HOTROCKS: AN ANALYTICAL SOLAR ENERGY PACKED BED THERMAL STORAGE MODEL

by

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ABSTRACT

HOTROCKS: AN ANALYTICAL SOLAR ENERGY
PACKED BED THERMAL STORAGE MODEL

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A need for a packed bed heat storage model for solar energy heat storage applications that is both computationally efficient and accurate enough to properly simulate packed bed transient response has been shown to exist. HOTROCKS, a FORTRAN-coded packed bed model based upon analytical solutions to the Schumann heat transfer equations, is presented and is shown to satisfy both requirements. HOTROCKS is accurate for a wide variety of bed parameters corresponding to typical solar heat storage beds both for the step input and sinusoidal input air temperature cases. HOTROCKS may be used as a stand-alone model, or as part of an integrated solar energy system, computer-aided design package.

ACKNOWLEDGMENTS

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TABLE OF CONTENTS

PAGE

CHAPTER

LIST OF SYMBOLS

SYMBOLS	DEFINITION	UNITS
a	Alexander-Taft model parameter; $a = -h_v A\ell/\dot{m}\, c_a\, n$	
A	Cross-sectional area of a packed bed	m^2
$\bar{A}$	Alexander-Taft model coefficient matrix	
A_s	Surface area of the outside wall of a packed bed	m^2
A^*	Ratio of packed bed rock surface area to total bed volume	m^{-1}
AT	Alexander-Taft model	
b	Alexander-Taft model parameter; $b = c_f\, f\, \rho_f/(1-f)\, c_s\, \rho_s$	
$\bar{B}$	Alexander-Taft model coefficient matrix	
C	Centigrade degrees	
c_a	Specific heat of air	joule/kg•C
c_s	Specific heat of rock	joule/kg•C
dx	Incremental element of distance	m
dt	Incremental element of time	sec
DBK	Duffie-Beckman-Klein model	
E	Modified Nusselt number; E = wall heat transfer coefficient • bed radius/turbulent radial heat conductivity	
erf(x)	Error function of argument "x"; $\frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$	
f	Void fraction; the fraction of a packed bed volume filled with air	
F(y,z)	Heat transfer function; the amount of heat lost by the air or picked up by the solid in a packed bed as a function of time and position	

SYMBOLS	DEFINITION	UNITS
F	Fahrenheit degrees	
h	Film heat transfer coefficient	joule/m^2·sec·C
h_o	Wall heat transfer coefficient	joule/m^2·sec·C
h_v	Volumetric heat transfer coefficient	joule/m^3·sec·C
HR	HOTROCKS computer model	
$I_o(x)$	Modified Bessel function of argument "x"; $I_o(x) = 1 + \frac{x^2}{2^2(1!)^2} + \frac{x^4}{2^4(2!)^4} + \frac{x^6}{2^6(3!)^6} + \ldots$	
$J_o(x)$	Bessel function of the First Kind; $J_o(x) = 1 - \frac{x^2}{2^2} + \left(\frac{x}{2}\right)^4 \cdot \frac{1}{1^2 \cdot 2^2} - \ldots$	
k	Generalized thermal conductivity	watt/m·C
k_r	Radial turbulent conductivity	watt/m·C
k_x	Axial turbulent conductivity	watt/m·C
K_{rock}	Conductivity of rock	watt/m·C
K_{air}	Conductivity of air	watt/m·C
K	Kelvin degrees	
ℓ	Length of a packed bed	m
$M_n(x)$	Modified Bessel function; $M_n(x) = \frac{d^n}{dx^n}\left[\overline{J_o}\ (2i\sqrt{x})\right]$	
n	Number of bed segments in a discrete segment model	
P_i	Variables introduced as a result of LaPlace transform	
q	Heat power transferred between two regions	watts
q_{in}	Heat carried into a region	watts
q_{out}	Heat removed from a region	watts
Δq_c	Change in heat due to conduction	watts

SYMBOLS	DEFINITION	UNITS
$\Delta q_{s \to a}$	Heat transferred from rock to air in a packed bed	watts
r	Radial coordinate in cylindrical coordinates	
R	Radius of the "Standard" Packed Bed	m
s	Unitless variable proportional to distance	
t	Time	sec
T_a	Air temperature	C
T_o	Initial (or input) temperature	C
T_s	Rock temperature	C
$\bar{T}(k)$	Air/rock temperature vector	
ΔT	Change in temperature	C
u	Bed outlet air temperature	C
U	Total bed wall heat transfer coefficient	joule/sec·C
v	Mean air velocity	m/sec
V	Bed volume	m^3
W_{nms}	Complex function of time and bed position	
x	Distance along the axis of a packed bed	m
y	Unitless coordinate proportional to distance; $y = \frac{hA^*}{\rho_a c_a f} \cdot \frac{x}{v}$	
z	Unitless coordinate proportional to time; $z = \frac{hA^*}{\rho_s s_s (1-f)} \cdot t$	
α	Variable of integration	
α_m	Root of the transcendental equation; $\alpha_m \cos \alpha_m + \gamma/2 \sin \alpha_m = 0$	
β	Variable of integration	

SYMBOLS	DEFINITION	UNITS
γ	Laplace transform of the input air temperature function	
η	Variable of integration	
Θ_a	Input air temperature function	
Θ_s	Initial rock temperature distribution function	
ξ	Variable of integration	
ξ_n	Roots of the equation $\xi_n J_1(\xi_n) = E J_o(\xi_n)$	
ρ_a	Air density	kg/m^3
ρ_s	Rock density	kg/m^3
τ	Unitless variable proportional to time	
ϕ	Angular coordinate in cylindrical coordinates	
ω	Heat energy	joules

LIST OF FIGURES

LIST OF FIGURES (continued)

LIST OF TABLES

CHAPTER I

INTRODUCTION

The concept of obtaining limitless, free heat energy from the sun for a variety of low temperature (less than 200°F) applications, including domestic space and water heating, is not new. Solar heat has been used with varying degrees of success from antiquity to the present day. Yet at present, solar heated dwellings comprise only a small percentage of those in existence. Further, few new house builders are willing to "take a chance" on solar. Why has such a readily available energy source not been utilized more fully in this time of energy uncertainty?

There are a variety of reasons, but the one concern cited more often than any other is that while solar energy may be free, its collection is difficult and collection hardware is expensive. Since in the past many solar heat designs were effected using incomplete data both about the insolation available and about the components comprising the collection and storage systems, many systems were designed and built with high margins of safety. This technique produced systems that performed well enough but were sized larger than optimum. Such designs consequently cost more (sometimes much more) than necessary. Other solar systems were designed to closer tolerances but due to the incomplete nature of the data set upon which the design was based, they failed to work as well as predicted. Mistakes such as these are costly when solar system hardware is involved.

Recently the need to expand the solar system design data base has been recognized. Along with obtaining more complete information about the amount of solar energy available at different locations around the world, analytic tools are being developed to more accurately predict the behavior of the various components of a solar heat system. Such mathematical models allow designers to optimize their systems without resorting to "best guess" engineering techniques.

One of the commonly used methods of heat storage in a solar energy heat system is a packed bed. The packed bed is simply a room full of loosely packed solid material (usually rocks of several inches diameter) through which air, or some other fluid medium, can be made to flow freely. Heat is stored in the bed when the fluid being pumped through is at a temperature higher than that of the surrounding solid. Heat is then transferred into the solid. If lower temperature fluid is subsequently pumped through the bed, heat is extracted by the moving fluid.

This intuitively simple process is not easy to model analytically. Due to the complexity of the problem, all packed bed models developed in the past suffer from one or more of several shortcomings. Either they have simplified the problem to the point where the model does not accurately predict the behavior of a real world packed bed, or they only solve part of the problem--the steady state for example, or they remain so complex in solution as to be totally impractical for use as a design aid.

A clear need exists for a packed bed thermal model that is of high enough resolution to accurately describe heat transfer within a bed used for solar energy heat storage and yet remains computationally

simple enough to serve as a system design optimizing tool. The FORTRAN coded HOTROCKS packed bed computer model fulfills this need.

HOTROCKS is developed from analytical expressions recognized as the physically "correct" formulation of the packed bed heat transfer problem. The theoretical development of this formulation is presented in detail in Chapter 2 of this paper. Also in Chapter 2 is a discussion of other packed bed models that have been developed in the past along with a description of the shortcomings of each.

Some simplifying assumptions had to be made in order to ensure that HOTROCKS remained computationally efficient, however each assumption was investigated thoroughly to guarantee that its inclusion would not significantly degrade the resultant model. Chapter 3 describes the sensitivity analysis performed for each assumption considered.

Chapter 4 relates the development of HOTROCKS as a model to be implemented on a computer. Structured programming techniques have been used in the coding of HOTROCKS, to facilitate model validation. The actual verification and validation process is presented in Chapter 5. As part of the validation procedure HOTROCKS has been compared with two other packed bed models. Results of the comparison are also in Chapter 5.

Finally, Chapter 6 summarizes the results of the process that led to the creation of HOTROCKS, discusses how HOTROCKS compares with other models and suggests some potential approaches to take if it is determined that further refinement of the HOTROCKS model should ever be necessary.

CHAPTER II

PARTICLE BED HEAT TRANSFER: A HISTORICAL OVERVIEW

A detailed summary of significant work accomplished in the past on packed particle bed heat transfer models is presented in this chapter. Although solar energy heat storage was not the application intended by the designers of most of these models, their results are more or less applicable to solar energy heat storage systems. Where it has been discovered that two or more authors working independently have developed similar models, the earliest model developed will be discussed in this text. A complete list of all applicable models researched are contained in the References.

The Schumann Model

The earliest analytical packed bed heat transfer model was developed by T. Schumann and published in 1929 [1]. Schumann considers a fluid with a mean velocity "v" flowing through a prism (a polyhedron having parallel polygons as ends and rectangles as sides) or a cylinder of great length. The prism is filled with loosely packed solid material, as shown in figure 1.

Schumann makes six assumptions about his system. Most of these assumptions are (as will be shown in a subsequent chapter) with minor changes completely compatible with a model of rock bed storage for solar heat systems. The assumptions are:

