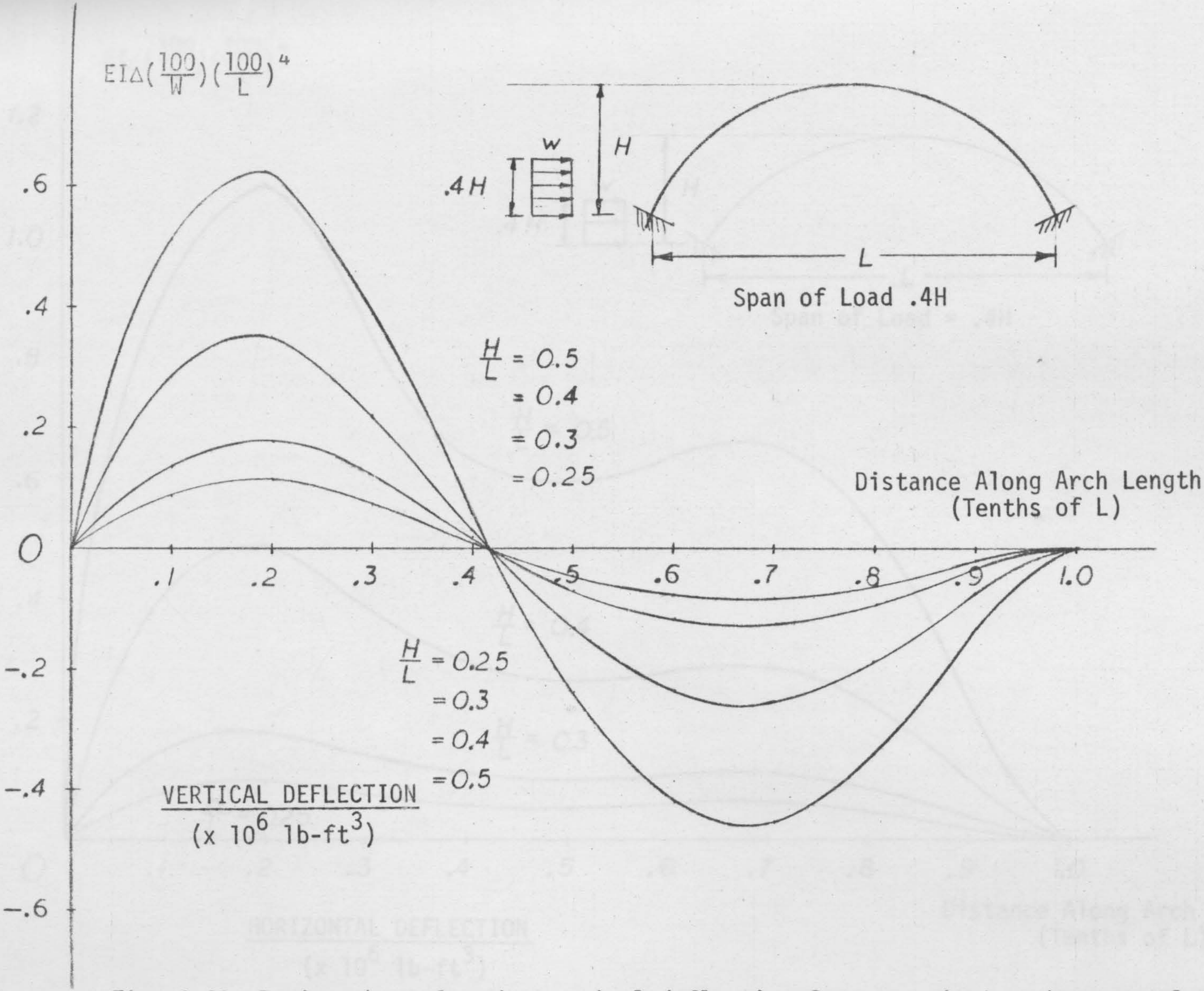


Fig. 3.39 Design chart for the vertical deflection for any point on the span of an arch with a span of load equal .4 the rise of the arch.

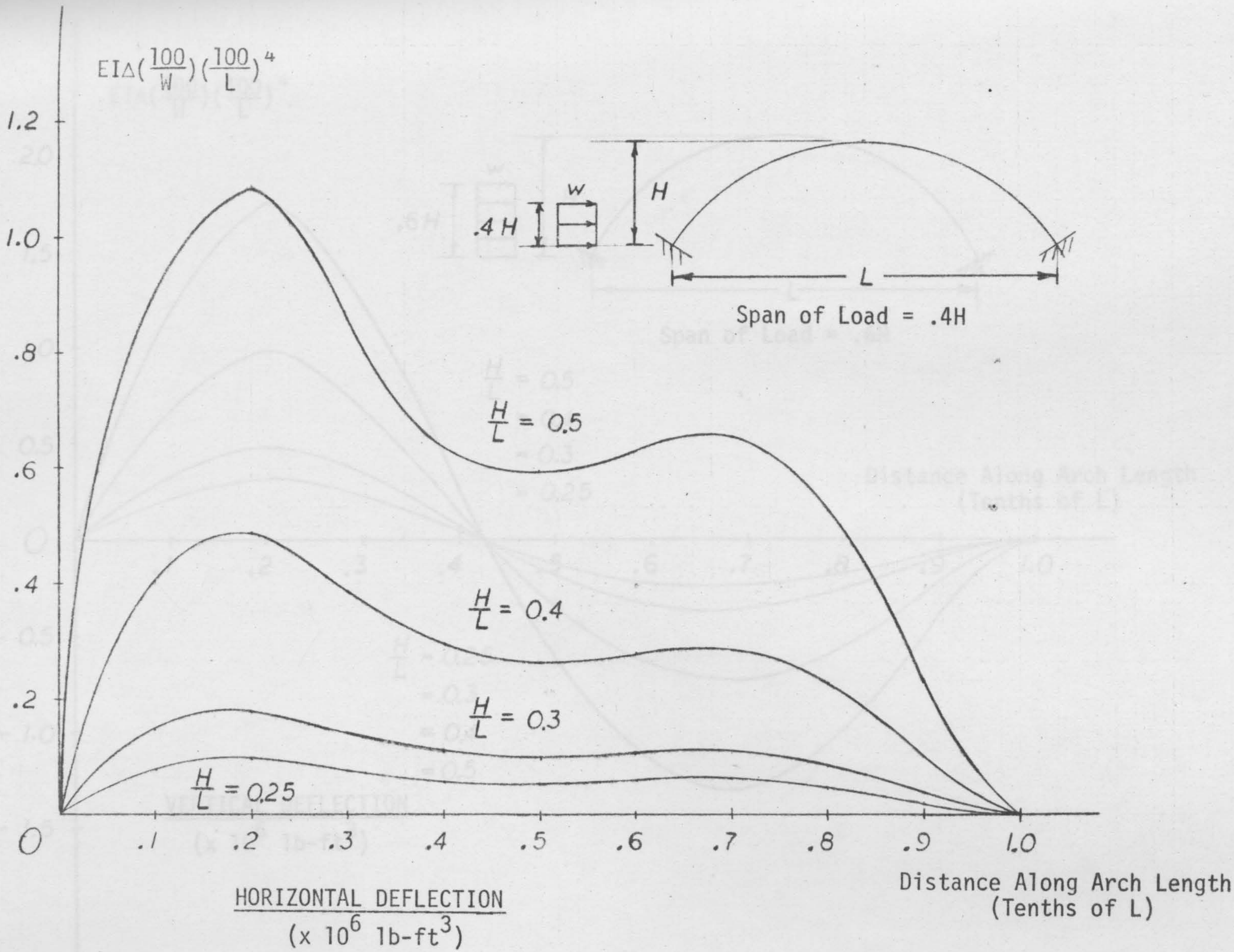


Fig. 3.40 Design chart for the horizontal deflection for any point on the span of an arch with a span of load equal .4 the rise of the arch.

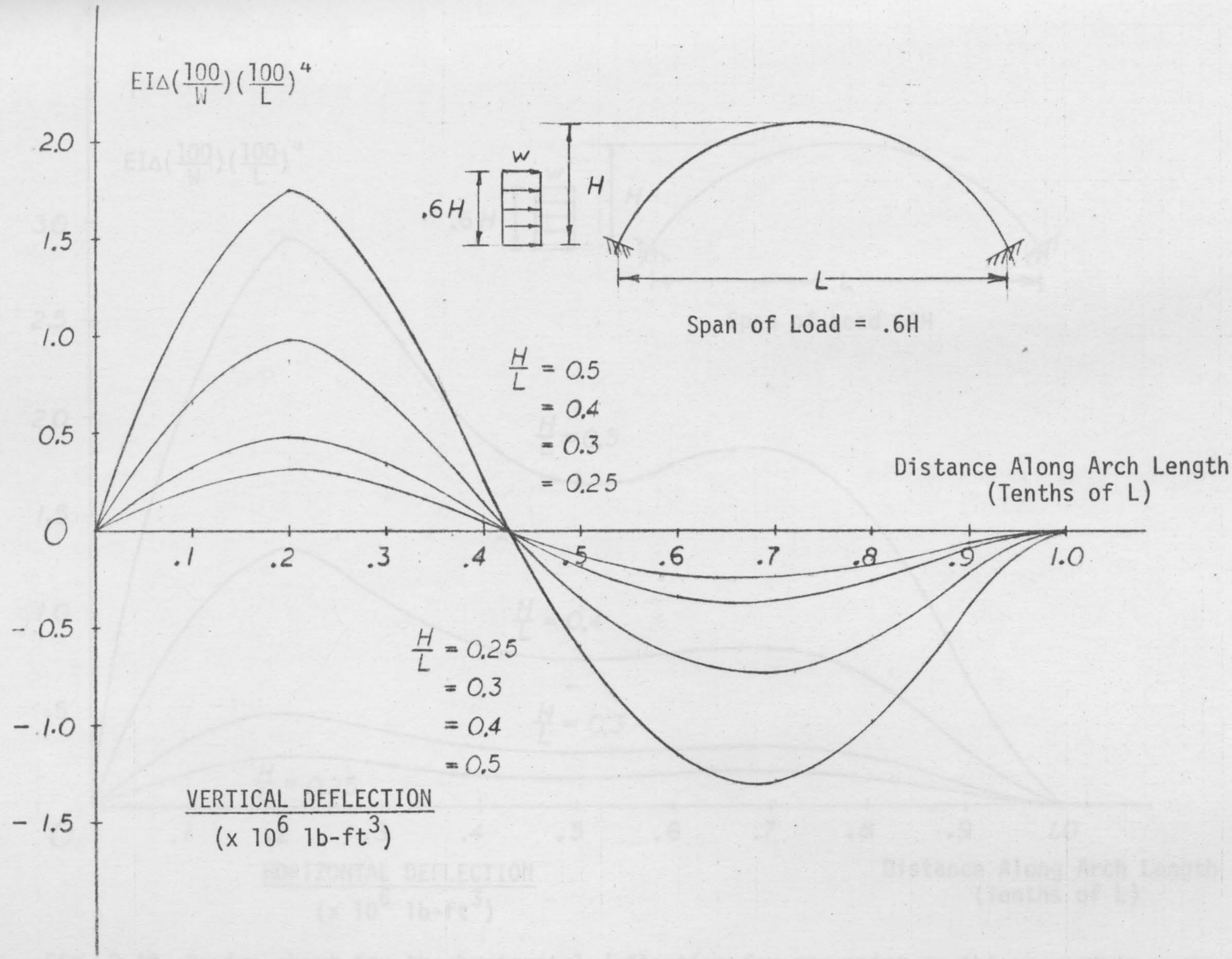


Fig. 3.41 Design chart for the vertical deflection for any point on the span of an arch with a span of load equal .6 the rise of the arch.

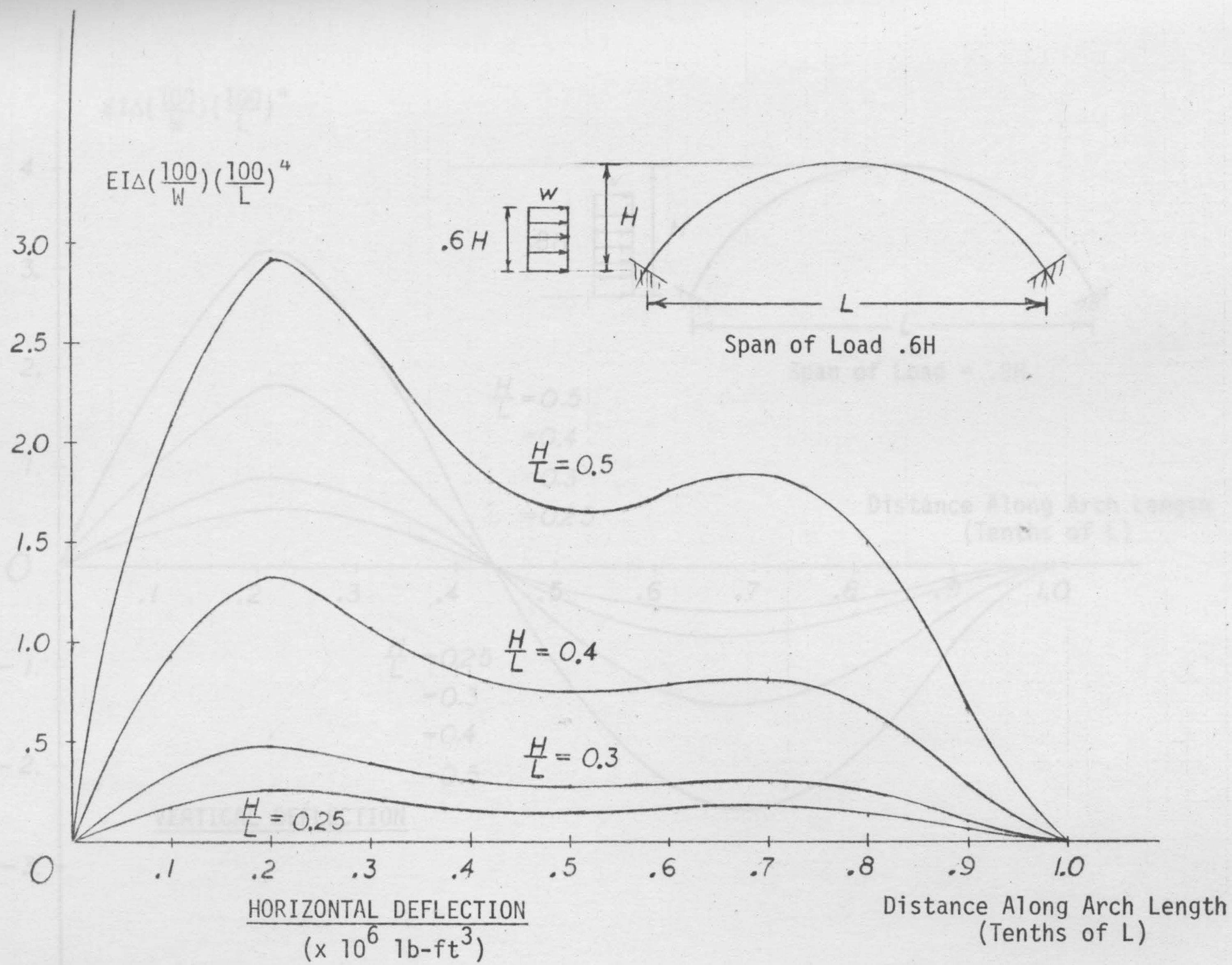


Fig. 3.42 Design chart for the horizontal deflection for any point on the span of an arch with a span of load equal .6 the rise of the arch.

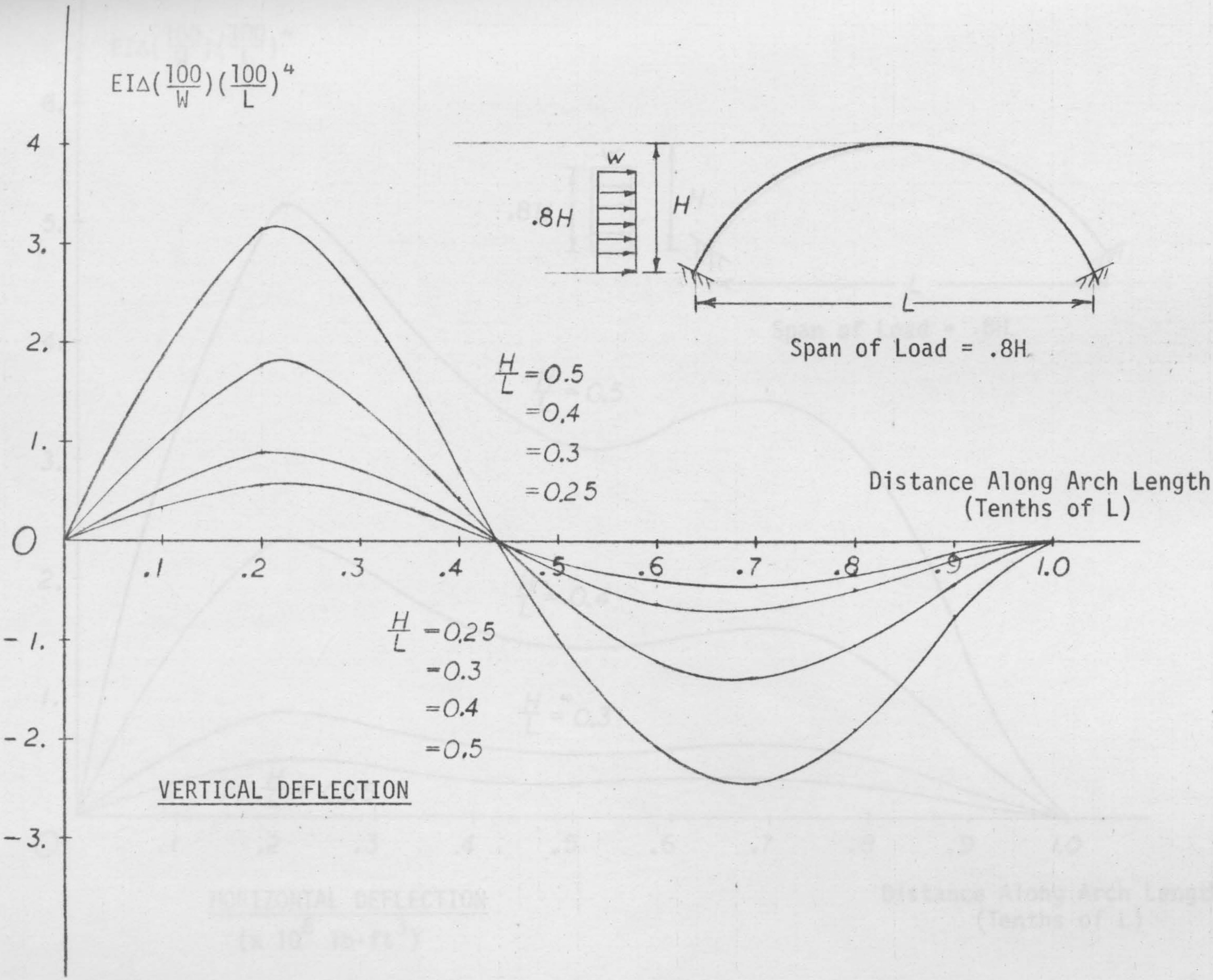


Fig. 3.43 Design chart for the vertical defection for any point on the span of an arch with a span of load equal .8 the rise of the arch.

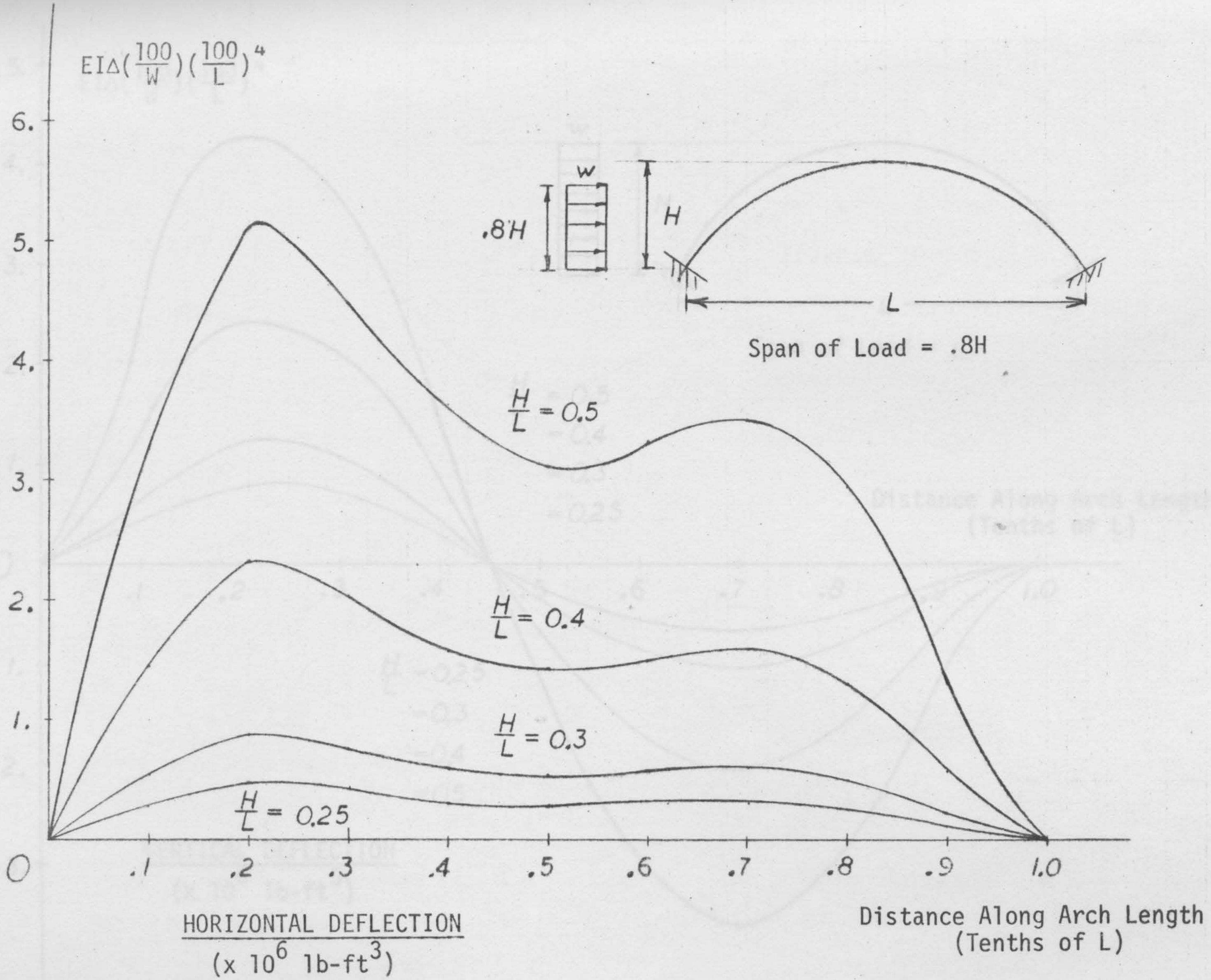


Fig. 3.44 Design chart for the horizontal deflection for any point on the span of an arch with a span of load equal .8 the rise of the arch.

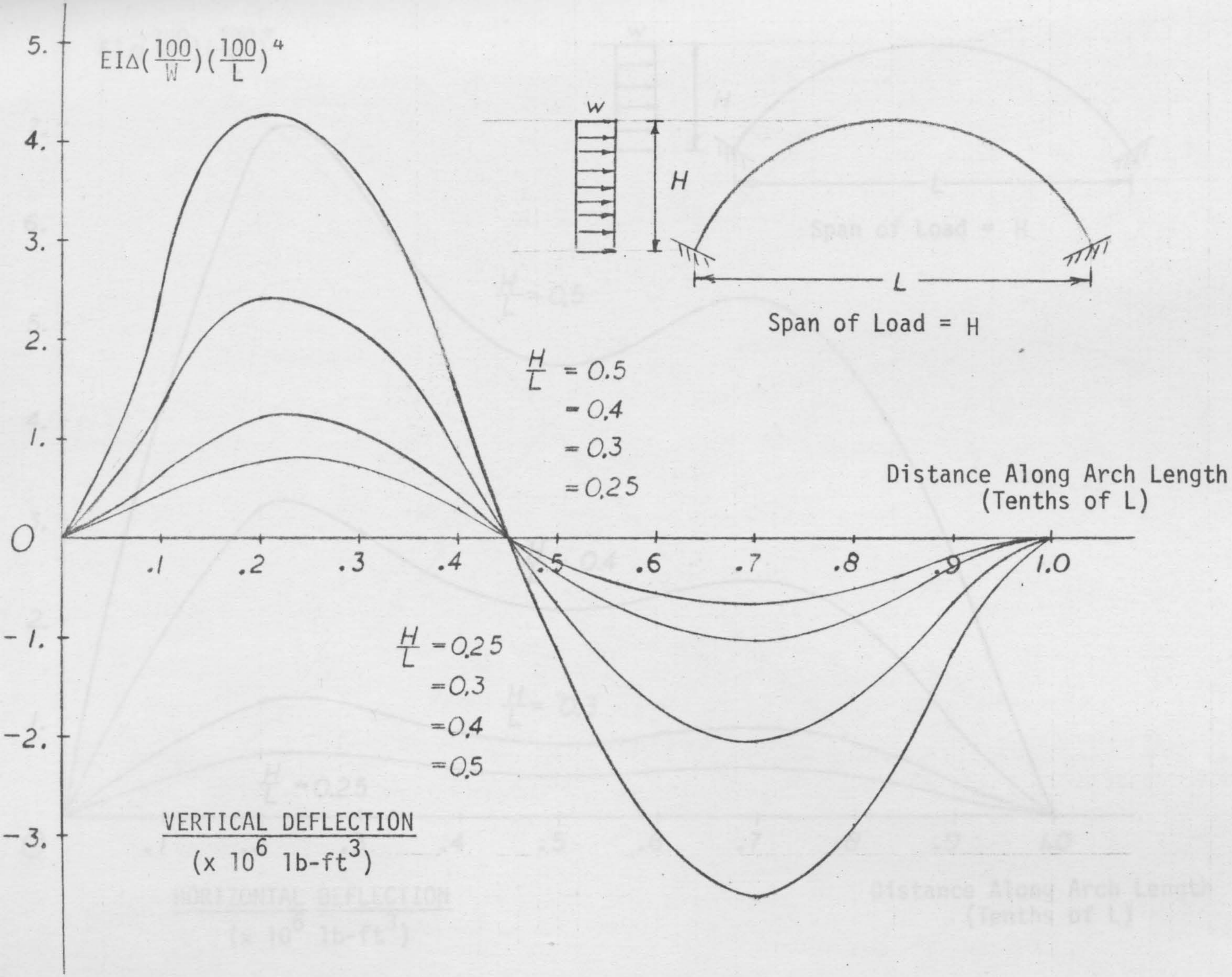


Fig. 3.45 Design chart for the vertical deflection for any point on the span of an arch with a span of load equal to the rise of the arch.

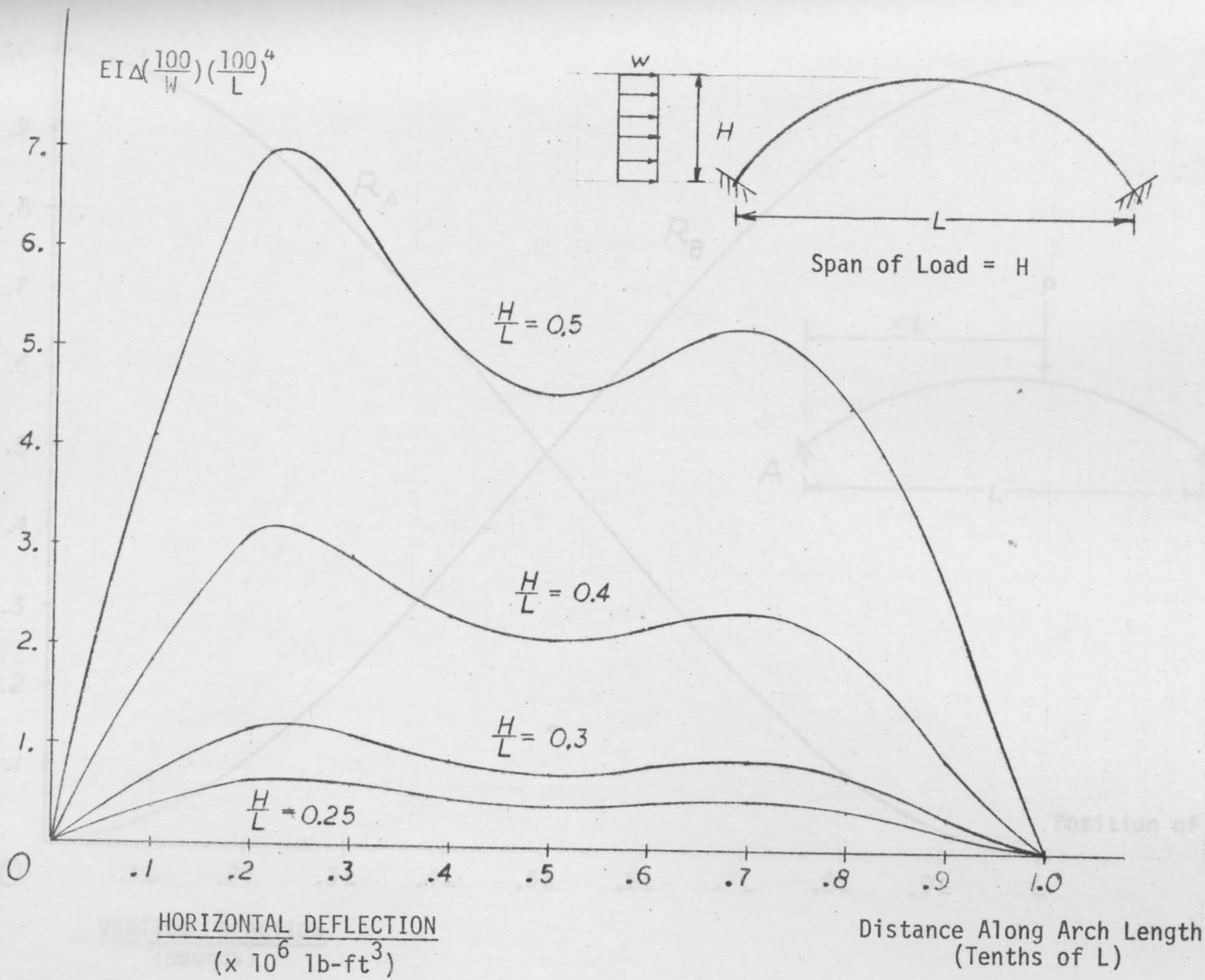
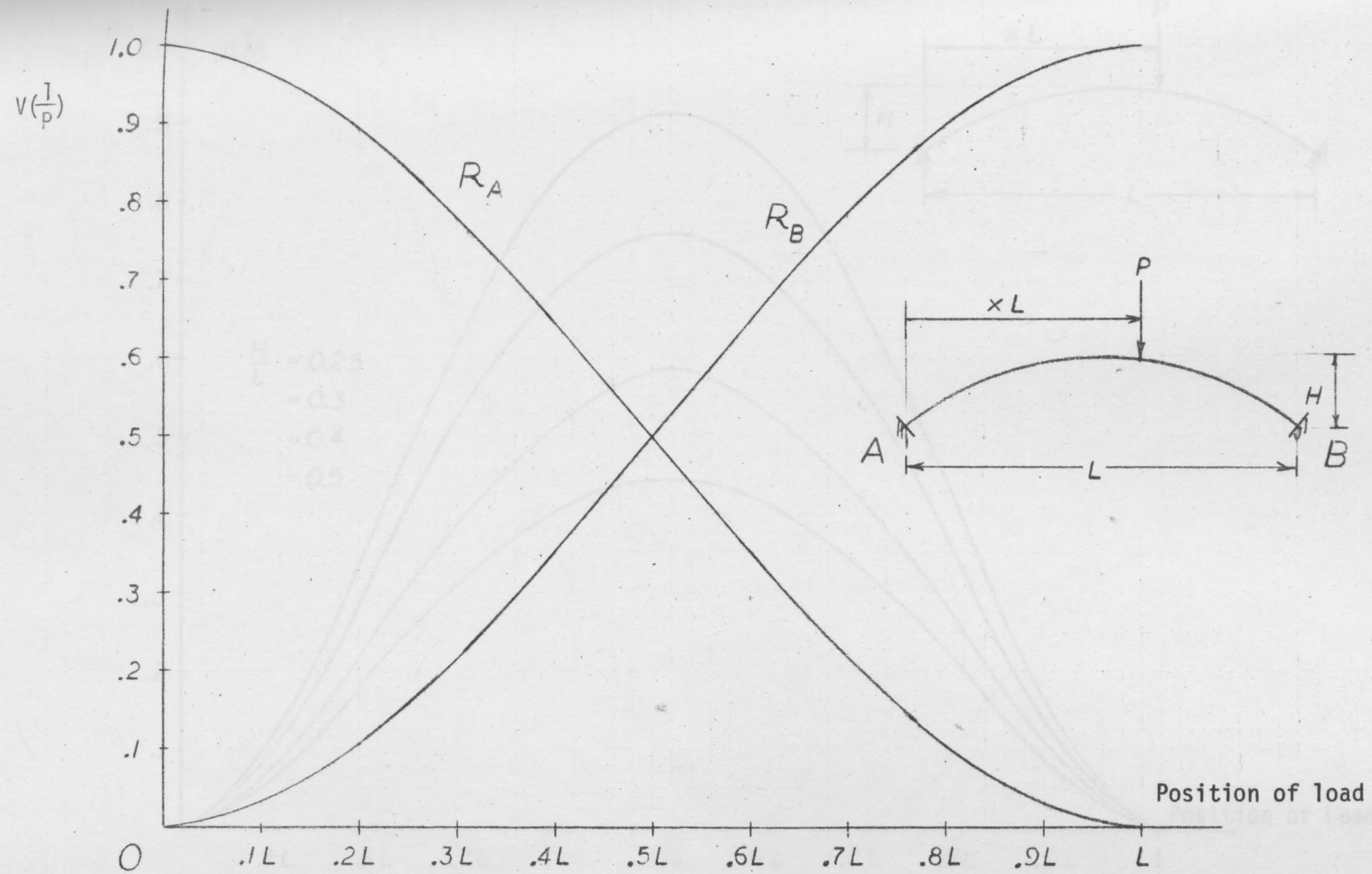


Fig. 3.46 Design chart for the horizontal deflection for any point on the span of an arch with a span of load equal to the rise of the arch.



