DESIGN AND ANALYSIS OF A DYNAMIC BALANCER

by

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ABSTRACT

DESIGN AND ANALYSIS OF A DYNAMIC BALANCER

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Master of Science

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This thesis establishes the design and analysis of a dynamic balancer. The balancing machine was developed as a Mechanical Engineering Graduate Project at Youngstown State University by the author during the winter of 1980.

The balancing machine rotates a wheel at 900 rpm. The wheel is supported by a shaft mounted on two soft support bearings. An accelerometer at the soft supports is used to detect vibrations caused by unbalance. By trial and error, balancing weights are attached to the wheel to annihilate vibrations at each support. Knowing the balancing weights added and their locations enables the determination of corrective balance weights to be established.

The analysis portion establishes the theory and requirements to determine the corrective balance weights.

Components and design of the machine are discussed. In addition, equipment limitations are established.

The actual operation procedure is discussed with an example given.

Recommendations for improving the operating range of the machine are discussed.

Conclusions deal with measuring vibrations and emphasize the importance of reducing drive-induced vibrations.

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LIST OF SYMBOLS

SYMBOL	DEFINITION
X, X_a, X_b, h, z, L_1, L_2	length (in.,cm.,etc)
Δ_{peak}	peak displacement (cm)
θ, θ_a, θ_b, θ_F, θ_M, θ_L, θ_R, θ_1, θ_2	angles (degrees,radians)
V peak	peak velocity (cm/sec)
r, r_a, r_b	radial distance (cm.,in.,etc.
a peak	peak acceleration (cm/sec^2)
F, F_x, F_y, F_b, F_a, $\tilde{F}$	force (lbs.,dyne,etc.)
$L_{V\Delta}$, L_{VV}, L_{Va}	recorded voltage levels, db
m, m_a, m_1, m_2, m_b	mass (slugs or grams)
V_Δ, V_V, V_a	voltage (volts)
M, M_x, M_y, $\bar{M}$	moments (in·lbs.,dyne·cm.)
R_L, R_R, R_{XR}, R_{YR}, R_{XL}, R_{YL}	reaction forces (lbs.,dyne)
$\theta_{\bar{F}}$	angles (degrees or radians)
t	time (seconds)
ζ	damping factor
ω, ω_n	angular frequency (rad/sec)
K	constant
CW_1, CW_2, CW_R, CW_F	correction weights (gm)
MF	magnification factor
$\bar{r}$	ω/ω_n

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CHAPTER I

INTRODUCTION

Problem Statement

The problem is to establish the analysis and design of a dynamic balancer. The machine should have the capability of detecting a minimum unbalance of 0.1 oz. in., and provide sufficient room for a wheel up to 10 in. in diameter and 3 pounds of weight. The operating speed should be greater than 360 rpm. In addition, limitations of the equipment should be identified.

Purpose

The purpose of this thesis is to identify the analysis, design, and operating procedure of the dynamic balancer developed as a Mechanical Engineering Graduate Project at Youngstown State University by the author during the winter of 1980. This balancing machine may be used by students investigating the theory of dynamic unbalance.

Procedure

The balancing machine is composed of two soft bearing supports, a drive motor, and a wheel support. An acceleromotor is attached to the soft bearings for taking vibration readings. By trial and error, balancing weights are attached to the unbalanced wheel until minimal vibration readings are obtained at each support. Knowing the balancing weights added and their location, total corrective balancing weights can be added.

CHAPTER II

MACHINE DESCRIPTION

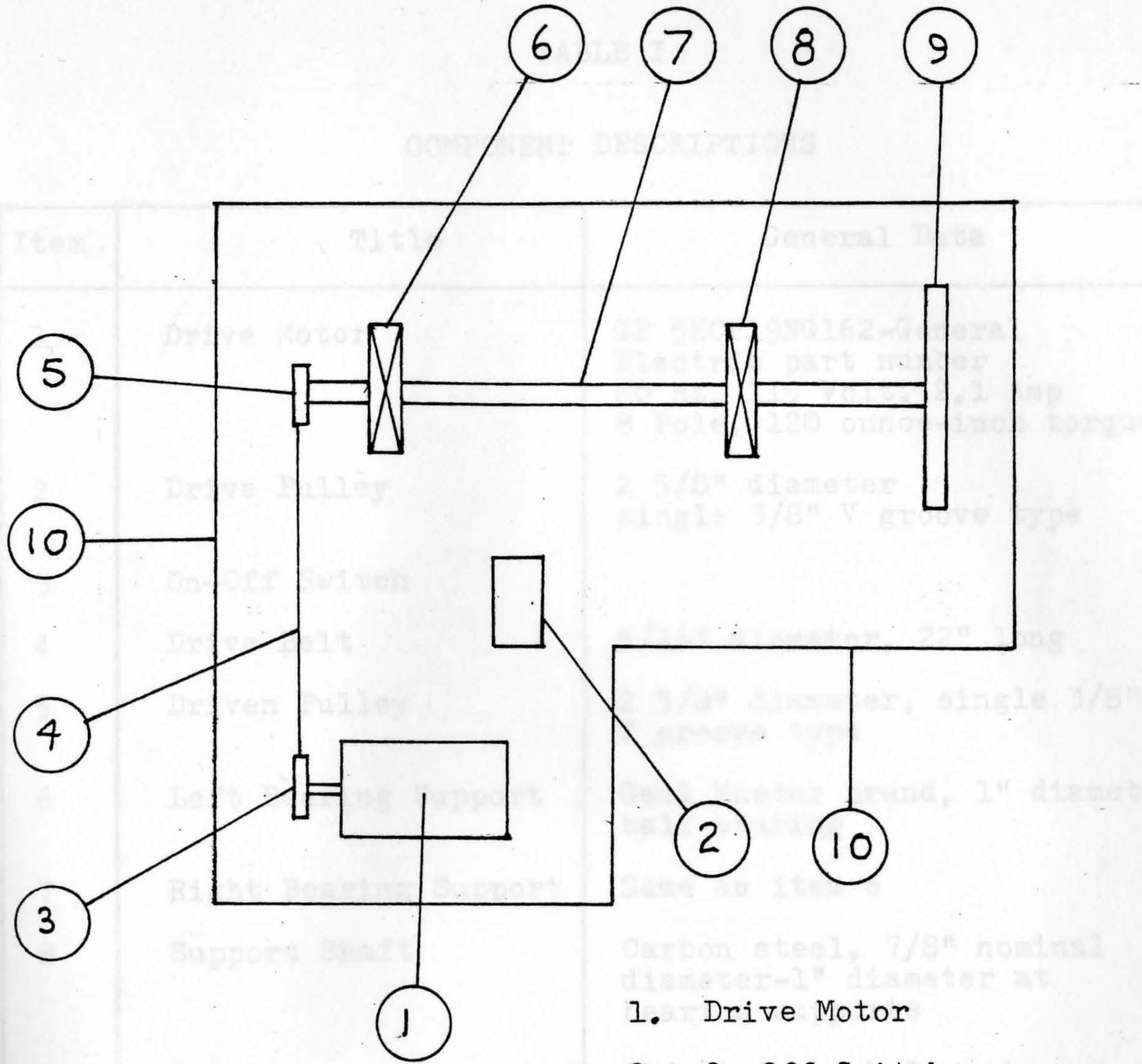
This section describes the balancing machine arrangement and identifies the various components.

Refer to Figure 1 for the overall machine arrangement and component identifications. Also, refer to Table 1 for general component data.

The bearing supports are mounted on special so-called soft supports. This type of support is intended to provide flexible support in the vertical direction only. An acceleromotor is attached to each support to provide vibration measurements.

The round rubber drive belt is used to provide rotation of the support shaft without adding significant error vibrations.

Operating speed of the shaft was selected to be compatible with recording equipment. This speed is 900 rpm.



1. Drive Motor
2. On-Off Switch
3. Drive Pulley
4. Drive Belt
5. Driven Pulley
6. R_L (Left Soft Support)
7. Support Shaft
8. R_R (Right Soft Support)
9. Object to be Balanced
10. Base

Fig. 1.. "Arrangement and Equipment Identification"

Plan view of the balancing machine with component identifications.

TABLE I

COMPONENT DESCRIPTIONS

Item	Title	General Data
1	Drive Motor	GE 5KCP19NG162-General Electric part number 60 Hz, 115 Volt, 2.1 Amp 8 Pole, 120 ounce-inch torque
2	Drive Pulley	2 5/8" diameter single 3/8" V groove type
3	On-Off Switch	
4	Drive Belt	5/16" diameter, 22" long
5	Driven Pulley	2 5/8" diameter, single 3/8" V groove type
6	Left Bearing Support	Seal Master brand, 1" diameter ball bearing
7	Right Bearing Support	Same as item 6
8	Support Shaft	Carbon steel, 7/8" nominal diameter-1" diameter at bearing supports
9	Wheel	Object to be balanced
10	Base	Welded ⊏8 x 11.5*

* ⊏8 x 11.5 is an American standard channel 8.0 inches in height weighing 11.5 lbs/Foot.

CHAPTER III

THEORY OF ANALYSIS

Unbalance of the Part

Refer to Figure 2 for identification of coordinates.

Let us assume that the part to be balanced has a center of mass not located at z=0, r=0 or, $(0,\theta,0)$. In addition, the principal axis of inertia is not parallel or perpendicular to the z axis.

It should be noted that when the center of mass is located at $(0,\theta,0)$, the wheel is said to be statically balanced. However, not until the wheel is both statically balanced and has its principal axis of inertia parallel or perpendicular to the axis of spin is the part considered dynamically balanced.[1]

At this point in time we shall assume that two masses at two different locations can be arranged to produce the equivalent unbalance generated by the wheel.[2] These masses shall be designated m_a for the first mass with location (r_a,θ_a,z_a) and m_b for the second mass with location (r_b,θ_b,z_b).

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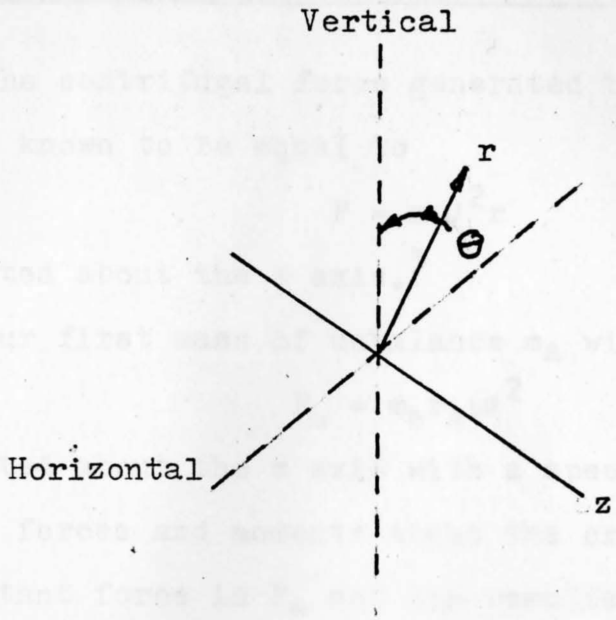
[1]Sanford Walton Groesberg, Advanced Mechanics (New York, N.Y.: John Wiley & Sons Inc. 1968) p. 162.