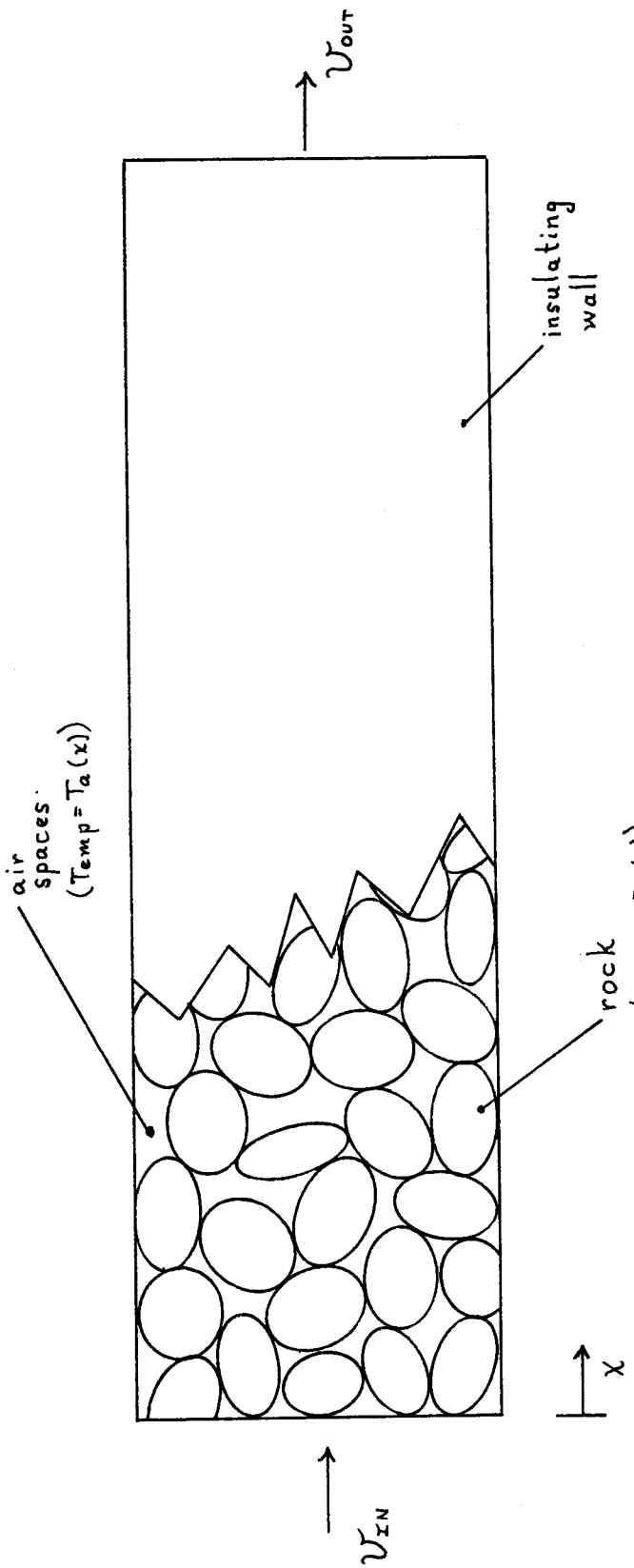


Fig. 1. The Schumann Packed Particle Bed

a) Each solid particle is at any time at a uniform temperature (thermal gradients within individual particles do not exist);

b) Conductive heat transfer between particles or within the fluid may be ignored;

c) The rate of heat transfer from fluid to solid is proportional to the temperature difference between them (according to the Fourier rate equation);

d) The prism (or cylinder) walls are perfect thermal insulators;

e) Changes in the volume of the fluid or solid due to temperature change may be neglected (the fluid and solid are "incompressible");

f) The thermal characteristics of the fluid and the solid (i.e., specific heats, densities, etc.) are constant.

With these six assumptions, Schumann models the response of his system (which is initially at uniform temperature) to a step input temperature change; for convenience in calculation the initial bed temperature is referenced to zero. To generate the equations describing his system, he applies the concept of conservation of energy.

The amount of heat transferred to the fluid in a region dx in time dt must be proportional to the difference in temperature between the solid and the fluid in that region,

$$\Delta q_{s \to a} = hA^*(T_s - T_a)dx\ dt\ . \qquad (1)$$

The heat carried into dx by the moving fluid is,

$$q_{in} = -vC_a\rho_a f \frac{\partial T_a}{\partial x} dx\, dt \,. \tag{2}$$

These quantities must sum to yield the heat that will be carried out of the region by the air in time dt,

$$q_{out} = C_a\rho_a f \frac{\partial T_a}{\partial t} dx\, dt \,. \tag{3}$$

Thus,

$$q_{in} + \Delta q_{s\to a} = q_{out} \,, \tag{4}$$

or,

$$-vC_a\rho_a f \frac{\partial T_a}{\partial x} + hA^*(T_s-T_a) = C_a\rho_a f \frac{\partial T_a}{\partial t} \,; \tag{5}$$

dx and dt, common to all three terms, have been dropped.

Equation (5) describes the air temperature in the bed as a function of position and time. By again employing the conservation of energy to the heat within the solid in a region dx, the solid temperature function can be modeled. Of course, the solid is stationary in a fixed bed, so any heat transfer within the region is simply proportional to the temperature difference between fluid and solid;

$$C_s\rho_s(1-f) \frac{\partial T_s}{\partial t} = -hA^*(T_s-T_a) \,. \tag{6}$$

Equations (5) and (6) must be solved simultaneously to yield the functions $T_a(x,t)$ and $T_s(x,t)$, which describe fluid and solid temperatures as functions of location in the bed and of time. Schumann could not find a general analytical solution of (5) and (6) for any arbitrary input conditions. However, he did find an analytic solution for a bed of uniform initial temperature and a step change in temperature input. Schumann's results are expressed in terms of infinite series

$$\frac{T_s}{T_o} = \left(\exp-(y+z)\right) \sum_{n=1}^{\infty} z^n M_n(yz) \tag{7}$$

and

$$\frac{T_a}{T_o} = \left(\exp-(y+z)\right) \sum_{n=0}^{\infty} z^n M_n(yz) \tag{8}$$

The variables y and z are transformations of the position variable x and the time variable t and the function $M_n(yz)$ is a special form of Bessel function of order n (see the "List of Symbols" section). Schumann uses two transformations of the variables x and t to arrive at equations for T_a and T_s which are forms of Bessel's equation, and are readily solvable. Figures 2 and 3 graphically depict Schumann's results for various values of y and z (and hence x and t).

The Schumann solution is exact for the constraints and conditions noted, though it is not expressible in closed form. The solution agrees very well with experimental data derived from systems

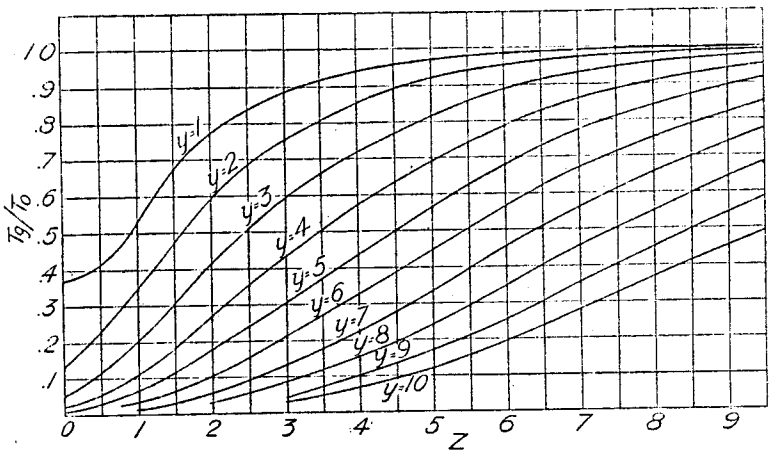


Fig. 2. Air Temperature Response to Step Input (Schumann Model) [1]

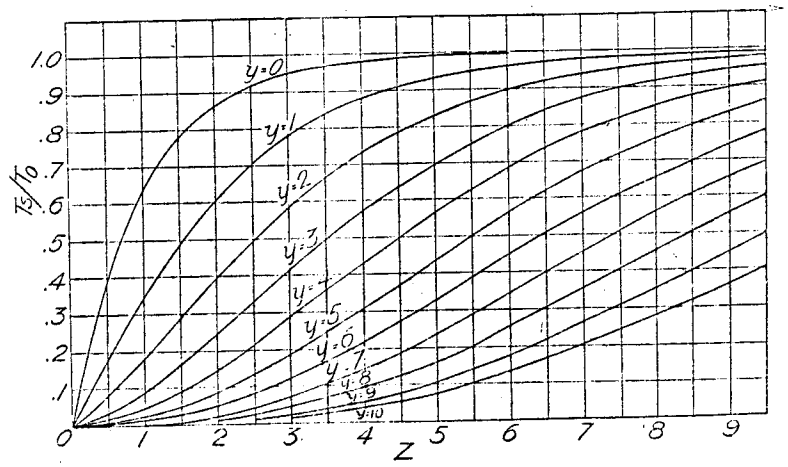


Fig. 3. Solid Temperature Response to Step Input (Schumann Model) [2]

[1,2] T. E. W. Schumann, "Heat Transfer: A Liquid Flowing Through a Porous Prism," Journal of the Franklin Institute, CCVIII (September 1929), p. 407.

built to Schumann's constraints, and it is generally accepted as the reference standard model to which other models are compared.

The Schumann model is not without problems. Its most severely limiting disadvantage is that it only predicts response to a step function input. Also, the infinite series solutions do not converge rapidly. Hence, calculations designed to produce numerical results are slow and tedious, even if done with the aid of a computer. These difficulties make the Schumann model impractical for use in a real time energy storage simulation routine.

The Amundson Model

Given the same particle bed geometry as Schumann's model, a more accurate representation of heat transfer within the bed is obtained when the effects of axial and radial conduction are considered. Obviously for radial heat conduction to occur, there must be a radial temperature gradient across the bed. This implies that there must be some heat loss through the walls of the bed. Axial conduction implies that heat may be propagated through the bed in the x-direction even when the mean fluid velocity is zero (see figure 4). Amundson [2] has considered just such a model in which both axial and radial conduction are allowed, and in which turbulent flow is assumed.

Thermal conduction is described by the Laplace equation, $\nabla^2 T = 0$ for steady state and $\nabla^2 T = \partial T/\partial t$ for the transient case. In cylindrical coordinates for the transient case,

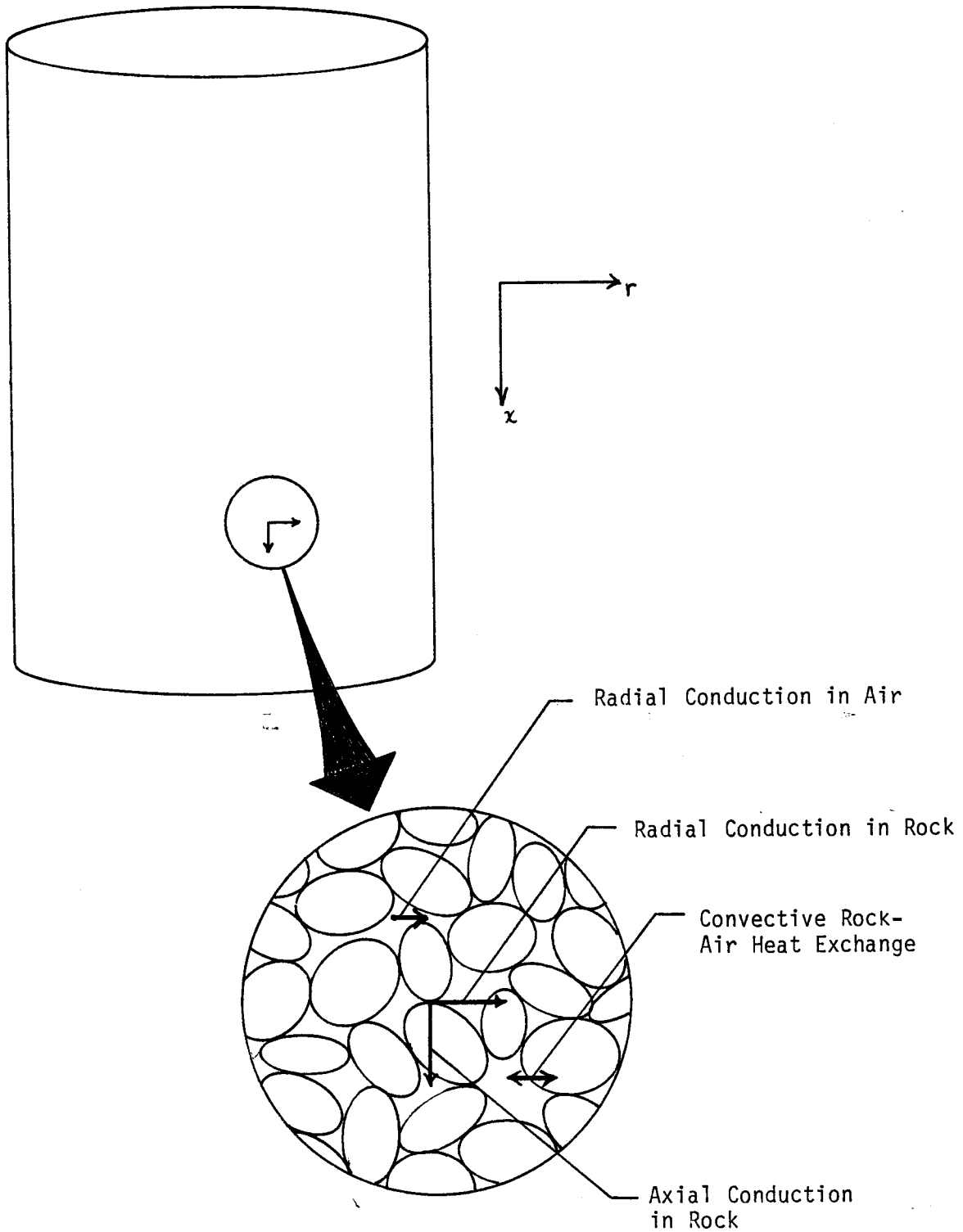


Fig 4. Methods of Heat Transfer in a Packed Bed

$$\nabla^2 T_a = \frac{\partial^2 T_a}{\partial r^2} + \frac{1}{r}\frac{\partial T_a}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T_a}{\partial \phi^2} + \frac{\partial^2 T_a}{\partial x^2} = \left.\frac{\partial T_a}{\partial t}\right|_{\text{conduction}} . \tag{9}$$