VERTICAL REACTION
(pounds)

Fig. 3.47 Design chart for the vertical reactions for any arch.
*Note: Positive values indicate upward direction.

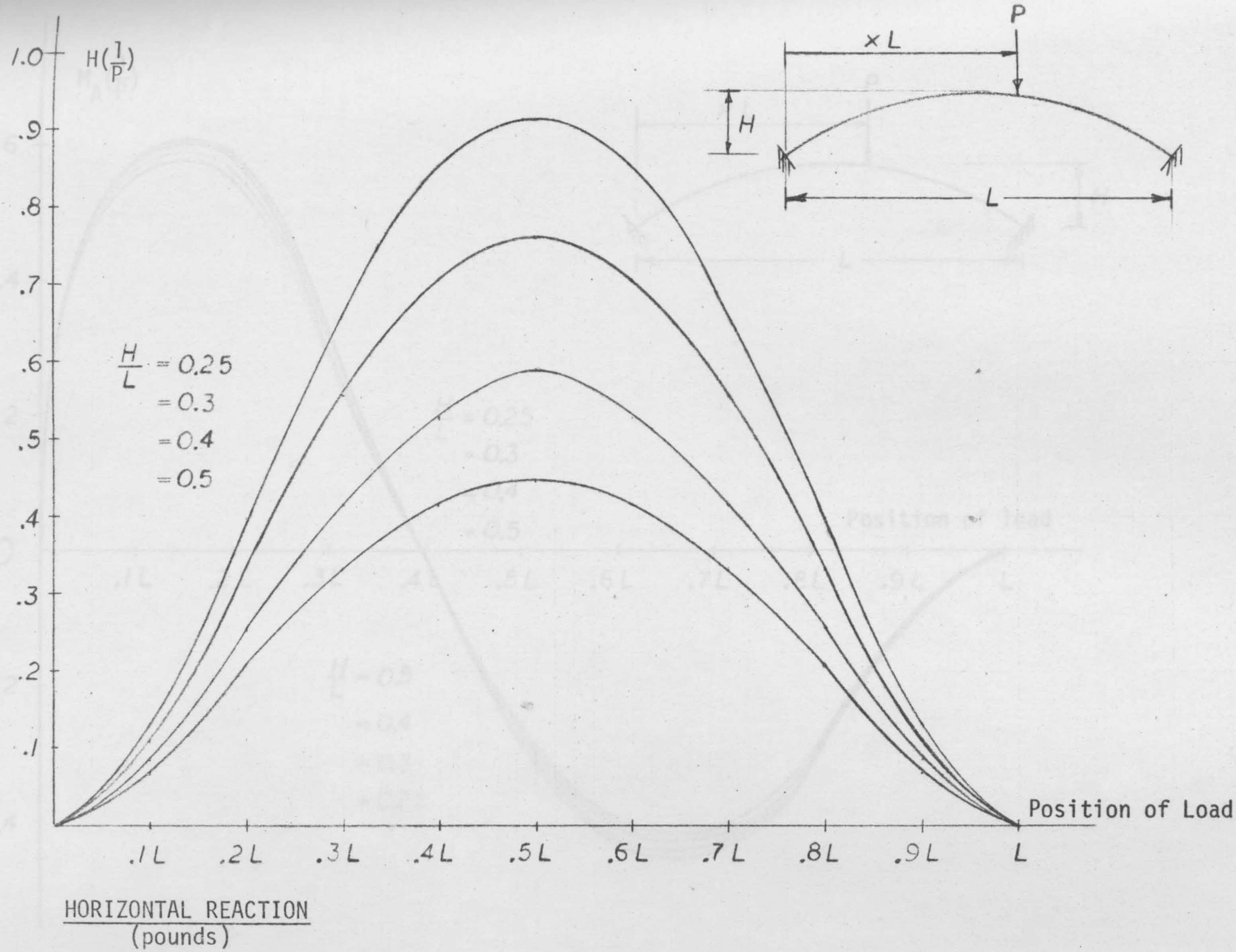


Fig. 3.48 Design chart for the horizontal reaction (at the left fixed-end) for any arch as a function of the height to span ratio of the arch.
*Note: Positive values indicate rightward direction.

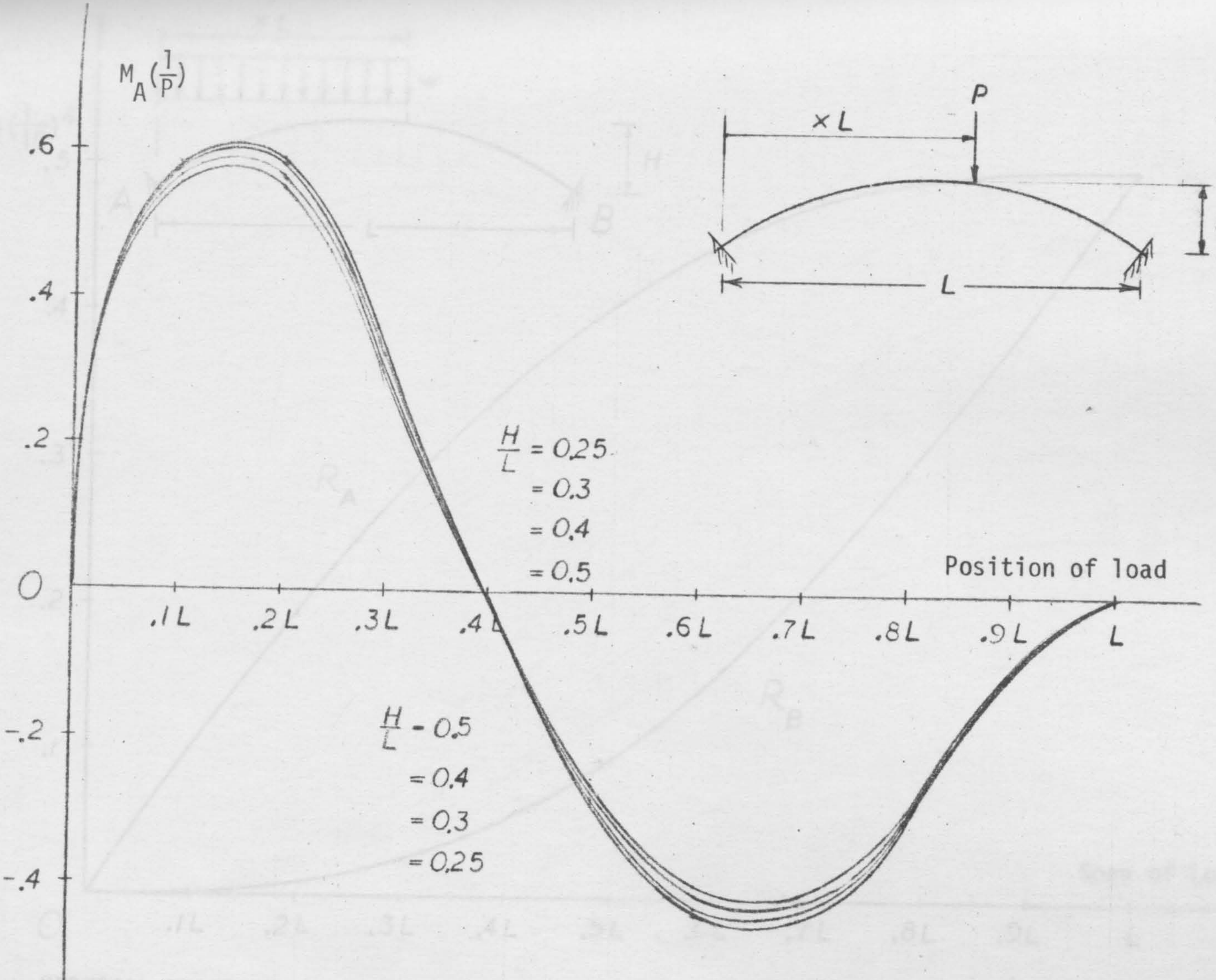


Fig. 3.49 Design chart for the left fixed-end moment for any arch as a function of the height to span ratio of the arch.
*Note: Positive values indicate anti-clockwise rotation.

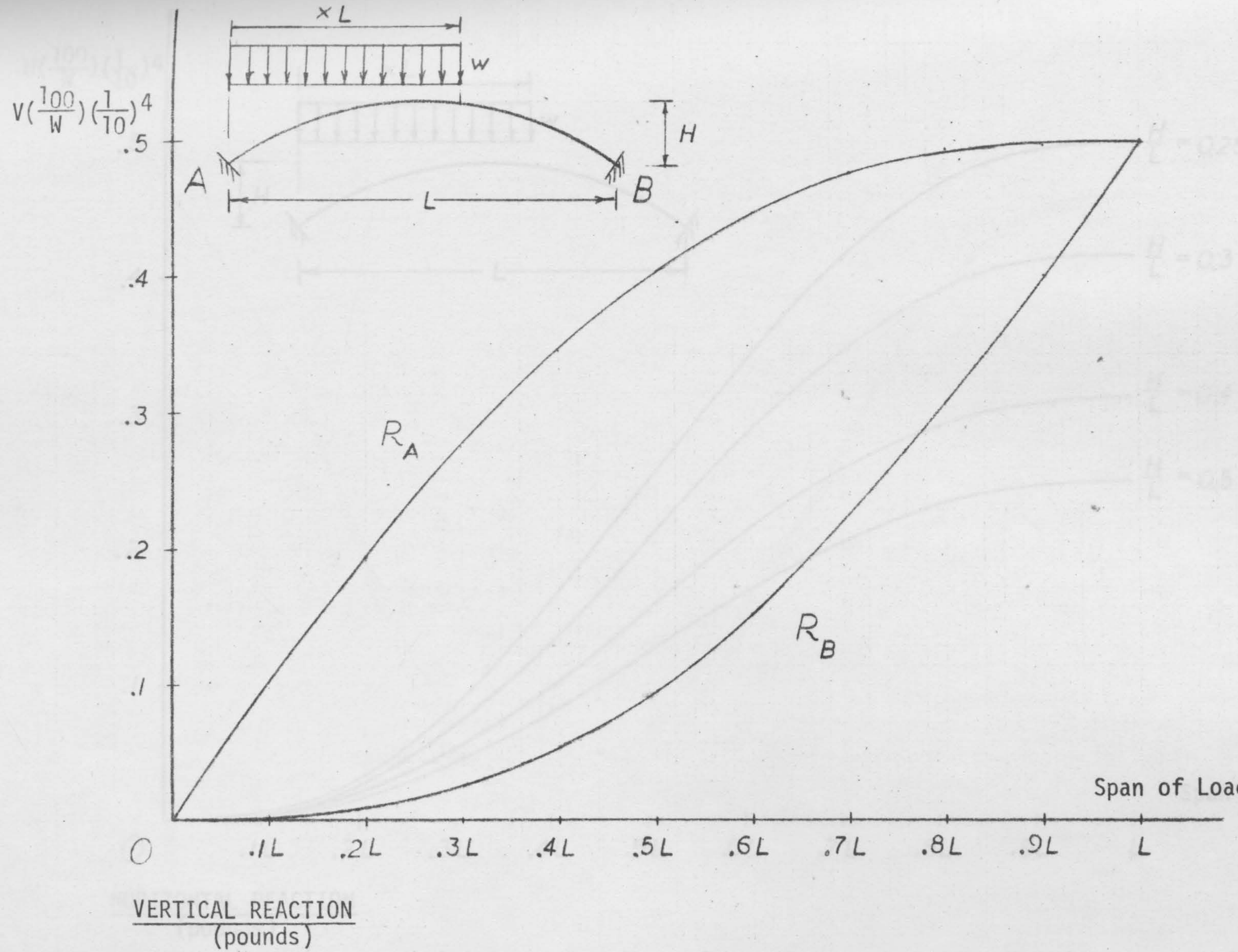


Fig. 3.50 Design chart for the vertical reactions for any arch with the span of load in tenths of the arch span.
*Note: Positive values indicate upward direction.

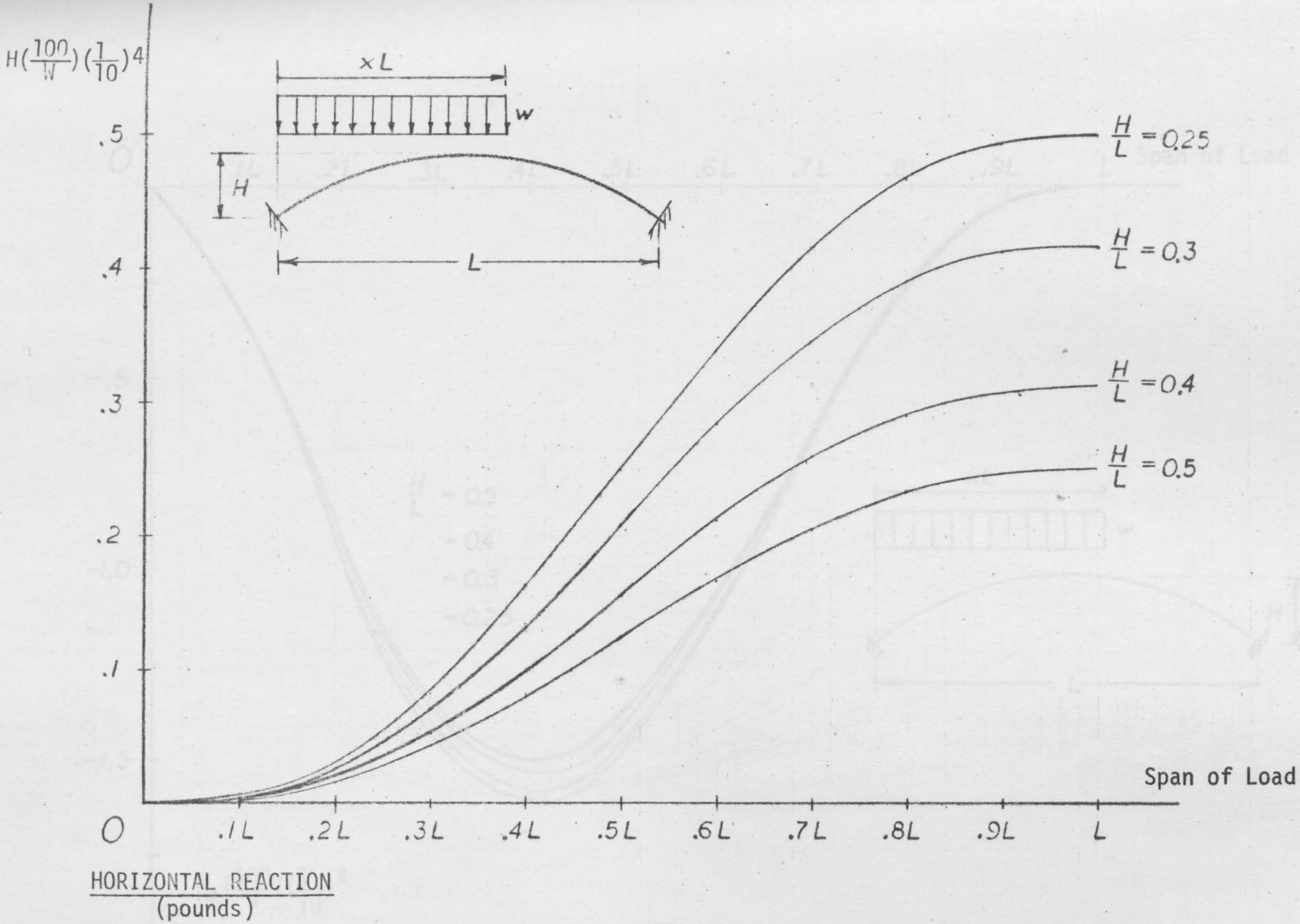


Fig. 3.51 Design chart for the horizontal reactions (at the left fixed-end) for any arch as a function of the height to span ratio of the arch and the span of load in tenths of the arch span.
*Note: Positive values indicate rightward direction.

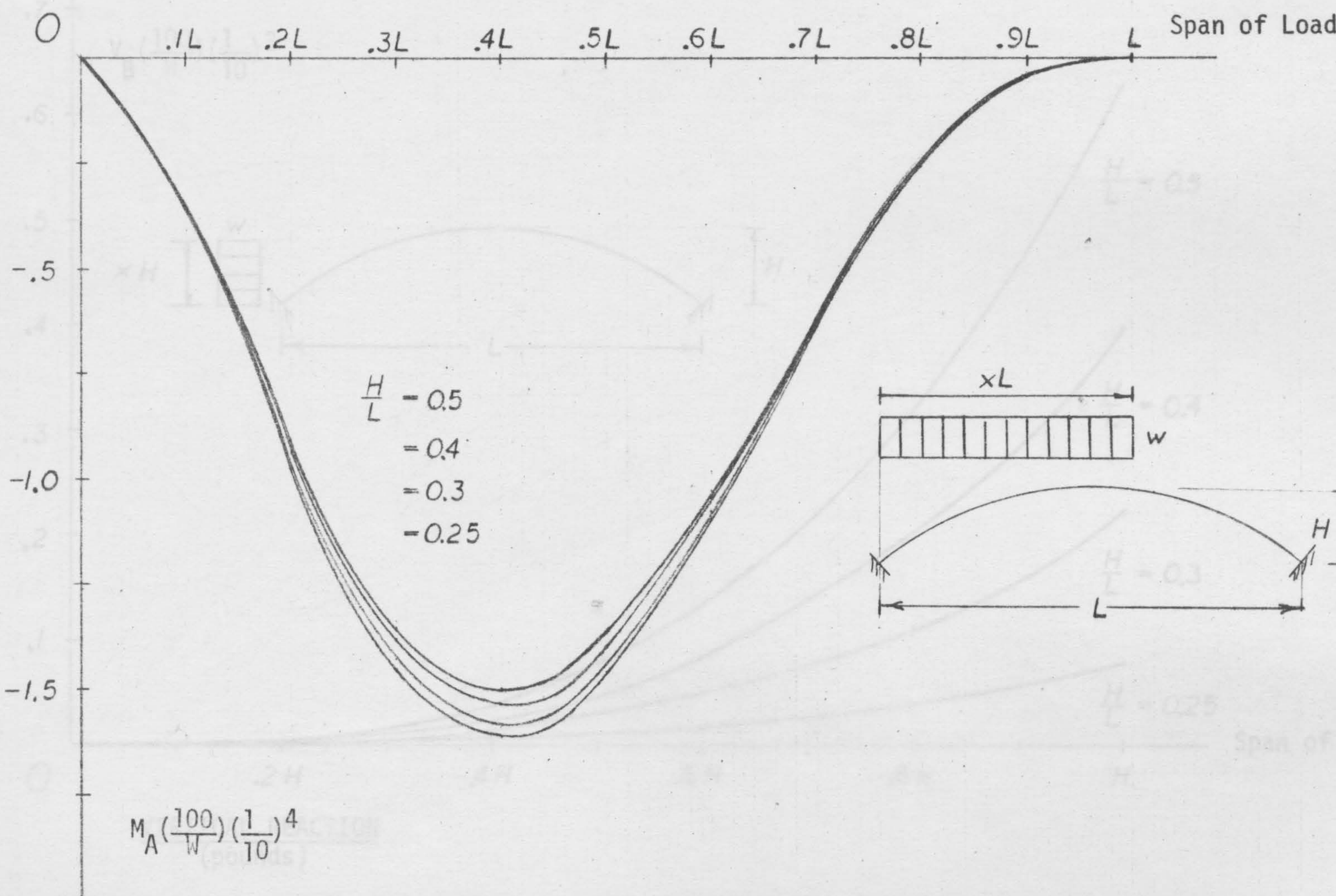


Fig. 3.52 Design chart for the left fixed-end moment for any arch as a function of the height to span ratio of the arch and the span of load in tenths of the arch span.
*Note: Negative values indicate anti clockwise rotation.

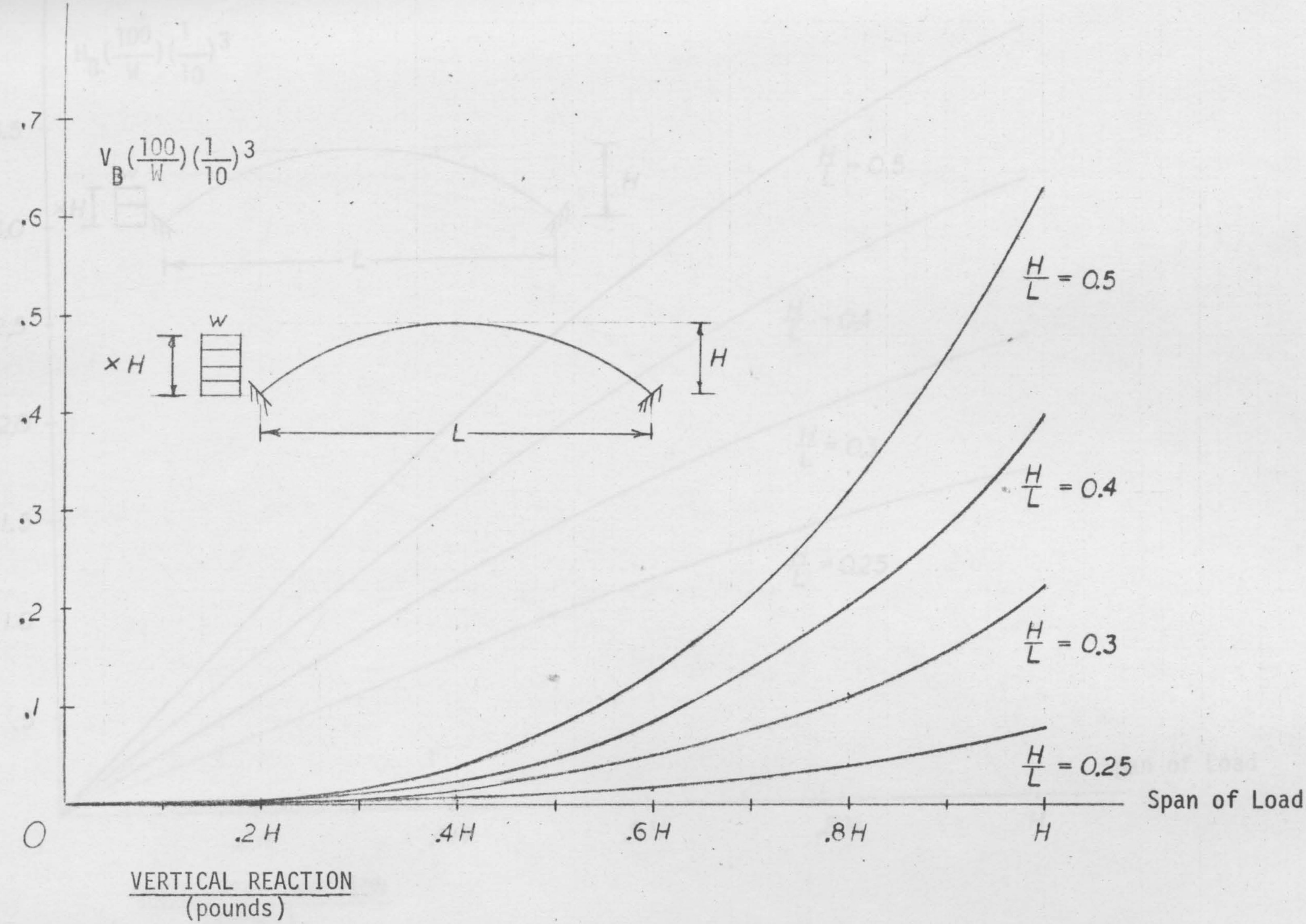


Fig. 3.53 Design chart for the right fixed-end vertical reaction for any arch as a function of the height to span ratio of the arch and the span of load in tenths of the arch rise.
*Note: Positive values indicate upward direction.

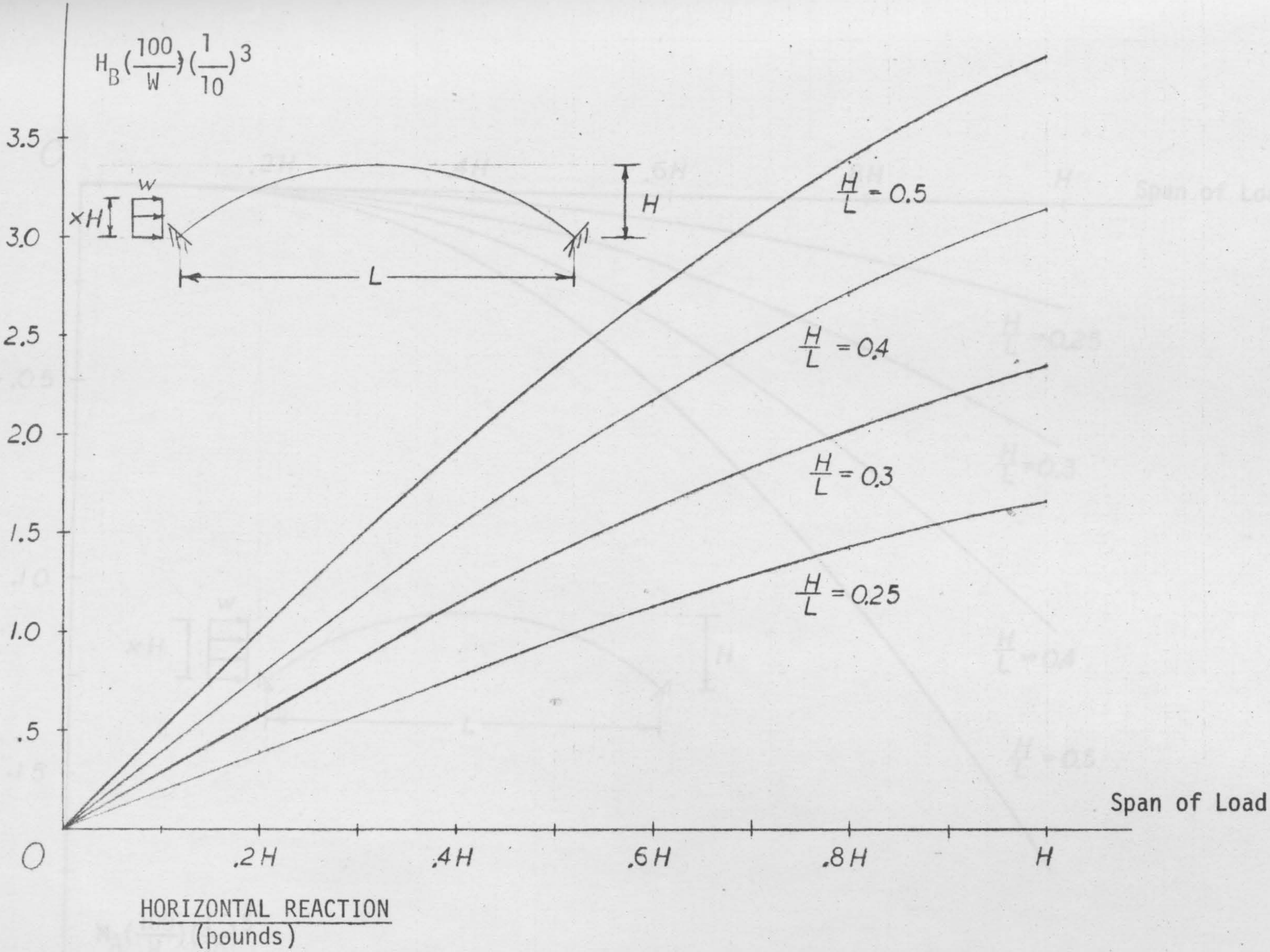


Fig. 3.54 Design chart for the right fixed-end horizontal reaction for any arch as a function of the height to span ratio of the arch and the span of load in tenths of the arch rise.
*Note: Positive values indicate leftward direction.