[2]S. Timoshenko, Vibration Problems in Engineering rev. by D.H. Young and S. Timoshenko (3rd. ed.: Princeton, New Jersey: D. Van Nostrand Company Inc. 1955) p. 60.



z coordinate-the z axis line along the center of the support shaft with an origin located at the center of the wheel to be balanced.

θ coordinate-is the angular measurement from an arbitrarily established line marked on the wheel to be balanced.

r coordinate-is the actual distance measured radially from the z axis to the point to be identified.

Fig. 2 "Identification of Coordinates"

Theory of Applied Forces and Moments at the Origin

The centrifugal force generated by a rotation of an object is known to be equal to

$$F = m\omega^2 r \quad (1)$$

when rotated about the z axis.[3]

Our first mass of unbalance m_a will produce a force

$$F_a = m_a r_a \omega^2 \quad (2)$$

when rotated about the z axis with a speed of rotation ω. If we sum forces and moments about the origin, we see that the resultant force is F_a and the resultant moment is $F_a z_a$ both with an angular orientation Θ_a.

$$M_a = F_a z_a \quad (3)$$

Similarly, our second mass m_b will produce a force F_b and resultant moment M_b.

$$F_b = m_b r_b \omega^2 \quad (4)$$

$$M_b = F_b z_b \quad (5)$$

at the origin.

The next step leads us to the next concept of applied force and moment. We simply add the two resultant forces F_a and F_b to obtain an equivalent applied force F with an angle Θ_F.

$$F = F_a \overset{\rightarrow}{+} F_b \quad (6)$$

Similarly, we can combine the moments to obtain an equivalent applied moment M with an angle Θ_M. It is obvious that the assumption of two particles made in Unbalance of the Part

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[3]Ferdinand L. Singer, Engineering Mechanics (3rd. ed.; New York, N.Y.: Harper & Row Publishers, 1975) p. 396.

section could be extended to an infinite number of particles. This leads us to the problem of determining these forces, moments, and applied angles. Once these values are known we can simply add balancing weights to produce a state of dynamic balance.

In an effort to simplify our calculations we shall take vibration measurements in the vertical direction only. It is therefore important to express both F and M in a convenient form. Refer to Figures 3 and 4.

$$\bar{F} = F_x \cos\omega t + F_y \sin\omega t \quad (7)$$

$$\bar{M} = M_x \cos\omega t + M_y \sin\omega t \quad (8)$$

$$F = \sqrt{F_x^2 + F_y^2} \quad (9)$$

$$M = \sqrt{M_x^2 + M_y^2} \quad (10)$$

$$\theta_F = \arctan\left\{\frac{F_y}{F_x}\right\} \quad (11)$$

$$\theta_M = \arctan\left\{\frac{M_y}{M_x}\right\} \quad (12)$$

Machine Analysis

The assumptions of the analysis are as follows:

1.) The effect of gravity is small in comparison to the centrifugal acceleration.

2.) The support shaft is very stiff and will not deflect.

3.) Motion of the soft support not being measured is zero due to fixation.

Refer to Figure 5 for a free body diagram of the

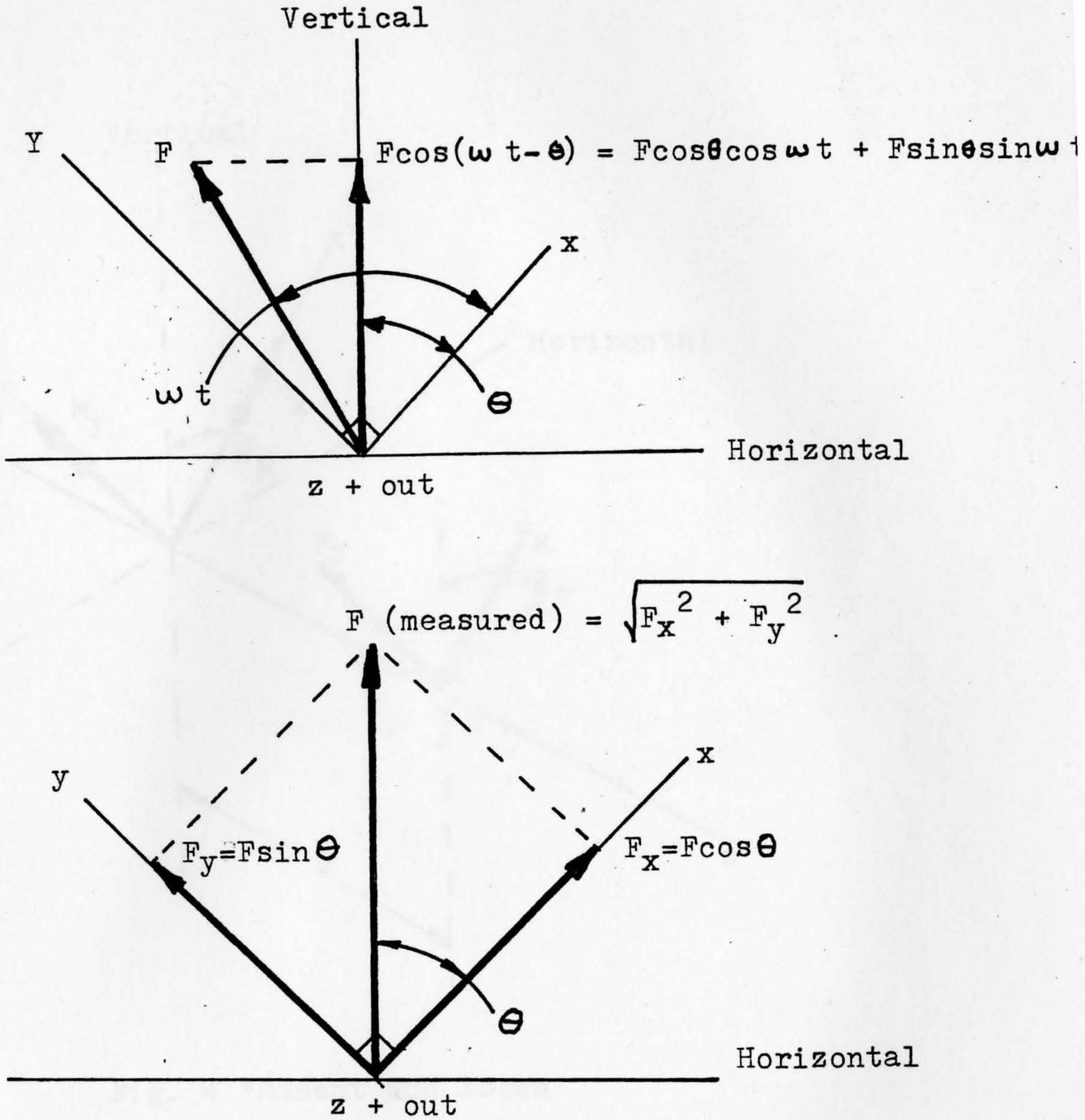

Fig. 3 "Force Coordinates"

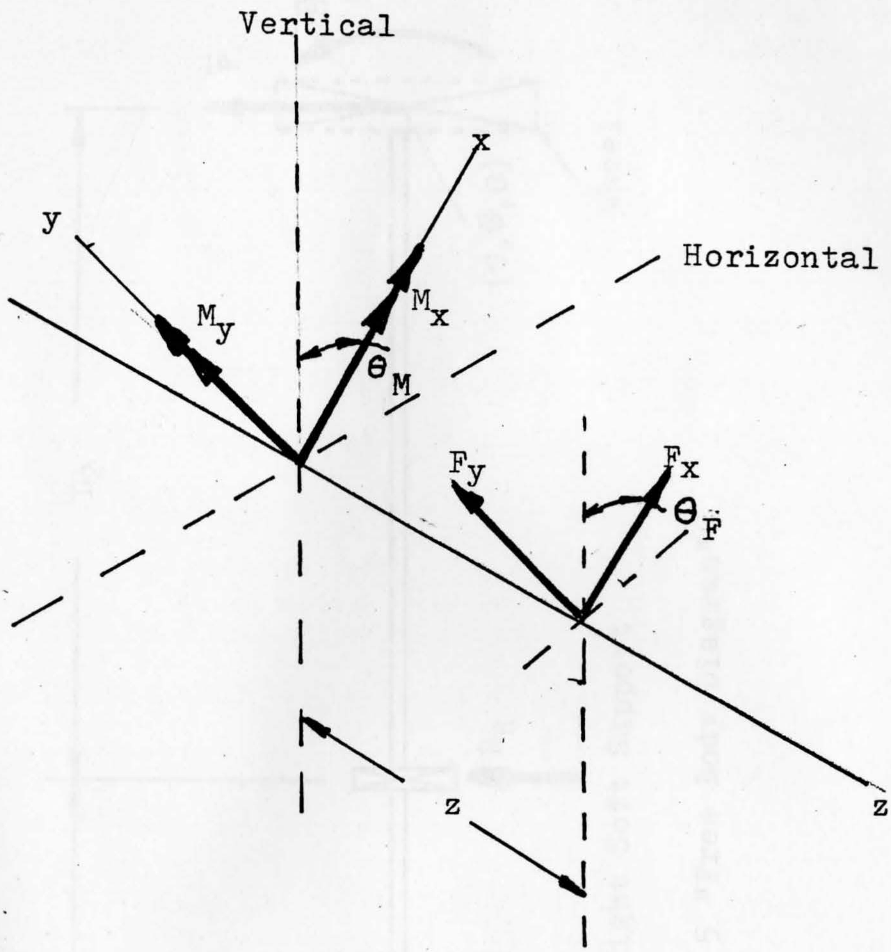

Fig. 4 "Moment and Force"