With a cylindrical bed given homogeneous insulation in the walls and uniform ambient outside temperature, there is no variation of T_a with ϕ. Equation (9) then becomes,

$$\frac{\partial T_a}{\partial t} = \left(\frac{\partial^2 T_a}{\partial r^2} + \frac{1}{r}\frac{\partial T_a}{\partial r}\right) + \frac{\partial^2 T_a}{\partial x^2} . \tag{10}$$

Equation (10) can be used to describe the energy transfer in a region dx due to conduction alone;

$$\Delta q_c = k_r\left(\frac{\partial^2 T_a}{\partial r^2} + \frac{1}{r}\frac{\partial T_a}{\partial r}\right) + k_x \frac{\partial^2 T_a}{\partial x^2} . \tag{11}$$

Now, using (11), the energy balance relationship for air according to Amundson is,

$$q_{in} + \Delta q_{s\to a} + \Delta q_c = q_{out} \tag{12}$$

or,

$$-vc_a\rho_a f\frac{\partial T_a}{\partial x} + hA^*(T_s - T_a) + k_r\left(\frac{\partial^2 T_a}{\partial r^2} + \frac{1}{r}\frac{\partial T_a}{\partial r}\right)$$

$$+ k_x\left(\frac{\partial^2 T_a}{\partial x^2}\right) = c_a\rho_a f\frac{\partial T_a}{\partial t} . \tag{13}$$

Since the rocks only contact each other at points of extremely small surface area, conduction from rock-to-rock is assumed negligible so the rock temperature equation remains,

$$c_s\rho_s(1-f)\frac{\partial T_s}{\partial t} = -hA^*(T_s-T_a) \ . \qquad (14)$$

Amundson solves (13) and (14) for appropriate boundary conditions;

a) $-k_r\ \partial T_a/\partial r|_{r=R} = U\ T_a|_{r=R}$, heat loss through the wall is a function of wall temperature,

b) $T_a(0,r,0) = T_f(r)$, where $T_f(r)$ is an initial fluid input temperature at the top of the bed (a function of r),

c) $T_s(x,r,0) = T_{s,0}(x,r)$, an initial rock temperature distribution in the bed (a function of x and r),

d) $T_a(x,r,0) = T_{a,0}(x,r)$, an initial fluid temperature distribution in the bed (a function of x and r),

e) $\partial T_a/\partial x|_{x=\ell} = 0$, a condition that assumes that fluid is removed from the bottom of storage instantaneously.

Amundson transforms the equations into relations involving dimensionless quantities (for ease of manipulation). Then he transforms the resultant equations into a form on which a Finite Fourier Transform can be applied. The resultant expressions are first order partial differential equations, to which he applies a Laplace transform. Solving the resultant polynominal expressions and subjecting the result to an inverse transform yields the final solution for both fluid temperature and solid temperature. The fluid temperature result is,

$$T_a = T_a'\Big|_{t=0} + 4\, e^{\frac{\gamma y}{2}} \left[\sum_{n=1}^{\infty} \frac{\xi_n^2\, J_0(s\,\xi_n)}{(\xi_n^2+E^2)\, \overline{J_0(\xi_n)^2}} \right.$$

$$\left. \cdot \sum_{n=1}^{\infty} \frac{\gamma^2+4\alpha_m^2}{2\gamma+\gamma^2+4\alpha_m^2}\, W_{nms}\, \sin(\alpha_m y) \right] + 4\, e^{\frac{\gamma y}{2}} \left[\sum_{n=1}^{\infty} \frac{J_0(s\,\xi_n)}{(\xi_n^2+E^2)\, \overline{J_0(\xi_n)^2}} \right.$$

$$\left. \cdot \sum_{n=1}^{\infty} \frac{(\gamma^2+4\alpha_m^2)\, \sin(\alpha_m y)}{2\gamma+\gamma^2+4\alpha_m^2} \cdot \sum_{i=1}^{2} e^{P_i \tau} \right] . \qquad (15)$$

The heretofore unused symbols represent combinations of the various constants in (13) and (14) with the variables x, r and t (see the "List of Symbols").

The Amundson model is applicable for all physically possible initial conditions, making it perhaps the most all-encompassing particle bed heat transfer model available with an exact analytic solution. However, from the form of (15) it is seen that the model is extremely cumbersome to evaluate as series of transforms of transformed quantities must be calculated in order to obtain a solution. Amundson's model then, like Schumann's, cannot be used practically in a real time heat storage simulation routine.

Interpretation of Hughes, Klein and Close

It is clear from the previous two summaries that an absolutely accurate model, while obtainable, is going to require much computational effort and hence is impractical for simulation purposes. It

is advisable to consider, alternatively, a simplication of the model or an approximate method of solution. Care must be taken to insure that such simplification or approximation does not degrade the model's accuracy by a significant amount. Hughes, Klein and Close have suggested that the wall losses in a particle bed used for solar energy heat storage can be made nearly insignificant [3]. Further, they maintain that since the air, the fluid in the particle bed, is of such a low density with respect to the rock that at any given instant most of the heat energy in storage is contained within the rock; the air has near zero thermal capacitance. Consequently, the Hughes, Klein and Close equations closely resemble Schumann's model with the air energy term set to zero ($fc_a\rho_a\ \partial T_a/\partial t = 0$);

$$fvc_a\rho_a \frac{\partial T_a}{\partial x} = hA^*(T_s-T_a) - h_oA_s(T_a-T_o) \tag{16}$$

and,

$$c_s\rho_s(1-f) \frac{\partial T_s}{\partial t} = -hA^*(T_s-T_a) \ . \tag{17}$$

Notice that the radial and axial transfer terms from Amundson's model have been replaced by an overall wall loss term. Implicit in this simplification is the assumption that wall losses, if they exist, are but a very small amount of the system's energy and can be averaged over the entire bed without significant loss in accuracy. At this point the authors suggest (but do not develop) a finite difference numerical approach to the solution of their equations.

They also state that for most practical systems, experimental data have shown that the effect of arbitrarily increasing the parameter $hA^*/fvc_a\rho_a$ in (16) and (17) has a negligible influence on the performance of the bed. If $hA^*/fvc_a\rho_a$ is allowed to become infinite, the difference between T_s and T_a must become zero for the equations to remain meaningful. So they propose, as do Duffie and Beckman [4] and Riaz [5], [6] that for the low air velocities encountered in a solar energy packed bed, $hA^*/fvc_a\rho_a$ can be treated as infinite, reducing (16) and (17) to one first order equation,

$$-A^*\rho_s c_s \frac{\partial T}{\partial t} = f\rho_a c_a v \frac{\partial T}{\partial x} + h_o A_s (T-T_o) \ . \qquad (18)$$

There is a clear disadvantage to simplifying the model this extensively; one is constrained to operating only within the region where $hA^*/fvc_a\rho_a$ approaches infinity. This limits what values the indicated parameters may take on, and necessitates an error analysis be performed on the model for all cases where $hA^*/fv\rho_a c_a$ drifts out of "practical system" limits. Since none of the authors offer a satisfactory definition of "practical system" limits, one is required to perform the error analysis for almost every new case, until a working knowledge of the sensitivity of the model to change is established.

Numerical Methods of Solution

Most of the recent particle bed heat transfer models used to describe solar energy heat storage systems are based upon numerical methods of solution. These numerical methods share a common drawback; since they are based upon breaking the bed into a finite number of dis-

crete elements and solving the resultant difference equations, they are inherently iterative. That is, all computations are based upon calculating temperatures in each of the elements of the bed, over relatively small time increments. Therefore, to get the air temperature of a certain bed segment after, say, fifteen time increments have elapsed, a minimum of fifteen different calculations are required. All but one of these numerical models use a "brute force" approach to the calculation that is accurate, but extremely time consuming. Duffie, Beckman and Klein [4], Mumma and Marvin [7], Clark, Nabozny and Heetderks [8], and Eshleman, Baird and Mears [9] have all developed such "brute force" schemes, with minor variations. The Duffie, Beckman and Klein (DBK) model is perhaps best representative of the numerical "finite difference" approach. DBK uses all the approximations of Hughes, et. al. discussed earlier to simplify Schumann's equations to the form of equation (18).

Given that all the simplifications performed are valid, equation (18) will describe both the air and rock temperature in the bed. To this equation an axial mixing term is added, because DBK believe axial mixing in the bed is significant;

$$-A^*c_s\rho_s \frac{\partial T}{\partial t} = f\rho_a c_a v \frac{\partial T}{\partial x} + h_o A_s (T-T_o) - k_x A^* \frac{\partial^2 T}{\partial x^2} \quad . \qquad (19)$$

The bed is then considered to be divided into N segments and the governing difference equations are expressed,

$$\frac{A^*\ell}{N} \rho_s c_s \frac{dT_1}{dt} = fc_a\rho_a v(T_{in}-T_1) - \frac{h_o A_s \ell}{N} (T_1-T_o) \frac{k_x A^*}{\ell} (T_1-T_2) \qquad (20)$$

for the first segment, or

$$\frac{A^*\ell}{N}\rho_s c_s \frac{dT_i}{dt} = fc_a\rho_a v(T_{i-1}-T_i) - \frac{h_o A_s \ell}{N}(T_i-T_o)$$

$$+ \frac{k_x A^*}{\ell}(T_{i-1}-T_i) - \frac{k_x A^*}{\ell}(T_i-T_{i+1}) \qquad (21)$$

for the intermediate segments (i = 2 to i = N-1) or,

$$\frac{A^*\ell}{N}\rho_s c_s \frac{dT_N}{dt} = fc_a\rho_a v\,(T_{N-1}-T_n) - \frac{h_o A_s \ell}{N}(T_N-T_o)$$

$$+ \frac{k_x A^*}{\ell}(T_{N-1}-T_N) \qquad (22)$$

for the final segment.

This technique produces a set of N simultaneous first order differential equations, which are solved for successive time increments to yield an air/rock temperature profile as a function of time. Obviously, since the time increments must be kept small to preserve accuracy, a large amount of computational time is necessary for solution.

The one rather unique, discrete numerical method of solution has been developed by Taft, Bailey and Alexander [10]. They start from a difference equation model of the air and rock temperature derived directly from Schumann's conservation of energy concept,

$$T_a(m)(k+1) = (1-a)\,T_a(m-1)(k) + a\,T_s(m)(k)\,, \qquad (23)$$

and

$$T_s(m)(k+1) = T_s(m)(k) + b[T_a(m-1)(k) - T_a(m)(k+1)] . \quad (24)$$

Combining equations (23) and (24) one obtains

$$T_s(m)(k+1) = a\ b\ T_a(m-1)(k) + (1-ab)\ T_s(m)(k) , \quad (25)$$

which is now of the same form as equation (22), but with the distinction between air and rock temperatures maintained.

