METHOD OF MOMENTS CAPACITANCE MODEL FOR MULTICONDUCTOR SYSTEMS

by

WILLIAM T. PHILLIPS, JR.

Submitted in Partial Fulfillment of the Requirements

for the Degree of

Master of Science

in the

Electrical Engineering

Program

Salvatore R. Pansino 6/22/89

Advisor Date

Sally M. Hotchkiss July 25, 1989

Dean of the Graduate School Date

YOUNGSTOWN STATE UNIVERSITY

June, 1989

ABSTRACT

METHOD OF MOMENTS CAPACITANCE MODEL
FOR MULTICONDUCTOR SYSTEMS

WILLIAM T. PHILLIPS, JR.

YOUNGSTOWN STATE UNIVERSITY, 1989

Industry would like an analytical model that can predict various electrical parameters to characterize the performance of their components rather than building prototypes and testing. Of importance are parameters such as capacitive loading effects, inductance, delay characteristics, characteristic impedance, signal bandwidth and distortion, system stability, radiated emissions, passive filtering, and crosstalk (electromagnetic coupling). These parameters need to be evaluated for various geometries such as multiconductor ribbon cables, wire bundles, coax cables, shielded wire bundles, twisted pairs, and wire bundles over a ground plane. Also of importance is how to handle all of the above conditions with discontinuities in geometry.

The primary objective of this discussion is to develop a mathematical model which will determine the capacitance of various multiconductor systems, the model being a FORTRAN program. It can be shown that once the capacitance is known all other parameters can be obtained.

The capacitance model developed in this document uses a

Fourier series approximation for the charge density on the conductor and dielectric surface. Using the charge density described above a near field potential function is developed for cylindrical conductors. The potential function is descritized and place in matrix form using the "method of moments", which was first introduced by R. F. Harrington.

When the wires are coated with a dielectric it is necessary to determine the electric field intensity. The electric field intensity is needed to completely specify or determine all the unknown charge densities on the conductor and dielectric surfaces. This is accomplished by using the potential function developed above and using Laplace's equation.

The capacitance matrix models which are presented in this document include dielectric coated multiconductor ribbon cables, dielectric coated multiconductor wire bundles with different radii and permittivities, shielded multiconductor wire bundles, multiconductor coax cables, and dielectric coated multiconductor wire bundles over a ground plane.

This report contains a discussion of the theory for the determination of the capacitance for the various configurations discussed above as well as some of the anomalies associated with various models and the FORTRAN program itself. Wherever possible, comparison of the capacitance model using the method of moments is made to that of the closed form solution, specifically, that of an uncoated (bare) 2-wire system, coax cable, shielded wire, and one bare wire over a ground plane. When discussing the capacitance of dielectric coated wires and those over a ground plane, the model is compared with results that are obtained from testing.

ACKNOWLEDGEMENTS

I wish to acknowledge Craig Reeves for acquiring test data for dielectric coated ripcord, Test Labs for test data on a dielectric coated wire over a ground plane, and Dave Bailey for his comments and time.

I also wish to thank my thesis advisor Dr. S. R. Pansino for his advice and comments in preparing this thesis and Dr. M. Siman and professor R. E. Kramer for being on my thesis committee and their comments.

TABLE OF CONTENTS

TABLE OF CONTENTS (cont.)

LIST OF SYMBOLS

SYMBOL	DEFINITION	UNITS OR REFERENCE
Δ	small displacement	mm.
σ_0	avg. charge density	Coulombs/sq. area
σ_m	charge density associated with cosine terms of the Fourier series expansion	Coulombs/sq. area
$\hat{\sigma}_m$	charge density associated with sine terms of the Fourier series expansion	Coulombs/sq. area
$\iint$	integral over a surface	
$\oiint$	integral over a closed surface	
[ ]	matrix or vector	
$\sum$	summation of all terms	
ϕ	absolute potential	Volts
λ	line charge	Coulombs/unit length
$\lvert r - r' \rvert$	measured distance between source matchpoints and potential matchpoints	meters
ϵ_0	permittivity in air	pF/meter
ϵ_r	relative permittivity	none
β_c	source angle of matchpoint with respect to horizontal	radians
θ	potential angle of matchpoint with respect to horizontal	radians
∇	del operator	

LIST OF FIGURES

WILLIAM F. MAAG LIBRARY
YOUNGSTOWN STATE UNIVERSITY

LIST OF TABLES

LIST OF TABLES (cont.)

TABLE PAGE

LIST OF TABLES (cont.)

LIST OF TABLES (cont.)

TABLE PAGE

LIST OF TABLES (cont.)

TABLE PAGE

LIST OF TABLES (cont.)

LIST OF TABLES (cont.)

TABLE PAGE

CHAPTER 1

INTRODUCTION

The capacitance of cylindrical conductors with or without dielectric coating is one of the principal parameters to be determined when calculating the electromagnetic coupling (crosstalk) of multi-conductor systems. Few attempts have been made to predict the capacitance of multiconductor systems which contain cylindrical conductors. The capacitance for a bare 2-wire system is well documented but little effort has gone into determining the capacitance of dielectric coated multiconductor systems. One attempt by Higgins and Black uses the concepts of Maxwell's "method of subareas".[1] In this procedure the cylindrical conductor is subdivided into an n-sided polygon which is infinitely long and each side is assumed to have an unknown constant charge density of σ_i. A set of matrix equations is formed to determine the charge density from the known potential boundary conditions. Once the charge density is determined, the capacitance is approximated from the area under the charge density curve. This procedure, however, has the drawback that it requires a large number of sides of the polygon to get a capacitance value which is only accurate to one significant digit. Mathematically this procedure will give a capacitance value but for large multiconductor systems is impractical.

Another attempt was to eliminate the error due to the polygon approximation. This was done by subdividing the cross-sectional area of the conductor into circular pie sections where the surface was continuous from one subsection to another.[2] This procedure also ran into difficulties since the integrals had to be approximated by numerical integration. Again, comparing the capacitance value found by this procedure to that of the closed form solution for a bare 2-wire system showed that it required a large number of subsections to approximate the capacitance to one significant digit.

Another attempt by Clayton Paul, as suggested by Arlon Adams, was to use a Fourier series approximation for the charge density and then apply this approximation using the "method of moments" to calculate the capacitance. The "method of moments" was first used to calculate electromagnetic problems by R. F. Harrington.[3,4]

The method of moments is used to express a function as a matrix from which available matrix techniques can be employed to solve for the unknown quantities, namely the charge densities σ's. Clayton Paul demonstrated that this technique gave extremely good results when using a small number of terms of the Fourier series. This application, however, was only used to determine the capacitance of dielectric coated multiconductor ribbon cables where the solid conductors had the same conductor and dielectric radii as well as the same relative permittivity for all wires.

The procedure which Clayton Paul used is also the one which is used to calculate the capacitance in this document. The procedure is extended to calculate the capactance of not only ribbon cables but multiconductor wire bundles with conductors having different radii and dielectrics having different permittivities. This procedure is also used to determine the capacitance of a multiconductor coaxial cables, shielded dielectric coated multiconductor wire bundles and multiconductor wire bundles over a ground plane.

Once the parasitic shunt capacitance is determined, other important parameters can be calculated such as inductance and characteristic impedance. Also a multitude of questions can be answered such as the effect capacitance has on signal bandwidth, delay characteristics of transmission lines, stability, radiated emissions, and electromagnetic coupling (crosstalk). The determination of the capacitance matrix is easily understood using an example. This example is illustrated in Arlon Adam's book and is reproduced here to show the basic procedure.[5] The example is a parallel plate capacitor as shown in figure 1.1.

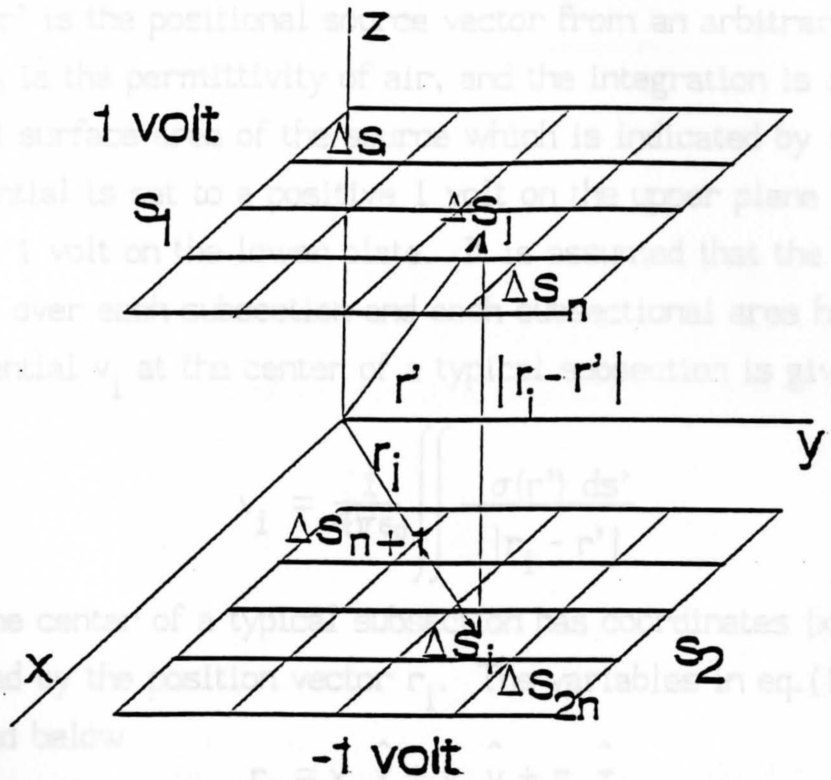


FIG. 1.1 PARALLEL PLATE CAPACITOR

First the plates are broken into 2n subsectional areas (n subsections for each plate) where S_1 is the total surface area of plate one and S_2 is the total surface area of plate two. The surface areas are subdivided as follows

$$S_1 = \Delta s_1 + \Delta s_2 + \Delta s_3 + \ldots + \Delta s_n \qquad \text{eq.(1.1)}$$

$$S_2 = \Delta s_{n+1} + \Delta s_{n+2} + \Delta s_{n+3} + \ldots + \Delta s_{2n} \qquad \text{eq.(1.2)}$$

where the total surface area S is given as follows

$$S = S_1 + S_2 \qquad \text{eq.(1.3)}$$

Now the potential function which describes the system is given by the following

$$v = \frac{1}{4\pi\epsilon_0} \iint \frac{\sigma(r')\, ds'}{|r - r'|} \qquad \text{eq.(1.4)}$$

where $\sigma(r')$ is the surface charge density, r is the positional field vector, r' is the positional source vector from an arbitrary reference point, ϵ_0 is the permittivity of air, and the integration is carried over the total surface area of the source which is indicated by ('). Next the potential is set to a positive 1 volt on the upper plane and a negative 1 volt on the lower plate. It is assumed that the potential is constant over each subsection and each subsectional area has unit area. The potential v_i at the center of a typical subsection is given by

$$v_i = \frac{1}{4\pi\epsilon_0} \iint \frac{\sigma(r')\, ds'}{|r_i - r'|} \qquad \text{eq.(1.5)}$$

where the center of a typical subsection has coordinates (x_i, y_i, z_i) and is located by the position vector r_i. The variables in eq.(1.5) are described below

$$r_i = x_i\,\hat{x} + y_i\,\hat{y} + z_i\,\hat{z}$$

$$r' = x'\,\hat{x} + y'\,\hat{y} + z'\,\hat{z}$$

$$ds' = dx'dy'$$

$$|r_i - r'| = \sqrt{(x_i - x')^2 + (y_i - y')^2 + (z_i - z')^2}$$

A typical subsectional area Δs_j in which the charge density is integrated over is shown in figure 1.2.

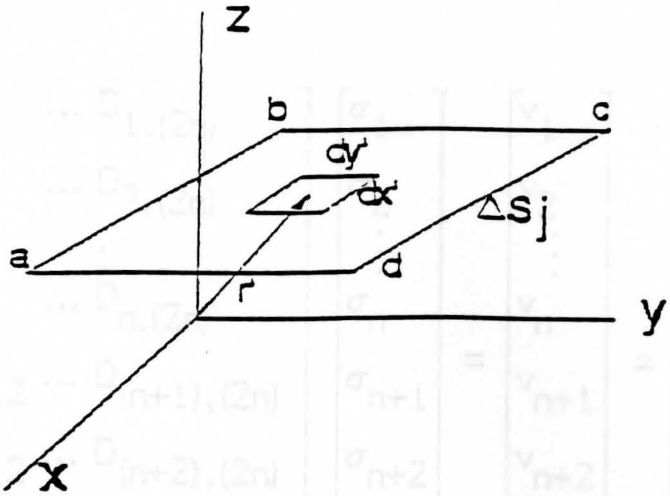


FIG. 1.2 SUBSECTIONAL AREA

The total potential of the i-th subsection due to the j-th unit surface charge density from each of the 2n subsections is given below

$$v_i = \sum_{j=1}^{2n} \sigma_j \underbrace{\frac{1}{4\pi\epsilon_0} \iint_{\Delta s_j} \frac{ds'}{|r_i - r'|}}_{D_{ij}} \qquad \text{eq.(1.6)}$$

where D_{ij} is a dummy variable used to discretize the function so that it can place in matrix form, thus the potential function can be written as follows

$$v_i = \sum_{j=1}^{2n} D_{ij}\sigma_j \qquad \text{eq.(1.7)}$$

where $D_{ij}\sigma_j$ represents the potential at a point i due to a source on subsection j and i=1,2,3,...,2n. The system has 2n equations with 2n unknowns, thus the equation above can be place in matrix form as follows

$$\begin{bmatrix} D_{1,1} & D_{1,2} & \cdots D_{1,(2n)} \\ D_{2,1} & D_{2,2} & \cdots D_{2,(2n)} \\ \vdots & \vdots & \vdots \\ D_{n,1} & D_{n,2} & \cdots D_{n,(2n)} \\ D_{(n+1),1} & D_{(n+1),2} & \cdots D_{(n+1),(2n)} \\ D_{(n+2),1} & D_{(n+2),2} & \cdots D_{(n+2),(2n)} \\ \vdots & \vdots & \vdots \\ D_{(2n),1} & D_{(2n),2} & \cdots D_{(2n),(2n)} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_n \\ \sigma_{n+1} \\ \sigma_{n+2} \\ \vdots \\ \sigma_{2n} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \\ v_{n+1} \\ v_{n+2} \\ \vdots \\ v_{2n} \end{bmatrix} = \begin{bmatrix} +1 \\ +1 \\ \vdots \\ +1 \\ -1 \\ -1 \\ \vdots \\ -1 \end{bmatrix}$$

or eq.(1.8)

$$[\,D\,][\,\sigma\,] = [\,v\,] \qquad \text{eq.(1.9)}$$

Solving the above matrix equation for the surface charge density, the matrix equation becomes

$$[\,\sigma\,] = [\,D\,]^{-1}\;[\,v\,] \qquad \text{eq.(1.10)}$$

Since it was assumed that each subsection is of unit area the charge on one plate, say the upper plate, is given by

$$q = \sum_{j=1}^{n} \sigma_j \qquad \text{eq.(1.11)}$$

From the definition of capacitance which is the ratio of charge on one plate to the potential difference of the plates, the capacitance is obtained as follows

$$c = \frac{q}{v_1 - v_2} = \frac{1}{2}\sum_{j=1}^{n} \sigma_j \qquad \text{eq.(1.12)}$$

where $v_1 = 1$ volt , $v_2 = -1$ volt, and σ_j = coulombs/unit area

It should be noted that v_1 and v_2 are the voltages over the surface of plates one and two respectively and the voltages shown in eq.(1.8) are the same voltages only at discrete points.

CHAPTER 2

APPLICATION OF THE METHOD OF MOMENTS IN DETERMINING THE POTENTIAL IN FREE SPACE FROM A CHARGE ON A CYLINDRICAL CONDUCTOR

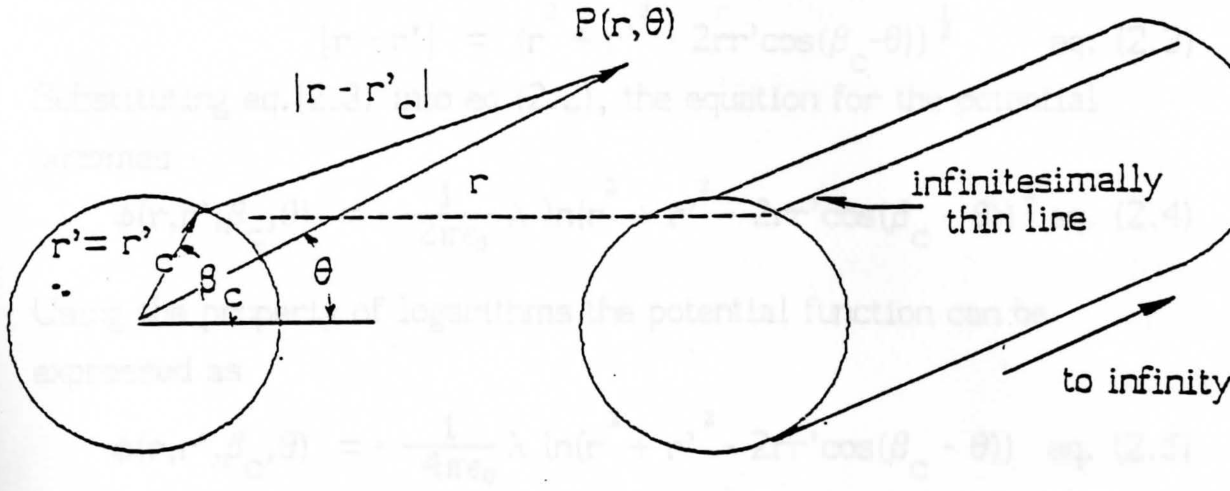


FIG 2.1 CYLINDRICAL CONDUCTOR WITH MATCHPOINTS

Before developing the model for the capacitance, it is necessary to determine the near field potential function due to a Fourier series approximation of the charge around a single cylindrical conductor as based on Clayton Paul's work.[2]

Recall the potential from an infinitely long, infinitesimally thin wire

$$\phi(r,\theta) - \phi(r_0,\theta) = \frac{1}{2\pi\epsilon_0}\lambda \ln(r/r_0) \qquad \text{eq.(2.1)}$$

where r is the distance to a field point in space from a source and r_0 is the distance from an arbitrary reference point in space.[5] It can be shown that if the total charge of a system is zero and the reference point is at infinity the potential can be expressed without the reference vector r_0 as shown in appendix A. Thus the potential from an infinitesimally thin wire is expressed as

$$\phi(r,\theta) = -\frac{1}{2\pi\epsilon_0}\lambda \ln(|r - r'|/1 \text{ meter}) \qquad \text{eq. (2.2)}$$

where $|r - r'|$ is the distance from the field point given by the positional vector r and the source point given by the positional vector r'. Using the law of cosines, the distance between the source point and the potential point can be expressed as

$$|r - r'| = (r^2 + r'^2 - 2rr'\cos(\beta_c - \theta))^{\frac{1}{2}} \qquad \text{eq. (2.3)}$$

Substituting eq.(2.3) into eq.(2.2), the equation for the potential becomes

$$\phi(r,r',\beta_c,\theta) = -\frac{1}{2\pi\epsilon_0}\lambda \ln(r^2 + r'^2 - 2rr'\cos(\beta_c - \theta))^{\frac{1}{2}} \qquad \text{eq. (2.4)}$$

Using the property of logarithms the potential function can be expressed as

$$\phi(r,r',\beta_c,\theta) = -\frac{1}{4\pi\epsilon_0}\lambda \ln(r^2 + r'^2 - 2rr'\cos(\beta_c - \theta)) \qquad \text{eq. (2.5)}$$

If an infinite number of infinitesimally thin line charges are place around the conductor surface, the charge becomes a surface charge. Thus summing all λ_n's, the potential function can be expressed as follows

$$\phi(r,r',\beta_c,\theta) = -\frac{1}{4\pi\epsilon_0}\sum_{n=1}^{\infty}\lambda_n(\ln(r^2 + r'^2 - 2rr'\cos(\beta_c - \theta)) \qquad \text{eq. (2.6)}$$

The infinite number of line charges is now replaced by the surface charge σ. Since these line charges are infinitely long, the surface area would be infinite so the integration will be done on a per unit length bases. This reduces the 3-dimensional problem to a 2-dimensional problem. Thus, replacing the sigma sumation with an integral and λ_n with σ, the potential function becomes

$$\phi(r,r',\beta_c,\theta) = -\frac{1}{4\pi\epsilon_0}\int_s \sigma \ln(r^2 + r'^2 - 2rr'\cos(\beta_c - \theta))\, ds \qquad \text{eq. (2.7)}$$

Assuming that the charge distribution on the surface of the conductor can be expressed as a Fourier series the surface charge distribution σ

in eq. (2.7) becomes

$$\sigma(\beta_c) = \sigma_o + \sum_{m=1}^{k} (\sigma_m \cos(m\beta_c) + \hat{\sigma}_m \sin(m\beta_c)) \qquad \text{eq. (2.8)}$$

Substituting eq. (2.8) into eq. (2.7) and integrating around the conductor surface the potential ϕ becomes

$$\phi = -\frac{1}{4\pi\epsilon_o} \int_s \left[\sigma_o + \sum_{m=1}^{k} (\sigma_m \cos(m\beta_c) + \hat{\sigma}_m \sin(m\beta_c)) \right] \ln(r^2 + r'^2 - 2rr'\cos(\beta_c - \theta))\, ds \qquad \text{eq. (2.9)}$$

or

$$\phi = -\frac{1}{4\pi\epsilon_o} \int_0^1 \int_0^{2\pi} \left[\sigma_o + \sum_{m=1}^{k} (\sigma_m \cos(m\beta_c) + \hat{\sigma}_m \sin(m\beta_c)) \right] \ln(r^2 + r'^2 - 2rr'\cos(\beta_c - \theta))\, r' d\beta_c dl \qquad \text{eq. (2.10)}$$

Separating eq. (2.10) into three integrals and assuming the charge density is uniform for a small subarea the potential ϕ can be expressed as

$$\phi = -\frac{1}{4\pi\epsilon_o} \sigma_o \int_0^1 \int_0^{2\pi} \ln(r^2 + r'^2 - 2rr'\cos(\beta_c - \theta))\, r' d\beta_c dl$$

$$-\frac{1}{4\pi\epsilon_o} \sum_{m=1}^{k} \sigma_m \int_0^1 \int_0^{2\pi} \cos(m\beta_c) \ln(r^2 + r'^2 - 2rr'\cos(\beta_c - \theta))\, r' d\beta_c dl$$

$$-\frac{1}{4\pi\epsilon_o} \sum_{m=1}^{k} \hat{\sigma}_m \int_0^1 \int_0^{2\pi} \sin(m\beta_c) \ln(r^2 + r'^2 - 2rr'\cos(\beta_c - \theta))\, r' d\beta_c dl \qquad \text{eq. (2.11)}$$

With a change of vairables, let the integrals equal the following

$$D_o = -\frac{1}{4\pi\epsilon_o}\int_0^1\int_0^{2\pi} \ln(r^2+r'^2-2rr'\cos(\beta_c-\theta))\; r'd\beta_c dl \qquad \text{eq.(2.12)}$$

$$D_m = -\frac{1}{4\pi\epsilon_o}\int_0^1\int_0^{2\pi} \cos(m\beta_c)\,\ln(r^2+r'^2-2rr'\cos(\beta_c-\theta))\; r'd\beta_c dl \qquad \text{eq.(2.13)}$$

$$\hat{D}_m = -\frac{1}{4\pi\epsilon_o}\int_0^1\int_0^{2\pi} \sin(m\beta_c)\,\ln(r^2+r'^2-2rr'\cos(\beta_c-\theta))\; r'd\beta_c dl \qquad \text{eq.(2.14)}$$

Substituting equations (2.12-2.14) into equation (2.11) the potential ϕ can be rewritten as

$$\phi = D_o\sigma_o + \sum_{m=1}^{k} D_m\sigma_m + \sum_{m=1}^{k} \hat{D}_m\hat{\sigma}_m \qquad \text{eq.(2.15)}$$

Using Dwight (handbook of integral tables) the solutions for eqs. (2.12-2.14) when $r \geq r'$ become [6]

$$D_o = -\frac{r'\ln(r)}{\epsilon_o} \qquad r \geq r' \qquad \text{eq.(2.16)}$$

$$D_m = \frac{(r')^{m+1}\cos(m\theta)}{2\epsilon_o\, mr^m} \qquad r \geq r' \qquad \text{eq.(2.17)}$$

$$\hat{D}_m = \frac{(r')^{m+1}\sin(m\theta)}{2\epsilon_o\, mr^m} \qquad r \geq r' \qquad \text{eq.(2.18)}$$

When $r < r'$ the solution for eqs.(2.12-2.14) become

$$D_o = -\frac{r'\ln(r')}{\epsilon_o} \qquad r < r' \qquad \text{eq.(2.19)}$$

$$D_m = \frac{(r')^{m-1}\cos(m\theta)}{2\epsilon_o\, mr^m} \qquad r<r' \qquad \text{eq.(2.20)}$$

$$\hat{D}_m = \frac{(r')^{m-1}\sin(m\theta)}{2\epsilon_o\, mr^m} \qquad r<r' \qquad \text{eq.(2.21)}$$

Substituting eqs(2.16 - 2.18) into eq.(2.15) the solution for the potential ϕ for one bare wire when $r \geq r'$ becomes

$$\phi = -\frac{r'\ln(r)}{\epsilon_o}\sigma_o + \frac{1}{2\epsilon_o}\sum_{m=1}^{k}\frac{(r')^{m+1}\cos(m\theta)}{mr^m}\sigma_m + \frac{1}{2\epsilon_o}\sum_{m=1}^{k}\frac{(r')^{m+1}\sin(m\theta)}{mr^m}\hat{\sigma}_m \qquad \text{eq.(2.22)}$$

Substituting eqs.(2.19-2.21) into eq.(2.15) the solution for the potential ϕ for one bare wire when $r<r'$ becomes

$$\phi = -\frac{r'\ln(r')}{\epsilon_o}\sigma_o + \frac{1}{2\epsilon_o}\sum_{m=1}^{k}\frac{(r')^{m-1}\cos(m\theta)}{mr^m}\sigma_m + \frac{1}{2\epsilon_o}\sum_{m=1}^{k}\frac{(r')^{m-1}\sin(m\theta)}{mr^m}\hat{\sigma}_m \qquad \text{eq.(2.23)}$$

Equations 2.22 and 2.23 are the near field potential functions due to a Fourier series representation of the charge around a cylindrical conductor.

CHAPTER 3

APPLICATION OF THE METHOD OF MOMENTS IN DETERMINING THE CAPACITANCE OF A BARE 2-CONDUCTOR SYSTEM

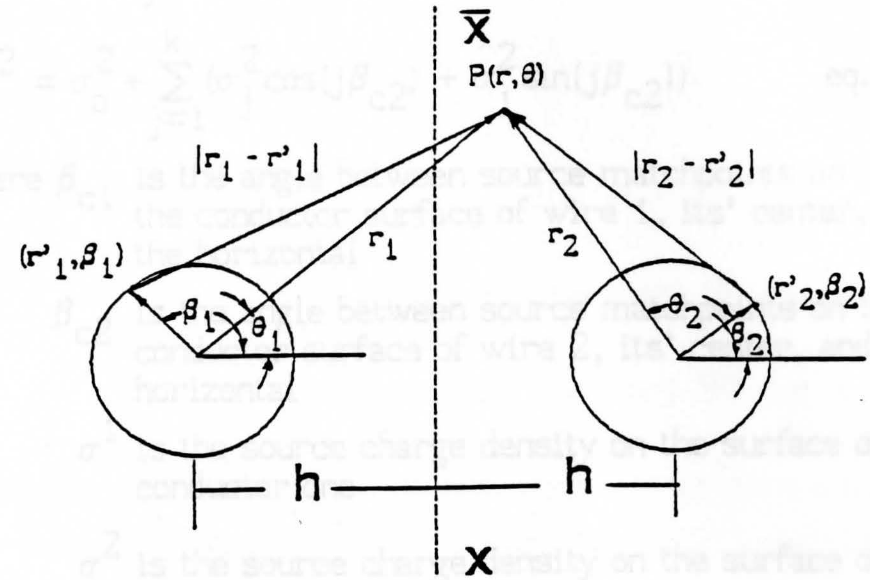


FIG 3.1 BARE 2-CONDUCTOR SYSTEM

Observing the plane of symmetry, (Figure 3.1), the charge distribution on wire 2 for a 2-conductor system is the image of that on wire 1 thus the contribution of the sine terms will be identically equal to zero. Even though it is not necessary to keep the sine terms in the Fourier series representation of the charge density expression it is done here to maintain continuity for developing the multiconductor system. Using the sine terms allows non-restricted orientation of the wires as will become more evident when deriving the multiconductor wire bundle system.

A note about notation, for the remainder of this report superscripts are indices used to indicate whether the source or potential is place on a particular wire and not exponents. For example σ^2 indicates the charge is placed on the surface of wire 2. Powers are expressed by using parentheses such as $(\sigma)^2$. Subscripts are used to indicate the i-th potential matchpoint or the j-th source point.

The charge distribution on each wire (see figure 3.1) can be expressed as follows

$$\sigma^1 = \sigma_o^1 + \sum_{j=1}^{k} (\sigma_j^1 \cos(j\beta_{c1}) + \hat{\sigma}_j^1 \sin(j\beta_{c1})) \qquad \text{eq. (3.1)}$$

$$\sigma^2 = \sigma_o^2 + \sum_{j=1}^{k} (\sigma_j^2 \cos(j\beta_{c2}) + \hat{\sigma}_j^2 \sin(j\beta_{c2})) \qquad \text{eq. (3.2)}$$

where β_{c1} is the angle between source matchpoints on the conductor surface of wire 1, its' center, and the horizontal

β_{c2} is the angle between source matchpoints on the conductor surface of wire 2, its' center, and the horizontal

σ^1 is the source charge density on the surface of conductor one

σ^2 is the source charge density on the surface of conductor two

Matchpoints are corresponding points of source and potential points on each conductor and the number of matchpoints depends on the number of harmonics that are used in the Fourier series. For example, if one harmonic is used there will be 3 unknowns σ_0, σ_j, $\hat{\sigma}_j$ for each conductor. Therefore, to solve for the unknowns uniquely there must be 3 matchpoints around each conductor. If k (number of harmonics) is 2 then there will 5 unknowns so 5 matchpoints must be selected around each conductor. In other words discrete points are selected to represent the source and potential distribution around each conductor. Equations 3.1 and 3.2 can be rewritten in short form sigma notation as follows

$$\sigma^n = \sigma_o^n + \sum_{j=1}^{k} (\sigma_j \cos(j\beta_{cn}) + \hat{\sigma}_j \sin(j\beta_{cn})) \qquad \text{eq. (3.3)}$$

where n = 1,2

The variable 'n' is used to denote the wire on which the charge is placed. Since the system is linear, superposition can be applied, thus the total potential at a point 'P' (see figure 3.1) from a source on wires one and two is given by

$$\phi_p^t = \phi_p^1 + \phi_p^2 = \sum_{n=1}^{2} \phi_p^n \qquad \text{eq. (3.4)}$$

where ϕ_p^t = total potential at point 'P'

ϕ_p^1 = the potential a point 'P' due to a source on wire 1

ϕ_p^2 = the potential a point 'P' due to a source on wire 2

Calculating the potential from a source charge from wire one as defined in eq.(3.1), the potential ϕ_p^1 (using eq.(2.22) becomes

$$\phi_p^1 = -\frac{r'_1 \ln(r_1)}{\epsilon_o}\sigma_o^1 + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\frac{(r'_1)^{j+1}\cos(j\theta_1)}{jr_1^j}\sigma_j^1 + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\frac{(r'_1)^{j+1}\sin(j\theta_1)}{jr_1^j}\hat{\sigma}_j^1$$

eq. (3.5)

The potential ϕ_p^2 from a source charge on wire 2 as defined in eq.(3.2) is expressed as

$$\phi_p^2 = -\frac{r'_2 \ln(r_2)}{\epsilon_o}\sigma_o^2 + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\frac{(r'_2)^{j+1}\cos(j\theta_2)}{jr_2^j}\sigma_j^2 + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\frac{(r'_2)^{j+1}\sin(j\theta_2)}{jr_2^j}\hat{\sigma}_j^2$$

eq. (3.6)

where θ_1 is the angle between potential point 'P', the center of source wire 1, and the horizontal

θ_2 is the angle between potential point 'P', the center of source wire 2, and the horizontal

r'_1 is the vector from center to source matchpoints on wire 1

r'_2 vector from center to source matchpoints on wire 2

r_1 vector from center of source wire 1 to point 'P'

r_2 vector from center of source wire 2 to point 'P'

Substituting eqs. (3.5-3.6) into eq. (3.4), the total potential at a point 'P' can be written in sigma notation as

$$\phi_p^t = \sum_{n=1}^{2}\left[-\frac{r'_n \ln(r_n)}{\epsilon_o}\sigma_o^n + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\frac{(r'_n)^{j+1}\cos(j\theta_n)}{j r_n^j}\sigma_j^n + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\frac{(r'_n)^{j+1}\sin(j\theta_n)}{j r_n^j}\hat{\sigma}_j^n \right] \quad \text{eq. (3.7)}$$

where $r \geq r'$

Now the potential function can be descritized using the following change of variables

$$D_{i1}^{1} = -\frac{r'_1 \ln(r_1)}{\epsilon_o} \quad \text{eq. (3.8)}$$

$$D_{i1}^{2} = -\frac{r'_2 \ln(r_2)}{\epsilon_o} \quad \text{eq. (3.9)}$$

$$D_{i(j+1)}^{1} = \frac{(r'_1)^{j+1}\cos(j\theta_1)}{2\epsilon_o j r_1^j} \quad \text{eq. (3.10)}$$

$$\hat{D}_{i(j+1)}^{1} = \frac{(r'_1)^{j+1}\sin(j\theta_1)}{2\epsilon_o j r_1^j} \quad \text{eq. (3.11)}$$

$$D_{i(j+1)}^{2} = \frac{(r'_2)^{j+1}\cos(j\theta_2)}{2\epsilon_o j r_2^j} \quad \text{eq. (3.12)}$$

$$\hat{D}_{i(j+1)}^{2} = \frac{(r'_2)^{j+1}\sin(j\theta_2)}{2\epsilon_o j r_2^j} \quad \text{eq. (3.13)}$$

The problem now becomes one of bookkeeping, since it is necessary to keep track of the potential points as well as the source points on the surface of each conductor and which wire has the source points and which one has the potential points. Substituting equation (3.8-3.13) into eq.(3.7) the total potential at each matchpoint 'i' due to corresponding unit sources from each matchpoint 'j' on wires one and two is given by

$$\phi_i^t = D_{i1}^1\sigma_o^1 + \sum_{j=1}^{k}(D_{i(j+1)}^1\sigma_j^1 + \hat{D}_{i(j+1)}^1\hat{\sigma}_j^1) + D_{i1}^2\sigma_o^2 + \sum_{j=1}^{k}(D_{i(j+1)}^2\sigma_j^2 + \hat{D}_{i(j+1)}^2\hat{\sigma}_j^2)$$

eq.(3.14)

or in shorthand sigma notation

$$\phi_i^t = \sum_{n=1}^{2}\left[D_{i1}^n\sigma_o^n + \sum_{j=1}^{k}(D_{i(j+1)}^n\sigma_j^n + \hat{D}_{i(j+1)}^n\hat{\sigma}_j^n\right].$$

eq.(3.15)

where i=1,2,...,(2k+1)

Let 'm' designate the wire on which the potential points are evaluated then eq.(3.15) becomes

$$\phi_i^m = \sum_{n=1}^{2}\left[D_{i1}^{mn}\sigma_o^n + \sum_{j=1}^{k}(D_{i(j+1)}^{mn}\sigma_j + \hat{D}_{i(j+1)}^{mn}\hat{\sigma}_j)\right]$$

eq.(3.16)

where m=1,2

i=1,2,3,..,(2k+1) for each wire

ϕ_i^m is defined as the potential on wire m at matchpoint i

The term D_{ij}^{mn} is defined as the potential at matchpoint 'i' on wire m due to a unit source charge at matchpoint 'j' on wire n.

Since the number of wires in the system is 2, n = m = 2, eq.(3.16) has 2(2k+1) unknowns which implies 2k+1 matchpoints should be selected around each wire to solve for the unknowns, namely σ_0, σ_j, $\hat{\sigma}_j$. The set of matrix equations that represent eq.(3.16) is shown in eq.(3.17) in partitioned form.

$$
\left[\begin{array}{c|c}
D^{11}_{11}\cdots D^{11}_{1(k+1)}\hat{D}^{11}_{12}\cdots\hat{D}^{11}_{1(k+1)} & D^{12}_{11}\cdots D^{12}_{1(k+1)}\hat{D}^{12}_{12}\cdots\hat{D}^{12}_{1(k+1)} \\
D^{11}_{21}\cdots D^{11}_{2(k+1)}\hat{D}^{11}_{22}\cdots\hat{D}^{11}_{2(k+1)} & D^{12}_{21}\cdots D^{12}_{2(k+1)}\hat{D}^{12}_{22}\cdots\hat{D}^{12}_{2(k+1)} \\
\vdots & \vdots \\
D^{11}_{(k+1)1}\quad\cdots\quad\hat{D}^{11}_{(k+1)(k+1)} & D^{12}_{(k+1)1}\quad\cdots\quad\hat{D}^{12}_{(k+1)(k+1)} \\
D^{11}_{(k+2)1}\quad\cdots\quad\hat{D}^{11}_{(k+2)(k+1)} & D^{12}_{(k+2)1}\quad\cdots\quad\hat{D}^{12}_{(k+2)(k+1)} \\
\vdots & \vdots \\
D^{11}_{(2k+1)1}\quad\cdots\quad\hat{D}^{11}_{(2k+1)(k+1)} & D^{12}_{(2k+1)1}\quad\cdots\quad\hat{D}^{12}_{(2k+1)(k+1)} \\
\hline
D^{21}_{11}\cdots D^{21}_{1(k+1)}\hat{D}^{21}_{12}\cdots\hat{D}^{21}_{1(k+1)} & D^{22}_{11}\cdots D^{22}_{1(k+1)}\hat{D}^{22}_{12}\cdots\hat{D}^{22}_{1(k+1)} \\
D^{21}_{21}\cdots D^{21}_{2(k+1)}\hat{D}^{21}_{22}\cdots\hat{D}^{21}_{2(k+1)} & D^{22}_{21}\cdots D^{22}_{2(k+1)}\hat{D}^{22}_{22}\cdots\hat{D}^{22}_{2(k+1)} \\
\vdots & \vdots \\
D^{21}_{(k+1)1}\quad\cdots\quad\hat{D}^{21}_{(k+1)(k+1)} & D^{22}_{(k+1)1}\quad\cdots\quad\hat{D}^{22}_{(k+1)(k+1)} \\
D^{21}_{(k+2)1}\quad\cdots\quad\hat{D}^{21}_{(k+2)(k+1)} & D^{22}_{(k+2)1}\quad\cdots\quad\hat{D}^{22}_{(k+2)(k+1)} \\
\vdots & \vdots \\
D^{21}_{(2k+1)1}\quad\cdots\quad\hat{D}^{21}_{(2k+1)(k+1)} & D^{22}_{(2k+1)1}\quad\cdots\quad\hat{D}^{22}_{(2k+1)(k+1)}
\end{array}\right]
\left[\begin{array}{c}
\sigma^{1}_{0} \\ \sigma^{1}_{1} \\ \vdots \\ \sigma^{1}_{k} \\ \hat{\sigma}^{1}_{1} \\ \vdots \\ \hat{\sigma}^{1}_{k} \\ \hline \sigma^{2}_{0} \\ \sigma^{2}_{1} \\ \vdots \\ \sigma^{2}_{k} \\ \hat{\sigma}^{2}_{1} \\ \vdots \\ \hat{\sigma}^{2}_{k}
\end{array}\right]
=
\left[\begin{array}{c}
\phi^{1}_{1} \\ \phi^{1}_{2} \\ \vdots \\ \phi^{1}_{(k+1)} \\ \phi^{1}_{(k+2)} \\ \vdots \\ \phi^{1}_{(2k+1)} \\ \hline \phi^{2}_{1} \\ \phi^{2}_{2} \\ \vdots \\ \phi^{2}_{(k+1)} \\ \phi^{2}_{(k+2)} \\ \vdots \\ \phi^{2}_{(2k+1)}
\end{array}\right]
\qquad \text{eq.(3.17)}
$$

Rewriting equation (3.17) as a single matrix equation the potential becomes

$$[D][\sigma] = [\phi] \qquad \text{eq.(3.18)}$$

Taking the inverse of the 'D' matrix, the solution of the charge density can be found using equation (3.19)

$$[\sigma] = [D]^{-1}[\phi] \qquad \text{eq.(3.19)}$$

Recall the charge density around conductor 1 was given by

$$\sigma^1(\beta_{c1}) = \sigma_o^1 + \sum_{j=1}^{k}(\sigma_j^1\cos(j\beta_{c1}) + \hat{\sigma}_j^1\sin(j\beta_{c1})) \quad \text{eq. (3.20)}$$

where β_{c1} is the angle between source matchpoints on the conductor surface of wire 1, its' center, and the horizontal

Integrating over the total surface of the conductor 1 (2π radians) the cosine and sine terms vanish, thus the total charge on the conductor surface is given by

$$q_1 = \int_s \sigma^1(\beta_{c1})ds = \int_0^1\int_0^{2\pi} \sigma^1(\beta_{c1})\, r_{c1}d\beta_{c1}dl = 2\pi r_{c1}\sigma_o^1 \quad \text{eq. (3.21)}$$

where r_{c1} is the radius of conductor 1

By definition the capacitance is the ratio of static charge on one conductor divided by the potential between the conductors, thus

$$C = \frac{q_1}{\phi^1 - \phi^2} = \frac{2\pi r_{c1}\sigma_o^1}{\phi^1 - \phi^2} = 2\pi r_{c1}\sigma_o^1 \quad \text{eq. (3.22)}$$

where it has been assumed that ϕ^1 is equal to one volt and ϕ^2 is at ground potential or zero.

The value of capacitance derived for a two wire system using the method of moments compares favorably with the closed form solution eq.(3.23).[7]

$$C = \frac{\pi\,\epsilon_o}{\cosh^{-1}(d/2r)} \quad \text{eq. (3.23)}$$

where d is defined as the distance between wire centers
r is defined as the radius of the conductor
ϵ_0 is defined as the permittivity of air which is equal to 8.85 pFm^{-1}

The tabulated results of the transmission line capacitance computed by the method of moments vs the closed form solution is shown in table 3.1. Analysis of the table indicates that the ratio of center-to-center separation to conductor radius (d/r) requires more Fourier terms or harmonics when the ratio is small. For larger separations fewer harmonics are required to approximate the closed form solution. The table also indicates that only a few harmonics are required before the approximate method (the method of moments) converges to that of the closed form solution. Table 3.1 also shows the difference in the number of Fourier terms and the CPU time requirements when the system under consideration is a ribbon cable (R) or a wire bundle (B). The reason for this difference will become clearer when multi-conductor systems are studied in the next section but for now the reason for the difference is in the number of Fourier terms.

Since there is symmetry about the line shown in figure 3.1, the sine terms cancel out. Therefore, when working with ribbon cables only the average terms plus the cosine terms are used in evaluating the charge density around the conductor. To be more precise 2k+1 terms (matchpoints) are used around the conductor when working with wire bundles whereas only k+1 terms (matchpoints) of the Fourier series are used when working with ribbon cables. Using this symmetry reduces the number of terms as well as the amount of CPU time required to calculate the capacitance.

The input data requirements to run the FORTRAN program for a 2-wire ribbon cable or ripcord with no dielectric insulation surrounding the conductors are listed below.

1. Type of configuration [R]
2. Option [2]
3. Number of wires [2]
4. Number of harmonics NHC = [1,2,3,...,19]
5. Are all wires solid ? [y]
6. Do all wires have the same radius [y]
7. Enter the radius of the conductors XRC = [1.0E-3]
8. Enter the center-to-center wires separation [XSEP]

The input data requirements to run the FORTRAN program for a 2-wire wire bundle with no dielectric insulation surrounding the conductors is listed below

1. Type of configuration [B]
2. Option [2]
3. Number of wires [2]
4. Number of harmonics NHC = [1,3,5,...,15]
5. Are all wires solid ? [y]
6. Do all wires have the same radius [y]
7. Enter the radius of the conductors XRC = [1.0E-3]
8. Enter the horizontal distance between wire (1) and wire (2) y(1,2) = (0,0)
9. Enter the vertical distance between wire(1) and wire (2) y(1,2) = (0,0)
10. Is the reference number the same as the ground reference conductor? Enter y/n, PROMPT = [y]

Inside the brackets [ ] requires user input in the form of a value or response to the program.