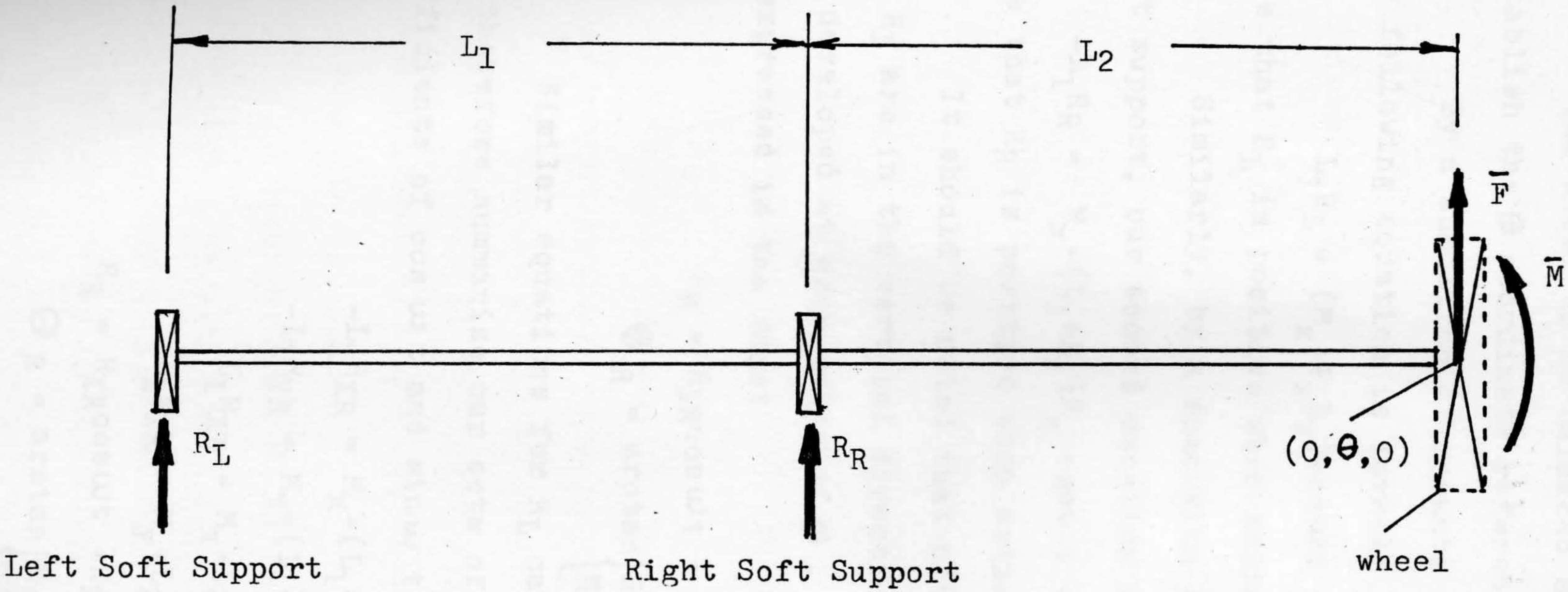

Fig. 5 "Free Body Diagram"

analysis.

The wheel to be balanced should first be marked to establish the Θ coordinate reference.

By a summation of moments about the right soft support the following equation is formed.

$$L_1R_L = (M_x+F_xL_2)\cos\omega t + (M_y+L_2F_y)\sin\omega t \qquad (13)$$

Note that R_L is positive when acting upward.

Similarly, by a summation of moments about the left soft support, our second equation is formed.

$$-L_1R_R = M_x+(L_1+L_2)F_x \cos\omega t + M_y+(L_1+L_2)F_y\sin\omega t \qquad (14)$$

Note that R_R is positive when acting upward.

It should be noted that although our readings of R_R and R_L are in the vertical direction, their maximum values are developed at some value of Θ. Therefore, R_R and R_L can be expressed in the form:

$$R_R = R_{XR}\cos\omega t + R_{YR}\sin\omega t \qquad (15)$$

$$\Theta_R = \arctan\left\{\frac{R_{YR}}{R_{XR}}\right\} \qquad (16)$$

Similar equations for R_L can also be written. We can therefore summarize our sets of equations by grouping coefficients of $\cos\omega t$ and $\sin\omega t$ as follows:

$$-L_1R_{XR} = M_x+(L_1+L_2)F_x \qquad (17)$$

$$-L_1R_{YR} = M_y+(L_1+L_2)F_y \qquad (18)$$

$$L_1R_{XL} = M_x+L_2F_x \qquad (19)$$

$$L_1R_{XR} = M_y+L_2F_y \qquad (20)$$

$$R_R = R_{XR}\cos\omega t + R_{YR}\sin\omega t \qquad (21)$$

$$\Theta_R = \arctan\left\{\frac{R_{YR}}{R_{XR}}\right\} \qquad (22)$$

$$R_L = R_{XL}\cos\omega t + R_{YL}\sin\omega t \quad (23)$$

$$\theta_L = \arctan\left\{\frac{R_{YR}}{R_{XR}}\right\} \quad (24)$$

There are now eight equations and twelve unknowns. It will be necessary to obtain four pieces of information. The information used by the vibration readings will enable R_L, θ_L, R_R and θ_R to be obtained.

The procedure for determining these values shall be to add balancing weights until vibration readings at the soft bearing support under investigation approaches zero. For the left soft support, the addition of balancing weights on the rotating wheel will result in an additional equation which is the summation of moments about the right soft support.

$$-\cos(\omega t-\theta_1)L_2 m_1 \omega^2 r_1 = |R_L| L_1 \cos(\omega t-\theta_L) \quad (25)$$

The subscript 1 on m, r, θ will apply to the mass added to minimize R_L.

From this data both R_L and θ_L can be determined. A similar method shall be used to determine R_R and θ_R. We simply add balancing weights to the rotating part until vibration readings at the right soft support approach zero and then sum moments about the left soft support. The following equation develops:

$$R_R L_1 \cos(\omega t-\theta_R) = (L_1+L_2)m_2 \omega^2 r_2 \cos(\omega t-\theta_2) \quad (26)$$

The subscript 2 on m, r, θ will apply to the mass added to minimize R_R. Thus we can now determine R_R and θ_R.

The next step in the analysis is to solve for our unknown quantities $F_x, F_y, M_x, M_y, R_{RX}, R_{RY}, R_{LX}$ and R_{LY} which total eight. By using equations 21, 22, 23, and 24, R_{RX}, R_{RY}, R_{LX}, and R_{LY} can be expressed directly as:

$$R_{RX} = R_R \cos\theta_R \tag{27}$$

$$\theta_R = \theta_2 \tag{28}$$

$$R_{RY} = R_R \sin\theta_R \tag{29}$$

$$R_{LX} = R_L \cos\theta_L \tag{30}$$

$$\theta_L = \theta_1 + \pi \tag{31}$$

$$R_{LY} = R_L \sin\theta_L \tag{32}$$

We can now solve for M_x, F_x, M_y, and F_y using equations 17, 18, 19 and 20. In addition, M, F, θ_F and θ_M can be determined using equations 9, 10, 11, and 12.

Simplification of the preceeding equations will allow us to solve for M, F, θ_M and θ_F directly by using the following equations:

$$M_x = \frac{\begin{vmatrix} -(L_1+L_2)m_2\omega^2 r_2\cos\theta_2 & (L_1+L_2) \\ -L_2 m_1\omega^2 r_1\cos\theta_1 & L_2 \end{vmatrix}}{\begin{vmatrix} 1 & (L_1+L_2) \\ 1 & L_2 \end{vmatrix}} \tag{33}$$

$$M_y = \frac{\begin{vmatrix} -(L_1+L_2)m_2\omega^2 r_2\sin\theta_2 & (L_1+L_2) \\ -L_2 m_1\omega^2 r_1\sin\theta_1 & L_2 \end{vmatrix}}{\begin{vmatrix} 1 & (L_1+L_2) \\ 1 & L_2 \end{vmatrix}} \tag{34}$$

$$F_x = \frac{\begin{vmatrix} 1 & -(L_1+L_2)m_2\omega^2 r_2\cos\theta_2 \\ 1 & -L_2 m_1\omega^2 r_1\cos\theta_1 \end{vmatrix}}{\begin{vmatrix} 1 & (L_1+L_2) \\ 1 & L_2 \end{vmatrix}} \quad (35)$$

$$F_y = \frac{\begin{vmatrix} 1 & -(L_1+L_2)m_2\omega^2 r_2\sin\theta_2 \\ 1 & -L_2 m_1\omega^2 r_1\sin\theta_1 \end{vmatrix}}{\begin{vmatrix} 1 & (L_1+L_2) \\ 1 & L_2 \end{vmatrix}} \quad (36)$$

Procedure for Determining Balance Weights

Refer to Figure 6 for a description of the unbalanced part. To achieve the dynamic balance of the part, we shall use the two-plane method.

The first step of our solution is to determine the correction weights for balancing the F portion of the unbalance. By summation of forces the correction weight, CW, can be found:

$$CW_1 = \frac{|F|}{2\omega^2 h} \quad (37)$$

One weight CW_1 would be placed at the front plane and one

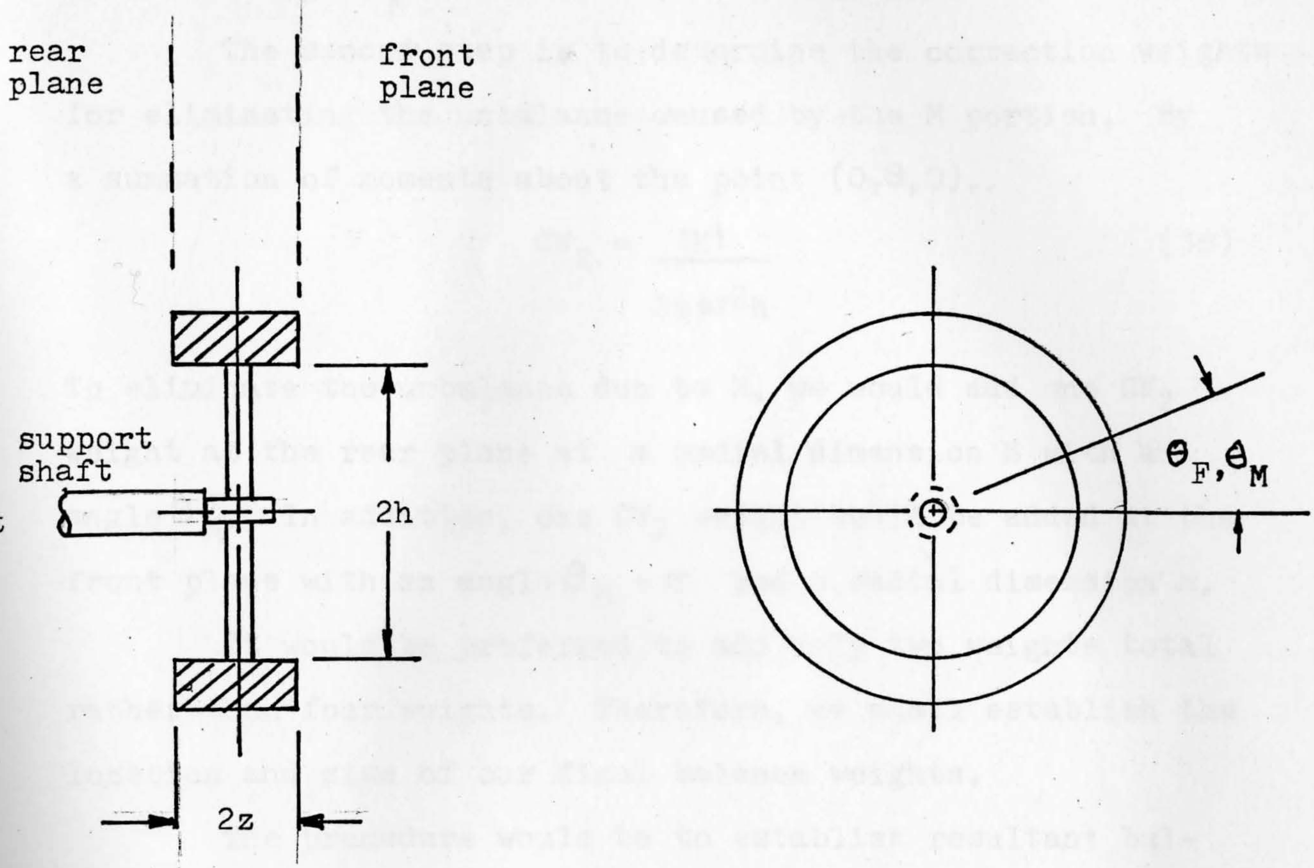

Fig. 6 "Part Description"

weight CW_1 would be placed at the rear plane at a dimension h with an angle $\theta_F + \pi$ to eliminate the unbalance F.