The equations (23) and (25) are also a system of 2N simultaneous difference equations and can be written in matrix form,

$$\bar{T}(k+1) = \bar{A}\ \bar{T}(k) + \bar{B}u . \quad (26)$$

In this form, u is the outlet temperature and

$$T(k+1) = \begin{bmatrix} T_a(1)(k+1) \\ \vdots \\ T_a(N)(k+1) \\ T_s(1)(k+1) \\ \vdots \\ T_s(N)(k+1) \end{bmatrix} . \quad (27)$$

$\bar{A}$, $\bar{T}(k)$ and $\bar{B}$ are also written to conform to this matrix format. The solution to such a time invariant system may be written,

$$\bar{T}(k) = (\bar{A})^k\ \bar{T}(0) + \left[\sum_{i=1}^{k} (A)^{i-1}\ \bar{B}\right]u . \quad (28)$$

What makes this technique so powerful is that once the matrices $(\bar{A})^k$ and $\sum_{i=1}^{k} (\bar{A})^{i-1}\bar{B}$ are computed (they need only be done once since A and B are time invariant) the solution profile $\bar{T}(k)$ can be had by merely performing a bit of matrix algebra, saving significant computational time over classical time step-by-time step numerical techniques. The only disadvantages of this technique are that the input temperature must be approximated by a series of step temperature changes (a minor concern), the solution temperature profile appears as a series of stepped layers rather than a continuous distribution, and the solution accuracy depends upon the number of layers (N) chosen. The more layers used, the more accurate the profile, but more layers require more computational time. Taft, Bailey and Alexander have shown that usable accuracy may be obtained using as few as five bed segments.

The Klinkenberg and Harmens Solution

A rather unique method of solution to the Schumann equations is presented by Klinkenberg and Harmens [11]. They use the variable transformations,

$$y = \frac{hA^*}{\rho_a c_a f}\frac{x}{v} \quad \text{and} \quad z = \frac{hA^*}{c_s\rho_s(1-f)}\left(t-\frac{x}{v}\right) \tag{29}$$

to transform equations (5) and (6) into,

$$-hA^*\frac{\partial T_a}{\partial y} + hA^*(T_s-T_a) = \frac{c_a\rho_a f}{c_s\rho_s(1-f)}hA^*\frac{\partial T_a}{\partial z} \tag{30}$$

and,

$$hA^* \frac{\partial T_s}{\partial z} = -hA^*(T_s - T_a) \ . \tag{31}$$

Now, by combining equation (31) with (30) and by making use of the Hughes, et. al., idea that since $\rho_a << \rho_s$, the air stores little heat compared with the rock, the two differential equations that remain to be solved are

$$\frac{\partial T_a}{\partial y} - \frac{\partial T_s}{\partial z} = 0 \tag{32}$$

and

$$\frac{\partial T_s}{\partial z} = -(T_s - T_a) \ . \tag{33}$$

These are minor variants of Schumann's equations and have been solved by Schumann (as have been shown) for a step input into a bed initially at uniform temperature. The interesting (and useful) fact about Klinkenberg's method is that it provides a solution for arbitrary initial bed temperature (as long as the distribution is a piecewise continuous function) and arbitrary inlet temperature. The solution is not in terms of relatively slowly converging Bessel functions (as with Amundson) but is expressed as simple integral relations. The only disadvantage of the method is a minor one; it ignores completely the effects of losses at the walls. However, this can easily be corrected if it is determined that wall losses are significant in the system under consideration.

The method is as follows. Let a function of y and z, called F(y,z) represent the total amount of heat transferred in the bed from air to rock. Notice that F(y,z) can be expressed as an equivalent function in terms of x and t, but in its present form it has no physical "units" associated with it. Conservation of energy requires that the heat lost by the air equal the heat picked up by the solid,

$$F(y,z) = z - \int_0^z T_a(y,\xi)d\xi = \int_0^y T_s(\eta,z)d\eta \ . \tag{34}$$

This integral equation is an equivalent representation of the governing differential equations (32) and (33). Equation (34) implies that there is a close relationship between the partial derivatives of F(y,z) and the functions $T_a(y,z)$ and $T_s(y,z)$;

$$\frac{\partial F}{\partial z} = 1-T_a \tag{35}$$

and

$$\frac{\partial F}{\partial y} = T_s \ . \tag{36}$$

Consider now the step input, initial uniform bed temperature case where,

$$T_a(0,z) = 1 \text{ (normalized)} \tag{37}$$

and

$$T_s(y,0) = 0 \text{ (initialized to reference zero)} \ . \tag{38}$$

It is obvious from equations (35) and (37) that

$$1 - \frac{\partial F}{\partial z} = \text{response of air temperature to a unit step input change}$$

and from equations (36) and (38),

$$\frac{\partial F}{\partial y} = \text{response of solid temperature to a unit step input change.}$$

Up until now nothing has been done that was not obvious from Schumann's equations. But Klinkenberg now uses F(y,z) in such a way that its exact form need not be found; only the properties of its derivations need to be determined.

First, substituting (35) and (36) into (33), one obtains

$$\frac{\partial^2 F}{\partial y \partial z} + \frac{\partial F}{\partial z} + \frac{\partial F}{\partial y} = 1 . \tag{39}$$

This produces a second order partial differential equation describing F(y,z). To obtain suitable conditions then substitute (35) and (36) into (37) and (38);

$$\left(\frac{\partial F}{\partial y}\right)_{y,0} = 0 \tag{40}$$

and

$$\left(\frac{\partial F}{\partial z}\right)_{0,z} = 0 \tag{41}$$

Now consider equation (39) at the boundary y = 0 (equivalent to setting x = 0). It becomes a simple first order differential equation due to condition (41);

$$\frac{\partial^2 F}{\partial y \partial z} + \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z} = 1$$

becomes

$$\frac{\partial}{\partial z}\left(\frac{\partial F}{\partial y}\right) + \frac{\partial F}{\partial y} = 1 .$$

Let $\partial F/\partial y = \gamma$ and solve,

$$\frac{d\gamma}{dz} + \gamma = 1$$

to yield

$$\gamma = 1-e^{-z}$$

or

$$\left(\frac{\partial F}{\partial y}\right)_{0,z} = 1-e^{-z} . \qquad (42)$$

Likewise it can be shown that

$$\left(\frac{\partial F}{\partial z}\right)_{y,0} = 1-e^{-y} . \qquad (43)$$

Equations (39) through (43), although they refer to the constant temperature step input problem, will be useful in solving the more general problem of arbitrary temperature distribution and arbitrary input air temperature.

To tackle the general problem, change the initial conditions to reflect arbitrary temperature distribution and input,

$$T_a(0,z) = \Theta_a(z) \tag{44}$$

and

$$T_s(y,0) = \Theta_s(y) \ . \tag{45}$$

Remember that these are at least piecewise continuous functions (it is not likely that a solar heat storage system will be subjected to a true "impulse" input). Thus, the principle of superposition may be employed to describe both air and rock temperatures in the bed. Consider the air passing point y, at time z . It will have lost (or gained) heat exponentially provided no other disturbances had entered the bed before;

$$T_a(y_1,z_1) = e^{-y_1} \Theta_a(z_1) \ . \tag{46}$$

But this transfer will be increased by the contributions of all air temperature peaks which have entered between z_0 and z_1 and by the contributions of all solid temperature peaks originally existing between y_0 and y_1. This way of visualizing the problem is called the method of superposition of "heat poles" and was first presented by Hausen in 1931. The general equation describing the air temperature at any point in the bed at any time can then be written

$$T_a(y,z) = e^{-y}\,\theta_a(z) - \int_0^z \theta_a(\alpha)\left[\frac{\partial^2 F}{\partial z^2}\right]_{y,(z-\alpha)} d\alpha$$

$$+ \int_0^y \theta_s(\beta)\left[\frac{\partial^2 F}{\partial y \partial z}\right]_{(y-\beta),z} d\beta \qquad (47)$$

and the solid temperature is described by,

$$T_s(y,z) = e^{-z}\,\theta_s(y) + \int_0^z \theta_a(\alpha)\left[\frac{\partial^2 F}{\partial y \partial z}\right]_{y,(z-\alpha)} d\alpha$$

$$- \int_0^y \theta_s(\beta)\left[\frac{\partial^2 F}{\partial y^2}\right]_{(y-\beta),z} d\beta\,. \qquad (48)$$

The order of the partial derivates within the integrals may be reduced by partial integration to yield,

$$T_a(y,z) = \theta_a(z) - [\theta_a(0)-\theta_s(0)]\left[\frac{\partial F}{\partial z}\right]_{y,z}$$

$$- \int_0^z \theta_a'(\alpha)\left[\frac{\partial F}{\partial z}\right]_{y,z-\alpha} d\alpha + \int_0^y \theta_s'(\beta)\left[\frac{\partial F}{\partial z}\right]_{(y-\beta),z} d\beta \qquad (49)$$

and

$$T_s(y,z) = \theta_s(y) + [\theta_a(0)-\theta_s(0)]\left[\frac{\partial F}{\partial y}\right]_{y,z}$$

$$+ \int_0^z \theta_a'(\alpha)\left[\frac{\partial F}{\partial y}\right]_{y,(z-\alpha)} d\alpha - \int_0^y \theta_s'(\beta)\left[\frac{\partial F}{\partial y}\right]_{(y-\beta),z} d\beta\,. \qquad (50)$$

Equations (49) and (50) are general solutions of the original problem ((32) and (33)). They are expressed in closed, definite integral form and depend upon knowing the partial derivatives $\partial F/\partial y$ and $\partial F/\partial z$. Klinkenberg has demonstrated that without having to determine $F(y,z)$ directly, the partial derivatives may be approximated by,

$$\frac{\partial F}{\partial y} = \frac{1}{2} + \frac{1}{2}\,\mathrm{erf}\left[\sqrt{z} - \sqrt{y} - \frac{1}{8\sqrt{z}} - \frac{1}{8\sqrt{y}}\right] \tag{51}$$

and

$$\frac{F}{y} = \frac{1}{2} + \frac{1}{2}\,\mathrm{erf}\left[\sqrt{z} - \sqrt{y} + \frac{1}{8\sqrt{z}} - \frac{1}{8\sqrt{y}}\right] \tag{52}$$

for values of y and $z > 2$ or,

$$\frac{\partial F}{\partial y} = \frac{1}{2} + \frac{1}{2}\,\mathrm{erf}\left[\sqrt{z} - \sqrt{y}\right] - \frac{z^{1/4}}{y^{1/4} + z^{1/4}} \cdot e^{-(y+z)}\, I_0(2\sqrt{yz}) \tag{53}$$

and

$$\frac{\partial F}{\partial z} = \frac{1}{2} - \frac{1}{2}\,\mathrm{erf}\left[\sqrt{z} - \sqrt{y}\right] - \frac{y^{1/4}}{y^{1/4} + z^{1/4}} \cdot e^{-(y+z)}\, I_0(2\sqrt{yz}) \tag{54}$$

for values of y and z between 1 and 2.