TABLE 3.1

Bare 2-wire ribbon cable capacitance computed by the method of moments vs closed form capacitance

ratio sep. to cond. radius (d/r)	closed form cap value pF/m	no. of harm.	# of Four. Terms B	R	trans. line cap. wire bundle (pF/m)	CPU time (sec)	trans. lin cap. ribbon cable (pF/m)	CPU time (sec)
2.02:1	196.85	1	3	2	76.658	10	60.055	10
		3	7	4	144.04	30	103.98	15
		5	1	6	176.99	80	137.41	25
		7	15	8	189.77	172	160.44	43
		9	19	10	194.34	322	175.19	70
		11	23	12	195.95	541	184.20	106
		13	27	14	196.52	872	189.54	152
		15	31	16	196.73	1229	192.66	216
		17	35	18			194.45	300
		19	39	20			195.48	384
2.5:1	40.133	1	3	2	38.514	10	36.918	10
		3	7	4	40.081	30	39.934	15
		5	11	6	40.128	80	40.119	25
		7	15	8	40.130	172	40.130	43
		9	19	10	40.130	322	40.130	70
3:1	28.902	1	3	3	28.602	10	28.245	10
		3	7	4	28.899	31	28.890	15
		5	11	6	28.902	82	28.902	25
4:1	21.122	1	3	3	21.082	10	21.032	10
		3	7	4	21.121	31	21.121	15
		5	11	6	21.122	82	21.122	25
5:1	17.754	1	3	3	17.743	10	17.730	10
		3	7	4	17.754	32	17.754	15
		5	11	6			17.754	25
6:1	15.780	1	3	3	15.776	10	15.772	10
		3	7	4	15.780	32	15.780	15

CHAPTER 4

APPLICATION OF THE METHOD OF MOMENTS IN DETERMINING THE CAPACITANCE OF A BARE MULTICONDUCTOR SYSTEM

Recall the definition of capacitance for a bare 2-wire system, i.e., the ratio of static charge on one conductor divided by the potential difference between the conductors

$$C = q/V \qquad \text{eq. (4.1)}$$

This is true provided the charge on conductor 1 (q_1) and the charge on conductor 2 (q_2) are equal in magnitude but opposite in sign, i.e.,

$$q_1 = -q_2 \qquad \text{eq. (4.2)}$$

If however q_1 and q_2 are arbitrary, the charge on each conductor is described in the following matrix equation

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \qquad \text{eq. (4.3)}$$

or

$$q_1 = c_{11}\phi_1 + c_{12}\phi_2 \qquad \text{eq. (4.4)}$$
$$q_2 = c_{21}\phi_1 + c_{22}\phi_2$$

where ϕ is the absolute potential with respect to infinity. Equation (4.3) can be expanded to the bare multiconductor case as shown in eq. (4.5)

$$\begin{bmatrix} q_1 \\ q_2 \\ \cdot \\ \cdot \\ \cdot \\ q_N \end{bmatrix} = \begin{bmatrix} C_{11} C_{12} \cdots & C_{1N} \\ C_{21} C_{22} \cdots & C_{2N} \\ \cdot \quad \cdot & \cdot \\ \cdot \quad \cdot & \cdot \\ \cdot \quad \cdot & \cdot \\ C_{1N} C_{2N} \cdots & C_{NN} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \cdot \\ \cdot \\ \cdot \\ \phi_N \end{bmatrix} \qquad \text{eq. (4.5)}$$

where C_{ij} are the terms of the generalized capacitance matrix. Equation (4.5) can be written as a matrix equation as follows

$$[q] = [C][\phi] \qquad \text{eq. (4.6)}$$

To determine the charge on an n-conductor system the procedure is similar to that of chapter 2, where the n-conductor system is described in figure 4.1

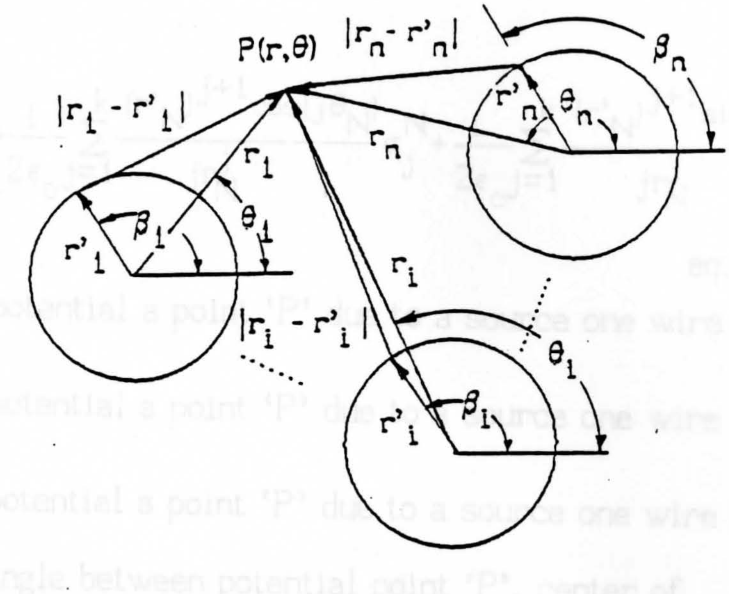


FIG. 4.1 BARE MULITICONDUCTOR SYSTEM

As before, assume a Fourier series representation of the charge distribution around each of the n conductors as described below

$$\sigma^n(\beta_{cn}) = \sigma_o^n + \sum_{j=1}^{k} \left(\sigma_j^n \cos(j\beta_{cn}) + \hat{\sigma}_j^n \sin(j\beta_{cn})\right) \qquad \text{eq. (4.7)}$$

where n=1,2,3,...,N

The potential at point 'P' from a source charge from each wire as described by equation (4.7) can be written as follows

$$\phi_p^1 = -\frac{r'_1 \ln(r_1)}{\epsilon_o}\sigma_o^1 + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\frac{(r'_1)^{j+1}\cos(j\theta_1)}{j r_1^j}\sigma_j^1 + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\frac{(r'_1)^{j+1}\sin(j\theta_1)}{j r_1^j}\hat{\sigma}_j^1$$

eq. (4.8)

$$\phi_p^2 = -\frac{r'_2 \ln(r_2)}{\epsilon_o}\sigma_o^2 + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\frac{(r'_2)^{j+1}\cos(j\theta_2)}{j r_2^j}\sigma_j^2 + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\frac{(r'_2)^{j+1}\sin(j\theta_2)}{j r_2^j}\hat{\sigma}_j^2$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

eq. (4.9)

$$\phi_p^N = -\frac{r'_N \ln(r_N)}{\epsilon_o}\sigma_o^N + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\frac{(r'_N)^{j+1}\cos(j\theta_N)}{j r_N^j}\sigma_j^N + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\frac{(r'_N)^{j+1}\sin(j\theta_N)}{j r_N^j}\hat{\sigma}_j^N$$

eq. (4.10)

where ϕ_p^1 is the potential a point 'P' due to a source one wire 1

ϕ_p^2 is the potential a point 'P' due to a source one wire 2

ϕ_p^N is the potential a point 'P' due to a source one wire N

θ_1 is the angle between potential point 'P', center of source wire 1, and the horizontal

θ_2 is the angle between potential point 'P', center of source wire 2, and the horizontal

θ_N is the angle between potential point 'P', center of source wire N, and the horizontal

r'_1 vector from center to source matchpoint on wire 1

r'_2 vector from center to source matchpoint on wire 2

r'_N vector from center to source matchpoint on wire N

r_1 vector from center of source wire 1 to point 'P'

r_2 vector from center of source wire 2 to point 'P'

r_N vector from center of source wire N to point 'P'

Using superposition, the potential at point 'P' due to sources from all n conductors is

$$\phi_p^t = \phi_p^1 + \phi_p^2 + \ldots + \phi_p^N = \sum_{n=1}^{N} \phi_p^n \qquad \text{eq. (4.11)}$$

where ϕ_p^t = total potential at point 'P' from each source wire

Summing the contributions to the potential from all wires, as described in eqs. (4.8-4.10), the total potential at a point 'P' can be as follows

$$\phi_p^t = \sum_{n=1}^{N}\left[-\frac{r'_n \ln(r_n)}{\epsilon_o}\sigma_o^n + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\frac{(r'_n)^{j+1}\cos(j\theta_n)}{j r_n^j}\sigma_j^n + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\frac{(r'_n)^{j+1}\sin(j\theta_n)}{j r_n^j}\hat{\sigma}_j^n \right] \qquad \text{eq.(4.12)}$$

The potential function can be descritized in terms of 'D' by the following change in variables

$$D_{i1}^1 = -\frac{r'_1 \ln(r_1)}{\epsilon_o} \qquad \text{eq.(4.13)}$$

$$D_{i1}^2 = -\frac{r'_2 \ln(r_2)}{\epsilon_o} \qquad \text{eq.(4.14)}$$

$$D_{i1}^N = -\frac{r'_N \ln(r_N)}{\epsilon_o} \qquad \text{eq.(4.15)}$$

$$D_{i(j+1)}^1 = \frac{(r'_1)^{j+1}\cos(j\theta_1)}{2\epsilon_o\, j r^j} \qquad \text{eq.(4.16)}$$

$$D_{i(j+1)}^2 = \frac{(r'_2)^{j+1}\cos(j\theta_2)}{2\epsilon_o\, j r^j} \qquad \text{eq.(4.17)}$$

$\vdots$

$$D_{i(j+1)}^N = \frac{(r'_N)^{j+1}\cos(j\theta_N)}{2\epsilon_o\, j r^j} \qquad \text{eq.(4.18)}$$

$$\hat{D}_{i(j+1)}^1 = \frac{(r'_1)^{j+1}\sin(j\theta_1)}{2\epsilon_o\, j r^j} \qquad \text{eq.(4.19)}$$

$$\hat{D}_{i(j+1)}^2 = \frac{(r'_2)^{j+1}\sin(j\theta_2)}{2\epsilon_o\, j r^j} \qquad \text{eq.(4.20)}$$

$\vdots$

$$\hat{D}^{N}_{i(j+1)} = \frac{(r'_N)^{j+1}\sin(j\theta_N)}{2\epsilon_o\, jr^j} \qquad \text{eq.}(4.21)$$

Substituting equations (4.13-4.21) into equation (4.12) the total potential at the i-th matchpoint due to a source on the j-th matchpoint is given by

$$\phi^t_i = \sum_{n=1}^{N}\left[D^n_{i1}\sigma^n_o + \sum_{j=1}^{k}(D^n_{i(j+1)}\sigma^n_j + \hat{D}^n_{i(j+1)}\hat{\sigma}^n_j)\right] \qquad \text{eq.}(4.22)$$

Again letting 'm' designate the wire on which the potential matchpoints are to be evaluated equation (4.22) becomes

$$\phi^m_i = \sum_{n=1}^{N}\left[D^{mn}_{i1}\sigma^n_o + \sum_{j=1}^{k}(D^{mn}_{i(j+1)}\sigma^n_j + \hat{D}^{mn}_{i(j+1)}\hat{\sigma}^n_j)\right] \qquad \text{eq.}(4.23)$$

Looking at eq.(4.23) there are Nx(2k+1) unknowns, therefore (2k+1) distinct matchpoints must be selected on each conductor to uniquely determine the charge density. A set of n matrix equations can be written for the n-conductor system as follows

$$\begin{bmatrix} D^{11} & D^{12} & \dots & D^{1N} \\ D^{21} & D^{22} & \dots & D^{2N} \\ \vdots & \vdots & & \vdots \\ D^{N1} & D^{N2} & \dots & D^{1N} \end{bmatrix} \begin{bmatrix} \sigma^1 \\ \sigma^2 \\ \vdots \\ \sigma^N \end{bmatrix} = \begin{bmatrix} \phi^1 \\ \phi^2 \\ \vdots \\ \phi^N \end{bmatrix} \qquad \text{eq. } (4.24)$$

Where each D^{mn} submatrix is a (2k+1)(2k+1) matrix which relates the sources on wire n to the potential matchpoints on wire m. A typical submatrix is given below

$$D^{mn} = \begin{bmatrix} D^{mn}_{11} & D^{mn}_{12} & \cdots & D^{mn}_{1(k+1)} & \hat{D}^{mn}_{12} & \cdots & \hat{D}^{mn}_{1(k+1)} \\ D^{mn}_{21} & D^{mn}_{22} & \cdots & D^{mn}_{2(k+1)} & \hat{D}^{mn}_{22} & \cdots & \hat{D}^{mn}_{2(k+1)} \\ \vdots & & & \vdots & \vdots & & \vdots \\ D^{mn}_{(2k+1)1} & & \cdots & D^{mn}_{(2k+1)(k+1)} & \hat{D}^{mn}_{(2k+1)2} & \cdots & \hat{D}^{mn}_{(2k+1)(k+1)} \end{bmatrix}$$

eq. (4.25)

where D^{mn}_{ij} is defined as before and vectors σ^n and ϕ^m are

$$\sigma^n = \begin{bmatrix} \sigma^n_o \\ \vdots \\ \sigma^n_k \\ \hat{\sigma}^n_1 \\ \vdots \\ \hat{\sigma}^n_k \end{bmatrix} \quad \text{eq.4.26)} \qquad \phi^m = \begin{bmatrix} \phi^m_1 \\ \vdots \\ \phi^m_{k+1} \\ \phi^m_{k+2} \\ \vdots \\ \phi^m_{2k+1} \end{bmatrix} \quad \text{eq. (4.27)}$$

where $n = 1,2,3,\ldots,N$ and $m = 1,2,3,\ldots N$

Rewriting eq. (4.24), the matrix equations in shorthand notation becomes

$$[D][\sigma] = [\phi] \qquad \text{eq. (4.28)}$$

Solving for the charge density, eq. (4.24) becomes

$$[\sigma] = [D]^{-1}[\phi] \qquad \text{eq.(4.29)}$$

Now let $T = D^{-1}$, then eq.(4.29) can be expresssed as

$$\begin{bmatrix} \sigma^1 \\ \sigma^2 \\ \vdots \\ \sigma^N \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & \cdots & T_{1N} \\ T_{21} & T_{22} & \cdots & T_{2N} \\ \vdots & \vdots & & \vdots \\ T_{N1} & T_{N2} & \cdots & T_{NN} \end{bmatrix} \begin{bmatrix} \phi^1 \\ \phi^2 \\ \vdots \\ \phi^N \end{bmatrix} \qquad \text{eq.(4.30)}$$

where the T_{mn} submatrix is a (2k+1)x(2k+1) matrix.

Recall the charge per unit length distribution on the i-th conductor as

$$\sigma^i(\beta_{ci}) = \sigma_o^i + \sum_{j=1}^{k} (\sigma_j^i \cos(j\beta_{ci}) + \hat{\sigma}_j^i \sin(j\beta_{ci}) \qquad \text{eq. (4.31)}$$

The total charge per unit length on the i-th conductor is given as

$$q_i = \int_s \sigma^i(\beta_{ci})\, ds = \int_0^1 \int_0^{2\pi} \sigma^i(\beta_{ci})\, r_{ci} d(\beta_{ci})\, dl \qquad \text{eq.(4.32)}$$

where r_{ci} = radius of the i-th conductor

σ^i = charge density of the i-th conductor

β_{ci} = angle between source matchpoints on the conductor surface, its center, and the horizontal

Substituting eq.(4.31) into eq.(4.32) and integrating around the conductor surface, the charge per unit length on the i-th conductor becomes

$$q_i = \int_0^1 \int_0^{2\pi} \left[\sigma_o^i + \sum_{j=1}^{k} (\sigma_j^i \cos(j\beta_{ci}) + \hat{\sigma}_j^i \sin(j\beta_{ci})) \right] r_{ci} d(\beta_{ci})\, dl \qquad \text{eq.(4.33)}$$

When the charge density is integrated around the conductor, the sine and cosine integrals drop out leaving the average charge density term in the Fourier series, thus the charge per unit length around the conductor is given by

$$q_i = \int_0^1 \int_0^{2\pi} \sigma_o^i \, r_{ci} \, d\beta_{ci} \, dl = 2\pi r_{ci} \, \sigma_o^i \qquad \text{eq. (4.34)}$$

where $i = 1,2,3,\ldots,N$

Thus the charge per unit length on each n conductors is given by

$$q_1 = 2\pi r_{c1} \, \sigma_o^1 \qquad \text{eq. (4.35)}$$

$$q_2 = 2\pi r_{c2} \, \sigma_o^2 \qquad \text{eq. (4.36)}$$

$$\vdots$$

$$q_N = 2\pi r_{cN} \, \sigma_o^N \qquad \text{eq. (4.37)}$$

Looking at equation (4.30) the charge density on the surface of the i-th conductor can be expressed in terms of T as follows

$$\sigma^i = T_{i1}\phi^1 + T_{i2}\phi^2 + \ldots + T_{iN}\phi^N \qquad \text{eq. (4.38)}$$

Since only the average charge density is requred to describe the charge on each wire, eqs. (4.35-4.37), only the first term of the σ^n vector is used, see eq. (4.26). Thus, only the first row of the T matrix is needed to determine the average charge density of the i-th conductor. Rewriting eq. (4.38), the average charge density is is expressed in terms of T as follows

$$\sigma_o^i = T^1_{i1}\phi^1 + T^1_{i2}\phi^2 + T^1_{i3}\phi^3 + \ldots + T^1_{iN}\phi^N \qquad \text{eq. (4.39)}$$

where T^1_{ij} is defined as a 1xn vector whose elements consist of the first row of the T_{ij} submatrix. Substituting eq. (4.39) into eq. (4.34) the charge per unit length on the i-th conductor becomes

$$q_i = 2\pi r_{ci} \sum_{j=1}^{N} T^1_{ij}\phi^j \qquad \text{eq. (4.40)}$$

where $i=1,2,3,\ldots,N$

Since there are (2k+1) matchpoints on each conductor, the charge per unit length on each conductor can be expressed as

$$q_1 = 2\pi r_{c1}\left[\sum_{m=1}^{2k+1} T_{11}^{1m}\phi^1 + \sum_{m=1}^{2k+1} T_{12}^{1m}\phi^2 + \dots \sum_{m=1}^{2k+1} T_{1N}^{1m}\phi^N\right] \quad \text{eq. (4.41)}$$

$$q_2 = 2\pi r_{c2}\left[\sum_{m=1}^{2k+1} T_{21}^{1m}\phi^1 + \sum_{m=1}^{2k+1} T_{22}^{1m}\phi^2 + \dots \sum_{m=1}^{2k+1} T_{2N}^{1m}\phi^N\right] \quad \text{eq. (4.42)}$$

$$\vdots$$

$$q_N = 2\pi r_{cN}\left[\sum_{m=1}^{2k+1} T_{N1}^{1m}\phi^1 + \sum_{m=1}^{2k+1} T_{N2}^{1m}\phi^2 + \dots \sum_{m=1}^{2k+1} T_{NN}^{1}\phi^N\right] \quad \text{eq. (4.43)}$$

or the charge per unit length on the i-th conductor can be expressed in sigma notation as

$$q_i = 2\pi r_{ci}\sum_{j=1}^{N}\left[\sum_{m=1}^{2k+1} T_{ij}^{1m}\phi^j\right] \quad \text{eq. (4.44)}$$

where T_{ij}^{pq} is an element of T_{ij} in the p-th row and q-th column
$i = 1,2,3,\dots,N$

Recall the matrix equation for determining the charge of an n-conductor system

$$\begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_N \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1N} \\ C_{21} & C_{22} & \cdots & C_{2N} \\ \vdots & \vdots & & \vdots \\ C_{N1} & C_{N2} & \cdots & C_{NN} \end{bmatrix} \begin{bmatrix} \phi^1 \\ \phi^2 \\ \vdots \\ \phi^N \end{bmatrix} \quad \text{eq. (4.45)}$$

Rewriting eq. (4.45) in sigma notation the equation for the charge becomes

$$q_i = \sum_{j=1}^{N} C_{ij}\phi^j \qquad \text{eq. (4.46)}$$

where $i = 1,2,3,\ldots,N$

Equating the two series, eqs. (4.46 and 4.44), term by term and assuming the potential is the same at all matchpoints for all n-conductors the expression becomes

$$\sum_{j=1}^{N} C_{ij}\,\phi^j = \sum_{j=1}^{N} \left[2\pi r_{ci} \sum_{m=1}^{2k+1} T_{ij}^{1m} \right] \phi^j \qquad \text{eq. (4.47)}$$

$$i=1,2,3,\ldots,n$$

Thus each term in the generalized capacitance matrix is found by

$$C_{ij} = 2\pi r_{ci} \sum_{m=1}^{2k+1} T_{ij}^{1m} \qquad \text{eq. (4.48)}$$

Simply stated, the elements of the generalized capacitance matrix C_{ij} can be found by adding all the terms in the first row of the ij-th submatrix of T.

CHAPTER 5

DETERMINATION OF THE TRANSMISSION LINE CAPACITANCE FROM THE GENERALIZED CAPACITANCE MATRIX

To determine the terms of the transmission line capacitance matrix from the generalized capacitance matrix, recall the matrix equation to determine the free charge (q_f) on each conductor for an n-conductor system.

$$\begin{bmatrix} q_1 \\ q_2 \\ \cdot \\ \cdot \\ \cdot \\ q_N \end{bmatrix} = \begin{bmatrix} q_{1f} \\ q_{2f} \\ \cdot \\ \cdot \\ \cdot \\ q_{Nf} \end{bmatrix} = \underbrace{\begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1N} \\ C_{21} & C_{22} & \cdots & C_{2N} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ C_{N1} & C_{N2} & \cdots & C_{NN} \end{bmatrix}}_{C} \begin{bmatrix} \phi^1 \\ \phi^2 \\ \cdot \\ \cdot \\ \phi^N \end{bmatrix} \qquad \text{eq. (5.1)}$$

where C is defined as the generalized capacitance matrix, q_{if} denotes the free charge on conductor i, and ϕ^i indicates the potential on on conductor i with respect to infinity. For this discussion, the n-th conductor will be chosen as the reference conductor and the transmission line voltages will be defined by [8,9,10]

$$V_i = (\phi^i - \phi^N) \qquad i=1,2,3,\ldots,N \qquad \text{eq. (5.2)}$$

Therefore the free charge in terms of transmission line voltage is given by

$$\begin{bmatrix} q_{1f} \\ q_{2f} \\ \cdot \\ \cdot \\ \cdot \\ q_{(N-1)f} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1(N-1)} \\ c_{21} & c_{22} & \cdots & c_{2(N-1)} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ c_{(N-1)1} & & \cdots & c_{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ \cdot \\ V_{(N-1)} \end{bmatrix} \text{eq. (5.3)}$$

Where c is defined as the transmission line capacitance matrix and it is assumed that the reference conductor satisfies the following constraint

$$q_{Nf} = -\sum_{i=1}^{N-1} q_{if} \quad \text{eq. (5.4)}$$

To express the charge on the i-th conductor in terms of transmission line voltages, subtract and add terms of eq. (5.1) as follows

$$q_{if} = (C_{i1}\phi^1 - C_{i1}\phi^N) + (C_{i2}\phi^2 - C_{i2}\phi^N) + \ldots + (C_{iN}\phi^N - C_{iN}\phi^N) + (C_{i1}\phi^N + \ldots + C_{iN}\phi^N) \quad \text{eq. (5.5)}$$

Factoring out like terms equation (5.5) becomes

$$q_{if} = C_{i1}(\phi^1 - \phi^N) + C_{i2}(\phi^2 - \phi^N) + \ldots + C_{iN}(\phi^N - \phi^N) + \sum_{m=1}^{N}(C_{im})\phi^N \quad \text{eq. (5.6)}$$

Subsituting eq. (5.2) into eq. (5.6) the free charge on the i-th conductor can be found as follows

$$q_{if} = C_{i1}V_1 + C_{i2}V_2 + \ldots + C_{i(N-1)}V_{(N-1)} + (\sum_{m=1}^{N} C_{im}\phi^N) \quad \text{eq. (5.7)}$$

To solve for ϕ^N express eq. (5.7) for each conductor and use eq. (5.4) as a constraint, thus

$$q_{1f} = C_{11}V_1 + C_{12}V_2 + \ldots + C_{1(N-1)}V_{(N-1)} + \sum_{m=1}^{N} C_{1m}\phi^N \quad \text{eq. (5.8)}$$

$$q_{2f} = C_{21}V_1 + C_{22}V_2 + \ldots + C_{2(N-1)}V_{(N-1)} + \sum_{m=1}^{N} C_{2m}\phi^N \quad \text{eq. (5.9)}$$

$$q_{3f} = C_{31}V_1 + C_{32}V_2 + \ldots + C_{3(N-1)}V_{(N-1)} + \sum_{m=1}^{N} C_{3m}\phi^N \quad \text{eq. (5.10)}$$

$$\vdots$$

$$q_{(N-1)f} = C_{(N-1)1}V_1 + C_{(N-1)2}V_2 + \ldots + C_{(N-1)(N-1)}V_{(N-1)} + \sum_{m=1}^{N} C_{(N-1)m}\phi^N \quad \text{eq. (5.11)}$$

$$-\sum_{i=1}^{N-1} q_{if} = q_{Nf} = C_{N1}V_1 + C_{N2}V_2 + \ldots + C_{N(N-1)}V_{(N-1)} + \sum_{m=1}^{N} C_{Nm}\phi^N \quad \text{eq. (5.12)}$$

Adding eqs. (5.8-5.12) and grouping like terms yields

$$0 = \left[\sum_{m=1}^{N} C_{m1}\right] V_1 + \left[\sum_{m=1}^{N} C_{m2}\right] V_2 + \left[\sum_{m=1}^{N} C_{m3}\right] V_3 + \ldots + \left[\sum_{m=1}^{N} C_{m(N-1)}\right] V_{(N-1)}$$

$$+ \left[\sum_{m=1}^{N} C_{1m} + \sum_{m=1}^{N} C_{2m} + \sum_{m=1}^{N} C_{3m} + \ldots + \sum_{m=1}^{N} C_{Nm}\right] \phi^N \qquad \text{eq. (5.13)}$$

Solving for ϕ^N yields

$$\phi^N = - \frac{\sum_{k=1}^{N-1} \left[\sum_{m=1}^{N} C_{mk} V_k\right]}{\sum_{p=1}^{N} \left[\sum_{m=1}^{N} C_{pm}\right]} \qquad \text{eq. (5.14)}$$

To determine the terms of the transmission line capacitance matrix, see eq. (5.3), from the generalized capacitance matrix, see eq. (5.1), substitute eq. (5.14) into eq. (5.7)

Let $A_i = \sum_{m=1}^{N} C_{im}$ and $D = \sum_{p=1}^{N} \left[\sum_{m=1}^{N} C_{pm}\right]$ in eqs. (5.7 and 5.14) respectively. Upon substituting D, A_i, and the value of ϕ into eq. (5.7), the charge on conductor one becomes

$$q_{1f} = C_{11} V_1 + C_{12} V_2 + \ldots + C_{1(N-1)} V_{(N-1)} - \frac{A_1}{D} \sum_{k=1}^{N-1} \left[\sum_{m=1}^{N} C_{mk} V_k\right]$$

eq. (5.15)

Expanding the last term of eq. (5.15) the free charge on conductor one becomes

$$q_{1f} = C_{11}V_1 + C_{12}V_2 + \dots + C_{1(N-1)}V_{(N-1)} - \frac{A_1}{D}\left[\sum_{m=1}^{N} C_{m1}V_1 + \dots + \sum_{m=1}^{N} C_{m(N-1)}V_{(N-1)}\right]$$

eq. (5.16)

Grouping like terms in eq. (5.16) the charge on conductor one is

$$q_{1f} = (C_{11} - \frac{A_1}{D}\sum_{m=1}^{N} C_{m1})V_1 + (C_{12} - \frac{A_1}{D}\sum_{m=1}^{N} C_{m2})V_2 + \dots + (C_{1(N-1)} - \frac{A_1}{D}\sum_{m=1}^{N} C_{1(N-1)})V_{(N-1)}$$

eq. (5.17)

Equation (5.17) can be expanded for the remaining conductors in matrix form as follows

$$\begin{bmatrix} q_{1f} \\ \vdots \\ q_{(N-1)f} \end{bmatrix} \begin{bmatrix} (C_{11} - \frac{A_1}{D}\sum_{m=1}^{N} C_{m1}) & \dots & (C_{1(N-1)} - \frac{A_1}{D}\sum_{m=1}^{N} C_{m(N-1)}) \\ \vdots & & \vdots \\ (C_{(N-1)1} - \frac{A_{(N-1)}}{D}\sum_{m=1}^{N} C_{m(N-1)} & \dots & (C_{(N-1)(N-1)} - \frac{A_{(N-1)}}{D}\sum_{m=1}^{N} C_{m(N-1)}) \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_{N-1} \end{bmatrix}$$

eq. (5.18)

In order for the matrix equations, eqs. (5.18 and 5.3), to be equal each term of the transmission line capacitance matrices must be equal, therefore

$$c_{ij} = C_{ij} - \frac{A_i}{D}\sum_{m=1}^{N} C_{mj} \quad \text{where } i,j = 1,2,3,\dots,(N-1)$$

eq. (5.19)

Substituting the values of A_i and D into eq. (5.19) produces the final equation for determining the terms of the transmission line capacitance matrix from the generalized capacitance matrix as shown in eqs (5.20).

$$c_{ij} = C_{ij} - \frac{\left[\sum_{m=1}^{N} C_{im}\right]\left[\sum_{m=1}^{N} C_{mj}\right]}{\sum_{p=1}^{N}\left[\sum_{m=1}^{N} C_{pm}\right]} \quad \text{eq. (5.20)}$$

where $i,j=1,2,\ldots,(N-1)$

It should be pointed out that the above development was based on the fact that the last wire in the system was selected as ground reference. The program, however, allows the user to start with another ground reference or to change the ground reference. Caution should be taken in interpreting the results of the transmission line capacitance matrix. For example, if the system consists of 10 wires and wire 4 is selected as reference, the elements of the transmission line capacitance matrix whose row and/or column indices (subscripts) are equal to or greater than that of the reference number must be increased by one. Those indices which are less than the reference number remain unchanged. In this example the position of the elements in the transmission line capacitance matrix are described as follows

position —> actual capacitance value

$c_{1,1} \rightarrow c_{1,1}$	$c_{1,2} \rightarrow c_{1,2}$	$c_{2,1} \rightarrow c_{2,1}$
$c_{2,2} \rightarrow c_{2,2}$	$c_{1,3} \rightarrow c_{1,3}$	$c_{3,1} \rightarrow c_{3,1}$
$c_{3,3} \rightarrow c_{3,3}$	$c_{1,4} \rightarrow c_{1,5}$	$c_{4,1} \rightarrow c_{5,1}$
$c_{4,4} \rightarrow c_{5,5}$	$c_{1,5} \rightarrow c_{1,6}$	$c_{5,1} \rightarrow c_{6,1}$
$\vdots$	$\vdots$	$\vdots$
$c_{9,9} \rightarrow c_{10,10}$	$c_{1,9} \rightarrow c_{1,10}$	$c_{9,1} \rightarrow c_{10,1}$

In other words, $c_{1,4}$ actually describes the capacitance between wires one and five and $c_{4,4}$ describes the self capacitance of wire 5, etc..

An actual run of the program where wire one is selected as reference is shown in appendix B tables(B.1-B.6). Table B.1 shows the generalized capacitance matrix for the four wire system and table B.2 shows the transmission line capacitance matrix. In this example, c_{11} in the transmission line capacitance matrix actually describes the self capacitance of wire 2. Element c_{12} of the transmission line capacitance matrix is actually the capacitance between wires 2 and 3, and element c_{13} is the capacitance between wires 2 and 4. The data listed in appendix B tables (B.1-B.6) are base on a bare wire system as shown in figure 5.1.

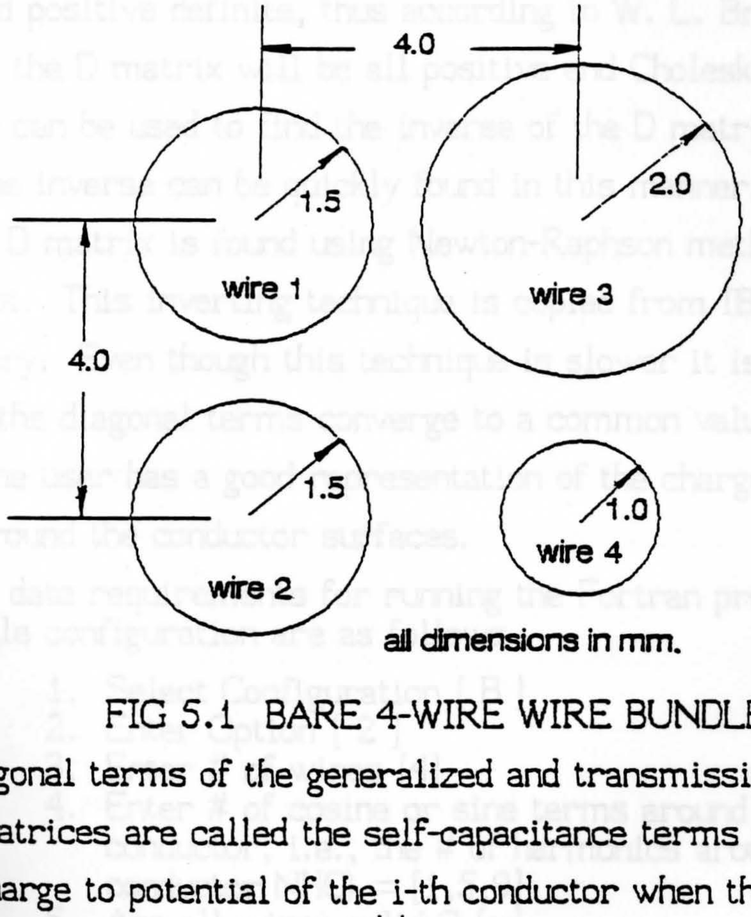


FIG 5.1 BARE 4-WIRE WIRE BUNDLE

The diagonal terms of the generalized and transmission line capacitance matrices are called the self-capacitance terms which is the ratio of charge to potential of the i-th conductor when the other conductors are present but grounded.[11] The diagonal terms will always be positive since the potential and the charge have the same sign. The off-diagonal terms are called mutual capacitance or coefficient of induction terms which are defined as the ratio of the induced charge on the i-th conductor to the potential of the j-th conductor when all conductors, except the j-th, are grounded. The induced charge is always opposite in sign to that of the inducing charge so the off-diagonal terms will always be negative or zero.

The reader may also note that the off-diagonal terms are not equal at first but converge to the same value when more harmonics are selected. The reason for this will be shown in chapter 6. It should be pointed out that the capacitance matrix should be and is symmetric and positive definite, thus according to W. L. Brogan the eigenvalues of the D matrix will be all positive and Cholesky's decomposition can be used to find the inverse of the D matrix.[16] Even though the inverse can be quickly found in this manner, the inverse of the D matrix is found using Newton-Raphson method with maximum pivot. This inverting technique is copied from IBM software library. Even though this technique is slower it is used because when the diagonal terms converge to a common value it implies that the user has a good representation of the charge distribution around the conductor surfaces.

The input data requirements for running the Fortran program for this wire bundle configuration are as follows

1. Select Configuration [B]
2. Enter Option [2]
3. Enter # of wires [4]
4. Enter # of cosine or sine terms around the conductor, i.e., the # of harmonics around the conductor NHC) = [1,5,9]
5. Are all wires solid ? [y]
6. Do all wires have the same radius [n]
7. Enter wire # no.= [1]
8. Enter radius of wire (1) RC(1)= [1.5E-3]
9. Enter wire # no.= [2]
10. Is RC(2)=RC(1)? Enter y/n [y]
11. Enter wire # no.= [3]

12. Is RC(3)=RC(2)? Enter y/n [n]
13. Enter radius of wire (3) RC(3)= [2.0E-3]
14. Enter wire # no.= [4]
15. Is RC(4)=RC(3)? Enter y/n [n]
16. Enter radius of wire (4) RC(4)= [1.0E-3]
17. Enter the horizontal distance between wire (1) and wire (2)
    X(1,2) = 0.0 (meters)
18. Enter the vertical distance between wire (1) and wire (2)
    Y(1,2) = -4.0E-3 (meters)
19. Enter the horizontal distance between wire (1) and wire (3)
    X(1,3) = 4.0E-3 (meters)
20. Enter the vertical distance between wire (1) and wire (3)
    Y(1,3) = 0.0 (meters)
21. Enter the horizontal distance between wire (1) and wire (4)
    X(1,4) = 4.0E-3 (meters)
22. Enter the vertical distance between wire (1) and wire (4)
    Y(1,4) = -4.0E-3 (meters)
23. Is the reference number the same as the ground reference conductor? Enter y/n, PROMPT = [y]

Note that all relative distances are measured from the reference wire, wire(1). The brackets [ ] indicate the actual value or response which must be entered in the FORTRAN program. It should be pointed out that the CPU time given in tables (B.3,B.5,B.7,B.9) are based on a VAX 750.

CHAPTER 6

APPLICATION OF THE METHOD OF MOMENTS IN DETERMINING THE CAPACITANCE OF A DIELECTRIC COATED MULITCONDUCTOR SYSTEM

When working with dielectric coated wires, additional boundary conditions are needed to solve for the unknown charge density residing on the conductor surface as well as the dielectric surface. When a dielectric is introduced into an electric field, it causes an additional charge to be present on the conductor surface. This charge is a results of the electric field passing through the dielectric. The charge on the dielectric, known as "bound charge", induces a charge on the conductor surface of equal magnitude but opposite in sign. This additional induced charge plus the "free charge" residing on the conductor surface must be taken into account when calculating the potential and electric fields from a dielectric coated wire. It is assumed that the dielectric coating is linear, homogeneous, and isotropic. An n-dielectric coated conductor system is shown below.

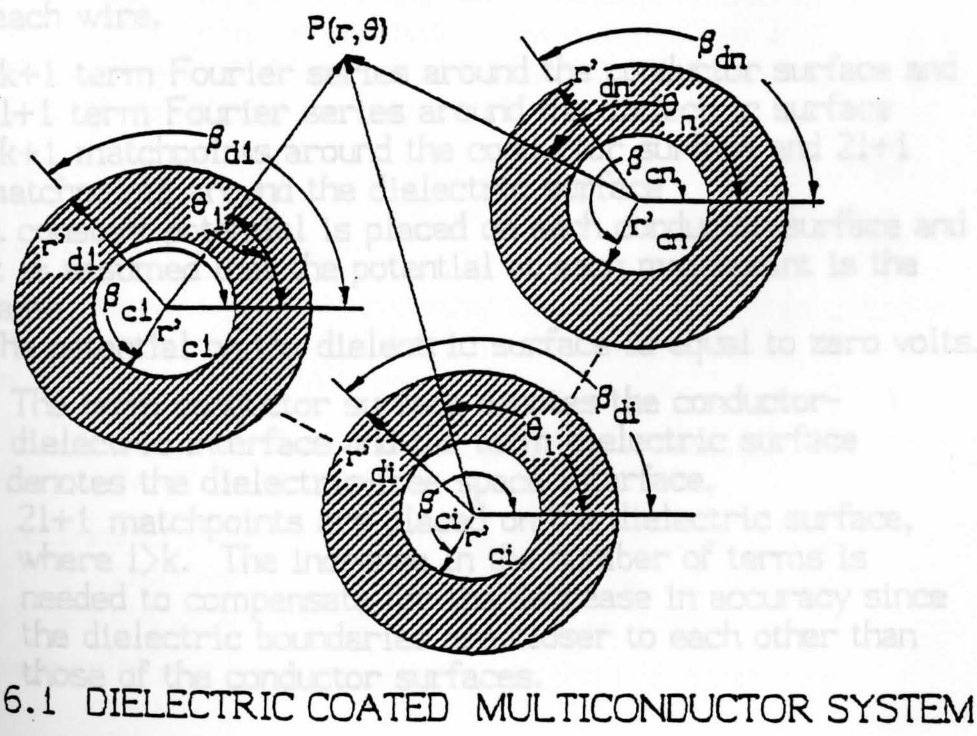


FIG. 6.1 DIELECTRIC COATED MULTICONDUCTOR SYSTEM

The variables in Figure 6.1 are defined as follows:

r_{ci} = radius of the i-th conductor

r_{di} = radius of the dielectric of the i-th conductor

β_{ci} = angle between matchpoints on the conductor surface, its' center, and the horizontal

β_{di} = angle between matchpoints on the dielectric surface, its' center, and the horizontal

θ_i = angle between point 'P', the center of source wire i, and the horizontal

P = potential field point

$\sigma(\beta_{di})$ = surface charge density from bound charge on the dielectric surface

$\sigma(\beta_{ci})$ = surface charge density from bound and free charge on the conduotor surface

q_i = the total static charge on the i-th conductor

ϕ_i = the absolute potential of the i-th conductor

ϵ_{ri} = the relative permittivity of the dielectric on the i-th wire

Based on the knowledge gained from the development of a bare multiconductor system the following boundary conditions are assumed to exist on each wire.

1. 2k+1 term Fourier series around the conductor surface and 2l+1 term Fourier series around the dielectric surface
2. 2k+1 matchpoints around the conductor surface and 2l+1 matchpoints around the dielectric surface
3. A constant potential is placed on each conductor surface and it is assumed that the potential at each matchpoint is the same.
4. The potential on the dielectric surface is equal to zero volts.

Note: 1. The term conductor surface denotes the conductor-dielectric interface and the term dielectric surface denotes the dielectric-free space interface.

2. 2l+1 matchpoints are placed on the dielectric surface, where l>k. The increase in the number of terms is needed to compensate for the decrease in accuracy since the dielectric boundaries are closer to each other than those of the conductor surfaces.

The matrix equation which describes a dielectric coated multi-conductor system is of the following form

$$\begin{bmatrix} D^{mn} & D^{mn'} \\ D^{m'n} & D^{m'n'} \end{bmatrix} \begin{bmatrix} \sigma^{n} \\ \sigma^{n'} \end{bmatrix} = \begin{bmatrix} \phi^{m} \\ 0^{m'} \end{bmatrix} \qquad \text{eq. (6.1)}$$

where $m,n,m',n'=1,2,\ldots,N$

The variables in equation (6.1) are defined as follows

D^{mn} is defined as a submatrix which contains the potentials at (2k+1) matchpoints on conductor m due to a unit charge at (2k+1) matchpoints on conductor n.

$D^{mn'}$ is defined as a submatrix which contains the potentials at (2k+1) matchpoints on conductor m due to a unit charge at (2L+1) matchpoints on dielectric surface on wire n.

$D^{m'n}$ is defined as a submatrix which contains the difference in the normal component of the displacement vector at (2l+1) matchpoints "just inside" and "just outside" the dielectric surface of wire m due to a unit charge at (2k+1) matchpoints on conductor surface n.

$D^{m'n'}$ is defined as a submatrix which contains the difference in the normal component of the displacement vector at (2l+1) matchpoints "just inside" and " just outside" the dielectric surface of wire m due to a unit charge at (2l+1) matchpoints on dielectric surface of wire n.

σ^{n} is defined as a vector containing the surface charge density (from free and bound charge) at (2k+1) matchpoints on conductor n

$\sigma^{n'}$ is defined as a vector containing the surface charge density (from bound charge) at (2l+1) matchpoints on the dielectric surface of wire n.

ϕ^{m} is defined as a vector containing the potentials at (2k+1) matchpoints on conductor surface m

$0^{m'}$ is defined as a vector containing the difference of the normal component of the displacement vector at (2l+1) matchpoints on the dielectric surface of wire m.

It is assumed that the charge density on the i-th conductor surface is defined mathematically by the Fourier series as follows

$$\sigma^{i} = \sigma^{i}_{o} + \sum_{j=1}^{k} \sigma^{i}_{j} \cos(j\beta_{ci}) + \sum_{j=1}^{k} \hat{\sigma}^{i}_{j} \sin(j\beta_{ci}) \qquad \text{eq. (6.2)}$$

and the charge density on the i-th dielectric surface is described by

$$\sigma^{i'} = \sigma^{i'}_{o} + \sum_{j=1}^{l} \sigma^{i'}_{j} \cos(j\beta_{di}) + \sum_{j=1}^{l} \hat{\sigma}^{i'}_{j} \sin(j\beta_{di}) \qquad \text{eq. (6.3)}$$

The submatrices D^{mn} and $D^{mn'}$ in the upper portion of eq. (6.1) denote the potentials on conductor m due to a unit magnitude charge on boundary n. The submatrices $D^{m'n}$ and $D^{m'n'}$ in the lower portion denote the difference between the normal component of the displacement vector just inside and just outside boundary m due to a unit charge on boundary n. The boundary condition denoting the difference between the normal component of the displacement vector just inside and just outside is described mathematically as follows

$$D^{i}_{n} - D^{o}_{n} = 0 \qquad \text{eq. (6.4)}$$

The displacement vector 'D' can be expressed in terms of the electric field intensity by the following equation.

$$D = \epsilon E \qquad \text{eq. (6.5)}$$

Substituting equation (6.5) into equation (6.4) the difference in the normal component of the displacement vector in terms of electric field intensity is

$$\epsilon E^{i}_{n} - \epsilon_0 E^{o}_{n} = \epsilon_r \epsilon_0 E^{i}_{n} - \epsilon_0 E^{o}_{n} = 0 \qquad \text{eq. (6.6)}$$

or

$$\epsilon_r E^{i}_{n} - E^{o}_{n} = 0 \qquad \text{eq. (6.7)}$$

where

D^{i}_{n} = normal component of the surface charge density "just inside" the dielectric

D_n^o = normal component of the surface charge density "just outside" the dielectric

ϵ = dielectric constant of the dielectric

ϵ_o = dielectric constant of air and equal to 8.85×10^{-12}

ϵ_r = relative dielectric constant

E_n^i = normal component of the electric field intensity "just inside" the dielectric

E_n^o = normal component of the electric field intensity "just outside" the dielectric

To determine the electric field from the potenital function when a dielectric is present, recall that when $r \geq r'$ the potential is given by

$$\phi(r,\theta) = -\sigma_0 \frac{r'\ln(r)}{\epsilon_0} + \frac{1}{2\epsilon_0}\sum_{j=1}^{k}\sigma_j \frac{(r')^{j+1}\cos(j\theta)}{jr^j} + \frac{1}{2\epsilon_0}\sum_{j=1}^{k}\hat{\sigma}_j \frac{(r')^{j+1}\sin(j\theta)}{jr^j}$$

eq. (6.8)

The electric field intensity is obtained from the potential function using Laplace's equation, i.e,

$$E(r,\theta) = -\nabla\phi(r,\theta) \qquad \text{eq.(6.9)}$$

The del operator '∇' can be expressed in cylindrical coordinates as follows

$$\nabla = \frac{\partial(\phi)}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial(\phi)}{\partial\theta}\hat{\theta} \qquad \text{eq.(6.10)}$$

Applying eq.(6.9-6.10) to eq.(6.8) the equation for the electric field intensity when $r \geq r'$ is as follows[4,8,9,10,12]

$$E(r,\theta) = \sigma_0 \frac{r'/r}{\epsilon_0}\hat{r} + \frac{1}{2\epsilon_0}\sum_{j=1}^{k}\sigma_j (r'/r)^{j+1}\left[\cos(j\theta)\,\hat{r} + \sin(j\theta)\,\hat{\theta}\right]$$

$$+ \frac{1}{2\epsilon_0}\sum_{j=1}^{k}\hat{\sigma}_j (r'/r)^{j+1}\left[\sin(j\theta)\,\hat{r} - \cos j\theta)\,\hat{\theta}\right]$$

eq. (6.11)

When $r < r'$ the equation describing the potential is given as

$$\phi(r,\theta) = -\sigma_0 \frac{r'\ln(r')}{\epsilon_0} + \frac{1}{2\epsilon_0}\sum_{j=1}^{k} \sigma_j \frac{(r)^j \cos(j\theta)}{j(r')^{j-1}} + \frac{1}{2\epsilon_0}\sum_{j=1}^{k} \hat{\sigma}_j \frac{(r)^j \sin(j\theta)}{j(r')^{j-1}} \qquad \text{eq. (6.12)}$$

Applying eqs.(6.9-6.10) to eq.(6.12) the equation for the electric field intensity when r<r' is a follows

$$E(r,\theta) = 0 - \frac{1}{2\epsilon_0}\sum_{j=1}^{k} \sigma_j (r/r')^{j-1}\left[\cos(j\theta)\,\hat{r} - \sin(j\theta)\,\hat{\theta}\right] - \frac{1}{2\epsilon_0}\sum_{j=1}^{k} \hat{\sigma}_j (r/r')^{j-1}\left[\sin(j\theta)\,\hat{r} + \cos(j\theta)\,\hat{\theta}\right]$$

eq.(6.13)

Rewriting equation (6.1) for all N-conductors, the matrix equation, in partitioned form becomes

$$\left[\begin{array}{cc|cc|c|cc} D^{11} & D^{11'} & D^{12} & D^{12'} & \ldots & D^{1N} & D^{1N'} \\ D^{1'1} & D^{1'1'} & D^{1'2} & D^{1'2'} & \ldots & D^{1'N} & D^{1'N'} \\ \hline D^{21} & D^{21'} & D^{22} & D^{22'} & \ldots & D^{2N} & D^{2N'} \\ D^{2'1} & D^{2'1'} & D^{2'2} & D^{2'2'} & \ldots & D^{2'N} & D^{2'N'} \\ \hline \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \hline D^{N1} & D^{N1'} & D^{N2} & D^{N2'} & \ldots & D^{NN} & D^{NN'} \\ D^{N'1} & D^{N'1'} & D^{N'2} & D^{N'2'} & & D^{N'N} & D^{N'N'} \end{array}\right] \left[\begin{array}{c} \sigma^1 \\ \sigma^{1'} \\ \hline \sigma^2 \\ \sigma^{2'} \\ \hline \vdots \\ \hline \sigma^N \\ \sigma^{N'} \end{array}\right] = \left[\begin{array}{c} \phi^1 \\ 0 \\ \hline \phi^2 \\ 0 \\ \hline \vdots \\ \hline \phi^N \\ 0 \end{array}\right]$$

eq(6.14)

Where the vectors in equation (6.14) are described below

$$\sigma^j = \begin{bmatrix} \sigma_o^j \\ \vdots \\ \sigma_k^j \\ \hat{\sigma}_1^j \\ \vdots \\ \hat{\sigma}_k^j \end{bmatrix} \quad \text{eq.(6.15)} \qquad \sigma^{j'} = \begin{bmatrix} \sigma_o^{j'} \\ \vdots \\ \sigma_l^{j'} \\ \hat{\sigma}_1^{j'} \\ \vdots \\ \hat{\sigma}_l^{j'} \end{bmatrix} \quad \text{eq.(6.16)} \qquad \phi^i = \begin{bmatrix} \phi_1^i \\ \phi_2^i \\ \phi_3^i \\ \vdots \\ \phi_{2k+1}^i \end{bmatrix} \quad \text{eq.(6.17)}$$

Where a typical submatrix in eq. (6.10) is defined as

$$D^{mn} = \begin{bmatrix} D_{11}^{mn} & D_{12}^{mn} & \cdots & D_{1(2k+1)}^{mn} \\ D_{21}^{mn} & D_{21}^{mn} & \cdots & D_{2(2k+1)}^{mn} \\ \vdots & \vdots & & \vdots \\ D_{(2K+1)1}^{mn} & & \cdots & D_{(2k+1)(2k+1)}^{mn} \end{bmatrix} \quad \text{eq.(6.18)}$$

$$D^{mn'} = \begin{bmatrix} D_{11}^{mn'} & \cdots & D_{1(2L+1)}^{mn'} \\ D_{21}^{mn'} & \cdots & D_{2(2L+1)}^{mn'} \\ \vdots & & \vdots \\ D_{(2k+1)1}^{mn'} & \cdots & D_{(2k+1)(2L+1)}^{mn'} \end{bmatrix} \quad \text{eq.(6.19)}$$

$$D^{m'n} = \begin{bmatrix} D_{11}^{m'n} & D_{12}^{m'n} & \cdots & D_{1(2k+1)}^{m'n} \\ D_{21}^{m'n} & D_{22}^{m'n} & \cdots & D_{2(2k+1)}^{m'n} \\ \vdots & \vdots & & \vdots \\ D_{(2L+1)1}^{m'n} & & \cdots & D_{(2L+1)(2k+1)}^{m'n} \end{bmatrix} \quad \text{eq.(6.20)}$$

$$D^{m'n'} = \begin{bmatrix} D_{11}^{m'n'} & \cdots & D_{1(L+1)}^{m'n'} \\ D_{21}^{m'n'} & \cdots & D_{2(L+1)}^{m'n'} \\ \vdots & & \vdots \\ D_{(2L+1)1}^{m'n'} & \cdots & D_{(2L+1)(2L+1)}^{m'n'} \end{bmatrix} \quad \text{eq.(6.21)}$$

Rewriting eq.(6.14) in shorthand matrix notation the potential becomes

$$[D][\sigma] = [\phi] \qquad \text{eq.(6.22)}$$

The solution for the charge density in eq.(6.22) becomes

$$[\sigma] = [D]^{-1}[\phi] \qquad \text{eq.(6.23)}$$

Let $T = D^{-1}$, then the partitioned set of matrix equations for finding the charge density for n-dielectric coated conductors becomes

$$
\begin{bmatrix} \sigma^{1} \\ \sigma^{1'} \\ \sigma^{2} \\ \sigma^{2'} \\ \vdots \\ \sigma^{N} \\ \sigma^{N'} \end{bmatrix}
=
\begin{bmatrix}
T_{11} & T_{11'} & T_{12} & T_{12'} & \cdots & T_{1N} & T_{1N'} \\
T_{1'1} & T_{1'1'} & T_{1'2} & T_{1'2'} & \cdots & T_{1'N} & T_{1'N'} \\
T_{21} & T_{21'} & T_{22} & T_{22'} & \cdots & T_{2N} & T_{2N'} \\
T_{2'1} & T_{2'1'} & T_{2'2} & T_{2'2'} & \cdots & T_{2'N} & T_{2'N'} \\
\vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
T_{N1} & T_{N1'} & T_{N2} & T_{N2'} & \cdots & T_{NN} & T_{NN'} \\
T_{N'1} & T_{N'1'} & T_{N'2} & T_{N'2'} & \cdots & T_{N'N} & T_{N'N'}
\end{bmatrix}
\begin{bmatrix} \phi^{1} \\ 0 \\ \phi^{2} \\ 0 \\ \vdots \\ \phi^{N} \\ 0 \end{bmatrix}
\qquad \text{eq. (6.24)}
$$

It should be noted that the charge densities in eq. (6.24 and 6.14) are the charge densities from "bound" and "free" charges. This combination of charges produces the potential and electric fields. Since the potential is not removed from the conductors the electric field intensity remains approximately the same.[5] Since the E-field remains the same Poisson's equation in integral form must be modified to incorporate the total charge enclosed. Thus, when a dielectric is present Poisson's equation becomes

$$\oint\!\!\oint E \cdot \hat{n}\, ds = \frac{q_{tot\ enc}}{\epsilon_0} \qquad \text{eq. (6.25)}$$

What this means is the free charge densities are increased, bound surface charges of opposite sign are induced, and the total surface charge densities remains unchanged. This also means that the potential and electric-field functions given by equations 6.8, 6.11, 6,12, and 6.13 are valid BOTH inside and outside the dielectric. In other words, the dielectric is replaced with an equivalent surface distribution.[5]

The relationship between the total charge on the i-th conductor surface and "free" and "bound" charge is given in eq.(6.26).

$$q_i = q_f - q_b \qquad \text{eq. (6.26)}$$

Recall the set of matrix equations which relates the free charge on a conductor to that of the potential, i.e.,

$$\begin{bmatrix} q_{1f} \\ q_{2f} \\ \vdots \\ q_{Nf} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1N} \\ C_{21} & C_{22} & \cdots & C_{2N} \\ \vdots & \vdots & & \vdots \\ C_{N1} & C_{N2} & \cdots & C_{NN} \end{bmatrix} \begin{bmatrix} \phi^1 \\ \phi^2 \\ \vdots \\ \phi^N \end{bmatrix} \qquad \text{eq.(6.27)}$$

Rewriting eq.(4.34), the equation for the "free" charge on the i-th conductor is given by

$$q_{if} = \int_0^1 \int_0^{2\pi} \sigma_o^i \, r_{ci} \, d\beta_{ci} \, dl = 2\pi \, r_{ci} \, \sigma_o^i \qquad \text{eq. (6.27)}$$

where r_{ci} = radius of the i-th conductor

σ_o^i = average charge density of the i-th conductor

β_{ci} is the angle between match-points on the conductor surface, its center, and the horizontal

Similarly, the bound charge on the i-th dielectric boundary is given by

$$q_b = q'_{ib} = \int_0^1 \int_0^{2\pi} \sigma_o^{i'} \, r_{di} \, d\beta_{di} \, dl = 2\pi \, r_{di} \, \sigma_o^{i'} \qquad \text{eq. (6.29)}$$

Where the variables in eq.(6.29) are defined as follows

r_{di} = the radius of the dielectric of the i-th wire measured from the center of the wire

$\sigma_o^{i'}$ = the average charge density on the dielectric surface of the i-th wire

β_{di} = the angle between matchpoints on the dielectric surface, the center of the conductor, and the horizontal

The total charge at the conductor interface is shown in eqs.(6.30-6.32)

$$q_1 = 2\pi r_{c1}\sigma_o^1 \quad \text{eq. (6.30)}$$

$$q_2 = 2\pi r_{c2}\sigma_o^2 \quad \text{eq. (6.31)}$$

$$\vdots$$

$$q_N = 2\pi r_{cN}\sigma_o^N \quad \text{eq. (6.32)}$$

The bound charge arising on each dielectric boundary is given by eqs.(6.33-6.35)

$$q'_{1b} = 2\pi r_{d1}\sigma_o^{1'} \quad \text{eq. (6.33)}$$

$$q'_{2b} = 2\pi r_{d2}\sigma_o^{2'} \quad \text{eq. (6.34)}$$

$$\vdots$$

$$q'_{Nb} = 2\pi r_{dN}\sigma_o^{N'} \quad \text{eq.(6.35)}$$

Thus the free charge on each conductor is

$$q_{1f} = q_1 + q'_{1b} \quad \text{eq. (6.36)}$$

$$q_{2f} = q_2 + q'_{2b} \quad \text{eq. (6.37)}$$

$$\vdots$$

$$q_{Nf} = q_N + q'_{Nb} \quad \text{eq.(6.38)}$$

or in general terms, the free charge on the i-th conductor is given by

$$q_{if} = q_i + q'_{ib} = 2\pi r_{ci}\sigma_o^i + 2\pi r_{di}\sigma_o^{i'} \quad \text{eq.(6.39)}$$

Looking at equation (6.24), the charge density on the i-th conductor can be written as

$$\sigma^i = T_{i1}\phi^1 + T_{i1},0 + T_{i2}\phi^2 + T_{i2},0 + \ldots + T_{iN}\phi^N + T_{iN},0$$

eq. (6.40)

or

$$\sigma^i = T_{i1}\phi^1 + T_{i2}\phi^2 + \ldots + T_{iN}\phi^N \qquad \text{eq. (6.41)}$$

Similarly, the charge density on the dielectric of the i-th conductor is found by

$$\sigma^{i'} = T_{i'1}\phi^1 + T_{i'2}\phi^2 + \ldots + T_{i'N}\phi^N \qquad \text{eq. (6.42)}$$

Since only the average term of either σ^i or $\sigma^{i'}$ vector is needed it is only necessary to look at the first row of either vector, thus the charge density on the i-th conductor is

$$\sigma_o^i = T_{i1}^1\phi^1 + T_{i2}^1\phi^2 + \ldots + T_{iN}^1\phi^N \qquad \text{eq. (6.43)}$$

Similarly, the charge density on the dielectric of the i-th conductor is found by

$$\sigma_o^{i'} = T_{i'1}^1\phi^1 + T_{i'2}^1\phi^2 + \ldots + T_{i'N}^1\phi^N \qquad \text{eq. (5.44)}$$

Substituting equations (6.43 and 6.44) into equation (6.39), the total charge of the i-th wire can be found by

$$q_{if} = 2\pi r_{ci}\left[T_{i1}^1\phi^1 + T_{i2}^1\phi^2 + \ldots + T_{iN}^1\phi^N\right] + 2\pi r_{di}\left[T_{i'1}^1\phi^1 + T_{i'2}^1\phi^2 + \ldots + T_{i'N}^1\phi^N\right]$$

eq. (6.45)

Equation (6.45) can be rewritten in sigma notation as

$$q_{if} = 2\pi r_{ci}\sum_{j=1}^{N} T_{ij}^1\phi^j + 2\pi r_{di}\sum_{j=1}^{N} T_{i'j}^1\phi^j \qquad \text{eq. (6.46)}$$

Since 2K+1 matchpoints were selected around the conductor and 2L+1 matchpoints around the dielectric equation, (6.45) becomes

$$q_{if} = 2\pi\sum_{j=1}^{N}\left[r_{ci}\sum_{p=1}^{2k+1} T_{ij}^{1p}\phi^j + r_{di}\sum_{q=1}^{2l+1} T_{i'j}^{1q}\phi^j\right] \qquad \text{eq. (6.47)}$$

Equation (6.47) can be expanded to a set of matrix equations as follows

$$\begin{bmatrix} q_{1f} \\ \vdots \\ q_{Nf} \end{bmatrix} = 2\pi \begin{bmatrix} \left[r_{c1}\sum_{p=1}^{2k+1} T_{11}^{1p} + r_{d1}\sum_{q=1}^{2l+1} T_{1'1}^{1q} \right] \cdots \left[r_{c1}\sum_{p=1}^{2k+1} T_{1N}^{1p} + r_{d1}\sum_{q=1}^{2l+1} T_{1'N}^{1q} \right] \\ \vdots \qquad\qquad \vdots \\ \left[r_{cN}\sum_{p=1}^{2k+1} T_{N1}^{1p} + r_{dN}\sum_{q=1}^{2l+1} T_{N'1}^{1q} \right] \cdots \left[r_{cN}\sum_{p=1}^{2k+1} T_{NN}^{1p} + r_{dN}\sum_{q=1}^{2l+1} T_{N'N}^{1q} \right] \end{bmatrix} \begin{bmatrix} \phi^1 \\ \vdots \\ \phi^N \end{bmatrix}$$

eq.(6.48)

Recall the matrix equation for determining the charge density of an n-conductor system

$$\begin{bmatrix} q_{1f} \\ q_{2f} \\ \vdots \\ q_{Nf} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1N} \\ C_{21} & C_{22} & \cdots & C_{2N} \\ \vdots & \vdots & & \vdots \\ C_{N1} & C_{N2} & \cdots & C_{NN} \end{bmatrix} \begin{bmatrix} \phi^1 \\ \phi^2 \\ \vdots \\ \phi^N \end{bmatrix}$$

eq.(6.49)

In order for equation (6.48) to be equal to equation (6.49), each term of the [C] matrix must be equal to the corresponding term in eq.(6.48). Thus, the terms in the generalized capacitance matrix can be found as follows

$$C_{ij} = 2\pi \left[r_{ci} \sum_{p=1}^{2k+1} T_{ij}^{1p} + r_{di} \sum_{q=1}^{2l+1} T_{ij}^{1q} \right]$$

eq.(6.50)

$i,j = 1,2,3,\ldots,N$

Where T_{ij}^{1p} is an element of T_{ij} submatrix in the first row and p-th column and T_{ij}^{1q} is an element of T_{ij} submatrix in the first row and q-th column. Note also that the first summation is around the conductor surface and the second summation is around the dielectric surface. To determine the actual capacitance between each wire, the transmission line capacitance, substitute eq.(6.50) into eq.(5.20).