The second step is to determine the correction weights for eliminating the unbalance caused by the M portion. By a summation of moments about the point $(0, \theta, 0)$..

$$CW_2 = \frac{|M|}{2z\omega^2 h} \tag{38}$$

To eliminate the unbalance due to M, we would add one CW_2 weight at the rear plane at a radial dimension h with an angle θ_M. In addition, one CW_2 weight would be added at the front plane with an angle $\theta_M + \pi$ and a radial dimension h.

It would be preferred to add only two weights total rather than four weights. Therefore, we shall establish the location and size of our final balance weights.

The procedure would be to establish resultant balancing forces at the front and rear planes. The following equations result by summation of forces at the front plane and rear plane:

$$CW_R = \left\{(CW_1\cos(\pi + \theta_F) + CW_2\cos(\theta_M))^2 + (CW_1\sin(\pi + \theta_F) + CW_2\sin(\theta_M))\right\}^{1/2} \tag{39}$$

$$\theta_R = \arctan\left\{\frac{CW_1\sin(\theta_F + \pi) + CW_2\sin(\theta_M)}{CW_1\cos(\theta_F + \pi) + CW_2\cos(\theta_M)}\right\} \tag{40}$$

$$CW_F = \left\{(CW_1\cos(\theta_F + \pi) + CW_2\cos(\theta_M + \pi))^2 + (CW_1\sin(\theta_F + \pi) + CW_2\sin(\theta_M + \pi))\right\}^{1/2} \tag{41}$$

$$\theta_F = \arctan\left\{\frac{CW_1\sin(\theta_F + \pi) + CW_2\sin(\theta_M + \pi)}{CW_1\cos(\theta_F + \pi) + CW_2\cos(\theta_M + \pi)}\right\} \tag{42}$$

CW_F is the correction weight to be added to the front plane at a radial distance h and angular position θ_F.

CW_R is the correction weight to be added to the rear plane at a radial distance h and angular position θ_R.

CHAPTER IV

MACHINE DESIGN

The dynamic balancing machine design was based on standard engineering criteria. This criteria is economics, availability of components, anticipated use, limitations of auxiliary equipment and knowledge.

The auxiliary equipment referred to above has a frequency range of 6.3 to 20,000 Hz. With this limitation in mind, as well as availability of components and economics, a 900 rpm, 110 volt drive motor was selected. With the 900 rpm speed, the lower filtering range of the frequency analyzer permitted this speed to lie within its 6.3 to 20 Hz range. Thus, higher frequency vibrations could be filtered out permitting the 900 rpm, 15 Hz, vibrations to be measured accurately.

The next consideration of the design was given to the maximum anticipated unbalance to be accounted for. This quantity was established after the purchase of a small unbalanced tire. Trial and error adjustment of the spring rate of the soft supports resulted in a maximum safe unbalance of 2.1 inch-ounces. The spring rate of the soft supports was adjusted to achieve detectable recordings at a level of 0.1 inch-ounce unbalance.

Figure 7 shows the arrangement of the soft supports. Deflection measurements of the soft support were taken with

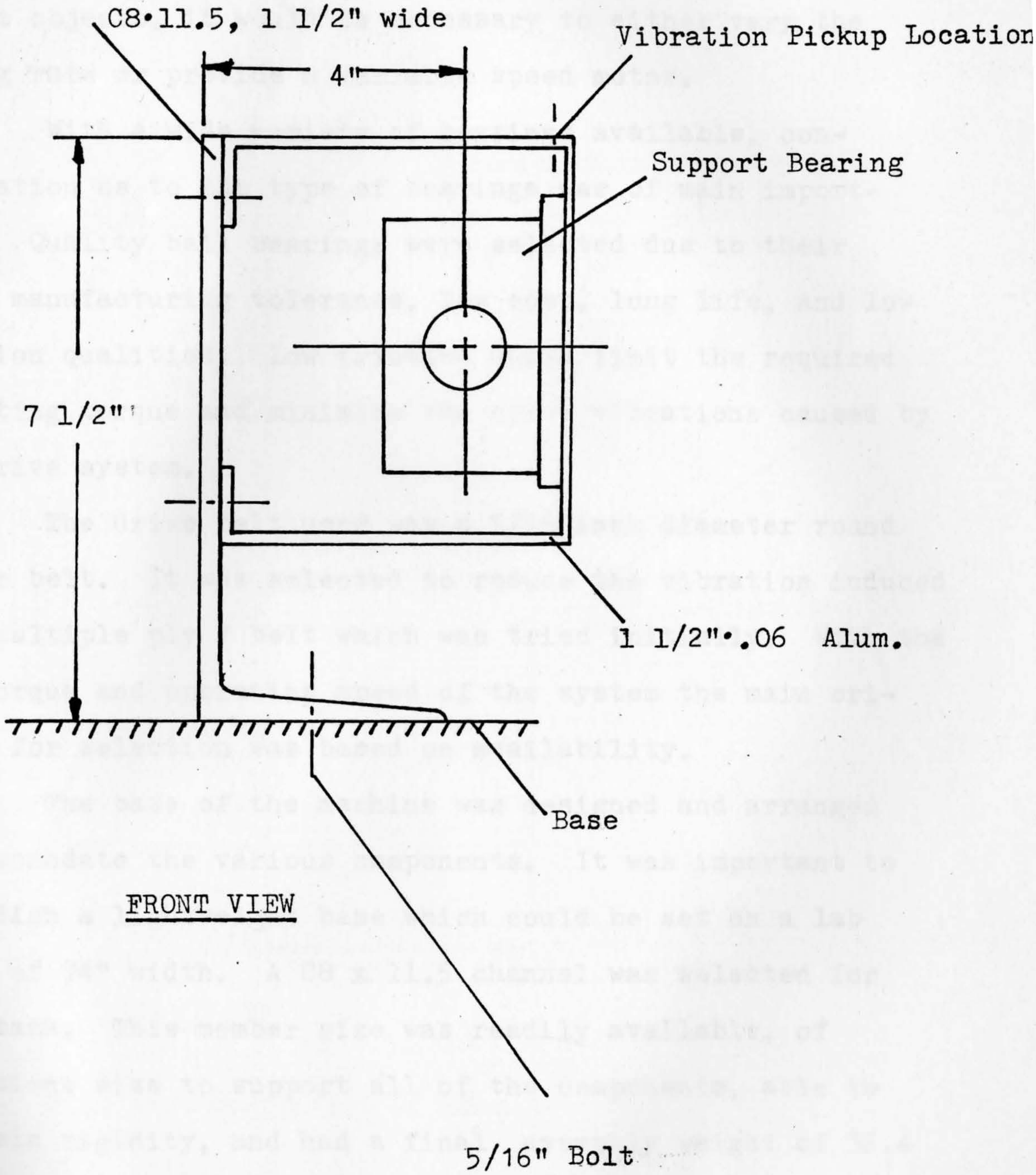

Fig. 7 "Soft Support Arrangement" Front view of the soft supports with component data.

a spring scale and a spring rate of 256 pounds per inch was measured. As previously mentioned, the spring rate was adjusted to optimize performance. In order to balance various weight objects, it would be necessary to either vary the spring rate or provide a variable speed motor.

With a wide variety of bearings available, consideration as to the type of bearings was of main importance. Quality ball bearings were selected due to their close manufacturing tolerance, low cost, long life, and low friction qualities. Low friction would limit the required operating torque and minimize the error vibrations caused by the drive system.

The drive belt used was a 5/16 inch diameter round rubber belt. It was selected to reduce the vibration induced by a multiple ply V belt which was tried initially. With the low torque and operating speed of the system the main criteria for selection was based on availability.

The base of the machine was designed and arranged to accomodate the various components. It was important to establish a lightweight base which could be set on a lab table of 34" width. A C8 x 11.5 channel was selected for this task. This member size was readily available, of sufficient size to support all of the components, able to maintain rigidity, and had a final assembly weight of 36.4 pounds. Figure 8 is an arrangement drawing of the base.

The next part of the design was to establish the dimensions L_1 and L_2. Refer to Figure 5. Due to space limitations, it was decided to limit $L_1 + L_2$ to 25". Refer