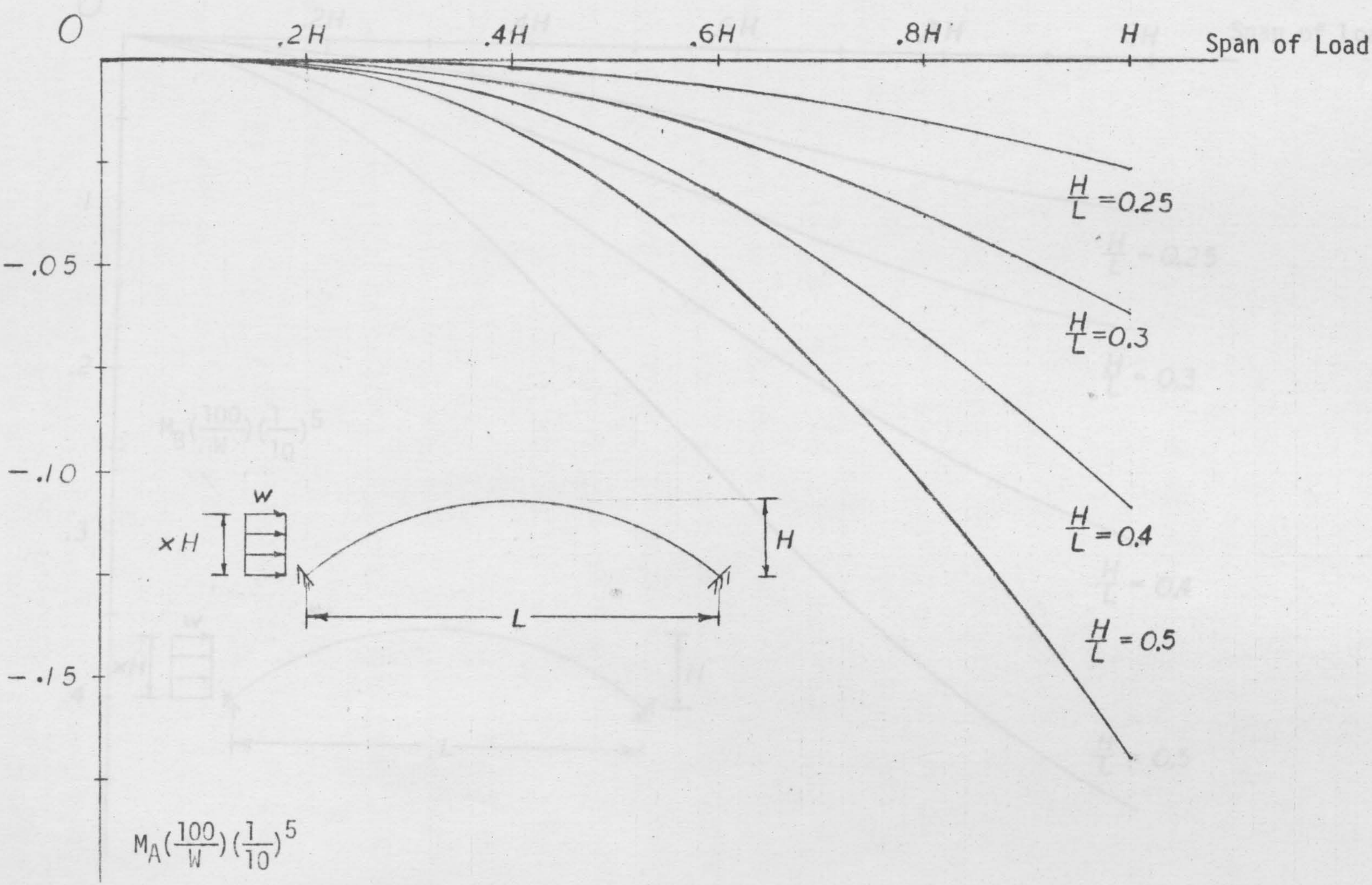


Fig. 3.55 Design chart for the left fixed-end moment for any arch as a function of the height to span ratio of the arch and the span of load in tenths of the arch rise.
*Note: Negative values indicate anti-clockwise rotation.

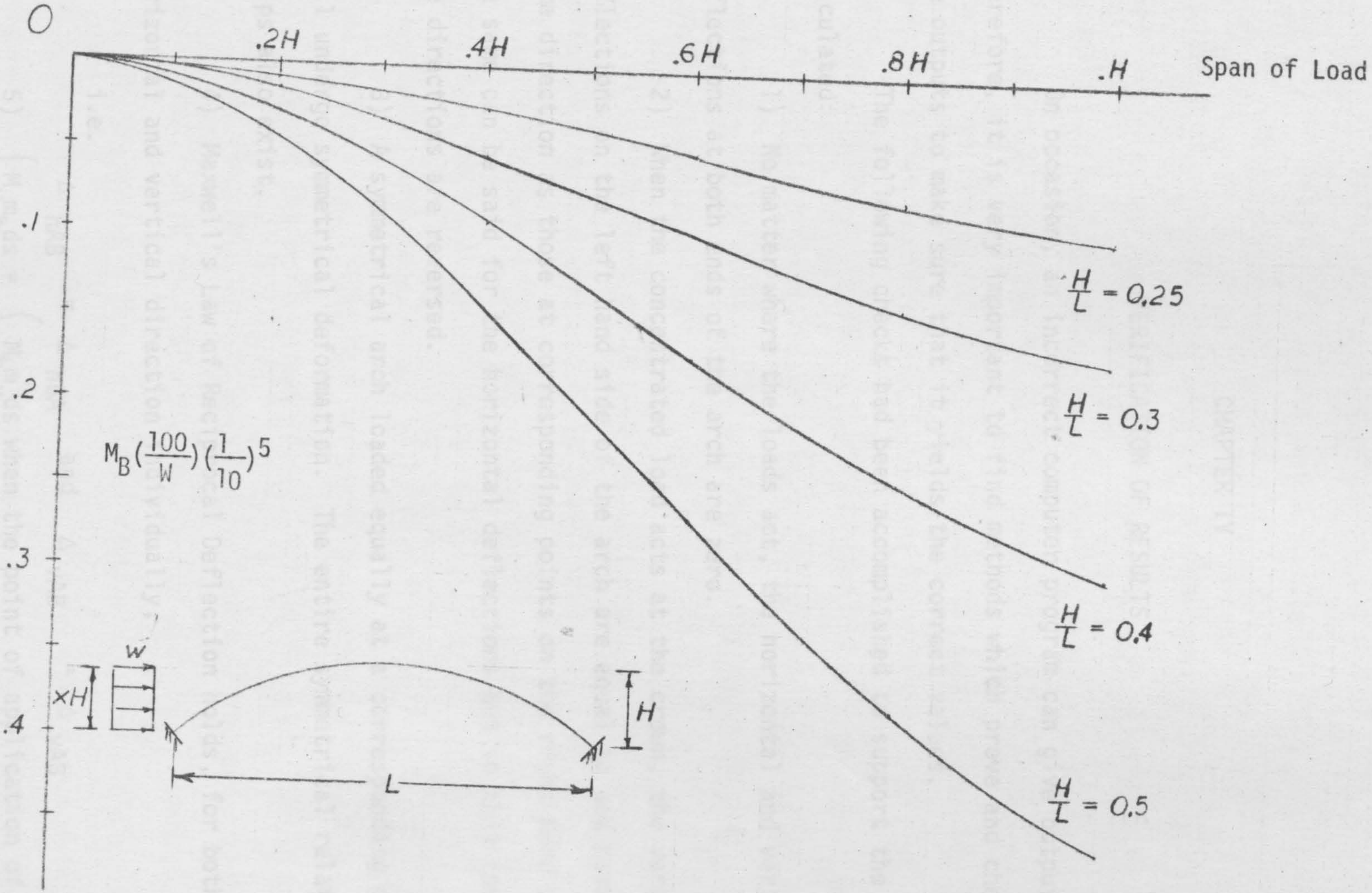


Fig. 3.56 Design chart for the right fixed-end moment for any arch as a function of the height to span ratio of the arch and the span of load in tenths of the arch rise.
*Note: Negative values indicate anti-clockwise rotation.

CHAPTER IV

VERIFICATION OF RESULTS

On occasion, an incorrect computer program can give output and, therefore, it is very important to find methods which prove and check the outputs to make sure that it yields the correct values.

The following checks had been accomplished to support the data calculated:

1) No matter where the loads act, the horizontal and vertical deflections at both ends of the arch are zero.

2) When the concentrated load acts at the crown, the vertical deflections on the left hand side of the arch are equal to and have the same direction as those at corresponding points on the right hand side. The same can be said for the horizontal deflections but in this instance, the directions are reversed.

3) A symmetrical arch loaded equally at a corresponding point will undergo symmetrical deformation. The entire symmetrical relationships also exist.

4) Maxwell's Law of Reciprocal Deflection holds, for both horizontal and vertical direction individually,

i.e. $\Delta_{hAB} = \Delta_{hBA}$ and $\Delta_{vAB} = \Delta_{vAB}$

5) $\int M_v m_h ds = \int M_h m_v ds$ when the point of application of the vertical load is at A, the horizontal deflection at B will come out equal in magnitude to the vertical deflection at A when the point of application of horizontal load is at B.

6) No deflection is obtained when the arch is loaded with a vertical uniformly distributed load over the entire span.

7) Another method used to check the results was to let the rise of the arch equal zero. The arch will then become an ordinary fixed-end beam. Formulas are available for the end reactions of such beams and, therefore, a convenient check was available. For flat arches, however, the axial energy must also be considered and containly an arch with a rise equal to zero qualifies as a flat arch. Appendix A presents an analysis of flat arches, i.e. $H/L < .2$ and the results were confirmed using the comparison between a fixed-end beam and an arch with a zero rise.

8) Leon Tovic (See Bibliography) has tabulated reactions for fixed-end parabolic arches. The computer results compared favorably with these tabulations.

9) The final check used was to assume that the results obtained from the action of a vertical concentrated load are right. Then using the results, influence lines were constructed and then used to check the results from the vertical uniformed load. For vertification purposes the midpoint of an arch (span length = 100 feet, rise = 25 feet) was selected as the test point. Influence lines were constructed using the design charts given in Chapter 3 (shown in Fig. 4.1 and Fig. 4.2) and with the aid of polar planimeter, it is possible to find the area under the influence lines. This results were then checked with those obtained for the numberical analysis for the case of vertical uniformly distributed load, since the area under the influence line between any two points multiplied by the intensity of the given uniform load will produce the same deflection as that by a uniform load acting between the same two points. The results of this comparison are shown in Table 4.1.

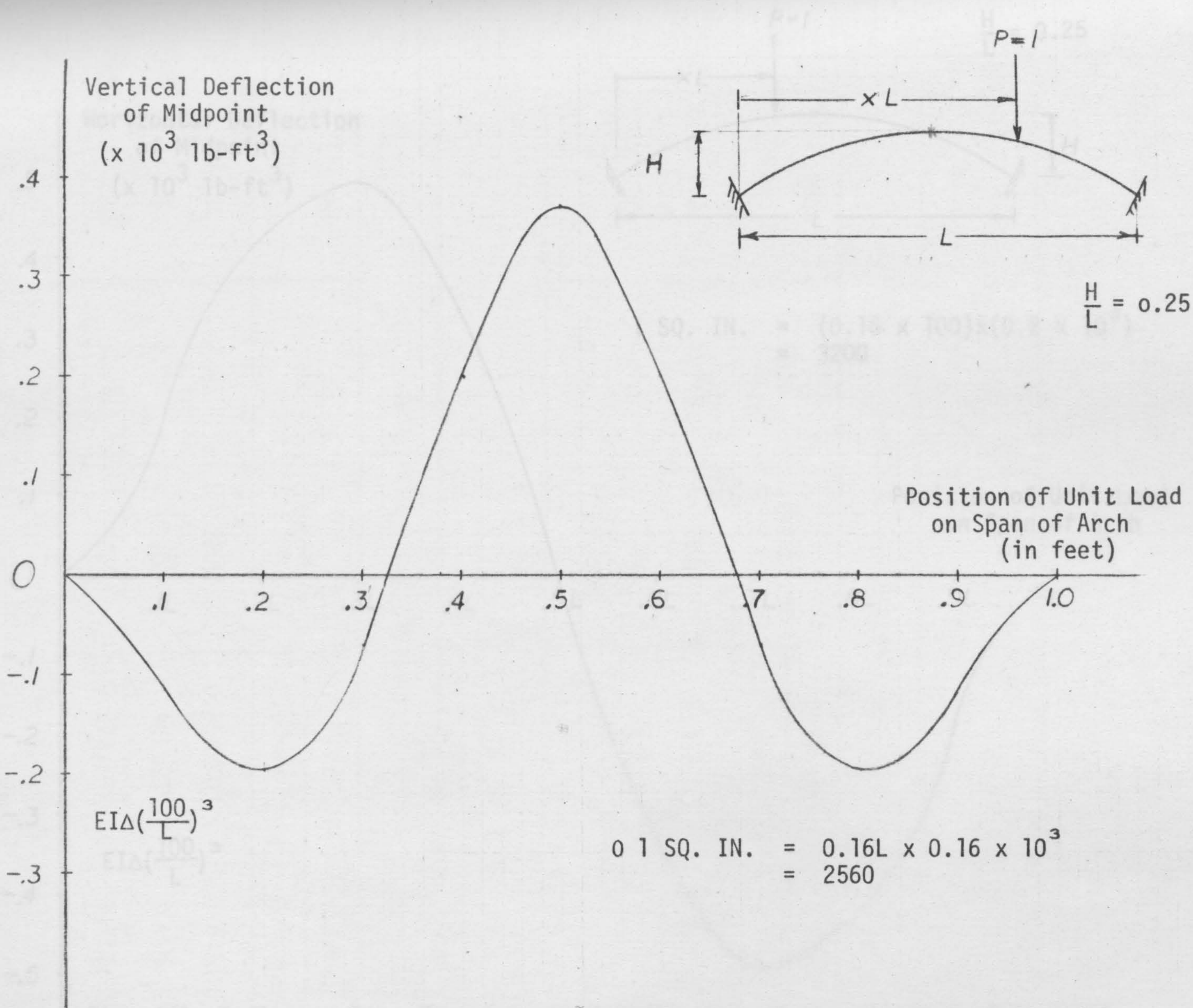


Fig. 4.1 Influence line for vertical deflection of the midpoint of the arch.

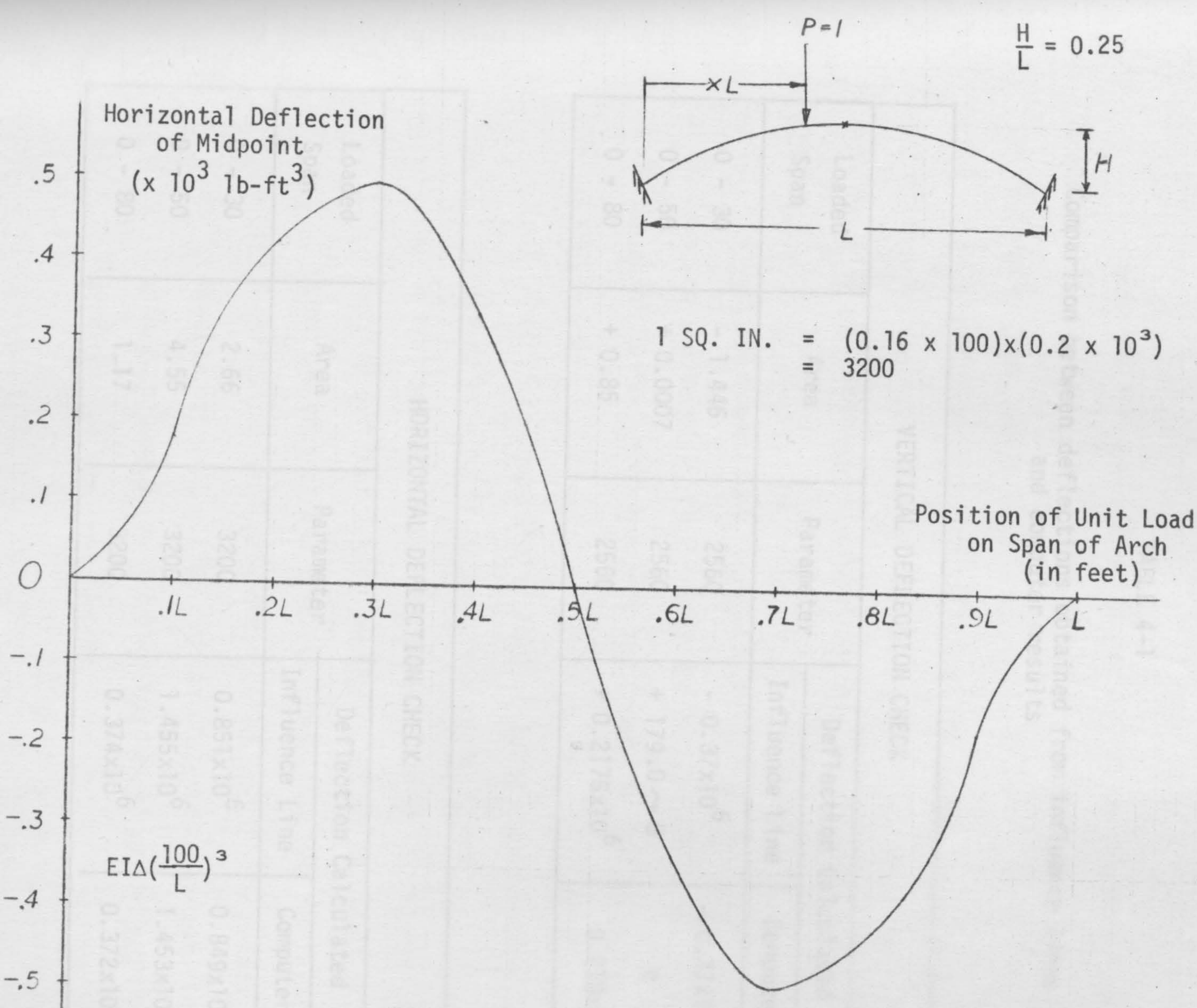


Fig. 4.2 Influence line for horizontal deflection of the midpoint of the arch.

TABLE 4-1

Comparison between deflections obtained from influence lines and computer results

VERTICAL DEFLECTION CHECK				
Loaded Span	Area	Parameter	Deflection Calculated	
			Influence Line	Computer
0 - 30	- 1.446	2560	$- 0.37\times10^6$	$- 0.37\times10^6$
0 - 50	+ 0.0007	2560	+ 179.0 ∼ 0	0
0 - 80	+ 0.85	2560	$+ 0.2175\times10^6$	0.218×10^6

HORIZONTAL DEFLECTION CHECK				
Loaded Span	Area	Parameter	Deflection Calculated	
			Influence Line	Computer
0 - 30	2.66	3200	0.851×10^6	0.849×10^6
0 - 50	4.55	3200	1.455×10^6	1.453×10^6
0 - 80	1.17	3200	0.374×10^6	0.372×10^6

CHAPTER V

APPLICATION

Under the assumptions in Chapter II, it can be shown that the preceeding design charts for the symmetrical, parabolic, fixed-end arch can be used without limits.

To show the application of the design charts in finding the deflections of the desired point in arbitrary arches, the following problems will be presented as demonstration:

Problem 1.

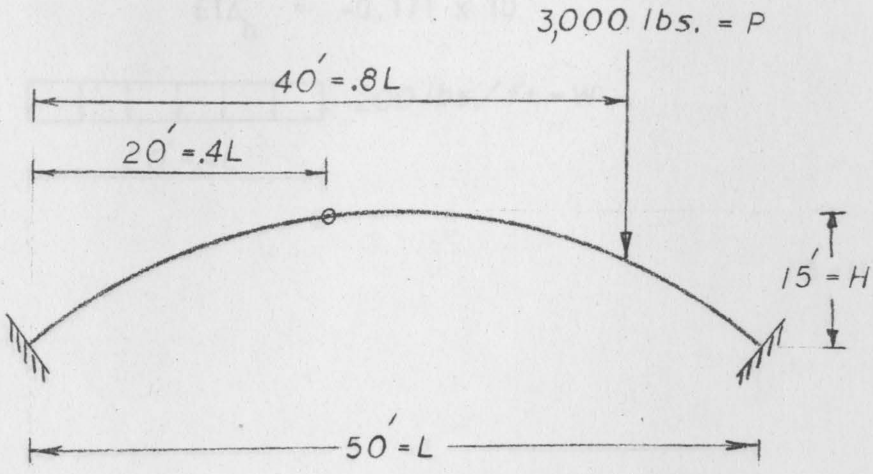
Given. A symmetrical, parabolic, fixed-end arch with a span of 50 feet and a rise of 15 feet. Find the deflections in terms of EI of the point 20 feet from the left end under the following individual loading conditions:

a) A vertical concentrated load of 3,000 pounds intensity acting at a point 40 feet from the left end;

b) A vertical uniformly distributed load of 200 pounds intensity per foot acting downward with a span equal to 0.4 time the span of the arch beginning from the left end;

c) A horizontal uniformly distributed load of 100 pounds intensity per foot acting rightward at the half-left side with a span equal to 0.2 time the height of the arch beginning from the left support.

a)



Solution: In this case we have

$$\frac{H}{L} = \frac{15}{50} = 0.3$$

$$\left(\frac{100}{L}\right)^3 = \left(\frac{100}{50}\right)^3 = 8$$

$$\frac{1000}{P} = \frac{1000}{3000} = \frac{1}{3}$$

(i) For vertical deflection:

From the preceeding chapter, design chart Fig. 3.15, the reading point on the horizontal axis is $\frac{20}{50} = 0.4L$ and the value is

$$EI\Delta v\left(\frac{1000}{P}\right)\left(\frac{100}{L}\right)^3 = -0.48 \times 10^6$$

Therefore

$$EI\Delta_V = \frac{-0.48 \times 10^6 \times 3}{8} = -0.18 \times 10^6$$

(ii) For horizontal deflection:

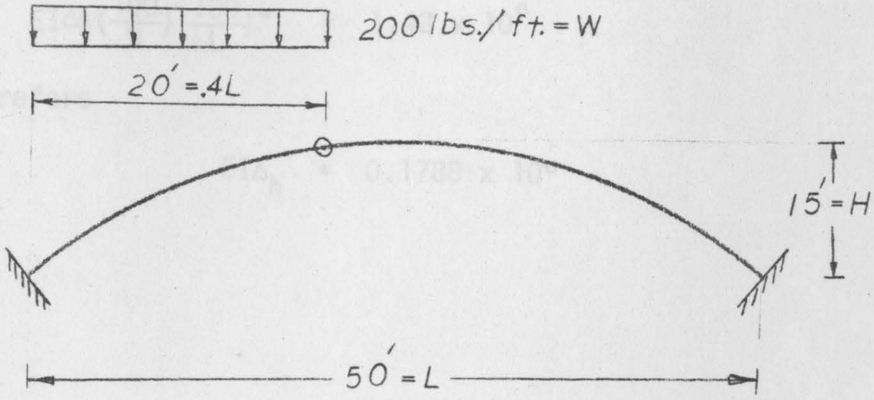
From design chart Fig. 3.16:

$$EI\Delta_h\left(\frac{1000}{P}\right)\left(\frac{100}{L}\right)^3 = -0.455 \times 10^6$$

Therefore

$$EI\Delta_h = -0.171 \times 10^6$$

b)



Solution: In this case we have

$$\frac{H}{L} = \frac{15}{50} = 0.3$$

$$(\frac{100}{L})^4 = (\frac{100}{50})^4 = 16$$

$$\frac{100}{W} = \frac{100}{200} = \frac{1}{2}$$

(i) For vertical deflection:

From design chart Fig. 3.25, on the horizontal axis $= \frac{20}{50} = 0.4L$;

the value reads

$$EI\Delta v(\frac{100}{W})(\frac{100}{L})^4 = 1.5 \times 10^6$$

Therefore

$$EI\Delta_v = 0.1875 \times 10^6$$

(ii) For horizontal deflection:

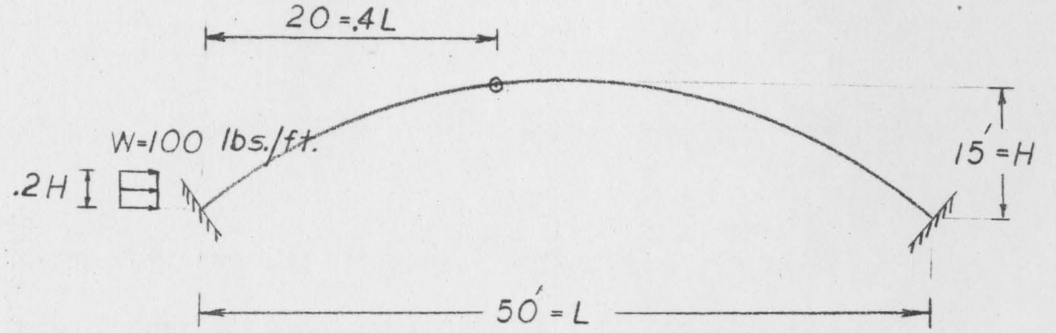
From design chart Fig. 3.26, at 0.4L

$$EI\Delta_h(\frac{100}{W})(\frac{100}{L})^4 = 1.43 \times 10^6$$

Therefore

$$EI\Delta_h = 0.1788 \times 10^6$$

c)



Solution: We have

$$\frac{H}{L} = \frac{15}{50} = 0.3$$

$$(\frac{100}{L})^4 = (\frac{100}{50})^4 = 16$$

$$\frac{100}{W} = \frac{100}{100} = 1$$

(i) For vertical deflection:

From design chart Fig. 3.37, on the horizontal axis = $\frac{20}{50}$ = .4L, the value reads

$$EI\Delta_v(\frac{100}{W})(\frac{100}{L})^4 = 0$$

Therefore

$$EI\Delta_v = 0$$

(ii) For horizontal deflection:

From design chart Fig. 3.38 at 0.4L

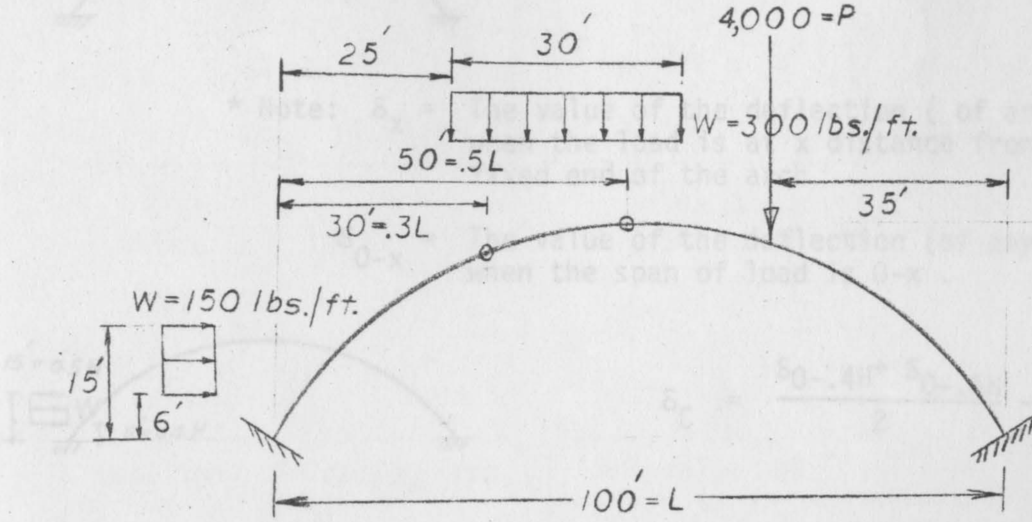
$$EI\Delta_h\left(\frac{100}{W}\right)\left(\frac{100}{L}\right)^4 = 0.015 \times 10^6$$

Therefore

$$EI\Delta_h = 937$$

Problem 2.

Given: A symmetrical, parabolic, fixed-end arch with a span of 100 feet and a rise of 30 feet. The loads acting on the arch are shown in the figure below.



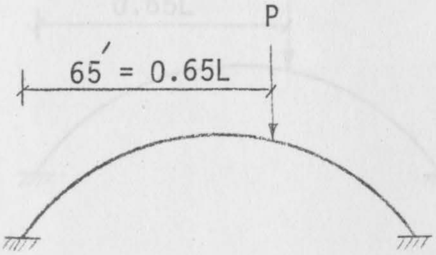
Find the vertical and horizontal deflections, in terms of EI for points 30 feet measured horizontally from the left end and at the crown.

Solution:

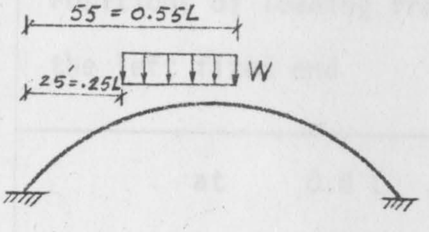
To solve this type of problem requires the principle of superposition and an interpolation of values between the charts. The method in general can be simplified thus:

(A) (B) (C)

And



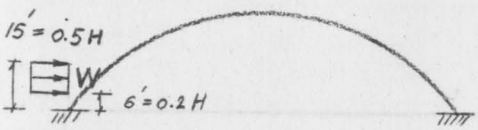
$$\delta_A = \frac{\delta_{0.6L} + \delta_{0.7L}}{2}$$



$$\delta_B = \frac{\delta_{0-.5L} + \delta_{0-.6L}}{2} - \frac{\delta_{0-.2L} + \delta_{0-.3L}}{2}$$

* Note: δ_x = The value of the deflection (of any point) when the load is at x distance from the left fixed end of the arch.