To show some of the anomalies of wire bundles an example is given in figure 6.2. Tabulated results of the wire configuration are shown in Appendix C tables C.1, C.2, and C.3. Table C.1 uses one harmonic when determining the capacitance, table C.2 uses 3 harmonics, and table C.3 uses 5 harmonics. The capacitance matrix should be symmetric but when only a small number of harmonics are used the off-diagonal terms are not the same as can be observed in tables C.1, C.2, and C.3. This dilemma is due to the matchpoint selection as shown in figures 6.3a and 6.3b. Recall that the potential and electric field intensity functions developed earlier are based on vectors r and r', where r' is the vector from the center of the source wire to the source matchpoint and r is the vector from the center of the source wire to the potential matchpoint on the potential wire. In these figures only the conductor surface is shown but the same analogy applies to the dielectric surface as well.

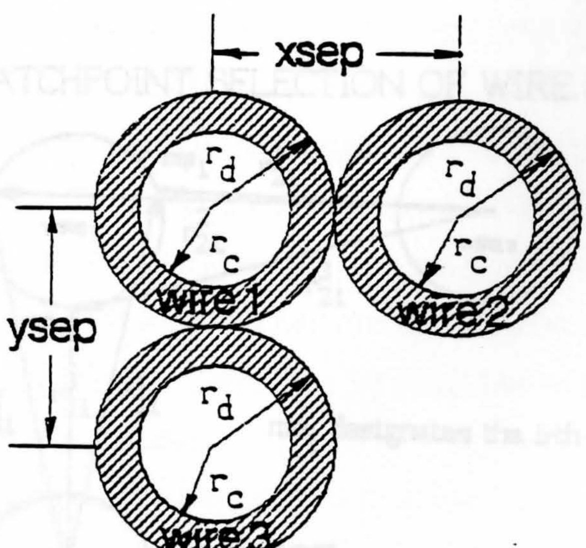


FIG. 6.2 3-WIRE WIRE BUNDLE

The wire data for figure 6.2 is given below

1. 18 AWG, 16 strands using 30 AWG wire
2. Relative permittivity 3.5 to 6.5
3. Equivalent conductor radius 0.6 mm
4. Dielectric radius 1.235 mm
5. Wire separation 2.59 mm

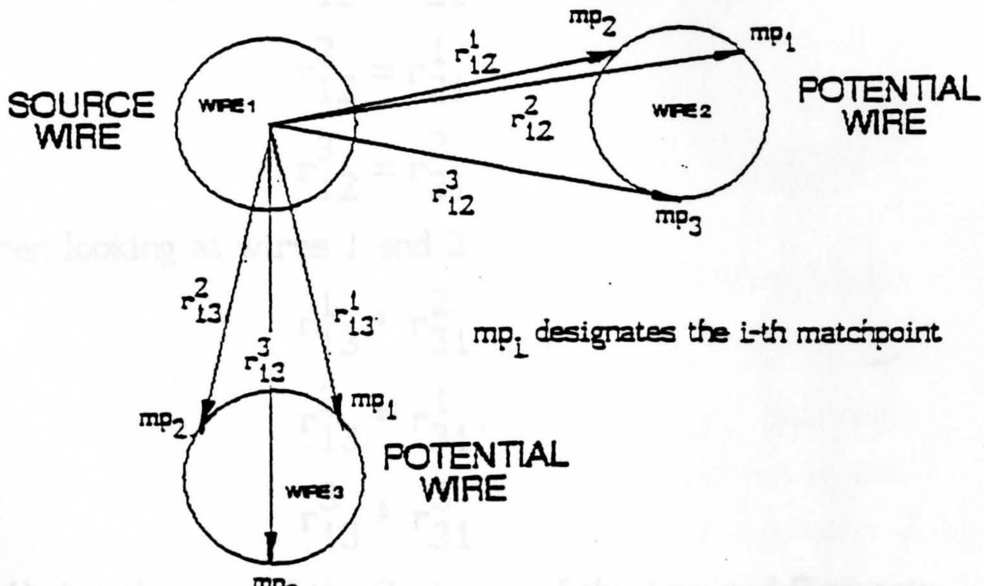


FIG. 6.3A MATCHPOINT SELECTION OF WIRE BUNDLES

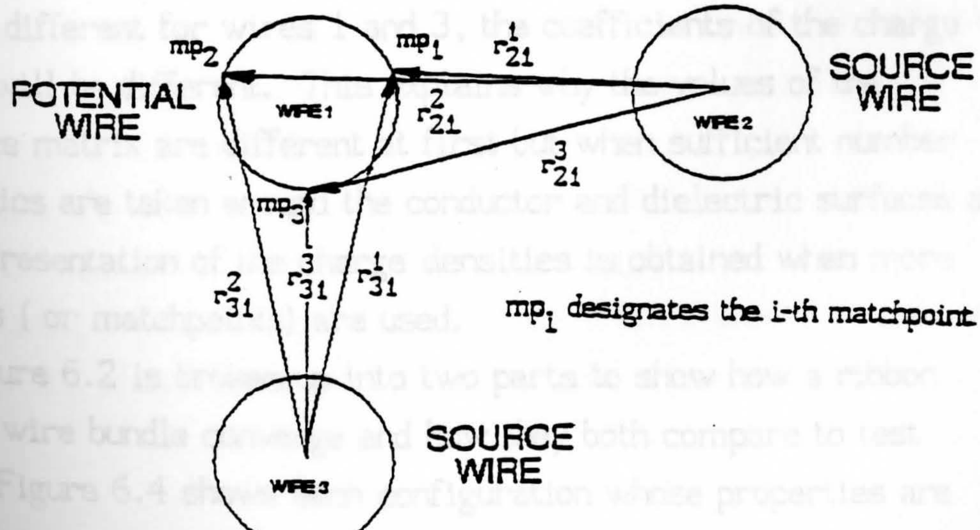


FIG. 6.3B MATCHPOINT SELECTION OF WIRE BUNDLES

From figures 6.3a and 6.3b, the horizontal wires (1 and 2) will be considered a ribbon cable and the vertical wires (1 and 3) will be considered a wire bundle. In ribbon cables, the matchpoint selection is such that when evaluating the charge the off-diagonal terms will be the same because the distance is the same as is evident from figures 6.3a and 6.3b, i.e.,

$$r_{12}^{1} = r_{21}^{2}$$

$$r_{12}^{2} = r_{21}^{1}$$

$$r_{12}^{3} = r_{21}^{3}$$

However when looking at wires 1 and 3

$$r_{13}^{1} \neq r_{31}^{2}$$

$$r_{13}^{2} \neq r_{31}^{1}$$

$$r_{13}^{3} \neq r_{31}^{3}$$

Recall that the sum of the first row of the inverted D matrix, the T matrix gives the capacitance values C_{ij} as was described in equations (6.40-6.50). These coefficients are based on r and r' and since r is different for wires 1 and 3, the coefficients of the charge densities will be different. This explains why the values of the capacitance matrix are different at first but when sufficient number of harmonics are taken around the conductor and dielectric surfaces a better representation of the charge densities is obtained when more harmonics (or matchpoints) are used.

Figure 6.2 is broken up into two parts to show how a ribbon cable and wire bundle converge and how they both compare to test results. Figure 6.4 shows each configuration whose properties are the same as those shown in figure 6.2.

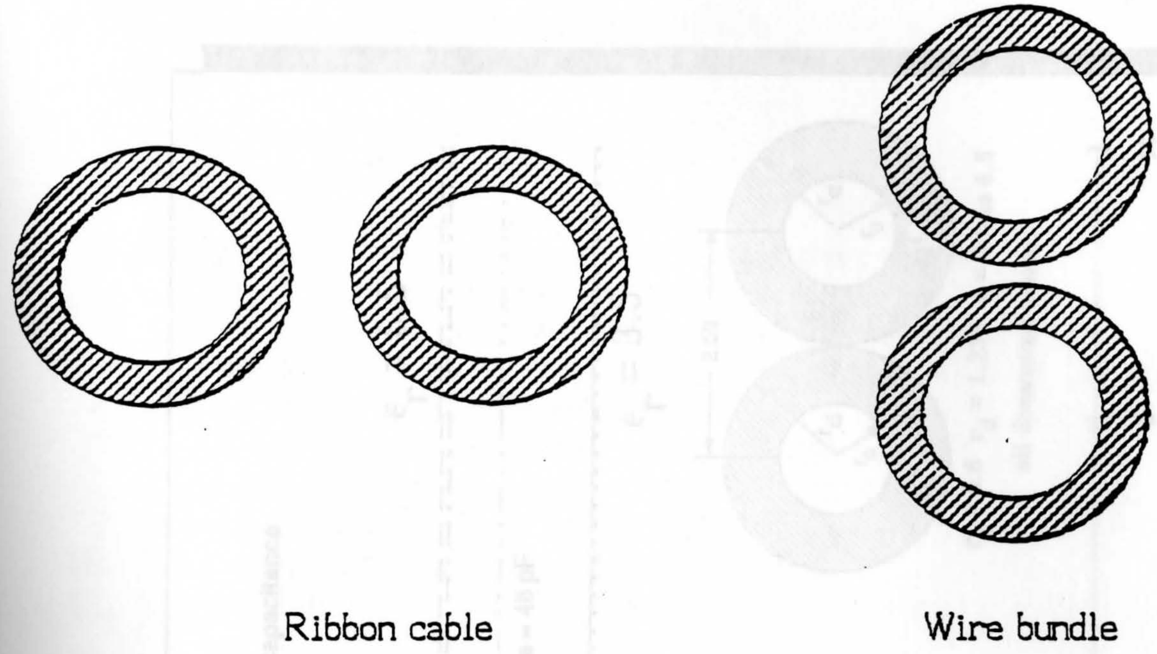


FIG. 6.4 RIBBON CABLE AND WIRE BUNDLE CONFIGURATIONS

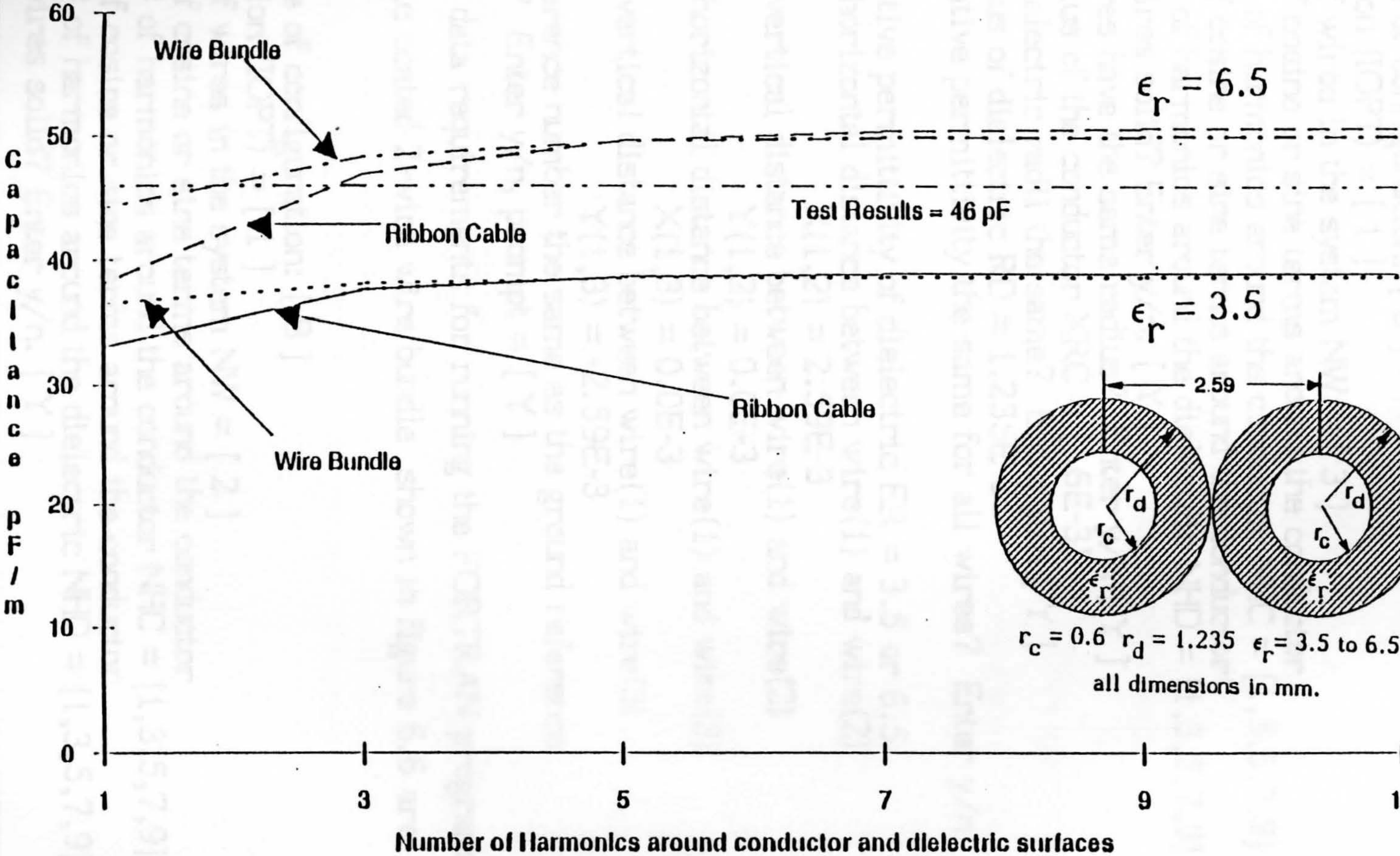
There are many varieties of dielectric coatings; polyvinyl chloride (PVC) is just one of them. According to Belden typical dielectric constant or permittivity of PVC can vary from 3.5 to 6.5.[15]

The measured value of capacitance was obtained using an HP 3577a network analyser and a 3 meter length of 2-wire ripcord as described above. The capacitance measurement was taken at 1 mHz. The calculated values for the ribbon cable and wire bundle are shown in Appendix D tables D.1 and D.2. These values are graphically shown in figure 6.5, page 56..

There are a lot of variables which contribute to the range of capacitance values beside the variance in permittivity. The manufacturing process itself, the thickness of the dielectric, and the actual pattern of the strands of wires are just a few examples.

FIGURE 6.5

Measured capacitance vs calculated capacitance



The input data requirements for running the FORTRAN program for the dielectric coated 3-wire wire bundle shown in figure 6.2 are listed below

1. Enter type of configuration: [B]
2. Enter option (IOPT) = [1]
3. Enter # of wires in the system NW = [3]
4. Enter # of cosine or sine terms around the conductor
   i.e. the # of harmonics around the conductor NHC = [1,3,5,7,9]
5. Enter # of cosine or sine terms around the conductor
   i.e. the # of harmonics around the dielectric NHD = [1,3,5,7,9]
6. Are all wires solid? Enter y/n. [Y]
7. Do all wires have the same radius? Enter y/n [Y]
8. Enter radius of the conductor XRC = [.6E-3]
9. Are all dielectric radii the same? Enter y/n [Y]
10. Enter radius of dielectric RD = 1.235E-3
11. Is the relative permittivity the same for all wires? Enter y/n [Y]
12. Enter relative permittivity of dielectric ER = 3.5 or 6.5
13. Enter the horizontal distance between wire(1) and wire(2)
    X(1,2) = 2.59E-3
14. Enter the vertical distance between wire(1) and wire(2)
    Y(1,2) = 0.0E-3
15. Enter the horizontal distance between wire(1) and wire(3)
    X(1,3) = 0.0E-3
16. Enter the vertical distance between wire(1) and wire(3)
    Y(1,3) = -2.59E-3
17. Is the reference number the same as the ground reference conductor? Enter y/n, prompt = [Y]

The input data requirements for running the FORTRAN program for the dielectric coated 2-wire wire bundle shown in figure 6.6 are listed below

1. Enter type of configuration: [B]
2. Enter option (IOPT) = [1]
3. Enter # of wires in the system NW = [2]
4. Enter # of cosine or sine terms around the conductor
   i.e. the # of harmonics around the conductor NHC = [1,3,5,7,9]
5. Enter # of cosine or sine terms around the conductor
   i.e. the # of harmonics around the dielectric NHD = [1,3,5,7,9]
6. Are all wires solid? Enter y/n. [Y]

7. Do all wires have the same radius? Enter y/n [Y]
8. Enter radius of the conductor XRC = [.6E-3]
9. Are all dielectric radii the same? Enter y/n [Y]
10. Enter radius of dielectric RD = 1.235E-3
11. Is the relative permittivity the same for all wires? Enter y/n [Y]
12. Enter relative permittivity of dielectric ER = 3.5 or 6.5
13. Enter the horizontal distance between wire(1) and wire(2)
    X(1,2) = 0.0
14. Enter the vertical distance between wire(1) and wire(2)
    Y(1,2) = -2.59E-3
15. Is the reference number the same as the ground reference conductor? Enter y/n, prompt = [Y]

The input data requirements for running the FORTRAN program for the dielectric coated 2-wire ribbon cable shown in figure 6.6 are listed below

1. Enter type of configuration: [R]
2. Enter option (IOPT) = [1]
3. Enter # of wires in the system NW = [2]
4. Enter # of cosine or sine terms around the conductor
   i.e. the # of harmonics around the conductor NHC = [1,3,5,7,9]
5. Enter # of cosine or sine terms around the conductor
   i.e. the # of harmonics around the dielectric NHD = [1,3,5,7,9]
6. Are all wires solid? Enter y/n. [Y]
7. Do all wires have the same radius? Enter y/n [Y]
8. Enter radius of the conductor XRC = [.6E-3]
9. Are all dielectric radii the same? Enter y/n [Y]
10. Enter radius of dielectric RD = 1.235E-3
11. Is the relative permittivity the same for all wires? Enter y/n [Y]
12. Enter relative permittivity of dielectric ER = 3.5 or 6.5
13. Is the reference number the same as the ground reference conductor? Enter y/n, prompt = [Y]

CHAPTER 7

APPLICATION OF THE METHOD OF MOMENTS IN DETERMINING THE CAPACITANCE OF MULTICONDUCTOR COAX CABLES

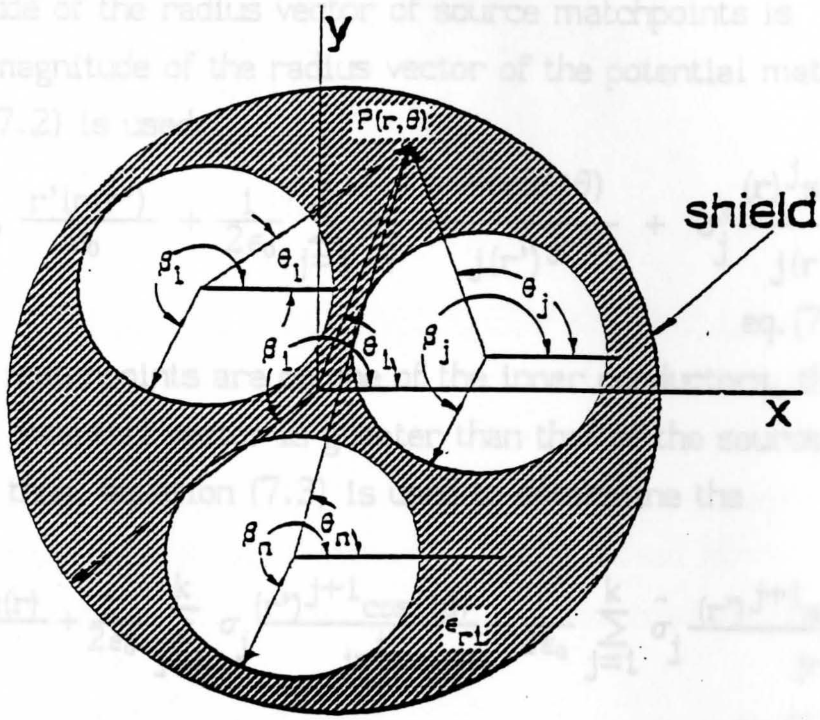


FIG.7.1 MULTICONDUCTOR COAX CABLE

In determining the capacitance for a multiconductor coax cables it is assumed that the charge on the shield is described by

$$q_1 = q_s = -\sum_{i=2}^{N} q_i \qquad \text{eq.(7.1)}$$

where q_s = the charge on the shield

q_i = the charge on the i-th conductor inside the shield

where i=2,3,4,...,N

In the program that determines the capacitance, it is assumed that the inner wires are bare and that they are surrounded by a dielectric which is linear, homogeneous, and isotropic, with a relative permittivity of ϵ_{r1}, see figure 7.1.

The capacitance is found in the same manner as that described in chapter 4 for bare multiconductor systems with one exception, that being, when the charge is on the shield the potential on the inner wires is found using eq.(7.2). The implication here is that $r<r'$, i.e., the magnitude of the radius vector of source matchpoints is larger than the magnitude of the radius vector of the potential matchpoints, thus eq.(7.2) is used.

$$\phi(r,\theta) = -\sigma_0 \frac{r'\ln(r')}{\epsilon_0} + \frac{1}{2\epsilon_0}\sum_{j=1}^{k} \sigma_j \frac{(r)^j\cos(j\theta)}{j(r')^{j-1}} + \sigma'_j \frac{(r)^j\sin(j\theta)}{j(r')^{j-1}}$$

eq.(7.2)

When the source matchpoints are on one of the inner conductors, the magnitude of the potential vector is greater than that of the source vector, i.e. $r \geq r'$ thus, equation (7.3) is used to determine the potential

$$\phi(r,\theta) = -\sigma_0 \frac{r'\ln(r)}{\epsilon_0} + \frac{1}{2\epsilon_0}\sum_{j=1}^{k} \sigma_j \frac{(r')^{j+1}\cos(j\theta)}{jr^j} + \frac{1}{2\epsilon_0}\sum_{j=1}^{k} \hat{\sigma}_j \frac{(r')^{j+1}\sin(j\theta)}{jr^j}$$

eq.(7.3)

Once the exception is implemented, the generalized capacitance and the transmission line capacitance are found as that described in chapters 4 and 5. The closed form solution for the capacitance of a coax cable with a single wire at the center of the shield, see figure 7.2, is given by eq.(7.4).[13]

$$C/l = \frac{55.6\epsilon_r}{\ln(b/a)}$$

eq.(7.4)

where b = inside radius of outer conductor
a = radius of the inner conductor
ϵ_r = relative permittivity of dielectric between conductors
C/l = per unit length capacitance (pF/m)

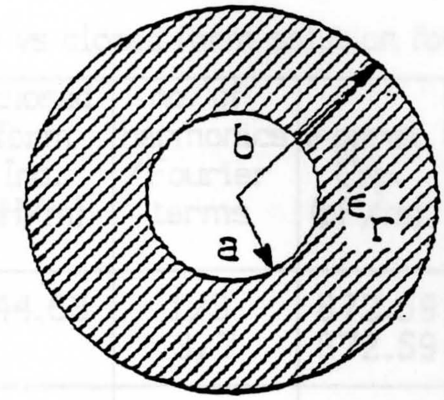


FIG. 7.2 SINGLEWIRE COAX CALBLE

The transmission line capacitance computed by the method of moments compares favorably with the closed from solution in eq. (7.4). The tabulated results of the transmission line capacitance matrix computed by the method of moments vs the closed form solution is shown in table 7.1. The dielectric is assumed to have a permittivity of 3.5.

TABLE 7.1

Approximate vs closed form solution for coax cable

ratio b/a	closed form cap. (pF/m)	closed form Ind. (nH/m)	no. of harmonics /Fourier terms	Approx Cap. (pF/m)	Approx Ind. (nH/m)	Approx CPU time (sec)
1.25:1	872.16	44.63	1/3 3/7	872.59 872.59	44.63 44.63	010 030
1.5:1	479.94	81.09	1/3 3/7	480.22 480.22	81.09 81.09	010 030
2.0:1	280.75	138.63	1/3 3/7	280.91 280.91	138.63 138.63	010 030
3.0:1	177.13	219.72	1/3 3/7	177.24 177.24	219.72 219.72	010 030
4.0:1	140.37	277.26	1/3 3/7	140.46 140.46	277.26 277.26	010 030
5.0:1	120.91	321.89	1/3 3/7	120.98 120.98	321.89 321.89	010 030

Another important parameter in the discussion of wires is the inductance. The closed form solution for inductance can be found in many texts and is reproduced here for comparative analysis.[13]

$$L/1 = \frac{\mu_o}{2\pi} \ln(b/a) = .2 \ln(b/a) \qquad \text{eq. (7.5)}$$

where b = inside diameter of the outer shield
a = the radius of the inner conductor
μ_o = permeability of dielectric
L/1 = per unit length inductance (nH/m)

It is assumed in equation 7.5 that the dielectric is non-ferrous. Therefore the permeability is independent of the medium and has the value of that in air, that is, $4\pi x 10^{-7}$ Henries. The approximate values of inductance are based on the development of W. T. Weeks, from IBM.[14] It was found that the matrices [C], [G], and [L], for a homogeneous medium are related to a matrix [K] which is independent of the medium and depends only on the geometry of the configuration. Stated mathematically, the matrices are related to the [K] matrix as follows

$$[\,C\,] = \epsilon[\,K\,] \qquad \text{eq. (7.6)}$$

$$[\,G\,] = \sigma[\,K\,] = (\sigma/\epsilon)[\,C\,] \qquad \text{eq. (7.7)}$$

$$[\,L\,] = \mu[\,K\,]^{-1} = (\mu\epsilon)[\,C\,]^{-1} \qquad \text{eq. (7.8)}$$

where μ = permeability of air 0.4π nH/m
ϵ = permittivity of medium (pF/m)
σ = conductivity of medium (Ω^{-1}/m)
C = capacitance/meter (pF/m)
G = conductance/meter (Ω^{-1}/m)
L = inductance/meter (nH/m)

What this means is to calculate the inductance simply remove the dielectric and calculate the capacitance of the bare wires, take the inverse and multiply by $\mu\epsilon$, where ϵ is actually ϵ_0 for bare wires.

Due to the symmetry of the coax configuration in figure 7.2, the values of capacitance and inductance remain unchanged for any number of harmonics.

To test whether or not the coax model using the method of moments compares favourably with known test results, various samples were taken from Belden.[15] Comparisions were done on the following types of cables

1. Broadcast and computer cables type 9889 with cellular polyethylene as a dielectric
2. Broadcast and computer cables type 9259 with cellular polyethylene as a dielectric

3. MATV cables type 8212 with cellular polyethlene as a dielectric
4. Broadcast and computer cables type 8267 with solid polyethylene as a dielectric

Cellular polyethylene has a relative permittiviy of 1.6 and solid polyethylene has a relative permittivity of 2.3. The pertinent data for the above examples is shown in Appendix E. A comparision of the test data from Belden and that using the method of moments is shown in table 7.2.

TABLE 7.2
Calculated capacitance verses measured capacitance for coax cable

Type coax	measured cap. pF/m	calculated cap. pF/m	% error
9889	85.3	85.487	.22
9259	56.8	57.425	1.1
8212	56.8	58.626	3.2
8267	101	109.93	8.8

When dealing with multiconductor coax cables which do not exhibit this symmetry it becomes necessary to use more terms of the Fourier series. An example of a simple multiconductor coax cable which does not exhibit this symmetry is shown in figure 7.3.

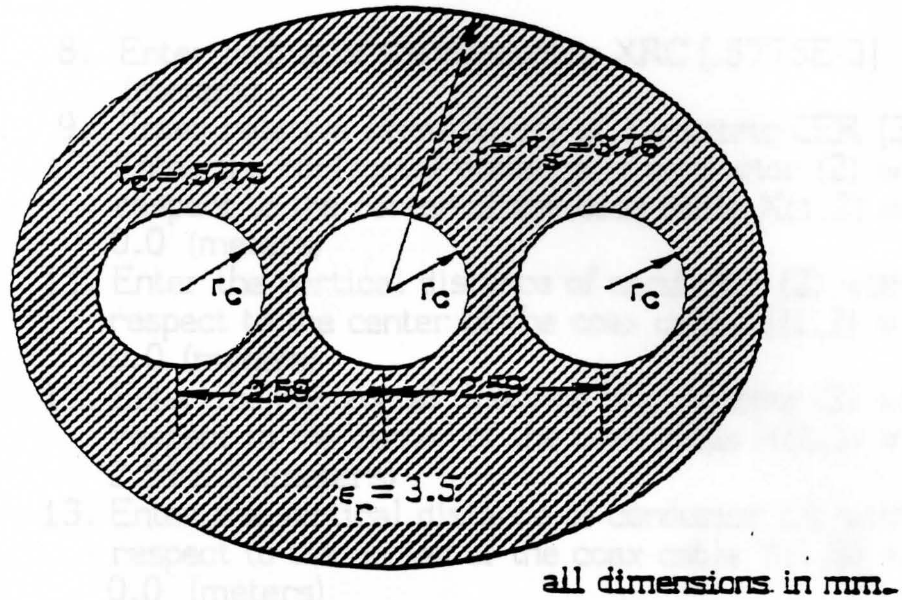


FIG. 7.3 MULTICONDUCTOR COAX CABLE

The output data for this configuration is shown in tables (F.2-F.11) in appendix F. Note that at first that the off-diagonal terms are not equal but with the addition of more Fourier terms the off-diagonal terms converge to a common value. This convergence is due simply to the increase in the number of matchpoints which more precisely represents the charges on the conductor and dielectric surfaces. The difference in the off-diagonal terms is compounded because of the relative closeness between the wires. To compensate for this condition more terms of the Fourier series will be required.

The input data requirements for running the Fortran program for this coax cable configuration are as follows

1. Configuration [C]
2. Option [2]
3. Number of wires [4]
4. Number of harmonics [1,3,5,7]
5. Are all wires solid ? [y]
6. Do all wires have the same radius [y]
7. Enter inside radius of coax shield RCX [3.76E-3] (meters)

8. Enter radius of the conductor XRC [.5775E-3] (meters)
9. Enter relative permittivity of dielectric CER [3.5]
10. Enter the horizontal distance of conductor (2) with respect to the center of the coax cable X(1,2) = 0.0 (meters)
11. Enter the vertical distance of conductor (2) with respect to the center of the coax cable Y(1,2) = 0.0 0.0 (meters)
12. Enter the horizontal distance of conductor (3) with respect to the center of the coax cable X(1,3) = 2.59E-3 (meters)
13. Enter the vertical distance of conductor (3) with respect to the center of the coax cable Y(1,3) = 0.0 (meters)
14. Enter the horizontal distance of conductor (4) with respect to the center of the coax cable X(1,4) = -2.59E-3 (meters)
15. Enter the vertical distance of conductor (4) with respect to the center of the coax cable Y(1,4) = 0.0 (meters)
16. Is the reference number the same as the ground reference conductor? Enter y/n, prompt = y

Note, in item 16 above it is ALWAYS assumed that the shield is the reference wire and the answer to the question is always 'y' when working with coax cables. Note also that all relative distances are measured from the reference wire, that being the shield.The brackets [ ] indicate the actual value or response which must be entered in the FORTRAN program.

It should be pointed out that the CPU time given in tables (F.3, F.5, F.7, F.9) are based on a VAX 750.

CHAPTER 8

APPLICATION OF THE METHOD OF MOMENTS IN DETERMINING THE CAPACITANCE FOR A SHIELDED MULTICONDUCTOR WIRE BUNDLE

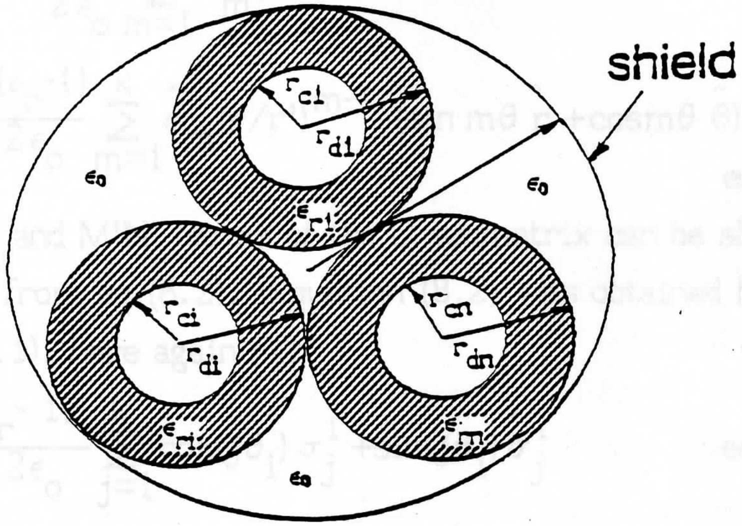


FIG. 8.1 SHIELDED MULTICONDUCTOR WIRE BUNDLE

The determination of the capacitance and inductance for a shielded multiconductor wire bundle is similar to that of a multiconductor coax cable. It is however, a little more complex. It is assumed that the charge on the reference wire, the shield, is that which is described in eq.(7.1). To complete the matrix for the shielded multiconductor case, it is assumed, in the program, that there is a dielectric on the shield having the same radius and a relative permittivity of 1. The program also requires the user to assume one more harmonic on this fictitious dielectric than that of the shield. This is done to insure that the matchpoints on the dielectric do not fall onto the same position as that of the conductive shield. Equation (7.2) is used when determining the MN and the MN' terms of the D submatrix when the source is on the shield. Equation (8.1) is required to satisfy the boundary condition at the dielectric to air interface, i.e., the difference in the normal component of the flux density when the source is on the shield. Equation 8.1 is used in determining the M'N and the M'N' terms of the D submatrix when the source is on the shield. Equation 8.1 was obtained from eq.(6.13).

$$0 = 0\,\hat{r} - \frac{(\epsilon_r-1)}{2\epsilon_o}\sum_{m=1}^{k}\sigma_m (r/r')^{m-1}\,(\cos m\theta\,\hat{r} - \sin m\theta\,\hat{\theta})$$

$$-\frac{(\epsilon_r-1)}{2\epsilon_o}\sum_{m=1}^{k}\hat{\sigma}_m (r/r')^{m-1}\,(\sin m\theta\,\hat{r} + \cos m\theta\,\hat{\theta})$$

eq.(8.1)

The diagonal M'N and M'N' terms of the D submatrix can be shown to be those obtained from eq.(8.2). Equation (8.2) was obtained from eqs.(6.13 and 6.11), here again r<r'

$$0 = -1\frac{\sigma_o}{\epsilon_o} - \frac{(\epsilon_r-1)}{2\epsilon_o}\sum_{j=1}^{k}\cos(j\theta_i)\,\sigma_j^i + \sin(j\theta_i)\,\hat{\sigma}_j^i \qquad \text{eq.(8.2)}$$

When the source is on the inner wires eq.(8.3) is used, equation (8.3) was obtained using eq. (6.11), here r≥r'.

$$0 = ((\epsilon_r-1)/\epsilon_0)\,\sigma_o (r/r')\,\hat{r} + \frac{(\epsilon_r-1)}{2\epsilon_o}\sum_{m=1}^{k}\sigma_m (r'/r)^{m+1}\,(\cos m\theta\,\hat{r} + \sin m\theta\,\hat{\theta})$$

$$\frac{(\epsilon_r-1)}{2\epsilon_o}\sum_{m=1}^{k}\hat{\sigma}_m (r'/r)^{m+1}\,(\sin m\theta\,\hat{r} - \cos m\theta\,\hat{\theta})$$

eq.(8.3)

Once the above considerations have been implemented, the generalized and the transmission line capacitance matrices can be found using the same techniques developed in chapters 4 and 5.

To verify that the program works an example is used to compare the approximate method, the method of moments, with that of a closed form solution. The configuration is shown in figure 8.2

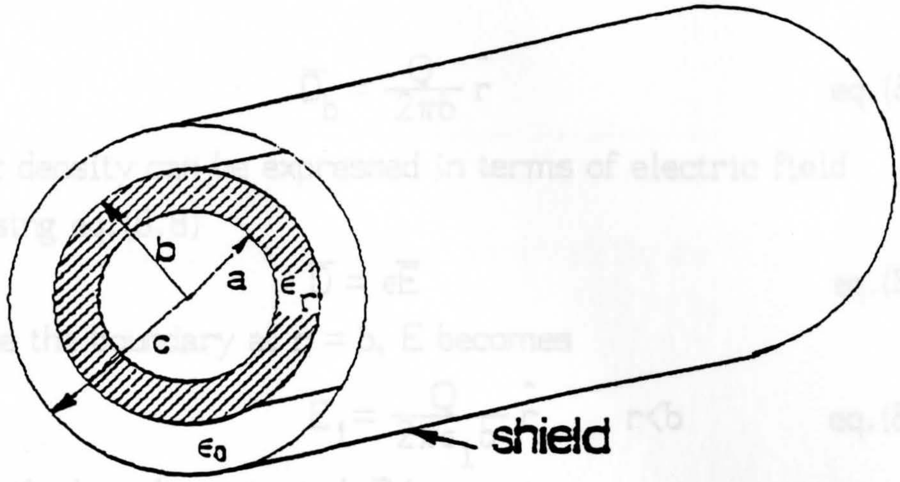


FIG. 8.2 SHIELDED DIELECTRIC COATED WIRE

The equation which describes the charge on the conductor is given by Gauss's law which states that the surface integral of the normal component of the electric flux density D over any closed surface equals the charge enclosed. Mathematically this is written as follows

$$\int_s \overline{D}\cdot\overline{ds} = Q \qquad \text{eq. (8.4)}$$

where $\overline{D}$ = the flux density

$\overline{ds} = r\cdot d\phi\cdot dz\ \hat{r}$

therefore

$$\int_0^1\int_0^{2\pi} (\overline{D})\cdot(r\,d\phi\,dz\,\hat{r}) = Q \qquad \text{eq. (8.5)}$$

The solution becomes

$$\overline{D} = \frac{Q}{2\pi r}\hat{r} \qquad \text{eq. (8.6)}$$

Applying eq. (8.6) to the boundary shown in figure (8.2) the flux density at a radius b from the center of the conductor becomes

$$\overline{D}_b = \frac{Q}{2\pi b}\hat{r} \quad \text{eq. (8.7)}$$

but the flux density can be expressed in terms of electric field intensity using eq. (8.8)

$$\overline{D} = \epsilon\overline{E} \quad \text{eq. (8.8)}$$

Thus, inside the boundary at r = b, E becomes

$$E_1 = \frac{Q}{2\pi\epsilon_1 b}\hat{r} \qquad r<b \quad \text{eq. (8.9)}$$

and outside the boundary at r = b E becomes

$$E_2 = \frac{Q}{2\pi\epsilon_2 b}\hat{r} \qquad r>b \quad \text{eq. (8.10)}$$

The potential is given by

$$\int_V^0 dV = -\int_a^c \overline{E}\cdot\overline{dl} = -\left[\int_a^b \frac{Q}{2\pi\epsilon_1 r}\hat{r}\cdot dr\,\hat{r} + \int_b^c \frac{Q}{2\pi\epsilon_2 r}\hat{r}\cdot dr\,\hat{r}\right] \quad \text{eq. (8.11)}$$

Solving eq. (8.11) the potential becomes

$$V = \frac{Q}{2\pi\epsilon_1}\ln\left(\frac{b}{a}\right) + \frac{Q}{2\pi\epsilon_2}\ln\left(\frac{c}{b}\right) \quad \text{eq. (8.12)}$$

By definition the capacitance is defined as follows

$$C = \frac{Q}{V} \quad \text{eq. (8.13)}$$

Substituting eq. (8.12) into eq. (8.13) and assuming the charge is 1 coulomb the capacitance can be found as follows

$$C = \frac{1}{\frac{1}{2\pi\epsilon_1}\ln(b/a) + \frac{1}{2\pi\epsilon_2}\ln(c/b)} \quad \text{eq. (8.14)}$$

or in terms of relative permittivity the capacitance C becomes

$$C = \frac{2\pi\epsilon_o}{(1/\epsilon_{r1})\ln(b/a) + (1/\epsilon_{r2})\ln(c/b)} \qquad \text{eq.(8.15)}$$

The following values for the variables shown in figure 8.2 are listed below

$$a = 1 \text{ mm}$$
$$b = 2 \text{ mm}$$
$$c = 3 \text{ mm}$$
$$\epsilon_{r1} = 3.5$$
$$\epsilon_{r2} = 1.0$$

The resulting capacitance value for this configuration is

$$C = 92.18198 \text{ pf}$$

Using the program results in the same value with just one harmonic selected.

An example of a shielded multiconductor system is shown in figure 8.3. This configuration is simply a 3-wire ripcord or ribbon cable placed inside a piece of convoluted conduit which was wrapped with aluminum backed cellophane to form the shield. The output data for this configuration is shown in tables (G.1-G.8) in appendix G.

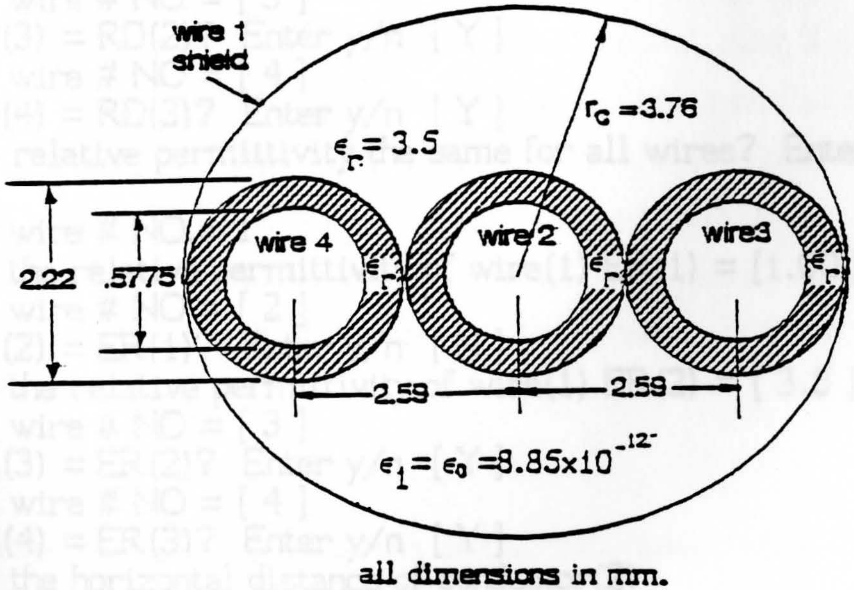


FIG. 8.3 SHIELDED 3-WIRE RIPCORD

Again note, the off-diagonal terms are not equal but with additional

Fourier terms, they will converge to a common value. Note also in tables (G.2,G.4,G.6,G.8) the CPU time requirements for running the shielded multiconductor configuration.