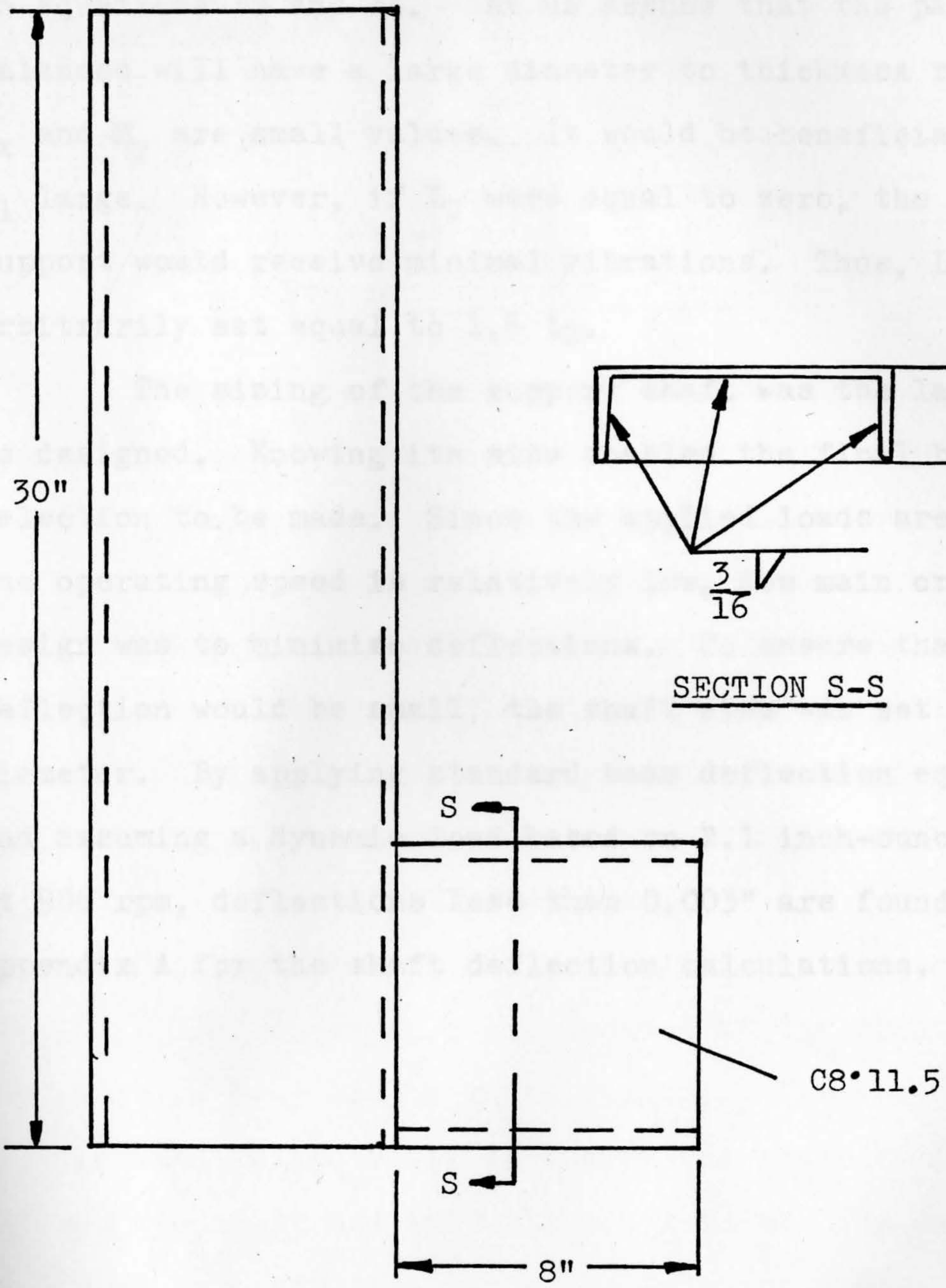

Fig. 8 "Arrangement of Machine Base"

to equations 33 and 34. Let us assume that the part to be balanced will have a large diameter to thickness ratio. Thus, M_x and M_y are small values. It would be beneficial to have L_1 large. However, if L_2 were equal to zero, the left soft support would receive minimal vibrations. Thus, L_1 was arbitrarily set equal to 1.5 L_2.

The sizing of the support shaft was the last item to be designed. Knowing its size enables the final bearing selection to be made. Since the applied loads are small and the operating speed is relatively low, the main criteria of design was to minimize deflections. To ensure that the shaft deflection would be small, the shaft size was set at a 1" diameter. By applying standard beam deflection equations, and assuming a dynamic load based on 2.1 inch-ounces unbalance at 900 rpm, deflections less than 0.003" are found.[4] See Appendix A for the shaft deflection calculations.

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[4]M.F. Spotts, Design of Machine Elements (4th. ed.: Englewood Cliffs, New Jersey: Prentice-Hall, 1971) p.21.

CHAPTER V

OPERATION PROCEDURE

Standard Operating Procedure

This section defines the standard operating procedure for balancing a wheel using the dynamic balancing machine. The procedure is broken into eleven main steps.

As with operating any machine, the user should first familiarize himself with the equipment and its limitations. Safety glasses should be worn at all times. Loose clothing should not be worn. Hands and measuring equipment should be kept away from rotating parts.

Step One

The first step is to securely mount the unbalanced wheel. A 1/2" diameter bolt, located at the right end of the shaft is used as the wheel support. The bolt should be tightened sufficiently to ensure the wheel bears evenly against the shaft end and will not rotate. The bolt should be locked into position using the bolt set screw.

Step Two

The wheel and shaft should be marked to establish the θ coordinate axis. By marking both the shaft and wheel, slippage can be detected during operation.

Step Three

Mount a previously calibrated accelerometer onto the right soft support. The analyzer should be set to measure peak displacement for a frequency of 15 Hz. A tapped hole at the bearing side of the support is provided for attaching the accelerometer.

Ensure the accelerometer lead wire is kept away from any of the rotating parts.

Step Four

Turn on the machine and record the displacement reading of the unbalanced part.

Step Five

Attach a 1.0 gram balance weight to the wheel rim. Attachment holes are provided on the rim for this purpose. Note the location, Θ , of the weight and the displacement reading. Repeat this procedure with the 1.0 gram weight mounted at 60°, 120°, 180°, 240°, and 300° from the first attachment weight. Note the displacement readings at each weight location. You will notice that some displacement readings are smaller and some are larger than the unbalanced wheel initial reading. The location with the smallest displacement reading is the closest to the final correction weight location.

Step Six

Remove all balancing weights from the tire and attach a 2 gram weight at the location of the weight with the lowest displacement reading of Step Five. Increase the balancing weight, 1 gram at a time, until the displacement reading is approximately 50 percent of the displacement reading of the unbalanced wheel.

Step Seven

Place the balancing weight developed in Step Six at different locations on the wheel and establish the location of minimum displacement. Do not position the weight more than 60^{o} from the location developed in Step Five.

Step Eight

Increase the balancing weight, with a location as indicated by Step Seven, 1.0 gram at a time until the displacement reading approaches zero. This weight and its location are defined as m_2 and θ_2.

Step Nine

Remount the accelerometer onto the left soft support, and repeat Steps Four thru Eight. The weight and its location are defined as m_1 and θ_1.

Step Ten

Solve for CW_R, θ_R, CW_F, and $\theta_{\bar{F}}$ using equations 9-12 and 33-42.

Step Eleven

Mount the rear correction weight, CW_R, at location θ_R and mount the front correction weight, CW_F, at location $\theta_{\bar{F}}$.

Take displacement readings at both the left and right soft supports. Both supports should have very small displacement vibrations indicating the wheel is dynamically balanced.

Example Wheel Balance

Refer to Figure 9 for a description of the wheel which was dynamically balanced for this example. The lock-

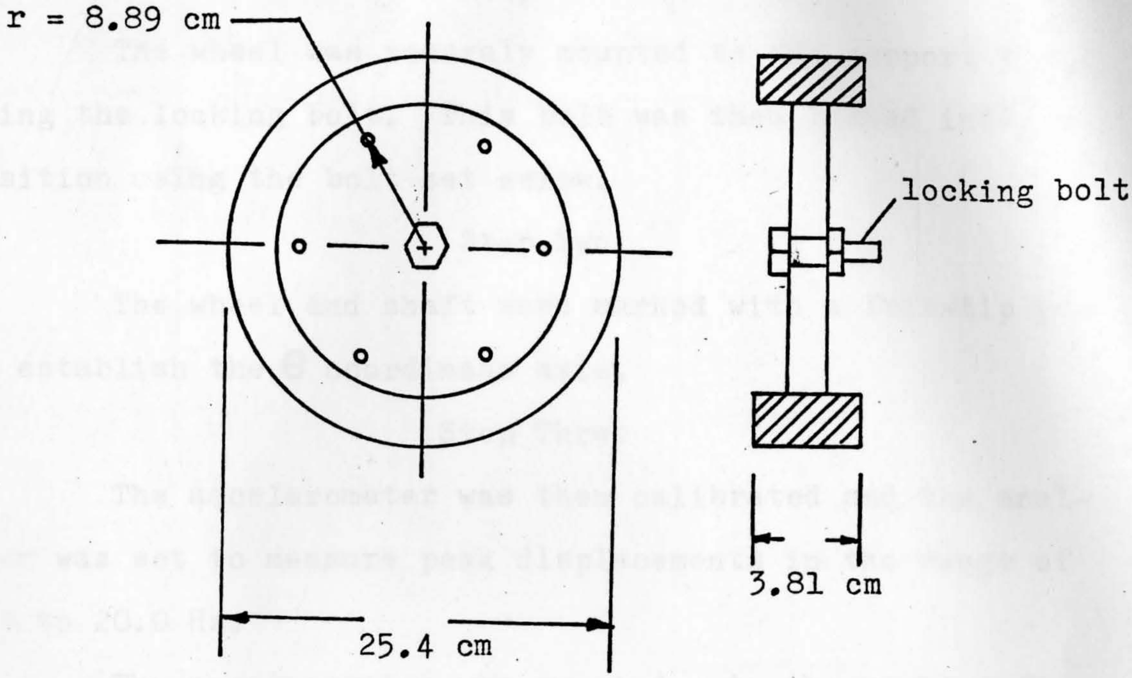

Fig. 9 "Wheel Description"

ing bolt was attached to the wheel to eliminate the play in the hub bearing. All displacement readings are listed in Table 2.

Step One

The wheel was securely mounted to the support shaft using the locking bolt. This bolt was then locked into position using the bolt set screw.

Step Two

The wheel and shaft were marked with a felt-tip pen to establish the θ coordinate axis.

Step Three

The accelerometer was then calibrated and the analyzer was set to measure peak displacements in the range of 6.3 to 20.0 Hz.

The accelerometer was mounted onto the right soft support and positioned with the lead wire input facing away from the support shaft.

Step Four

Reading one, 0.06 cm, was taken with the wheel fully unbalanced. Significant vibrations were noticed during startup. In addition, accurate vibration readings at large displacements are difficult to make due to significant variations.

Step Five

A 2.5 gram weight was attached to the wheel at the location θ=0 and reading 2 was taken. The weight was then placed at location 60^{o}, 120^{o}, 180^{o}, 240^{o}, and 300^{o} with readings 3 thru 7 taken repectively. The minimum displace-

TABLE 2

PEAK DISPLACEMENT READINGS

Reading Number	Reading at Support	Mass Added (grams)	Location (degrees)	Reading (cm)
1	R_R	0.0	0.0	0.06
2	R_R	2.5	0.0	.055
3	R_R	2.5	60.0	.049
4	R_R	2.5	120.0	.056
5	R_R	2.5	180.0	.062
6	R_R	2.5	240.0	.07
7	R_R	2.5	300.0	.067
8	R_R	7.3	60.0	.033
9	R_R	7.3,2.5	60.0,120.0	.032
10	R_R	7.3,2.5	60.0,0.0	.025
11	R_R	3.4,13.4	120.0,60.0	.002
12	R_L	0.0	0.0	.026
13	R_L	2.5	0.0	.024
14	R_L	2.5	60.0	.023
15	R_L	2.5	120.0	.025
16	R_L	2.5	180.0	.028
17	R_L	2.5	240.0	.031
18	R_L	2.5	300.0	.028
19	R_L	8.2	60.0	.012
20	R_L	8.2,2.5	60.0,0.0	.013

TABLE 2 (continued)

21	R_L	8.2,2.5	60.0,120.0	.011
22	R_L	3.4,14.2	120.0,60.0	.004
23	R_L	14.8,1.8	78.9,348.6	.002
24	R_L	14.8,1.8	78.9,348.6	.004

ment reading, 0.049 cm, occurred at the 60^o location.