Hence δ_{0-x} = The value of the deflection (of any point) when the span of load is 0-x .



$$\delta_C = \frac{\delta_{0-.4H} + \delta_{0-.6H}}{2} - \delta_{0-.2H}$$

Hence the result will be

$$\delta = \delta_A + \delta_B + \delta_C$$

from the given, we have

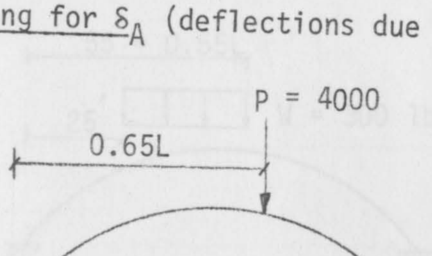
$$\frac{H}{L} = \frac{30}{100} = 0.3$$

and the positions of the determined points are

$$\frac{30}{100} = 0.3L$$

and 0.5L horizontally

Solving for δ_A (deflections due to vertical concentrated load)



$$\left(\frac{100}{L}\right)^3 = \left(\frac{100}{100}\right)^3 = 1$$

$$\frac{1000}{P} = \frac{1000}{4000} = \frac{1}{4}$$

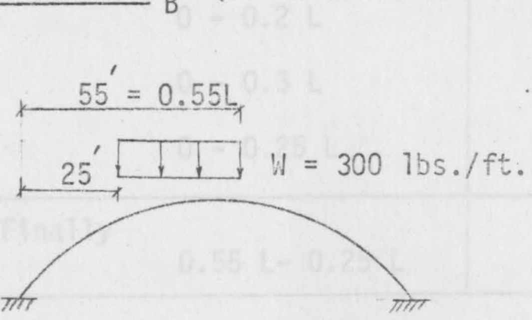
Positions of loading from the left fixed end	The value of $EI\Delta_v(\frac{1000}{P})(\frac{100}{L})^3$ at	
	0.3 L (horizontally)	0.5 L (horizontally)
at 0.6 L	-0.495×10^6	$.21 \times 10^6$
at 0.7 L	-0.677×10^6	-0.07×10^6
at 0.65 L	-0.586×10^6	0.07×10^6

			at 0.3 L	at 0.5 L
Hence	$EI\Delta_v$	=	$-0.586 \times 10^6 \times 4$	$0.07 \times 10^6 \times 4$
		=	-2.344×10^6	0.28×10^6

Positions of loading from the left fixed end	The value of $EI\Delta_h(\frac{1000}{P})(\frac{100}{L})^3$ at	
	0.3 L (horizontally)	0.5 L (horizontally)
at 0.6 L	-0.47×10^6	-0.335×10^6
at 0.7 L	-0.60×10^6	-0.50×10^6
at 0.65 L	-0.535×10^6	-0.4175×10^6

			at 0.3 L	at 0.5 L
Hence	$EI\Delta_h$	=	$-0.535 \times 10^6 \times 4$	$-0.4175 \times 10^6 \times 4$
		=	-2.14×10^6	-1.67×10^6

<u>Solve for δ_B</u> (deflections due to vertical uniformly distributed load)



$$(\frac{100}{L})^4 = (\frac{100}{100})^4 = 1$$

$$\frac{100}{W} = \frac{100}{300} = \frac{1}{3}$$

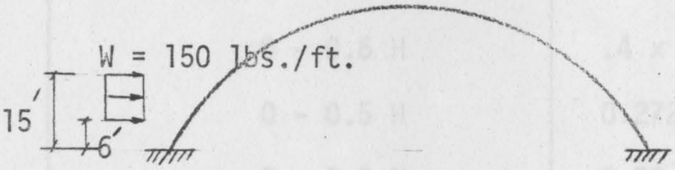
Span of loading (horizontally)	The value of $EI\Delta_v(\frac{100}{W})(\frac{100}{L})^4$ at	
	0.3 L (horizontally)	0.5 L (horizontally)
0 - .50 L	2.0×10^6	0
0 - .60 L	$.172 \times 10^6$	0.033×10^6
0 - .55 L	1.086×10^6	0.0165×10^6
0 - .20 L	0.475×10^6	-0.22×10^6
0 - .30 L	1.2×10^6	-0.39×10^6
0 - 0.25 L	0.8375×10^6	-0.305×10^6
Finally 0.55 L- 0.25 L	0.2485×10^6	0.3215×10^6

			at 0.3 L	at 0.5 L
Hence	$EI\Delta_v$	=	$0.2485 \times 10^6 \times 3$	$0.3215 \times 10^6 \times 3$
		=	0.7455×10^6	0.9645×10^6

Span of loading (horizontally)	The value of $EI\Delta_h(\frac{100}{W})(\frac{100}{L})^4$ at	
	0.3 L (horizontally)	0.5 L (horizontally)
0 - 0.5 L	1.83×10^6	1.46×10^6
0 - 0.6 L	1.2×10^6	1.3×10^6
0 - 0.55 L	1.515×10^6	1.38×10^6
0 - 0.2 L	0.535×10^6	0.37×10^6
0 - 0.3 L	0.095×10^6	0.086×10^6
0 - 0.25 L	0.315×10^6	0.228×10^6
Finally 0.55 L- 0.25 L	1.2×10^6	1.152×10^6

			at .3 L	at 0.5 L
Hence	$EI\Delta_h$	=	$1.2 \times 10^6 \times 3$	$1.152 \times 10^6 \times 3$
		=	3.6×10^6	3.456×10^6

Solving for δ_c (deflections due to horizontal uniformly distributed load)



$$\left(\frac{100}{L}\right)^4 = \left(\frac{100}{100}\right)^4 = 1$$

$$\frac{100}{W} = \frac{100}{150} = \frac{2}{3}$$

Span of loading (vertically)	The value of $EI\Delta_V\left(\frac{100}{W}\right)\left(\frac{100}{L}\right)^4$ at 0.3 L (horizontally)	0.5 L (horizontally)
0 - 0.4 H	$.12 \times 10^6$	$-.07 \times 10^6$
0 - 0.6 H	0.35×10^6	-0.19×10^6
0 - 0.5 H	0.235×10^6	-0.13×10^6
0 - 0.2 H	$.0009 \times 10^6$	$-.011 \times 10^6$
Finally 0.5 H - 0.2 H	0.226×10^6	-0.119×10^6

			at 0.3 L	at 0.5 L
Hence	$EI\Delta_V$	=	$0.226 \times 10^6 \times \frac{3}{2}$	$-0.119 \times 10^6 \times \frac{3}{2}$
		=	0.339×10^6	-0.1785×10^6

Span of loading (vertically)	The value of EI $(\frac{100}{W})(\frac{100}{L})$ at 0.3 L (horizontally)	0.5 L (horizontally)
0 - 0.4 H	$.145 \times 10^6$	$.1 \times 10^6$
0 - 0.6 H	$.4 \times 10^6$	$.28 \times 10^6$
0 - 0.5 H	0.2725×10^6	$.19 \times 10^6$
0 - 0.2 H	0.02×10^6	$.014 \times 10^6$
Finally 0.5 H - 0.2 H	0.2525×10^6	0.176×10^6

			at 0.3 L	at 0.5 L
Hence	$EI\Delta_h$	=	$0.2525 \times 10^6 \times \frac{3}{2}$	$0.176 \times 10^6 \times \frac{3}{2}$
		=	0.3788×10^6	0.264×10^6

From $\delta = \delta_A + \delta_B + \delta_C$

			at 0.3L	at 0.5L
i.e. total	$EI\Delta_v$	=	-2.344×10^6	0.28×10^6
			0.7455×10^6	0.9645×10^6
			0.339×10^6	-0.1785×10^6
		=	-1.2595×10^6	1.066×10^6
and total	$EI\Delta_h$	=	-2.14×10^6	1.67×10^6
			3.6×10^6	3.456×10^6
			0.3788×10^6	0.264×10^6
		=	1.8388×10^6	2.05×10^6

The combination of the deflections will follow algebric rules.

The previous examples show how utilize the design charts to find deflections. The results of reactions and moments can be calculated (from the included charts in Chapter III) in a similar manner.

CHAPTER VI

CONCLUSIONS

With the use of all the preceeding charts, it is believed that the analysis of the symmetrical, parabolic, fixed-end arches can be easily accomplished. Other types of symmetrical fixed-end arches, e.g. circular arches, can be analyzed in the same manner by changing the equation of the arch axis in the computer programs.

It is possible to include two-hinged arch design charts with those of the fixed-end, and thus, the analysis of the symmetrical, parabolic arches will become more complete and effective. Finally, by the same procedure, it is hoped that in the future some other types of arches, symmetrical and unsymmetrical, can also be simplified and represented in diagrams or charts, which in turn clarify the analysis of arches.

It is hoped that these design charts can be made available to the practicing engineer to aid in his designs. Secondary moments set up in the arch by the arch deflecting away from the thrust line can be significant when the thrust line lies close to the arch axis. The charts presented in this study will enable the design to consider these secondary moments since the deformed shape of the arch can be determined.

APPENDIX A

In the analysis of the flat arches, ($\frac{H}{L} \leq 0.2$) the effect of axial deformation becomes substantial and the energy method used previously must be modified accordingly. The basic procedure remains essentially the same with the deflection expression taking the following form;

$$\Delta = \int \frac{Mmds}{EI} + \int \frac{Nnds}{EA}$$

where

N represents the axial force of the desired point in the member caused by the real loads;

n represents the axial force of the desired point in the member caused by the dummy unit load.

A represent the cross-section of the arch member at the desired point.

The design charts are constructed by keeping the height-to-span ratio constant and varying the values of R,

where R = square of radius of gyration of the cross section of the arch

$$= \frac{I}{A}$$

hence $$EI\Delta = \int Mmds + \int (NnR)\, ds$$

The following design charts show the values of the deflections due to a vertical applied concentrated unit load.

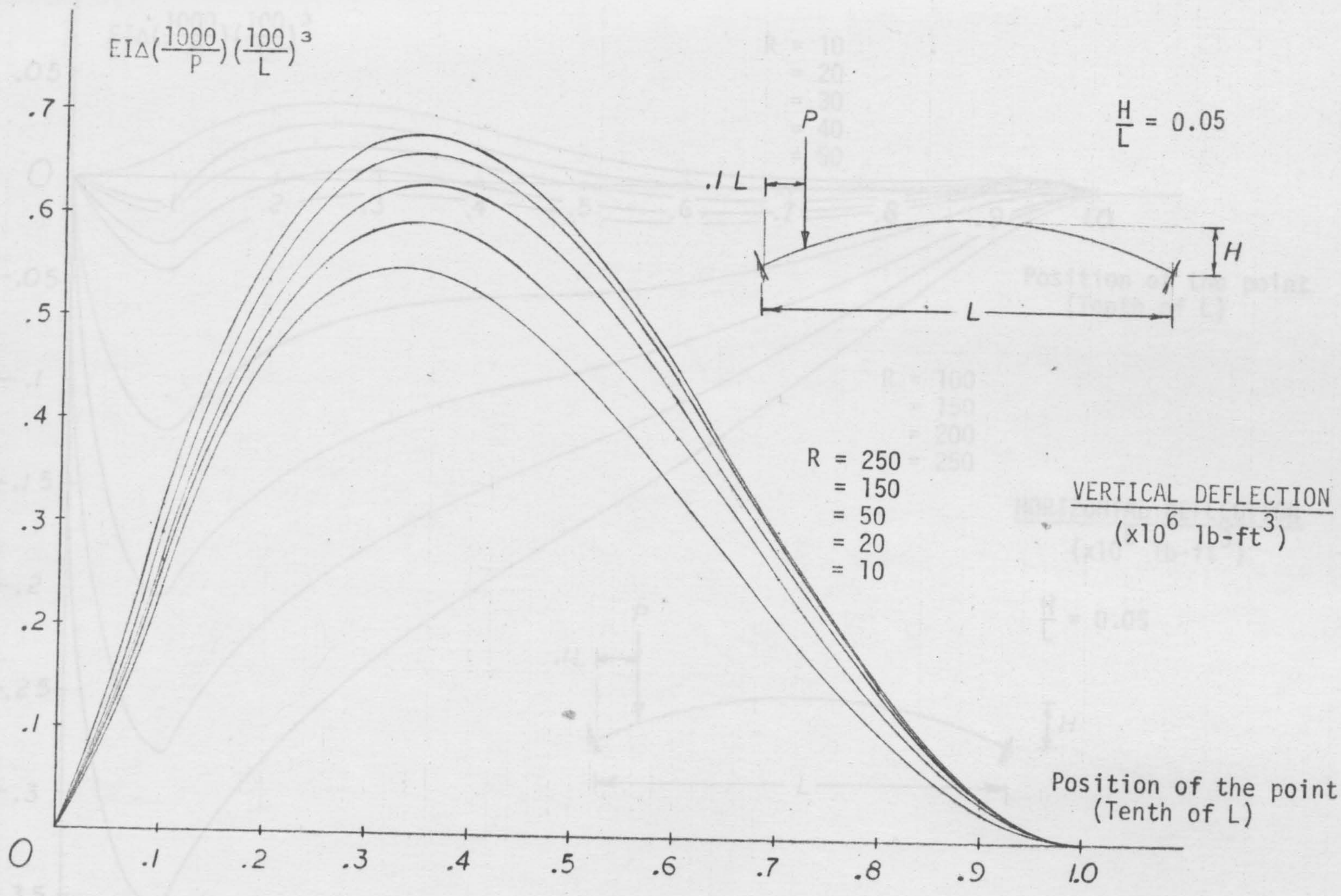


Fig. A.1 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.05) with the position of load at .1 the span of the arch.

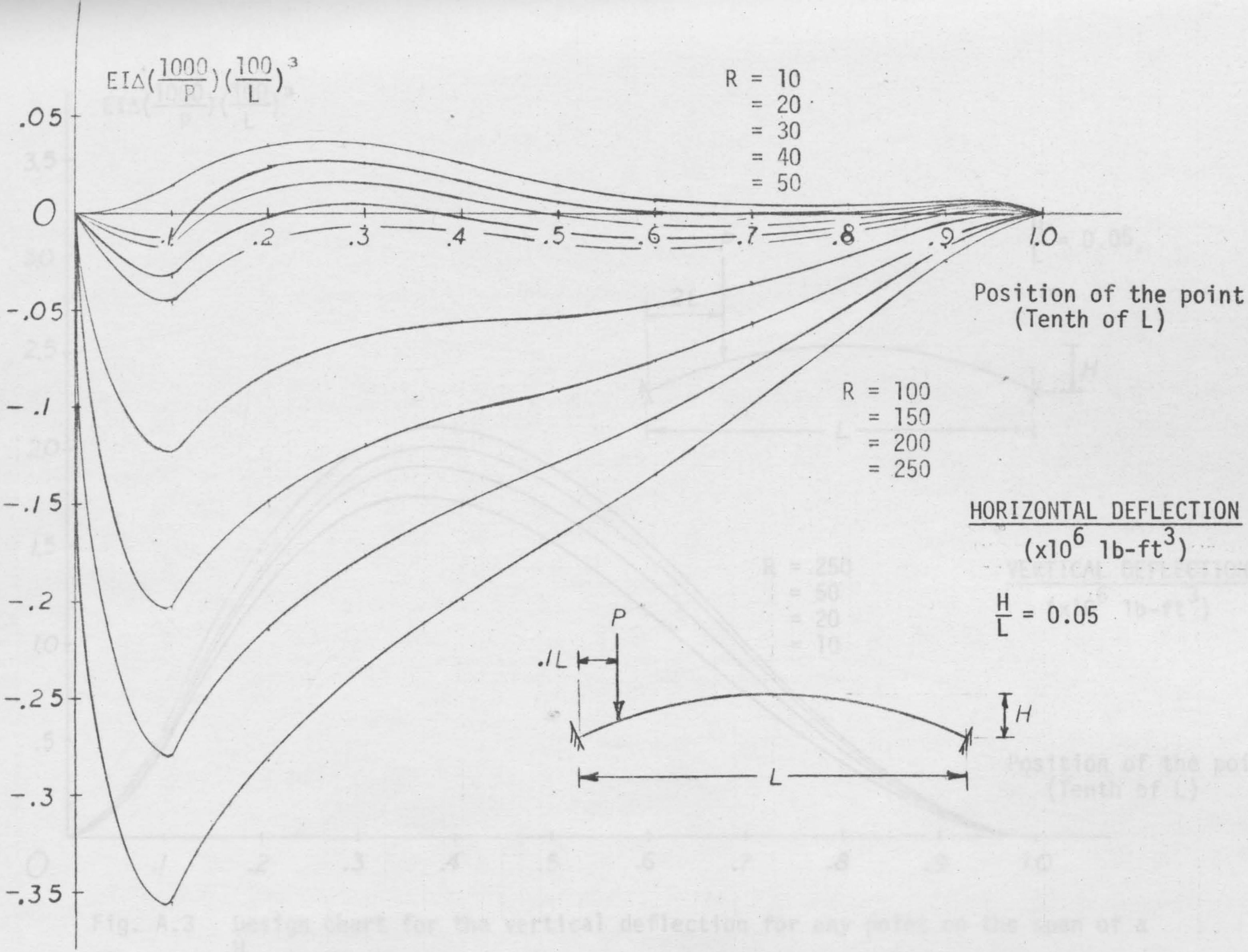


Fig. A.2. Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.05) with the position of load at .1 the span of the arch.

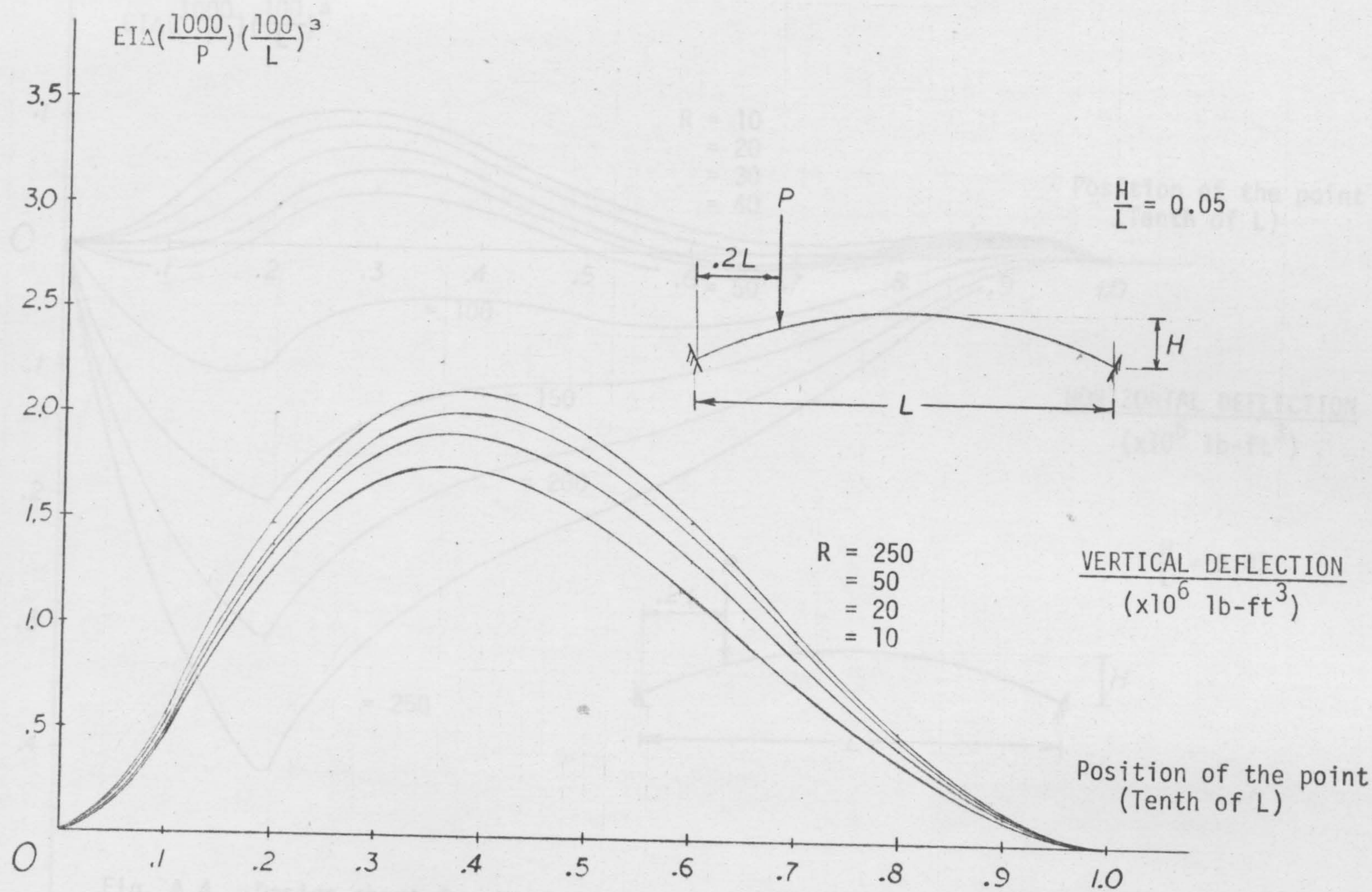


Fig. A.3 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.05) with the position of load at .2 the span of the arch.

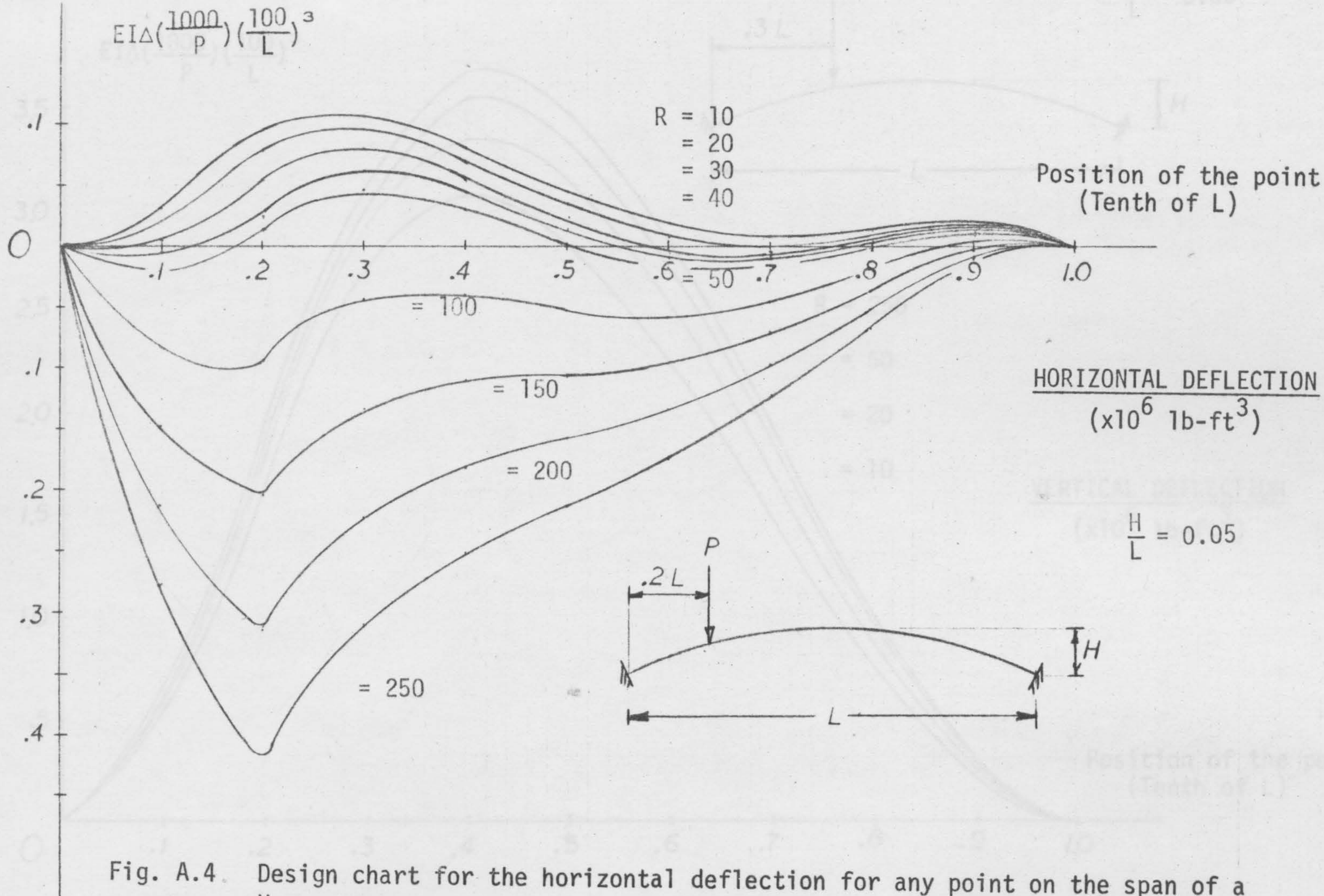


Fig. A.4 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.05) with the position of load at .2 the span of the arch.

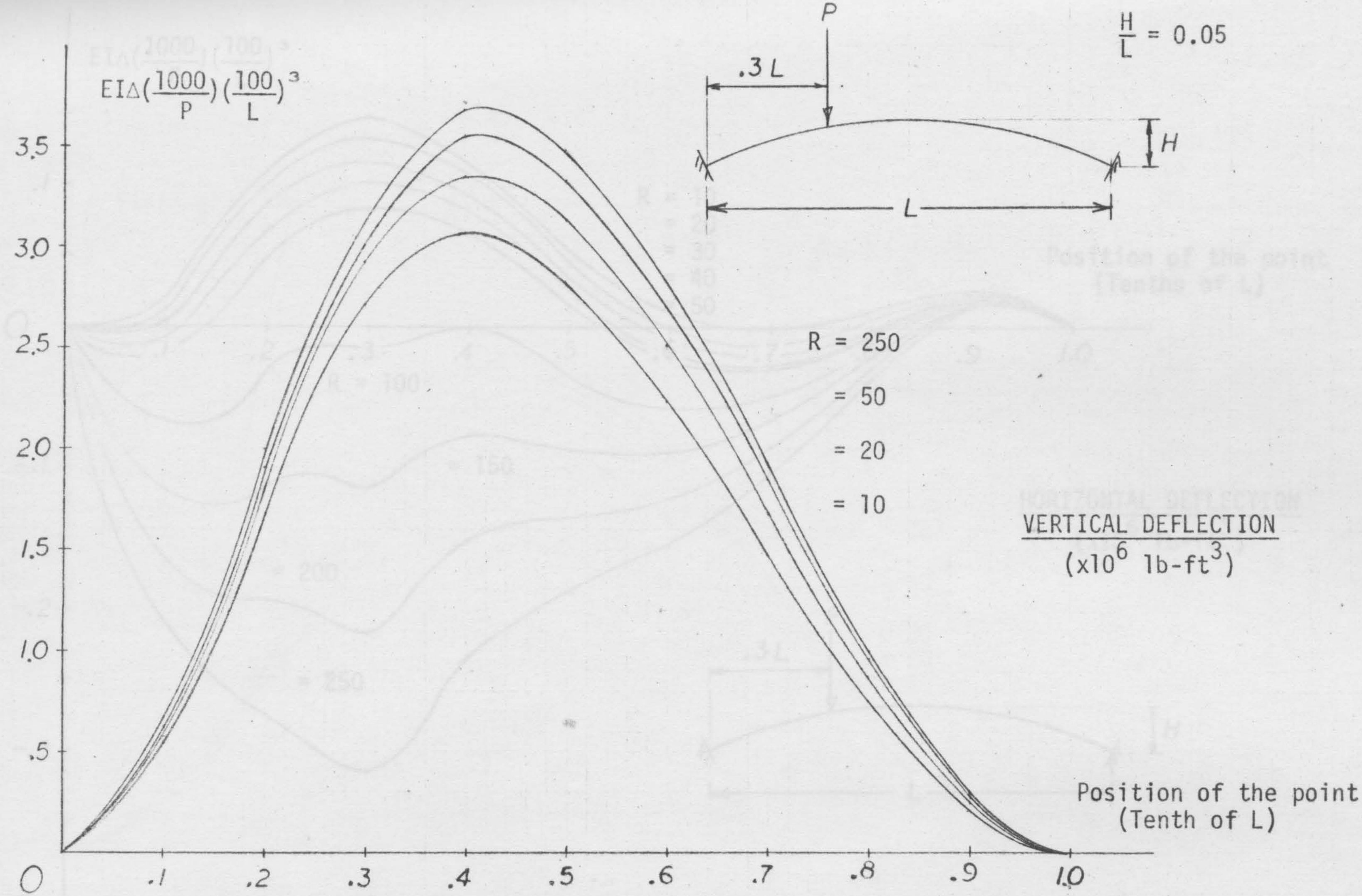


Fig. A.5 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.05) with the position of load at .3 the span of the arch.

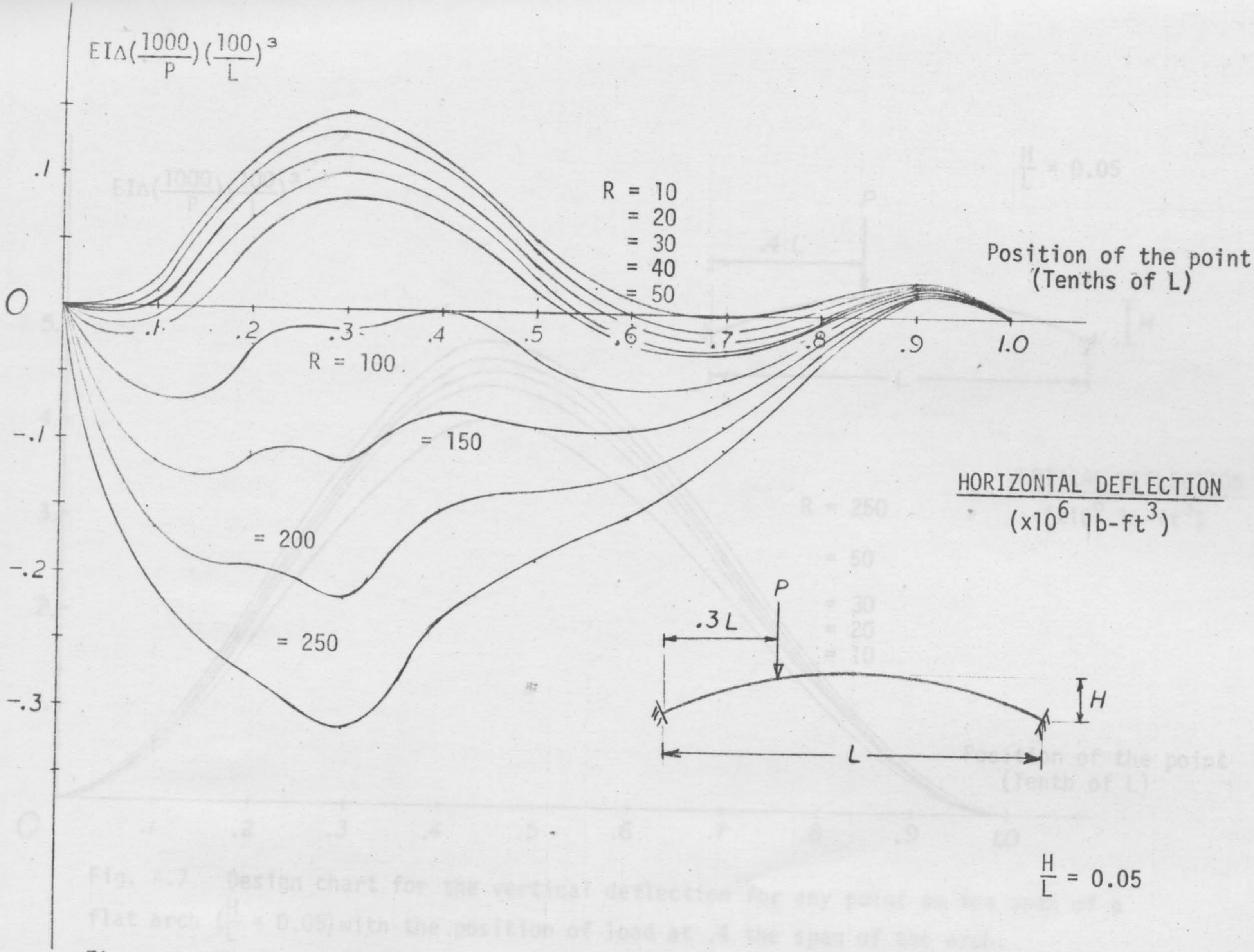


Fig. A.6 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.05) with the position of load at .3 the span of the arch.

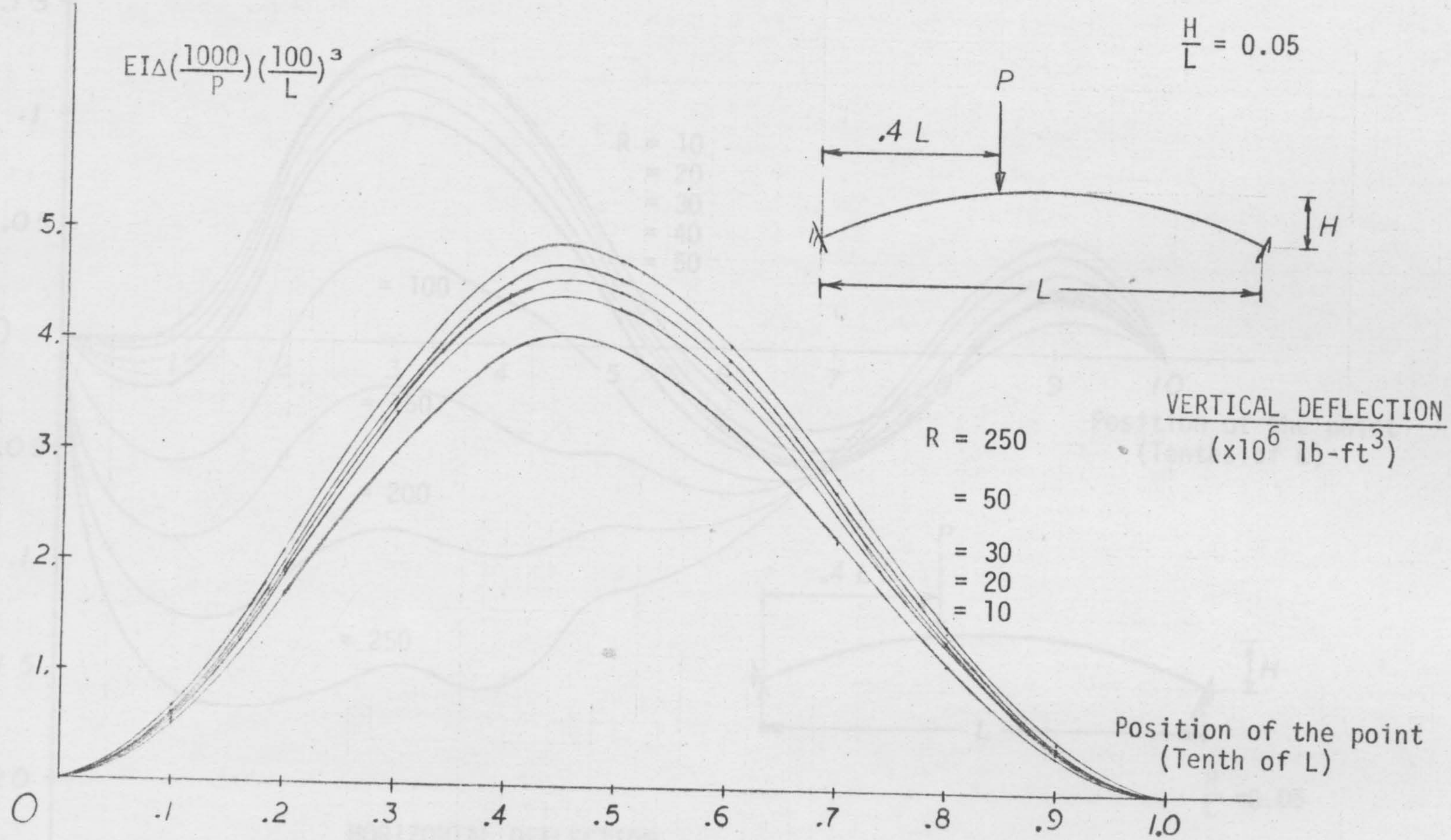


Fig. A.7 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.05) with the position of load at .4 the span of the arch.