The input data requirements for running the Fortran program for the shielded multiconductor configuration in this example are listed below

1. Enter type of configuration: [S]
2. Enter option (IOPT)= [1]
3. Enter # of wires including shield NW= [4]
4. Enter # of cosine or sine terms around the conductor
   i.e. the # of harmonics around the conductor NHC = [1,3,5,7]
5. Enter # of cosine or sine terms around the dielectric
   i.e. the # of harmonics around the dielectric NHD = [2,4,6,8]
6. Are all wires solid inside shield? Enter y/n [y]
7. Do all wires have the same radius? Enter y/n [y]
8. Enter inside radius of shield RCX = [3.76E-3]
9. Enter radius of the conductor XRC = [.5775E-3]
10. Are all dielectric radii the same? Enter y/n [N]
11. Enter wire # NO.= 1
12. Enter radius of dielctric of wire (1) RD(1) = [3.76E-3]
13. Enter wire # NO = [2]
14. Is RD(2) = RD(1)? Enter y/n [N]
15. Enter radius of dielctric of wire (2) RD(2) = [1.11E-3]
13. Enter wire # NO = [3]
14. Is RD(3) = RD(2)? Enter y/n [Y]
15. Enter wire # NO = [4]
16. Is RD(4) = RD(3)? Enter y/n [Y]
17. Is the relative permittivity the same for all wires? Enter y/n [N]
18. Enter wire # NO.= 1
19. Enter the relative permittivity of wire(1) ER(1) = [1.00]
20. Enter wire # NO = [2]
21. Is ER(2) = ER(1)? Enter y/n [N]
19. Enter the relative permittivity of wire(1) ER(2) = [3.5]
20. Enter wire # NO = [3]
21. Is ER(3) = ER(2)? Enter y/n [Y]
22. Enter wire # NO = [4]
23. Is ER(4) = ER(3)? Enter y/n [Y]
24. Enter the horizontal distance of conductor(2)
    with respect to the center of the shield X(1,2) = [0.0]

25. Enter the vertical distance of conductor(2)
    with respect to the center of the shield Y(1,2) = [0.0]
26. Enter the horizontal distance of conductor(3)
    with respect to the center of the shield X(1,3) = [2.59E-3]
27. Enter the vertical distance of conductor(3)
    with respect to the center of the shield Y(1,2) = [0.0]
28. Enter the horizontal distance of conductor(4)
    with respect to the center of the shield X(1,4) = [-2.59E-3]
29. Enter the vertical distance of conductor(4)
    with respect to the center of the shield Y(1,4) = [0.0]
30. Is the reference number the same as the ground
    reference conductor? Enter y/n, PROMPT = [Y]

Note in item 30 the shield is considered ground as well as the reference conductor. The answer to item 30 will always be Y for shielded multiconductor wire bundles. Caution, when interpreting the data in tables (G.2,G.4,G.6,G.8,G.10) the reference wire is the shield which is wire 1 so C_{11} is actually the self capacitance of wire two, C_{22}, and C_{12} is the transmission line capacitance between wires 2 and 3, i.e., C_{23}. The remaining transmission line capacitance terms must be interpreted accordingly.

A simple test to see if the capacitance terms of the shielded 3-wire ripcord are within the proper limits is to check it against the coax model using ϵ_r equal to 1.0 and 3.5. The results using the coax model when $\epsilon_r = 1.0$ are shown in appendix H and those results using $\epsilon_r = 3.5$ in appendix F. The output data files for the shielded 3-wire ripcord are in appendix G. Comparing the results of all three appendices shows that the capacitance values of the shielded 3-wire ripcord are within the limits of the coax model when $\epsilon_r = 1.0$ and $\epsilon_r = 3.5$. In other words

$$C_{ij}^{1.0} < C_{ij}^{s} < C_{ij}^{3.5}$$

where $C_{ij}^{1.0}$ are the capacitance terms of the coax model when $\epsilon_r = 1$.

C^{s}_{ij} are the capacitance terms of the shielded 3-wire ripcord

$C^{3.5}_{ij}$ are the capacitance terms of the coax model when $\epsilon_r = 3.5$

It should also be pointed out that the program can only handle shield which are circular. To handle elliptical shields or other types of geometries the program would have to be modified.

CHAPTER 9

APPLICATION OF THE METHOD OF MOMENTS IN DETERMINING THE CAPACITANCE MATRIX FOR A DIELECTRIC COATED WIRE BUNDLE OVER A GROUND PLANE

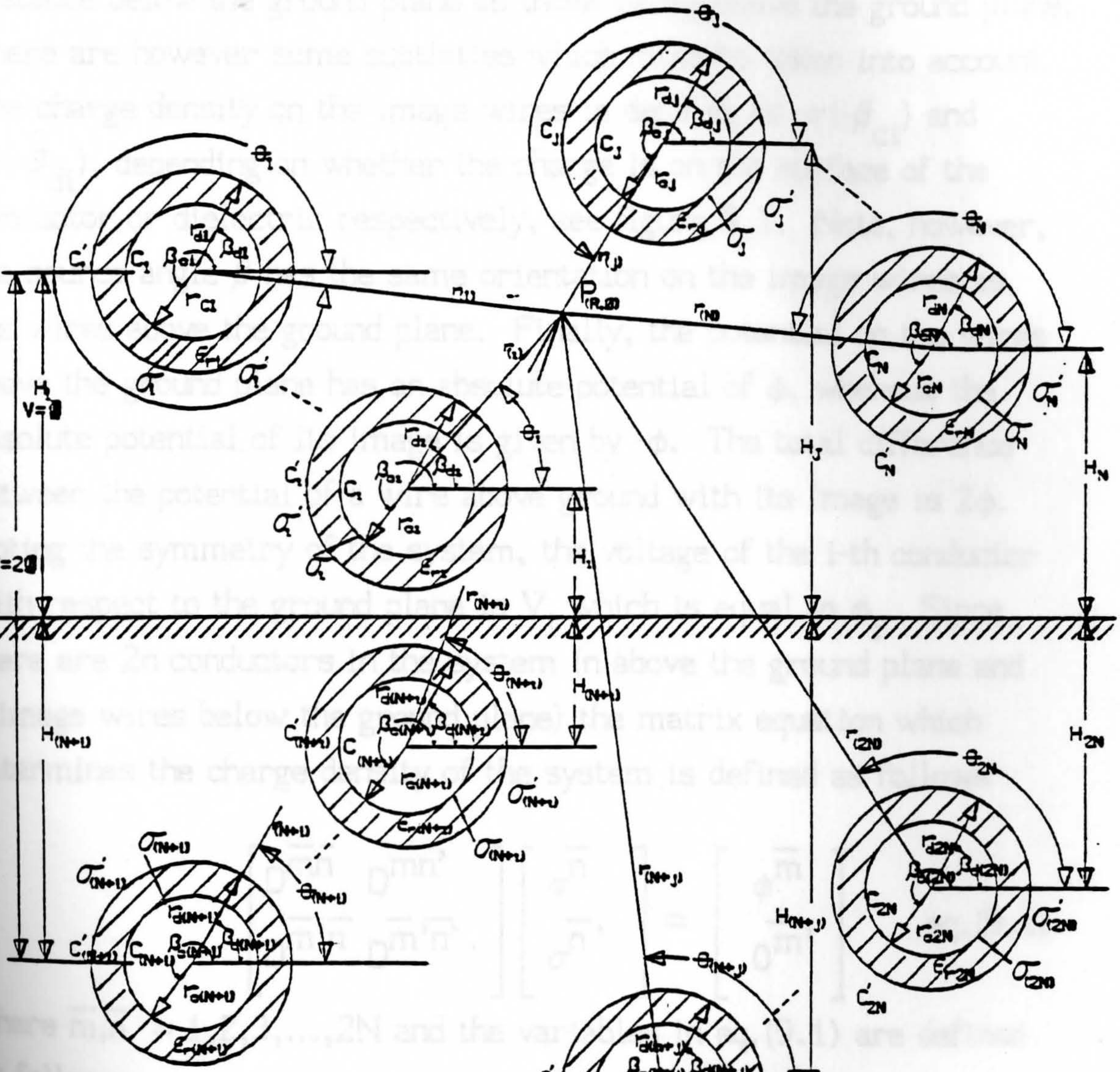


FIG. 9.1 DIELECTRIC COATED MULTICONDUCTOR SYSTEM OVER A GROUND PLANE

To determine the capacitance of dielectric coated wires over a ground plane requires the use of the "method of images". This method replaces the ground plane with image wires which are the same distance below the ground plane as those wires above the ground plane. There are however some subtleties which must be taken into account. The charge density on the image wires is defined as $-\sigma(-\beta_{ci})$ and $-\sigma(-\beta_{di})$, depending on whether the charge is on the surface of the conductor or dielectric respectively, see figure 9.1.. Note, however, the source angle β has the same orientation on the image wires as the wires above the ground plane. Finally, the potential on the wires above the ground plane has an absolute potential of ϕ, whereas the absolute potential of its image is given by $-\phi$. The total difference between the potential of a wire above ground with its image is 2ϕ. Noting the symmetry of the system, the voltage of the i-th conductor with respect to the ground plane is V_i which is equal to ϕ_i. Since there are 2n conductors in the system (n above the ground plane and n image wires below the ground plane) the matrix equation which determines the charge density of the system is defined as follows

$$\begin{bmatrix} D^{\overline{m}\overline{n}} & D^{\overline{m}\overline{n}'} \\ D^{\overline{m}'\overline{n}} & D^{\overline{m}'\overline{n}'} \end{bmatrix} \begin{bmatrix} \sigma^{\overline{n}} \\ \sigma^{\overline{n}'} \end{bmatrix} = \begin{bmatrix} \phi^{\overline{m}} \\ 0^{\overline{m}'} \end{bmatrix} \qquad \text{eq. (9.1)}$$

where $\overline{m},\overline{n} = 1,2,3,\ldots,2N$ and the variables in eq. (9.1) are defined as follows:

$D^{\overline{m}\overline{n}}$ is defined as the potential at 2k+1 matchpoints on the conductor surface of wire $\overline{m}$ above and below the ground plane due to 2k+1 source charges on the conductor surface of wire $\overline{n}$

$D^{\overline{m}\overline{n}'}$ is defined as the potential at 2k+1 matchpoints on the conductor surface of wire $\overline{m}$ above and below the ground plane due to 2l+1 source charges on the dielectric surface of wire $\overline{n}$.

$D^{\overline{m}'\overline{n}}$ is defined as the difference in the normal component of the flux density "just inside" and "just outside" at 2l+1 matchpoints on the dielectric surface of wire $\overline{m}$ above and below the ground plane due to 2k+1 source charges on the conductor surface on wire $\overline{n}$.

$D^{\overline{m}'\overline{n}'}$ is defined as the difference in the normal component of the flux density "just inside" and "just outside" at 2l+1 matchpoints on the dielectric surface of wire $\overline{m}$ above and below the ground plane due to 2l+1 source charges on the dielectric surface of wire $\overline{n}$.

$\sigma^{\overline{n}}$ is defined as the charge density a 2k+1 matchpoints on the conductor surface of wire $\overline{n}$ above and below the ground plane

$\sigma^{\overline{n}'}$ is defined as the charge density at 2l+1 matchpoints on the dielectric surface of wire $\overline{n}$ above and below the ground plane

$\phi^{\overline{m}}$ is defined as the absolute potential with respect to infinity at 2k+1 matchpoints on the conductor surface of wire $\overline{m}$ above and below the ground plane

$0^{\overline{m}'}$ is defined as a vector containing the difference of the normal component of the displacement vector at (2l+1) matchpoints on the dielectric surface of wire $\overline{m}$ above and below the ground plane

The submatrices $D^{\overline{m}\overline{n}}$, $D^{\overline{m}\overline{n}'}$, $D^{\overline{m}'\overline{n}}$, and $D^{\overline{m}'\overline{n}'}$ in eq.(9.1) are given more precisely as follows

$$D^{\overline{m}\overline{n}} = \begin{bmatrix} D^{(1)(1)} & \cdots & D^{(1)(n)} & D^{(1)(n+1)} & \cdots & D^{(1)(2n)} \\ D^{(2)(1)} & \cdots & D^{(2)(n)} & D^{(2)(n+1)} & \cdots & D^{(2)(2n)} \\ \vdots & & \vdots & \vdots & & \vdots \\ D^{(n)(1)} & \cdots & D^{(n)(n)} & D^{(n)(n+1)} & \cdots & D^{(n)(2n)} \\ D^{(n+1)(1)} & \cdots & D^{(n+1)(n)} & D^{(n+1)(n+1)} & \cdots & D^{(n+1)(2n)} \\ D^{(n+2)(1)} & \cdots & D^{(n+2)(n)} & D^{(n+2)(n+1)} & \cdots & D^{(n+2)(2n)} \\ \vdots & & \vdots & \vdots & & \vdots \\ D^{(2n)(1)} & \cdots & D^{(2n)(n)} & D^{(2n)(n+1)} & \cdots & D^{(2n)(2n)} \end{bmatrix}$$

eq.(9.2)

$$D^{\overline{m}\overline{n}'} = \begin{bmatrix} D^{(1)(1)'} & \cdots & D^{(1)(n)'} & D^{(1)(n+1)'} & \cdots & D^{(1)(2n)'} \\ D^{(2)(1)'} & \cdots & D^{(2)(n)'} & D^{(2)(n+1)'} & \cdots & D^{(2)(2n)'} \\ \vdots & & \vdots & \vdots & & \vdots \\ D^{(n)(1)'} & \cdots & D^{(n)(n)'} & D^{(n)(n+1)'} & \cdots & D^{(n)(2n)'} \\ D^{(n+1)(1)'} & \cdots & D^{(n+1)(n)'} & D^{(n+1)(n+1)'} & \cdots & D^{(n+1)(2n)'} \\ D^{(n+2)(1)'} & \cdots & D^{(n+2)(n)'} & D^{(n+2)(n+1)'} & \cdots & D^{(n+2)(2n)'} \\ \vdots & & \vdots & \vdots & & \vdots \\ D^{(2n)(1)'} & \cdots & D^{(2n)(n)'} & D^{(2n)(n+1)'} & \cdots & D^{(2n)(2n)'} \end{bmatrix}$$

eq.(9.3)

$$D^{\overline{m}'\overline{n}} = \begin{bmatrix} D^{(1)'(1)} & \cdots & D^{(1)'(n)} & D^{(1)'(n+1)} & \cdots & D^{(1)'(2n)} \\ D^{(2)'(1)} & \cdots & D^{(2)'(n)} & D^{(2)'(n+1)} & \cdots & D^{(2)'(2n)} \\ \vdots & & \vdots & \vdots & & \vdots \\ D^{(n)'(1)} & \cdots & D^{(n)'(n)} & D^{(n)'(n+1)} & \cdots & D^{(n)'(2n)} \\ D^{(n+1)'(1)} & \cdots & D^{(n+1)'(n)} & D^{(n+1)'(n+1)} & \cdots & D^{(n+1)'(2n)} \\ D^{(n+2)'(1)} & \cdots & D^{(n+2)'(n)} & D^{(n+2)'(n+1)} & \cdots & D^{(n+2)'(2n)} \\ \vdots & & \vdots & \vdots & & \vdots \\ D^{(2n)'(1)} & \cdots & D^{(2n)'(n)} & D^{(2n)'(n+1)} & \cdots & D^{(2n)'(2n)} \end{bmatrix}$$

eq. (9.4)

$$D^{\overline{m}'\overline{n}'} = \begin{bmatrix} D^{(1)'(1)'} & \cdots & D^{(1)'(n)'} & D^{(1)'(n+1)'} & \cdots & D^{(1)'(2n)'} \\ D^{(2)'(1)'} & \cdots & D^{(2)'(n)'} & D^{(2)'(n+1)'} & \cdots & D^{(2)'(2n)'} \\ \vdots & & \vdots & \vdots & & \vdots \\ D^{(n)'(1)'} & \cdots & D^{(n)'(n)'} & D^{(n)'(n+1)'} & \cdots & D^{(n)'(2n)'} \\ D^{(n+1)'(1)'} & \cdots & D^{(n+1)'(n)'} & D^{(n+1)'(n+1)'} & \cdots & D^{(n+1)'(2n)'} \\ D^{(n+2)'(1)'} & \cdots & D^{(n+2)'(n)'} & D^{(n+2)'(n+1)'} & \cdots & D^{(n+2)'(2n)'} \\ \vdots & & \vdots & \vdots & & \vdots \\ D^{(2n)'(1)'} & \cdots & D^{(2n)'(n)'} & D^{(2n)'(n+1)'} & \cdots & D^{(2n)'(2n)'} \end{bmatrix}$$

eq. (9.5)

Each term in the submatrices described in eqs. (9.2-9.5) is also a submatrix which contains the matchpoints around a particular boundary illustrated below

$$D^{mn} = \begin{bmatrix} D^{mn}_{11} & D^{mn}_{12} \cdots D^{mn}_{1(2k+1)} \\ D^{mn}_{21} & D^{mn}_{21} \cdots D^{mn}_{2(2k+1)} \\ \vdots & \vdots \quad \vdots \\ D^{mn}_{(2K+1)1} & \cdots D^{mn}_{(2k+1)(2k+1)} \end{bmatrix} \qquad D^{mn'} = \begin{bmatrix} D^{mn'}_{11} & \cdots & D^{mn'}_{1(2L+1)} \\ D^{mn'}_{21} & \cdots & D^{mn'}_{2(2L+1)} \\ \vdots & & \vdots \\ D^{mn'}_{(2k+1)1} & \cdots & D^{mn'}_{(2k+1)(2L+1)} \end{bmatrix}$$

eq. (9.6) eq. (9.7)

$$D^{m'n} = \begin{bmatrix} D^{m'n}_{11} & D^{m'n}_{12} \cdots D^{m'n}_{1(2k+1)} \\ D^{m'n}_{21} & D^{m'n}_{22} \cdots D^{m'n}_{2(2k+1)} \\ \vdots & \vdots \quad \vdots \\ D^{m'n}_{(2L+1)1} & \cdots D^{m'n}_{(2L+1)(2k+1)} \end{bmatrix} \qquad D^{m'n'} = \begin{bmatrix} D^{m'n'}_{11} & \cdots & D^{m'n'}_{1(L+1)} \\ D^{m'n'}_{21} & \cdots & D^{m'n'}_{2(L+1)} \\ \vdots & & \vdots \\ D^{m'n'}_{(2L+1)1} & \cdots & D^{m'n'}_{(2L+1)(2L+1)} \end{bmatrix}$$

eq. (9.8) eq. (9.9)

The charge density on the i-th conductor surface above the ground plane is given by tthe Fourier series in equation (9.10) and that of its image in eq. (9.11)

$$\sigma^i_c = \sigma^i_{co} + \sum_{j=1}^{k} \sigma^i_{cj} \cos(j\beta_{ci}) + \sum_{m=1}^{k} \hat{\sigma}^i_{cj} \sin(j\beta_{ci}) \qquad \text{eq. (9.10)}$$

$$\tilde{\sigma}^i_c = -\tilde{\sigma}^i_{co} - \sum_{j=1}^{k} \tilde{\sigma}^i_{cj} \cos(-j\beta_{ci}) - \sum_{j=1}^{k} \hat{\tilde{\sigma}}^i_{cj} \sin(-j\beta_{ci}) \qquad \text{eq. (9.11)}$$

Using the properties of even and odd functions eq. (9.11) becomes

$$\tilde{\sigma}^i_c = -\tilde{\sigma}^i_{co} - \sum_{j=1}^{k} \tilde{\sigma}^i_{cj} \cos(j\beta_{ci}) + \sum_{j=1}^{k} \hat{\tilde{\sigma}}^i_{cj} \sin(j\beta_{ci}) \qquad \text{eq. (9.12)}$$

In a similiar manner, the charge density on the i-th dielectric surface and its image is given by eqs. (9.13-914) respectively

$$\sigma_d^i = \sigma_{do}^i + \sum_{j=1}^{l} \sigma_{dj}^i \cos(j\beta_{di}) + \sum_{j=1}^{l} \hat{\sigma}_{dj}^i \sin(j\beta_{di}) \qquad \text{eq.(9.13)}$$

$$\tilde{\sigma}_d^i = -\tilde{\sigma}_{do}^i - \sum_{j=1}^{l} \tilde{\sigma}_{dj}^i \cos(j\beta_{di}) + \sum_{j=1}^{l} \hat{\tilde{\sigma}}_{dj}^i \sin(j\beta_{di}) \qquad \text{eq.(9.14)}$$

The vectors $\overline{\sigma^n}$, $\overline{\sigma^{n'}}$, $\overline{\phi^m}$, and $\overline{0^{m'}}$ can now be expanded as shown below

$$\overline{\sigma^n} = \begin{bmatrix} \sigma_o^n \\ \sigma_1^n \\ \vdots \\ \sigma_k^n \\ \hat{\sigma}_1^n \\ \vdots \\ \hat{\sigma}_k^n \\ -\tilde{\sigma}_o^n \\ -\tilde{\sigma}_1^n \\ \vdots \\ -\tilde{\sigma}_k^n \\ -\hat{\tilde{\sigma}}_1^n \\ \vdots \\ -\hat{\tilde{\sigma}}_k^n \end{bmatrix} \begin{array}{l} \text{average terms} \\ \text{cosine terms} \\ \\ \\ \text{sine terms} \\ \\ \\ \text{avg. image terms} \\ \text{cosine image terms} \\ \\ \\ \text{sine image terms} \\ \\ \\ \end{array} \qquad \text{eq.(9.15)}$$

$$\overline{\sigma^{n'}} = \begin{bmatrix} \sigma_o^{n'} \\ \sigma_1^{n'} \\ \vdots \\ \sigma_l^{n'} \\ \hat{\sigma}_1^{n'} \\ \vdots \\ \hat{\sigma}_l^{n'} \\ -\tilde{\sigma}_o^{n'} \\ -\tilde{\sigma}_1^{n'} \\ \vdots \\ -\tilde{\sigma}_l^{n'} \\ -\hat{\tilde{\sigma}}_1^{n'} \\ \vdots \\ -\hat{\tilde{\sigma}}_l^{n'} \end{bmatrix} \begin{array}{l} \text{average terms} \\ \text{cosine terms} \\ \\ \\ \text{sine terms} \\ \\ \\ \text{avg. image terms} \\ \text{cosine image terms} \\ \\ \\ \text{sine image terms} \\ \\ \\ \end{array} \qquad \text{eq.(9.16)}$$

$$\overline{\phi^m} = \begin{bmatrix} \phi_1^m \\ \phi_2 \\ \vdots \\ \phi_{k+1}^m \\ \hat{\phi}_{k+2}^m \\ \vdots \\ \hat{\phi}_{2k+1}^i \\ -\tilde{\phi}_1^m \\ -\tilde{\phi}_2^m \\ \vdots \\ -\tilde{\phi}_{k+1}^m \\ -\hat{\tilde{\phi}}_{k+2}^m \\ \vdots \\ -\hat{\tilde{\phi}}_{2k+1}^m \end{bmatrix} \begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \text{image wires} \\ \\ \\ \\ \\ \\ \\ \end{array} \qquad \text{eq.(9.17)}$$

$$\overline{0^{m'}} = \begin{bmatrix} 0_1^{m'} \\ 0_2^{m'} \\ \vdots \\ 0_{l+1}^{m'} \\ 0_{l+2} \\ \vdots \\ 0_{2l+1}^{m'} \\ 0_1^{m'} \\ 0_2^{m'} \\ \vdots \\ 0_{l+1}^{m'} \\ 0_{l+2}^{m'} \\ \vdots \\ 0_{2l+1}^{m'} \end{bmatrix} \begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \text{image wires} \\ \\ \\ \\ \\ \\ \\ \end{array} \qquad \text{eq.(9.18)}$$

Equations(9.15-9.18) have incorporated the fact that the charge on the image wire is the negative of that of the wire above ground and the potential of the image wire is the negative of that of the wire above the ground plane. Mathematically this is expressed in eqs.(9.18-9.24).

$$\sigma^{N+1} = -\sigma^{1} \qquad \text{eq.(9.19)}$$

$$\sigma^{N+2} = -\sigma^{2} \qquad \text{eq.(9.20)}$$

$$\vdots$$

$$\sigma^{2N} = -\sigma^{N} \qquad \text{eq.(9.21)}$$

$$\phi^{N+1} = -\phi^{1} = -V^{1} \qquad \text{eq.(9.22)}$$

$$\phi^{N+2} = -\phi^{2} = -V^{2} \qquad \text{eq.(9.23)}$$

$$\vdots$$

$$\phi^{2N} = -\phi^{N} = -V^{N} \qquad \text{eq.(9.24)}$$

The potential at a point 'P' due to a unit source charge at 2k+1 matchpoints on the i-th conductor surface above the ground plane is described by eq.(9.25)

$$\phi_p(r_i,\theta_i) = -\sigma_o^i \frac{r'_i \ln(r_i)}{\epsilon_o} + \frac{1}{2\epsilon_o}\sum_{j=1}^{k} \sigma_j^i \frac{(r'_i)^{j+1}\cos(j\theta_i)}{j r_i^j} + \frac{1}{2\epsilon_o}\sum_{j=1}^{k} \hat{\sigma}_j^i \frac{(r'_i)^{j+1}\sin(j\theta_i)}{j r_i^j}$$

where $i=1,2,3,\ldots N$ $\quad r_i \geq r'_i$ eq.(9.25)

$r'_i = r_{ci}$ or r_{di} depending on the boundary the unit source is residing

r_i = vector length from center of source wire to potential point 'P'

whereas the potential at point 'P' due to a unit source charge at 2k+1 matchpoints on the conductor surface of the i-th image wire is given by eq. (9.26)

$$\tilde{\phi}_p(r_i,\theta_i) = +\tilde{\sigma}_o^i \frac{r'_i \ln(r_i)}{\epsilon_o} - \frac{1}{2\epsilon_o}\sum_{j=1}^{k} \tilde{\sigma}_j^i \frac{(r'_i)^{j+1}\cos(j\theta_i)}{j r_i^j} + \frac{1}{2\epsilon_o}\sum_{j=1}^{k} \tilde{\hat{\sigma}}_j^i \frac{(r'_i)^{j+1}\sin(j\theta_i)}{j r_i^j}$$

eq.(9.26)

where $i=N+1,N+2,\ldots,2N$ $\quad r_i \geq r'_i$

$r'_i = r_{ci}$ or r_{di} depending on the boundary the unit source is residing

r_i = vector length from center of source wire to potential point 'P'

In a similar manner the potential at a point 'P' due to unit source charges at 2l+1 matchpoints on the i-th dielectric surface above the ground plane is described by eq.(9.25) and those of its image by eq.(9.26) after substituting l for k.

When $r_i < r'_i$ the potential due to a unit charge on the i-th wire above the ground plane is described as follows

$$\phi_p(r_i,\theta_i) = -\sigma_o^i \frac{r'_i \ln(r'_i)}{\epsilon_o} + \frac{1}{2\epsilon_o}\sum_{j=1}^{k} \sigma_j^i \frac{(r_i)^j \cos(j\theta_i)}{j r_i'^{\,j-1}} + \frac{1}{2\epsilon_o}\sum_{j=1}^{k} \hat{\sigma}_j^i \frac{(r_i)^j \sin(j\theta_i)}{j r_i'^{\,j-1}}$$

eq.(9.27)

where $i=1,2,3,\ldots N$ $\quad r<r'$

$r'_i = r_{ci}$ or r_{di} depending on the boundary the unit source is residing

r_i = vector length from center of source wire to potential point 'P'

When $r_i < r'_i$ the potential function of the i-th image wire is described as follows

$$\tilde{\phi}_p(r_i,\theta_i) = +\tilde{\sigma}_o^i \frac{r'_i \ln(r'_i)}{\epsilon_o} - \frac{1}{2\epsilon_o}\sum_{j=1}^{k} \tilde{\sigma}_j^i \frac{(r_i)^j \cos(j\theta_i)}{j r_i'^{\,j-1}} + \frac{1}{2\epsilon_o}\sum_{j=1}^{k} \tilde{\hat{\sigma}}_j^i \frac{(r_i)^j \sin(j\theta_i)}{j r_i'^{\,j-1}}$$

eq.(9.28)

where $i=N+1,N+2,\ldots,2N$ $\quad r<r'$

$r'_i = r_{ci}$ or r_{di} depending on the boundary the unit source is residing

r_i = vector length from center of source wire to potential point 'P'

The electric field intensity can be obtained from the potential functions using Laplace's equation shown in eq. (9.29)

$$E(r,\theta) = -\nabla \phi(r,\theta) \qquad \text{eq. (9.29)}$$

The del operator (∇) in a cylindrical coordinate system is expressed in eq. (9.30)

$$\nabla = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta} \qquad \text{eq. (9.30)}$$

Applying eq. (9.29) to eq. (9.25), the electric field intensity from the i-th wire above the ground plane when $r \geq r'$ is given by eq. (9.31)

$$E(r_i,\theta_i) = \sigma_0^i \frac{r_i'/r_i}{\epsilon_0}\hat{r}_i + \frac{1}{2\epsilon_0}\sum_{j=1}^{k} \sigma_j^i (r_i'/r_i)^{j+1}\left[\cos(j\theta_i)\hat{r}_i + \sin(j\theta_i)\hat{\theta}_i\right]$$

$$+ \frac{1}{2\epsilon_0}\sum_{j=1}^{k} \hat{\sigma}_j^i (r_i'/r_i)^{j+1}\left[\sin(j\theta_i)\hat{r}_i - \cos j\theta_i)\hat{\theta}_i\right]$$

where $i=1,2,3,\ldots,N$ eq. (9.31)

When $r<r'$ the equation describing the electric intensity from i-th wire above the ground plane is described by eq. (9.32)

$$E(r_i,\theta_i) = 0 - \frac{1}{2\epsilon_0}\sum_{j=1}^{k} \sigma_j^i (r_i/r_i')^{j-1}\left[\cos(j\theta_i)\hat{r}_i - \sin(j\theta_i)\hat{\theta}_i\right]$$

$$- \frac{1}{2\epsilon_0}\sum_{j=1}^{k} \hat{\sigma}_j^i (r_i/r_i')^{j-1}\left[\sin(j\theta_i)\hat{r}_i + \cos(j\theta_i)\hat{\theta}_i\right]$$

where $i=1,2,3,\ldots N$ eq. (9.32)

The electric field intensity from the i-th image wire when $r \geq r'$ is described by eq. (9.33)

$$\tilde{E}(r_i,\theta_i) = -\tilde{\sigma}_0^i \frac{r_i'/r_i}{\epsilon_0}\hat{r}_i - \frac{1}{2\epsilon_0}\sum_{j=1}^{k}\tilde{\sigma}_j^i (r_i'/r_i)^{j+1}\left[\cos(j\theta_i)\,\hat{r}_i + \sin(j\theta_i)\,\hat{\theta}_i\right]$$

$$+\frac{1}{2\epsilon_0}\sum_{j=1}^{k}\hat{\tilde{\sigma}}_j^i (r_i'/r_i)^{j+1}\left[\sin(j\theta_i)\,\hat{r}_i - \cos j\theta_i)\,\hat{\theta}_i\right]$$

where $i=N+1,N+2,\ldots,2N$ eq. (9.33)

When $r<r'$, the equation describing the electric intensity from the i-th image wire is given by eq.(9.34)

$$\tilde{E}(r_i,\theta_i) = 0 + \frac{1}{2\epsilon_0}\sum_{j=1}^{k}\tilde{\sigma}_j^i (r_i/r_i')^{j-1}\left[\cos(j\theta_i)\,\hat{r}_i - \sin(j\theta_i)\,\hat{\theta}_i\right]$$

$$-\frac{1}{2\epsilon_0}\sum_{j=1}^{k}\hat{\tilde{\sigma}}_j^i (r_i/r_i')^{j-1}\left[\sin(j\theta_i)\,\hat{r}_i + \cos(j\theta_i)\,\hat{\theta}_i\right]$$

where $i=N+1,N+2,\ldots,2N$ eq.(9.34)

Knowing the above information, the various terms of the $D^{\overline{m}\overline{n}}$, $D^{\overline{m}\overline{n}'}$, $D^{\overline{m}'\overline{n}}$, and $D^{\overline{m}'\overline{n}'}$ submatrices can be determined.

The terms of the $D^{\overline{m}\overline{n}}$ submatrix in eq.(9.2) are obtained from the coefficients of the charge densities of the potential function described in eq.(9.25), if the wire is above the ground plane, and eq.(9.26) if it is the image wire. The following substitutions are for the diagonal terms

$$r_i' = r_i = r_{ci} \qquad \text{eq.(9.35)}$$

and for the off-diagonal terms

$$r_i' = r_{ci} \text{ and } r_i \text{ to be determined} \qquad \text{eq.(9.36)}$$

The variable r_i is calculated using the Pythagorean theorem where the sides are determined from the wire separation and the radius of the conductor of the potential wire. The potential wire is the wire in which the terms of the potential function are evaluated.

The terms of the $D^{\overline{m}\overline{n}'}$ submatrix in eq.(9.3) are obtained in a similar manner. The diagonal terms are obtained using eq.(9.27) when the wire is above the ground plane and eq.(9.28) when the wire is an image wire. To calculate the diagonal terms, substitute the following into the appropriate equation

$$r'=r_{di} \text{ and } r_i=r_{ci} \qquad \text{eq.(9.37)}$$

The off-diagonal terms, however, are determined by using eq. (9.25) (for wires above the ground plane) and eq.(9.26) (for image wires) after substituting the following change of variables

$$r'_i = r_{di} \text{ and } r_i \text{ to be determined} \qquad \text{eq.(9.38)}$$

The variable r_i is calculated using the Pythagorean theorem where the sides are determined from the wire separation and the radius of the of the dielectric of the potential wire.

The off-diagonal terms of the $D^{\overline{m}'\overline{n}}$ submatrix in eq.(9.4) are obtained by substituting eq.(9.31) into eq.(6.7) which is rewritten here as eq.(9.39) the results of which are shown in eq.(9.40)

$$\epsilon_r E^i_n - E^o_n = 0 \qquad \text{eq.(9.39)}$$

where E^i_n = normal component of the electric intensity vector just inside the boundary

E^o_n = normal component of the electric intensity vector just outside the boundary

ϵ_r = relative permittivity of the dielectric

$$0 = \frac{(\epsilon_r-1)}{\epsilon_o}(r_i'/r_i)\sigma_o^i \hat{r}_i + \frac{(\epsilon_r-1)}{2\epsilon_o}\sum_{j=1}^{k}(r_i'/r_i)^{j+1}\left[\cos(j\theta_i)\,\hat{r} + \sin(j\theta_i)\,\hat{\theta}_i\right]\sigma_j^i$$

$$+\frac{(\epsilon_r-1)}{2\epsilon_o}\sum_{j=1}^{k}(r_i'/r_i)^{j+1}\left[\sin(j\theta_i)\,\hat{r}_i - \cos j\theta_i)\,\hat{\theta}_i\right]\hat{\sigma}_j^i$$

eq. (9.40)

where ϵ_r is the relative permittivity of the dielectric
$i=1,2,3,\ldots,N$

The off-diagonal image terms of the $D^{\overline{m},\overline{n}}$ submatrix are obtained by substituting eq. (9.33) into eq. (9.39) the results of which are shown in eq. (9.41)

$$0 = -\tilde{\sigma}_0^i\,\frac{(\epsilon_r-1)(\frac{r_i'}{r_i})}{\epsilon_0}\hat{r}_i - \frac{(\epsilon_r-1)}{2\epsilon_0}\sum_{j=1}^{k}\tilde{\sigma}_j^i\,(r_i'/r_i)^{j+1}\left[\cos(j\theta_i)\,\hat{r}_i + \sin(j\theta_i)\,\hat{\theta}_i\right]$$

$$+\frac{(\epsilon_r-1)}{2\epsilon_0}\sum_{j=1}^{k}\hat{\tilde{\sigma}}_j^i\,(r_i'/r_i)^{j+1}\left[\sin(j\theta_i)\,\hat{r}_i - \cos j\theta_i)\,\hat{\theta}_i\right]$$

where $i=N+1,N+2,N+3,\ldots,2N$ eq. (9.41)

Thus the off-diagonal terms are obtained by using eq. (9.41) replacing the variables as shown below

$r_i' = r_{ci}$ and r_i= to be determined eq. (9.43)

The variable r_i is calculated using the Pythagorean theorem where the sides are determined from the wire separation and the radius of the of the dielectric of the potential wire.

To obtain the diagonal terms of the $D^{\overline{m},\overline{n}}$ submatrix replace the variables shown in eqs. (9.40-9.41) with those shown in eq. (9.42).

$$r_i' = r_{ci} \text{ and } r_i = r_{di} \qquad \text{eq.(9.42)}$$

then take the dot product with respect to a unit normal vector to the boundary.

The diagonal terms of the $D^{\overline{m}'\overline{n}'}$ submatrix in eq.(9.5) are obtained by substituting eqs(9.31and 9.32) into eq.(9.39). Equation (9.32) is used for E_n^i and eq.(9.31) is used for E_n^o. The results are shown in eq.(9.44)

$$0 = -\frac{(r_i'/r_i)}{\epsilon_o}\sigma_o^i \hat{r}_i + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\left\{\left[-\epsilon_r(r_i/r_i')^{j-1} - (r_i'/r_i)^{j+1}\right]\cos(j\theta_i)\,\hat{r}_i\right.$$
$$\left.+\left[\epsilon_r(r_i/r_i')^{j-1} - (r_i'/r_i)^{j+1}\right]\sin(j\theta_i)\,\hat{\theta}_i\right\}\sigma_i^j + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\left\{\left[-\epsilon_r(r_i/r_i')^{j-1} -\right.\right.$$
$$\left.\left.(r_i'/r_i)^{j+1}\right]\sin(j\theta_i)\,\hat{r}_i + \left[-\epsilon_r(r_i/r_i')^{j-1} + (r_i'/r_i)^{j+1}\right]\cos(j\theta_i)\,\hat{\theta}_i\right\}\sigma_i^j.$$

eq.(9.44)

This expression, eq.(9.44), can further be simplified by noting the fact that at the boundary, the source, and potential matchpoints have the same radius, i.e.

$$r_i' = r_i = r_{di} \qquad \text{eq.(9.45)}$$

Substituting eq.(9.45) into eq.(9.44) reduces the equation to the following

$$0 = -1\frac{\sigma_o}{\epsilon_o}\hat{r}_i + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\left[-(\epsilon_r+1)\cos(j\theta_i)\,\hat{r}_i + (\epsilon_r-1)\sin(j\theta_i)\,\hat{\theta}_i \right]\sigma_j^i$$

$$+ \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\left[-(\epsilon_r+1)\sin(j\theta_i)\,\hat{r}_i - (\epsilon_r-1)\cos(j\theta_i)\,\hat{\theta}_i \right]\hat{\sigma}_j^i$$

eq.(9.46)

Since only the normal component is desired, i.e. $\hat{n} = \hat{r}_i$, the dot product of of the unit normal vector with eq.(9.46) results in a further simplification as shown in eq.(9.47)

$$0 = -1\frac{\sigma_o}{\epsilon_o} - \frac{(\epsilon_r+1)}{2\,\epsilon_o}\sum_{j=1}^{k}\cos(j\theta_i)\sigma_j^i + \sin(j\theta_i)\hat{\sigma}_j^i \qquad \text{eq.(9.47)}$$

The off-diagonal $D^{\overline{m}',\overline{n}'}$ terms, however, are obtained by using eq.(9.40) by replacing the following variables

$$r'_i = r_{di} \text{ and } r_i = \text{to be determined} \qquad \text{eq.(9.48)}$$

Again r_i is found by using the Pythagorean theorem where one side is the center-to-center separation of the wires and the other side is the dielectric radius of the potential wire.

The diagonal image wire terms, of the $D^{\overline{m}',\overline{n}'}$ submatrix are found by substituting eqs.(9.33 and 9.34) into eq. (9.39) where eq. (9.33) is E_n^i and eq.(9.34) is E_n^o. The results are shown in eq.(9.49)

$$0 = \frac{(r'_i/r_i)\tilde{\sigma}^i_o}{\epsilon_o}\hat{r}_i + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\left\{\left[+\epsilon_r(r_i/r'_i)^{j-1} + (r'_i/r_i)^{j+1}\right]\cos(j\theta_i)\,\hat{r}_i\right.$$

$$\left.+\left[-\epsilon_r(r_i/r'_i)^{j-1} + (r'_i/r_i)^{j+1}\right]\sin(j\theta_i)\,\hat{\theta}_i\right\}\tilde{\sigma}^j_i + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\left\{\left[-\epsilon_r(r_i/r'_i)^{j-1} - \right.\right.$$

$$\left.(r'_i/r_i)^{j+1}\right]\sin(j\theta_i)\,\hat{r}_i + \left.\left[-\epsilon_r(r_i/r'_i)^{j-1} + (r'_i/r_i)^{j+1}\right]\cos(j\theta_i)\,\hat{\theta}_i\right\}\hat{\tilde{\sigma}}^j_i$$

eq. (9.49)

The variables r_i and r'_i in eq. (9.49) for the image wires of the diagonal $D^{\overline{m}'\overline{n}'}$ are replaced by the following change of variables

$$r_i = r_{di} = r'_i = r_{di} \qquad \text{eq. (9.50)}$$

Substituting eq. (9.50) into eq. (9.49) results in the following reduction.

$$0 = 1\frac{1}{\epsilon_o}\tilde{\sigma}^j_o\hat{r}_i + \frac{1}{2\epsilon_o}\sum_{j=1}^{k}\left[(\epsilon_r+1)\cos(j\theta_i)\,\hat{r}_i - (\epsilon_r-1)\sin(j\theta_i)\,\hat{\theta}_i\right]\tilde{\sigma}^j_i$$

$$\frac{1}{2\epsilon_o}\sum_{j=1}^{1}\left[-(\epsilon_r+1)\sin(j\theta_i)\,\hat{r}_i - (\epsilon_r-1)\cos(j\theta_i)\,\hat{\theta}_i\right]\hat{\tilde{\sigma}}^j_i$$

eq. (9.51)

Since only the normal component is needed in eq. (9.51), $\hat{n} = \hat{r}_i$, eq. (9.51) finally reduces to

$$0 = 1\frac{1}{\epsilon_o}\tilde{\sigma}^j_o + \frac{(\epsilon_r+1)}{2\epsilon_o}\sum_{j=1}^{k}\cos(j\theta_i)\,\tilde{\sigma}^j_i - \sin(j\theta_i)\,\hat{\tilde{\sigma}}^j_i \qquad \text{eq. (9.52)}$$

The off-diagonal image wire terms of the $D^{\overline{m}',\overline{n}'}$ submatrix are found by using eq.(9.53)

$$0 = \frac{-(\epsilon_r - 1)}{\epsilon_o}(r_i'/r_i)\tilde{\sigma}_o^i \hat{r}_i - \frac{(\epsilon_r - 1)}{2\epsilon_o}\sum_{j=1}^{k}(r_i'/r_i)^{j+1}(\cos(j\theta_i)\hat{r} + \sin(j\theta_i)\hat{\theta}_i)\tilde{\sigma}_j^i + \frac{(\epsilon_r - 1)}{2\epsilon_o}\sum_{j=1}^{k}(r_i'/r_i)^{j+1}(\sin(j\theta_i)\hat{r}_i - \cos j\theta_i)\hat{\theta}_i)\tilde{\hat{\sigma}}_j^i$$

eq.(9.53)

where $r'_i = r_{di}$ and r_i to be determined by using the Pythagorean thereom where one side is the center-to-center separation of the wires and the other side is the dielectric radius of the potential wire.

Now that all the elements of the matrix equation, eq.(9.1), have been identified and defined, the charge density can be determined by first rewriting the set of equations described in eq.(9.1) in matrix notation as follows

$$[D][\sigma] = [\phi] \qquad \text{eq.(9.55)}$$

then, as before, invert the D matrix to obtain the charge density from which the generalized capacitance matrix can be determined as described in chapter 6. Before inverting the D matrix, symmetry can be used to reduce the order of each of the submatrices to an NxN submatrix, i.e., an NxN submatrix around the conductor and an NxN submatrix around the dielectric-air interface. This can be accomplished by using the equations described in eqs.(9.19-9.24) which takes into account all the charges of the system but reduces the the number of equations needed to describe it to only N equations and and N unknowns. The elements of the new $\overset{\star}{D}$ matrix are determined by adding the coefficients of like terms of the image wire from its counterpart above the ground plane. Thus the new matrix equation is of the form shown in eq.(9.55)

$$\left[\begin{array}{ccc|ccc}
\overset{\star}{D}{}^{mn}_{1,1} & \cdots & \overset{\star}{D}{}^{mn}_{1,2k+1} & \overset{\star}{D}{}^{mn'}_{1,1} & \cdots & \overset{\star}{D}{}^{mn'}_{1,2l+1} \\
\overset{\star}{D}{}^{mn}_{2,1} & \cdots & \overset{\star}{D}{}^{mn}_{2,2k+1} & \overset{\star}{D}{}^{mn'}_{2k+1,1} & \cdots & \overset{\star}{D}{}^{mn'}_{2k+1,2l+1} \\
\vdots & & \vdots & \vdots & & \vdots \\
\overset{\star}{D}{}^{mn}_{2k+1,1} & \cdots & \overset{\star}{D}{}^{mn}_{2k+1,2k+1} & \overset{\star}{D}{}^{mn'}_{2k+1,1} & \cdots & \overset{\star}{D}{}^{mn'}_{2k+1,2l+1} \\
\hline
\overset{\star}{D}{}^{m'n}_{1,1} & \cdots & \overset{\star}{D}{}^{m'n}_{1,2k+1} & \overset{\star}{D}{}^{m'n'}_{1,1} & \cdots & \overset{\star}{D}{}^{m'n'}_{1,2l+1} \\
\overset{\star}{D}{}^{m'n}_{2,1} & \cdots & \overset{\star}{D}{}^{m'n}_{2,2k+1} & \overset{\star}{D}{}^{m'n'}_{2,1} & \cdots & \overset{\star}{D}{}^{m'n'}_{2,2l+1} \\
\vdots & & \vdots & \vdots & & \vdots \\
\overset{\star}{D}{}^{m'n}_{2l+1,1} & \cdots & \overset{\star}{D}{}^{m'n}_{2l+1,2k+1} & \overset{\star}{D}{}^{m'n'}_{2l+1,1} & \cdots & \overset{\star}{D}{}^{m'n'}_{2l+1,2l+1}
\end{array}\right]
\left[\begin{array}{c}
\sigma^{n}_{1} \\ \sigma^{n}_{2} \\ \vdots \\ \sigma^{n}_{2k+1} \\ \hline \sigma^{n'}_{1} \\ \sigma^{n'}_{2} \\ \vdots \\ \sigma^{n'}_{2l+1}
\end{array}\right]
=
\left[\begin{array}{c}
\phi^{m}_{1} \\ \phi^{m}_{2} \\ \vdots \\ \phi^{m}_{2k+1} \\ \hline \phi^{m'}_{1} \\ \phi^{m'}_{2} \\ \vdots \\ \phi^{m'}_{2l+1}
\end{array}\right]$$

where $m,n,m',n'=1,2,3,\ldots,N$ eq. (9.55)

The $\overset{\star}{D}$ matrix is obtained as follows: $\overset{\star}{D}{}^{ij} = D^{ij} + D^{i(N+j)}$, $\overset{\star}{D}{}^{ij'} = D^{ij'} + D^{i(N+j)'}$, $\overset{\star}{D}{}^{i'j} = D^{i'j} + D^{i'(N+j)}$, and $\overset{\star}{D}{}^{i'j'} = D^{i'j'} + D^{i'(N+j)'}$. More exactly, the terms of the $\overset{\star}{D}{}^{mn}$ submatrix can be found by adding the cofficients of like charge densities of the image wires to their respective counterparts above the plane. The coefficients of σ_0, $\sigma_{j'}$, and $\hat{\sigma}_j$ for the $\overset{\star}{D}{}^{mn}$ submatrix for the i-th conductor are determined by adding eq. (9.26) to eq. (9.25)

What has been shown in the above analysis is that the number of unknowns has been reduced from 2N(2k+1) to N(2k+1), a significant reduction of the order of the matrix which is to be inverted. The reduction of the order of the potential matrix D to $\overset{\star}{D}$ reduces the CPU time to invert substantially. Placing all these terms in the $\overset{\star}{D}$ matrix and inverting produces the generalized capacitance matrix from which transmission line capacitance matrix can be found as described in chapters 4 and 5.

Since there exists a closed form solution for one bare wire over a ground plane, it is compared here to the approximate method developed above. The capacitance model for one bare wire over a ground plane is well known. The formula for calculating it is shown in eq.(9.85).

$$C = \frac{2\pi\epsilon}{\cosh^{-1}(d/2r)} \qquad \text{eq.(9.85)}$$

A comparison of the closed form solution and the approximate solution using the method of moments is shown in table 9.1.

TABLE 9.1
Approximate vs closed form solution of capacitance for one bare wire over a ground plane

ratio of height to conductor radius	closed form Cap. (pF/m)	no. of harmonics /Fourier terms	Approx. Cap. (pF/m)
1.5:1	57.805	1/2	54.791
		5/6	57.803
2.0:1	42.243	1/2	41.666
		5/6	42.243
3.0:1	31.560	1/2	31.477
		5/6	31.560
4.0:1	26.961	1/2	26.937
		5/6	26.961
5.0:1	24.268	1/2	24.258
		5/6	24.268

Since a closed form solution for a dielectric coated wire is not available, the method of moments approximation is compared to results obtained from testing. The results of this comparison are shown in table 9.2 and figure 9.3, page 95. Table 9.2 also shows a comparison of the method of moments to that of test results using a bare wire approximation.