Unfortunately, the values of y and z for solar energy storage packed beds are very often <1. This necessitates the development of an alternate form for the partial derivatives of F(y,z). Nusselt [12] has developed a series representation for F(y,z) which is

$$F(y,z) = \sum_{n=}^{\infty} \left[\left(1-e^{-z} \cdot \sum_{k=0}^{n} \frac{z^k}{k!} \right) \left(1-e^{-y} \cdot \sum_{k=0}^{n} \frac{y^k}{k!} \right) \right] \tag{55}$$

Initially, this looks like a rather cumbersome form to work with, but with an added restriction applied and a little manipulation performed the partial derivations of F(y,z) can be had in a usable form. The added restriction is that neither y nor z can equal zero. Then F(y,z) may be rewritten,

$$F = (1-e^{-z})(1-e^{-y}) + \sum_{n=1}^{\infty} \left[1-e^{-y} - e^{-y} \sum_{k=1}^{n} \frac{y^k}{k!} \right] \left[1-e^{-z} \cdot \sum_{k=0}^{n} \frac{z^k}{k!} \right] . \tag{56}$$

Then $\partial F/\partial y$ becomes,

$$\frac{\partial F}{\partial y} = (1-e^{-z})(e^{-y}) + \sum_{n=1}^{\infty} \left[1-e^{-z} \cdot \sum_{k=0}^{n} \frac{z^k}{k!} \right] \cdot \left[e^{-y} - \left[-e^{-y} \cdot \sum_{k=1}^{n} \frac{y^k}{k!} + e^{-y} \cdot \sum_{k=1}^{n} \frac{k_y^{(k-1)}}{k!} \right] \right] \tag{57}$$

which when simplified is,

$$\frac{\partial F}{\partial y} = e^{-y} \left[(1-e^{-z}) + \sum_{n=1}^{\infty} \left[1-e^{-z} \sum_{k=0}^{n} \frac{z^k}{k!} \right] \left(\frac{y^n}{n!} \right) \right] . \tag{58}$$

and,

$$\frac{\partial F}{\partial z} = e^{-z}\left[(1-e^{-y}) + \sum_{n=1}^{\infty}\left[1-e^{-y}\sum_{k=0}^{n}\frac{y^k}{k!}\right]\left[\frac{z^n}{n!}\right]\right] . \qquad (59)$$

These expressions for $\partial F/\partial y$ and $\partial F/\partial z$, while still in series form, are very rapidly convergent for all values of y and z not equal to zero. This makes the calculation of T_a and T_s using equations (49) and (50) a relatively quick process. In addition, these partial derivative forms do not suffer from the weakness of the Klinkenberg approximations; equations (58) and (59) are exact expressions, so no error due to approximation is introduced into the final solution.

Klinkenberg and Harmens' solution is a good compromise between the analytic solutions of Amundson which are so complex as to be impractical and the solution of Schumann which is too limited to be of wide applicability. It is interesting to note that all models presently used to describe solar heat storage in packed rock beds are based upon numerical techniques of solution to difference equations. The contention of the other models' authors is that the governing equations cannot be practically solved by analytical methods. One can only assume that the work of Klinkenberg (which was published in a rather obscure British journal) has escaped widespread notice. The packed bed model to be developed in this paper will use the Klinkenberg technique, with partial derivatives developed from Nusselt's function for F(y,z) as a starting point.

CHAPTER III

AN INVESTIGATION OF THE PHYSICAL SYSTEM-DETERMINATION OF VALID ASSUMPTIONS

In the last chapter several different packed bed modeling schemes were discussed; each model had certain assumptions either expressed outright or implied by the nature of the model. In this chapter a "standard packed bed" (one that in size and shape closely resembles the packed rock beds used in solar heat storage systems) shall be closely examined to determine which assumptions, if any, are not valid. Axial and radial conduction in the standard bed shall be investigated, as shall the effect of losing energy through the walls of the bed due to turbulent convective transfer at the bed-wall interface. Finally, the heat content of a standard bed volume of air shall be compared to the heat content of a standard bed filled entirely with rock, to determine just what fraction of the heat in a packed bed system is likely to be stored in the air.

The Standard Model

For purposes of analysis and comparison the "standard model" shall be taken to be as depicted in figure 5. It is cylindrical with a radius of 1 meter and height of 2 meters. During the charging mode, hot air is forced down through the bed at a mean velocity, v. For analysis, the assumed air velocity shall be 1.0 meters/second unless otherwise noted. This is faster than the speed at which air normally moves through a packed bed, but it shall be treated as an upper limiting case here. The rock within the bed is dry sandstone, with a thermal conductivity of

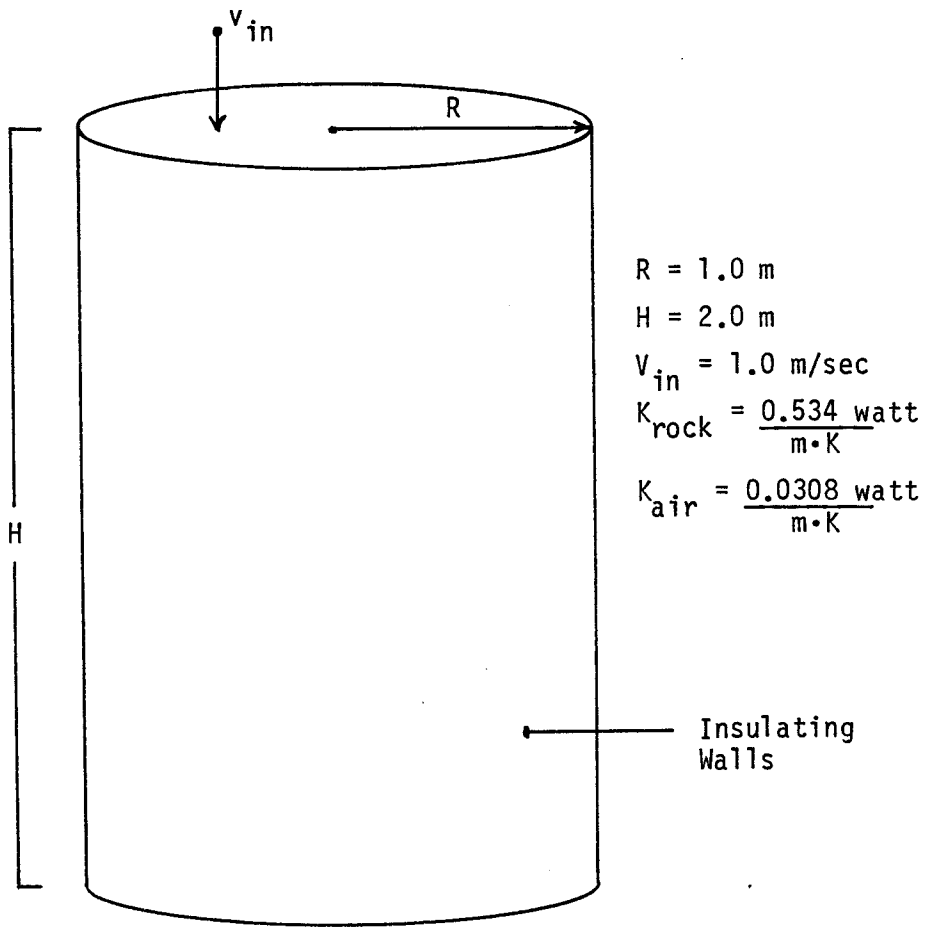


Fig. 5. A "Standard Packed Bed" for Analysis

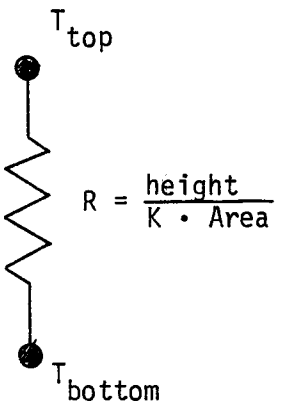


Fig. 6. Thermal Resistance Model of Steady-State Heat Conduction

0.534 watt/m·K [13]. The air within the bed, over the temperature range of interest and at the low velocities encountered within the bed, is taken to be an incompressible fluid [14]. The thermal conductivity of air at 1 atm pressure and 360°K is 0.0308 watt/m·K [15]. The convective film heat transfer coefficient is a function of air velocity through the bed, but a reasonable guess as to its upper bound is 25 watt/m^2·K [16].

Axial Conduction Through Rock

As an initial assumption, let the bed be composed entirely of rock (no air) and let the side walls be lossless. Then the only mode of heat transfer is via conduction through the rock from the top to the bottom. As absolute maximum steady-state conditions consider the temperature at the top of the bed to be 80°C (176°F) and the bottom temperature to be 10°C (50°F). Heat conduction through a solid is described by Laplace's equation, here taken in only one dimension ($\partial^2 T/\partial x^2 = 0$). The solution to this form of Laplace's equation gives the temperature through the bed as a linear function of the distance down from the top of the bed, so the power transferred from the top to the bottom of the bed may be modeled using a simple thermal resistance between the two temperature regions; this model is shown in figure 6.

The power transferred is given by the equation

$$q = \frac{\Delta T}{R} \tag{60}$$

where

ΔT = the temperature difference between the top and bottom of the bed (in C°),

q = power transferred (in watts).

The thermal resistance for sandstone is 1.192 K/watt, hence the net power flow in this standard bed is 58.72 watts, or in terms of power density for any sized bed is q/Area = 18.69 watts/m^2.

Of course the axial condition through rocks in a typical rock bed would be even less than the above power density term indicates, since the rocks only touch each other over an extremely small area, and the entire bed is not made of rock. It is then safe to assume that axial conduction within the rocks of a packed bed is a negligible component of the heat transfer process within the bed.

Axial Conduction Through Air

Again make the same limiting assumptions as in the rock axial conduction case, but consider a standard bed filled entirely with air. Of course in any such system thermal convection will occur, but constrain the system to have no forced convection; then convection's effect is contained within the previously mentioned air conductivity term (it is a turbulent conductivity term). The resulting air thermal resistance is 20.67 K/watt, making the net power flow in the standard bed just 30.3 watts. More generally, the power density is 1.08 watts/m^2. Thus, it is also safe to assume that axial conduction through air is insignificant in the overall heat transfer picture. In fact, the worst case air conduction power density value is only 1/17 the magnitude of the rock axial conduction power density.

Radial Conduction Through Rock

As suitable limiting case assumptions for this analysis it can be said that the storage bed is composed entirely of solid rock and there is a hole of radius 0.2 m (7.5 in., a figure selected arbitrarily; any small radius hole will serve) drilled axially through the center of the bed (see the cross section, figure 7). If the radius of the hole is called r_h, and that of the bed r_b and if it is now assumed that the top and bottom of the bed are lossless, heat transfer can only take place radially through the bed. If the hole medium is kept at a temperature different from that surrounding the outside of the bed, the Fourier Heat Transfer equation [17] can be used to determine the power flow from the hole to the outside wall in the steady-state case. The applicable equation is

$$q = -kA \frac{dT}{dr} \tag{61}$$

where

k = the thermal conductivity,

A = the surface area of either the inner wall or the outer wall of the bed (= $2\pi r\ell$),

and

dT/dr = the radial change of temperature in the bed.

So,

$$q = -2\pi r\ell k \frac{dT}{dr}$$

or after solving this first order differential equation,

$$q = \frac{2\pi \ell k \, (T_{rh}-T_{rb})}{\ln \, (rb/rh)} \tag{62}$$

where

T_{rh} = temperature within the hole,

and

T_{rb} = temperature outside the bed.

For T_{rh} = 80°C and T_{rb} = 10°C, q = 282.8 watts in the standard bed. The power density at the outer wall is 22.5 watts/m^2. This analysis shows that while the power lost through radial conduction in rock is greater than rock axial conduction power loss, it is still an insignificantly small value.

Radial Conduction Through Air

The same procedure, with the same resultant equation, is used to obtain air radial conduction. Again, the bed is assumed to be entirely made up of dry air at 1 atm. For the standard bed at the outer wall the power lost is 16.42 watts, resulting in a power density through the outer wall of 1.31 watts/m^2.

This is so small that it can also be considered insignificant, especially since the effect of insulating bed walls (which will reduce all of the above power densities still further) has not been investigated.