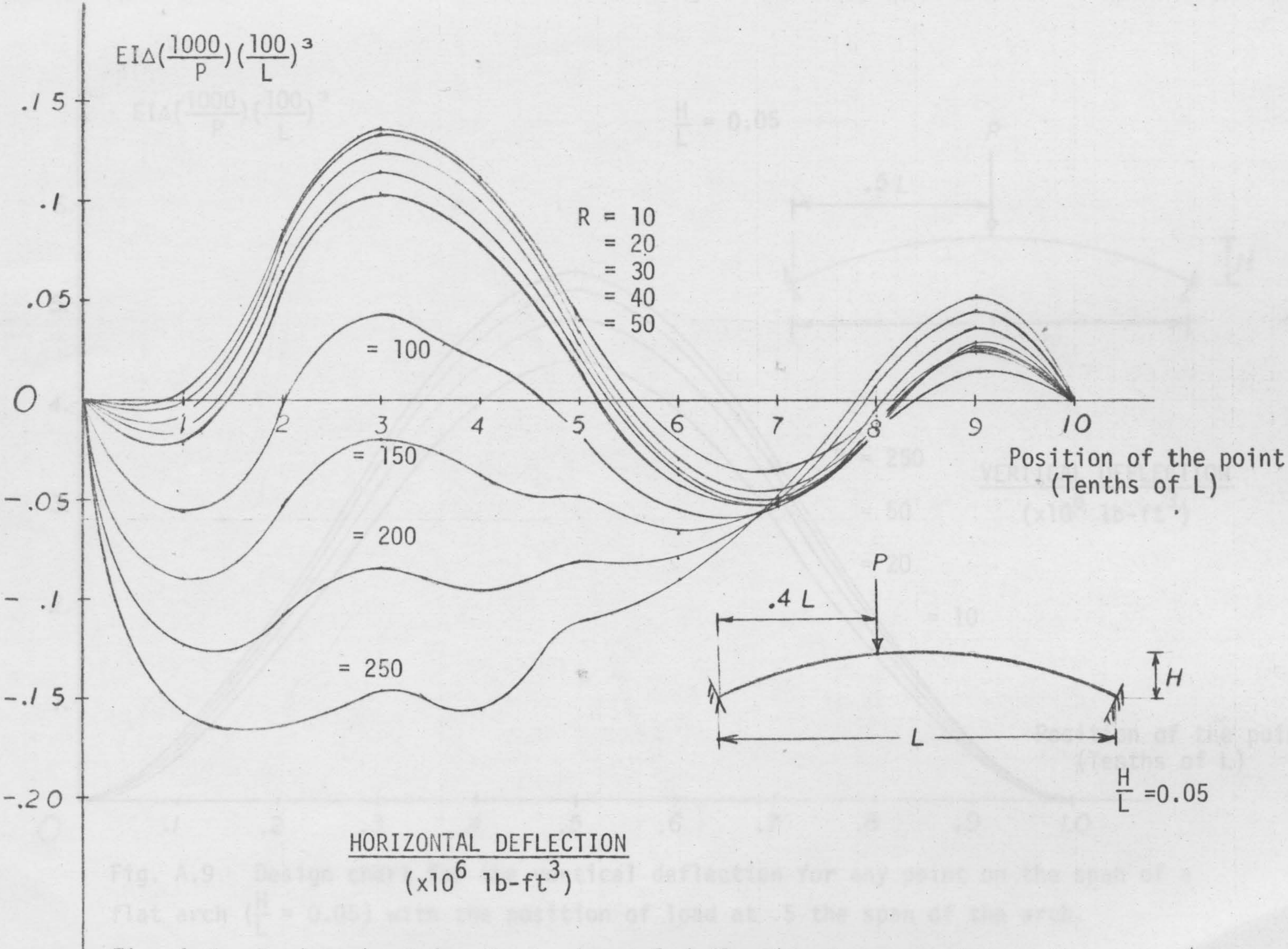


Fig. A.8. Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.05) with the position of load at .4 the span of the arch.

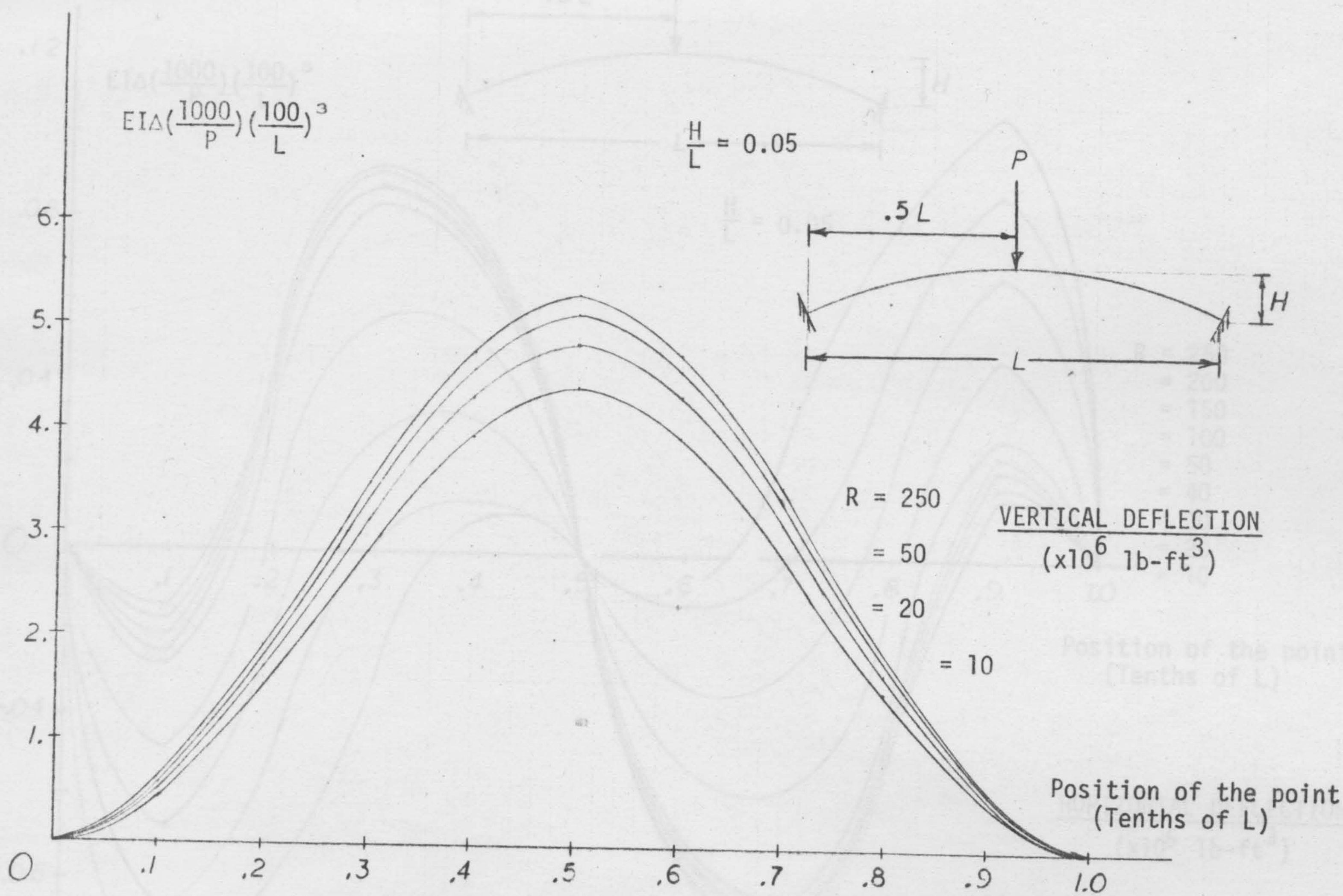


Fig. A.9 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.05) with the position of load at .5 the span of the arch.

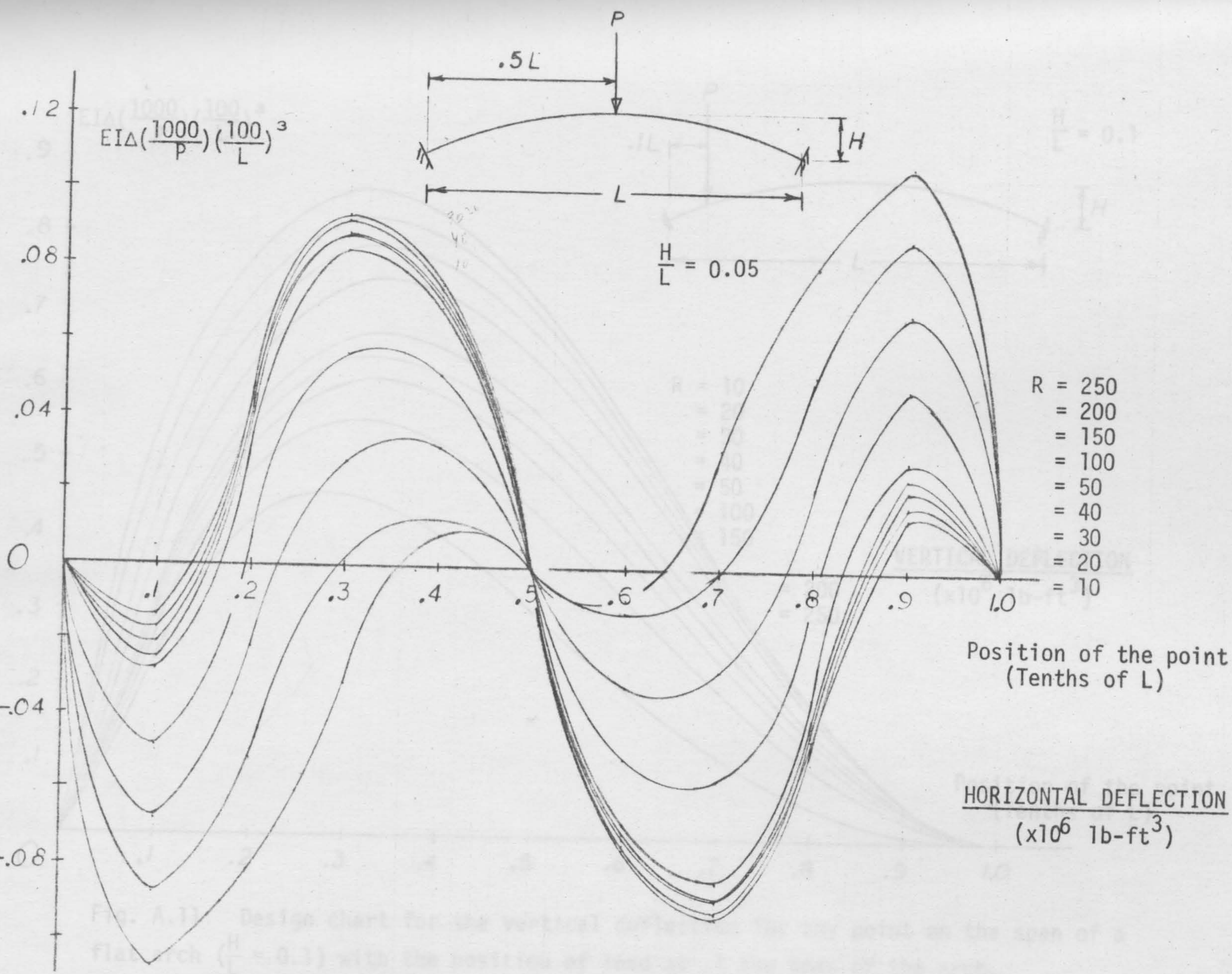


Fig. A.10 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.05) with the position of loat at .5 the span of the arch.

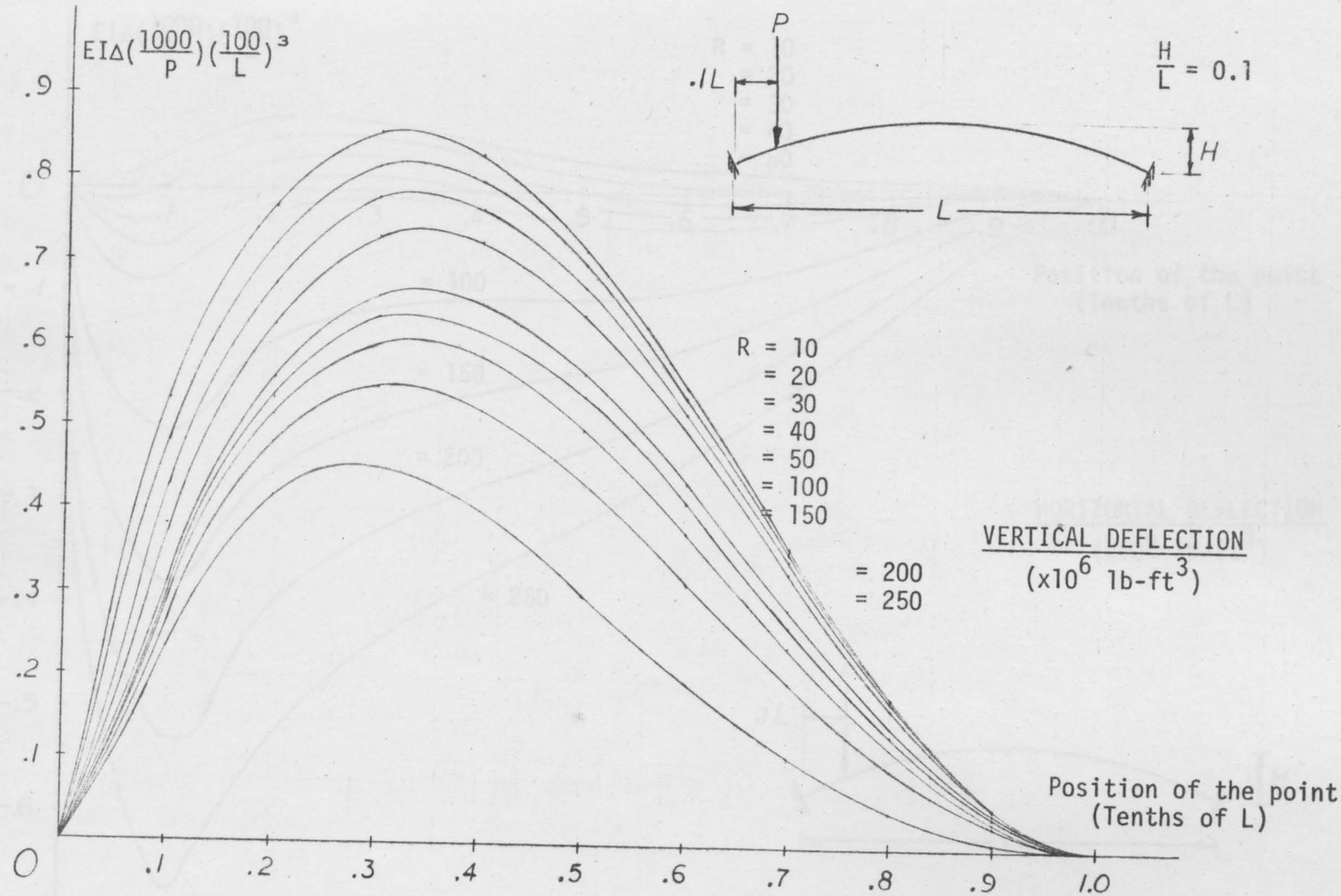


Fig. A.11 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L} = 0.1$) with the position of load at .1 the span of the arch.

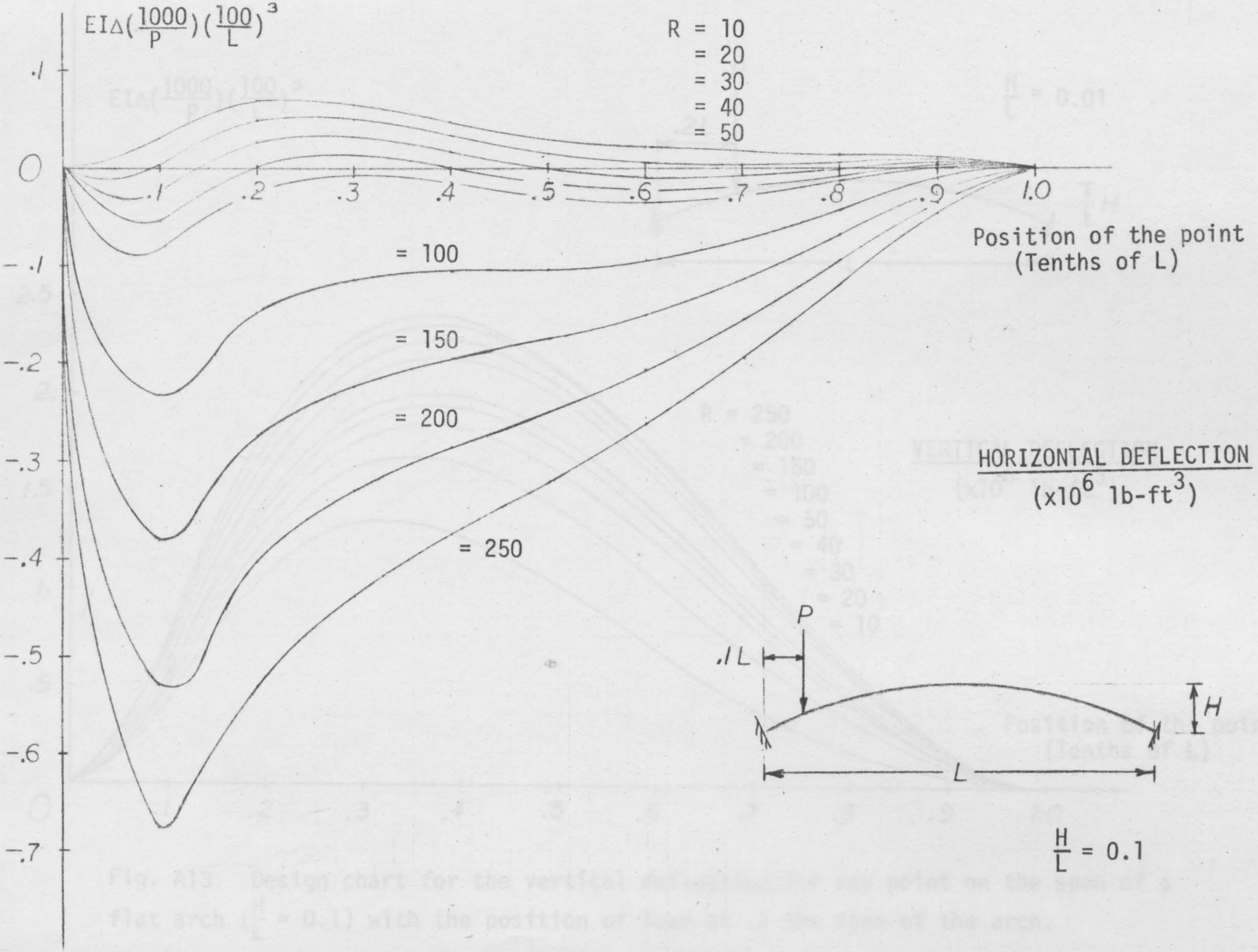


Fig. A.12 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.1) with the position of load at .1 the span of the arch.

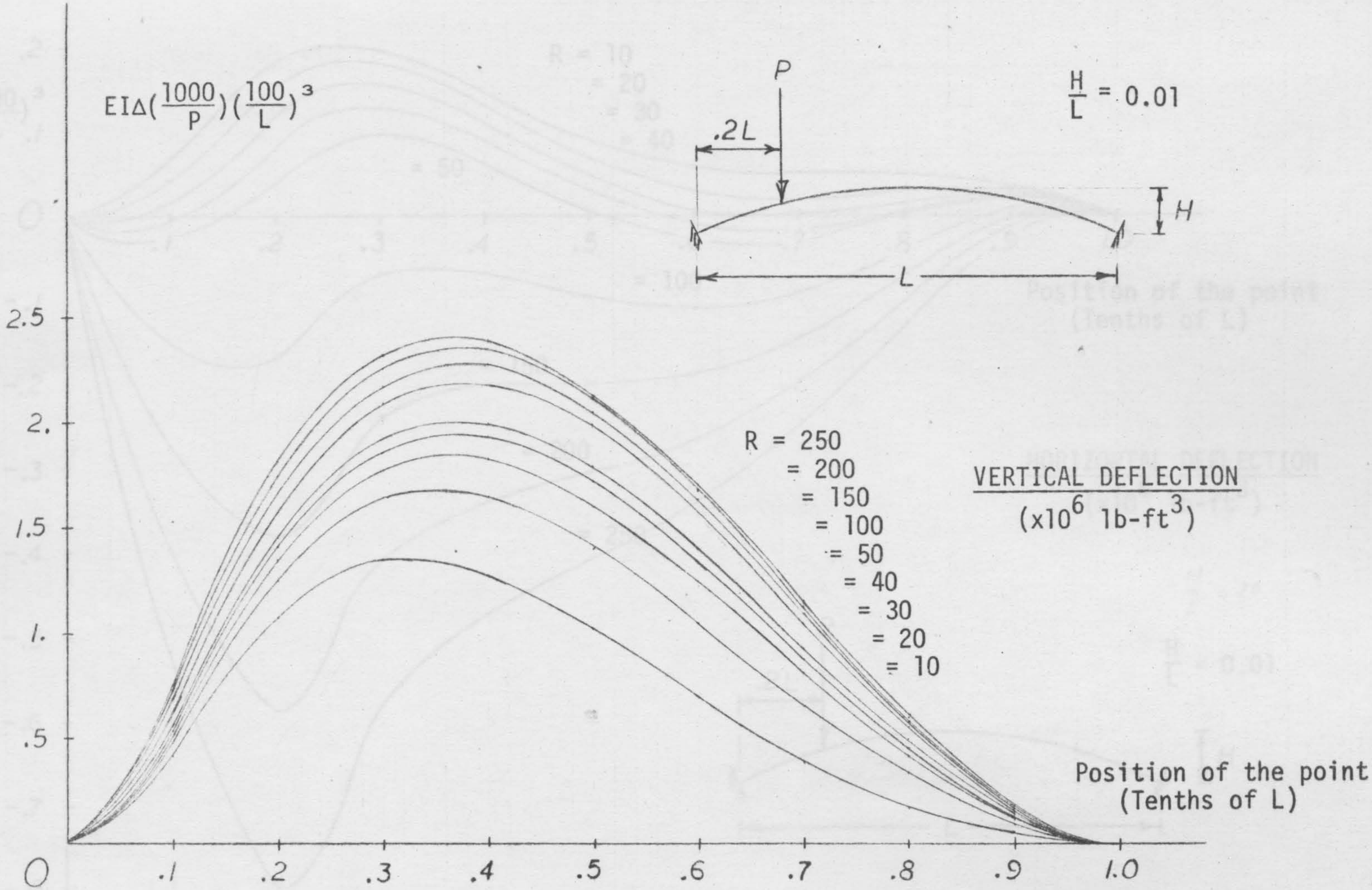


Fig. A13. Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.1) with the position of load at .2 the span of the arch.

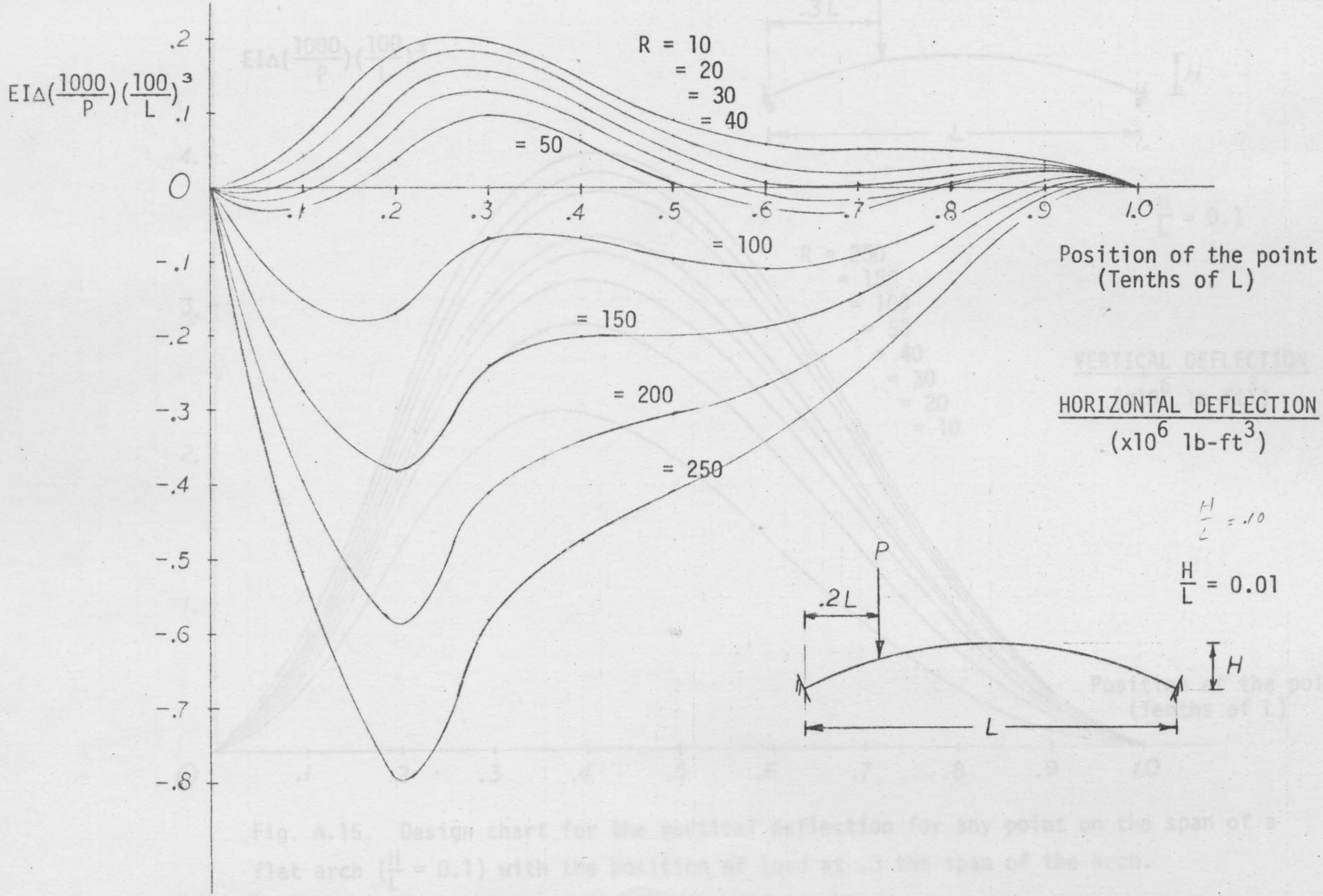


Fig. A.14. Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.1) with the position of load at .2 the span of the arch.

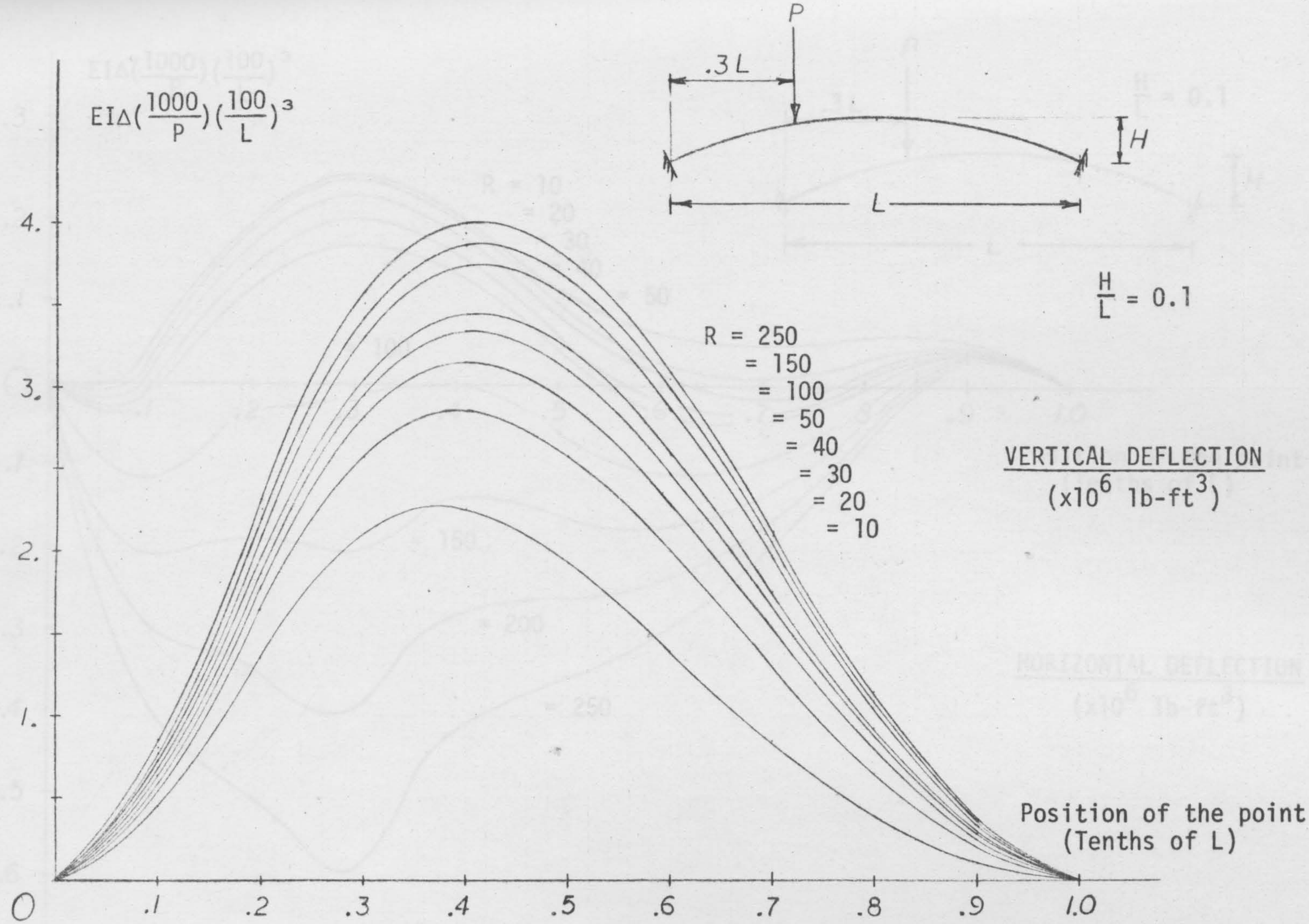


Fig. A.15. Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.1) with the position of load at .3 the span of the arch.