TABLE 9.2

Test vs bare apprpoximation vs dielectric approximation

height above gnd. (mm)	test cap/m value (pf/m)	bare cap/m value (pf/m)	dielectric cap/m value (pf/m)
3.5	70.0	27.344	65.666
12.5	25.7	16.819	23.801
19.5	21.4	14.826	19.945
28.5	18.5	13.464	17.542
53.5	14.4	11.683	14.629
103.5	10.8	10.261	12.465
153.5	8.4	9.5659	11.453

The test setup is shown in figure 9.2

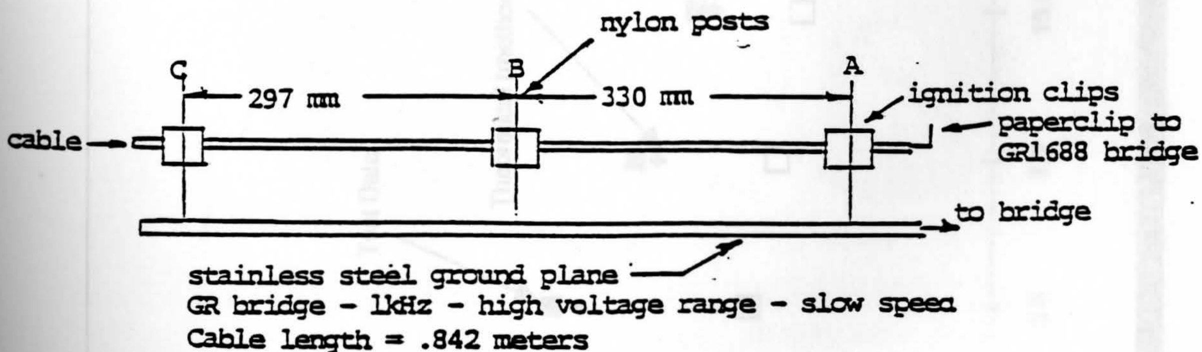
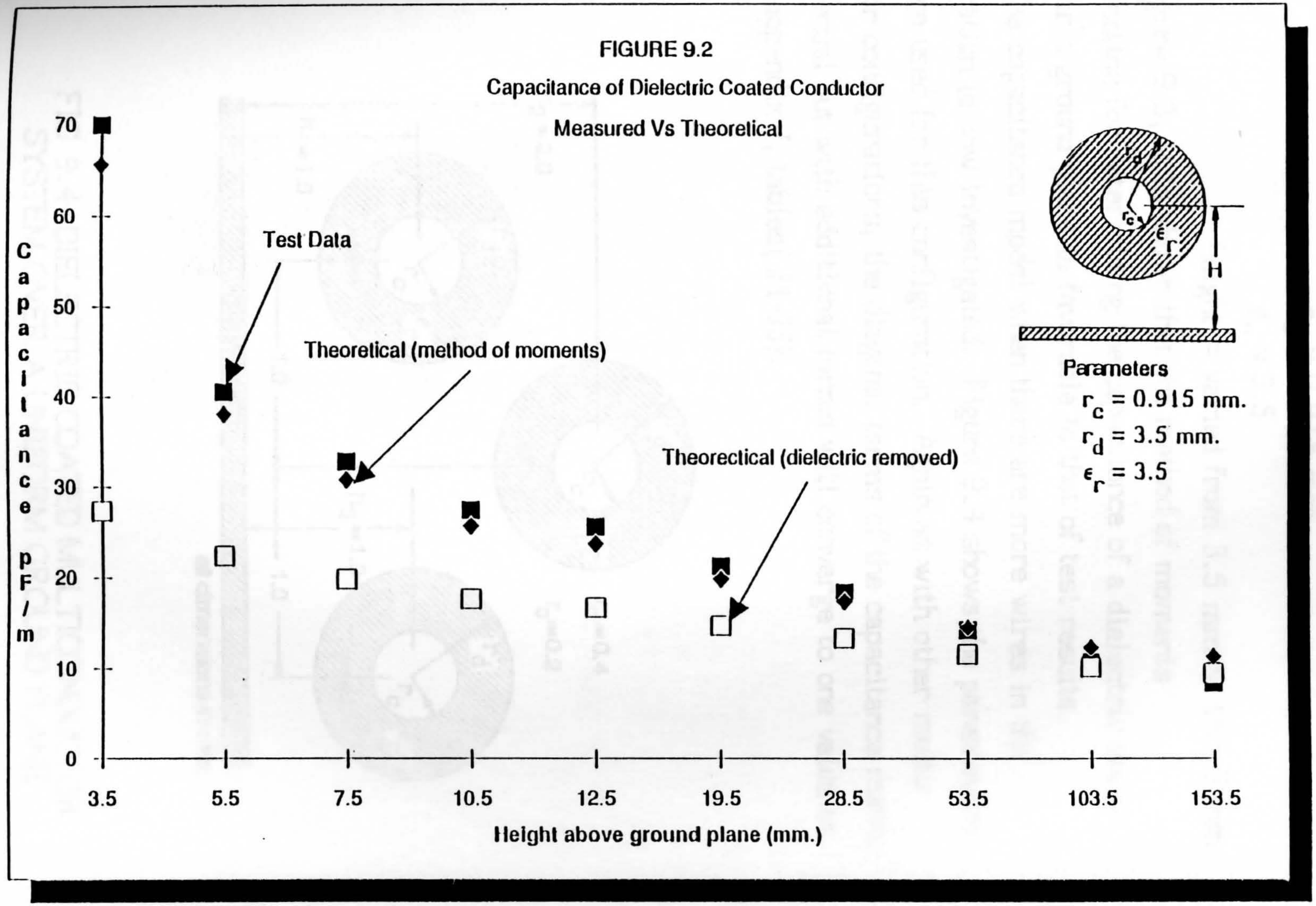


Fig. 9.2 TEST SETUP TO MEASURE CAPACITANCE

FIGURE 9.2

Capacitance of Dielectric Coated Conductor

Measured Vs Theoretical



The dielectric coated wire under test had the following parameters

$$r_d = 3.5 \text{ mm}$$
$$r_c = 0.915 \text{ mm}$$
$$\epsilon_r = 3.5$$
$$\text{height} = \text{varied from } 3.5 \text{ mm to } 153.5 \text{ mm}$$

From figure 9.3, it is shown that the method of moments approximation for determining the capacitance of a dielectric coated wire over a ground plane is favorable to that of test results.

The capacitance model when there are more wires in the configuration is now investigated. Figure 9.4 shows the parameters which are used for this configuration. Again as with other multi-conductor configurations, the diagonal terms of the capacitance matrix are not equal but with additional terms will converge to one value as seen in appendix J, tables(J1-J5).

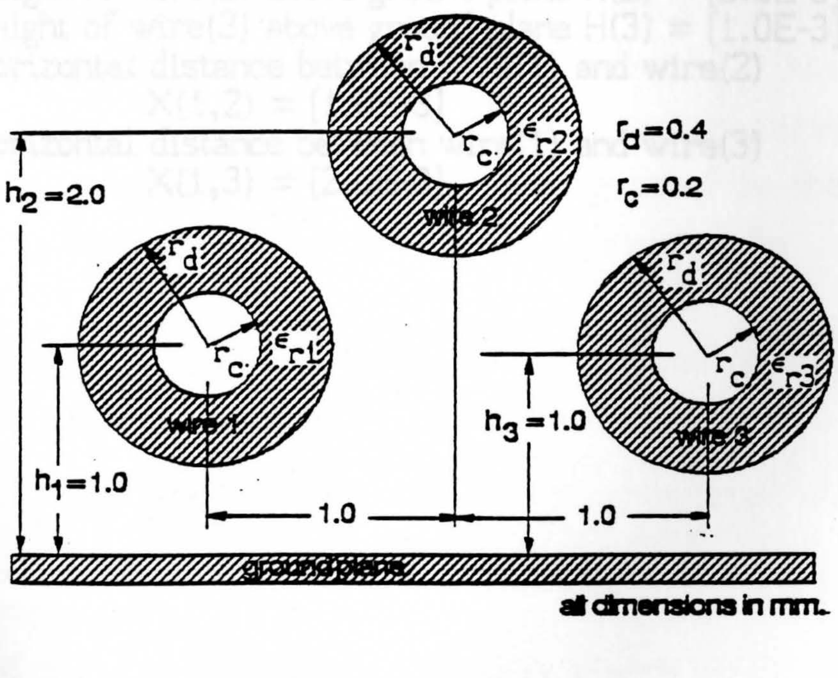


FIG 9.4 DIELECTRIC COATED MULTICONDUCTOR SYSTEM OVER A UNIFORM GROUND PLANE

The input data requirements for running the Fortran program for the dielectric coated multiconductor configuration of the above example are listed below

1. Enter type of configuration: [P]
2. Enter option (IOPT) = [1]
3. Enter # of wires in the system NW = [3]
4. Enter # of cosine or sine terms around the conductor
   i.e. the # of harmonics around the conductor NHC = [1,3,5,7,9]
5. Enter # of cosine or sine terms around the dielectric
   i.e. the # of harmonics around the dielectric NHD = [1,3,5,7,9]
6. Are all wires solid? Enter y/n [y]
7. Do all wires have the same radius? y/n [y]
8. Enter radius of the conductor XRC = [.2E-3]
9. Are all dielectric radii the same? Enter y/n [y]
10. Enter radius of dielectric RD = [.4E-3]
11. Is the relative permittivity the same for all wires? Enter y/n [Y]
12. Enter relative permittivity of dielectric ER = [3.5]
13. Enter height of wire(1) above ground plane H(1) = [1.0E-3]
14. Enter height of wire(2) above ground plane H(2) = [2.0E-3]
15. Enter height of wire(3) above ground plane H(3) = [1.0E-3]
16. Enter horizontal distance between wire(1) and wire(2)
    X(1,2) = [1.0E-3]
17. Enter horizontal distance between wire(1) and wire(3)
    X(1,3) = [2.0E-3]

CONCLUDING REMARKS

An approximate solution for finding the capacitance of multi-conductor systems such as ribbon cables, wire bundles, multi-conductor coax cables, shielded wire bundles, and dielectric coated wire bundles over a ground plane has been developed based on the method of moments. The results show that for known solutions as well as some test data that this method gives a good approximation. It has been shown that several computer runs may be necessary before the desired degree of accuracy becomes apparent. The speed of convergence is dependent on the configuration, the fastest being a single wire coax cable. The more symmetrical the configuration, the faster it will converge. Convergence also depends highly on the relative closeness of the conductors. The closer the conductors are to one another the more terms of the Fourier series will be needed before convergence is realized. The configuration which converged the slowest was the shielded dielectric coated wire bundle. This was due to the fact that all terms of the Fourier series were required, i.e., the average, cosine, and sine terms. Convergence of the shielded wire bundles was slow also due to the relative closness of the conductors.

The following is a list of observations and conclusions made for the various types of systems discussed in this report.

1. A computer model has been developed for the computation of the capacitance for a closely spaced multiconductor system in a linear, homogeneous or nonhomogeneous, isotropic medium assuming TEM mode of propagation.
2. The method of moments approach for a bare 2-wire system is a very good approximation to that of the closed form solution.
3. The method of moments using a Fourier series approximation for the charge distribution around a conductor converges rapidly for only a few matchpoints (or terms of the Fourier series).

4. Care must be taken in selecting matchpoints so as not to destroy the symmetry of the matrix or to cause one of its rows or columns to become dependent.
5. A larger number of Fourier series terms (or matchpoints) should be used around the dielectric surface than around the conductor surface in order to maintain the same degree of accuracy.
6. The speed of convergence is directly proportional to the geometry of the configuration. Those geometries which possess a high degree of symmetry converge faster than those systems which do do not. The speed of convergence is also directly related to the relative closeness of the conductors. The closer the conductors are to one another the more harmonics are needed to have the off-diagonal terms of the capacitance matrix converge to the same value. The speed of convergence of configurations which have wires that do not all lie in a horizontal plane converge slower than those that do due to the matchpoint selection around the conductors and dielectrics.
7. A larger matrix is necessary to describe multiconductor wire bundles, coax cables, shielded wire bundles, and wire bundles over a ground plane than those describing ribbon cables. The order of the matrix for coax cables, etc, is n(2k+1) compared to n(k+1) for ribbon cables. "n" is the number of wires in the system and k is the number of harmonics around the conductor or dielectric surface.
8. The method of moments approach for determining the capacitance of a coax cables compares extremely well to that of the closed form solution.
9. The method of moments approach for determining the capacitance of a shielded wire compares extremely well to that of the closed form solution.
10. The method of moments approach for determining the capacitance of one bare wire over a ground plane compares favorably to that of the closed form solution.
11. The method of moments approach for determining the capacitance of one dielectric coated wire over a ground plane compares favorably to that of test results.
12. The program presented in this report runs in quad precision on a VAX-750. CPU times are presented in tables along with output data for the respective configurations. Also along with the CPU

time is the elapsed time. This time is the actual time required before results are avaliable and depends on the number of users on the system. The VAX-750 is also limited as to the size matrix it can invert. The dimensions in the program are set to these limits, the limits being 15 wires and 10 harmonics around the conductor and 10 around the dielectric. Ten harmonics implies that 21 Fourier terms are used around the conductor as well as 21 Fourier terms around the dielectric. These dimensions are based on configurations which contain wire bundles, coax, and shielded wire bundles. Using the above limitation implies that the largest matrix the VAX-750 can invert is a 630x630 matrix. Any combination which does not exceed this limit will work. For example, if the configuration contains 30 dielectric coated wires then the user will have to redimension the program. If for example 5 harmonics are selected around the conductor and dielectric surfaces the order of the potential matrix, "D1" in the the program, will be described as follows.

no. of harmonics around conductor NHC=5
no. of harmonics around dielectric NHD=5
no. of Fourier terms around conductor = NFC = 2xNHC+1=11
no. of Fourier terms around dielectric = NFD = 2xNHD+1=11
total no. of Fourier terms = NF = NFC + NFD = 22
order of the D1 matrix = NWxNF= 30x22=660

Notice in this example the size of the matrix is 660x660 which is too large for the VAX-750 to invert. If the user attempts to dimension the variables in the program an error message will appear when a link is attempted. The message will be "insufficient virtual address space to complete the link, image file not created". To run a 30 wire configuration that has a dielectric coating on the wires requires the user to limit the number of harmonics selected to only 4, i.e., NHC and NHD cannot exceed 4 in this example.

A program listing of the capacitance model is given in appendix K and a listing of the subprograms, their function, and description of the variables in each subroutine is given in appendix J.

Some topics for future interest involve obtaining valid models of the structures which contain discontinuities in the geometry, noncylindrical conductors, as well as determining other system parameters such as: system stability, signal bandwidth, delay characteristics, radiated emissions, and crosstalk for PC boards and connectors. Also crosstalk due to various sources such as: high frequency sinusoidal, pulsed, transients, and impluse signals, and various loads containing not only resistive loads but combinations of capacitve and inductive loading as well.

Another point of interest would be to compare the results obtained by the method of moments to that using finite element analysis. Finite element codes can be obtained from MacNeal-Schwendler Corporation in Milwaukee, Wisconsin or from Ansoft Corporation in Pittsburgh, Pennsylvania. These are just two of of the finite element codes that are available today.

APPENDIX A
ELIMINATION OF REFERENCE AT INFINITY

Recall eq.(1.1) for determining the potential at an arbitrary point in space from an arbitrary reference point.

$$\phi(r,\theta) - \phi(r_o,\theta) = -\frac{1}{2\pi\epsilon_o}\lambda \ln(r/r_o) \qquad \text{eq.(A.1)}$$

If the charges add up to zero, then the system is balanced. That is for every positive charge in the system there is an equal an opposite charge. Now consider the balanced system shown in figure A.1, where the reference point is denoted by r_0 and r_p is the potential point from sources $+\lambda$ and $-\lambda$.

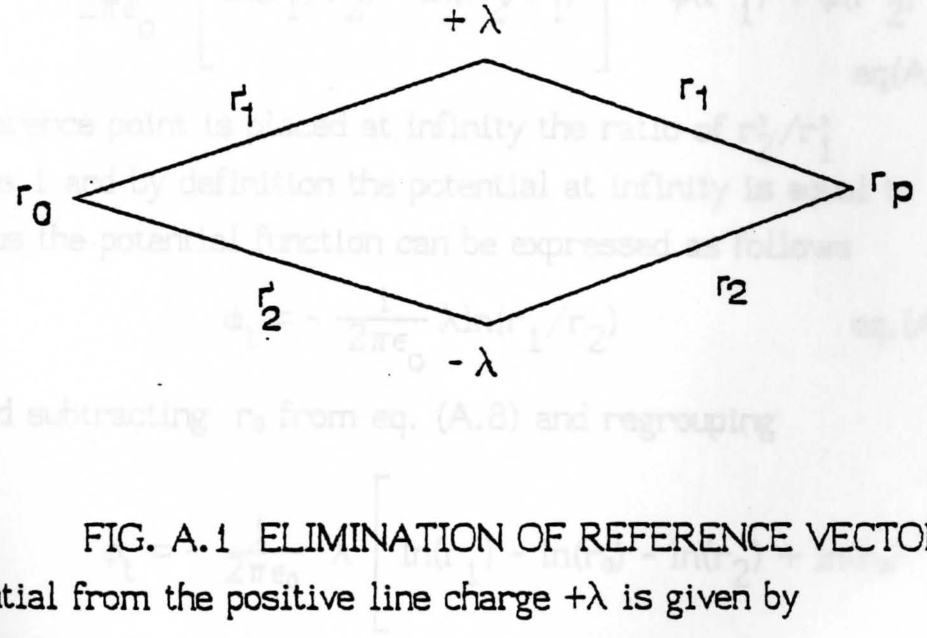


FIG. A.1 ELIMINATION OF REFERENCE VECTOR

The potential from the positive line charge $+\lambda$ is given by

$$\phi_1(r_1) - \phi(r'_1) = -\frac{1}{2\pi\epsilon_o}\lambda\ln(r_1/r'_1) \qquad \text{eq.(A.2)}$$

and the potential from the negative charge $-\lambda$ is given by

$$\phi_2(r_2) - \phi(r'_2) = +\frac{1}{2\pi\epsilon_o}\lambda\ln(r_2/r'_2) \qquad \text{eq.(A.3)}$$

The total potential at point r is found by using superposition as follows

$$\phi_t(r) = \phi_1(r) + \phi_2(r) \qquad \text{eq.(A.4)}$$

Substituting eqs(A.2-A.3) into A.4 , the equation for the potential function becomes

$$\phi_t = -\frac{1}{2\pi\epsilon_0}\lambda\ln(r_1/r'_1) + \frac{1}{2\pi\epsilon_0}\lambda\ln(r_2/r'_2) + \phi(r'_1) + \phi(r'_2) \qquad \text{eq(A.5)}$$

or

$$\phi_t = -\frac{1}{2\pi\epsilon_0}\lambda\left[\ln(r_1/r'_1) - \ln(r_2/r'_2)\right] + \phi(r'_1) + \phi(r'_2) \qquad \text{eq(A.6)}$$

$$\phi_t = -\frac{1}{2\pi\epsilon_0}\lambda\left[\ln(r_1/r_2) - \ln(r'_2/r'_1)\right] + \phi(r'_1) + \phi(r'_2) \qquad \text{eq(A.7)}$$

If the reference point is placed at infinity the ratio of r'_2/r'_1 approaches 1 and by definition the potential at infinity is equal to zero. Thus the potential function can be expressed as follows

$$\phi_t = -\frac{1}{2\pi\epsilon_0}\lambda\ln(r_1/r_2) \qquad \text{eq.(A.8)}$$

Adding and subtracting r_0 from eq. (A.8) and regrouping

$$\phi_t = -\frac{1}{2\pi\epsilon_0}\lambda\left[\ln(r_1) - \ln(r_0) - \ln(r_2) + \ln(r_0)\right] \qquad \text{eq.(A.9)}$$

or

$$\phi_t = -\frac{1}{2\pi\epsilon_0}\lambda\left[\ln(r_1/r_0) - \ln(r_2/r_0)\right] \qquad \text{eq.(A.10)}$$

Now let r_0 equal 1 meter since r is measured in meters the result is

$$\phi_t = -\frac{1}{2\pi\epsilon_0}\lambda\left[\ln(r_1) - \ln(r_2)\right] \qquad \text{eq. (A.11)}$$

APPENDIX B

Tables (B.1-B.10) are output files for a 4-wire bare wire bundle.

TABLE B.1
Output data for the generalized capacitance matrix with 1 harmonic on the conductor and 1 harmonics around the dielectric (units F/m)

row \ column	1	2	3	4
1	6.5948E-11	-2.3354E-11	-3.5531E-11	-4.1886E-12
2	-1.9404E-11	5.3082E-11	-4.5056E-12	-1.3299E-11
3	-3.3532E-11	-5.1901E-13	7.7349E-11	-2.5548E-11
4	-4.1925E-13	-1.4337E-11	-2.0169E-11	4.5547E-11

TABLE B.2
Output data for the transmission line capacitance matrix with 1 harmonic on the conductor and 1 harmonic around the dielectric (units (F/m)

row \ column	1	2	3
1	4.8766E-11	-9.6929E-12	-1.6253E-11
2	-4.5899E-12	7.1209E-11	-2.9044E-11
3	-1.7225E-11	-2.3640E-11	4.3570E-11

CPU time 00:00:30.39 elapsed time 00:00:01.74

TABLE B.3
Output data for the generalized capacitance matrix with 3 harmonics on the conductor and 3 harmonics around the dielectric (units F/m)

row \ column	1	2	3	4
1	7.2839E-11	-2.1440E-11	-3.8113E-11	-3.0190E-13
2	-2.1795E-11	5.3714E-11	-2.3643E-12	-1.3922E-11
3	-3.8101E-11	-2.6032E-12	8.0868E-11	-2.1175E-11
4	-5.8883E-13	-1.4030E-11	-2.1359E-11	4.5260E-11

TABLE B.4
Output data for the transmission line capacitance matrix with 3 harmonics on the conductor and 3 harmonics around the dielectric (units (F/m)

row \ column	1	2	3
1	4.9546E-11	-7.4352E-12	-1.6710E-11
2	-7.6652E-12	7.4708E-11	-2.4562E-11
3	-1.6819E-11	-2.4752E-11	4.3395E-11

CPU time 00:02:37.08 elapsed time 00:05:40.20

TABLE B.5

Output data for the generalized capacitance matrix with 5 harmonics on the conductor and 5 harmonics around the dielectric (units F/m)

row \ column	1	2	3	4
1	7.3299E-11	-2.1740E-11	-3.8552E-11	-5.6686E-13
2	-2.1754E-11	5.3737E-11	-2.3847E-12	-1.3972E-11
3	-3.8527E-11	-2.4053E-13	8.1161E-11	-2.1220E-11
4	-5.5029E-13	-1.3964E-11	-2.1212E-11	4.5079E-11

TABLE B.6

Output data for the transmission line capacitance matrix with 5 harmonics on the conductor and 5 harmonics around the dielectric (units (F/m)

row \ column	1	2	3
1	4.9574E-11	-7.4491E-12	-1.6757E-11
2	-7.4693E-12	7.5000E-11	-2.4608E-11
3	-1.6749E-11	-2.4600E-11	4.3216E-11

CPU time 00:08:25.83 elapsed time 00:19:08.79

TABLE B.7
Output data for the generalized capacitance matrix with 7 harmonics on the conductor and 7 harmonics around the dielectric (units F/m)

row \ column	1	2	3	4
1	7.3339E-11	-2.17454-11	-3.8566E-11	-5.4859E-13
2	-2.1752E-11	5.3741E-11	-2.3951E-12	-1.3966E-11
3	-3.8568E-11	-2.3936E-12	8.1166E-11	-2.1191E-11
4	-5.4966E-13	-1.3966E-11	-2.1192E-11	4.5062E-11

TABLE B.8

Output data for the transmission line capacitance matrix with 7 harmonics on the conductor and 7 harmonics around the dielectric (units (F/m)

row \ column	1	2	3
1	4.9578E-11	-7.4601E-12	-1.6751E-11
2	-7.4588E-12	7.5004E-11	-2.4579E-11
3	-1.6751E-11	-2.4580E-11	4.3199E-11

CPU time 00:19:35.63 elapsed time 00:52:07.33

TABLE B.9

Output data for the generalized capacitance matrix with 9 harmonics on the conductor and 9 harmonics around the dielectric (units F/m)

row \ column	1	2	3	4
1	7.3343E-11	-2.1754E-11	-3.8572E-11	-5.4969E-13
2	-2.1753E-11	5.3741E-11	-2.3943E-12	-1.3966E-11
3	-3.8572E-11	-2.3940E-13	8.1171E-11	-2.1191E-11
4	-5.4967E-13	-1.3966E-11	-2.1191E-11	4.5060E-11

TABLE B.10

Output data for the transmission line capacitance matrix with 9 harmonics on the conductor and 9 harmonics around the dielectric (units (F/m)

row \ column	1	2	3
1	4.9578E-11	-7.4595E-12	-1.6751E-11
2	-7.4592E-12	7.5008E-11	-2.4579E-11
3	-1.6751E-11	-2.4579E-11	4.3198E-11

CPU time 00:36:02.26 elapsed time 00:40:46.02

APPENDIX C

Tables (C.1-C.10) are output files for a 3-wire wire bundles as shown in figure 5.2

TABLE C.1

Output data for the generalized capacitance matrix with 1 harmonic on the conductor and 1 harmonic around the dielectric (units F/m)

row \ column	1	2	3
1	5.1479E-11	-3.0863E-11	-3.7512E-11
2	-3.7212E-11	3.2074E-11	-2.0561E-11
3	-3.3288E-11	-2.6090E-11	3.3236E-11

TABLE C.2

Output data for the transmission line capacitance matrix with 1 harmonic on the conductor and 1 harmonic around the dielectric (units F/m)

row \ column	1	2
1	4.1376E-11	-1.1275E-11
2	-1.6628E-11	4.2682E-11

CPU time 00:00:54.42 elapsed time 00:01:19.43

TABLE C.3

Output data for the generalized capacitance matrix with 3 harmonics on the conductor and 3 harmonics around the dielectric (units F/m)

row \ column	1	2	3
1	5.6586E-11	-3.7982E-11	-3.7227E-11
2	-3.7924E-11	3.5010E-11	-2.1848E-11
3	-3.7187E-11	-2.1806E-11	3.4311E-11

TABLE C.4

Output data for the transmission line capacitance matrix with 3 harmonics on the conductor and 3 harmonics around the dielectric (units F/m)

row \ column	1	2
1	4.4024E-11	-1.2839E-11
2	-1.2821E-11	4.3291E-11

CPU time 00:07:02.63 elapsed time 00:09:57.58

TABLE C.5

Output data for the generalized capacitance matrix with 5 harmonics on the conductor and 5 harmonics around the dielectric (units F/m)

row \ column	1	2	3
1	5.6842E-11	-3.7827E-11	-3.7582E-11
2	-3.7758E-11	3.4912E-11	-2.1922E-11
3	-3.7628E-11	-2.1849E-11	3.4741E-11

TABLE C.6

Output data for the transmission line capacitance matrix with 5 harmonics on the conductor and 5 harmonics around the dielectric (units F/m)

row \ column	1	2
1	4.3922E-11	-1.2912E-11
2	-1.2850E-11	4.3740E-11

CPU time 00:25:21.92 elapsed time 00:35:19.43

TABLE C.7

Output data for the generalized capacitance matrix with 7 harmonics on the conductor and 7 harmonics around the dielectric (units F/m)

row \ column	1	2	3
1	5.6855E-11	-3.7718E-11	-3.7680E-11
2	-3.7719E-11	3.4865E-11	-2.1912E-11
3	-3.7676E-11	-2.1914E-11	3.4826E-11

TABLE C.8

Output data for the transmission line capacitance matrix with 7 harmonics on the conductor and 7 harmonics around the dielectric (units F/m)

row \ column	1	2
1	4.3875E-11	-1.2901E-11
2	-1.2904E-11	4.3835E-11

CPU time 00:59:20.96 elapsed time 01:25:47.86

TABLE C.9

Output data for the generalized capacitance matrix with 9 harmonics on the conductor and 9 harmonics around the dielectric (units F/m)

row \ column	1	2	3
1	5.6847E-11	-3.7697E-11	-3.7689E-11
2	-3.7697E-11	3.4851E-11	-2.1920E-11
3	-3.7690E-11	-2.1920E-11	3.4844E-11

APPENDIX D

This is a list of output data files for the ribbon cable and wire bundle shown in figure 6.4

TABLE D.1

OUTPUT DATA FOR RIBBON CABLE

no. of harmonics	cap. ϵ_r=3.5 (Pf/m)	cap. ϵ_r=6.5 (Pf/m)	CPU time (sec.)
1	33.108	38.149	15
3	37.704	46.951	44
5	38.734	49.684	108
7	38.962	50.448	220
9	39.015	50.655	400
11	39.027	50.711	644

TABLE D.2

OUTPUT DATA FOR WIRE BUNDLE

no. of harmonics	cap. ϵ_r=3.5 (Pf/m)	cap. ϵ_r=6.5 (Pf/m)	CPU time (sec.)
1	36.548	44.198	24
3	38.238	48.352	88
5	38.630	49.622	480
7	38.706	49.919	1142
9	38.721	49.988	2187
11	38.724	50.005	3602

APPENDIX E

Cable data from Belden

Broadcast and Computer Cables

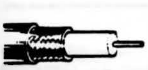
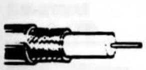
Description	Trade & U.L. Type Number	Standard Lengths ft.	Standard Lengths m	Std. Unit Lbs. ea.	AWG (Stranding) Dia. in In. Nom. D.C.R.	Insulation & Nominal Core O.D.	Core O.D. Inch	Core O.D. mm	Nominal O.D. Inch	Nominal O.D. mm	No. of Shields & Material Nom. D.C.R.	Nom. Imp. Ω	Nom. Vel. of Prop.	Nominal Capacitance pF/ft.	Nominal Capacitance pF/m	MHz	Nominal Attenuation db/100 ft.	Nominal Attenuation db/100 m
 RG-58/U Type	**9889** UL 1354 60C	500 1000 2000	152.4 304.8 609.6	15.6 29.6 60.5	18 (Solid) .041 bare copper 6.5Ω/M' 21.3Ω/km	Cellular Poly-ethylene	.116	2.95	.216	5.49	Duofoil® with 4-24 AWG tinned copper drain wires 25Ω/M' 82Ω/km	50 Black PVC jacket.	78%	26	85.3	50 100 300 500 1000	3.3 4.9 9.3 13.6 18.8	10.8 16.1 30.5 44.6 61.7
 Dual RG59/U Type	**9555** UL 20063 300V 80C	100 500 1000	30.5 152.4 304.8	8.1 39.0 74.7	23 (Solid) .023 bare copper covered steel 47Ω/M' 154.2Ω/km	Poly-ethylene	.146	3.71	.238 x .478	6.04 x 12.14	Bare copper 2.6Ω/M' 8.5Ω/km 95% shield coverage	75 Black PVC jacket. **For Plenum version, see 89555 on page 132.**	66%	20.5	67.3	100 200 400 700 900 1000	3.4 5.1 7.5 11.4 12.0 12.7	11.5 16.7 24.6 37.4 39.4 41.7
 RG-59/U Type	**9259** UL 1354 80C	50 100 U-500 500 U-1000 1000	15.2 30.5 U-152.4 152.4 U-304.8 304.8	2.0 4.0 18.4 19.1 35.7 36.7	22 (7x30) .031 bare copper 15.0Ω/M' 49.2Ω/km	Cellular Poly-ethylene	.146	3.71	.242	6.15	Bare copper 2.6Ω/M' 8.5Ω/km 95% shield coverage	75 Black PVC jacket. For CCTV applications. 100% Sweep Tested 5-300 MHz **For Plenum version, see 89259 on page 131.**	78%	17.3	56.8	50 100 200 400 700 900 1000	2.1 3.0 4.5 6.6 8.9 10.1 10.9	6.9 9.8 14.8 21.7 29.2 33.1 35.8
 RG-59/U JAN-C-17A	**8241** UL 1354 60C	25 50 100 U-500 500 U-1000 1000 5000	7.6 15.2 30.5 U-152.4 152.4 U-304.8 304.8 1524.0	1.3 2.2 4.3 18.5 19.2 36.0 36.9 192.0	22 (Solid) .025 bare copper covered steel 55Ω/M' 180.5Ω/km	Poly-ethylene	.146	3.71	.242	6.15	Bare copper 2.7Ω/M' 8.9Ω/km 95% shield coverage	73 Black PVC jacket. **For Plenum version, see 88241 on page 131.**	66%	21.0	68.9	50 100 200 400 700 900 1000	2.4 3.4 4.9 7.1 9.5 10.9 12.0	7.9 11.2 16.1 23.3 31.2 35.6 39.4

Broadcast Cables

Description	Trade & U.L. Type Number	Standard Lengths ft.	Standard Lengths m	Std. Unit Lbs. ea.	AWG (Stranding) Dia. in In. Nom. D.C.R.	Insulation & Nominal Core O.D.	Core O.D. Inch	Core O.D. mm	Nominal O.D. Inch	Nominal O.D. mm	No. of Shields & Material Nom. D.C.R.	Nom. Imp. Ω	Nom. Vel. of Prop.	Nominal Capacitance pF/ft.	Nominal Capacitance pF/m	MHz	Nominal Attenuation db/100 ft.	Nominal Attenuation db/100 m
 RG-59B/U MIL-C-17D	**8263†** 80C	U-500 500 U-1000 1000	U-152.4 152.4 U-304.8 304.8	18.1 18.8 35.2 38.1	23 (Solid) .023 bare copper covered steel 47Ω/M' 154.2Ω/km	Poly-ethylene	.146	3.71	.242	6.15	Bare copper 2.6Ω/M' 8.5Ω/km 95% shield coverage	75 Black non-contaminating PVC jacket.	66%	20.5	67.3	50 100 200 400 700 900 1000	2.4 3.4 4.9 7.0 9.7 11.1 12.0	7.9 11.2 16.1 23.0 31.8 36.4 39.4
 MIL-C-17F M17/29-RG59 QPL	**9204†** 80C	U-500 500 U-1000 1000	U-152.4 152.4 U-304.8 304.8	18.1 18.7 35.2 36.0	23 (Solid) .023 bare copper covered steel 47Ω/M' 154.2Ω/km	Poly-ethylene	.146	3.71	.242	6.15	Bare copper 2.6Ω/M' 8.5Ω/km 95% shield coverage	75 Black non-contaminating PVC jacket.	66%	20.5	67.3	50 100 200 400 700 900 1000	2.4 3.4 4.9 7.0 9.7 11.1 12.0	7.9 11.2 16.1 23.0 31.8 36.4 39.4
 RG-59/U Type	**New** **9659†** UL 1354 80C	U-500 500 U-1000 1000	U-152.4 152.4 U-304.8 304.8	17.0 17.5 32.6 33.6	22 (7x30) .031 bare copper 15.0Ω/M' 49.2Ω/km	Cellular Poly-ethylene	.146	3.71	.242	6.15	Bare copper 2.6Ω/M' 8.5Ω/km 95% shield coverage	75 Black non-contaminating PVC jacket. For CCTV applications. 100% Sweep Tested 5-300 MHz	78%	17.3	56.8	50 100 200 400 700 900 1000	2.1 3.0 4.5 6.6 8.9 10.1 10.9	6.9 9.8 14.8 21.7 29.2 33.1 35.8

†Passes the VW-1 Vertical Wire Flame Test.
Request quotations of RG U cables not listed

MATV Cables

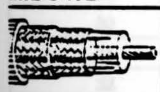
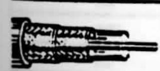
Description	Trade & U.L. Type Number	Standard Lengths ft.	Standard Lengths m	Std. Unit Lbs. ea.	AWG (Stranding) Dia. in In. Nom. D.C.R.	Insulation & Nominal Core O.D. Inch	Insulation & Nominal Core O.D. mm	Nominal O.D. Inch	Nominal O.D. mm	No. of Shields & Material Nom. D.C.R.	Nom. Imp. Ω	Nom. Vel. of Prop.	Nominal Capacitance pF/ft.	Nominal Capacitance pF/m	Nominal Attenuation MHz	db/100 ft.	db/100 m
 RG-59/U Type	**9100** Replaces 9282 80C	U-500♦ Black White Lt. Beige	152.4	11.2	20 (Solid) .032 copper covered steel	Cellular Poly-ethylene .146	3.71	.242	6.15	Duobond II® + 40% aluminum braid 100% shield coverage	75 100% Sweep Tested 5-450 MHz	78%	17.3	56.8	50 100 200 500 900	1.8 2.6 3.8 6.2 8.4	5.9 8.5 12.5 20.3 27.6
		1000♦ Black	304.8	22.3													
		U-1000♦ Black White Lt. Beige	U-304.8	21.3													
		2000♦ Black	609.6	48.2													
 RG-59/U Type	**9101** Replaces 9377 80C	1000♦	304.8	40.4	20 (Solid) .032 copper covered steel	Cellular Poly-ethylene .146	3.71	.242 x .385	6.15 x 9.78	Duobond II + 40% aluminum braid 100% shield coverage	75 Black PVC jacket. 100% Sweep Tested 5-450 MHz Messengered—.051" (1.3 mm) galvanized steel messenger.	78%	17.3	56.8	50 100 200 500 900	1.8 2.6 3.8 6.2 8.4	5.9 8.5 12.5 20.3 27.6
 RG-59/U Type	**8212** 80C	U-500♦ 500♦ U-1000♦ 1000♦ 2000♦	U-152.4 152.4 U-304.8 304.8 609.2	15.5 16.3 30.0 30.9 65.5	20 (Solid) .032 bare copper covered steel 34.5Ω/M' 113.2Ω/km	Cellular. Poly-ethylene .146	3.71	.242	6.15	Bare copper braid. 2.6Ω/M' 8.5Ω/km 95% shield coverage	75 Black polyethylene jacket. 100% Sweep Tested 5-450 MHz	78%	17.3	56.8	50 100 200 500 900	1.8 2.6 3.8 6.2 8.4	5.9 8.5 12.5 20.3 27.6
 RG-59/U Type	**9274** 80C	500♦ U-1000♦ 1000♦ 2000♦	152.4 U-304.8 304.8 609.6	16.4 30.3 31.2 66.1	20 (Solid) .032 bare copper covered steel 61.5Ω/M' 201.8Ω/km	Cellular Poly-ethylene .146	3.71	.242	6.15	Bare copper braid 3.5Ω/M' 11.5Ω/km 95% shield coverage	75 Black PVC jacket. 100% Sweep Tested 5-450 MHz	78%	17.3	56.8	50 100 200 500 900	1.8 2.6 3.8 6.2 8.4	5.9 8.5 12.5 20.3 27.6
 Flooded Burial Cable RG-59/U Type	**9590** 80C	1000♦	304.8	22.2	20 (Solid) .032 copper covered steel 61.5Ω/M' 201.8Ω/km	Cellular Poly-ethylene .146	3.71	.242	6.15	Duofoil® + 53% aluminum braid + flooding 12.5Ω/M' 41.0Ω/km 100% shield coverage	75 Black polyethylene jacket. 100% Sweep Tested 5-450 MHz	78%	17.3	56.8	50 100 200 500 900	1.8 2.6 3.8 6.2 8.4	5.9 8.5 12.5 20.3 27.6

♦ Spools are one piece, but length may vary ± 10% from length shown.
Request quotations of RG/U cables not listed.

Broadcast and Computer Cables

Description	Trade & U.L. Type Number	Standard Lengths ft.	Standard Lengths m	Std. Unit Lbs. ea.	AWG (Stranding) Dia. in In. Nom. D.C.R.	Insulation & Nominal Core O.D. Inch	Insulation & Nominal Core O.D. mm	Nominal O.D. Inch	Nominal O.D. mm	No. of Shields & Material Nom. D.C.R.	Nom. Imp. Ω	Nom. Vel. of Prop.	Nominal Capacitance pF/ft.	Nominal Capacitance pF/m	Nominal Attenuation MHz	db/100 ft.	db/100 m
 MIL-C-17F M17/157-00001 (RG122/U) QPL	9252 UL 1354 60C	100 U-500 500 U-1000 1000	30.5 U-152.4 152.4 U-304.8 304.8	2.1 9.7 9.4 18.4 19.0	22 (27x36) .028 tinned copper 17.1Ω/M' 56.1Ω/km	Polyethylene .096	2.44	.160	4.06	Tinned copper 5.2Ω/M' 17.1Ω/km 97% shield coverage	50 Black non-contaminating PVC jacket.	66%	30.8	101.0	50 100 200 400 700 900 1000	4.5 7.0 10.0 15.2 21.2 25.0 26.5	14.8 23.0 32.8 49.9 69.6 82.0 87.0
 RG-174/U MIL-C-17D	8216 UL 1354 60C	100 500 1000	30.5 152.4 304.8	.8 4.4 8.1	26 (7x34) .019 bare copper covered steel 97Ω/M' 318.3Ω/km	Polyethylene .060	1.52	.101	2.56	Tinned copper 10.3Ω/M' 33.8Ω/km 88% shield coverage	50 Black PVC jacket.	66%	30.8	101.0	50 100 200 400 700 900 1000	6.6 8.9 12.0 17.5 24.1 28.2 30.0	21.7 29.2 39.4 57.4 79.1 92.5 98.4

Broadcast Cables

Description	Trade & U.L. Type Number	Standard Lengths ft.	Standard Lengths m	Std. Unit Lbs. ea.	AWG (Stranding) Dia. in In. Nom. D.C.R.	Insulation & Nominal Core O.D. Inch	Insulation & Nominal Core O.D. mm	Nominal O.D. Inch	Nominal O.D. mm	No. of Shields & Material Nom. D.C.R.	Nom. Imp. Ω	Nom. Vel. of Prop.	Nominal Capacitance pF/ft.	Nominal Capacitance pF/m	Nominal Attenuation MHz	db/100 ft.	db/100 m
 RG-213/U MIL-C-17D	8267† UL 1354 60C	500 1000	152.4 304.8	55.0 108.0	13 (7x21) .089 bare copper 1.87Ω/M' 6.1Ω/km	Polyethylene .285	7.24	.405	10.29	Bare copper 1.2Ω/M' 3.9Ω/km 97% shield coverage	50 Black non-contaminating. PVC jacket.	66%	30.8	101.0	50 100 200 400 700 900 1000 4000	1.6 2.2 3.2 4.7 6.9 8.0 8.9 21.5	5.2 7.2 10.5 15.4 22.6 26.3 29.2 70.5
 MIL-C-17F M17/164-00001 (RG214/U) QPL	8268† UL 1354 60C	500 1000	152.4 304.8	67.5 133.0	13 (7x.0296) .089 silver coated copper 1.73Ω/M' 5.7Ω/km	Polyethylene .285	7.24	.425	10.80	2 silver coated copper .7Ω/M' 2.3Ω/km 98% shield coverage	50 Black non-contaminating PVC jacket.	66%	30.8	101.0	50 100 200 400 700 900 1000 4000	1.6 2.2 3.2 4.7 6.9 8.0 8.9 21.5	5.2 7.2 10.5 15.4 22.6 26.3 29.2 70.5
 MIL-C-17F M17/167-00001 (RG223/U) QPL	9273 UL 1354 60C	100 U-500 500 U-1000 1000	30.5 U-152.4 152.4 U-304.8 304.8	3.9 18.5 19.1 35.9 36.7	19 (Solid) .035 silver coated copper 8.05Ω/M' 26.4Ω/km	Polyethylene .116	2.95	.212	5.38	2 silver coated copper 2.5Ω/M' 8.3Ω/km 97% shield coverage	50 Black non-contaminating PVC jacket.	66%	30.8	101.0	50 100 200 400 700 900 1000	3.1 4.5 6.4 9.2 12.5 14.3 16.3	10.1 14.8 21.0 30.2 41.0 46.9 53.5

†Passes the VW-1 Vertical Wire Flame Test.
Request quotations of RG/U cables not listed.

Solid Bare Copper Wire American Wire Gage

Gage (AWG) or (B & S)	Nominal Diameter		Circular Mil Area	Weight Pounds per M'	Resistance at 68°F Ohms per M'
	Inches	mm			
10	.1019	2.60	10380.	31.43	.9989
11	.0907	2.30	8234.	24.92	1.260
12	.0808	2.05	6530.	19.77	1.588
13	.0720	1.83	5178.	15.68	2.003
14	.0641	1.63	4107.	12.43	2.525
15	.0571	1.45	3260.	9.858	3.184
16	.0508	1.29	2583.	7.818	4.016
17	.0453	1.15	2050.	6.200	5.064
18	.0403	1.02	1620.	4.917	6.385
19	.0359	.912	1200.	3.899	8.051
20	.0320	.813	1020.	3.092	10.15
21	.0285	.724	812.1	2.452	12.80
22	.0253	.643	640.4	1.945	16.14
23	.0226	.574	511.5	1.542	20.36
24	.0201	.511	404.0	1.223	25.67
25	.0179	.455	320.4	.9699	32.37
26	.0159	.404	253.0	.7692	40.81
27	.0142	.361	201.5	.6100	51.47
28	.0126	.320	159.8	.4837	64.90
29	.0113	.287	126.7	.3836	81.83
30	.0100	.254	100.5	.3042	103.2
31	.0089	.226	79.7	.2413	130.1
32	.0080	.203	63.21	.1913	164.1
33	.0071	.180	50.13	.1517	206.9
34	.0063	.160	39.75	.1203	260.9
35	.0056	.142	31.52	.09542	331.0
36	.0050	.127	25.00	.07568	414.8
37	.0045	.114	19.83	.0613	512.1
38	.0040	.102	15.72	.04759	648.6
39	.0035	.089	12.20	.03774	847.8
40	.0031	.079	9.61	.02993	1080.0

Information from National Bureau of Standards Copper Wire Tables—Handbook 100.

Stranded Tinned Copper Wire American Wire Gage

AWG Size	Stranding	Nominal O.D. of Strand	Approximate O.D. Inch	Approximate O.D. mm	Circular Mil Area	Weight per 1000'	Ohms per 1000'
36	7/44	.002	.006	.153	28.00	.085	371.0
34	7/42	.0025	.0075	.191	43.75	.132	237.0
32	7/40	.0031	.008	.203	67.27	.203	164.0
32	19/44	.002	.009	.229	76.00	.230	136.4
30	7/38	.004	.012	.305	112.00	.339	103.2
30	19/42	.0025	.012	.305	118.75	.359	87.3
28	7/36	.005	.015	.381	141.75	.529	64.9
28	19/40	.0031	.016	.406	182.59	.553	56.7
27	7/35	.0056	.018	.457	219.52	.664	51.47
26	7/34	.0063	.019	.483	277.83	.841	37.3
26	10/36	.0050	.021	.533	250.00	.757	41.48
26	19/38	.0040	.020	.508	304.00	.920	34.43
24	7/32	.008	.024	.610	448.00	1.356	23.3
24	10/34	.0063	.023	.584	396.90	1.201	26.09
24	19/36	.0050	.024	.610	475.00	1.430	21.08
24	41/40	.0031	.023	.584	384.40	1.160	25.59
22	7/30	.0100	.030	.762	700.00	2.120	14.74
22	19/34	.0063	.031	.787	754.11	2.28	13.73
22	26/36	.0050	.030	.762	650.00	1.97	15.94
20	10/30	.0100	.035	.890	1,000.00	3.025	10.32
20	19/32	.0080	.037	.940	1,216.00	3.68	8.63
20	26/34	.0063	.036	.914	1,031.94	3.12	10.05
20	41/36	.0050	.036	.914	1,025.00	3.10	10.02
18	7/26	.0159	.048	1.22	1,769.60	5.36	5.86
18	16/30	.0100	.047	1.20	1,600.00	4.84	6.48
18	19/30	.0100	.049	1.24	1,900.00	5.75	5.46
18	41/34	.0063	.047	1.20	1,627.29	4.92	6.37
18	65/36	.0050	.047	1.20	1,625.00	4.91	6.39
16	7/24	.0201	.060	1.52	2,828.00	8.56	3.67
16	19/29	.0113	.058	1.47	2,426.30	7.35	4.27
16	26/30	.0100	.059	1.50	2,600.00	7.87	4.00
16	65/34	.0063	.059	1.50	2,579.85	7.81	4.02
16	105/36	.0050	.059	1.50	2,625.00	7.95	3.99
14	7/22	.0253	.073	1.85	4,480.0	13.56	2.31
14	19/27	.0142	.073	1.85	3,830.4	11.59	2.70
14	41/30	.0100	.073	1.85	4,100.0	12.40	2.53
14	105/34	.0063	.073	1.85	4,167.5	12.61	2.49
12	7/20	.0320	.096	2.44	7,168.0	21.69	1.45
12	19/25	.0179	.093	2.36	6,087.6	18.43	1.70
12	65/30	.0100	.095	2.41	6,500.0	19.66	1.75
12	165/34	.0063	.095	2.41	6,548.9	19.82	1.58
10	37/26	.0159	.115	2.92	9,353.6	28.31	1.11
10	49/27	.0142	.116	2.95	9,878.4	29.89	1.09
10	105/30	.0100	.116	2.95	10,530.0	31.76	.98

[illegible] from National Bureau of Standards Copper Wire Tables—Handbook 100.

APPENDIX F

Tables (F.1-F.10) are output data files for a multiconductor coax cable

TABLE F.1

Output data for the generalized capacitance matrix with 1 harmonic or 3 Fourier terms (units F/m)

row \ column	1	2	3	4
1	1.3876E-10	-6.5517E-11	-1.1013E-10	-1.1013E-10
2	-6.4507E-11	1.2015E-10	-2.7824E-11	-2.7824E-11
3	-1.0903E-10	-2.7548E-11	1.4515E-10	-8.5744E-12
4	-1.0903E-10	-2.7548E-11	-8.5744E-12	1.4515E-10

TABLE F.2

Output data for the transmission line capacitance matrix with 1 harmonic or 3 Fourier terms (units F/m)

row \ column	1	2	3
1	1.2015E-10	-2.7824E-11	-2.7824E-11
2	-2.7548E-11	1.4515E-10	-8.5744E-12
3	-2.7548E-11	-8.5744E-12	1.4515E-10

CPU time 00:00:26.42 elapsed time 00:00:30.21

TABLE F.3

Output data for the generalized capacitance matrix with 3 harmonics or 7 Fourier terms (units F/m)

row \ column	1	2	3	4
1	1.8661E-10	-5.9798E-11	-1.3692E-10	-1.3692E-10
2	-5.9880E-11	1.2140E-10	-3.0758E-11	-3.0758E-11
3	-1.3687E-10	-3.0810E-11	1.7015E-10	-2.4647E-12
4	-1.3687E-10	-3.0810E-11	-2.4647E-12	1.7015E-10

TABLE F.4

Output data for the transmission line capacitance matrix with 3 harmonics or 7 Fourier terms (units F/m)

column / row	1	2	3
1	1.2140E-10	-3.0758E-11	-3.0758E-11
2	-3.0810E-11	1.7015E-10	-2.4647E-12
3	-3.0810E-11	-2.4647E-12	1.7015E-10

CPU time 00:02:32.78 elapsed time 00:04:08.32

TABLE F.5

Output data for the generalized capacitance matrix with 5 harmonics or 11 Fourier terms (units F/m)

column / row	1	2	3	4
1	1.9523E-10	-5.9212E-11	-1.4152E-10	-1.4152E-10
2	-5.9218E-11	1.2145E-10	-3.1116E-11	-3.1116E-11
3	-1.4152E-10	-3.1119E-11	1.7387E-10	-1.2294E-12
4	-1.4152E-10	-3.1119E-11	-1.2294E-12	1.7387E-10

TABLE F.6

Output data for the transmission line capacitance matrix with 5 harmonics or 11 Fourier terms (units F/m)

column / row	1	2	3
1	1.2145E-10	-3.1116E-11	-3.1116E-11
2	-3.1119E-11	1.7387E-10	-1.2294E-12
3	-3.1119E-11	-1.2294E-12	1.7387E-10

CPU time 00:07:57.40 elapsed time 00:09:46.38

TABLE F.7

Output data for the generalized capacitance matrix with 7 harmonics or 15 Fourier terms (units F/m)

row \ column	1	2	3	4
1	1.9690E-10	-5.9155E-11	-1.4238E-10	-1.4238E-10
2	-5.9154E-11	1.2145E-10	-3.1149E-11	-3.1149E-11
3	-1.4238E-10	-3.1148E-11	1.7452E-10	-9.9034E-13
4	-1.4238E-10	-3.1148E-11	-9.9034E-13	1.7452E-10

TABLE F.8

Output data for the transmission line capacitance matrix with 7 harmonics or 15 Fourier terms (units F/m)

row \ column	1	2	3
1	1.2145E-10	-3.1149E-11	-3.1149E-11
2	-3.1148E-11	1.7452E-10	-9.9034E-13
3	-3.1148E-11	-9.9034E-13	1.7452E-10

CPU time 00:19:13.63 elapsed time 00:37:55.38

TABLE F.9

Output data for the generalized capacitance matrix with 9 harmonics or 19 Fourier terms (units F/m)

row \ column	1	2	3	4
1	1.9726E-10	-5.9151E-11	-1.4256E-10	-1.4256E-10
2	-5.9151E-11	1.2145E-10	-3.1150E-11	-3.1150E-11
3	-1.4256E-10	-3.1150E-11	1.7465E-10	-9.4112E-13
4	-1.4256E-10	-3.1150E-11	-9.4112E-13	1.7465E-10

TABLE F.10

Output data for the transmission line capcitance matrix with 9 harmonics or 19 Fourier terms (units F/m)

row \ column	1	2	3
1	1.2145E-10	-3.1150E-11	-3.1150E-11
2	-3.1150E-11	1.7465E-10	-9.4112E-13
3	-3.1150E-11	-9.4114E-13	1.7465E-10

CPU time 00:37:27.85 elapsed time 01:23:50.15

APPENDIX G

Tables (G.1-G.10) are output files for a shielded 3-wire ripcord.