Step Six

The 2.5 gram weight located at $\theta = 60^o$ was increased one gram at a time until the recorded displacement was reduced approximately 50%. This weight was 7.3 grams with the displacement reading 8 with 0.033 cm recorded.

Step Seven

The 7.3 gram weight was kept at the 60^o location. An additional 2.5 gram weight was added at a 120^o location and reading 9 was taken.

With the 7.3 gram weight at 60^o and a 2.5 gram weight at 0^o reading 10 was taken. It was noted that reading 10 was less than reading 9. Thus the true location of the correction weight lies between location 60^o and location 120^o.

Step Eight

The weights at 60^o and 120^o were then varied until the minimum possible vibration reading, reading 11, was obtained. The displacement vibration was now reduced from 0.06 cm to .002 cm. This occurred with 3.4 grams at location 120^o and 13.4 grams at location 60^o.

The correction weight added, m_2, is established by adding vectorially the above weights to obtain 15.38 grams at location, θ_2, 71.4^o.

Step Nine

The accelerometer was then remounted onto the left soft support. Again, the accelerometer lead wire was oriented away from any moving parts. Secondly, corrective weights added during Step Eight were removed.

A displacement vibration measurement, reading 12, was then taken of the unbalanced wheel.

Steps Five thru Eight were then repeated. The minimum vibration occurred with 3.4 grams at location 120^o and 14.2 grams at location 60^o.

The correction weight added, m_1, was found to be 16.17gm at location $\theta_1 = 70.5^o$.

Step Ten

Using equations 9-12 and 33-42, CW_R, and θ_R can be found. See Appendix B for their determination. CW_F was found to be 14.76 grams, CW_R was 1.78 grams, θ_R was 348.59^o, and θ_F was 78.92^o.

Step Eleven

The correction weights were attached to the wheel. Displacement vibration readings were then taken at the right soft support, reading 23, and at the left soft support, reading 24. Both the right and left soft supports had small displacement vibrations which indicates the wheel was dynamically balanced.

CHAPTER VI

SUMMARY

Findings

The major findings of experimenting with the machine are grouped into three areas. First, the effects of a constant rate soft support coupled with a constant speed drive limit the operating range of balanced part weights. Secondly, the vibration recordings which measure displacement, acceleration, and velocity are shown. Thirdly, alignment of components minimized the input torque thus reducing drive error vibrations.

Limitations of Operation

This machine has both a constant spring rate at the soft supports and a constant speed of rotation. The only damping in the system would be due to the component materials. During operation of the balance machine, it was noticed that from start to operating speed an indication of passing through the first natural frequency was experienced at the right soft support. As noted earlier, the spring rate of the soft bearing supports was measured to be 256 pounds per inch. The effective mass supported by the right soft support is approximately 0.47 slugs. Knowing the spring rate and mass of the system, the natural frequency of the right support would be 12.9 cycles per second. Where

$$\omega_n = \sqrt{\frac{k}{m}} \tag{43}$$

The above analysis assumes a single degree of freedom. A two degree of freedom analysis is given in Appendix C. This results in the natural frequency occurring at 774 rpm. With no damping, $\zeta = 0$, and a ratio of operating speed to the natural frequency speed, of 1.16, the magnification of static displacement, MF, would be approximately four.[5] Therefore, variation of the weight of the part to be balanced would cause a change in the natural frequency.

$$MF = \frac{\bar{r}^2}{\sqrt{(1-\bar{r}^2)^2 + (2\zeta\bar{r})^2}} \tag{44}$$

Vibration Measurement

Vibration recordings were made using an accelerometer, frequency analyzer and recorder manufactured by Bruel and Kjaer. Figures 10, 11, and 12 show the peak voltage measurements for displacement, velocity, and acceleration versus frequency taken at the right soft support with a three pound balanced tire being rotated. Recordings of these graphs were taken with a 45 db per octave selection.

To establish actual peak displacement values, 90 db was subtracted from the db readings of Figure 10. The recorded voltage, V_Δ, can be established knowing the rela-

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[5]Francis S. Tse, Ivan E. Morse, and Rolland T. Henkle, Mechanical Vibrations (2nd. ed.: Boston: Allyn and Bacon, Inc. 1978) p. 38.

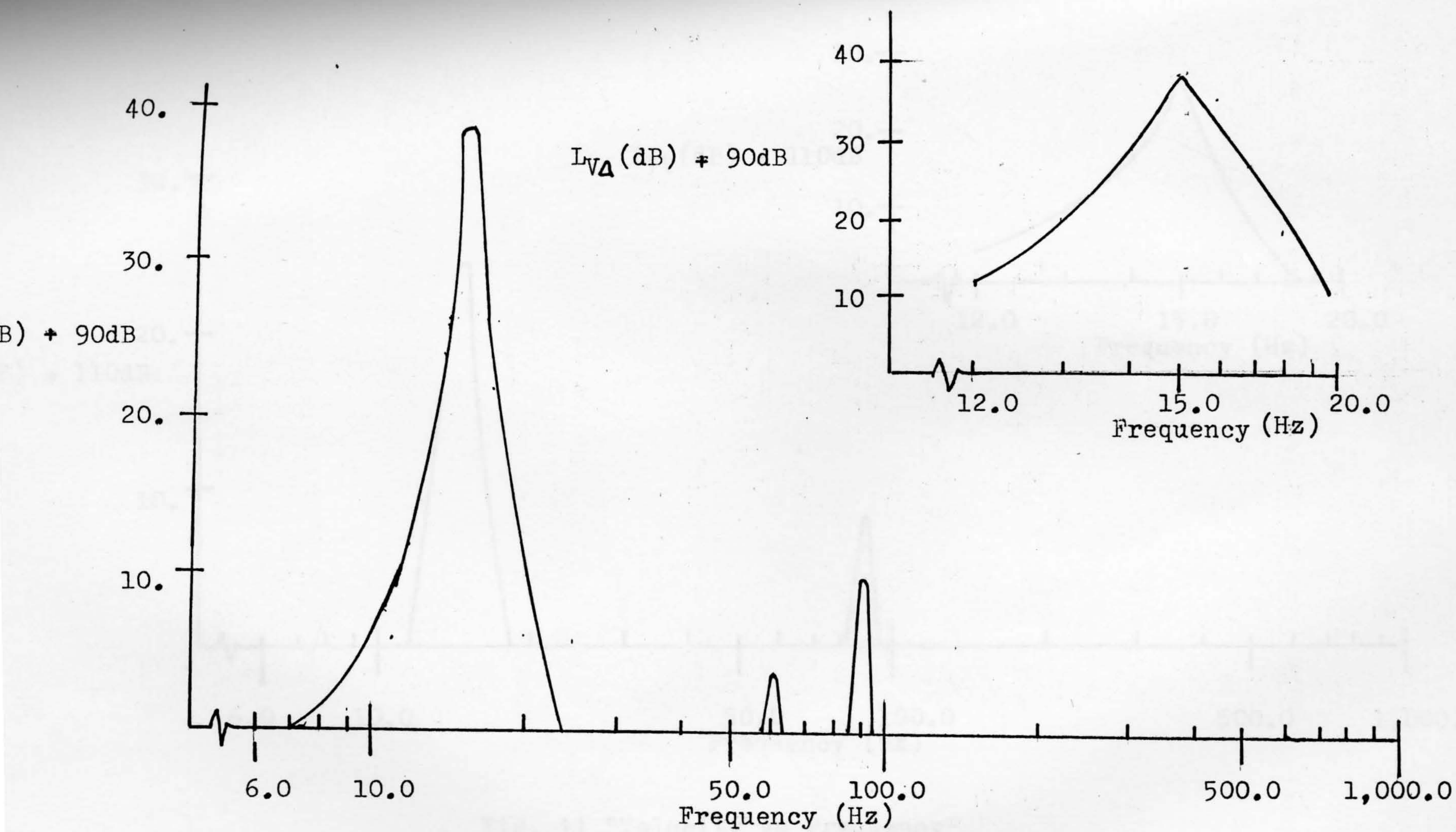

Fig. 10 "Displacement vs Frequency"

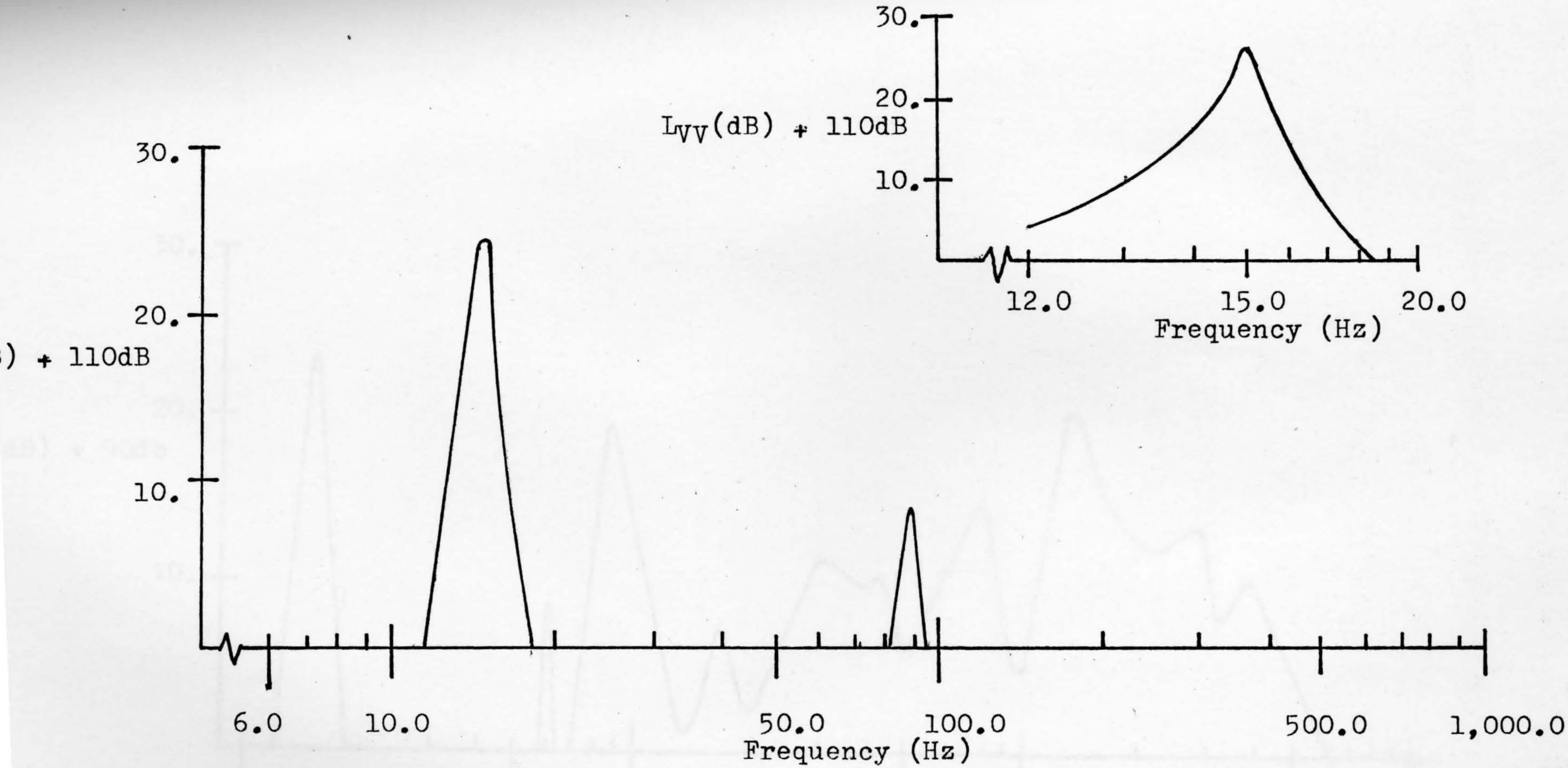

Fig. 11 "Velocity vs Frequency"