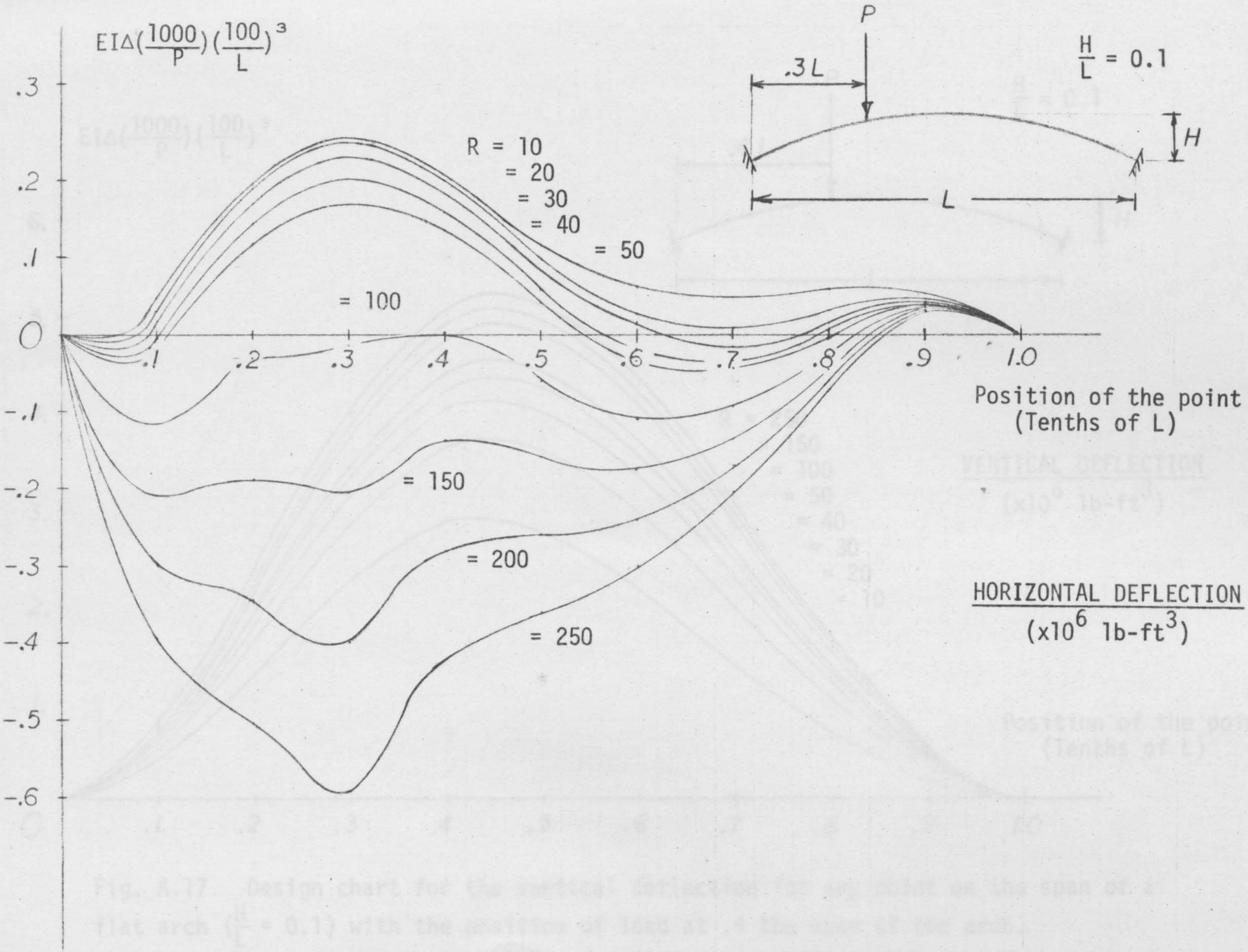


Fig. A.16 Design chart for the horizontal deflection for any point on the spain of a flat arch ($\frac{H}{L}$ = 0.1) with the position of load at .3 the span of the arch.

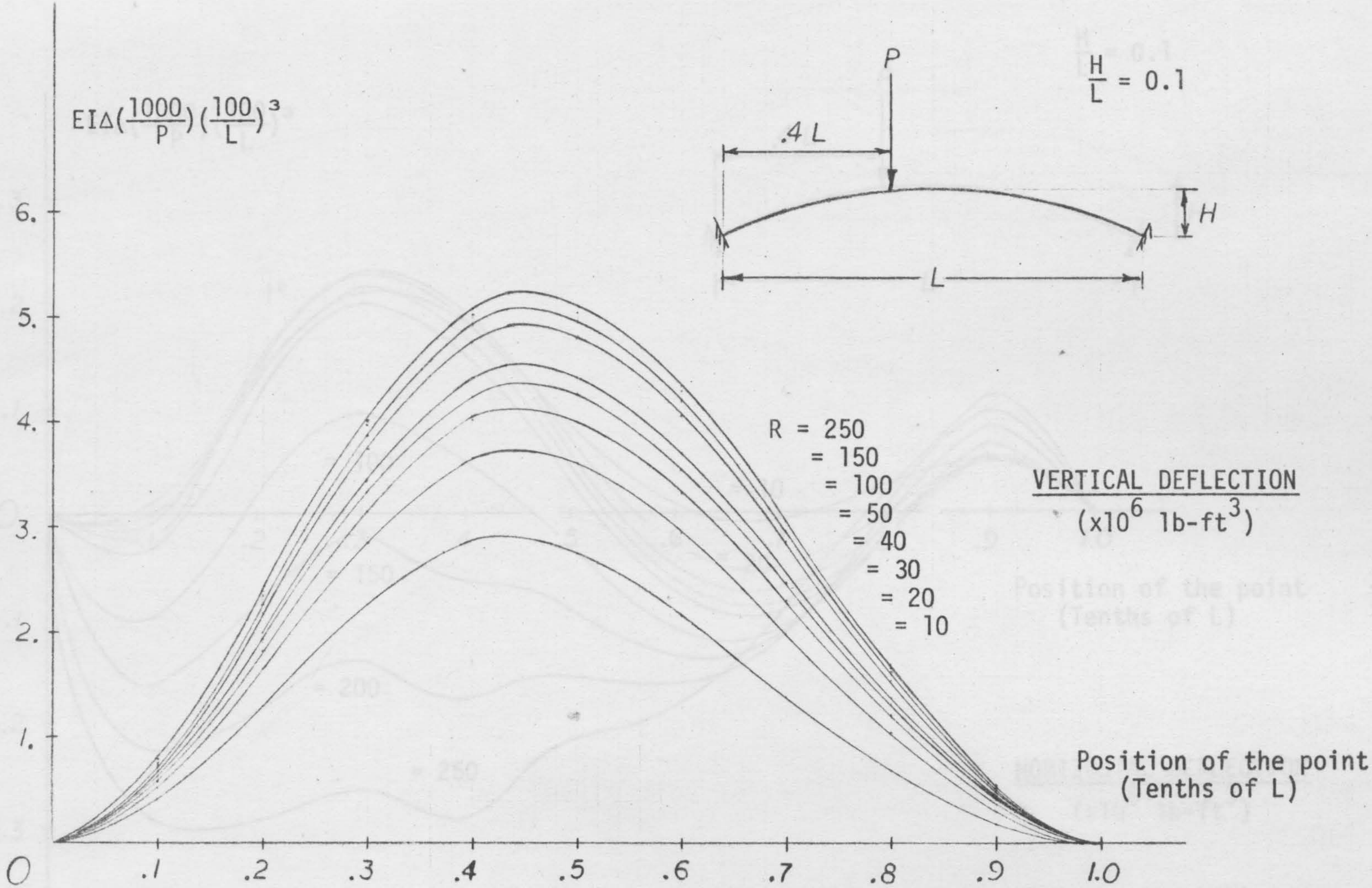


Fig. A.17. Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.1) with the position of load at .4 the span of the arch.

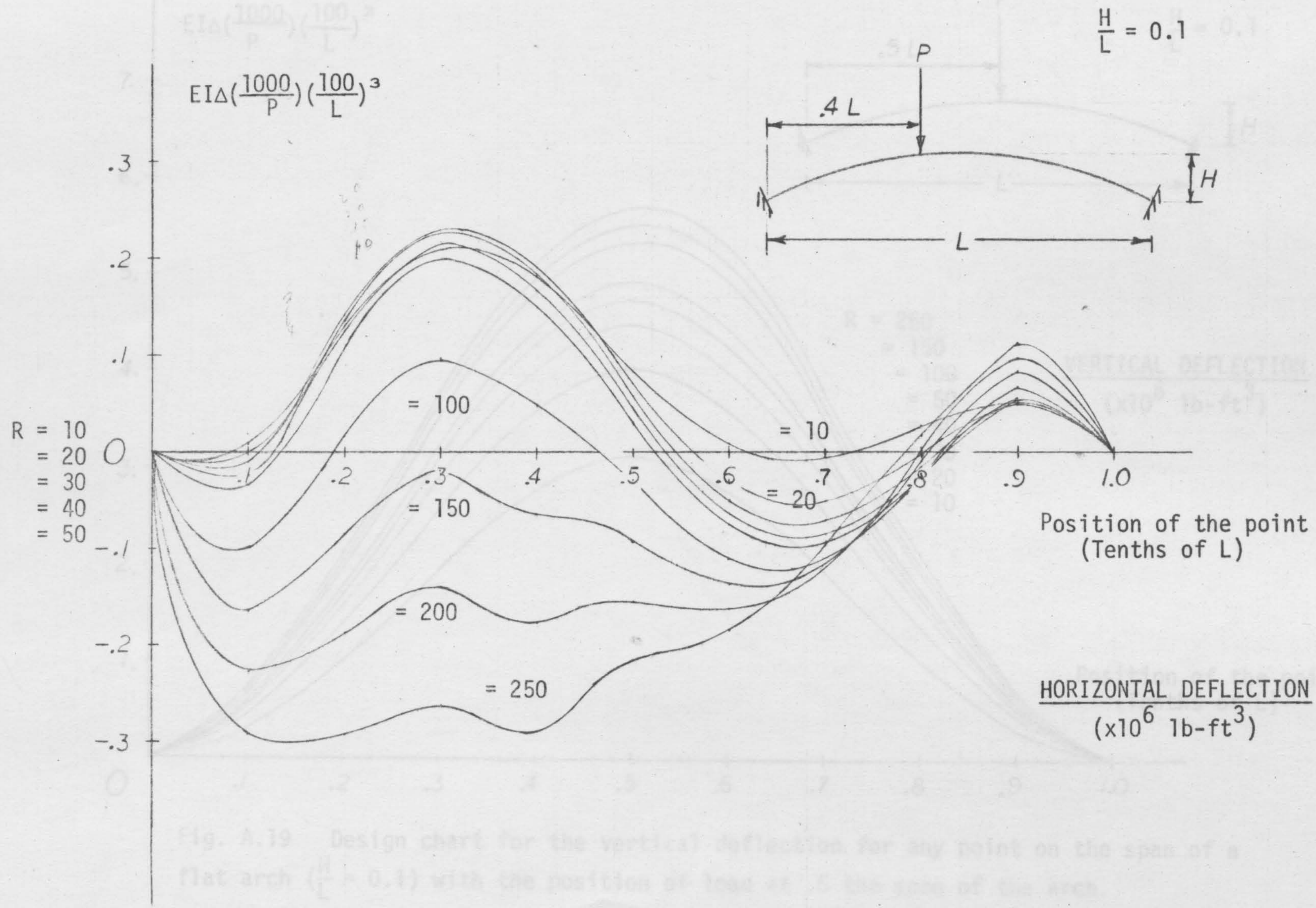


Fig. A.18 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.1) with the position of load at .4 the span of the arch.

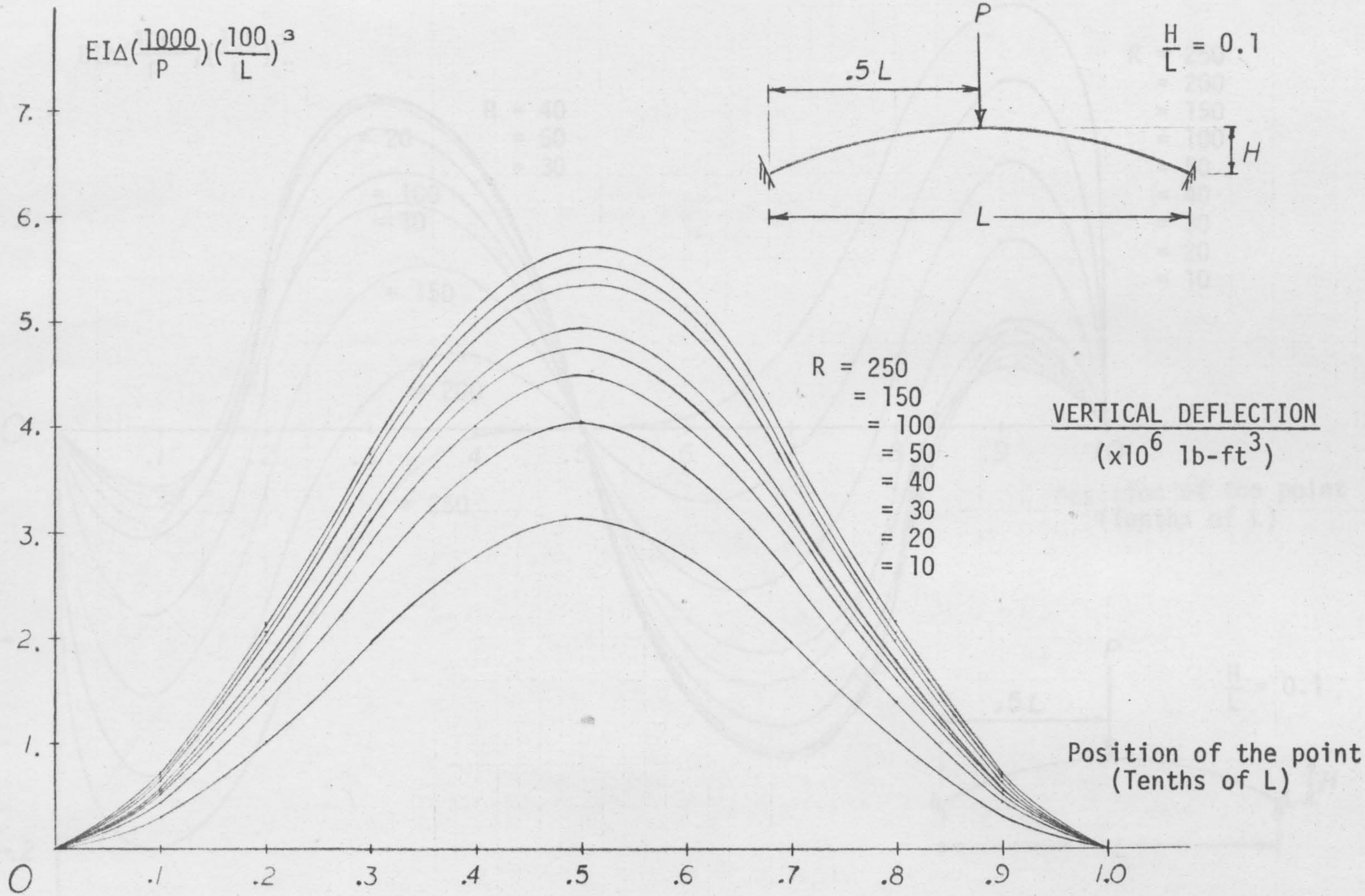


Fig. A.19 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.1) with the position of load at .5 the span of the arch.

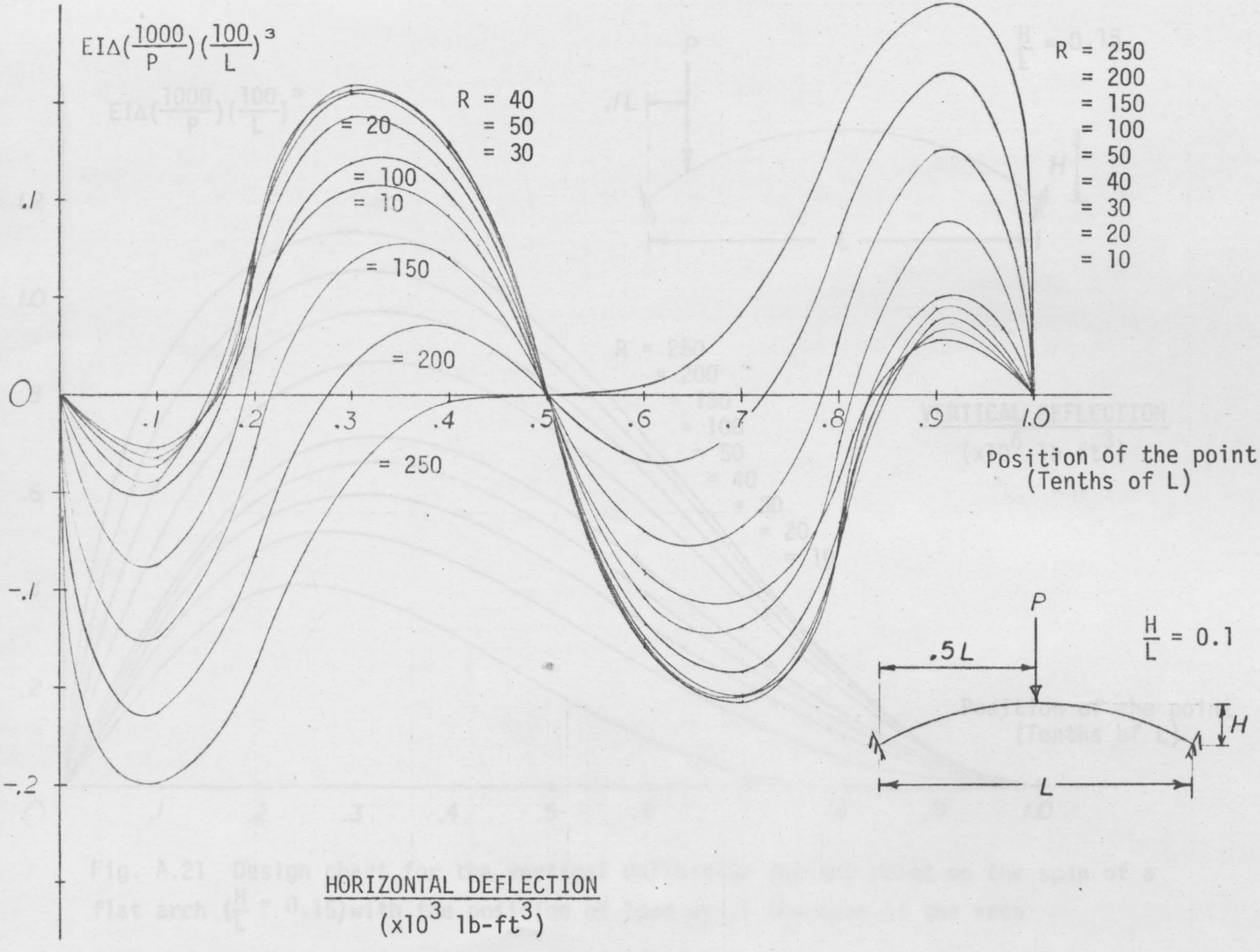


Fig. A.20. Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.1) with the position of load at .5 the span of the arch.

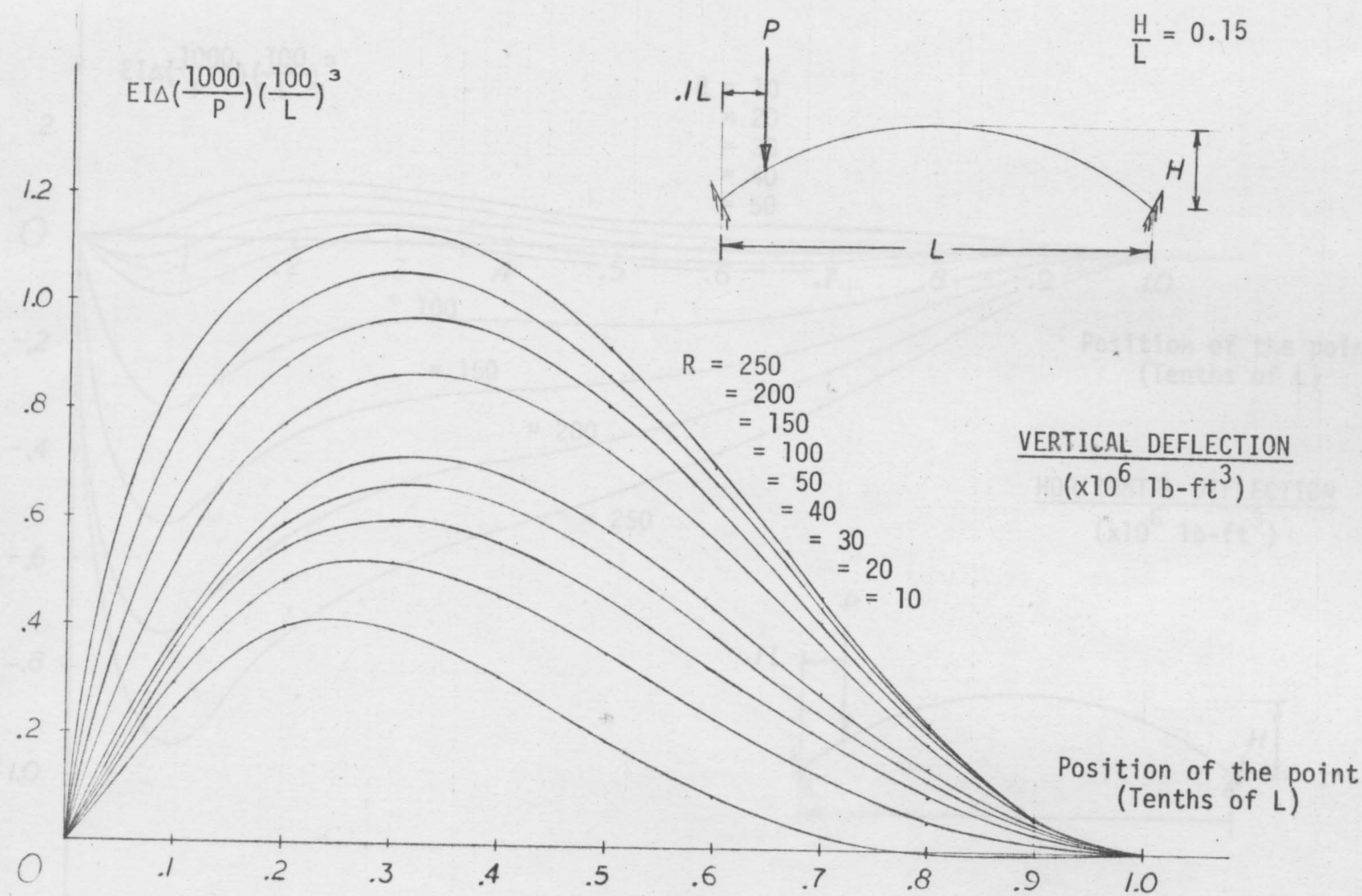
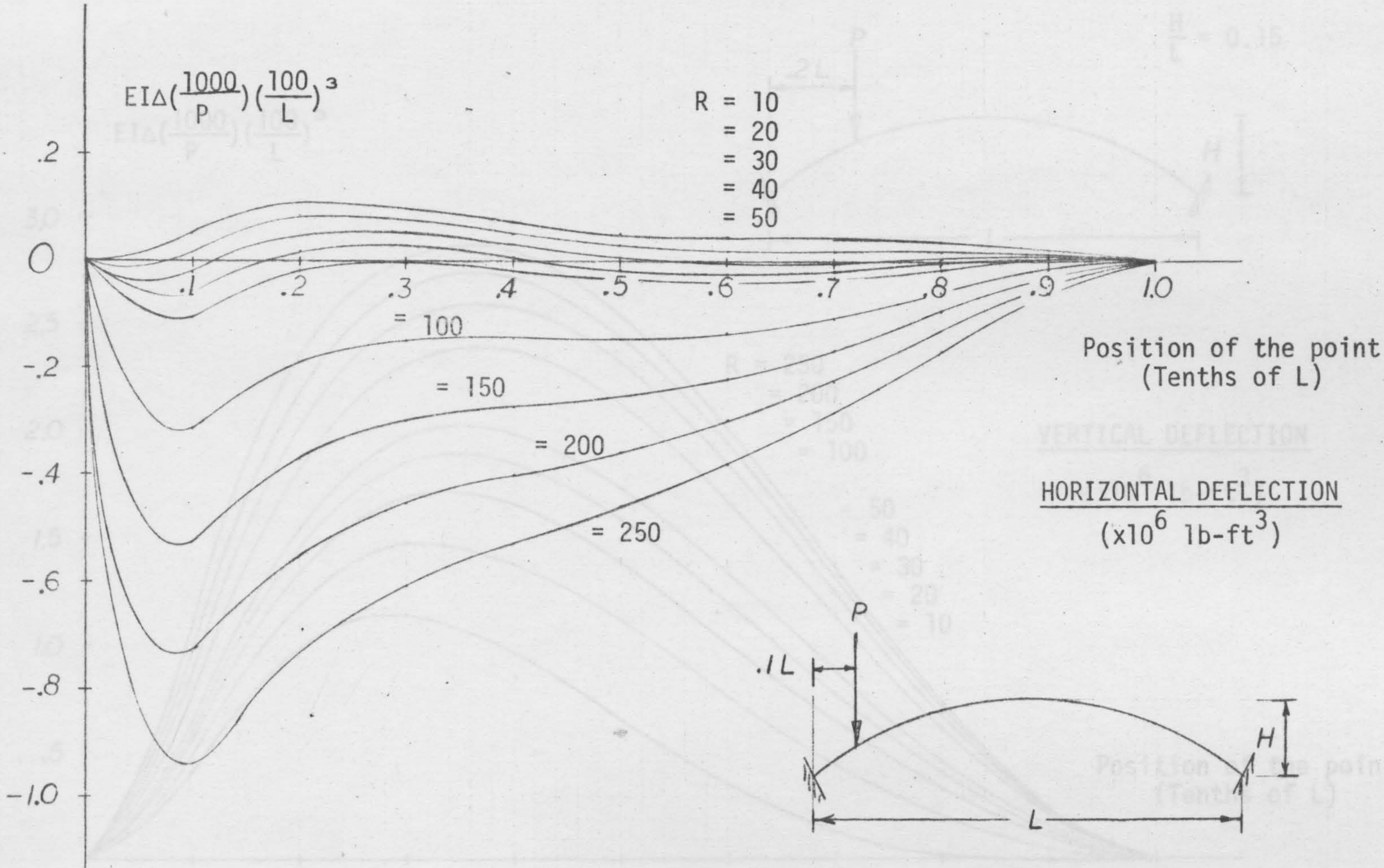


Fig. A.21 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.15) with the position of load at .1 the span of the arch



$\frac{H}{L} = 0.15$

Fig. A. 22 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.15) with the position of load at .1 the span of the arch.

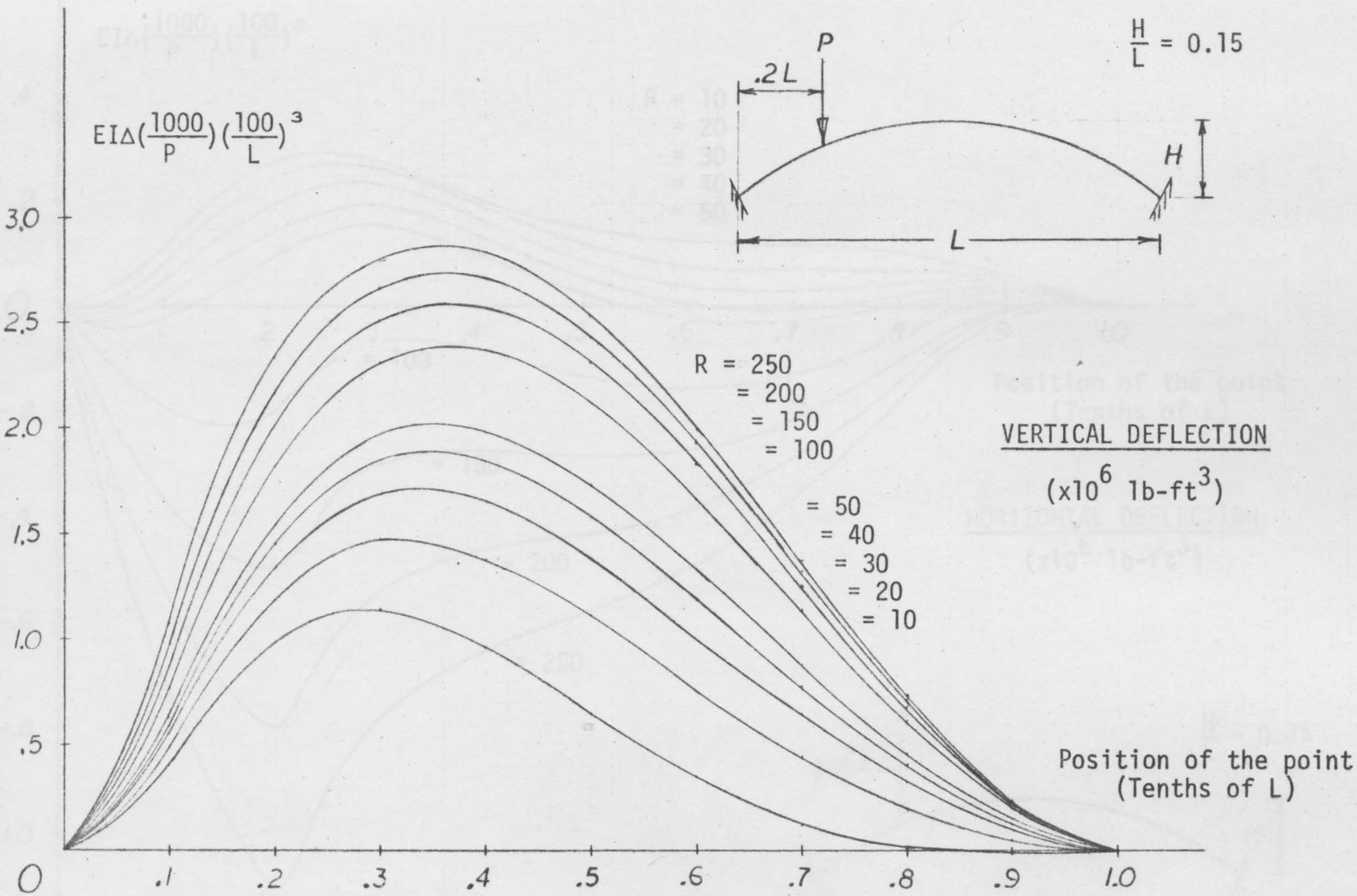


Fig. A.23 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.15) with the position of load at .2 the span of the arch.