Convective Wall Losses

It should be noted that (as was seen in Chapter 2) of all the models developed presently, most neglect the effects of axial and radial conduction. About half of them use some term within the modeling equations to account for convective losses out of the walls of the bed. Obviously it is of extreme importance to know if these losses are significant because if they are not, allowing for them in the model is inserting an extra measure of complication into a problem that is already very complex. Also, if much energy is lost out the side walls, the simple "average loss" terms used in many of those models that allow for convective losses may not be adequate to describe what really goes on in the bed.

Again performing a worst case analysis, consider the standard bed model. Assume the bed is filled entirely with air. Let the air temperature within the main body of the bed equal 80°C; let the outside air temperature equal 10°C. The typical solar energy packed bed wall is constructed of concrete block. Assume it is lined on the inside by a 2 inch thickness of styrofoam covered by a 1 inch thickness of plywood. The thermal conductivity of concrete is 1.20 watt/m•K. The conductivities of styrofoam and plywood are 0.043 watt/m•K and 0.029 watt/m•K, respectively. Figure 7 presents the wall in cross section. Assume air is flowing into the bed, so the convective film heat transfer coefficient at the wall is 40 watt/m^2•K. Assume natural convection is taking place outside the bed, so the heat transfer coefficient on the outside is 4 watt/m•K. The air space in normal concrete block shall be neglected, so the block is modeled as a 2 inch thickness of concrete. Ignoring any

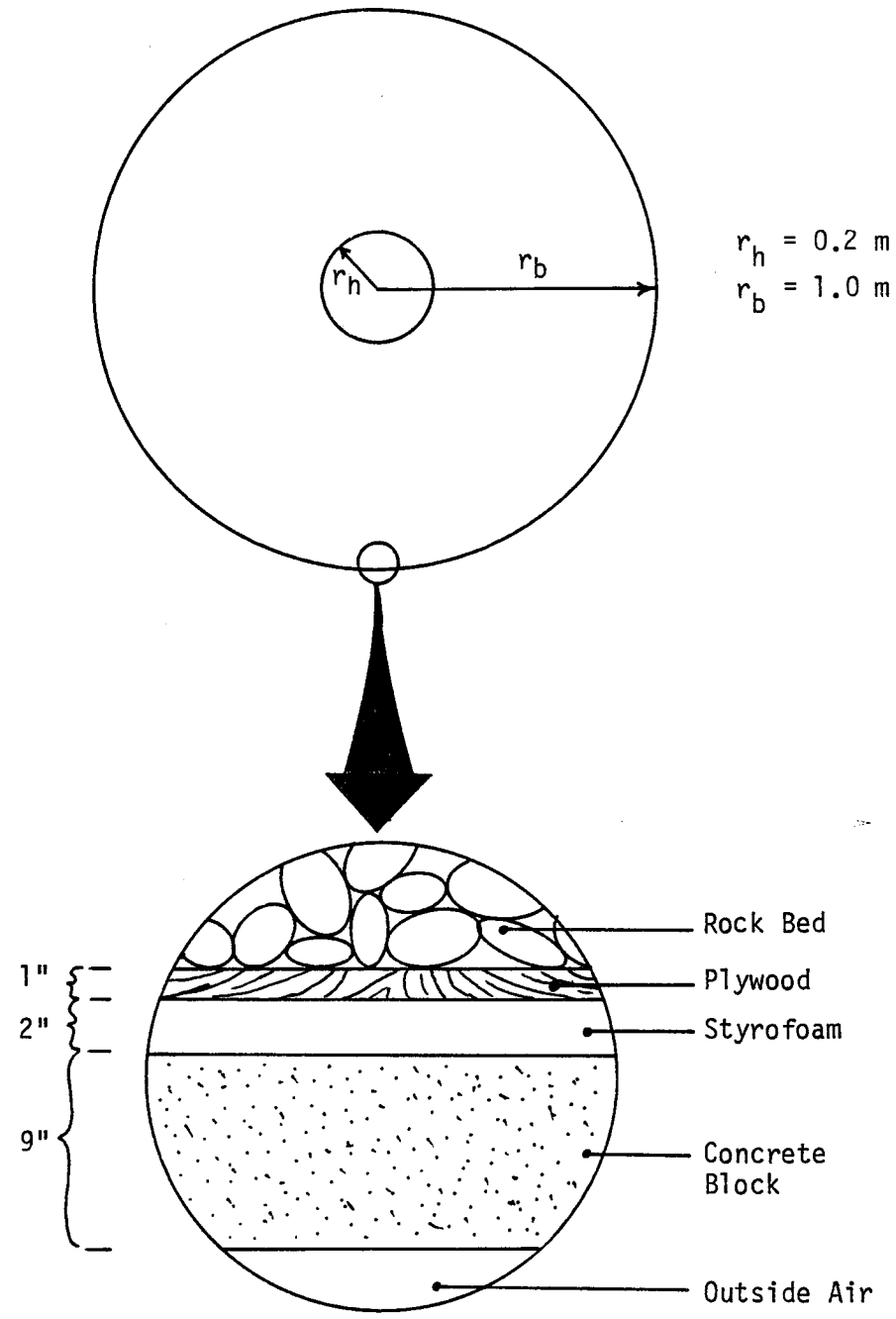


Fig. 7. Conduction/Convection Analytic Model

possible shape factor arising from the fact that the wall is cylindrical, the heat transfer model is again a simple resistive network. (See figure 7.) The surface area of the wall is 2π x wall radius x wall height, which equals (for the standard bed) 12.57 m^2. The resistance of the wood, styrofoam and concrete are given by the equation:

$$R_{thermal} = \frac{\text{thickness of wall}}{(\text{conductivity})(\text{area})} . \tag{63}$$

So, R_{wood} = 0.0157 K/watt, $R_{styrofoam}$ = 0.0940 K/watt, $R_{concrete}$ = 0.0034 K/watt. The equivalent thermal resistance of convective air is given in the equation:

$$R_{convective} = \frac{1}{(\text{heat transf. coeff.})(\text{area})} \tag{64}$$

Thus, $R_{conv.\ inside}$ = 0.002 K/watt and $R_{conv.\ outside}$ = 0.020 K/watt. The total thermal resistance in the network is R_{total} = 0.1351 K/watt. The amount of heat escaping from the side walls of the bed is given by:

$$q = \frac{\Delta T}{R} \tag{65}$$

So the "standard bed" is losing q = 518 watts. The power density through the side walls in the q/A = 41.22 watts/m^2.

This is clearly quite a low power density, yet it is by no means insignificant. It indicates that a fully charged (i.e., $T_A = T_S$ = 80°C everywhere) bed will lose about 1.85 x 10^6 joules (about 1800 Btu) in a period of one hour just out of the side walls. Whether or not this is a significant fraction of the energy that is stored in the bed will be investigated in the following section.

Heat Content Of Air/Rock In The Bed

This section serves two purposes. Herein will be determined how much energy is stored within a fully charged packed bed, and what percentage of that heat energy is stored in each of the two phases, air and rock, which make up the system. Previous sections dealt with temperature limits of 80°C and 10°C: 80°C because that is very near to the maximum temperature the collectors in a practical solar energy home heating system will supply, and 10°C because that is approximately the temperature of the earth ten feet below the surface, hence it is the lowest temperature one would expect to encounter in an unheated basement. Yet, the average ambient temperature of a house is about 20°C (68°F), hence the useful, extractable, energy within the packed bed will be at a temperature of about 20°C. Thus, the reference temperature limit for this analysis is 20°C.

The amount of useful heat energy stored in a substance depends upon its temperature above the reference (here 20°C), its specific heat, and its density according the the relation,

$$\omega = \rho V c \, \Delta T \tag{66}$$

where

ω = heat energy in a given amount of material,

ρ = density of the material,

V = volume of the material,

c = specific heat of the material, and

ΔT = temperature above the reference.

Assuming the standard bed is filled completely with air at 80°C, the heat contained in that volume of air is

$$\omega_{air} = \left(0.98 \frac{kg}{m^3}\right)\left(2\pi\ m^3\right)\left(1.01\times10^3 \frac{joules}{kg\cdot°C}\right)\left(60°C\right)$$

or

$$\omega_{air} = 3.73\times10^5 \text{ joules (about 355 Btu).}$$

Now consider the standard bed filled completely with rock at 80°C;

$$\omega_{rock} = \left(2400 \frac{kg}{m^3}\right)\left(2\pi\ m^3\right)\left(1.05\times10^3 \frac{joules}{kg\cdot°C}\right)\left(60°C\right)$$

or

$$\omega_{rock} = 9.5\times10^8 \text{ joules (90,000 Btu).}$$

Obviously for a bed void fraction of 0.5 (50 percent of the bed is rock, 50 percent is air) the air component of stored energy is only 0.4 percent of the total bed energy. Also one may conclude that the bed is losing only about 5 percent of its stored energy per given hour to the outside, including losses both from the side walls and the top and bottom.

Results

The analyses performed in the preceding sections of this chapter have shown that axial and radial conduction within a packed bed used for solar heat energy storage may be neglected in the modeling of the bed. Also, losses to the outside due to convection within the bed may also be neglected without significant degradation in the performance of the model, provided adequate thermal insulation is assumed to exist. However, for the model to describe a real system most accurately, some provision to allow for convective losses may be made.

CHAPTER IV

THE HOTROCKS MODEL

The HOTROCKS particle bed heat storage computer model, which will be described in detail within the body of this chapter, is developed from the analytical work done by Klinkenberg. Klinkenberg's solutions are used as a starting point for several good reasons. Initially, his solutions are analytical; they are readily checked merely by substitution into the original system equations. Also, Klinkenberg avoids unnecessary complication of the model. In Chapter III it was shown that many of the second order mechanisms of heat transfer in a packed bed (such as axial and radial conduction transfer) are an insignificant fraction of the complete heat transfer phenomenon. Ignoring these second order effects greatly simplifies the model, and one is justified in doing so. Further, Klinkenberg makes no simplifying assumptions that would tend to degrade the accuracy of the model, as is the case with some of the discrete segment models discussed in Chapter II.

The HOTROCKS Model, Step by Step

The HOTROCKS computer model is designed to provide an hourly simulation of the thermal transfer between air and rock within a packed bed solar heat energy storage system. The HOTROCKS routine furnishes as output a hard copy table of air and rock temperatures of ten equispaced locations within the bed, updated every hour. The

resultant temperature profile may be used (using methods similar to those detailed in Chapter III) to determine the amount of thermal energy stored within the bed as a function of time. The output may also be used to furnish hourly energy availability information for a real time solar energy system simulation model (i.e., with minor modifications, HOTROCKS can be easily incorporated into a larger simulation routine). In addition, the time increment of calculation is not constrained to be one hour; it may be varied to suit any constraints imposed by the larger routine.

Figure 8 shows a block diagram of HOTROCKS. To make validation of the final program easy, a modular structured programming approach was used in developing the model. Detailed flow charts, using the IBM-MIT conceived structured flow symbology, describing the logic behind each of the HOTROCKS routines, are contained in Appendix A. Appendix B includes a complete FORTRAN listing of the HOTROCKS program, and Appendix C contains a listing of each of the test modules built to verify the operation of each of the HOTROCKS routines.

The routine MAIN is nothing more than a supervisor. It ensures that all computational routines are called in the proper order. Since the model is designed as an hour-by-hour simulation, MAIN must also determine (from input data) just how many hours the simulation is to run.

The routine INPUT reads from input data all the values of the physical constants and initial parameters necessary for the operation of the simulator. When a current simulation run is complete, OUTPRT prints an hourly listing of air and rock temperatures within the rock bed. A table of the initially input constants is also printed.