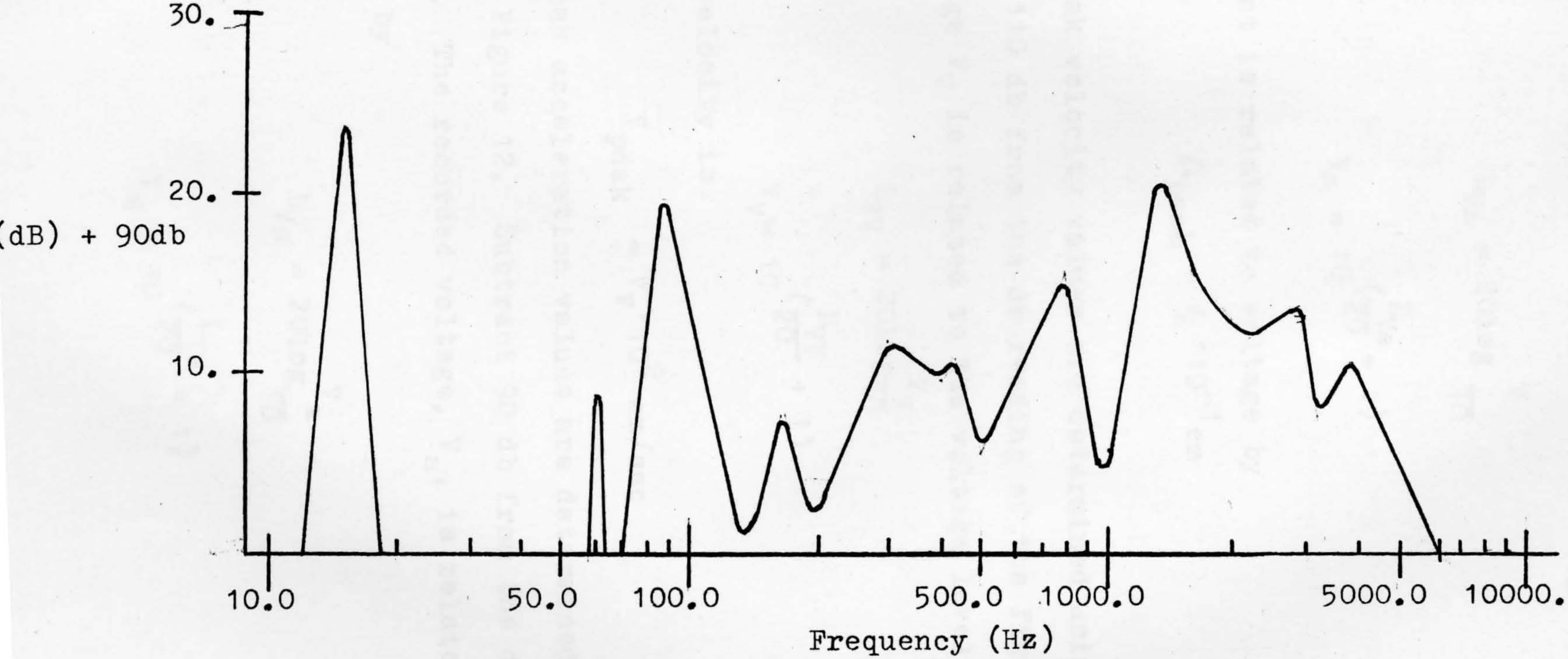

Fig. 12 "Acceleration vs Frequency"

tionship between the recorded voltage level, $L_{V\Delta}$ in db,

$$L_{V\Delta} = 20\log \frac{V}{10} \tag{45}$$

$$V_{\Delta} = 10^{\left(\frac{L_{V\Delta}}{20} + 1\right)} \tag{46}$$

The displacement is related to voltage by

$$\Delta_{peak} = V_{\Delta} \cdot 10^{-1}\,\text{cm} \tag{47}$$

The peak velocity values are determined using Figure 11. Subtract 110 db from the db reading of the figure. The recorded voltage V_V is related to the voltage level by

$$L_{VV} = 20\log \frac{V_V}{10} \tag{48}$$

$$V_V = 10^{\left(\frac{L_{VV}}{20} + 1\right)} \tag{49}$$

and the peak velocity is

$$V_{peak} = V_V \cdot 10^{2}\ \text{cm/sec} \tag{50}$$

The peak acceleration values are determined from the db reading of Figure 12. Subtract 90 db from the db reading of the figure. The recorded voltage, V_a, is related to the voltage level by

$$L_{Va} = 20\log \frac{V_a}{10} \tag{51}$$

$$V_a = 10^{\left(\frac{L_{Va}}{20} + 1\right)} \tag{52}$$

and the peak acceleration is

$$a_{peak} = V_a \cdot 10^3 \text{ cm/sec}^2 \qquad (53)$$

Table 3 gives values of peak displacement, peak velocity, and peak acceleration at selected frequencies. These values were calculated using equations 45-52.

Component Alignment

The initial machine setup required the components to be in alignment. First, the shaft was made level. Secondly, the motor shaft was leveled. Next, the shaft of the motor was set parallel to the support shaft. This procedure of alignment assured the drive pulley would be angularly aligned with the driven pulley.

It was noticed that smooth, vibration free operation required fine tuning the alignment. By inspection of vibrations at the supports, fine adjustment of the alignment was accomplished. This resulted in a reduced torque requirement.

Conclusion

The balancing machine developed can balance a rotating three pound object to within 0.1 ounce-inch. With the constant speed of rotation and fixed soft support spring rates, a wheel weighing between 3 lbs. and 5 lbs. can be balanced. However, an unbalance greater than 0.1 ounce-inch will occure.

Although use of frequency filters reduce high frequency error vibrations, vibration recordings should be

TABLE 3

RECORDED VIBRATIONS OF A BALANCED PART

Frequency cps	Acceleration cm/sec^2	Velocity cm/sec	Displacement cm
15	5.011	.0501	.0022
60	.794		.00005
90	3.16	.0089	.00011
300	3.98		
450	3.54		
800	1.99		
1300	5.011		
2000	6.310		
3300	5.623		
6.3 - 20000	22.38	.093	.0028

measured by peak displacement. The acceleration recorded at 15 Hz was 5.011 cm/sec^2 and 22.38 cm/sec^2 was recorded with filtering vibrations not in the range of 6.3 Hz to 20000.0 Hz. The velocity recorded at 15 Hz was 0.0501 cm/sec and 0.093 cm/sec was recorded with filtering. The displacement recorded at 15 Hz was 0.0022 cm. and 0.0028 was recorded with filtering. Therefore, the peak displacement has the least error due to the high frequency vibrations.

The drive system induced the largest error vibrations into the system. Isolation of the drive from the system was greatly enhanced by the use of a round rubber drive belt. Drive belts with plies tend to cause binding and introduced significant vibrations.

Although the trial and error approach to finding the balancing weights and their locations was difficult, experience gained with repeated balancing greatly reduced the amount of time expended. It should be noted that phase location is much easier to establish after the vibrations are significantly reduced.

APPENDIX A

Shaft Deflection Calculations

SHAFT DEFLECTION CALCULATIONS

Given

Refer to Figure 5 for a free body diagram of the shaft loading.

The following information is known.

1" OD shaft, carbon steel

$L_1 = 15"$ $\omega = 94.25$ rad/sec

$L_2 = 10"$ $mr = 2.1$ inch ounces

$$F = m\omega^2 r = \frac{2.1 \cdot 94.25^2}{16 \cdot 386.4} = 3.02 \text{ lbs.}$$

$M = 0$

Find

Determine the maximum shaft deflection.

Solution

Assume the deflection due to shear is very small and can be neglected. Secondly, the modulus of elasticity, E, is $30 \cdot 10^6$ psi.

The moment is related to deflection, Y, by the following equation.

$$EI\ddot{Y} = -M$$

X = 0, Y = 0 is located at R_L. X is positive to the right and Y is positive downward.

By a summation of moments about R_L we find

$$15R_R = 25F$$

$$R_R = 5.03\text{lbs}$$

By summation of forces in the vertical direction we find

$$R_R + R_L + F = 0$$

$$R_L = -2.01 \text{ lbs}$$

For $0 \leq X \leq 25"$

$$EI\ddot{Y} = -R_L X + R_R \langle X - L_1 \rangle$$

$$EI\ddot{Y} = -2.01X - 5.03 \langle X - 15 \rangle$$

$$EI\dot{Y} = -1.005X^2 - 2.515 \langle X - 15 \rangle^2 + C_1$$

$$EIY = -.335X^3 - .8383 \langle X - 15 \rangle^3 + C_1 X + C_2$$

$Y = 0$ at $X = 0 \therefore C_2 = 0$

$Y = 0$ at $X = 15" \therefore C_1 = 75.375$ lb in^3

$\dot{Y} = 0$ at $X = 8.66"$

The moment of inertia, I, is

$$I = \frac{\pi d^4}{64} = .05 in^4$$

The deflections are

$Y\ (8.66) = .0003"$

$Y\ (25) = .003"$ (maximum)

Therefore, the maximum shaft deflection is .003" at the wheel location.

APPENDIX B

Calculation of Example Correction Weights

CALCULATION OF EXAMPLE CORRECTION WEIGHTS

Known Values

The following values were determined from the example balancing as noted in Chapter V.