TABLE G.1

Output data for the generalized capacitance matrix with 1 harmonic on the conductor and 2 harmonics around the dielectric (units F/m)

row \ column	1	2	3	4
1	7.2590E-11	-2.2850E-11	-4.5873E-11	-4.5873E-11
2	-2.2992E-11	5.8842E-11	-1.7925E-11	-1.7925E-11
3	-4.5298E-11	-1.8167E-11	6.8645E-11	-5.1797E-12
4	-4.5298E-11	-1.8167E-11	-5.1797E-12	6.8645E-11

TABLE G.2

Output data for the transmission line capacitance matrix with 1 harmonic on the conductor and 2 harmonics around the dielectric (units F/m)

row \ column	1	2	3
1	5.8842E-11	-1.7925E-11	-1.7925E-11
2	-1.8167E-11	6.8645E-11	-5.1797E-12
3	-1.8167E-11	-5.1797E-13	6.8645E-11

CPU time 00:03:36.83 elapsed time 00:09:52.75

TABLE G.3

Output data for the generalized capacitance matrix with 3 harmonic on the conductor and 4 harmonics around the dielectric (units F/m)

row \ column	1	2	3	4
1	1.2038E-10	-1.9746E-11	-7.1318E-11	-7.1318E-11
2	-1.9791E-11	5.9705E-11	-1.9957E-11	-1.9957E-11
3	-7.1458E-11	-1.9980E-11	9.3560E-11	-2.1218E-12
4	-7.1458E-11	-1.9980E-11	-2.1218E-12	9.3560E-11

TABLE G.4

Output data for the transmission line capacitance matrix with 3 harmonic on the conductor and 4 harmonics around the dielectric (units F/m)

row \ column	1	2	3
1	5.9705E-11	-1.9957E-11	-1.9957E-11
2	-1.9980E-11	9.3560E-11	-2.1218E-12
3	-1.9980E-11	-2.1218E-12	9.3560E-11

CPU time 00:22:23.18 elapsed time 00:39:39.22

TABLE G.5

Output data for the generalized capacitance matrix with 5 harmonic on the conductor and 6 harmonics around the dielectric (units (F/m)

row \ column	1	2	3	4
1	1.3668E-10	-1.9507E-11	-7.9590E-11	-7.9590E-11
2	-1.9460E-11	5.9701E-11	-2.0120E-11	-2.0120E-11
3	-7.9699E-11	-2.0095E-11	1.0068E-10	-8.8310E-13
4	-7.9699E-11	-2.0095E-11	-8.8310E-13	1.0068E-10

TABLE G.6

Output data for the transmission line capacitance matrix with 5 harmonic on the conductor and 6 harmonics around the dielectric (units F/m)

row \ column	1	2	3
1	5.9701E-11	-2.0120E-11	-2.0120E-11
2	-2.0095E-11	1.0068E-10	-8.8310E-13
3	-2.0095E-11	-8.8310E-13	1.0068E-10

CPU time 01:12:26.54 elapsed time 02:17:40.03

TABLE G.7

Output data for the generalized capacitance matrix with 7 harmonic on the conductor and 8 harmonics around the dielectric (units F/m)

column / row	1	2	3	4
1	1.4163E-10	-1.9518E-11	-8.2061E-11	-8.2061E-11
2	-1.9499E-11	5.9700E-11	-2.0101E-11	-2.0101E-11
3	-8.2106E-11	-2.0091E-11	1.0269E-10	-4.8951E-13
4	-8.2106E-11	-2.0091E-11	-4.8951E-13	1.0269E-10

TABLE G.8

Output data for the transmission line capacitance matrix with 7 harmonic on the conductor and 8 harmonics around the dielectric (units (F/m)

column / row	1	2	3
1	5.9700E-11	-2.0101E-11	-2.0101E-11
2	-2.0091E-11	1.0269E-10	-4.8951E-13
3	-2.0091E-11	-4.8951E-13	1.0269E-10

CPU time 02:45:09.82 elapsed time 05:20:15.67

TABLE G.9

Output data for the generalized capacitance matrix with 9 harmonic on the conductor and 10 harmonics around the dielectric (units F/m)

column / row	1	2	3	4
1	1.4322E-10	-1.9529E-11	-8.2847E-11	-8.2847E-11
2	-1.9524E-11	5.9700E-11	-2.0088E-11	-2.0088E-11
3	-8.2864E-11	-2.0085E-11	1.0331E-10	-3.6071E-13
4	-8.2864E-11	-2.0085E-11	-3.6071E-13	1.0331E-10

TABLE G.10

Output data for the transmission line capacitance matrix with 9 harmonic on the conductor and 10 harmonics around the dielectric (units (F/m)

column / row	1	2	3
1	5.9700E-11	-2.0088E-11	-2.0088E-11
2	-2.0085E-11	1.0331E-10	-3.6071E-13
3	-2.0085E-11	-3.6071E-13	1.0331E-10

CPU time 05:05:03.75 elapsed time 07:39:28.47

Tables (H.1-H.10) are output data files for a multiconductor coax cable when $\epsilon_r = 1.0$

TABLE H.1

Output data for the generalized capacitance matrix with 1 harmonic or 3 Fourier terms (units F/m)

row \ column	1	2	3	4
1	3.9647E-11	-1.8719E-11	-3.1466E-11	-3.1466E-11
2	-1.8431E-11	3.4330E-11	-7.9497E-12	-7.9497E-12
3	-3.1152E-11	-7.8709E-12	4.1472E-11	-2.4498E-12
4	-3.1152E-11	-7.8709E-12	-2.4498E-12	4.1472E-11

TABLE H.2

Output data for the transmission line capacitance matrix with 1 harmonic or 3 Fourier terms (units F/m)

row \ column	1	2	3
1	3.4330E-11	-7.9497E-12	-7.9497E-12
2	-7.8709E-12	4.1472E-11	-2.4498E-12
3	-7.8709E-12	-2.4498E-12	4.1472E-11

CPU time 00:00:28.70 elapsed time 00:00:39.86

TABLE H.3

Output data for the generalized capacitance matrix with 3 harmonics or 7 Fourier terms (units F/m)

row \ column	1	2	3	4
1	3.3318E-11	-1.7085E-11	-3.9119E-11	-3.9119E-11
2	-1.7109E-11	3.4685E-11	-8.7880E-12	-8.7880E-12
3	-3.9107E-11	-8.8029E-12	4.8614E-11	-7.0421E-13
4	-3.9107E-11	-8.8029E-12	-7.0421E-13	4.8614E-11

TABLE H.4

Output data for the transmission line capacitance matrix with 3 harmonic or 7 Fourier terms (units F/m)

row \ column	1	2	3
1	3.4685E-11	-8.7880E-12	-8.7880E-12
2	-8.8029E-12	4.8614E-11	-7.0421E-12
3	-8.8029E-12	-7.0421E-13	4.8614E-11

CPU time 00:02:32.59 elapsed time 00:03:35.64

TABLE H.5

Output data for the generalized capacitance matrix with 5 harmonics or 11 Fourier terms (units F/m)

row \ column	1	2	3	4
1	5.5780E-11	-1.6918E-11	-4.0434E-11	-4.0434E-11
2	-1.6919E-11	3.4700E-11	-8.8903E-12	-8.8903E-12
3	-4.0435E-11	-8.8912E-12	4.9677E-11	-3.5127E-13
4	-4.0435E-11	-8.8912E-12	-3.5127E-13	4.9677E-11

TABLE H.6

Output data for the transmission line capacitance matrix with 5 harmonic or 11 Fourier terms (units F/m)

row \ column	1	2	3
1	3.4700E-11	-8.8903E-12	-8.8903E-12
2	-8.8912E-12	4.9677E-11	-3.5127E-13
3	-8.8912E-12	-3.5127E-13	4.9677E-11

CPU time 00:08:04.68 elapsed time 00:11:52.35

TABLE H.7

Output data for the generalized capacitance matrix with 7 harmonics or 15 Fourier terms (units F/m)

column / row	1	2	3	4
1	5.6256E-11	-1.6901E-11	-4.0680E-11	-4.0680E-11
2	-1.6901E-11	3.4700E-11	-8.8996E-12	-8.8996E-12
3	-4.0681E-11	-8.8996E-12	4.9863E-11	-2.8295E-13
4	-4.0681E-11	-8.8996E-12	-2.8295E-13	4.9863E-11

TABLE H.8

Output data for the transmission line capacitance matrix with 7 harmonic or 15 Fourier terms (units F/m)

column / row	1	2	3
1	3.4700E-11	-8.8996E-12	-8.8996E-12
2	-8.8996E-12	4.9863E-11	-2.8295E-13
3	-8.8996E-12	-2.8295E-13	4.9863E-11

CPU time 00:18:51.82 elapsed time 00:52:19.69

TABLE H.9

Output data for the generalized capacitance matrix with 9 harmonics or 19 Fourier terms (units F/m)

column / row	1	2	3	4
1	5.6359E-11	-1.6900E-11	-4.0732E-11	-4.0732E-11
2	-1.6900E-11	3.4700E-11	-8.9001E-12	-8.9001E-12
3	-4.0732E-11	-8.9000E-12	4.9901E-11	-2.6889E-13
4	-4.0732E-11	-8.9000E-12	-2.6889E-13	4.9901E-11

TABLE H.9

Output data for the transmission line capacitance matrix with 9 harmonic or 19 Fourier terms (units F/m)

column / row	1	2	3
1	3.4700E-11	-8.9001E-12	-8.9001E-12
2	-8.9000E-12	4.9901E-11	-2.6889E-13
3	-8.9000E-12	-2.6889E-13	4.9901E-11

CPU time 00:37:24.70 elapsed time 00:54:05.02

APPENDIX I

Tables (I.1-I.5) are output data files for a dielectric wire bundle over a ground plane.

TABLE I.1

Output data for the transmission line capacitance matrix with 1 haromonic around the conductor and 1 haromonic around the dielectric for a 3-wire diectric coated configuration over a ground plane

row \ column	1	2	3
1	3.6935E-11	-1.1119E-11	-2.5560E-12
2	-1.0808E-11	3.0903E-11	-1.0808E-11
3	-2.5560E-12	-1.1119E-11	3.6935E-11

CPU time 00:01:56.48 elapsed time 00:02:36.43

TABLE I.2

Output data for the transmission line capacitance matrix with 3 haromonics around the conductor and 3 haromonics around the dielectric for a 3-wire diectric coated configuration over a ground plane

row \ column	1	2	3
1	3.7027E-11	-1.0970E-11	-2.4675E-12
2	-1.0970E-11	3.1089E-11	-1.0970E-11
3	-2.4675E-12	-1.0970E-11	3.7027E-11

CPU time 00:12:11.77 elapsed time 00:15:47.50

TABLE I.3

Output data for the transmission line capacitance matrix with 5 haromonics around the conductor and 5 haromonics around the dielectric for a 3-wire diectric coated configuration over a ground plane

column / row	1	2	3
1	3.7029E-11	-1.0969E-11	-2.4685E-12
2	-1.0969E-11	3.1087E-11	-1.0969E-11
3	-2.4685E-12	-1.0969E-11	3.7029E-11

CPU time 00:36:21.29 elapsed time 00:49:50.18

APPENDIX J

The following is a listing of the individual subprograms which make up the program for the capacitance model. A program flow diagram is provided in figure J.1 to show which subroutines are called by the calling program or subroutines.

Main - The main program is used in controlling the input from the terminal and the output to data files.

Winfo - Subroutine which allows the user to input wire data while minimizing the input requirements from the user.

Samstd - Subroutine which allows the user to input stranded wire information while minimizing the input requirements and then calculates and equivalent conductor radius when the strands have the same radius.

Difstd - Subroutine which allows the user to input stranded wire information while minimizing the input requirements and then calculates and equivalent conductor radius when the wires have a different strand radii.

Samrad - Subroutine which allows the user to input wire radius information while minimizing the input requirements when the conductors have the same radius.

Difrad - Subroutine which allows the user to input conductor information while minimizing the input requirements when the conductors have different radii.

Dinfo - Subroutine which allows the user to input various dielectric radii.

Rpinfo - Subroutine which allows the user to input various permittivities for the dielectric coating around conductors.

Size - Subroutine which multiplies all pertinent variables by a constant to minimize computational errors when inverting the D matrix.

Rgenxy - Subroutine which allows the user to input x,y coordinate information based on the ground reference wire and then calculates all remaining pertinent reference data for ribbon cables.

Bgenxy - Subroutine which allows the user to input x,y coordinate information based on the ground reference wire and then calculates all remaining pertinent reference data for a wire bundle

Cgenxy - Subroutine which allows the user to input x,y coordinate information based on the ground reference wire in this case the shield and then calculates all remaining pertinent reference data for a coax cable or shielded wire bundle

Pgenxy - Subroutine which allows the user to input x,y coordinate information based on the ground reference plane and then calculates all remaining pertinent reference data for a wire bundle over a ground plane

Newref - Subroutine which allows user to change ground reference wire and then calculates all x,y coordinate information based on the new reference wire (not used when a ground plane is present)

Cap - Subroutine which calculates the generalized capacitance matrix

Dia - Subroutine which calculates the diagonal terms of the D submatrix

Ofdia - Subroutine which caluclates the off-diagonal terms of the D submatrix

Place - Subroutine which places the calculated values of the various D submatrices into the large D1 matrix

Sum - Subroutine which calculates the generalized capacitance matrix by summing certain rows of the large D1 matrix

Trans - Subroutine which calculates the transmission line capacitance matrix from the generalized capacitance matrix.

Plane - Subroutine which places the values of the generalized capacitance matrix into the transmission line capacitance matrix when a ground plane is present

P1 - Subroutine which used to reduce the order of the D1 matrix from 2N(2k+1) to N(k+1)

Minv - Subroutine which is a canned IBM program for inverting a matrix using Gauss illimation with max pivoting. The program has been modified to run in quad precision

Mprt - Subroutine which places all output data in matrix form

A list of variable names which are passed from and to the various subroutines is shown in tables (J1-J23). The tables consist of variable names and descriptions of variable names, and their type, i.e., integer, character, or real. The tables also tell if the variable is an input and/or output variable from or to other subroutines, screen, or file.

The various abbreviations used in the tables are listed below

1. TO - Terminal output to screen
2. TI - Terminal input from screen
3. PO - Program output to screen
4. PI - Program input from screen
5. PV - Program variable that is not passed to other subroutines
6. PC - Program counter which is used only in that program or subroutine
7. PI/O- Program input variable which is from another subroutine and then passed to another
9. FO- Program output to a file

FIG J.1
PROGRAM FLOW DIAGRAM

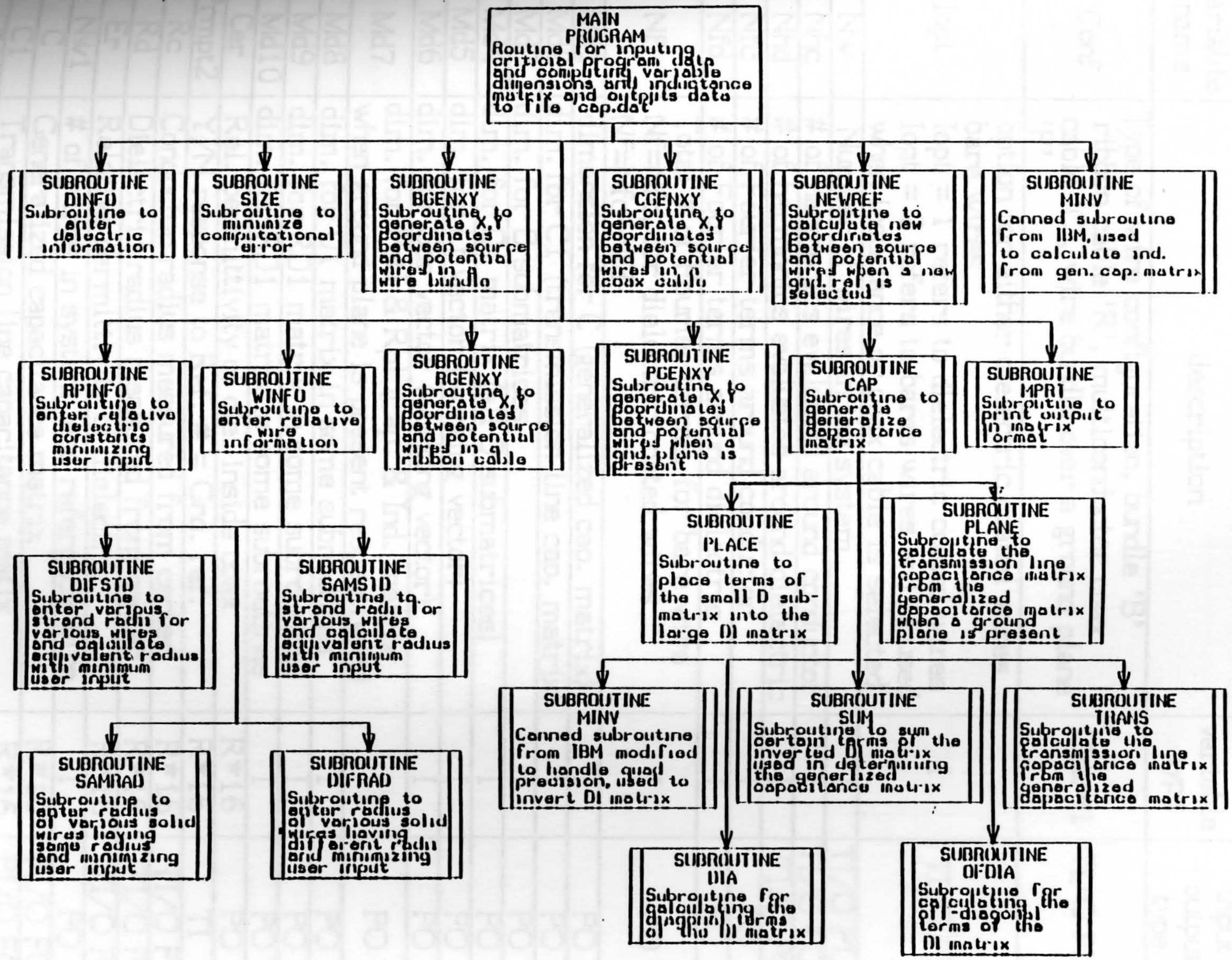


TABLE J.1
Variables used in Subroutine Main

variable name	description	variable type	input output type
Conf	Type of wire configuration, bundle 'B' ribbon cable 'R', multiconductor coax cable 'C', wire bundle over a ground plane 'P'	Char*1	TI/O PI/O
Iopt	option for either dielectric coated wires bare wires Iopt = 1 refers to dielectric coated wires Iopt = 2 refers to bares wires and is used when inductance or coax cable is selected	I	TI/O PI/O
Nw	Number of wires in the system	I	TI/O PI/O
Nhc	# of harmonics evaluated around conductor	I	TI/O PO
Nhd	# of harmonics evaluated around dielectric	I	TI/O PO
Nfc	# of Fourier terms around conductor	I	PO
Nfd	# of Fourier terms around dielectric	I	PO
Nf	Total # of Fourier terms, for bare wire Nf=Nfc, for dielectric coated wires Nf=Nfc+Nfd	I	PO
Md1	dimension for C (generalized cap. matrix)	I	PO
Md2	dim. for C1 (transmission line cap. matrix)	I	PO
Md3	dim. for D submatrices	I	PO
Md4	dim. for D1 matrix (all D submatrices)	I	PO
Md5	dim. for Lt vector (working vector)	I	PO
Md6	dim. for Scr vector (working vector)	I	PO
Md7	dim. for Pctl & Rpl (Cap. & Ind. matrix when ground plane is present respectively)	I	PO
Md8	dim. for D1 matrix in some subroutines	I	PO
Md9	dim. for RD1 matrix in some subroutines	I	PO
Md10	dim. for RD1 matrix in some subroutines	I	PO
Cer	Rel. permittivity of die. inside coax	R*16	PO
Prmpt2	Y/N response to Ref. # = Gnd. ref.	R*16	TI
Rc	Conductor radius measured from center	R*16	TI/O PO
Rd	Dielectric radius measured from center	R*16	TI/O PO
Er	Relative permittivity of dielectric	R*16	TI/O PO
Nw1	# of wires in system less reference wire	I	PO
C	Generalized capacitance matrix	R*16	PI/O FO
C1	Transmission line capacitance matix	R*16	PI/O FO
Rl	Transmission line inductance matrix	R*16	PI/O FO

TABLE J.1
Variables used in program Main (continued)

variable name	description	variable type	input output type
NWH	wires above gnd. when gnd. plane present	I	SV
PCTL	Transmission line Cap. matrix when gnd. plane is present	R*16	PI/O FO
D	submatrices making up the potential matrix	R*16	PI/O FO
D1	large matrix comprised of D submatrices	R*16	TO PI/O
SCR	working vector	R*16	TI/O PO
SAC1	saved transmission line cap. matrix	R*16	PO FO
LT	working vector	R*16	PO FO
RD1	reduced D1 matrix when ground present	R*16	PO FO
PCG	generalized capacitance matrix gnd present	R*16	PO FO
SAPCTL	saved transmission line matrix gnd present	R*16	PO FO
PD1	D1 matrix when gnd plane is present	R*16	PO FO
NS	number of strands in a wire	I	PI/O
NX	number of remaining solid wires in sys.	I	PI/O
RCL	Inductance matrix for coax cable	R*16	PO FO
SMRC	smallest conductor radius in system	R*16	PO FO
WRC	conductor radius before sizing	R*16	PO FO
AA1	sizing constant to reduce inversion error	R*16	PO FO
X	relative horizontal distance between wires	R*16	PO FO
Y	relative vertical distance between wires	R*16	PO FO
SEP	wire-to-wire sep. measured from center	R*16	PO FO
IREF	reference wire number	R*16	PO FO
H	vertical height from ground plane	R*16	PO FO
ON	integer program counter	I	PC
KO	integer program counter	I	PC
KU	integer program counter	I	PC
KP	integer program counter	I	PC
MN	integer program counter	I	PC
KR	integer program counter	I	PC
KT	integer program counter	I	PC
KS	integer program counter	I	PC
KV	integer program counter	I	PC

TABLE J.2
Variables used in Subroutine Winfo

variable name	description	variable type	input output type
Prmpt1	Y/N response to all wires solid	char*1	TI
Prmpt2	Y/N response to all wires stranded	char*1	TI
Ns	# of wires that are strand	char*1	TI PO
Prmpt3	Y/N response to all wires have same rad.	char*1	TI
Nx	Remaining solid wires in system	I	PO
Nw	Total no. of wires in the system	I	PI
Smrc	Smallest conductor radius in system	R*16	PI/O
Rc	Conductor radius	R*16	PI/O
Conf	Type of configuration selected	char*1	PI/O
WRC	conductor radius before sizing	R*16	PI/O
Prmpt4	Y/N response to remaining wires have the same conductor radius	char*1	TI
Prmpt5	Y/N response to all strands have the same conductor radius	char*1	TI
Prmpt6	Y/N response to all wires have the same conductor radius	char*1	TI

TABLE J.3
Variables used in Subroutine Dinfo

variable name	description	variable type	input output type
Prmpt8	Y/N response to are all dielectric radii the same	char*1	TI
Xrd	Radius of the dielectric	R*16	TI
Nw	Total number of wires in the system	I	PI
Rd	Dielectric radius measured from center	R*16	TI/O PO
No	no. of particular wire in system	I	TI PV
Ino	Previous wire number	I	PV
Prmpt9	Y/N response to previous dielectric radius = to present dielectric radius	char*1	TI
WRD	dielectric radius before sizing	R*16	PO

TABLE J.4
Variables used in Subroutine Rpinfo

variable name	description	variable type	input output type
Prmpt10	Y/N response to is relative permittivity of the wires the same	char*1	TI
Xer	Relative permittivity of the dielectric	R*16	TI
Er	Relative permittivity of the dielectric	R*16	TI/O PO
Nw	No. of wires in the system	I	PI
No	no. of particular wire in the system	I	TI PV
Ino	Previous number of wire in system	I	PV
Prmpt11	Y/N response to previous relative permittivity = to present value	char*1	TI

TABLE J.5
Variables used in Subroutine Size

variable name	description	variable type	input output type
Smrc	Smallest strand or conductor radius	R*16	PI
Nj	Power of sizing factor to min. error	I	PV
AA1	Sizing factor to minimize error	R*16	PO
Iref	# of reference wire set to 1	I	PO
Nw	Total no. of wires in the system	I	PI
Rc	Conductor radius	R*16	PI/O
Rd	Dielectric radius measured form center	R*16	PI/O
Iopt	Option as to either dielectric coated wires or bare wires selected for system	I	PI

TABLE J.6
Variables used in Subroutine Difstd

variable name	description	variable type	input output type
Nw	Total no. of wires in the system	I	PI
Ns	No. of wires that are stranded in system	I	PI
Smrc	Smallest stand or conductor radius	R*16	PO
Rc	Conductor radius	R*16	PO
Conf	Configuration selected	char*1	PI
N	Dummy variable for no. of stranded wires	I	PV
Rcx	Inside radius of shield of coax cable	R*16	TI PO
No	Current wire no.	I	PV
Ino	Previous wire no.	I	PV
Rstd	Radius of one strand of wire	I	TI
Nstd	No. of strands comprising the wire	I	TI
Prmpt7	Y/N response to previous wire radius equal present wire radius	char*1	TI
WRC	conductor radius before sizing	R*16	Po
XRC	equivalent conductor radius	R*16	PV

TABLE J.7
Variables used in Subroutine Samstd

variable name	description	variable type	input output type
Nw	Total no. of wires in the system	I	PI
Ns	No. of wires that are stranded in system	I	PI
Smrc	Smallest stand or conductor radius	R*16	PI
Rc	Conductor radius	R*16	PO
Conf	Configuration selected	char*1	PI
K	Dummy wire counter when coax or shielded wire bundle configuration is selected	I	PV
Rcx	Inside radius of shield of coax cable	R*16	TI PO
Rstd	Radius of one strand of wire	R*16	TI
Nstd	No. of strands comprising the wire	I	TI
Xrc	Equivalent conductor radius	R*16	PV
N	dummy variable for no. of stranded wires	I	PV
WRC	conductor radius before sizing	R*16	PV

TABLE J.8
Variables used in Subroutine Difrad

variable name	description	variable type	input output type
CONF	configuration selected	char*1	PI
NX	number of wires that are solid	I	PI
NW	number of wires in the system	I	PI
RCX	inside radius of coax shield	R*16	TI/O
N	dummy counter	I	PV
K	dummy counter	I	PV
RC	radius of conductor	R*16	TI/O PO
WRC	radius of conductor before sizing	R*16	PO
SMRC	smallest conductor radius in system	R*16	PO
NO	present wire number	I	TI
INO	previous wire number	I	PV

TABLE J.9
Variables used in Subroutine Samrad

variable name	description	variable type	input output type
CONF	configuration selected	char*1	PI
NX	number of wires that are solid	I	PI
NW	number of wires in the system	I	PI
RCX	inside radius of coax shield	R*16	TI/O
N	dummy counter	I	PV
K	dummy counter	I	PV
RC	radius of conductor	R*16	TI/O PO
WRC	radius of conductor before sizing	R*16	PO
SMRC	smallest conductor radius in system	R*16	PO

TABLE J.10
Variables used in Subroutine RGENXY

variable name	description	variable type	input output type
SEP	wire-to-wire separation	R*16	TI/O
NW	number of wires in the system	I	PI
X	horizontal cntr-to-cntr wire separation	R*16	PO
Y	vertical cntr-to-cntr wire separation	R*16	PO
AA1	sizing constant	R*16	PI

TABLE J.11
Variables used in Subroutine BGENXY

variable name	description	variable type	input output type
NW	number of wires in the system	I	PI
X	horizontal cntr-to-cntr wire separation	R*16	TO PO
Y	vertical cntr-to-cntr wire separation	R*16	TO PO
XVALUE	horizontal cntr-to-cntr wire separation	R*16	TI
YVALUE	vertical cntr-to-cntr wire separation	R*16	TI
IREF	number given to reference wire	I	PI
AA1	sizing constant	R*16	PI

TABLE J.12
Variables used in Subroutine CGENXY

variable name	description	variable type	input output type
NW	number of wires in the system	I	PI
X	horizontal cntr-to-cntr wire separation	R*16	TO PO
Y	vertical cntr-to-cntr wire separation	R*16	TO PO
IREF	number given to reference wire	I	PI
XVALUE	horizontal cntr-to-cntr wire separation	R*16	TI
YVALUE	vertical cntr-to-cntr wire separation	R*16	TI
AA1	sizing constant	R*16	PI

TABLE J.13
Variables used in Subroutine PGENXY

variable name	description	variable type	input output type
NWH	number of wires in the system above gnd	I	PV
NW	number of wires in the system	I	PI
H	vert. height of wire measured from cntr	R*16	TI
X	horizontal cntr-to-cntr wire separation	R*16	TO PO
Y	vertical cntr-to-cntr wire separation	R*16	TO PO
XVALUE	horizontal cntr-to-cntr wire separation	R*16	TI
IREF	number given to reference wire	I	PI

TABLE J.14
Variables used in Subroutine NEWREF

variable name	description	variable type	input output type
IGREF	ground reference wire number	R*16	TI
NW	number of wires in the system	I	PI
X	horizontal cntr-to-cntr wire separation	R*16	PO
Y	vertical cntr-to-cntr wire separation	R*16	PO
IREF	number given to reference wire	I	PO

TABLE J.15
Variables used in Subroutine CAP

variable name	description	variable type	input output type
PI	variable assigned to value of pi	R*16	PV
NW	number of wires in the system	I	PI
NW1	no. of wires in system less ref. wire	I	PO
NW12	NW! squared	I	PO
NFC	number Fourier terms around conductor	I	P1
NFC1	no. of Fourier trms around cond.+avg.term	I	PI/O
NFD	number of Fourier terms around dielectric	I	PI
NFD1	no. of Fourier trms around die. + avg term	I	PI/O
LD	total trms around cond & die.*no. of wires	I	PI/O
NF	total Fourier terms around cond & die	I	PI/O
AC	angle of matchpoint on conductor surface	R*16	PI/O
AD	angle of matchpoint on dielectric surface	R*16	PI/O
DELTC	offset angle of matchpoint on conductor	R*16	PI/O
DELTD	offset angle of matchpoint on dielectric	R*16	PI/O
NSW	present source wire	I	PI/O
NPW	present potential wire	I	PI/O
XSEP	hoizontal wire separation	R*16	PO
YSEP	vertical wire separation	R*16	PO
X	horizontal wire separation	R*16	PI
Y	vertical wire separation	R*16	PI
ER1	permittivity of dielectric - 1	R*16	PI/O
ER2	permittivity of dielectric + 1	R*16	PI/O
ER	permittivity of dielectric	R*16	PI/O

TABLE J.15 (cont.)
Variables used in Subroutine CAP

variable name	description	variable type	input output type
CONF	configuration selected	char*1	PI/O
RC	conductor radius	R*16	PI/O
IREF	number of reference wire	I	PI/O
NHC	number of harmonics around conductor	I	PI/O
IOPT	option either bare or dielectric coated	I	PI/O
NHD	number of harmonics around dielectric	I	PI/O
RD	dielectric radius measured from center	R*16	PI/O
D	potential submarix	R*16	PI/O
MD1	dim. of generalized cap. matrix	I	PI/O
MD2	dim. of transmission line cap. matrix	I	PI/O
MD3	dim. of D submarices	I	PI/O
MD4	dim. of large D1 matrix(potential matrix)	I	PI/O
MD5	dim. of Lt vector (working vector)	I	PI/O
MD6	dim. of SCR vector (working vector)	I	PI/O
MD7	dim. of transmission line cap matrix plane	I	PI/O
MD8	dim. of D1 matrix in some subroutines	I	PI/O
MD9	dim. of reduced RD1 matrix	I	PI/O
MD10	dim. of reduced RD1 matrix	I	PI/O
MM	dummy variable (same as NPW)	I	PI/O
MN	dummy variable (same as NSW)	I	PI/O
D1	large potential matrix(contains D's)	R*16	PI/O
NWH	number of wires above the ground plane	I	PO
MLD	order of RD1 matrix for sub. MINV	I	PO
RD1	reduced D! matrix when gnd. plane present	R*16	PI/O
LT	working vector	R*16	PO
ISTP	number given to step where MINV fails	I	PI/O
CG	generalized cap. matrix	R*16	PI/O
PCG	generalized cap. matrix when gnd. present	R*16	PI/O
CER	permittivity of dielectric inside coax	R*16	PI/O
PCTL	transmission line cap. matrix gnd. present	R*16	PI/O

TABLE J.16

Variables used in Subroutine OFDIA

variable name	description	variable type	input output type
BETA	angle matchpts makes with cntr.+offset	R*16	PV
MPP	program counter max=NFC	I	PC
AC	angle matchpoint makes with wire cntr	R*16	PI
DELTC	offset angle matchpoints make with cntr	R*16	PI
CANG	cosine of BETA	R*16	PV
SANG	sine of BETA	R*16	PV
Q1	total horiz. dis between potential wire matchpoint and cntr of source wire	R*16	PV
XSEP	cntr.-to-cntr. distance between wires	R*16	PI
RC	conductor radius measured from center	R*16	PI
NPW	no. of present potential wire	I	PI
Q12	Q1 squared	R*16	PV
Q2	total vert. dis. between potential wire matchpoint and cntr of source wire	R*16	PV
Q22	Q2 squared	R*16	PV
RO	dis. to matchpt. from cntr. of source wire	R*16	PV
THETA	angle matchpt,source wire and horizontal	R*16	PV
B1	dummy varialble	R*16	PV
CONF	variable used for type of configuration	char*1	PI
NSW	present source wire	I	PI
IREF	no. of wire that is selected as reference	I	PI
RC	conductor radius measured from center	R*16	PI
IOPT	option either bare or dielectric coated	I	PI
NFC1	no. of Fourier terms less avg. term	I	PI
RD	dielectric radius measured from center	R*16	PI
D	small potential submatrix	R*16	PO
A1	dummy variable	R*16	PV
A3	dummy varialble	R*16	PV
NHC	no. of harmonics around conductor	I	PI
J1	program counter (matrix index)	I	PC
J2	program counter (matrix index)	I	PC
B2	dummy variable	R*16	PC
B3	dummy variable	R*16	PV
J	program counter	I	PV

TABLE J.16 (cont)
Variables used in Subroutine OFDIA

variable name	description	variable type	input output type
NHD	no. of harmonics around dielectric	I	PI
AD	angle of matchpt, cntr, and horizontal	R*16	PI
DELTD	of set of matchpt. cntr. and horizontal	R*16	PI
GMA	angle E normal makes with r direction	R*16	PV
RHATN	unit normal vector in r directon	R*16	PV
THETN	unit normal vector in theta direction	R*16	PV
JJ	program counter (matrix index)	I	PC
ER1	permittivity of dielectric - 1	R*16	PI
J4	program counter (matrix index)	I	PC
B4	dummy variable	I	PV

TABLE J.17
Variables used in Subroutine DIA

variable name	description	variable type	input output type
CONF	variable used for type of configuration	char*1	PI
NPW	present potential wire	I	PI
B1	dummy variable	R*16	PV
RC	conductor radius	R*16	PI
NSW	present source wire	I	PI
J	program counter	I	PC
NFC	no. of Fourier terms around dielectric	I	PI
D	small potential submatrix	R*16	PO
IOPT	option either bare or dielectric coating	I	PI
BB1	dummy variable	R*16	PV
RD	dielectric radius measured from center	R*16	PI
NFC1	no. of Fourier tmrs + avg. term	I	PI
BBB1	dummy variable	R*16	PV
ER1	permittiviyt of dielectric + 1	R*16	PI
BBB2	dummy variable	R*16	PV

TABLE J.17 (cont)
Variables used in Subroutine DIA

variable name	description	variable type	input output type
J1	program counter (matrix index)	I	PC
J2	program counter (matrix index)	I	PC
A1	dummy variable	R*16	PV
NHD	no. of harmonics around dielectric	I	PI
J3	program counter (matrix index)	I	PC
J4	program counter (matrix index)	I	PC
A12	A1 squared	R*16	PC
AD	angle of matchpt. cntr. & horizontal	R*16	PI
DELTD	offset angle of matchpt,cntr,& horizontal	R*16	PI
B2	dummy variable	R*16	PV
B3	dummy variable	R*16	PV
ER2	permittivity of dielectric +1	R*16	PI
B4	dummy variable	R*16	PV

TABLE J.18
Variables used in Subroutine PLACE

variable name	description	variable type	input output type
NP	program counter (matrix index)	I	PC
NF	total number of Fourier terms	I	PI
MM	no. of present potential wire	I	PI
NN	no. of present source wire	I	PI
LD	prod of NF * no. of wires	I	PI
D1	large potential matrix comprised of D's	R*16	PO
J	program counter (matrix index)	I	PC

TABLE J.19
Variables used in Subroutine SUM

variable name	description	variable type	input output type
PI	variable assigned to pi	R*16	PV
EPS	variable assinged to value of permittivity	R*16	PV
A3	dummy variable	R*16	PV
NWH	no. of wires above the gnd. plane	I	PV
CONF	variable assigned to type of configuration	char*1	PI
MNW	dummy var. for wires (depends on conf)	I	PV
NW	number of wires in the system	I	PI
IROW	dummy variable assigned to row	I	PC
NF	no. of Fourier terms	I	PI
IL	program counter (matrix index)	I	PV
J	program counter (matrix index)	I	PV
LD	no. of Fourier terms * no. of wires	I	PI
A1	dummy variable	R*16	PV
A2	dummy variable	R*16	PV
K	program counter (matrix index)	I	PC
NFC	no. of Fourier terms around the conductor	I	PI
RD1	reduced D1 matrix (gnd. present)	R*16	PI
D1	potential matrix	R*16	PI
IOPT	variable assigned to type of configuration	char*1	PI
PCG	general capacitance matrix (gnd. present)	R*16	PO
CER	permittivity of dielectric in coax cable	R*16	PI
RC	conductor radius	R*16	PI
RD	dielectric radius	R*16	PI
MSUM	used to add certain terms in matrix	R*16	PV
I	program counter(matrix index)	I	PC
CG	generalized capacitance matrix	R*16	PO
II	program counter (matrix index)	I	PC
IREF	variable assigned to reference wire	I	PI
A3	dummy variable	R*16	PV
PD1	potential matrix when gnd. plane is presnt	R*16	PV
J	program counter (matrix index)	I	PC
NW12	NW1 squared	I	PI
NW1	no. of wires less reference wire	I	PI
CTL	transmission line capacitance matrix	R*16	PO
JJ	program counter (matrix index)	I	PC

TABLE J.20
Variables used in Subroutine P1

variable name	description	variable type	input output type
NWH	no. of wires above ground plane	I	PV
NW	no. of wires (real and image)	I	PI
NP	program counter (matrix index)	I	PC
NF	total no. of Fourier terms	I	PI
NK	program counter (matrix index)	I	PC
NN	program counter (matrix index)	I	PC
N	program counter (matrix index)	I	PC
MP	program counter (matrix index)	I	PC
I	program counter	I	PC
J	program counter	I	PC
RD1	reduced D1 matrix when gnd plane present	R*16	PO
D1	potential matrix comprixed of D's	R*16	PI

TABLE J.21
Variables used in Subroutine PLANE

variable name	description	variable type	input output type
K	program counter (matrix index)	I	PC
I	program counter (matrix index)	I	PC
J	program counter (matrix index)	I	PC
NWH	no. of wires above the gnd. plane	I	PI
PCTL	transmission line cap matrix gnd. present	R*16	PO
PCG	generalized cap. matrix gnd. present	R*16	PI

TABLE J.22
Variables used in Subroutine MINV

variable name	description	variable type	input output type
D	resultant of determinant	R*16	PO
NK	negative of the order of input matrix	I	PC
N	order of input matrix	I	PI
K	program counter	I	PC
L	vector storing indices	I	PI
M	vector storing indices	I	PI
A	input matrix which inverted then outputed	R*16	PI/O
BIGA	largest value in matrix (pivot)	R*16	PV
IZ	program counter (matrix index)	I	PC
IJ	program counter (matrix index)	I	PC
J	program counter	I	PC
KI	program counter (maltrix index)	I	PC
HOLD	temporarily hold term of input matrix	R*16	PV
JI	program counter (matrix index)	I	PC
I	program counter (matrix index)	I	PC
JK	program counter (matrix index)	I	PC

TABLE J.23
Variables used in Subroutine MPRT

variable name	description	variable type	input output type
A	input matrix to be printed	R*16	PI FO
M	number of rows in A	I	PI
N	number of columns in A	I	PI
B	literal input of title (hollerith form)	CHAR	PV
J	no. of characters in B	I	PC
I	program counter	I	PC
LL	program counter (matrix index)	I	PC
LU	max values of program counter	I	PC
K	program variable (matrix counter)	I	PC