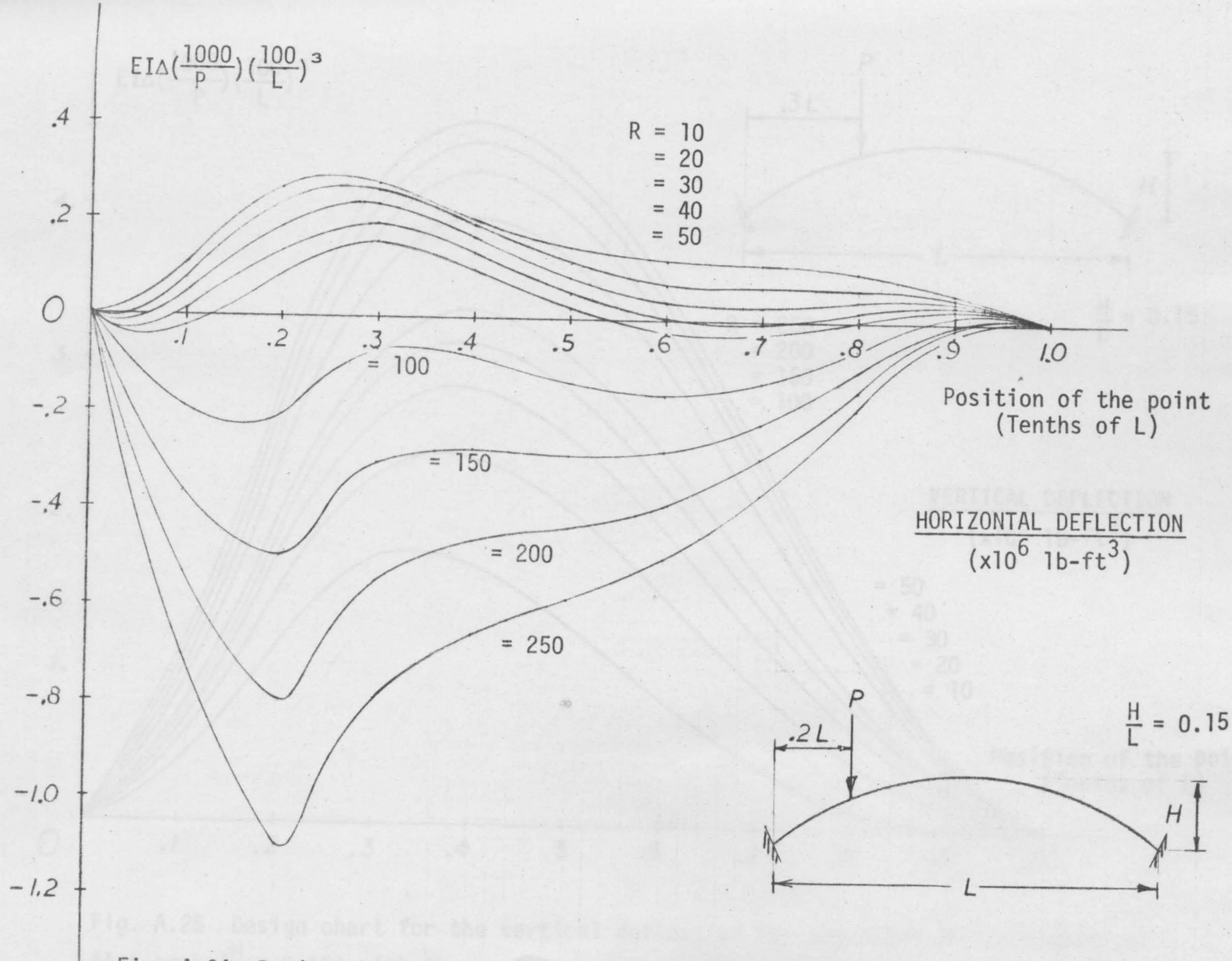


Fig. A.24 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.15) with the position of load at .2 the span of the arch.

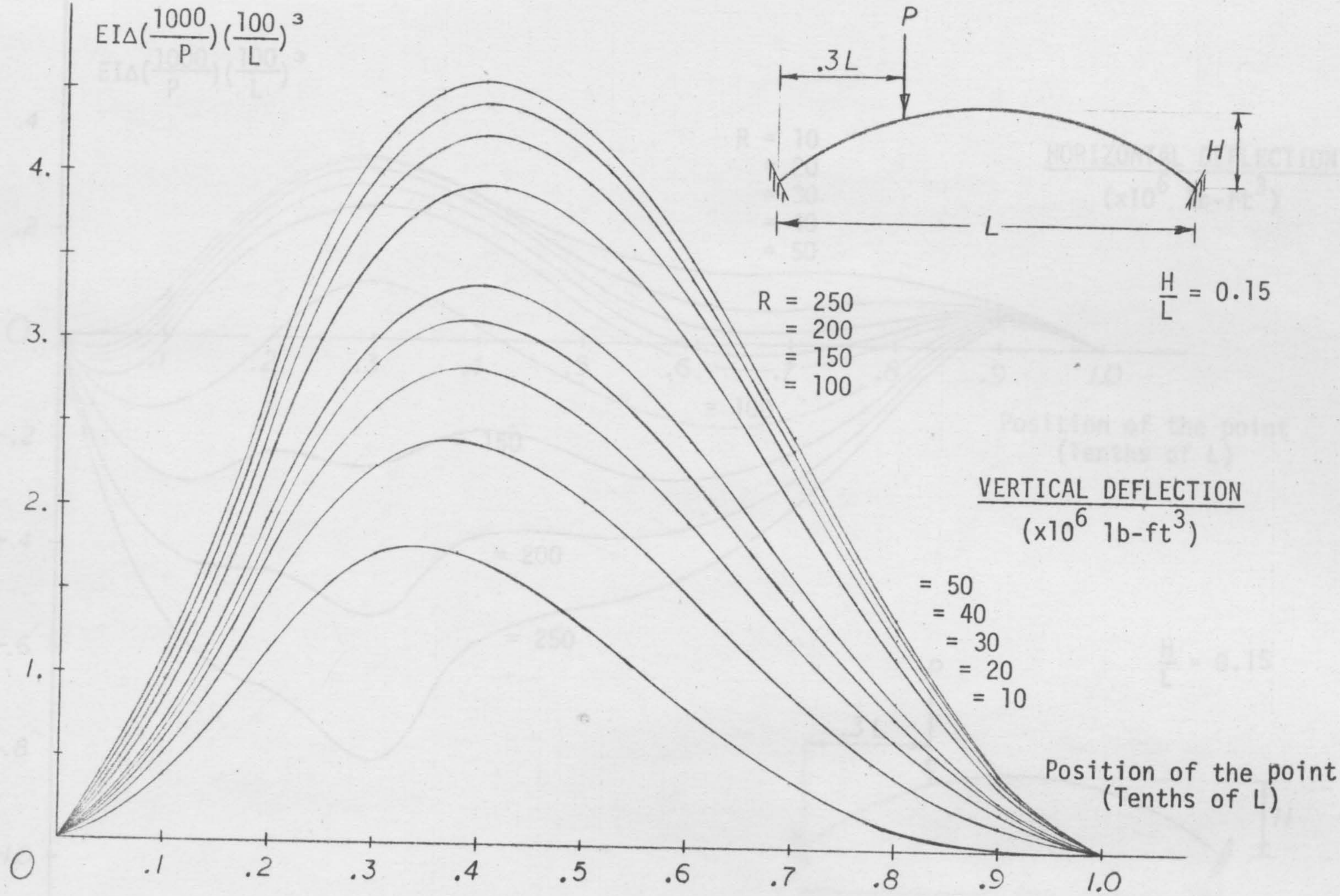


Fig. A.25 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L} = 0.15$) with the position of load at .3 the span of the arch.

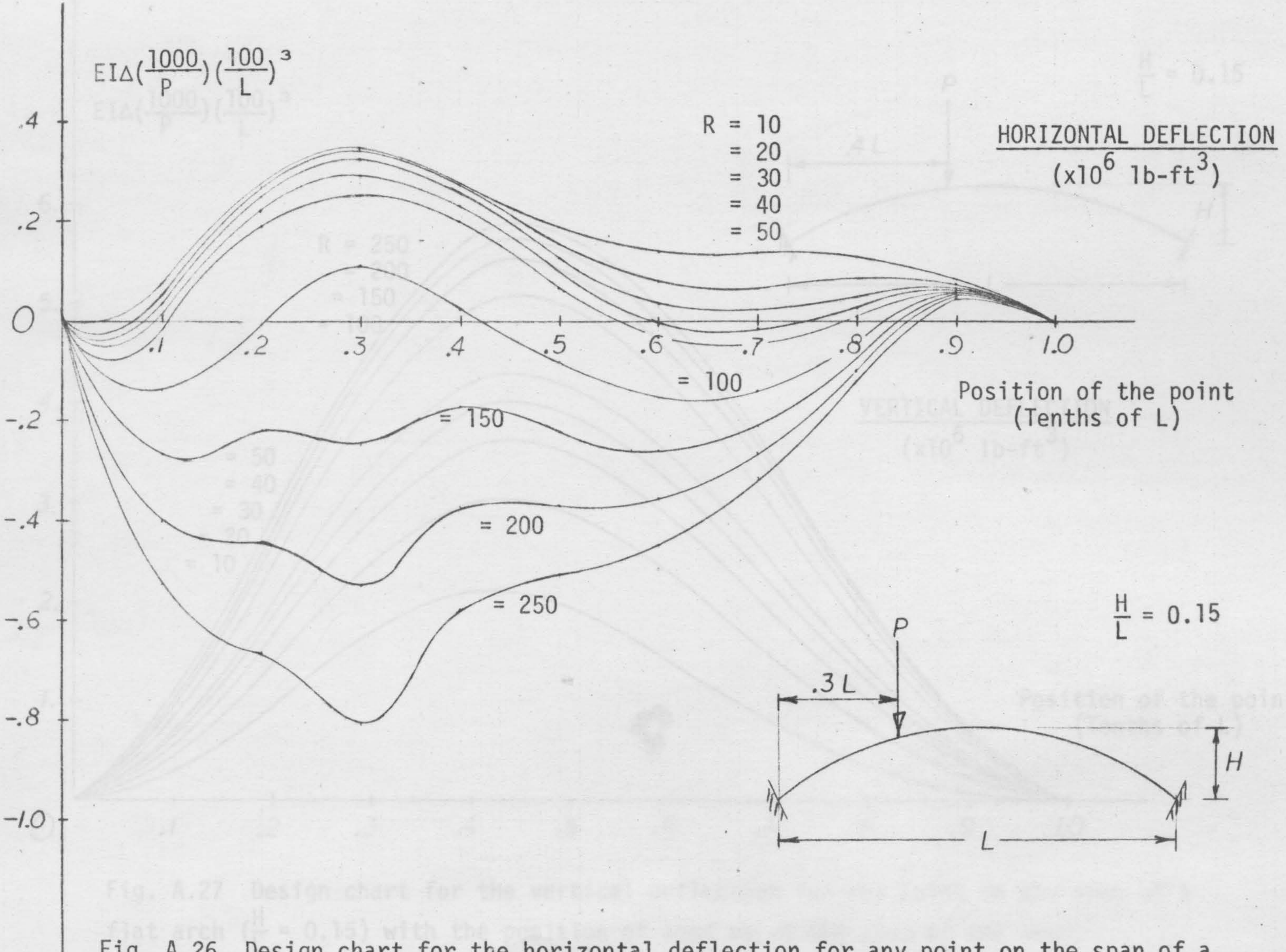


Fig. A.26 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.15) with the position of load at .3 the span of the arch.

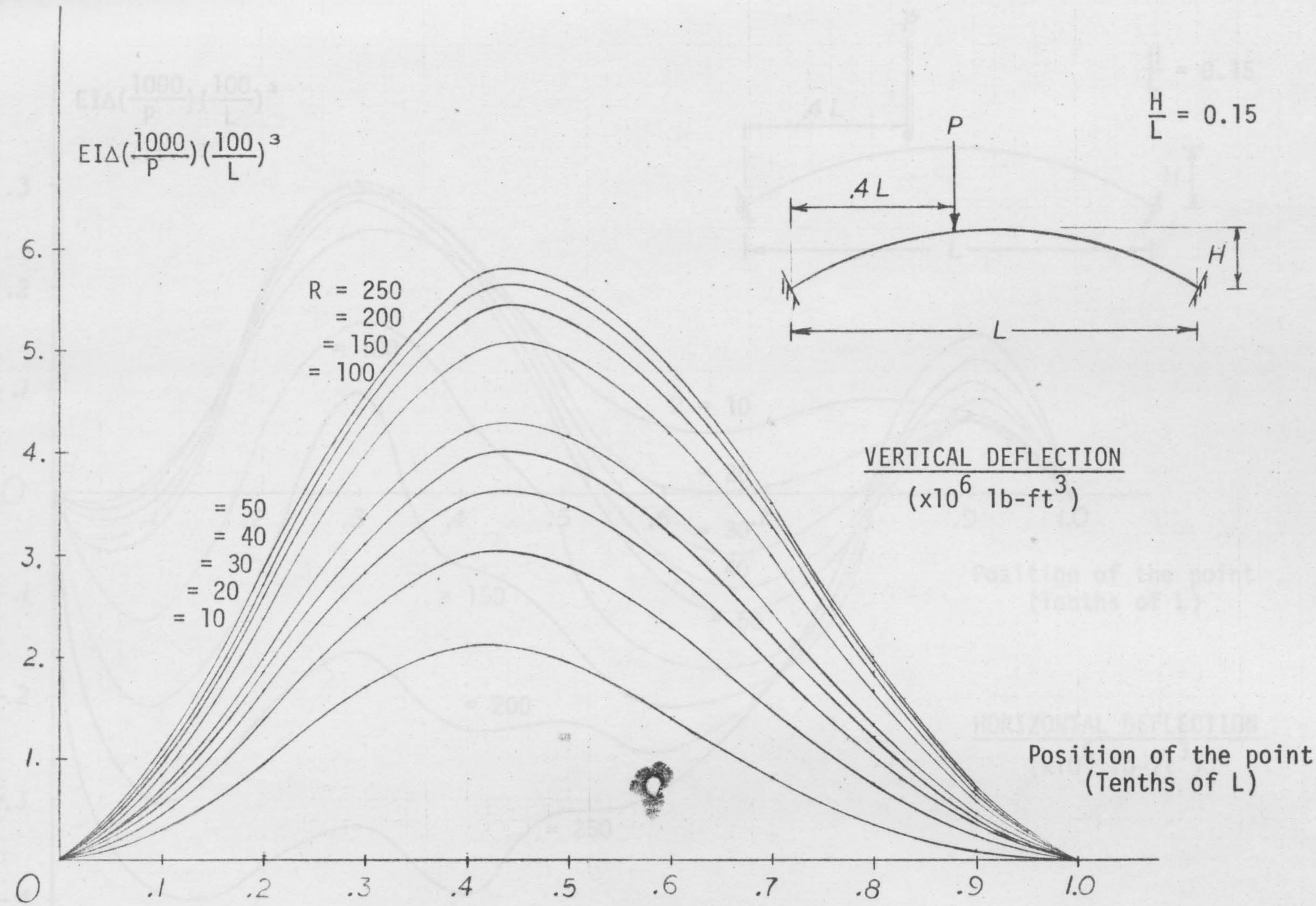


Fig. A.27 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.15) with the position of load at .4 the span of the arch.

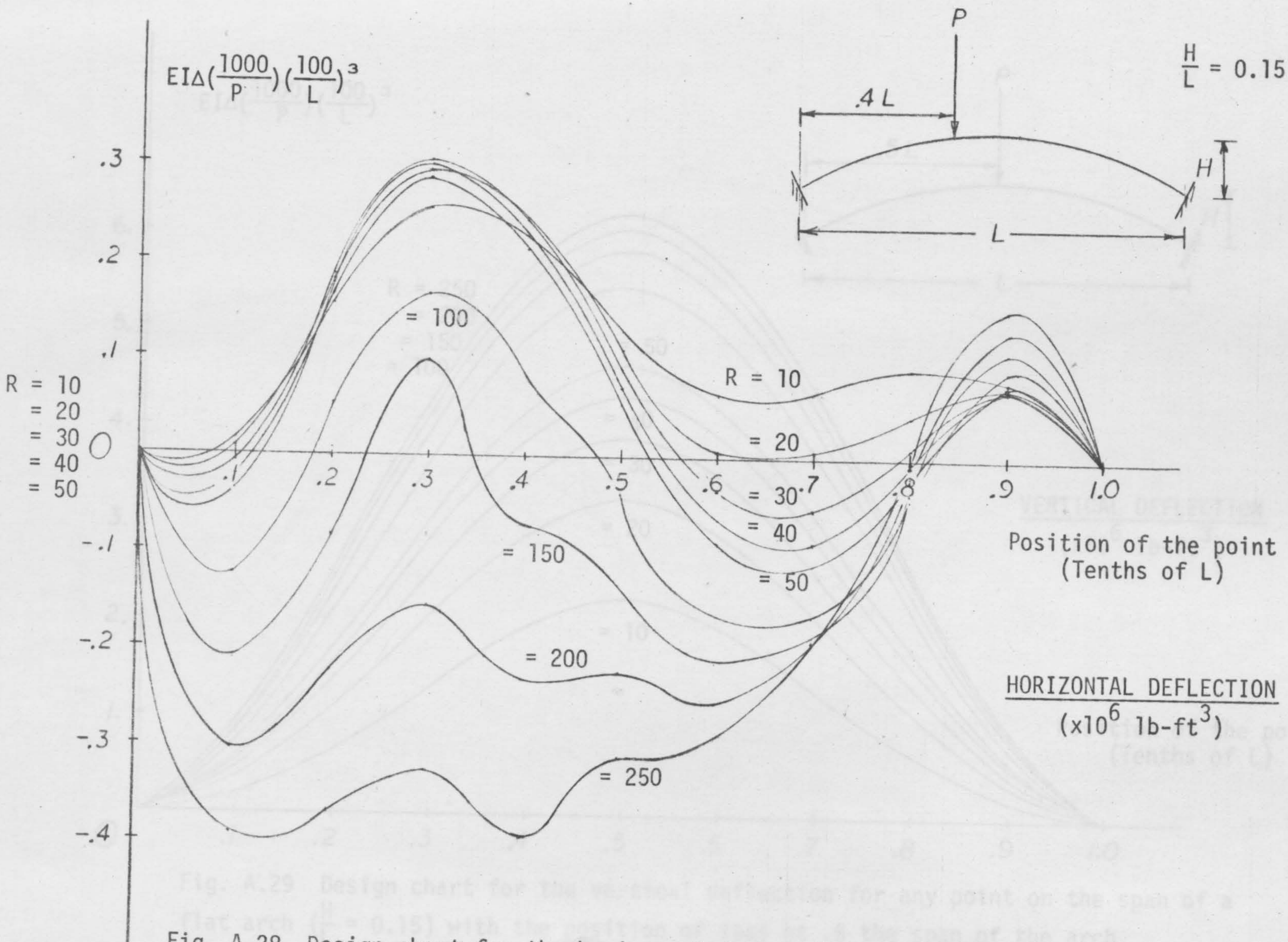


Fig. A.28 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.15) with the position of load at .4 the span of the arch.

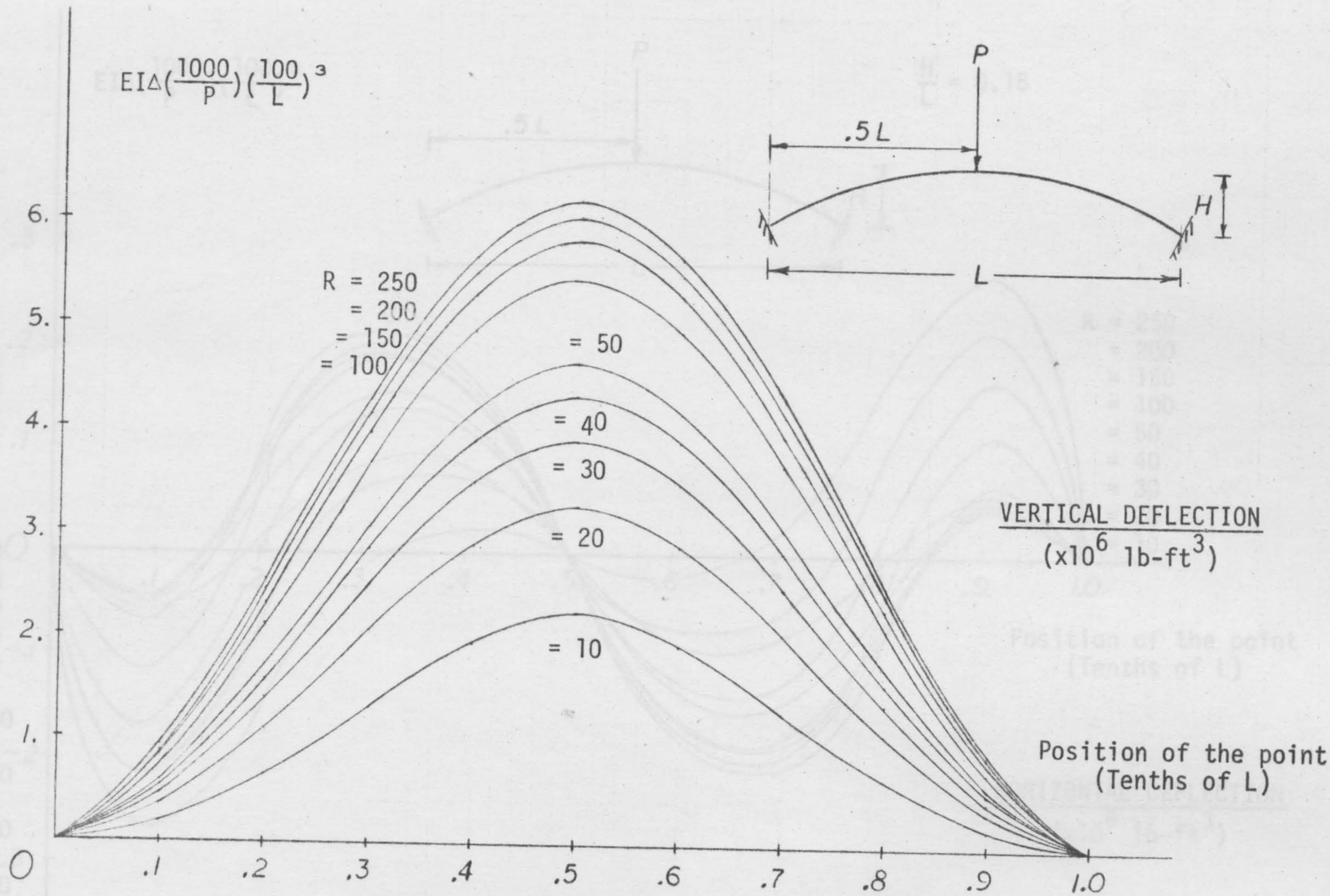


Fig. A.29 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.15) with the position of load at .5 the span of the arch.

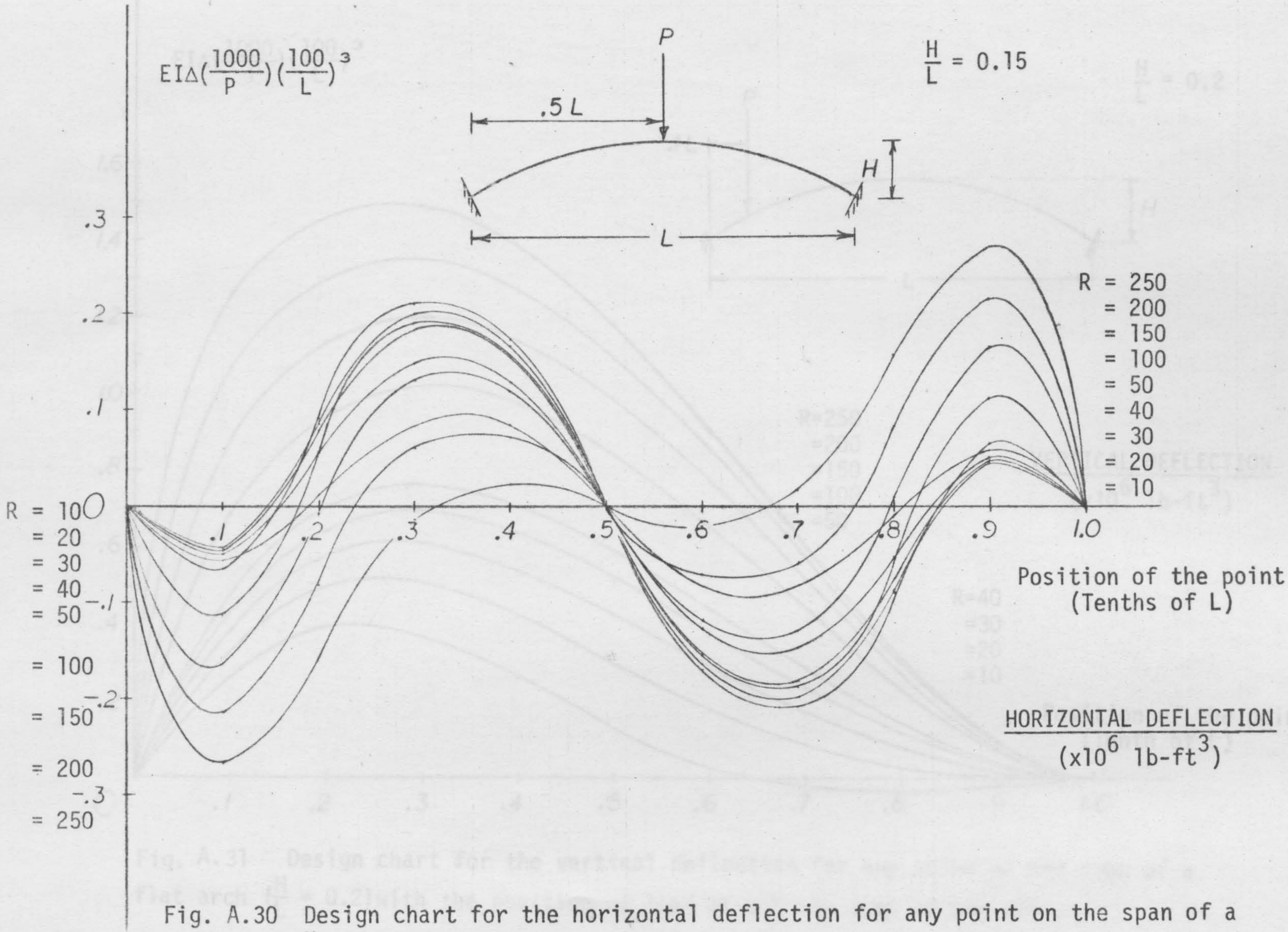


Fig. A.30 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.15) with the position of load at .5 the span of the arch.

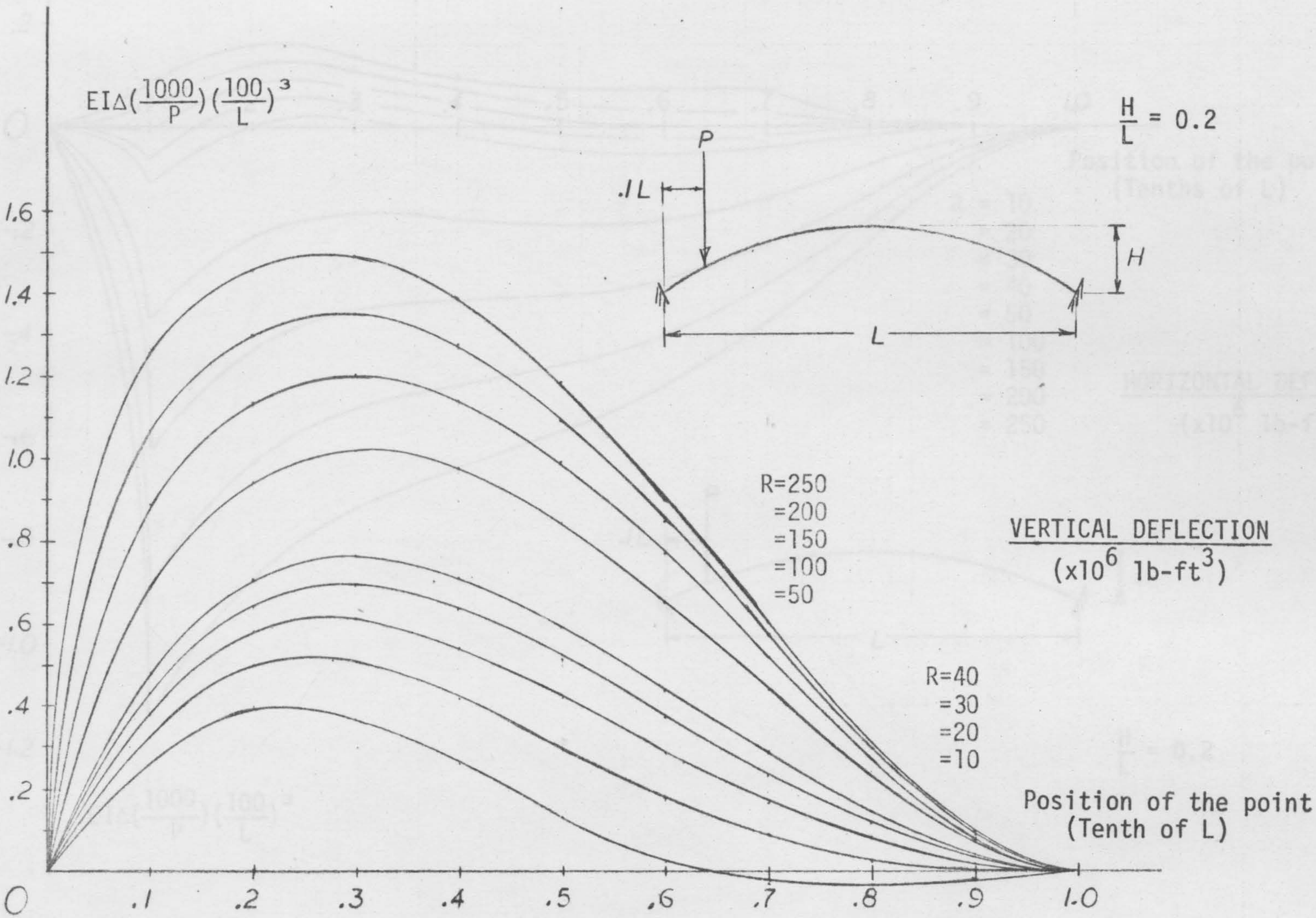


Fig. A.31 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L} = 0.2$) with the position of load at 0.1 the span of the arch.

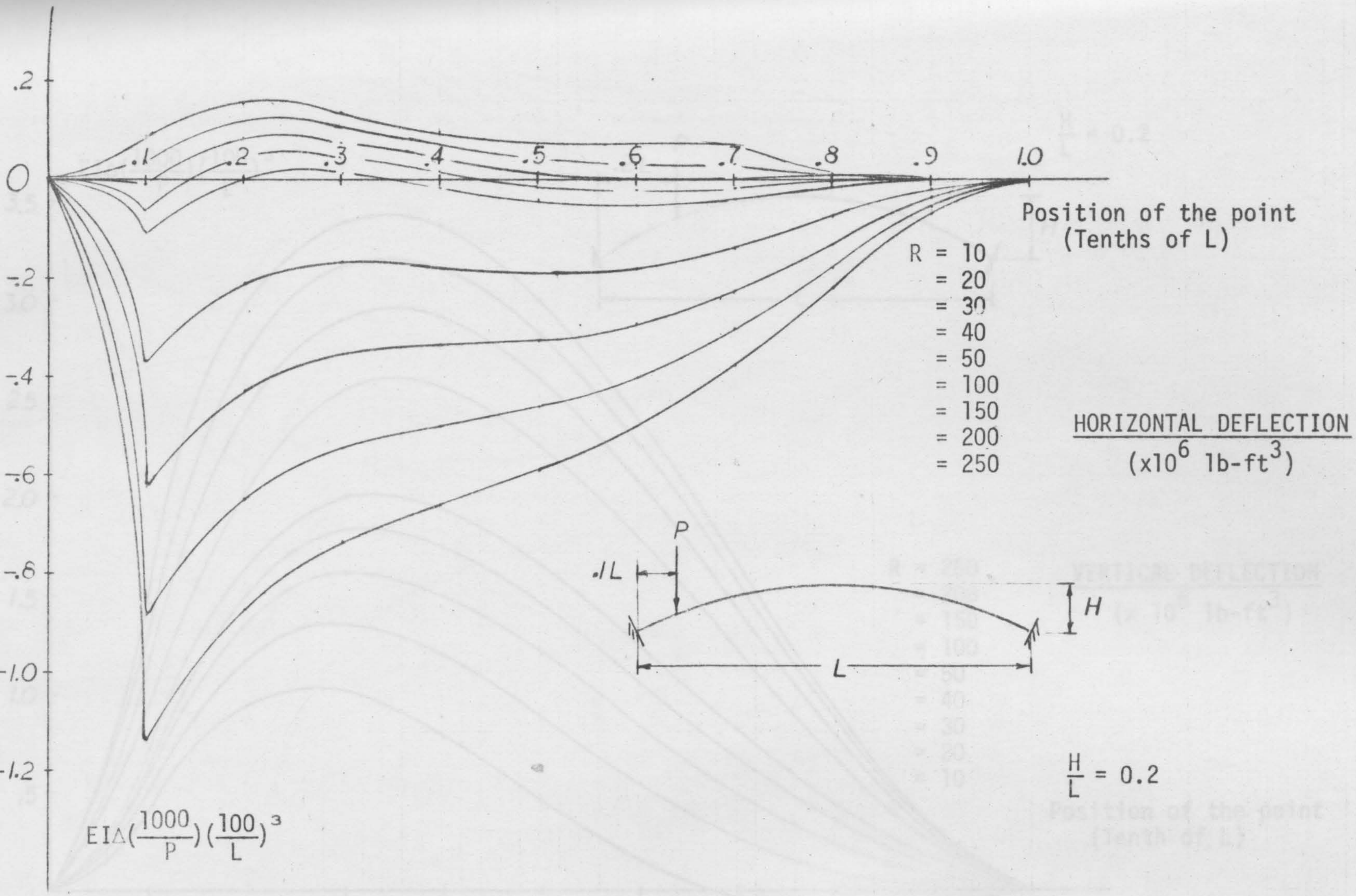


Fig. A.32 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.2) with the position of load at .1 the span of the arch.