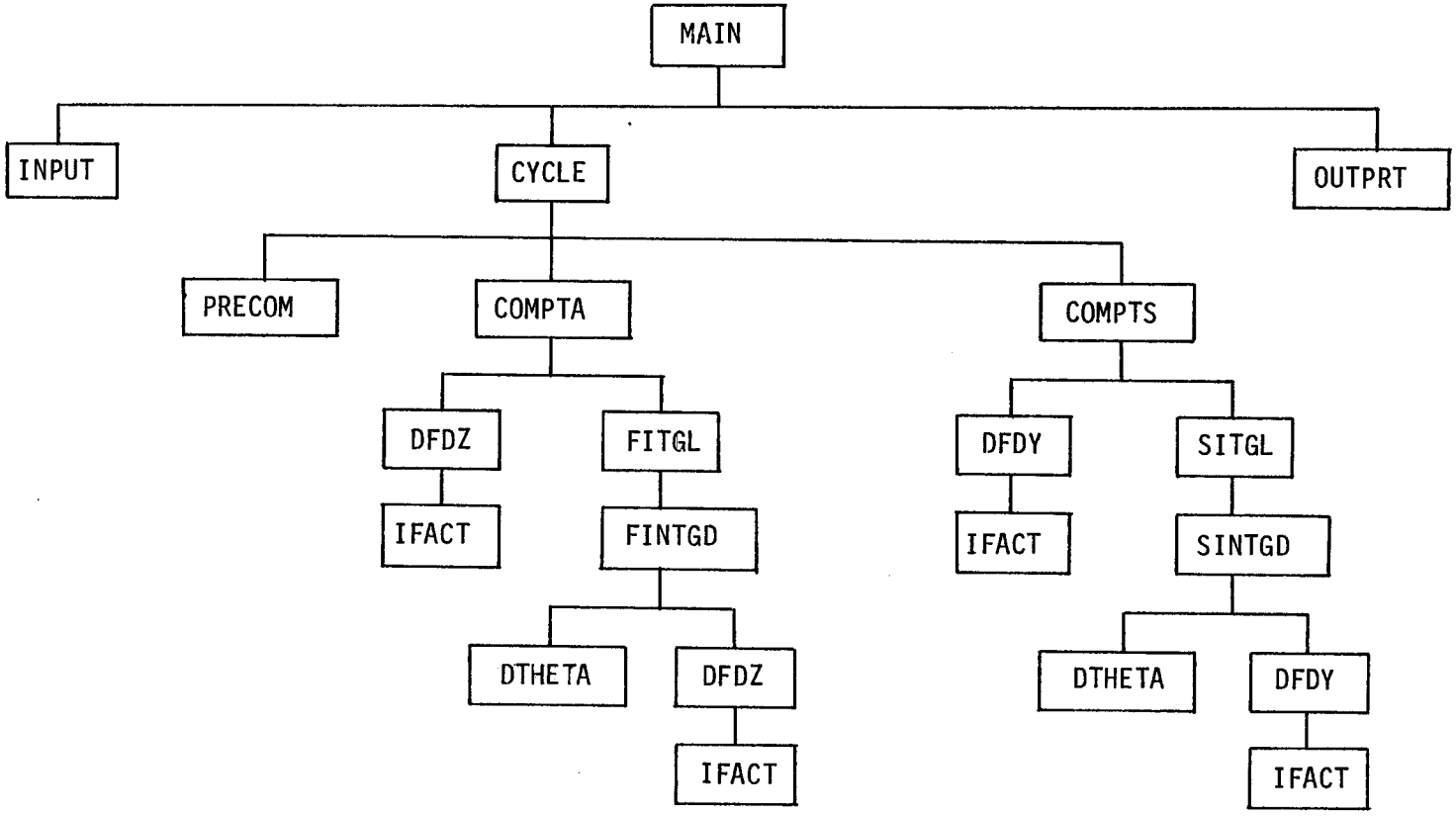


Fig. 8. Block Diagram of the HOTROCKS Program

Calculation of the solutions to Klinkenberg's heat transfer equations is greatly simplified when dimensionless parameters (i.e., parameters which have no physical units associated with them, but are proportional to quantities such as position or time) are used. HOTROCKS uses the transformations for position ($x \rightarrow y$) and time ($t \rightarrow z$) devised by Klinkenberg to provide working dimensionless parameters. The PRECOM routine precomputes numerical values for y and z at the start of each hour's calculations.

Just as the MAIN routine ensures that each of the other routines of HOTROCKS gets executed in the proper order, CYCLE ensures that each hourly calculation of air and rock temperature is performed using the correct set of initial data for each hour. CYCLE keeps track of each hour of calculations as they are being performed. CYCLE also determines, from the value of input air velocity, whether the system is in the charge, discharge or off mode of operation for each given hour. Positive air velocity corresponds to flow from the top of the bed down (charge), whereas negative velocity indicates flow from the bottom up (discharge). A different routine within CYCLE is used for each case. If during any given hour there is no air flow (velocity equals zero), the system is shut off and quasi-steady-state (no change in segment temperatures) is assumed. The air and rock temperatures that are returned from routines called by CYCLE are placed in a matrix to be output after the simulation is complete.

A close examination of the Klinkenberg solutions (equations 49 and 50, Chapter II) reveals that they can be even further simplified for the case where the input air temperature is constant over a par-

ticular time increment. Since insignificant loss in accuracy occurs by approximating the continuous input into a packed by by a series of step inputs of sufficiently short time duration, and since the thermal inertia of a solar energy packed bed is great enough to allow one-hour increments to be considered as sufficiently short, appropriately simplified Klinkenberg solutions are adequate for HOTROCKS. These solutions are,

$$T_a(y,z) = \theta_a - [\theta_a - \theta_s(0)]\left(\frac{\partial F}{\partial z}\right)_{y,z} + \int_0^y \theta_s'(\eta)\left(\frac{\partial F}{\partial z}\right)_{y-\eta,z} d\eta \qquad (67)$$

and

$$T_s(y,z) = \theta_s(y) + [\theta_a - \theta_s(0)]\left(\frac{\partial F}{\partial y}\right)_{y,z} + \int_0^y \theta_s'(\eta)\left(\frac{\partial F}{\partial y}\right)_{y-\eta,z} d\eta \qquad (68)$$

These are the equations that are solved by the HOTROCKS routines COMPTA and COMPTS. Inputs to these routines include the initial rock temperature distribution (as ten discrete elements, $\theta_s(y)$), the constant input air temperature (θ_a), the time increment of the calculation (z) and the position coordinates (as ten elements, y) of the locations in the bed for which calculations are to be performed. At the end of the execution of these routines, a list of ten elements corresponding to air temperature T_a (or rock temperature T_s) is returned to CYCLE.

The partial derivative functions $\partial F/\partial y$ and $\partial F/\partial z$ are computed in routines DFDY and DFDZ using the variation of Nusselt's heat transfer function developed in Chapter 2 (equations 58 and 59). To avoid

excessive computation time, the series are only expanded to the number of terms that will insure accuracy of $\partial F/\partial y$ or $\partial F/\partial z$ to ± 0.001 percent; at no time will more than the first twelve terms of the series be needed. Since factorials must be evaluated as part of DFDY and DFDZ, and since many FORTRAN compilers do not have available a factorial library function, the routine IFACT was created. IFACT will calculate the factorial of any integer n, where $0 \leq n \leq 12$. The modified Nusselt series do not work at values of y or z equal to zero. So, if $y = 0$ or $z = 0$ is input to either DFDY or DFDZ, the routines replace those zero values with a very small positive number. Since $\partial F/\partial y$ and $\partial F/\partial z$ are well-behaved functions, this substitution will not degrade the accuracy of the resultant air and rock temperature calculations.

Both the air and rock temperature calculations require the evaluation of a definite integral. The routines FITGL and SITGL perform the integration function. Both routines use a Romberg integration algorithm, similar to the algorithm presented in Burden, Faires and Reynolds [18], but modified somewhat to fit a structured FORTRAN form. The Romberg algorithm was chosen as a good compromise between computational speed and accuracy. FITGL (or SITGL) will calculate the numerical value of its integrand to within ± 0.01 percent.

The integrands needed for the evaluation performed by FITGL and SITGL are produced by the routines FINTGD and SINTGD. These routines require that the variation in rock temperature as a function of bed position (i.e., $\theta_s'(y)$) be known. This derivative is calculated from the ten initial rock bed segment temperatures by the routine DTHETA. DTHETA simply calculates the slope of the line between the two known rock temperature values nearest to the input bed position, as an approximation to the actual derivative at that input position.

Those are the modules that make up HOTROCKS. The HOTROCKS program has been subjected to an extensive verification process, both at the modular and the complete program level. These verification and testing procedures are the subject of Chapter 5.

CHAPTER V

HOTROCKS MODEL VERIFICATION

Module Verification

Although the structure of HOTROCKS was planned from the "top down", actual coding and verification of each of the modules was accomplished from the "bottom up"; each module, beginning with IFACT, was built and verified before the next module above it in the hierarchy was begun. As was noted in Chapter 4, the test routines used to verify each of the modules are listed in Appendix C; only the results of those tests will be presented in this chapter.

The routine IFACT was the first built and the first tested. It is a simple integer calculation algorithm, and as table 1 shows, the comparison between actual factorials and those calculated by IFACT are exact for at least those values of $N \leq 13$. Those values of $N > 13$ result in incorrect data only because the integer precision of the machine used to test the module is exceeded for $N > 13$.

Next to be written and tested were the modules DFDY and DFDZ. The test routine built for these two modules was designed as a reasonableness check. A range of y and z values representative of all the values that HOTROCKS is likely to encounter was input, with the results shown in table 2. Several values of $\partial F/\partial y$ and $\partial F/\partial z$ were calculated by hand, using an HP 25 calculator; these are listed as expected values in the table. Notice that the computed module solutions compare favor-

TABLE 1

RESULTS OF MODULE IFACT VERIFICATION

N	N! Computed	N! Expected
0	1	1
1	1	1
2	2	2
3	6	6
4	24	24
5	120	120
6	720	720
7	5040	5040
8	40320	40320
9	362880	362880
10	3628800	3628800
11	39916800	39916800
12	479001600	479001600
13	6227020800	6227020800

TABLE 2

RESULTS OF MODULES DFDY/DFDZ VERIFICATION

Y	Z	DFDY HOTROCKS	DFDY Expected	Δ DFDY (%)
1.0	1.0	.34574	.34574	0
0.1	10.0	.99900	.99995	-0.09%
1.0	10.0	.99792	.99812	-0.02%

Y	Z	DFDZ HOTROCKS	DFDZ Expected	Δ DFDZ (%)
1.0	1.0	.34575	.34574	+0.01%
0.1	10.0	6.8610×10^{-6}	6.860×10^{-6}	+0.01%
1.0	10.0	5.7267×10^{-4}	5.7270×10^{-4}	-0.01%

ably with the expected values in every case. The complete DFDY and DFDZ test outputs along with a sample $\partial F/\partial y$ expected value calculation are included with the DFDY test listing in Appendix C.

The routine DTHETA was verified by comparing its output against expected values for both a linear and sinusoid input. These two test temperature functions were chosen because the actual rock temperature distribution in a packed bed will be somewhere in between the two, for any realistic input. Table 3 graphically depicts the linear and sinusiod functions, and lists both the module's generated and expected values of Θ_s' for ten positions in the bed. Initial inspection suggests that the DTHETA response to sinusoidal input may be a potential problem. But since the Θ_s' term is only used in the integral portion of the T_a and T_s solutions, and since the integral portion is a relatively small component of the total expressions for T_a and T_s, the module DTHETA can afford to sacrifice some accuracy in return for speed of computation.