APPENDIX K

The following is a listing of the FORTRAN capacitance model.

```
C               CONFIGURATIONS CONSIDERED IN THIS PROGRAM ARE RIBBON
C               CABLES, WIRE BUNDLES, RIBBON AND WIRE BUNDLES OVER A
C               GROUND PLANE, MULTICONDUCTOR COAX CABLES, AND SHIELD
C               WIRE BUNDLES
C
C               C(NW**2) SIZE OF THE GENERALIZED CAPACITANCE VECTOR
C               FOR ALL, CONFIGURATIONS OTHER THAN WHEN A GROUND PLANE
C               IS PRESENT
C
C               C1((NW-1)**2) SIZE OF TRANSMISSION LINE CAPACITANCE
C               VECTOR FOR EVERY CONFIGURATION OTHER THAN WHEN A
C               GROUND PLANES
C
C               NHC EQUALS THE NUMBER OF HARMONICS AROUND THE
C               CONDUCTOR SURFACE
C               NHD EQUALS THE NUMBER OF HARMONICS AROUND THE
C               DIELECTRIC SURFACE
C               NFC EQUALS THE TOTAL NUMBER OF FOURIER TERMS AROUND
C               THE CONDUCTOR SURFACE
C               NFD EQUALS THE TOTAL NUMBER OF FOURIER TERMS AROUND
C               THE DIELECTRIC SURFACE
C
C                       NOTE: NFC MUST BE LESS THAN OR EQUAL TO NFD
C
C               NF EQUALS THE TOTAL NUMBER OF FOURIER TERMS AROUND
C               BOTH THE CONDUCTOR AND DIELECTRIC, I.E., NF=NFC+NFD
C               IS DIELECTRIC IS PRESENT OTHERWISE NF+NFC
C
C               D(NF,NF)  SIZE OF SUBMATRIX OF THE LARGER D1 MATRIX
C
C               D1(NW*NF,NW*NF) LARGE MATRIX DIMENSION IN MAIN PROGRAM
C               BUT IS DIMENSIONED AS D1((NW*NF)**2) IN SUBROUTINE CAP
C
C               LT(2*NF*NW) SIZE OF WORKING VECTOR
C               SCR((NF*NW+1)/2) SIZE OF SCRATCH VECTOR
C               SCR AND LT ARE SCRATCH VECTORS OF DIFFERENT TYPE BUT
C               CAN SHARE STORAGE LOCATIONS
C
C               X(NW,NW) CNTR-TO-CNTR SEPARATION IN X DIRECTION
C               Y(NW,NW) CNTR-TO-CNTR SEPARATION IN Y DIRECTION
C
C               RPL(NW**2) SIZE OF TRANSMISSION LINE INDUCTANCE MATRIX
C               WHEN A GROUND PLANE IS PRESENT WHERE NW IS THE NUMBER
C               OF WIRES ABOVE THE GROUND PLANE
C
C               PCTL(NW**2) SIZE OF THE TRANSMISSION LINE CAPACITANCE
C               MATRIX WHEN A GROUND PLANE IS PRESENT
C
C               RL((NW-1)**2) SIZE OF THE TRANSMISSION LINE
C               INDUCTANCE MATRIX FOR ALL CONFIGURATIONS OTHER THAN
C               WHEN A GROUND PLANE IS PRESENT.
```

```
C
C           NOTE: THE DIMENSIONS OF X,Y,RC,RD,ER,H, AND  WILL BE
C           DOUBLED WHEN WHEN A GROUND PLANE IS PRESENT.  FOR
C           EXAMPLE IF THERE ARE ARE FIVE WIRES ABOVE A GROUND
C           PLANE THEN THE DIMENSIONS WILL BE AS FOLLOWS:
C           X(10,10),Y(10,10),RC(10),RD(10),ER(10),H(10)
C
C           THE DIMENSIONS PROVIDED IN THIS PROGRAM FOR THE
C           VARIOUS MATRICES AND VECTORS ARE FOR UP TO 8 WIRES
C           WITH UP TO 10 HARMONICS ON THE CONDUCTOR AND 10
C           HARMONICS ON THE DIELECTRIC.  ANOTHER WAY OF SAYING
C           IT IS 8 WIRES WITH 42 FOURIER TERMS ON THE CONDUCTOR
C           AND 42 FOURIER TERMS ON THE DIELECTRIC. ANY NUMBER OF
C           WIRES OR FOURIER COEFFICIENTS GREATER THAN THOSE
C           PROVIDED REQUIRE A CHANGE IN THE DIMENSION STATEMENTS
C
C
      REAL*16 C(64),C1(49),D(82,82),D1(496,496)
      REAL*16 RL(49),RPL(64),X(8,8),Y(8,8),PD1(8)
      REAL*16 ER(8),H(8),PCTL(64),WRD(8)
      REAL*16 RC(8),RD(8),CER,DET,WRC(8)
      REAL*16 SAC1(49),Z(49),RLB(49),ZB(49),SAPCTL(64),ZP(64)
      REAL*16 RPLB(64),ZPB(64),RCL(49),ZC(49),ZCB(49),RCLB(49)
      REAL*16 RCB(49),RD1(88,88),PCG(8,8)
      DIMENSION LT(992),SCR(248)
      CHARACTER*1 CONF
      EQUIVALENCE (SCR(1),LT(1))
C
C     FOLLOWING ROUTINE INPUTS PARAMETERS TO THE PROGRAM FROM THE
C     SCREEN
C
C     THIS ROUTINE ENTERS CONFIGURATION OPTIONS
C
5     WRITE(6,10)
10    FORMAT(/,15X,'SELECT WIRING CONFIGURATION FROM
     + THE FOLLOWING LIST',
     +/,5X,'WIRE BUNDLE (B)',
     +/,5X,'MULTI-CONDUCTOR COAX CABLE (C)',
     +/,5X,'WIRE BUNDLE OR RIBBON CABLE OVER A GROUND PLANE (P).'
     +/,5X,'RIBBON CABLE (R)',
     +/,5X,'SHIELDED RIBBON CABLE OR WIRE BUNDLE (NOT YET AVAILABLE
     + (S)',
     +/,5X,'TWISTED PAIR (NOT YET AVAILABLE) (T)',
     +/,5X,'ENTER TYPE OF CONFIGURATION: (B),(C),(P),(R),(S),OR,(T)
     + CONF= ',$)
      READ(5,15)CONF
15    FORMAT(A1)
      WRITE(6,20)CONF
20    FORMAT(//,25X,'CONF= ',A1)
      IF(CONF.EQ.'B'.OR.CONF.EQ.'C'.OR.CONF.EQ.'P'.OR.
     &CONF.EQ.'R'.OR.CONF.EQ.'S'.OR.CONF.EQ.'T') THEN
      GO TO 30
      ELSE
      WRITE(6,25)
25    FORMAT(//,25X,'INCORRECT INPUT SELECT AGAIN')
      GO TO 5
      END IF
30    WRITE(6,35)
35    FORMAT(/,15X,'NOTE: SELECT FROM THE FOLLOWING OPTIONS',//
```

```
        +,10X,'IOPT=1 IMPLIES DIELECTRIC COATED WIRES',/,10X,'NOTE:
        + SELECT THIS OPTION IF',/17X,'(B),(P),(R),(S), OR (T) WAS
        +SELECTED',//
        +,10X,'IOPT=2 IMPLIES BARE WIRES',/,10X,'NOTE: SELECT THIS OPTION
        + IF (C) WAS SELECTED',/,17X,' OR INDUCTANCE MATRIX IS DESIRED',/)
              WRITE(6,40)
   40         FORMAT(5X,'ENTER OPTION (IOPT)=  ',$)
              READ(5,*)IOPT
              IF(IOPT.LT.1.OR.IOPT.GT.2) THEN
              WRITE(6,45)
   45         FORMAT(//,25X,'INCORRECT INPUT SELECT AGAIN')
              GO TO 30
              END IF
   50         IF(CONF.EQ.'C'.OR.CONF.EQ.'S') THEN
              WRITE(6,55)
   55         FORMAT(/,25X,'NOTE: THE SHIELD OF THE COAX CABLE OR WIRE BUNDLE
        &',/,25X,'IS ASSUMED TO BE A WIRE THAT IS NOT STRANDED')
              WRITE(6,60)
   60         FORMAT(//,5X,'ENTER # OF WIRES INCLUDING SHIELD NW= ',$)
              READ(5,*)NW
              ELSE
              WRITE(6,65)
   65         FORMAT(//,5X,'ENTER # OF WIRES (NW)=  ',$)
              READ(5,*)NW
              END IF
              IF(CONF.EQ.'P') NW=2*NW
              IF(CONF.NE.'P'.AND.NW.LE.1) THEN
               WRITE(6,70)
   70         FORMAT(/,' ***ERROR***  NUMBER OF WIRES LESS THAN TWO.
        +     ENTER AGAIN.',/)
              GO TO 50
              ELSE
              IF(NW.GT.10) THEN
              WRITE(6,75)
   75         FORMAT(5X,'NUMBER OF WIRES SELECTED OUT OF RANGE.',/,5X,
        &'PROGRAM DIMENSION STATEMENTS MUST BE MODIFIED.')
              GO TO 50
              END IF
              END IF
   80         WRITE(6,85)
   85         FORMAT(5X,'ENTER # OF COSINE OR SINE TERMS AROUND THE CONDUCTOR
        +',/,5X,'(I.E. THE # OF HARMONICS AROUND THE CONDUCTOR NHC) = ',$)
              READ(5,*)NHC
              IF(NHC.GT.20) THEN
              WRITE(6,90)
   90         FORMAT(//,15X,'INCORRECT INPUT, VALUE SELECTED EXCEEDS
        & PROGRAM DIMENSIONS.',/,
        &15X,'PROGRAM DIMENSION STATEMENTS MUST BE MODIFIED')
              GO TO 80
              END IF
              IF(IOPT.EQ.1) THEN
  100         WRITE(6,105)
  105         FORMAT(5X,'ENTER # OF COSINE OR SINE TERMS AROUND THE
        +DIELECTRIC',/,5X,'(I.E. THE # OF HARMONICS AROUND THE DIELECTRIC
        +NHD)= ',$)
              READ(5,*)NHD
              IF(CONF.EQ.'S') THEN
              IF(NHD.EQ.NHC) THEN
```

```
            WRITE(6,106)
      106   FORMAT(5X,'NOTE: IF NHC IS AN ODD NUMBER THEN NHD SHOULD',/,5X,
     &'BE EVEN AND ONE GREATER THAN NHC. REENTER NEW VALUES')
            GO TO 80
            END IF
            END IF
            IF(NHD.GT.20) THEN
            WRITE(6,110)
      110   FORMAT(//,15X,'INCORRECT INPUT, VALUE SELECTED EXCEEDS
     &DIMENSION.',/,15X,'PROGRAM DIMENSION STATEMENTS MUST BE MODIFIED')
            GO TO 100
            END IF
            END IF
      C
      C     NFC=NO. OF COSINE + SINE TERMS + AVG. TERM AROUND CONDUCTOR
      C     NFD=NO. OF COSINE + SINE TERMS + AVG. TERM AROUND DIELECTRIC
      C
            IF(CONF.EQ.'R') THEN
            NFC=NHC+1
            NFD=NHD+1
            ELSE
            NFC=2*NHC+1
            NFD=2*NHD+1
            END IF
            IF(IOPT.EQ.1) THEN
            NF=NFC+NFD
            ELSE IF(IOPT.EQ.2) THEN
            NF=NFC
            END IF
      C
      C           MD1=DIMENSION FOR C MATRIX
      C           MD2=DIMENSION FOR C1,RL,SAC1,Z,RLB,ZB,RCL,ZC,ZCB,RCLB,
      C               RCB MATRICES
      C           MD3=DIMENSION FOR D SUBMATRIX
      C           MD4=DIMENSION FOR D1 MATRIX  NOTE: THAT D1 IS ALSO
      C               DIMENSION AS (NW*NF)**2 WHICH IS MD8
      C           MD5=DIMENSION FOR LT VECTOR
      C           MD6=DIMENSION FOR SCR VECTOR
      C           MD7=DIMENSION FOR RPL,PCTL,CAPCTL,ZP,RPLB,ZPB MATRICES
      C           MD8=DIMENSION OF D1 MATRIX IN SOME SUBROUTINES
            MD1=NW**2
            MD2=(NW-1)**2
            MD3=NF
            MD4=NW*NF
            MD5=2*MD4
            MD6=(MD4+1)/2
            MD7=(NW/2)**2
            MD8=MD4**2
            MD9=(NW/2)*NF
            MD10=MD9**2
            NWH=NW/2
            CALL WINFO(RC,RD,NW,CONF,NS,NX,SMRC,WRC,WRD)
            CER=1.0Q0
            IF(IOPT.EQ.1) THEN
             CALL DINFO(NW,RD,WRD)
             CALL RPINFO(NW,ER)
             END IF
             IF(CONF.EQ.'C') THEN
             WRITE(6,115)
```

```
115        FORMAT(5X,'ENTER THE RELATIVE PERMITIVITY OF THE DIELECTRIC',
      +/,5X,'INSIDE THE COAX CABLE. CER= ',$)
           READ(5,*)CER
          END IF
          CALL SIZE(SMRC,AA1,NW,RC,RD,IOPT,IREF)
          IF(CONF.EQ.'R') THEN
          CALL RGENXY(NW,AA1,X,Y,SEP)
          ELSE IF(CONF.EQ.'B') THEN
          CALL BGENXY(NW,AA1,X,Y,IREF)
          ELSE IF(CONF.EQ.'C'.OR.CONF.EQ.'S') THEN
          CALL CGENXY(NW,AA1,X,Y,IREF)
          ELSE IF(CONF.EQ.'P') THEN
          CALL PGENXY(NW,AA1,X,Y,IREF,H)
          END IF
          IF(CONF.EQ.'P') GO TO 130
          WRITE(6,120)
120       FORMAT(5X,'IS THE REFERENCE NUMBER THE SAME AS THE GROUND',/
      +,5X,' REFERENCE CONDUCTOR?  ENTER Y/N, PROMPT=   ',$)
          READ(5,125)PRMPT2
125       FORMAT(A1)
          IF(PRMPT2.EQ.'N') THEN
          CALL NEWREF(NW,X,Y,IREF)
          END IF
130       IF(IOPT.EQ.1) THEN
          WRITE(6,135)NW,NHC,NHD,NHC,NHD,NFC,NFD
135       FORMAT('1',80('*'),//,6X,'GENERALIZED AND TRANSMISSION LINE CAP
      +ACITANCE MATRIX PARAMETERS',//,6X,'CONDITIONS:',/
      +,11X,I3,' WIRES ',/
      +,11X,I3,' COSINE TERMS AROUND THE CONDUCTOR',/
      +,11X,I3,' COSINE TERMS AROUND THE DIELECTRIC',/
      +,11X,I3,' SINE TERMS AROUND THE CONDUCTOR',/
      +,11X,I3,' SINE TERMS AROUND THE DIELECTRIC',/
      +,11X,I3,' FOURIER COEFFICIENTS AROUND THE CONDUCTOR',/
      +,11X,I3,' FOURIER COEFFICIENTS AROUND THE DIELECTRIC',/)
          IF(NFD-NFC)140,150,150
140       WRITE(6,145)NFC
145       FORMAT(/,' ***ERROR*** NUMBER OF COEFFICIENTS AROUND CONDUCTOR G
      +IVEN GREATER THAN NUMBER AROUND DIELECTRIC.',/,14X,'BOTH SET EQU
      +AL TO ',I3,'.',/)
          NFD=NFC
150       WRITE(6,155)
155       FORMAT(/,24X,'PHYSICAL CHARACTERISTICS:',//,10X,'CONDUCTOR
      +',15X,'DIELECTRIC',19X,'RELATIVE',/,12X,'RADIUS',19X,'RADIUS
      +',20X,'DIELECTRIC',/,63X,' CONSTANT',/)
          DO 165 J=1,NW
          WRITE(6,160)J,WRC(J),J,WRD(J),J,ER(J)
160      FORMAT(3X,'RC(',I2,')= ',1PE10.3,' (METERS)',1X,
      +'RD(',I2,')= ',E10.3,' (METERS)',1X,
      +'ER(',I2,')= ',1X,E10.3,/)
165       CONTINUE
          ELSE IF(IOPT.EQ.2) THEN
          WRITE(6,170)NW,NHC,NHC,NFC
170       FORMAT('1',80('*'),//,6X,'GENERALIZED AND TRANSMISSION LINE
      +CAPACITANCE MATRIX PARAMETERS',//,6X,'CONDITIONS:',/
      +,11X,I3,' WIRES ',/
      +,11X,I3,' COSINE TERMS AROUND THE CONDUCTOR',/
      +,11X,I3,' SINE TERMS AROUND THE CONDUCTOR',/
      +,11X,I3,' FOURIER COEFFICIENTS AROUND THE CONDUCTOR')
          WRITE(6,175)
```

```
175        FORMAT(/,24X,'PHYSICAL CHARACTERISTICS:',/,10X,'CONDUCTOR RAD
     +IUS',/)
           DO 185 J=1,NW
           WRITE(6,180)J,WRC(J)
180     FORMAT(3X,'RC(',I2,')= ',1PE11.4,' (METERS)',/)
185        CONTINUE
           END IF
           CALL CAP(NW,NFC,NFD,NF,RC,RD,ER,IREF,IOPT,C,C1,D,D1,SCR,LT
     +,NHC,NHD,X,Y,MD1,MD2,MD3,MD4,MD5,MD6,MD7,MD8,MD9,MD10,PCTL
     +,CONF,PD1,CER,RD1,NWH,PCG)
C
C           RESULTS HAVE BEEN CALCULATED - MATRICES ARE PRINTED OUT
C
           NW1=NW-1
           IF(CONF.EQ.'R'.OR.CONF.EQ.'B') THEN
           IF(IOPT.EQ.1) THEN
           CALL MPRT(C,NW,NW,'GENERALIZED CAPACITANCE',23)
           CALL MPRT(C1,NW1,NW1,'TRANSMISSION LINE CAPACITANCE',29)
           KN=1
           DO 190 I=1,NW1
           DO 190 J=1,NW1
           SAC1(KN)=C1(KN)
           KN=KN+1
190        CONTINUE
           ELSE
           CALL MPRT(C,NW,NW,'GENERALIZED CAPACITANCE FOR BARE WIRES',38)
           CALL MPRT(C1,NW1,NW1,'TRANSMISSION LINE CAPACITANCE FOR
     &BARE WIRES',44)
           ON=1
           DO 205 I=1,NW1
           DO 205 J=1,NW1
           SAC1(ON)=C1(ON)
           ON=ON+1
205        CONTINUE
           CALL MINV(C1,NW1,DET,LT(1),LT(NW1+1))
           MN=1
           DO 210 I=1,NW1
           DO 210 J=1,NW1
           RLB(MN)=1.1126497Q-17*C1(MN)
           MN=MN+1
210        CONTINUE
           CALL MPRT(RLB,NW1,NW1,'TRANSMISSION LINE INDUCTANCE',28)
           END IF
           ELSE IF(CONF.EQ.'P') THEN
           NWH=NW/2
           IF(IOPT.EQ.1) THEN
           CALL MPRT(PCTL,NWH,NWH,'TRANSMISSION LINE CAPACITANCE
     + OVER A GROUND PLANE',49)
           KL=1
           DO 220 I=1,NWH
           DO 220 J=1,NWH
           SAPCTL(KL)=PCTL(KL)
           KL=KL+1
220        CONTINUE
           ELSE IF(IOPT.EQ.2) THEN
           NWH=NW/2
           CALL MPRT(PCTL,NWH,NWH,'TRANSMISSION LINE CAPACITANCE OF BARE
     &     WIRES OVER A GND. PLANE',61)
           KP=1
```

```
      DO 235 I=1,NWH
      DO 235 J=1,NWH
      SAPCTL(KP)=PCTL(KP)
      KP=KP+1
235   CONTINUE
      CALL MINV(PCTL,NWH,DET,LT(1),LT(NWH+1))
      KO=1
      DO 240 I=1,NWH
      DO 240 J=1,NWH
      RPLB(KO)=1.1126497Q-17*PCTL(KO)
      KO=KO+1
240   CONTINUE
      CALL MPRT(RPLB,NWH,NWH,'BARE WIRE INDUCTANCE OVER A GROUND
     &PLANE',40)
      END IF
      ELSE IF(CONF.EQ.'S') THEN
      CALL MPRT(C,NW,NW,'GENERALIZED SHIELDED WIRE CAPACITANCE',37)
      CALL MPRT(C1,NW1,NW1,'SHIELDED WIRE CAPACITANCE MATRIX',34)
      IF(IOPT.EQ.1) THEN
      KR=1
      DO 250 I=1,NW1
      DO 250 J=1,NW1
      SAC1(KR)=C1(KR)
      KR=KR+1
250   CONTINUE
      ELSE
      KS=1
      DO 251 I=1,NW1
      DO 251 J=1,NW1
      SAC1(KS)=C1(KS)
      KS=KS+1
251   CONTINUE
      CALL MINV(C1,NW1,DET,LT(1),LT(NW1+1))
      KT=1
      DO 252 I=1,NW1
      DO 252 J=1,NW1
      C1(KT)=C1(KT)
      RCL(KT)=1.1126497Q-17*C1(KV)
      KT=KT+1
252   CONTINUE
      CALL MPRT(RCL,NW1,NW1,'SHIELDED TRANS. LINE INDUCTANCE MATRIX'
     &,38)
      END IF
      ELSE IF(CONF.EQ.'C') THEN
      CALL MPRT(C,NW,NW,'GENERALIZED COAX CAPACITANCE',28)
      CALL MPRT(C1,NW1,NW1,'COAX CAPACITANCE MATRIX',23)
      KU=1
      DO 265 I=1,NW1
      DO 265 J=1,NW1
      SAC1(KU)=C1(KU)
      KU=KU+1
265   CONTINUE
```

```
      CALL MINV(C1,NW1,DET,LT(1),LT(NW1+1))
      KV=1
      DO 270 I=1,NW1
      DO 270 J=1,NW1
      C1(KV)=C1(KV)*CER
      RCLB(KV)=1.1126497Q-17*C1(KV)
      KV=KV+1
270   CONTINUE
      CALL MPRT(RCLB,NW1,NW1,'COAX TRANS. LINE INDUCTANCE MATRIX',34)
      END IF
      END
```

```
C
C        THIS SUBROUTINE ALLOWS THE USER TO ENTER INFORMATION CONCERNING
C        THE VARIOUS TYPES OF WIRES TO THE PROGRAM FROM THE SCREEN
C
         SUBROUTINE WINFO(RC,RD,NW,CONF,NS,NX,SMRC,WRC,WRD)
         REAL*16 SMRC,AA1
         REAL*16 RC(NW),RD(NW),WRC(NW),WRD(NW)
         CHARACTER*1 PRMPT1,PRMPT2,PRMPT3
         CHARACTER*1 PRMPT5,PRMPT6,CONF
         IF(CONF.EQ.'S'.OR.CONF.EQ.'C') THEN
          WRITE(6,5)
5         FORMAT(/5X,'ARE ALL WIRES SOLID INSIDE SHIELD? ENTER Y/N ',$)
          READ(5,6)PRMPT1
6         FORMAT(A1)
         ELSE
          WRITE(6,10)
10        FORMAT(/5X,'ARE ALL WIRES SOLID? ENTER Y/N ',$)
          READ(5,20)PRMPT1
20        FORMAT(A1)
         END IF
         IF(PRMPT1.EQ.'N') THEN
          WRITE(6,30)
30        FORMAT(5X,'ARE ALL WIRES STRANDED? ENTER Y/N ', $)
          READ(5,40)PRMPT2
40        FORMAT(A1)
          IF(PRMPT2.EQ.'N') THEN
           WRITE(6,50)
50         FORMAT(5X,'ENTER # OF WIRES THAT ARE STRANDED NS= ',$)
           READ(5,60)NS
60         FORMAT(I2)
           WRITE(6,70)
70         FORMAT(5X,'DO ALL STRANDS HAVE SAME RADIUS? ENTER
     &Y/N ', $)
            READ(5,80)PRMPT3
80          FORMAT(A1)
            IF(PRMPT3.EQ.'N') THEN
             CALL DIFSTD(NW,NS,SMRC,RC,CONF,WRC)
            ELSE
             CALL SAMSTD(NW,NS,SMRC,RC,CONF,WRC)
            END IF
            NX=NW-NS
            IF(NX.EQ.NW) GO TO 130
            WRITE(6,90)
90          FORMAT(5X,'DO REMAINING SOLID WIRES HAVE THE SAME
     &RADIUS? ENTER Y/N ', $)
            READ(5,100)PRMPT4
100         FORMAT(A1)
            IF(PRMPT4.EQ.'Y') THEN
             CALL SAMRAD(NW,NX,NS,RC,SMRC,CONF,WRC)
            ELSE
             CALL DIFRAD(NW,NX,RC,SMRC,CONF,WRC)
            END IF
          ELSE
           WRITE(6,110)
110        FORMAT(5X,'DO STRANDS HAVE THE SAME RADIUS? ENTER Y/N ', $)
           READ(5,120)PRMPT5
120        FORMAT(A1)
            NS=NW
           IF(PRMPT5.EQ.'Y') THEN
            CALL SAMSTD(NW,NS,SMRC,RC,CONF,WRC)
```

```
            ELSE
             CALL DIFSTD(NW,NS,SMRC,RC,CONF,WRC)
            END IF
           END IF
           ELSE
          WRITE(6,140)
140       FORMAT(5X,'DO ALL WIRES HAVE THE SAME RADIUS? ENTER Y/N ', $)
          READ(5,150)PRMPT6
150       FORMAT(A1)
          NX=NW
          IF(PRMPT6.EQ.'Y') THEN
          CALL SAMRAD(NW,NX,NS,RC,SMRC,CONF,WRC)
          ELSE
          CALL DIFRAD(NW,NX,RC,SMRC,CONF,WRC)
          END IF
          END IF
130       RETURN
          END
```

```
C
C         SUBROUTINE FOR INPUTING THE DIELECTRIC RADIUS
C
          SUBROUTINE DINFO(NW,RD,WRD)
          CHARACTER*1 PRMPT8,PRMPT9
          REAL*16 RD(NW),WRD(NW)
          INTEGER NW,NO,INO
          REAL*16 XRD
          WRITE(6,10)
10        FORMAT(/,5X,'ARE ALL DIELECTRIC RADII THE SAME?  ENTER Y/N ',$)
          READ(5,20)PRMPT8
20        FORMAT(A1)
          IF(PRMPT8.EQ.'Y') THEN
C
C         ROUTINE FOR ENTERING DIELECTRIC RADII IF THEY ARE THE SAME
C
          WRITE(6,30)
30        FORMAT(/,15X,'NOTE: THE RADIUS OF DIELECTRIC IS THE
     +',/,21X,'RADIUS FROM THE CENTER OF THE
     +',/,21X,'CONDUCTOR TO THE OUTER EDGE OF
     +',/,21X,'THE DIELECTRIC',/)
          WRITE(6,40)
40        FORMAT(5X,'ENTER RADIUS OF DIELECTRIC (RD)=  ',$)
          READ(5,*)XRD
          DO 60 I=1,NW
          RD(I)=XRD
          WRD(I)=XRD
          WRITE(6,50)I,RD(I)
50        FORMAT(5X,'RD(',I2,')= ',E11.4)
60        CONTINUE
          ELSE
C
C         ROUTINE ENTERING DIELECTRIC RADIUS IF THEY ARE DIFFERENT
C
          DO 160 II=1,NW
           IF(II.EQ.1) THEN
            WRITE(6,70)
70          FORMAT(/,5X,'ENTER WIRE # NO.= ',$)
            READ(5,80)NO
80          FORMAT(I2)
            WRITE(6,90)NO,NO
90          FORMAT(5X,'ENTER RADIUS OF DIELECTRIC OF WIRE(',I2,')'
     &,/,25X,' RD(',I2,')= ',$)
            READ(5,*)RD(NO)
            WRD(NO)=RD(NO)
           ELSE
            INO=NO
            WRITE(6,100)
100         FORMAT(5X,'ENTER WIRE # NO.= ',$)
            READ(5,110)NO
110         FORMAT(I2)
            WRITE(6,120)NO,INO
120         FORMAT(5X,'IS RD(',I2,')=RD(',I2,') ?',/,5X,'
     + ENTER Y/N ',$)
            READ(5,130)PRMPT9
130         FORMAT(A1)
            IF(PRMPT9.EQ.'Y')THEN
             RD(NO)=RD(INO)
            ELSE
             WRITE(6,140)NO,NO
```

```
140       FORMAT(5X,'ENTER RADIUS OF DIELECTRIC OF WIRE(',I2,')'
     &,/,25X,' RD(',I2,')= ',$)
          READ(5,*)RD(NO)
          WRD(NO)=RD(NO)
          END IF
        END IF
          WRITE(6,150)NO,RD(NO)
150       FORMAT(5X,'RD(',I2,')= ',E10.3)
160     CONTINUE
        END IF
        RETURN
        END
```

```
C
C       SUBROUTINE RPINFO FOR ENTERING THE RELATIVE PERMITTIVITY
C       OF EACH WIRE
C
        SUBROUTINE RPINFO(NW,ER)
        REAL*16 ER(NW)
        INTEGER INO,NO
        CHARACTER*1 PRMPT10,PRMPT11
        REAL*16 XER
        WRITE(6,10)
10      FORMAT(/,5X,'IS THE RELATIVE PERMITIVITY THE SAME
     + FOR ALL WIRES?  ENTER Y/N ',$)
        READ(5,20)PRMPT10
20      FORMAT(A1)
        IF(PRMPT10.EQ.'Y') THEN
C
C       THIS ROUTINE FOR ENTERING RELATIVE PERMITTIVITY IF IT IS
C       THE SAME
C
        WRITE(6,30)
30      FORMAT(5X,'ENTER RELATIVE PERMITIVITY OF DIELECTRIC ER= ',$)
        READ(5,*)XER
        DO 50 I=1,NW
        ER(I)=XER
        WRITE(6,40)I,ER(I)
40      FORMAT(5X,'ER(',I2,')= ',E11.4)
50      CONTINUE
        ELSE
C
C       THIS ROUTINE FOR ENTERING THE RELATIVE PERMITTIVITY IF DIFFERENT
C
        DO 150 II=1,NW
         IF(II.EQ.1) THEN
          WRITE(6,60)
60        FORMAT(/,5X,'ENTER WIRE # NO.= ',$)
          READ(5,70)NO
70        FORMAT(I2)
          WRITE(6,80)NO,NO
80        FORMAT(5X,'ENTER THE RELATIVE PERMITIVITY OF ',/,5X,'
     &WIRE(',I2,') ER(',I2,')= ',$)
          READ(5,*)ER(NO)
         ELSE
          WRITE(6,90)
90        FORMAT(5X,'ENTER WIRE # NO.= ',$)
          READ(5,100)NO
100       FORMAT(I2)
          WRITE(6,110)NO,INO
110       FORMAT(5X,'IS ER(',I2,')=ER(',I2,') ?',/,5X,'
     & ENTER Y/N ',$)
          READ(5,120)PRMPT11
120       FORMAT(A1)
          IF(PRMPT11.EQ.'Y')THEN
           ER(NO)=ER(INO)
           WRITE(6,125)NO,ER(NO)
125        FORMAT(5X,'ER(',I2,')= ',E10.3)
          ELSE
           WRITE(6,130)
130     FORMAT(5X,'ENTER RELATIVE PERMITIVITY OF WIRE(',I2,')
     & ER(',I2,')= ',$)
           READ(5,*)ER(NO)
```

```
            WRITE(6,140)NO,ER(NO)
 140        FORMAT(5X,'ER(',I2,')= ',E10.3)
            END IF
          END IF
          INO=NO
 150    CONTINUE
        END IF
        RETURN
        END
```

```
C
C       SUBROUTINE SIZE SIZES THE RELATIVE MEASUREMENTS TO THAT OF
C       THE SMALLEST RADIUS OF THE CONDUCTORS TO MINIMIZE COMPUTAT-IONAL
C       ERRORS
C
        SUBROUTINE SIZE(SMRC,AA1,NW,RC,RD,IOPT,IREF)
        INTEGER NW,NJ
        REAL*16 SMRC,AA1,RC(NW),RD(NW)
        NJ=-1.0Q0*QLOG10(SMRC)
          AA1=10**NJ
          IREF=1
        DO 10 K=1,NW
          RC(K)=RC(K)*AA1
          RD(K)=RD(K)*AA1
10      CONTINUE
        RETURN
        END
```

```
C
C        SUBROUTINE SAMSTD IS USED FOR ENTERING STRANDED WIRE
C        INFORMATION IF THE STRANDS HAVE THE SAME DIMENSIONS
C
         SUBROUTINE SAMSTD(NW,NS,SMRC,RC,CONF,WRC)
         REAL*16 RSTD,RC(NS),SMRC,XRC,WRC(NS)
         INTEGER NS,NO,INO,NW,NSTD
         CHARACTER*1 CONF
         WRITE(6,10)NW,NS
10       FORMAT(//,5X,'TOTAL NUMBER OF WIRES NW= ',I2,/,
     &5X,'TOTAL NUMBER OF STRANDED WIRES NS= ',I2)
         IF(CONF.EQ.'C') THEN
         WRITE(6,20)
20       FORMAT(/,5X,'ENTER INSIDE RADIUS OF COAX SHIELD RCX= ',$)
         READ(5,*)RCX
         END IF
         IF(CONF.EQ.'S') THEN
         WRITE(6,21)
21       FORMAT(/,5X,'ENTER INSIDE RADIUS OF SHIELD RCX= ',$)
         READ(5,*)RCX
         END IF
         IF(NS.LE.0) NS=NW-1
         WRITE(6,30)
30       FORMAT(//,5X,'ENTER STRANDED WIRE INFORMATION')
           WRITE(6,40)
40         FORMAT(/,5X,'ENTER RADIUS OF ONE STRAND OF WIRE
     & RSTD= ',$)
           READ(5,*)RSTD
50         WRITE(6,60)
60         FORMAT(5X,'ENTER # OF STRANDS NSTD= ',$)
           READ(5,*)NSTD
         IF(NSTD.EQ.7.OR.NSTD.EQ.10.OR.NSTD.EQ.19.
     &OR.NSTD.EQ.26.OR.NSTD.EQ.37.OR.NSTD.EQ.41.
     &OR.NSTD.EQ.65) THEN
         GO TO 80
         ELSE
         WRITE(6,70)
70       FORMAT(//,20X,'INCORRECT INPUT FOR THE NUMBER OF STRANDS.',/,
     &20X,'INPUT SHOULD BE 7,10,19,26,37,41,OR 65.  IF VALUE',/,
     &20X,'ENTERED IS DIFFERENT TRY AGAIN, OTHERWISE PRGRAM',/,
     &20X,'MUST BE MODIFIED.',//)
         GO TO 50
         END IF
80         IF(NSTD.EQ.7)  XRC=3.0*RSTD
           IF(NSTD.EQ.10) XRC=4.0*RSTD
           IF(NSTD.EQ.19) XRC=5.0*RSTD
           IF(NSTD.EQ.26) XRC=6.0*RSTD
           IF(NSTD.EQ.37) XRC=7.0*RSTD
           IF(NSTD.EQ.41) XRC=8.0*RSTD
           IF(NSTD.EQ.65) XRC=9.0*RSTD
         IF((NW-NS).EQ.0) THEN
         N=NW
         ELSE
         N=NS
         END IF
         IF(CONF.EQ.'C'.OR.CONF.EQ.'S') THEN
         RC(1)=RCX
         WRC(1)=RCX
         IF(CONF.EQ.'C') THEN
         WRITE(6,90)RC(1)
```

```
 90     FORMAT(/,5X,'INSIDE RADIUS OF COAX CABLE RC(1)= ',E10.3
        ELSE
        WRITE(6,91)RC(1)
 91     FORMAT(/,5X,'INSIDE RADIUS OF SHIELD RC(1)= ',E10.3)
        END IF
        END IF
        DO 110 JJ=1,N
          IF(CONF.EQ.'C'.OR.CONF.EQ.'S') THEN
           K=JJ+1
          ELSE
           K=JJ
          END IF
           RC(K)=XRC
           WRC(K)=XRC
           WRITE(6,100)K,RC(K)
 100       FORMAT(5X,'RC(',I2,')= ',E10.3)
           SMRC=RC(K)
 110    CONTINUE
        RETURN
        END
```

```
C
C       SUBROUTINE DIFSTD IS USED FOR ENTERING STRANDED WIRE
C       INFORMATION IF THE DIMENSIONS OF THE STRANDS ARE OF
C       DIFFERENT DIMENSIONS
C
        SUBROUTINE DIFSTD(NW,NS,SMRC,RC,CONF,WRC)
        REAL*16 RSTD,RC(NS),SMRC,WRC(NS)
        INTEGER NS,NO,INO,NSTD
        CHARACTER*1 PRMPT7,CONF
        IF((NW-NS).EQ.0) THEN
        N=NW
        ELSE
        N=NS
        END IF
        DO 170 JJ=1,N
        IF(JJ.EQ.1) THEN
        IF(CONF.EQ.'C'.OR.CONF.EQ.'S') THEN
        IF(CONF.EQ.'C') THEN
        WRITE(6,10)
10      FORMAT(5X,'ENTER INSIDE RADIUS OF COAX CABLE RCX= ',$)
        READ(5,*)RCX
        ELSE
        WRITE(6,11)
11      FORMAT(5X,'ENTER INSIDE RADIUS OF SHIELD RCX= ',$)
        READ(5,*)RCX
        END IF
        RC(JJ)=RCX
        WRC(JJ)=RCX
        WRITE(6,20)JJ,RC(JJ)
20      FORMAT(5X,'RC(',I2,')= ',E10.3)
        END IF
          WRITE(6,30)
30        FORMAT(/,5X,'ENTER WIRE # NO.= ',$)
          READ(5,40)NO
40        FORMAT(I2)
         ELSE
          INO=NO
          WRITE(6,50)
50        FORMAT(5X,'ENTER WIRE # NO.= ',$)
          READ(5,60)NO
60        FORMAT(I2)
          WRITE(6,70)NO,INO
70        FORMAT(5X,'IS THE CHARACTERISTICS OF WIRE(',I2,')=
     +WIRE(',I2,').',/,5X,'ENTER Y/N ', $)
          READ(5,80)PRMPT7
80        FORMAT(A1)
          IF(PRMPT7.EQ.'Y') THEN
           RC(NO)=RC(INO)
           WRC(NO)=RC(INO)
           GO TO 130
          END IF
        END IF
          WRITE(6,90)NO
90        FORMAT(5X,'ENTER RADIUS OF ONE STRAND OF WIRE(',I2,')
     &RSTD= ',$)
          READ(5,*)RSTD
100       WRITE(6,110)NO
110       FORMAT(5X,'ENTER # OF STRANDS OF WIRE(',I2,') NSTD=
     & ',$)
          READ(5,*)NSTD
```

```
                IF(NSTD.EQ.7)  RC(NO)=3.0*RSTD
                IF(NSTD.EQ.10) RC(NO)=4.0*RSTD
                IF(NSTD.EQ.19) RC(NO)=5.0*RSTD
                IF(NSTD.EQ.26) RC(NO)=6.0*RSTD
                IF(NSTD.EQ.37) RC(NO)=7.0*RSTD
                IF(NSTD.EQ.41) RC(NO)=8.0*RSTD
                IF(NSTD.EQ.65) RC(NO)=9.0*RSTD
              IF(NSTD.EQ.7.OR.NSTD.EQ.10.OR.NSTD.EQ.19.
           &OR.NSTD.EQ.26.OR.NSTD.EQ.37.OR.NSTD.EQ.41.
           &OR.NSTD.EQ.65) THEN
              GO TO 130
              ELSE
              WRITE(6,120)
   120        FORMAT(5X,'INCORRECT INPUT FOR THE NUMBER OF STRANDS.',/,
           &5X,'INPUT SHOULD BE 7,10,19,26,37,41,OR 65.  IF VALUE',/,
           &5X,'ENTERED IS DIFFERENT TRY AGAIN, OTHERWISE PRGRAM',/,
           &5X,'MUST BE MODIFIED.',//)
              GO TO 100
              END IF
   130          WRITE(6,140)NO,RC(NO)
   140          FORMAT(5X,'RC(',I2,')= ',E10.3)
                IF(RC(INO)-RC(NO))150,150,160
   150          SMRC=RC(INO)
   160            CONTINUE
   170         CONTINUE
              RETURN
              END
```

```
C
C      ROUTINE  DIFRAD IS USED FOR ENTERING SOLID WIRE INFORMATION
C      WHERE THE WIRES HAVE THE SAME RADIUS
C
       SUBROUTINE DIFRAD(NW,NX,RC,SMRC,CONF,WRC)
       REAL*16 RCX,XRC,SMRC,RC(NW),WRC(NW)
       INTEGER NW
       CHARACTER*1 CONF
       IF(CONF.EQ.'C'.OR.CONF.EQ.'S') THEN
       IF((CONF.EQ.'C'.OR.CONF.EQ.'S').AND.NX.LT.NW) THEN
       NX=NX-1
       WRITE(6,10)NW,NX
10     FORMAT(//,5X,'TOTAL NUMBER OF WIRES NW= ',I2,
     &/,5X,'TOTAL NUMBER OF SOLID WIRES NX= ',I2)
       GO TO 40
       ELSE
       WRITE(6,20)NW,NX
20     FORMAT(//,5X,'TOTAL NUMBER OF WIRES NW= ',I2,
     &/,5X,'TOTAL NUMBER OF SOLID WRIES NX= ',I2)
       IF(CONF.EQ.'C') THEN
       WRITE(6,30)
30     FORMAT(/,5X,'ENTER INSIDE RADIUS OF COAX SHIELD RCX= ',$)
       READ(5,*)RCX
       ELSE
       WRITE(6,31)
31     FORMAT(5X,'ENTER INSIDE RADIUS OF SHIELD RCX= ',$)
       READ(5,*)RCX
       END IF
       END IF
       END IF
40     IF((NW-NX).EQ.0) THEN
       N=NW
       ELSE
       N=NX
       END IF
       IF(CONF.EQ.'C'.OR.CONF.EQ.'S') THEN
        DO 150 I=1,N
       IF(NX.LT.NW) THEN
       K=I+NX+1
       ELSE
       K=I
       END IF
        IF(K.EQ.1) THEN
         RC(K)=RCX
         WRC(K)=RCX
       IF(CONF.EQ.'C') THEN
         WRITE(6,50)K,RC(K)
50       FORMAT(/,5X,'INSIDE RADIUS OF COAX SHIELD = RC(',I2,')=
     &',E10.3)
       ELSE
         WRITE(6,51)K,RC(K)
51       FORMAT(/,5X,'INSIDE RADIUS OF SHIELD = RC(',I2,')=  ',E10.3)
       END IF
         WRITE(6,60)I,RC(I)
60       FORMAT(/,37X,'RC(',I2,')= ',E10.3)
        ELSE
         WRITE(6,70)
70       FORMAT(5X,'ENTER WIRE # NO.= ',$)
         READ(5,80)NO
80       FORMAT(I2)
```

```
          WRITE(6,90)NO,K
  90      FORMAT(5X,'IS RC(',I2,')=RC(',I2,')? ENTER Y/N ',$)
          READ(5,100)PRMPT8
  100     FORMAT(A1)
          IF(PRMPT8.EQ.'Y')THEN
           RC(NO)=RC(K)
           WRC(NO)=RC(K)
          ELSE
           WRITE(6,110)NO,NO
  110      FORMAT(5X,'ENTER RADIUS OF WIRE(',I2,') RC(',I2,')= ',$)
           READ(5,*)RC(NO)
           WRC(NO)=RC(NO)
          END IF
         END IF
          NO=K
           WRITE(6,120)NO,RC(NO)
  120      FORMAT(5X,'RC(',I2,')= ',E10.3)
           IF(RC(K)-RC(NO))130,130,140
  130      SMRC=RC(K)
  140      CONTINUE
  150    CONTINUE
         ELSE
         DO 250 I=1,N
         IF(I.EQ.1) THEN
          WRITE(6,160)
  160     FORMAT(/,5X,'ENTER WIRE # NO.= ',$)
          READ(5,170)NO
  170     FORMAT(I2)
         ELSE
          INO=NO
          WRITE(6,180)
  180     FORMAT(5X,'ENTER WIRE # NO.= ',$)
          READ(5,190)NO
  190     FORMAT(I2)
          WRITE(6,200)NO,INO
  200     FORMAT(5X,'IS RC(',I2,')=RC(',I2,')? ENTER Y/N ',$)
          READ(5,210)PRMPT8
  210     FORMAT(A1)
          IF(PRMPT8.EQ.'Y') THEN
           RC(NO)=RC(INO)
           WRC(NO)=RC(INO)
          END IF
         END IF
           WRITE(6,220)NO,NO
  220      FORMAT(5X,'ENTER RADIUS OF WIRE(',I2,') RC(',I2,')= ',$)
           READ(5,*)RC(NO)
           WRC(NO)=RC(NO)
           WRITE(6,230)NO,RC(NO)
  230      FORMAT(5X,'RC(',I2,')= ',E10.3)
          IF(RC(INO)-RC(NO))240,240,250
  240     SMRC=RC(INO)
  250      CONTINUE
         END IF
         RETURN
         END
```

```
      C
      C        ROUTINE  SAMRAD IS USED TO ENTER SOLID WIRE INFORMATION IF THE
      C        SOLID WIRES HAVE THE SAME RADIUS
      C
               SUBROUTINE SAMRAD(NW,NX,NS,RC,SMRC,CONF,WRC)
               REAL*16 RCX,XRC,SMRC,RC(NW),WRC(NW)
               INTEGER NW
               CHARACTER*1 CONF
               IF(CONF.EQ.'C'.OR.CONF.EQ.'S') THEN
               IF((CONF.EQ.'C'.OR.CONF.EQ.'S').AND.NX.LT.NW) THEN
               NX=NX-1
               WRITE(6,10)NW,NXB
      10       FORMAT(//,5X,'TOTAL NUMBER OF WIRES NW= ',I2,
           &/,5X,'TOTAL NUMBER OF SOLID WIRES NX= ',I2)
               GO TO 40
               ELSE
               WRITE(6,20)NW,NX
      20       FORMAT(//,5X,'TOTAL NUMBER OF WIRES NW= ',I2,
           &/,5X,'TOTAL NUMBER OF SOLID WRIES NX= ',I2)
               IF(CONF.EQ.'C') THEN
               WRITE(6,30)
      30       FORMAT(/,5X,'ENTER INSIDE RADIUS OF COAX SHIELD RCX= ',$)
               READ(5,*)RCX
               ELSE
               WRITE(6,31)
      31       FORMAT(/,5X,'ENTER INSIDE RADIUS OF SHIELD RCX= ',$)
               READ(5,*)RCX
               END IF
               END IF
               END IF
      40       WRITE(6,50)
      50       FORMAT(5X,'ENTER RADIUS OF THE CONDUCTOR XRC= ',$)
                READ(5,*)XRC
               IF((NW-NX).EQ.0) THEN
               N=NW
               ELSE
               N=NX
               END IF
               IF(CONF.EQ.'C'.OR.CONF.EQ.'S') THEN
                DO 80 I=1,N
               IF(NX.EQ.NW) THEN
               K=I
               ELSE
               K=I+NS+1
               END IF
                IF(K.EQ.1) THEN
                 RC(K)=RCX
                 WRC(K)=RCX
               IF(CONF.EQ.'C') THEN
                 WRITE(6,60)K,RC(K)
      60         FORMAT(/,5X,'INSIDE RADIUS OF COAX SHIELD = RC(',I2,')=
           &',E10.3)
               ELSE
                 WRITE(6,61)K,RC(K)
      61         FORMAT(/,5X,'INSIDE RADIUS OF COAX SHIELD = RC(',I2,')=
           &',E10.3)
               END IF
                 WRITE(6,70)K,RC(K)
      70         FORMAT(/,37X,'RC(',I2,')= ',E10.3)
                ELSE
```

```
          RC(K)=XRC
          WRC(K)=XRC
          SMRC=XRC
          WRITE(6,75)K,RC(K)
   75     FORMAT(/,37X,'RC(',I2,')= ',E10.3)
         END IF
   80   CONTINUE
        ELSE
         DO 90 I=1,N
         K=(NW+1)-I
         RC(K)=XRC
         WRC(K)=XRC
         SMRC=RC(K)
   90    CONTINUE
         END IF
        RETURN
        END
```

```
C
C         THIS SUBROUTINE AUTOMATICALLY GENERATES THE X AND Y
C         DISTANCES BETWEEN WIRES FOR A RIBBON CABLE
C
          SUBROUTINE RGENXY(NW,AA1,X,Y,SEP)
          REAL*16 X(NW,NW),Y(NW,NW),AA1,SEP
          INTEGER NW
           WRITE(6,10)
10         FORMAT(5X,'ENTER CNTR-TO-CNTR SPACING OF CONDUCTORS (SEP)
     &=   ',$)
           READ(5,*)SEP
           DO 20 I=1,NW
           DO 20 J=1,NW
           IF(I.LT.J) THEN
            X(I,J)=-1.0Q0*SEP*AA1
            Y(I,J)=0.0Q0
            X(I,J)=X(I,J)+X(I,J-1)
           ELSE IF(I.EQ.J) THEN
            X(I,J)=0.0Q0
            Y(I,J)=0.0Q0
           ELSE
            X(I,J)=SEP*AA1
            Y(I,J)=0.0Q0
            X(I,J)=X(I,J)+X(I-1,J)
           END IF
20        CONTINUE
          RETURN
          END
```

```
C
C       THIS SUBROUTINE AUTOMATICALLY GENERATES THE X AND Y
C       DISTANCES BETWEEN WIRES FOR A WIRE BUNDLE CONFIGURATION
C
        SUBROUTINE BGENXY(NW,AA1,X,Y,IREF)
        REAL*16 X(NW,NW),Y(NW,NW),AA1,XVALUE,YVALUE
          WRITE(6,10)
10        FORMAT(/,15X,'NOTE: ARBITRARILY SELECT FROM THE WIRE BUNDLE',/
     +,21X,'A COUNTING SEQUENCE WHERE WIRE ONE IS',/,21X,'ASSIGNED
     + COORDINATES(0,0).  ENTER ALL',/,21X,'OTHER "X" AND "Y"
     +COORDINATES WITH',/,21X,'RESPECT TO WIRE ONE',//
     +,21X,'X(I,J) REPRESENTS THE CENTER-TO-CENTER DISTANCE',/
     +,21X,'IN THE X DIRECTION BETWEEN WIRES I AND J',//
     +,21X,'Y(I,J) REPRESENTS THE CENTER-TO-CENTER DISTANCE',/
     +,21X,'IN THE Y DIRECTION BETWEEN WIRES I AND J',//
     +,21X,'NOTE: LATER IN THE PROGRAM THE USER WILL BE PROMPTED',/
     +,28X,'TO SELECT A GROUND REFERENCE WIRE',//)
          DO 40 I=1,NW
          DO 40 J=1,NW
          IF(I.EQ.J) THEN
            X(I,J)=0.0Q0
            Y(I,J)=0.0Q0
           ELSE IF(I.GT.1.AND.J.LT.I) THEN
            X(I,J)=-1.0Q0*X(J,I)
            Y(I,J)=-1.0Q0*Y(J,I)
           ELSE IF(I.GT.1.AND.J.GT.I) THEN
            X(I,J)=X(IREF,J)-X(IREF,I)
            Y(I,J)=Y(IREF,J)-Y(IREF,I)
           ELSE
            WRITE(6,20)I,J,I,J
20          FORMAT(/,5X,'ENTER THE HORIZONTAL DISTANCE BETWEEN
     &WIRE(',I2,') AND WIRE(',I2,')',/,25X,'X(',I2,',',I2,')
     &= ',$)
            READ(5,*)XVALUE
            X(I,J)=-1.0Q0*AA1*XVALUE
            WRITE(6,30)I,J,I,J
30          FORMAT(/,5X,'ENTER THE VERTICAL DISTANCE BETWEEN
     &WIRE(',I2,') AND WIRE(',I2,')',/,25X,'Y(',I2,',',I2,'
     &)= ',$)
            READ(5,*)YVALUE
            Y(I,J)=-1.0Q0*AA1*YVALUE
          END IF
40          CONTINUE
        RETURN
        END
```

```
C
C           THIS SUBROUTINE AUTOMATICALLY GENERATES THE X AND Y
C           COORDINATES OF CONDUCTORS INSIDE A COAX CABLE
C
            SUBROUTINE CGENXY(NW,AA1,X,Y,IREF)
            REAL*16 X(NW,NW),Y(NW,NW),AA1,XVALUE,YVALUE
              WRITE(6,10)
 10           FORMAT(/,15X,'NOTE: ARBITRARILY SELECT A COUNTING SEQ
     +,/,21X,'FOR THE CONDUCTORS INSIDE THE COAX SHIELD.'
     +,/,21X,'THE CENTER OF THE COAX SHIELD IS ASSIGNED'
     +,/,21X,'COORDINATES(0,0).  ENTER ALL OTHER "X"'
     +,/,21X,'AND "Y" COORDINATES WITH RESPECT TO THE'
     +,/,21X,'CENTER OF CENTER OF THE COAX CABLE',//
     +,21X,'X(I,J) REPRESENTS THE CENTER-TO-CENTER DISTANCE',/
     +,21X,'IN THE X DIRECTION BETWEEN CONDUCTOR I AND',/
     +,21X,'CENTER OF THE COAX CABLE',//
     +,21X,'Y(I,J) REPRESENTS THE CENTER-TO-CENTER DISTANCE',/
     +,21X,'IN THE Y DIRECTION BETWEEN CONDUCTOR I AND',/
     +,21X,'CENTER OF THE COAX CABLE',//)
              DO 40 I=1,NW
              DO 40 J=1,NW
              IF(I.EQ.J) THEN
                X(I,J)=0.0Q0
                Y(I,J)=0.0Q0
               ELSE IF(I.GT.1.AND.J.LT.I) THEN
                X(I,J)=-1.0Q0*X(J,I)
                Y(I,J)=-1.0Q0*Y(J,I)
               ELSE IF(I.GT.1.AND.J.GT.I) THEN
                X(I,J)=X(IREF,J)-X(IREF,I)
                Y(I,J)=Y(IREF,J)-Y(IREF,I)
               ELSE
                WRITE(6,20)J,I,J
 20             FORMAT(/,5X,'ENTER THE HORIZONTAL DISTANCE OF CONDUC
     &(',I2,')',/,5X,'WITH RESPECT TO THE CENTER OF THE COAX CAB
     &/,25X,'X(',I2,',',I2,')= ',$)
                READ(5,*)XVALUE
                X(I,J)=-1.0Q0*AA1*XVALUE
                WRITE(6,30)J,I,J
 30             FORMAT(/,5X,'ENTER THE VERTICAL DISTANCE OF CONDUCTO
     &(',I2,')',/,5X,'RESPECT TO THE CENTER OF THE COAX CABLE',
     &/,25X,'Y(',I2,',',I2,')= ',$)
                READ(5,*)YVALUE
                Y(I,J)=-1.0Q0*AA1*YVALUE
              END IF
 40             CONTINUE
            RETURN
            END
```

```
C
C       THIS SUBROUTINE COMPUTES THE RELATIVE DISTANCES BETWEEN
C       WIRES ABOVE A GROUND PLANE.  IT ALSO DETERMINES THE DISTANCES
C       BETWEEN THE IMAGES OF THOSE WIRES AND THE GROUND PLANE
C
        SUBROUTINE PGENXY(NW,AA1,X,Y,IREF,H)
        REAL*16 X(NW,NW),Y(NW,NW),H(NW),AA1,XVALUE,YVALUE
        NWH=NW/2
        DO 20 I=1,NWH
          WRITE(6,10)I,I
 10     FORMAT(/,5X,'ENTER HEIGHT OF WIRE(',I2,') ABOVE GROUND PLANE'
     +,/,25X,'H(',I2,')= ',$)
        READ(5,*)H(I)
        H(I+NWH)=-1.0Q0*H(I)
        WRITE(6,333)I+NWH,H(I+NWH)
 333    FORMAT(/5X,'H(',I3,')= ',E13.4)
 20     CONTINUE
        DO 40 I=1,NW
        DO 40 J=1,NW
         IF(I.EQ.J) THEN
          X(I,J)=0.0Q0
          Y(I,J)=0.0Q0
        WRITE(6,334)I,J,X(I,J),I,J,Y(I,J)
 334    FORMAT(5X,'X(',I2,',',I2,')= ',E13.4,3X,'Y(',I2,',',I2,')= '
     +,E13.4)
         ELSE IF(I.EQ.1.AND.J.LE.NWH) THEN
          WRITE(6,30)I,J,I,J
 30       FORMAT(/,5X,'ENTER THE HORIZONTAL DISTANCE BETWEEN WIRE(',I2,
     +') AND WIRE(',I2,')',/,25X,'X(',I2,',',I2,')= ',$)
          READ(5,*)XVALUE
          X(I,J)=AA1*XVALUE
          Y(I,J)=AA1*(H(J)-H(I))
        WRITE(6,335)I,J,X(I,J),I,J,Y(I,J)
 335    FORMAT(5X,'X(',I2,',',I2,')= ',E13.4,3X,'Y(',I2,',',I2,')= '
     +,E13.4)
         ELSE IF(I.EQ.1.AND.J.GT.NWH) THEN
          X(I,J)=X(I,J-NWH)
          Y(I,J)=AA1*(H(J)-H(I))
        WRITE(6,336)I,J,X(I,J),I,J,Y(I,J)
 336    FORMAT(5X,'X(',I2,',',I2,')= ',E13.4,3X,'Y(',I2,',',I2,')= '
     +,E13.4)
         ELSE IF(I.GT.1.AND.J.LE.NWH.AND.J.LE.I) THEN
          X(I,J)=-X(J,I)
          Y(I,J)=-Y(J,I)
        WRITE(6,337)I,J,X(I,J),I,J,Y(I,J)
 337    FORMAT(5X,'X(',I2,',',I2,')= ',E13.4,3X,'Y(',I2,',',I2,')= '
     +,E13.4)
         ELSE IF(I.GT.1.AND.J.LE.NWH.AND.J.GT.I) THEN
          X(I,J)=X(IREF,J)-X(IREF,I)
          Y(I,J)=AA1*(H(J)-H(I))
        WRITE(6,338)I,J,X(I,J),I,J,Y(I,J)
 338    FORMAT(5X,'X(',I2,',',I2,')=',E13.4,3X,'Y(',I2,',',I2,
     +')=',E13.4)
        ELSE IF(I.GT.1.AND.J.GT.NWH) THEN
        X(I,J)=X(I,J-NWH)
```

```
        Y(I,J)=AA1*(H(J)-H(I))
      WRITE(6,339)I,J,X(I,J),I,J,Y(I,J)
339   FORMAT(5X,'X(',I2,',',I2,')= ',E13.4,3X,'Y(',I2,',',I2,')= '
     +,E13.4)
       ELSE IF(I.GT.NWH.AND.J.GT.NWH) THEN
        X(I,J)=X(I-NWH,J-NWH)
        Y(I,J)=AA1*(H(J)-H(I))
      WRITE(6,340)I,J,X(I,J),I,J,Y(I,J)
340   FORMAT(5X,'X(',I2,',',I2,')= ',E13.4,3X,'Y(',I2,',',I2,')= '
     +,E13.4)
        END IF
40      CONTINUE
      RETURN
      END
```

```
C
C       THIS SUBROUTINE ALLOWS THE USER TO CHANGE THE REFERENCE
C       NUMBER THAT HAS BEEN SELECTED TO A NEW REFERENCE PREFERABLY
C       THE GROUND WIRE.  WHEN THE CONFIGURATION (P) IS SELECTED THE
C       REFERENCE IS THE GROUND PLANE
C
        SUBROUTINE NEWREF(NW,X,Y,IREF)
        REAL*16 X(NW,NW),Y(NW,NW)
        WRITE(6,10)
10      FORMAT(5X,'ENTER REFERENCE GROUND CONDUCTOR IGREF=  ',$)
        READ(5,20)IGREF
20      FORMAT(I2)
          DO 30 I=1,NW
          DO 30 J=1,NW
          IF(J.EQ.I) THEN
          X(I,J)=0
          Y(I,J)=0
          ELSE
          X(I,J)=X(IGREF,J)-X(IGREF,I)
          Y(I,J)=Y(IGREF,J)-Y(IGREF,I)
          END IF
30      CONTINUE
        IREF=IGREF
        RETURN
        END
```

```
C
C         SUBROUTINE CAP IS USED TO CALCULATE THE GENERALIZED CAPACITANCE
C         MATRIX, AND FROM THAT THE TRANSMISSION LINE CAPACITANCE MATRIX
C         FOR THE VARIOUS CONFIGURATIONS SELECTED IN THE MAIN PROGRAM
C
C         DESCRIPTION OF PARAMETERS:
C            NW  - NUMBER OF WIRES SELECTED
C            NHC - NUMBER OF COSINE OR SINE TERMS AROUND THE CONDUCTOR
C            NHD - NUMBER OF COSINE OR SINE TERMS AROUND THE DIELECTRIC
C            NFC - NUMBER OF FOURIER TERMS SELECTED AROUND THE CONDUCTOR
C                  WHERE NFC = 2*NHC+1
C            NFD - NUMBER OF FOURIER TERMS SELECTED AROUND THE
C                  DIELECTRIC WHERE NFD = 2*NHD+1
C
C         NOTE: NFD SHOULD BE GREATER THAN OR EQUAL TO NFC
C
C            NF  - NFC+NFD FOR IOPT=2 OTHERWISE EQUAL TO NFC
C            RC  - RADIUS OF THE CONDUCTOR
C            RD  - RADIUS OF THE DIELECTRIC FROM THE CENTER OF THE WIRE
C            X   - CENTER-TO-CENTER SPACING BETWEEN THE CONDUCTORS
C                  IN THE X (HORIZONTAL) DIRECTION
C            Y   - CENTER-TO-CENTER SPACING BETWEEN THE CONDUCTORS
C                  IN THE Y (VERTICAL) DIRECTION
C
C                 NOTE: ALL DIMENSIONS MUST BE IN THE SAME UNITS.
C
C            ER   - RELATIVE DIELECTRIC CONSTANT OF THE INSULATION
C            IREF - NUMBER OF THE REFERENCE CONDUCTOR ("1" INDICATES THE
C                   FIRST WIRE.)  CAN NOT EXCEED THE NUMBER OF WIRES
C            IOPT - OPTION SELECTOR
C                   IOPT=1 COMPUTES CAPACITANCE OF DIELECTRIC COATED
C                          WIRES FOR VARIOUS CONFIGURATION
C                   IOPT=2 COMPUTES CAPACITANCE OF BARE WIRES OR FOR
C                          MULTICONDUCTOR COAX CABLES
C            CG   - CONTAINS THE  GENERALIZED CAPACITANCE MATRIX ON
C                   RETURN DIMENSION OF MATRIX IS NW BY NW
C            CTL  - CONTAINS THE TRANSMISSION LINE CAPACITANCE MATRIX
C                   ON RETURN DIMENSION IS (NW-1) BY (NW-1)
C            D    - WORKING SQUARE MATRIX OF ORDER (NFC+NFD)
C            D1   - WORKING VECTOR OF DIMENSION ((NFC+NFD)*NW)**2 IS
C                   IOPT=1 OTHERWISE THE DIMENSION IS (NFC*NW)**2
C            SCR  - SCRATCH VECTOR OF DIMENSION (NF*NW+1)/2
C            LT   - SCRATCH VECTOR OF DIMENSION 2*NF*NW
C
C         SUBROUTINES REQUIRED:
C               MINV - MATRIX INVERSION ROUTINE
C               MPC  - MATRIX MULTIPLICATION WITH A CONSTANT MULTIPLIED
C                      BY THE RESULT
C
          SUBROUTINE CAP(NW,NFC,NFD,NF,RC,RD,ER,IREF,IOPT,
     +CG,CTL,D,D1,SCR,LT,NHC,NHD,X,Y,MD1,MD2,MD3,MD4,MD5,
     +MD6,MD7,MD8,MD9,MD10,PCTL,CONF,PD1,CER,RD1,NWH,PCG)
          COMPLEX*16 MPR,MR
          REAL*16 D(MD3,MD3),D1(MD8),A12,ER(NW),PCTL(MD7)
          REAL*16 X(NW,NW),Y(NW,NW),PD1(NW),CG(NW,NW),CTL(MD2)
          REAL*16 RSTD,XNSTD,SMRC,SRC,SRD,XER,BETA
          REAL*16 XVALUE,YVALUE,PI,EPS,AC,AD,DELTC,DELTD
```

```
      REAL*16 GMA,RHATN,THETN,ER1,ER2,CER
      REAL*16 A1,A2,A3,A4,A5,RD1(MD10),PCG(NWH,NWH)
      REAL*16 B1,B2,B3,B4,B5,B12,BB1,BBB1
      REAL*16 RC(NW),RD(NW),CANG,SANG,RO,THETA
      REAL*16 Q1,Q12,Q2,Q22,XSEP,YSEP
      DIMENSION LT(MD5),SCR(MD6)
      CHARACTER*1 CONF
C
C       CONSTANTS AND COMMON COMPUTATIONS
C
      PI=3.1415927Q0
      EPS=8.854185Q-12
      NW1=NW-1
      NW12=NW1*NW1
      NFC1=NFC+1
      NFD1=NFD+1
      LD=NF*NW
C
C     ANGLE BETWEEN MATCH POINTS ON THE CONDUCTOR AND DIELECTRIC
C     SURFACES
C
      AC=2.0Q0*PI/NFC
      AD=2.0Q0*PI/NFD
C
C     ANGULAR ROTATION OF MATCHPOINTS FROM 0 DEGREES, I.E., THE
C     HORIZONTAL
C
      DELTC=PI/(2.0Q0*NFC)
      DELTD=PI/(2.0Q0*NFD)
C
C*********************************************************************
C     IS ROUTINE COMPUTES THE OFF-DIAGONAL "D" SUBMATRICES
C*********************************************************************
C      NPW = PRESENT WIRE IN WHICH THE POTENTIAL IS CALCULATED
C      NSW=PRESENT WIRE IN WHICH THE SOURCE IS RESIDING
C      MPP=PRESENT MATCH POINT ON THE BOUNDARY IN WHICH POTENTIAL IS
C      CALCUATED
C      XSEP=SEPARATION IN THE HORIZONTAL DIRECTON FROM NSW TO NPW
C      YSEP=SEPARATION IN THE VERTICAL DIRECTION FROM NSW TO NPW
C
      DO 10 NSW=1,NW
      DO 10 NPW=1,NW
      XSEP=X(NPW,NSW)
      YSEP=Y(NPW,NSW)
      MNPW=NPW
      MNPW=NSW
      ER1=ER(NPW)-1.0Q0
      ER2=ER(NPW)+1.0Q0
      IF((NSW-NPW).EQ.0)THEN
      CALL DIA(NSW,ER,CONF,NPW,NW,RC,NFC,D,IOPT,
     &RD,NFC1,ER1,NF,AC,DELTC,NHC,NHD,AD,DELTD,
     &MD1,MD2,MD3,MD4,MD5,MD6,MD7,MD8,ER2)
      ELSE
      CALL OFDIA(NFC,AC,DELTC,NPW,RC,CONF,NSW,IREF,NW,
     &NHC,IOPT,NHD,NFC1,RD,D,NFD,AD,DELTD,ER1,XSEP,YSEP,
     &MD1,MD2,MD3,MD4,MD5,MD6,MD7,MD8,ER2)
      END IF
C
```

```
C       THIS SECTION TELLS WHERE TO PUT THE SUBMATRIX "D" IN THE
C       LARGER "D1" MATRIX BEFORE IT IS INVERTED
C
        MM=NPW
        NN=NSW
        CALL PLACE(MM,NN,LD,NF,D,D1,MD8)
10      CONTINUE
C
C       THIS PORTION OF THE PROGRAM INSERTS THE SUBMATRIX ON THE
C       DIAGONAL OF THE LARGE "D1" MATRIX NW TIMES
C
C       INVERSTION OF "D1" AND COMPUTATION OF THE GENERLIZED
C       CAPACITANCE MATRIX "CG"
C
C       FULL MATRIX INVERSION OF THE "D1" MATRIX IS DONE
C       THEN THE TERMS OF THE GENERALIZED CAPACITANCE MATRIX ARE
C       COMPUTED
C
        IF(CONF.EQ.'P') THEN
        CALL P1(NW,NF,D1,MD8,LD,NFC,RD1,MD10)
        NWH=NW/2
        MLD=NWH*NF
        CALL MINV(RD1,MLD,DET,LT(1),LT(MLD+1))
        ELSE
        CALL MINV(D1,LD,DET,LT(1),LT(LD+1))
        END IF
        ISTP=1
        IF(DET)20,30,20
20      CALL SUM(NW,NF,LD,NFC,D1,RC,CG,PCG,RD,MD8,IOPT,CER,
     &CONF,RD1,MD10,NWH)
        IF(CONF.EQ.'R'.OR.CONF.EQ.'B'.OR.CONF.EQ.'C'.OR.CONF.
     &EQ.'S') THEN
        CALL TRANS(NW,CG,NW12,NW1,CTL,MD2,IREF,IOPT,PD1)
        ELSE IF(CONF.EQ.'P') THEN
        CALL PLANE(NWH,PCG,PCTL,MD7)
        END IF
        RETURN
C
C       AFTER THE PER-UNIT LENGTH GENERALIZED AND TRANSMISSION LINE
C       CAPACITANCE MATRICES HAVE BEEN CALCULATED, CONTROL RETURNS
C       TO THE CALLING PROGRAM
C
C       ERROR RETURN
C
30      WRITE(6,40)ISTP
40      FORMAT(' ** SINGULAR MATRIX AT STEP ',I1)
        NW=0
        WRITE(6,50)NW
50      FORMAT(5X,'SINGULAR PASS NW= ',I3)
        WRITE(6,60)MD1,MD2,MD2,MD3,MD3,MD4,MD4,MD5,MD6,MD7,MD8,
     +NW,NW,NW,NW,NW,NW,NW,NW,NW
60      FORMAT(///,10X,'VERIFY THAT VARIABLE ARRAYS AND VECTORS ARE',/,
     +10X,'DIMENSIONED PROPERLY IN THE PROGRAM.'
     +,/,10X,'THE DIMENSIONS IN THE PROGRAM MUST BE LESS THAN OR'
     +,/,10X,' EQUAL TO THE DIMENSIONS LISTED BELOW',//,
     +10X,'DIMENSION FOR C IS C(',I3,')',/,
     +10X,'DIMENSION OF C1 & RL ARE C1(',I3,'), RL(',I3,')',/,
     +10X,'DIMENSION OF D IS D(',I3,',',I3,') IN MAIN',/,
```

```
     +10X,'DIMENSION OF D1 IS D1(',I3,',',I3,')',/,
     +10X,'DIMENSION OF LT IS LT(',I3,')',/,
     +10X,'DIMENSION OF SCR IS SCR(',I3,')',/,
     +10X,'DIMENSION OF PCTL IS PCTL(',I3,')',/,
     +10X,'DIMENSION OF RPL IS RPL(',I3,')',/,
     +10X,'DIMENSION OF D1 IS D1(',I6,') IN SUBROUTINES GETCAP &
     +PLACE',/,
     +10X,'DIMENSIONS OF X & Y ARE X(',I3,',',I3,'),
     +Y(',I3,',',I3,')',/,
     +10X,'DIMENSIONS OF RC,RD,ER,H,& PD1 ARE',/,
     +10X,'RC(',I3,'), RD(',I3,'), ER(',I3,'), H(',I3,'),
     + & PD1(',I3,')',/)
      RETURN
      END
```

```
C
C        THIS ROUTINE COMPUTES THE POTENTIALS OF THE MATCH
C        WIRE DUE TO THE WIRE AND DIELECTRIC OF THE END WIRE
C        I.E. IT COMPUTES THE OFF-DIAGONAL TERMS OF THE SMALL
C        D SUBMATRIX
C
         SUBROUTINE OFDIA(NFC,AC,DELTC,NPW,RC,CONF,
     &NSW,IREF,NW,NHC,IOPT,NHD,NFC1,RD,D,NFD,AD,DELTD,ER1,XSEP,YSEP,
     &MD1,MD2,MD3,MD4,MD5,MD6,MD7,MD8,ER2)
         REAL*16 AC,DELTC,BETA,CANG,SANG,RC(NW)
         REAL*16 Q1,Q12,Q2,Q22,RO,THETA,B1,D(MD3,MD3)
         REAL*16 A1,A3,B2,B3,RD(NW),A2,B4,B5,AD,DELTD,GMA
         REAL*16 RHATN,THETN,ER1,XSEP,YSEP,ER2
         CHARACTER*1 CONF
         DO 60 MPP=1,NFC
          BETA=(MPP-1)*AC+DELTC
          CANG=QCOS(BETA)
          SANG=QSIN(BETA)
          Q1=XSEP+RC(NPW)*CANG
          Q12=Q1**2
          Q2=YSEP+RC(NPW)*SANG
          Q22=Q2**2
          RO=QSQRT(Q12+Q22)
          THETA=QATAN2(Q2,Q1)
          B1=QLOG(RO)
          IF((CONF.EQ.'C'.AND.NSW.EQ.IREF).OR.
     &(CONF.EQ.'S'.AND.NSW.EQ.IREF)) THEN
C
C        COMPUTE AVERAGE MN TERM FOR COAX OR SHIELD WHITH SOURCE
C        ON SHIELD R<R'
C
           D(MPP,1)=-RC(NSW)*QLOG(RC(NSW))
           IF(IOPT.EQ.1) THEN
C
C        COMPUTE AVERAGE MN' TERM FOR COAX OR SHIELD WHITH
C        SOURCE ON SHIELD R<R'
C
           D(MPP,NFC1)=-RD(NSW)*QLOG(RD(NSW))
           END IF
          ELSE IF(CONF.EQ.'P'.AND.NSW.GT.NW/2) THEN
C
C        COMPUTE AVERAGE MN TERM FOR WIRE BUNDLE WITH PLANE PRESENT
C
          D(MPP,1)=RC(NSW)*B1
           IF(IOPT.EQ.1) THEN
C
C        COMPUTE AVERAGE MN' TERM FOR WIRE BUNDLE WITH PLANE PRESENT
C
           D(MPP,NFC1)=RD(NSW)*B1
          END IF
         ELSE
C
C        COMPUTE AVERAGE MN TERMS FOR REMAINING CONFIGURATIONS
C
           D(MPP,1)=-RC(NSW)*B1
          IF(IOPT.EQ.1) THEN
C
C        COMPUTE AVERAGE MN' TERMS FOR REMAINING CONFIGURATIONS
C
           D(MPP,NFC1)=-RD(NSW)*B1
```

```
            END IF
            END IF
          IF((CONF.EQ.'C'.AND.NSW.EQ.IREF).OR.
     &(CONF.EQ.'S'.AND.NSW.EQ.IREF)) THEN
C
C         COMPUTE THE MN TERMS FOR CONFIGURATION COAX AND SHIELD WHEN
C         SOURCE IS ON THE SHIELD R<R'
C
            A1=RO
            A3=1.0Q0
            DO 10 J=1,NHC
             J1=J+1
             J2=J1+NHC
             B2=QCOS(J*THETA)/(2.0Q0*J*A3)
             B3=QSIN(J*THETA)/(2.0Q0*J*A3)
             D(MPP,J1)=B2*A1
             D(MPP,J2)=B3*A1
             A1=A1*RO
             A3=A3*RC(NSW)
10        CONTINUE
C
C         COMPUTE THE MN' TERMS FOR CONFIGURATION COAX AND SHIELD WHEN
C         SOURCE IS ON THE SHIELD R<R'
C
            IF(IOPT.EQ.1) THEN
            A1=RO
            A3=1.0Q0
            DO 11 J=1,NHD
             J1=J+NFC1
             J2=J1+NHD
             B2=QCOS(J*THETA)/(2.0Q0*J*A3)
             B3=QSIN(J*THETA)/(2.0Q0*J*A3)
             D(MPP,J1)=B2*A1
             D(MPP,J2)=B3*A1
             A1=A1*RO
             A3=A3*RD(NSW)
11        CONTINUE
             END IF
          ELSE IF(CONF.EQ.'B'.OR.(CONF.EQ.'C'.AND.
     &NSW.NE.IREF).OR.(CONF.EQ.'S'.AND.NSW.NE.IREF)
     &.OR.(CONF.EQ.'P'.AND.NSW.LE.NW/2)) THEN
C
C         COMPUTE THE MN TERMS FOR CONFIGURATION COAX AND SHIELD WHEN
C         SOURCE IS NOT ON THE SHIELD OR FOR PLANE (REAL WIRES), RIBBON
C         CABLES, WIRE BUNDLE, AND WIRE BUNDLE OF GROUND R>R'
C
            A1=RC(NSW)
            A3=RO*2.0Q0
            DO 20 J=1,NHC
             J1=J+1
             J2=J1+NHC
             B2=QCOS(J*THETA)/(J*A3)
             B3=QSIN(J*THETA)/(J*A3)
             A1=A1*RC(NSW)
             D(MPP,J1)=B2*A1
             D(MPP,J2)=B3*A1
             A3=A3*RO
20        CONTINUE
```

```
C
C        COMPUTE THE MN' TERMS FOR CONFIGURATION COAX AND SHIELD WHEN
C        SOURCE IS NOT ON THE SHIELD OR FOR PLANE (REAL WIRES), RIBBON
C        CABLES, WIRE BUNDLE, AND WIRE BUNDLE OF GROUND R>R'
C
           IF(IOPT.EQ.1) THEN
            A2=RD(NSW)
            A3=RO*2.0Q0
            DO 30 J=1,NHD
             J3=J+NFC1
             J4=J3+NHD
             B2=QCOS(J*THETA)/(J*A3)
             B3=QSIN(J*THETA)/(J*A3)
             A2=A2*RD(NSW)
             D(MPP,J3)=B2*A2
             D(MPP,J4)=B3*A2
             A3=A3*RO
 30      CONTINUE
            END IF
         ELSE IF(CONF.EQ.'R') THEN
           A1=RC(NSW)
           A3=RO*2.0Q0
           DO 31 J=1,NHC
            J1=J+1
            B2=QCOS(J*THETA)/(J*A3)
            A1=A1*RC(NSW)
            D(MPP,J1)=B2*A1
           A3=A3*RO
 31      CONTINUE
           IF(IOPT.EQ.1) THEN
            A2=RD(NSW)
            A3=RO*2.0Q0
            DO 32 J=1,NHD
             J3=J+NFC1
             B2=QCOS(J*THETA)/(J*A3)
             A2=A2*RD(NSW)
             D(MPP,J3)=B2*A2
             A3=A3*RO
 32      CONTINUE
         END IF
          ELSE IF(CONF.EQ.'P'.AND.NSW.GT.NW/2) THEN
C
C        COMPUTE MN TERMS FOR IMAGE WIRES
C
           A1=RC(NSW)
           A3=RO*2.0Q0
           DO 40 J=1,NHC
            J1=J+1
            J2=J1+NHC
            B2=QCOS(J*THETA)/(J*A3)
            B3=QSIN(J*THETA)/(J*A3)
            A1=A1*RC(NSW)
            D(MPP,J1)=-B2*A1
            D(MPP,J2)=B3*A1
            A3=A3*RO
 40         CONTINUE
           IF(IOPT.EQ.1) THEN
C
```

```
C         COMPUTE MN' TERMS FOR IMAGE WIRES
C
             A2=RD(NSW)
             A3=RO*2.0Q0
             DO 50 J=1,NHD
               J3=J+NFC1
               J4=J3+NHD
               B4=QCOS(J*THETA)/(J*A3)
               B5=QSIN(J*THETA)/(J*A3)
               A2=A2*RD(NSW)
               D(MPP,J3)=-B4*A2
               D(MPP,J4)=B5*A2
               A3=A3*RO
50             CONTINUE
             END IF
           END IF
60        CONTINUE
          IF(IOPT.EQ.1) THEN
C
C         THIS SECTION COMPUTES THE ELECTRIC FIELD COMPONENTS ON THE
C         MATCH WIRE DUE TO BOTH THE CONDUCTIOR AND DIELECTRIC OF THE
C         END WIRE
C
           DO 110 MPP=1,NFD
             BETA=(MPP-1)*AD+DELTD
             CANG=QCOS(BETA)
             SANG=QSIN(BETA)
            Q1=XSEP+RD(NPW)*CANG
            Q12=Q1**2
            Q2=YSEP+RD(NPW)*SANG
            Q22=Q2**2
            RO=QSQRT(Q12+Q22)
            THETA=QATAN2(Q2,Q1)
             GMA=BETA-THETA
             RHATN=QCOS(GMA)
             THETN=QSIN(GMA)
             B1=ER1*RHATN/RO
             JJ=NFC+MPP
          IF(CONF.EQ.'S'.AND.NSW.EQ.IREF) THEN
C
C         COMPUTE THE AVERAGE M'N TERM FOR SHIELD R<R' (SOURCE
C         ON SHIELD)
C
            D(JJ,1)=0.0Q0
C
C         COMPUTE THE AVERAGE M'N' TERMS FOR SHIELD R<R' (SOURCE
C         ON SHIELD)
C
            D(JJ,NFC1)=0.0Q0
          ELSE IF(CONF.EQ.'P'.AND.NSW.GT.NW/2) THEN
            D(JJ,1)=-RC(NSW)*B1
            D(JJ,NFC1)=-RD(NSW)*B1
          ELSE
C
C         COMPUTE REMAINING M'N TERMS FOR OTHER CONFIGURATION R>R'
C
            D(JJ,1)=RC(NSW)*B1
C
C         COMPUTE REMAINING M'N' TERMS FOR OTHER CONFIGURATION R>R'
```

```
                D(JJ,NFC1)=RD(NSW)*B1
             END IF
             IF(CONF.EQ.'S'.AND.NSW.EQ.IREF) THEN
      C
      C      COMPUTE THE M'N TERMS WHEN R<R' FOR SHIELDED WIRE BUNDLE
      C      WHEN SOURCE IS ON SHIELD AND POTENTIAL MATCHPOINTS ARE
      C      ON DIELECTRICS OF THE INNER WIRES. NOTE, ALSO THAT THESE
      C      ROUTINES WORK IF THERE IS A DIELECTRIC BETWEEN THE SHIELD
      C      AND THE INNER WIRES
      C
             A1=1.0Q0
             A3=1.0Q0
             DO 65 I=1,NHC
             J1=I+1
             J2=J1+NHC
             B2=-ER1*(QCOS(I*THETA)*RHATN-SIN(I*THETA)*THETN)
             B3=-ER1*(QSIN(I*THETA)*RHATN+COS(I*THETA)*THETN)
             D(JJ,J1)=A1*B2/(2.0Q0*A3)
             D(JJ,J2)=A1*B3/(2.0Q0*A3)
             A1=A1*RO
             A3=A3*RD(NSW)
      65     CONTINUE
      C
      C      COMPUTE THE M'N' TERMS WHEN R<R' NOTE: DIELECTRIC RADIUS
      C      OF SHIELD SMALLER THAN CONDUCTOR RADIUS.  THIS ALSO
      C      CALCULATES THE DIFFERENCE OF THE FLUX DENSITY WHEN SOURCE
      C      IS ON SHIELD AND POTENTIAL MATCHPOINTS ARE ON THE INNER
      C      WIRES
      C
                A2=1.0Q0
                A3=1.0Q0
               DO 66 J=1,NHD
                J3=J+NFC1
                J4=J3+NHD
                B2=-ER1*(QCOS(J*THETA)*RHATN-QSIN(J*THETA)*THETN)
                B3=-ER1*(QSIN(J*THETA)*RHATN+QCOS(J*THETA)*THETN)
                D(JJ,J3)=(A2*B2)/(2.0*A3)
                D(JJ,J4)=(A2*B3)/(2.0*A3)
                A2=A2*RO
                A3=A3*RD(NSW)
      66     CONTINUE
              ELSE IF(CONF.EQ.'B'.OR.(CONF.EQ.'S'.AND.NSW.NE.IREF)
          &.OR.(CONF.EQ.'P'.AND.NSW.LE.NW/2)) THEN
      C
      C      COMPUTE THE M'N TERMS WHEN R>R' FOR SHIELDED WIRE BUNDLE
      C      OR COAX CABLE WHEN SOURCE IS NOT ON THE SHIELD OR FOR
      C      THE REMAINING CONFIGURATIONS, I.E., RIBBON CABLES, WIRE
      C      BUNDLES, AD WIRE BUNDLES WITH GROUND PLANE.
      C
                A1=RC(NSW)
                A3=RO*2.0Q0
                DO 70, J=1,NHC
                 J1=J+1
                 J2=J1+NHC
                 A1=A1*RC(NSW)
                 A3=A3*RO
                 B2=ER1*(QCOS(J*THETA)*RHATN+QSIN(J*THETA)*THETN)
                 B3=ER1*(QSIN(J*THETA)*RHATN-QCOS(J*THETA)*THETN)
                 D(JJ,J1)=A1*B2/A3
```

```
              D(JJ,J2)=A1*B3/A3
   70       CONTINUE
   C
   C        COMPUTE THE M'N' TERMS WHEN R>R' FOR SHIELDED WIRE BUNDLE
   C        OR COAX CABLE WHEN SOURCE IS NOT ON THE SHIELD OR FOR THE
   C        REMAINING CONFIGURATIONS, I.E., RIBBON CABLES, WIRE
   C        BUNDLES, AD WIRE BUNDLES WITH GROUND PLANE.
   C
              A2=RD(NSW)
              A3=RO*2.0Q0
             DO 80 J=1,NHD
              J3=J+NFC1
              J4=J3+NHD
              A2=A2*RD(NSW)
              A3=A3*RO
              B2=ER1*(QCOS(J*THETA)*RHATN+QSIN(J*THETA)*THETN)
              B3=ER1*(QSIN(J*THETA)*RHATN-QCOS(J*THETA)*THETN)
              D(JJ,J3)=(A2*B2)/A3
              D(JJ,J4)=(A2*B3)/A3
   80       CONTINUE
            ELSE IF(CONF.EQ.'R') THEN
               A1=RC(NSW)
               A3=RO*2.0Q0
               DO 71, J=1,NHC
                J1=J+1
                A1=A1*RC(NSW)
                A3=A3*RO
                B2=ER1*(QCOS(J*THETA)*RHATN+QSIN(J*THETA)*THETN)
                D(JJ,J1)=A1*B2/A3
   71       CONTINUE
            IF(IOPT.EQ.1) THEN
                A2=RD(NSW)
                A3=RO*2.0Q0
               DO 72 J=1,NHD
                J3=J+NFC1
                A2=A2*RD(NSW)
                A3=A3*RO
                B2=ER1*(QCOS(J*THETA)*RHATN+QSIN(J*THETA)*THETN)
                D(JJ,J3)=(A2*B2)/A3
   72       CONTINUE
            END IF
              ELSE IF(CONF.EQ.'P'.AND.NSW.GT.NW/2) THEN
   C
   C        COMPUTE THE M'N TERMS FOR THE IMAGE WIRES
   C
               A1=RC(NSW)
               A3=RO*2.0Q0
               DO 90, J=1,NHC
                J1=J+1
                J2=J1+NHC
                A1=A1*RC(NSW)
                A3=A3*RO
                B4=ER1*(QCOS(J*THETA)*RHATN+QSIN(J*THETA)*THETN)
                B5=ER1*(QSIN(J*THETA)*RHATN-QCOS(J*THETA)*THETN)
                D(JJ,J1)=-(A1*B4)/A3
                D(JJ,J2)=(A1*B5)/A3
   90           CONTINUE
   C
   C
            COMPUTE THE M'N' TERMS FOR THE IMAGE WIRES
```

```
C
          IF(IOPT.EQ.1) THEN
             A2=RD(NSW)
             A3=RO*2.0Q0
             DO 100 J=1,NHD
               J3=J+NFC1
               J4=J3+NHD
               A2=A2*RD(NSW)
               A3=A3*RO
               B4=ER1*(QCOS(J*THETA)*RHATN+QSIN(J*THETA)*THETN)
               B5=ER1*(QSIN(J*THETA)*RHATN-QCOS(J*THETA)*THETN)
               D(JJ,J3)=-(A2*B4)/A3
               D(JJ,J4)=(A2*B5)/A3
100          CONTINUE
          END IF
            END IF
110        CONTINUE
          END IF
          RETURN
          END
```

```
C
C        CALCULATE DIAGONAL TERMS OF CAPACITANCE MATRIX
C
         SUBROUTINE DIA(NSW,ER,CONF,NPW,NW,RC,NFC,D,IOPT,
     &RD,NFC1,ER1,NF,AC,DELTC,NHC,NHD,AD,DELTD,
     &MD1,MD2,MD3,MD4,MD5,MD6,MD7,MD8,ER2)
         REAL*16 ER1,ER(NW),ER2,B1,RC(NW),D(MD3,MD3),BB1
         REAL*16 RD(NW),BBB1,BBB2,AC,DELTC,A3,BETA,A1,A12
         REAL*16 AD,DELTD,B2,B3,B4,B5
         CHARACTER*1 CONF
         IF(CONF.EQ.'P'.AND.NSW.GT.NW/2) THEN
         B1=RC(NSW)*QLOG(RC(NSW))
         ELSE
         B1=-RC(NSW)*QLOG(RC(NSW))
         END IF
         DO 10 J=1,NFC
          D(J,1)=B1
   10    CONTINUE
         IF(IOPT.EQ.1) THEN
          IF(CONF.EQ.'P'.AND.NSW.GT.NW/2) THEN
          BB1=RD(NSW)*QLOG(RD(NSW))
          ELSE
          BB1=-RD(NSW)*QLOG(RD(NSW))
          END IF
          DO 20 J=1,NFC
           D(J,NFC1)=BB1
   20    CONTINUE
          IF(CONF.EQ.'P'.AND.NSW.GT.NW/2) THEN
           BBB1=-ER1*RC(NSW)/RD(NSW)
           BBB2=1.0Q0
          ELSE
           BBB1=ER1*RC(NSW)/RD(NSW)
           BBB2=-1.0Q0
          END IF
          DO 30 J=NFC1,NF
           D(J,1)=BBB1
           D(J,NFC1)=BBB2
   30    CONTINUE
         END IF
         DO 80 MPP=1,NFC
          BETA=(MPP-1)*AC+DELTC
          A3=1.0Q0
         IF(CONF.EQ.'B'.OR.CONF.EQ.'C'.OR.CONF.EQ.'S'
     &.OR.(CONF.EQ.'P'.AND.NSW.LE.NW/2)) THEN
          DO 40 J=1,NHC
           J1=J+1
           J2=J1+NHC
           D(MPP,J1)=RC(NSW)*QCOS(J*BETA)/2.0D0/J
           D(MPP,J2)=RC(NSW)*QSIN(J*BETA)/2.0D0/J
   40    CONTINUE
          IF(IOPT.EQ.1) THEN
           A1=RC(NSW)/RD(NSW)
           A3=1.0Q0
           DO 50 J=1,NHD
            J3=J+NFC1
            J4=J3+NHD
            D(MPP,J3)=A3*RC(NSW)*QCOS(J*BETA)/2.0D0/J
            D(MPP,J4)=A3*RC(NSW)*QSIN(J*BETA)/2.0D0/J
            A3=A3*A1
   50    CONTINUE
```

```
                END IF
              ELSE IF(CONF.EQ.'R') THEN
                DO 51 J=1,NHC
                 J1=J+1
                 D(MPP,J1)=RC(NSW)*QCOS(J*BETA)/2.0D0/J
      51      CONTINUE
                IF(IOPT.EQ.1) THEN
                 A1=RC(NSW)/RD(NSW)
                 A3=1.0Q0
                 DO 52 J=1,NHD
                  J3=J+NFC1
                  D(MPP,J3)=A3*RC(NSW)*QCOS(J*BETA)/2.0D0/J
                  A3=A3*A1
      52      CONTINUE
                END IF
              ELSE IF(CONF.EQ.'P'.AND.NSW.GT.NW/2) THEN
               DO 60 J=1,NHC
                J1=J+1
                J2=J1+NHC
                D(MPP,J1)=-RC(NSW)*QCOS(J*BETA)/2.0D0/J
                D(MPP,J2)=RC(NSW)*QSIN(J*BETA)/2.0D0/J
      60        CONTINUE
               IF(IOPT.EQ.1) THEN
                A1=RC(NSW)/RD(NSW)
                A3=1.0Q0
                DO 70 J=1,NHD
                 J3=J+NFC1
                 J4=J3+NHD
                 D(MPP,J3)=-A3*RC(NSW)*QCOS(J*BETA)/2.0D0/J
                 D(MPP,J4)=A3*RC(NSW)*QSIN(J*BETA)/2.0D0/J
              A3=A3*A1
      70      CONTINUE
                END IF
              END IF
      80      CONTINUE
              IF(IOPT.EQ.1) THEN
      C
      C       THIS SECTION COMPUTES THE FIELD ON THE DIELECTRIC DUE TO THE
      C       DIELECTRIC ITSELF AND ALSO THE CONDUCTORS INSIDE THEM
      C
               A1=RC(NSW)/RD(NSW)
               A12=A1*A1
               DO 130 MPP=NFC1,NF
                BETA=(MPP-NFC1)*AD+DELTD
                 A3=1.0Q0
              IF(CONF.EQ.'B'.OR.CONF.EQ.'S'
           &.OR.(CONF.EQ.'P'.AND.NSW.LE.NW/2)) THEN
      C       COMPUTE THE M'N TERMS OF THE D SUBMATRIX THE SOURCE IS ON THE SH
IELD
      C       AND THE POTENTIAL MATCHPOINTS ARE ON THE SHIELD ALSO
      C       COMPUTE THE M'N TERMS OF THE D SUBMATRIX FOR OTHER REAL WIRES
                DO 90 J=1,NHC
                 J1=J+1
                 J2=J1+NHC
                 B2=QCOS(J*BETA)/2.0D0
                 B3=QSIN(J*BETA)/2.0D0
                 D(MPP,J1)=ER1*A3*A12*B2
                 D(MPP,J2)=ER1*A3*A12*B3
                 A3=A3*A1
```

```
90      CONTINUE
C       COMPUTE THE M'N' TERMS OF THE D SUBMATRIX FOR OTHER REAL WIRES
          DO 100 J=1,NHD
           J3=J+NFC1
           J4=J3+NHD
           B2=QCOS(J*BETA)/2.0Q0
           B3=QSIN(J*BETA)/2.0Q0
           D(MPP,J3)=-ER2*B2
           D(MPP,J4)=-ER2*B3
100     CONTINUE
          ELSE IF(CONF.EQ.'R') THEN
          DO 101 J=1,NHC
           J1=J+1
           B2=QCOS(J*BETA)/2.0D0
           D(MPP,J1)=ER1*A3*A12*B2
           A3=A3*A1
101     CONTINUE
          DO 102 J=1,NHD
           J3=J+NFC1
           B2=QCOS(J*BETA)/2.0Q0
           D(MPP,J3)=-ER2*B2
102     CONTINUE
          ELSE IF(CONF.EQ.'P'.AND.NSW.GT.NW/2) THEN
          A3=1.0Q0
           DO 110 J=1,NHC
            J1=J+1
            J2=J1+NHC
            B4=QCOS(J*BETA)/2.0Q0
            B5=QSIN(J*BETA)/2.0D0
            D(MPP,J1)=-ER1*A3*A12*B4
            D(MPP,J2)=ER1*A3*A12*B5
110         A3=A3*A1
           DO 120 J=1,NHD
            J3=J+NFC1
            J4=J3+NHD
            B4=QCOS(J*BETA)/2.0Q0
            B5=QSIN(J*BETA)/2.0Q0
            D(MPP,J3)=ER2*B4
            D(MPP,J4)=-ER2*B5
120         CONTINUE
           END IF
130     CONTINUE
        END IF
        RETURN
        END
```

```
C
C     SUBROUTINE PLACE PLACES THE D SUBMATRICES INTO THE LARG
C     D1 MATRIX
C
      SUBROUTINE PLACE(MM,NN,LD,NF,D,D1,MD8)
      REAL*16 D(NF,NF),D1(MD8)
      NP=NF*(MM-1+(NN-1)*LD)
      DO 20 I=1,NF
      DO 10 J=1,NF
      D1(NP+J)=D(J,I)
10    CONTINUE
20    NP=NP+LD
      RETURN
      END
```

```
C
C         SUBROUTINE SUM SELECTS CERTAIN ROWS ASSOCIATED WITH THE AVG
C         TERM OF THE FOURIER SERIES AND SUMS THEM TO DETERMINE THE
C         ELEMENTS IN THE CG MATRIX
C
          SUBROUTINE SUM(NW,NF,LD,NFC,D1,RC,CG,PCG,RD,MD8,IOPT,CER,
     &CONF,RD1,MD10,NWH)
          REAL*16 CG(NW,NW),D1(MD8),PI,EPS,A1,A2,A3
          REAL*16 RC(NW),RD(NW),CER,RD1(MD10),PCG(NWH,NWH)
          CHARACTER*1 CONF
          PI=3.1415927Q0
          EPS=8.854185Q-12
          A3=PI*EPS*2.0Q0
          NWH=NW/2
          IF(CONF.EQ.'P') THEN
           MNW=NWH
          ELSE
           MNW=NW
          END IF
          DO 20 I=1,MNW
           IROW=(I-1)*NF+1
          DO 20 J=1,MNW
          IF(CONF.EQ.'P') THEN
           IL=((J-1)*LD*NF/2)+IROW
          ELSE
           IL=(J-1)*LD*NF+IROW
          END IF
           A1=0.0Q0
           A2=0.0Q0
          DO 10 K=1,NFC
          IF(CONF.EQ.'P') THEN
           A1=A1+RD1(IL)
          ELSE
           A1=A1+D1(IL)
          END IF
          IF(CONF.EQ.'P') THEN
          END IF
           IF(IOPT.EQ.1) THEN
            IF(CONF.EQ.'P') THEN
            A2=A2+RD1(IL+NFC)
            ELSE
            A2=A2+D1(IL+NFC)
            END IF
           END IF
           IF(CONF.EQ.'P') THEN
           IL=IL+LD/2
           ELSE
           IL=IL+LD
           END IF
   10     CONTINUE
          IF(CONF.EQ.'P') THEN
          IF(IOPT.EQ.2) THEN
           PCG(I,J)=A1*RC(I)*A3*CER
          ELSE IF(IOPT.EQ.1) THEN
           PCG(I,J)=(A1*RC(I)+A2*RD(I))*A3
          END IF
          ELSE
          IF(IOPT.EQ.2) THEN
           CG(I,J)=A1*RC(I)*A3*CER
          ELSE IF(IOPT.EQ.1) THEN
```

```
            CG(I,J)=(A1*RC(I)+A2*RD(I))*A3
           END IF
           END IF
 20        CONTINUE
           RETURN
           END
```

```
C
C      THIS SUBROUTINE COMPUTES THE TRANSMISSION LINE CAPACITANCE
C      MATRIX FROM THE GENERALIZED CAPACITANCE MATRIX
C
C              MSUM IS THE MATRIX SUM
C              A2 IS THE RUNNING ROWSUM
C              A3 ISTHE COLUMN SUM - STORED IN D1(1=NW1)
C
       SUBROUTINE TRANS(NW,CG,NW12,NW1,CTL,MD2,IREF,IOPT,PD1)
       REAL*16 CG(NW,NW),CTL(MD2),PD1(NW),MSUM,A2,A3
       MSUM=0.0Q0
       DO 10 I=1,NW
10     MSUM=MSUM+CG(IREF,I)
       II=1
       DO 50 I=1,NW
       IF(I-IREF)20,50,20
20     A2=0.0Q0
       A3=0.0Q0
       DO 30 J=1,NW
       A2=A2+CG(I,J)
30     A3=A3+CG(J,I)
       MSUM=MSUM+A2
       PD1(II)=A3
       DO 40 J=II,NW12,NW1
40     CTL(J)=A2
       II=II+1
50     CONTINUE
       II=1
       JJ=1
       DO 90 J=1,NW
       IF(J-IREF)60,90,60
60     DO 80 I=1,NW
       IF(I-IREF)70,80,70
70     CTL(II)=CG(I,J)-CTL(II)*PD1(JJ)/MSUM
       II=II+1
80     CONTINUE
       JJ=JJ+1
90     CONTINUE
       RETURN
       END
```

```
C
C         THIS SUBROUTINE COMPUTES THE TERMS WHICH REDUCE THE CAPACITANCE
C         MATRIX WHEN A GROUND PLANE IS PRESENT FORM A 2NX2N MATRIX TO A
C         NXN MATRIX.  NOTE THE MATRIX RD1 IS THE REDUCED D1 MATRIX.
C
          SUBROUTINE P1(NW,NF,D1,MD8,LD,NFC,RD1,MD10)
          REAL*16 D1(MD8),RD1(MD10)
          NWH=NW/2
          NP=NF*NW
          NK=NF*NWH
          NN=NP*NK
          N=0
          MP=1
          DO 20 I=1,NK
          DO 10 J=1,NK
          RD1(MP)=D1(N+J)+D1(NN+N+J)
          MP=MP+1
10        CONTINUE
          N=N+LD
20        CONTINUE
          RETURN
          END
```

```
C
C     THIS SUBROUTINE COMPUTES THE CAPACITANCE MATRIX FOR
C     A WIRE BUNDLE OVER A GROUND PLANE FROM THE GENERALIZED
C     CAPACITANCE MATRIX
C
      SUBROUTINE PLANE(NWH,PCG,PCTL,MD7)
      REAL*16 PCG(NWH,NWH),PCTL(MD7)
      K=1
      DO 10 I=1,NWH
      DO 10 J=1,NWH
      PCTL(K)=PCG(I,J)
      K=K+1
10    CONTINUE
      RETURN
      END
```

```
C
C         SUBROUTINE MINV IS A PROGRAM TO INVERT A MATRIX
C
C              PARAMETERS
C              A=INPUT MATRIX,DESTROYED IN COMPUTATION AND REPLACED
C                BY RESULTANT INVERSE.
C              N=ORDER OF MATRIX A
C              D=RESULTANT DETERMINANT
C              L=WORK VECTOR OF LENGTH N
C              M=WORK VECTOR OF LENGTH N
C
          SUBROUTINE MINV (A,N,D,L,M)
          DIMENSION L(1),M(1)
          REAL*16 A(1),BIGA,HOLD,D
          D=1.0Q0
          NK=-N
          DO 19 K=1,N
          NK=NK+N
          L(K)=K
          M(K)=K
          KK=NK+K
          BIGA=A(KK)
          DO 3 J=K,N
          IZ=N*(J-1)
          DO 3 I=K,N
          IJ=IZ+I
1         IF(QABS(BIGA)-QABS(A(IJ))) 2,3,3
2         BIGA=A(IJ)
          L(K)=I
          M(K)=J
3         CONTINUE
C
C         INTERCHANGE ROWS
C
          J=L(K)
          IF(J-K) 6,6,4
4         KI=K-N
          DO 5 I=1,N
          KI=KI+N
          HOLD=-A(KI)
          JI=KI-K+J
          A(KI)=A(JI)
5         A(JI)=HOLD
C
C         INTERCHANGE COLUMNS
C
6         I=M(K)
          IF(I-K) 9,9,7
7         JP=N*(I-1)
          DO 8 J=1,N
          JK=NK+J
          JI=JP+J
          HOLD=-A(JK)
          A(JK)=A(JI)
8         A(JI)=HOLD
C
C         DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS
C         CONTAINED IN BIGA
C
9         IF(BIGA) 11,10,11
```

```
10      D=0.0Q0
        RETURN
11      DO 13 I=1,N
        IF(I-K) 12,13,12
12      IK=NK+I
        A(IK)=A(IK)/(-BIGA)
13      CONTINUE
C
C       REDUCE MATRIX
C
        DO 16 I=1,N
        IK=NK+I
        HOLD=A(IK)
        IJ=I-N
        DO 16 J=1,N
        IJ=IJ+N
        IF(I-K) 14,16,14
14      IF(J-K) 15,16,15
15      KJ=IJ-I+K
        A(IJ)=HOLD*A(KJ)+A(IJ)
16      CONTINUE
C
C       DIVIDE ROW BY PIVOT
C
        KJ=K-N
        DO 18 J=1,N
        KJ=KJ+N
        IF(J-K) 17,18,17
17      A(KJ)=A(KJ)/BIGA
18      CONTINUE
C
C       PRODUCT OF PIVOTS
C
        D=D*BIGA
C
C       REPLACE PIVOT BY RECIPROCAL
C
        A(KK)=1.0Q0/BIGA
19      CONTINUE
C
C       FINAL ROW AND COLUMN INTERCHANGE
C
        K=N
20      K=(K-1)
        IF(K) 27,27,21
21      I=L(K)
        IF(I-K) 24,24,22
22      JQ=N*(K-1)
        JR=N*(I-1)
        DO 23 J=1,N
        JK=JQ+J
        HOLD=A(JK)
        JI=JR+J
        A(JK)=-A(JI)
23      A(JI)=HOLD
24      J=M(K)
        IF(J-K) 20,20,25
25      KI=K-N
```

```
          DO 26 I=1,N
          KI=KI+N
          HOLD=A(KI)
          JI=KI-K+J
          A(KI)=-A(JI)
   26     A(JI)=HOLD
          GO TO 20
   27     RETURN
          END
```

```
C
C     SUBROUTINE MPRT IS USED TO PRINT OUT A MATRIX IN MATRIX
C     FORM WITH LABELING
C
C           DESCRIPTION OF PARAMETERS
C           A=INPUT MATRIX
C           M=NUMBER OF ROWS IN A
C           N=NUMBER OF COLUMNS IN A
C           B=LITERAL INPUT OF TITLE. HOLLERITH FORM.
C           J=NUMBER OF CHARACTERS,INCLUDING SPACES,ETC. IN B
C
      SUBROUTINE MPRT(A,M,N,B,J)
      DIMENSION B(J),C(18)
      REAL*16 A(M,N)
      LOGICAL*1 B,C/' ','M','A','T','R','I','X',' ','-',' '
   +  ,'-',' ','P','A','G','E',' ','1'/
      OPEN (UNIT=3,FILE='GOOD.DAT',STATUS='NEW')
      WRITE(3,3)B,C
      I=2
      LL=1
1     LU=MIN0(LL+5,N)
      WRITE(3,6)(L,L=LL,LU)
      WRITE(3,7)
      DO 2 K=1,M
2     WRITE(3,4)K,(A(K,L),L=LL,LU)
      IF(LU.EQ.N)RETURN
      WRITE(3,5)I
      CLOSE(UNIT=3)
      LL=LU+1
      I=I+1
      GO TO 1
3     FORMAT('1',70A1)
4     FORMAT(9X,'*',/,4X,I3,2X,'*',1P6E13.4)
5     FORMAT('1PAGE ',I2,' RIGHT - - - -.')
6     FORMAT(/,5X,'COLUMN',6(4X,I3,4X))
7     FORMAT(/,3X,'ROW',3X,67('*'))
      END
```