$L_1 = 38.1$ cm $m_1 = 16.17$ gm $\theta_1 = 70.5^\circ$ $\omega = 94.25$ rad/sec

$L_2 = 25.4$ cm $m_2 = 15.38$ gm $\theta_2 = 71.4^\circ$ $r_1 = r_2 = 8.89$ cm

$h = 8.89$ cm $z = 1.9$ cm

The equations to be solved are 9-12 and 33-42.

Unknown Values

F, M, F_x, F_y, M_x, M_y, θ_F, θ_M, CW_1 and CW_2 must be determined to establish the correction weights and their locations, CW_F, CW_R, θ_F and θ_R.

Solution

Using equation (33)

$$M_x = \frac{\begin{vmatrix} -(38.1+25.4)15.38\cdot 94.25^2\cdot 8.89\cos 71.4^\circ & 38.1+25.4 \\ -25.4\cdot 16.17\cdot 94.25^2\cdot 8.89\cos 70.5^\circ & 25.4 \end{vmatrix}}{\begin{vmatrix} 1 & 38.1+25.4 \\ 1 & 25.4 \end{vmatrix}}$$

$$M_x = \frac{\begin{vmatrix} -24599700.6 & 63.5 \\ -10826883.7 & 25.4 \end{vmatrix}}{\begin{vmatrix} 1 & 63.5 \\ 1 & 25.4 \end{vmatrix}}$$

$$M_x = \frac{62674719.7}{-38.1} = -164499.2 \text{ dyne-cm}$$

Using equation (34)

$$M_y = \frac{\begin{vmatrix} -(38.1+25.4)15.38\cdot 94.25^2\cdot 8.89\sin 71.4^\circ & (38.1+25.4) \\ -25.4\cdot 16.17\cdot 94.25^2\cdot 8.89\sin 70.5^\circ & 25.4 \end{vmatrix}}{\begin{vmatrix} 1 & (38.1+25.4) \\ 1 & 25.4 \end{vmatrix}}$$

$$M_y = \frac{\begin{vmatrix} -73096531.3 & 63.5 \\ -30574176.4 & 25.4 \end{vmatrix}}{\begin{vmatrix} 1 & 63.5 \\ 1 & 25.4 \end{vmatrix}}$$

$$M_y = \frac{84808306.4}{-38.1} = -2225939.8 \text{ dyne-cm}$$

Using equation (35)

$$F_x = \frac{\begin{vmatrix} 1 & -(38.1+25.4)15.38\cdot 94.25^2\cdot 8.89\cos 71.4^\circ \\ 1 & -25.4\cdot 16.17\cdot 94.25^2\cdot 8.89\cos 70.5^\circ \end{vmatrix}}{\begin{vmatrix} 1 & (38.1+25.4) \\ 1 & 25.4 \end{vmatrix}}$$

$$F_x = \frac{\begin{vmatrix} 1 & -2599700.6 \\ 1 & -10826883.7 \end{vmatrix}}{\begin{vmatrix} 1 & 63.5 \\ 1 & 25.4 \end{vmatrix}}$$

$$F_x = \frac{13772816.9}{-38.1} = -361491.3 \text{ dyne}$$

Using equation (36)

$$F_y = \frac{\begin{vmatrix} 1 & -(38.1+25.4)15.38\cdot 94.25^2\cdot 8.89\sin 71.4^\circ \\ 1 & -25.4\cdot 16.17\cdot 94.25^2\cdot 8.89\sin 70.5^\circ \end{vmatrix}}{\begin{vmatrix} 1 & (38.1+25.4) \\ 1 & 25.4 \end{vmatrix}}$$

$$F_y = \frac{\begin{vmatrix} 1 & -73096531.3 \\ 1 & -30574176.4 \end{vmatrix}}{\begin{vmatrix} 1 & 63.5 \\ 1 & 25.4 \end{vmatrix}}$$

$$F_y = \frac{42522354.9}{-38.1} = -1116072.3 \text{ dyne}$$

Using equation (11)

$$\theta_F = \arctan \left\{\frac{-1116072.3}{-361491.3}\right\} = 252.053^\circ$$

Using equation (12)

$$\theta_M = \arctan \left\{\frac{-2225939.8}{-164499.2}\right\} = 265.773^\circ$$

Using equation (9)

$$|F| = \sqrt{(361491.3)^2+(-1116072.3)^2} = 1173155.3 \text{ dyne}$$

Using equation (10)

$$|M| = \sqrt{(-164499.2)^2+(2225939.8)^2} = 2232009.8 \text{ dyne-cm}$$

Using equation (37)

$$CW_1 = \frac{1173155.3}{2\cdot 94.25^2\cdot 8.89} = 7.43 \text{ grams}$$

Using equation (38)

$$CW_2 = \frac{2232009.8}{2\cdot 1.9\cdot 94.25^2\cdot 8.89} = 7.44 \text{ grams}$$

Using equation (39)

$$CW_R = \left\{(7.43\cos 72.053^\circ + 7.44\cos 265.773^\circ)^2 + (7.43\sin 72.053^\circ + 7.44\sin 265.773^\circ)\right\}^{1/2} = 1.78 \text{ grams}$$

Using equation (40)

$$\theta_R = \arctan\left\{\frac{7.43\sin 72.053^\circ + 7.44\sin 265.773^\circ}{7.43\cos 72.053^\circ + 7.44\cos 265.773^\circ}\right\} = 348.59^\circ$$

Using equation (41)

$$CW_F = \left\{(7.43\cos 72.053^\circ + 7.44\cos 85.773^\circ)^2 + (7.43\sin 72.053^\circ + 7.44\sin 85.773^\circ)^2\right\}^{1/2} = 14.76 \text{ grams}$$

Using equation (42)

$$\theta_F = \arctan\left\{\frac{7.43\sin 72.053^\circ + 7.44\sin 85.773^\circ}{7.43\cos 72.053^\circ + 7.44\cos 85.773^\circ}\right\} = 78.92^\circ$$

Summary

$CW_R = 1.78$ grams

$\theta_R = 348.59^\circ$

$CW_F = 14.76$ grams

$\theta_F = 78.92^\circ$

APPENDIX C

Two Degree of Freedom Analysis

TWO DEGREE OF FREEDOM ANALYSIS

Refer to Figure 13 for a free body diagram of the system. This system consists of a shaft, two bearings, two soft supports, and a wheel.

This analysis assumes the motion of the shaft is in the vertical direction only.

The following data is known:

for the left bearing	$m = 1360$ grams	$J = 1671\ gm.cm^2$
	$x = 0.0$ cm.	
for the right bearing	$m = 1360$ grams	$J = 1671\ gm.cm^2$
	$x = 38.1$ cm.	
for the shaft	$m = 2527$ grams	$J = 13374\ gm.cm^2$
	$x = 31.75$ cm.	
for the wheel	$m = 1586$ grams	$J = 21614\ gm.cm^2$

The center of gravity is located at $x=D_1$. It can be found using the following equation.

$$D_1 = \sum \frac{mx}{m}$$

Therefore, D_1 is

$$D_1 = \frac{232732}{6833} = 34.06 \text{ cm.}$$

The interia of the system is known to be equal to

$$J = \sum J_0 + m(D_1 - x)^2$$

Therefore, the interia of the system is

$$J = 2988003.\ gm.cm^2$$

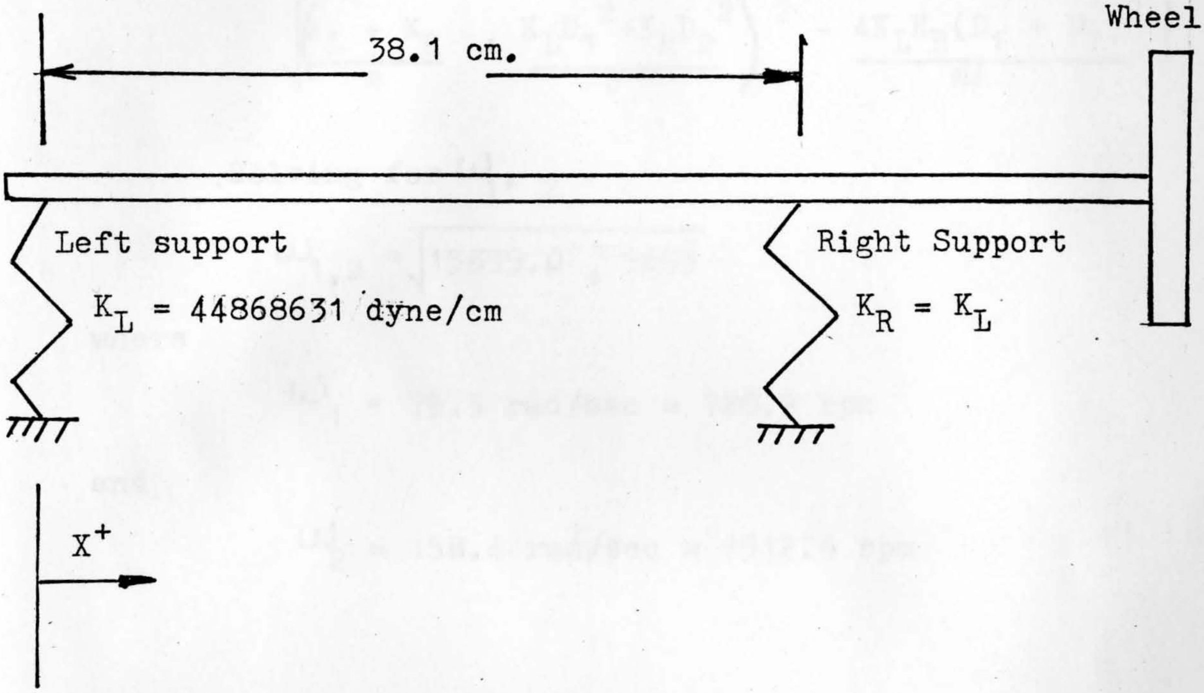

Fig. 13. "Free Body Diagram Two Degree of Freedom Analysis"

The total mass of the system, m, is 6833 grams.

The natural circular frequency of the system has a known solution.

$$\omega_{1,2}^{2} = \frac{1}{2}\left\{ \frac{K_L + K_R}{m} + \frac{K_L D_1^{\,2} + K_R D_2^{\,2}}{J} \mp \sqrt{\left(\frac{K_L + K_R}{m} + \frac{K_L D_1^{\,2} + K_R D_2^{\,2}}{J}\right)^2 - \frac{4K_L K_R (D_1 + D_2)^2}{mJ}} \right\}$$

Solving for $\omega_{1,2}$

$$\omega_{1,2} = \sqrt{15399.0 \mp 9695}$$

where

$$\omega_1 = 75.5 \text{ rad/sec} = 720.9 \text{ rpm}$$

and

$$\omega_2 = 158.4 \text{ rad/sec} = 1512.6 \text{ rpm}$$