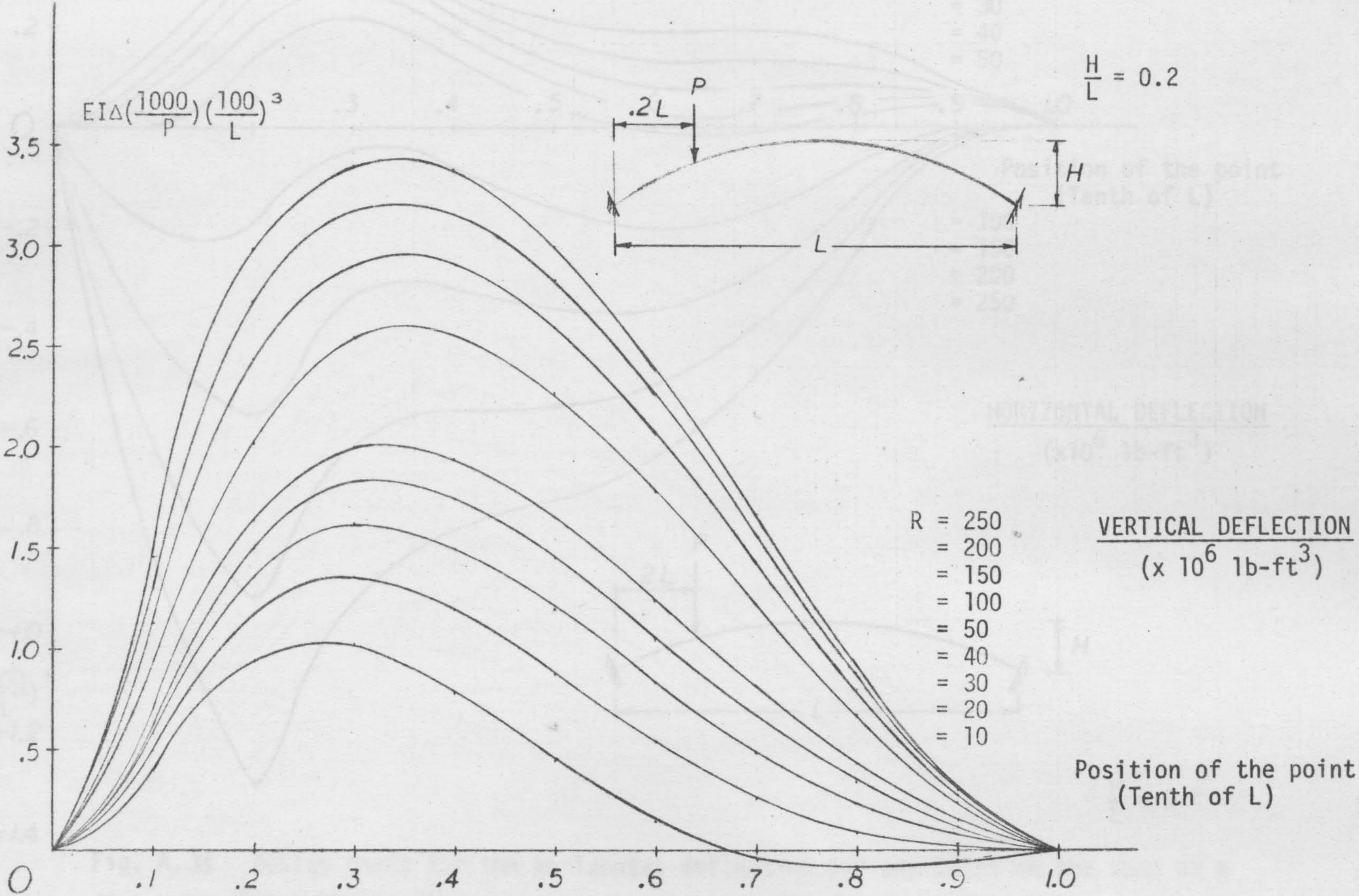


Fig. A.33 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.2)with the position of load at .2 the span of the arch.

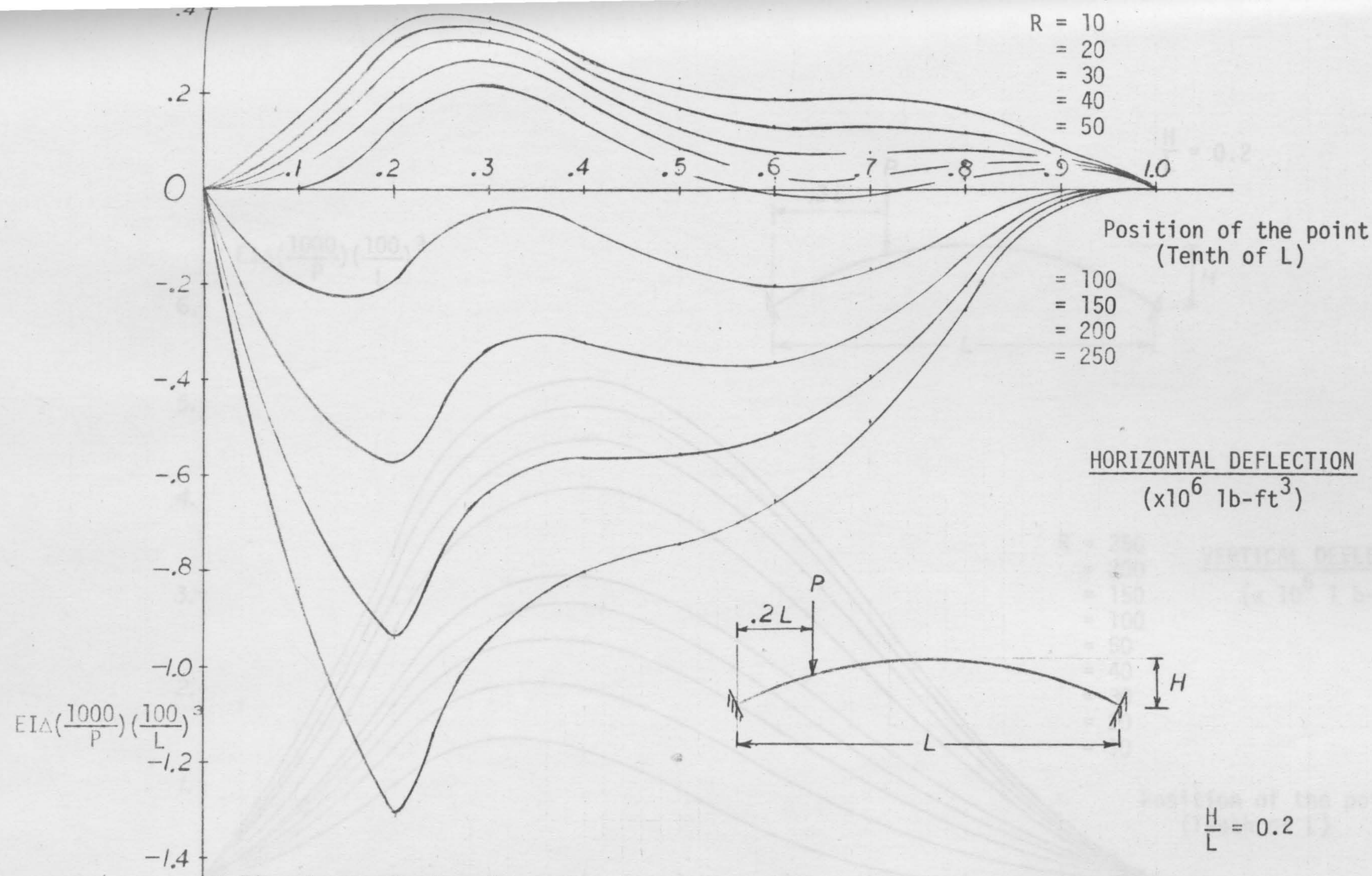


Fig. A. 34 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.2) with the position of load at .2 the span of the arch.

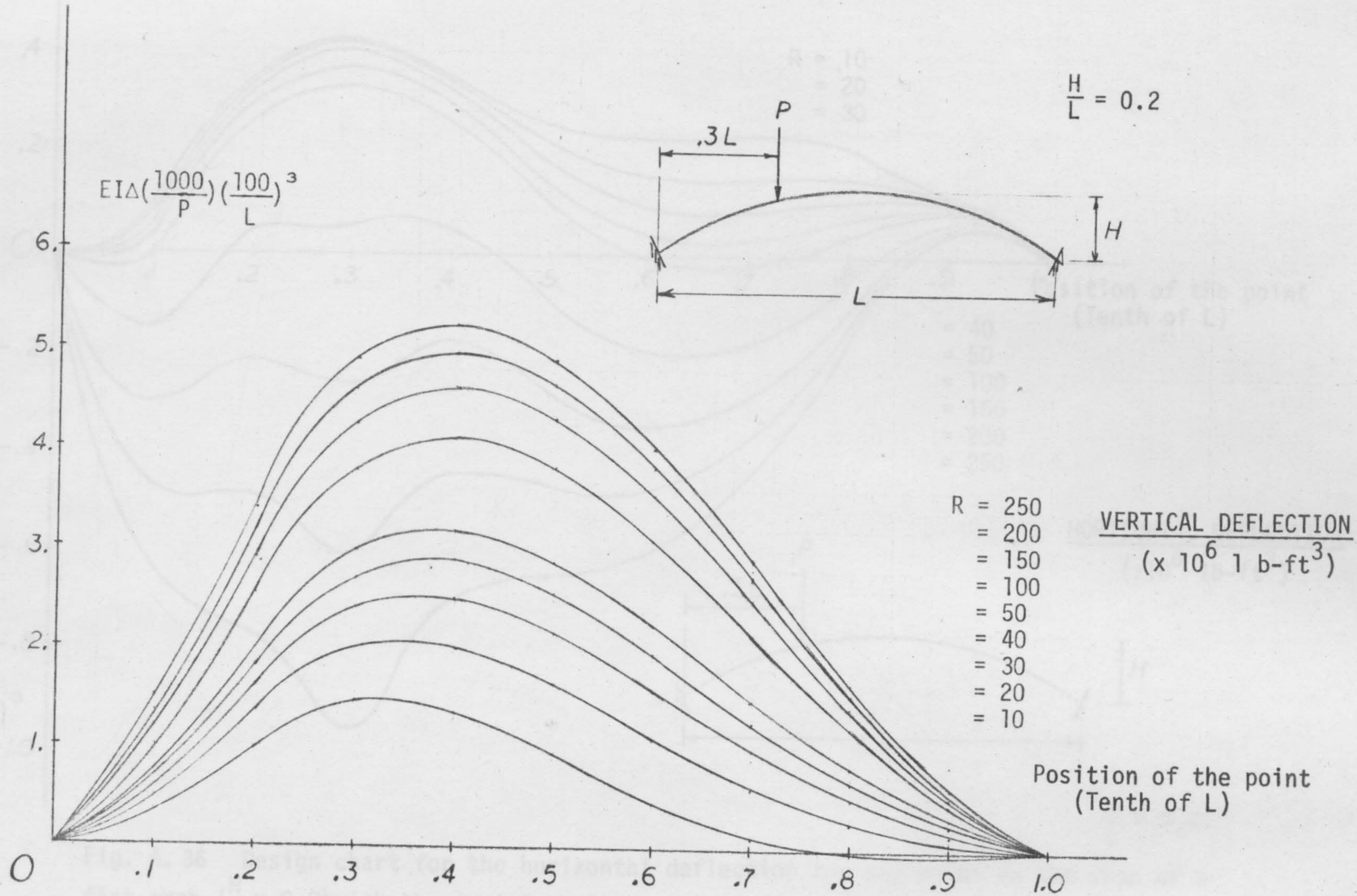


Fig. A. 35 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.2) with the position of load at .3 the span of the arch.

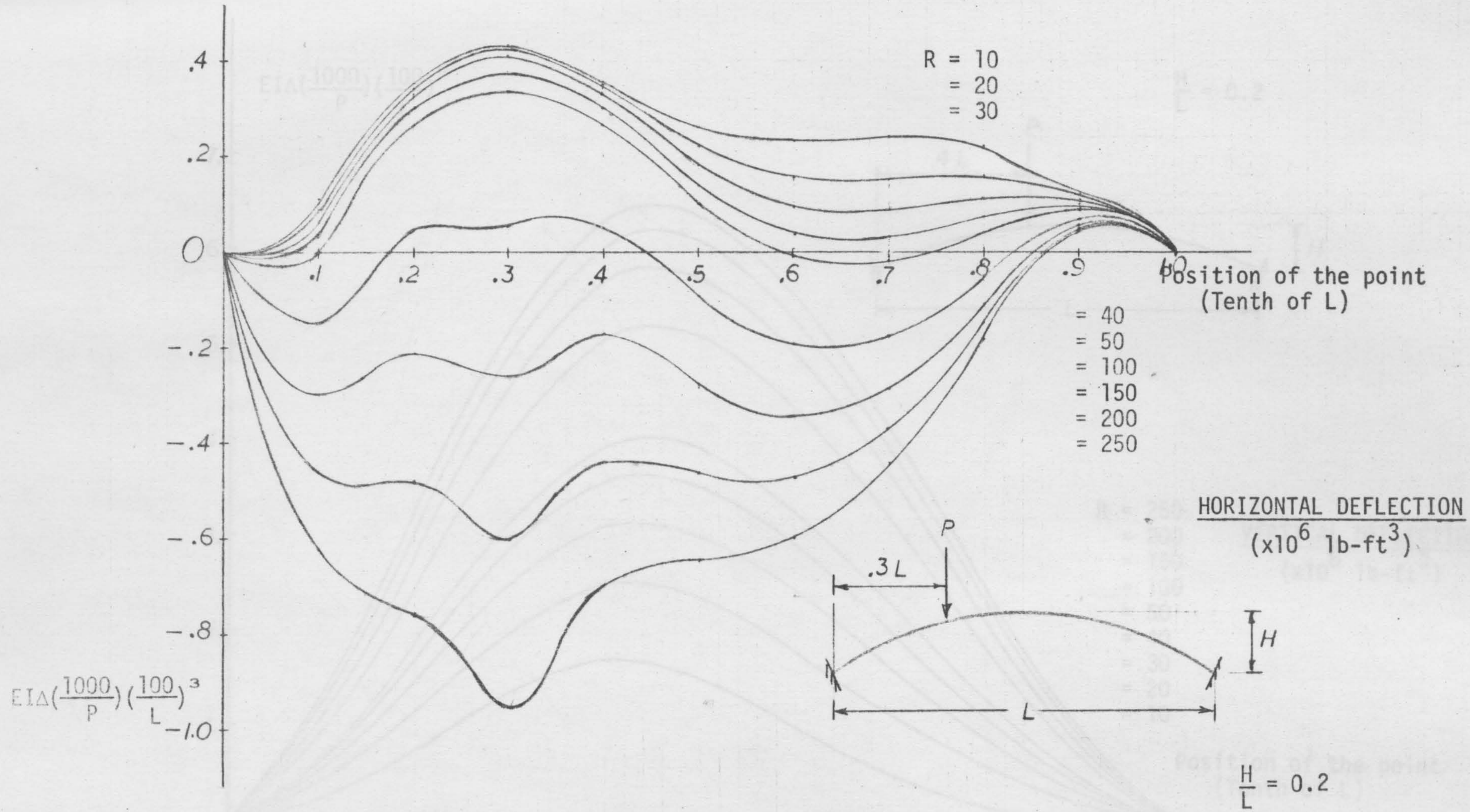


Fig. A. 36 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.2) with the position of load at .3 the span of the arch.

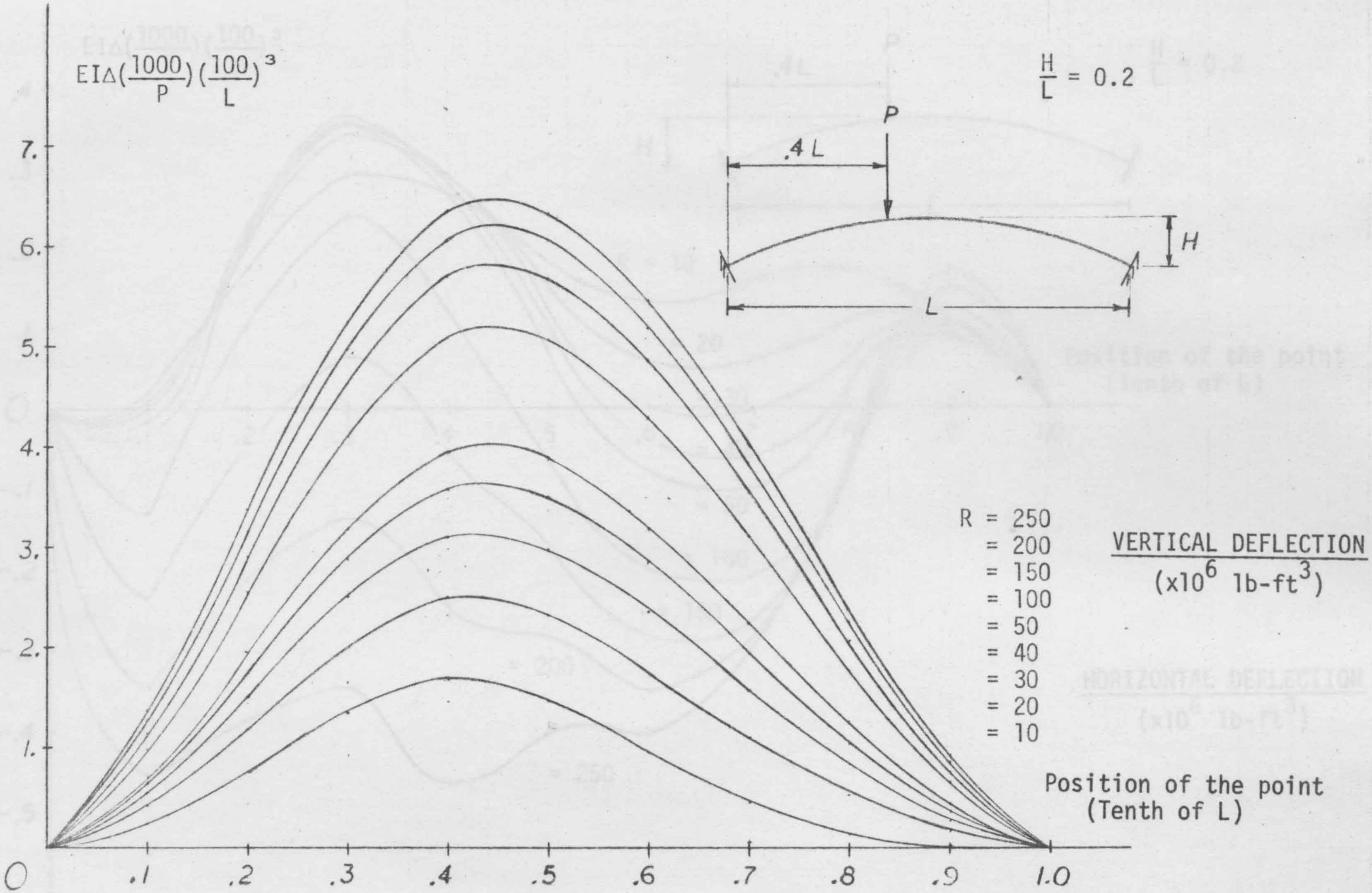


Fig. A.37 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.2) with the position of load at .4 the span of the arch.

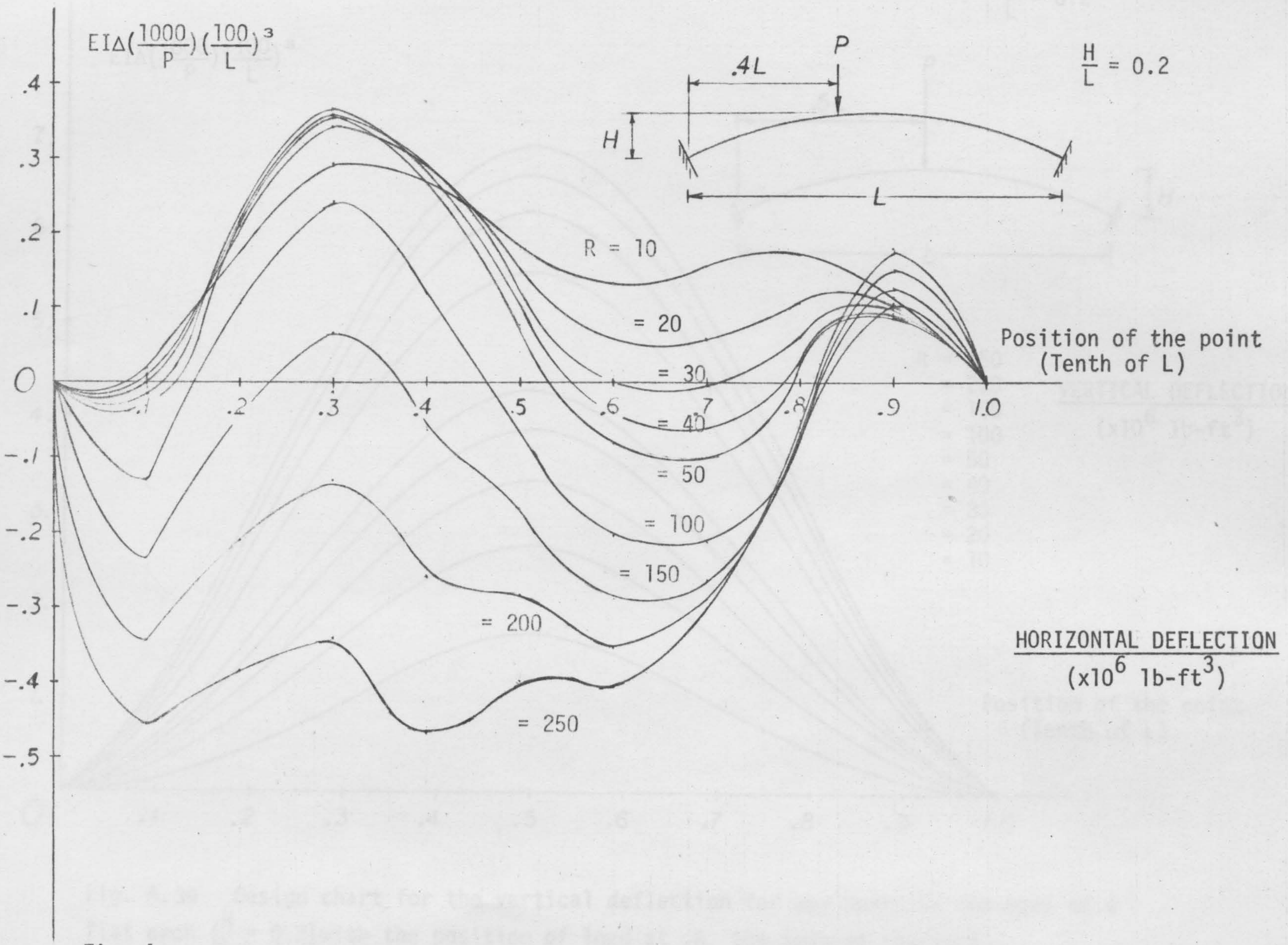


Fig. A.38 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.2) with the position of load at .4 the span of the arch.

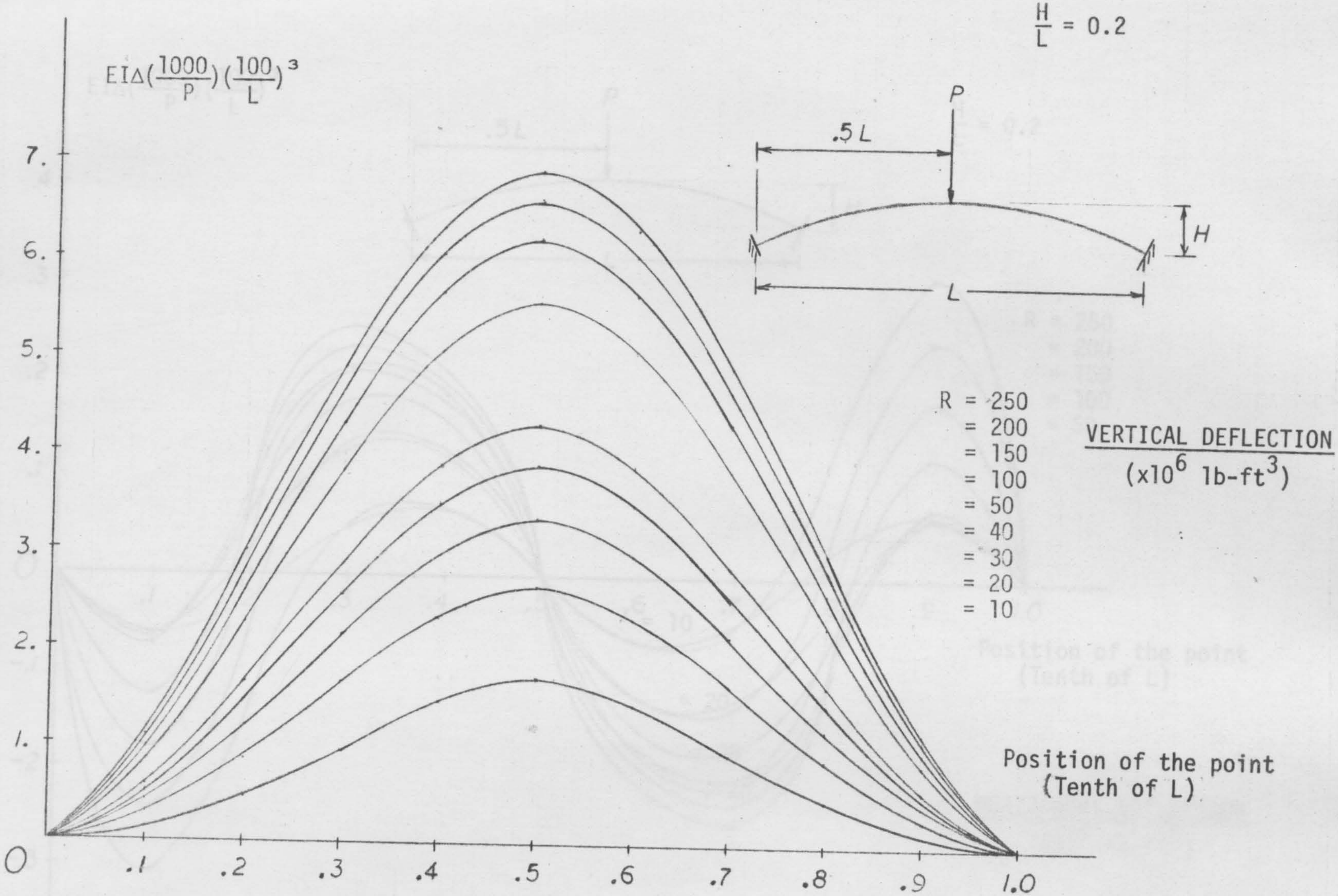


Fig. A.39 Design chart for the vertical deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.2) with the position of load at .5 the span of the arch.

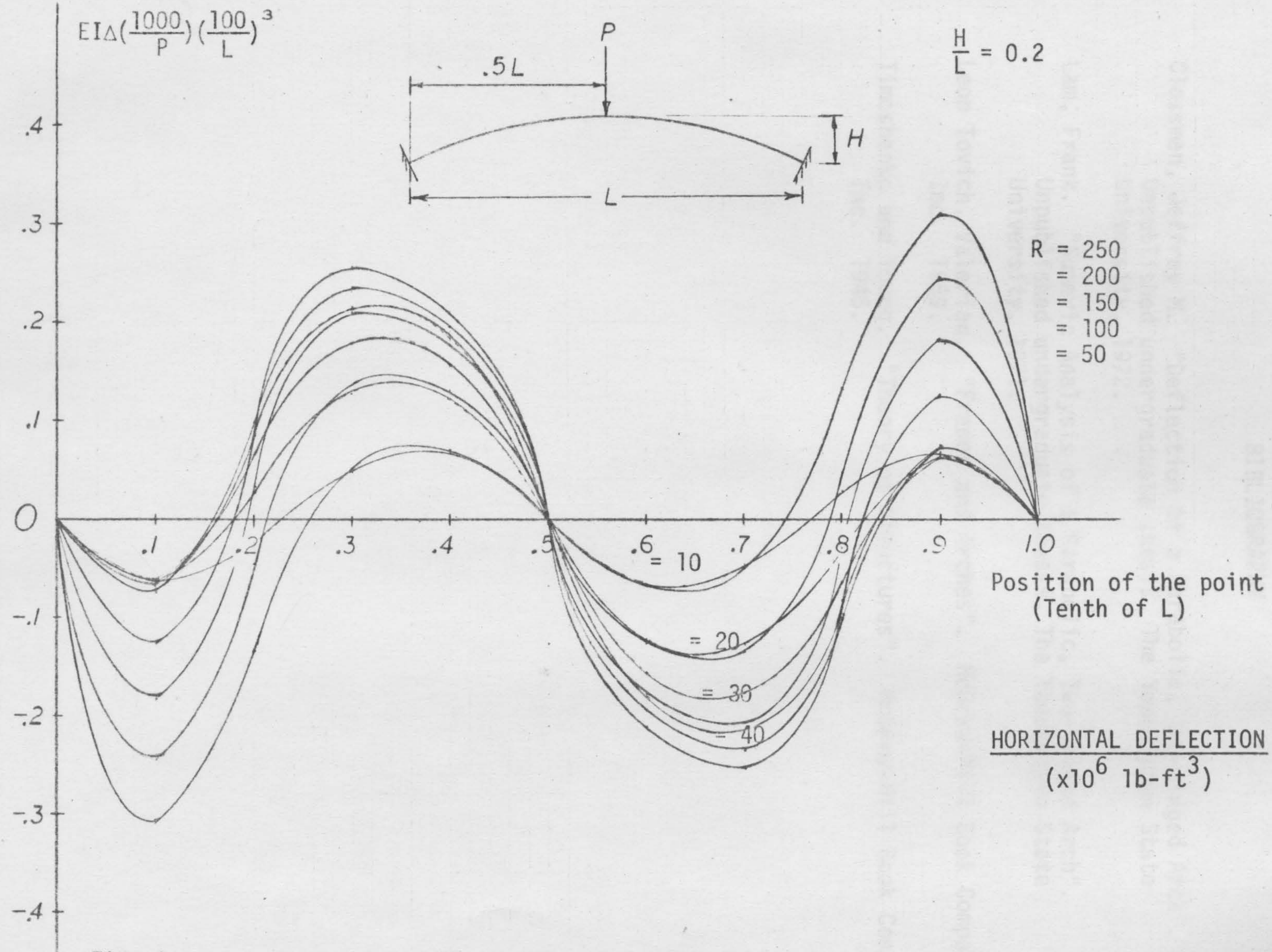


Fig. A.40 Design chart for the horizontal deflection for any point on the span of a flat arch ($\frac{H}{L}$ = 0.2) with the position of load at .5 the span of the arch.

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