REFERENCES

1. Higgins, Thomas J. and Black Kenneth G., "Rigorous Determination of The Parameters of Microstrip Transmission Lines," IRE Transactions on Microwave Theory and Techniques, Vol. MTT-3, pp.93-113, March 1955.

2. Paul, C. R., and Clements J. C., "Computation of the Capacitance Matrices for Dielectric-Coated Wires", Technical Report, Rome Air Development Center, Griffis AFB. N.Y.,, 1974

3. Harrington R. F., "Field Computation by Moment Methods." New York: Macmillan, 1968.

4. Harrington, R. F., "Matrix Methods for Field Problems", IEEE Trans. Electromag. Compat., Vol. 55, pp.136-149, February 1967

5. Adams, A. T., "Electromagnetics for Engineers.," New York: Ronald Press, 1971.

6. Dwight, H. B., "Tables of Integrals and Other Mathematical Data, New York: Macmillan, 1961.

7. King, Ronald Wyeth Percival, "Transmission -line theory," New York: McGraw-Hill, 1955

8. Paul, C. R. and Feather, A. E., "Applications of Multiconductor Transmission Line Theory to the Prediction of Cable Coupling. Vol. II, Computation of the Capacitance Matrices for Ribbon Cables", Technical Report, Rome Air Development Center, Griffis AFB. N.Y., 1976

9. Paul, C. R. and Feather, A. E., "Computation of the Transmission Line Inductance and Capacitance Matrices from the Generalized Capacitance Matrix," IEEE Trans. Electromag. Compat., Vol EMC-18, pp. 175-183, November 1976.

10. Paul, C. R., "Applications of Multiconductor Transmission Line Theory to the Prediction of Cable Coupling. Vol. I, Multiconductor transmission Line Theory, Technical Report, Rome Air Development Center, Griffis AFB. N.Y., 1976

11. Smythe, "Static and Dynamic Electricity". New York: McGraw Hill, 1950

12. Clements, J. C., Paul, C. R., and Adams, A.R., " Computation of the Capacitance Matrix for Systems of Dielectric Coated Cylindrical Conductors", IEEE Trans. Electromag. Compat., Vol. EMC-17, pp. 238-248, November 1975

13. Kraus, John D. and Carver, Keith R. "Electromagnetics". New York: McGraw Hill Book Company, 1973

14. Weeks, W. T., "Multiconductor Transmission Line Theory in the TEM Approximation", IBM Journal of Research and Development", pp. 604-611, November 1972.

15. Belden Electronc Wire and Cable master catalog 885, P.O. box 1980, Richmond, In. 47375

16. Brogan, William L., "Modern Control Theory", Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1985