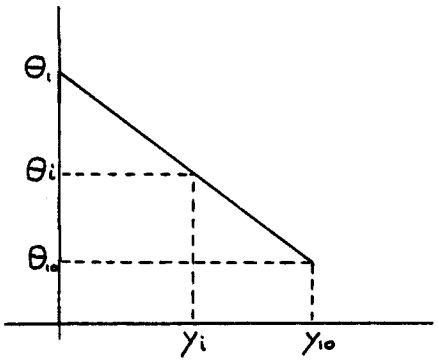
The modules FINTGD and SINTGD are FORTRAN coding of the algebraic function found in the integrand of the air temperature or rock temperature integral. These routines can be verified by inspection, so no formal test procedures were developed for them.

The modules FITGL and SITGL are identical with the exception of the integrands used in the routines. Thus, a test of FITGL will serve to validate the function of SITGL as well. The test chosen is basic; three functions, representing forms close to those likely to be encountered by the integrator during actual HOTROCKS execution, are entered in FITGL as the integrand FINTGD. Then FITGL is made to inte-

TABLE 3

RESULTS OF MODULE DTHETA VERIFICATION

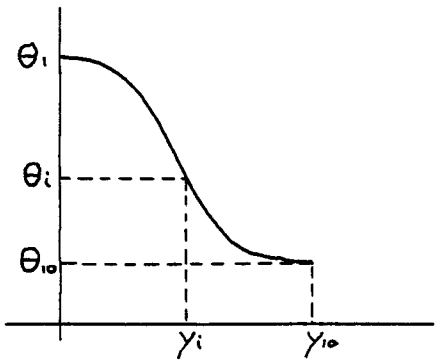
A. Linear Temperature Distribution



Linear Distribution:
$\theta = -25y + 70$

Input y	Input θ	DTHETA θ′	Expected θ′	Delta θ′
0.	70.	-25.	-25.	0.
0.2	65.	-25.	-25.	0.
0.4	60.	-25.	-25.	0.
0.6	55.	-25.	-25.	0.
0.8	50.	-25.	-25.	0.
1.0	45.	-25.	-25.	0.
1.2	40.	-25.	-25.	0.
1.4	35.	-25.	-25.	0.
1.6	30.	-25.	-25.	0.
1.8	25.	-25.	-25.	0.

B. Sinusoid Temperature Distribution



Sinusoid Distribution
$\theta = 22.5 \cos(1.745y) + 47.5$
$\theta' = -39.26 \sin(1.745y)$

Input y	Input θ	DTHETA θ′	Expected θ′	Delta θ′
0.	70.0	-7.0	0.	+7.0
0.2	68.6	-13.3	-13.4	-0.1
0.4	64.7	-24.8	-25.2	-0.4
0.6	58.8	-33.5	-34.0	-0.5
0.8	51.4	-38.0	-38.7	-0.7
1.0	43.6	-38.0	-38.7	-0.7
1.2	36.3	-37.0	-34.0	+3.0
1.4	30.3	-29.5	-25.3	+4.2
1.6	26.4	-13.0	-13.5	-0.5
1.8	25.0	-6.5	0.	+6.5

grate each of those functions three times, from zero to three different input argument values. The FITGL result is then compared to an analytically derived expected result. Results of this test are summarized in table 4. Obviously, the FITGL and SITGL routines will be accurate to at least five significant figures for continuous functions.

The routines COMPTA and COMPTS are algebraic statements, FORTRAN coded, which correspond to the equations used to calculate air and rock temperature. So again, no formal testing is required for these two modules.

PRECOM does several things; it determines the x coordinate value of the ten test points located within the bed, and it develops the dimensionless length and time parameters, y and z, for each of these ten locations. The test for PRECOM is as follows; input values for the constants used to calculate the parameters y and z, and confirm by an independent calculation that PRECOM is performing the proper calculation. As can be seen from table 5, PRECOM is performing satisfactorily.

The module CYCLE oversees the proper execution of most of the program. Therefore, all modes of CYCLE must be confirmed. The program used to test CYCLE checks all three operating modes: the $v > 0$ (charge) mode, the $v < 0$ (discharge) mode and the $v = 0$ (off) mode. As shown in table 6, all modes work properly; test values and expected values compare identically.

TABLE 4

RESULTS OF MODULE FITGL VERIFICATION

A. Linear Integration FINTGD = -4.5x + 70

Input (x)	FITGL Result	Expected Result	Delta
1.0	67.750	67.750	0
3.0	189.75	189.75	0
10.0	475.00	475.00	0

B. Exponential Integration FINTGD = $71.2\ e^{-(0.1x)} - 1.2$

Input (x)	FITGL Result	Expected Result	Delta
1.0	66.556	66.556	0
3.0	180.94	180.94	0
10.0	438.07	438.07	0

C. Sinusoid Integration FINTGD = $22.5 \cos(x\pi/10) + 47.5$

Input (x)	FITGL Result	Expected Result	Delta
1.0	69.632	69.632	0
3.0	200.44	200.44	0
10.0	474.97	474.97	0

TABLE 5

RESULTS OF MODULE PRECOM VERIFICATION

Input Constants: $h = 1$, $A^* = 2$, $\rho_f = 3$, $\rho_s = 4$, $C_f = 3$, $C_s = 2$, $F = 0.5$, $v = 1$, $x = 2$, $t = 3600.$

Output Parameter	Computed	Expected	Delta
Y(1) Top	0.0	0.0	0.0
Y(2)	0.09877	0.09877	0.0
Y(3)	0.1975	0.1975	0.0
Y(4)	0.2963	0.2963	0.0
Y(5)	0.3951	0.3951	0.0
Y(6)	0.4938	0.4938	0.0
Y(7)	0.5926	0.5926	0.0
Y(8)	0.6914	0.6914	0.0
Y(9)	0.7901	0.7901	0.0
Y(10)	0.8889	0.8889	0.0
Z	1.80×10^3	1.80×10^3	0.0

TABLE 6

RESULTS OF MODULE CYCLE VERIFICATION

(For test PRECOM, COMPTA, COMPTS as given in Appendix C.)

Mode	Hour	Bed Segment	TA Computed	TA Expected	TS Computed	TS Expected
v>0	1	1	90	90	80	80
v>0	1	2	110	110	90	90
v>0	1	3	130	130	100	100
v>0	1	4	150	150	110	110
v>0	1	5	170	170	120	120
v>0	1	6	190	190	130	130
v>0	1	7	210	210	140	140
v>0	1	8	230	230	150	150
v>0	1	9	250	250	160	160
v>0	1	10	270	270	170	170
v>0	2	1	150	150	110	110
v>0	2	2	250	250	160	160
v>0	2	3	370	370	220	220
v>0	2	4	510	510	290	290
v>0	2	5	670	670	370	370
v>0	2	6	850	850	460	460
v>0	2	7	1050	1050	560	560
v>0	2	8	1270	1270	670	670
v>0	2	9	1510	1510	790	790
v>0	2	10	1770	1770	920	920
v=0	3	1	150	150	110	110
v=0	3	2	250	250	160	160
v=0	3	3	370	370	220	220
v=0	3	4	510	510	290	290
v=0	3	5	670	670	370	370
v=0	3	6	850	850	460	460
v=0	3	7	1050	1050	560	560
v=0	3	8	1270	1270	670	670
v=0	3	9	1510	1510	790	790
v=0	3	10	1770	1770	920	920
v<0	4	1	180	180	125	125
v<0	4	2	390	390	230	230
v<0	4	3	730	730	400	400
v<0	4	4	1230	1230	650	650
v<0	4	5	1920	1920	995	995
v<0	4	6	2830	2830	1450	1450
v<0	4	7	3990	3990	2030	2030
v<0	4	8	5430	5430	2750	2750
v<0	4	9	7180	7180	3625	3625
v<0	4	10	9270	9270	9270	9270

All computed values compare with expected values identically.

The routine INPUT is a very convenient way to get values of the HOTROCKS input constants into the program; it needs no formal test procedure. OUTPRT consists only of WRITE and FORMAT statements; it can be verified by inspection of any HOTROCKS output. A sample of HOTROCKS output format is shown in tables 7 and 8. Note that the output shown is for test sinusoid input, and does not represent "real world" data. Finally, MAIN is a simple supervisor consisting of only a few CALL statements; it requires no formal verification.

Complete Model Verification

With the individual modules tested and verified, the next step in the verification procedure is to see how well HOTROCKS performs as a complete model. The standard of comparison chosen for the first complete test is the Schumann model, since it is universally accepted as the "correct" response of a packed bed to a step input. The constants chosen, ρ_f = 1.15 kg/m^3, ρ_s = 2400 kg/m^3, C_f = 1006 joule/kg·C, C_s = 1046 joule/kg·C, f = 0.5, x = 2 m, h = 7.076 joule/m^2·sec·C, and A^* = 23.62 m^{-1}, are taken from the work of Duffie, Beckmann and Klein [19] and represent typical solar energy packed bed values. Ambient temperature is set at 22°C (72°F) and input air temperature at 70°C (158°F). Since the Schumann model's computational inefficiency makes producing Schumann data a long process and since the input parameters for this comparison allow a significant portion of the bed to be fully charged after a few Hours, three hours was chosen as a good first simulation period. The Schumann model, of course, can only be used to calculate air and rock temperatures for the uniform initial condition, step input response

AIR AND ROCK TEMPERATURE PROFILE FOR EACH HOUR PERIOD, ON AN HOURLY BASIS, GIVEN THE LISTED PARAMETERS

TIME	MODE	VEL	INPUT	TEMP	TOP.....	AIR TEMPERATURE PROFILE.....						BOTTOM
1.00	1	0.051	-3.96	-3.96	-5.01	-5.21	-3.61	-0.30	4.22	9.50	15.20	21.13	27.21
2.00	1	0.051	8.99	8.99	6.38	2.72	-0.09	-1.08	-0.13	2.50	6.41	11.19	16.53
3.00	1	0.051	20.25	20.25	17.65	13.25	8.57	4.70	2.27	1.55	2.49	4.89	8.43
4.00	1	0.051	29.13	29.13	26.97	23.03	18.28	13.47	9.23	6.02	4.14	3.72	4.71
5.00	1	0.051	35.06	35.06	33.53	30.57	26.68	22.27	17.73	13.46	9.86	7.23	5.79
6.00	1	0.051	37.67	37.67	36.88	35.16	32.60	29.31	25.50	21.40	17.32	13.60	10.54
7.00	1	0.051	36.79	36.79	36.80	36.44	35.44	33.87	31.16	28.03	24.47	20.73	17.10
8.00	1	0.051	32.49	32.49	33.29	34.33	34.96	34.88	33.97	32.26	29.85	26.90	23.62
9.00	1	0.051	25.03	25.03	26.58	28.94	31.18	32.81	33.61	33.55	32.64	30.98	28.69
10.00	1	0.051	14.88	14.88	17.07	20.61	24.32	27.57	30.05	31.67	32.41	32.30	31.40
11.00	1	0.051	2.67	2.67	5.37	9.87	14.82	19.48	23.49	26.72	29.09	30.60	31.27
12.00	1	0.051	-10.82	-10.82	-7.78	-2.61	3.27	9.04	14.33	18.97	22.85	25.92	28.17
13.00	1	0.051	-24.74	-24.74	-21.55	-16.03	-9.60	-3.07	3.17	8.92	14.06	18.51	22.22
14.00	1	0.051	-38.23	-38.23	-35.09	-29.57	-22.99	-16.11	-9.31	-2.81	3.27	8.83	13.78
15.00	1	0.051	-50.43	-50.43	-47.54	-42.37	-36.05	-29.25	-22.33	-15.48	-8.84	-2.53	3.36
16.00	1	0.051	-60.58	-60.58	-58.12	-53.62	-47.96	-41.67	-35.06	-28.29	-21.51	-14.84	-8.39
17.00	1	0.051	-68.04	-68.04	-66.17	-62.62	-57.97	-52.59	-46.70	-40.44	-33.95	-27.33	-20.73
18.00	1	0.051	-72.35	-72.35	-71.18	-68.80	-65.46	-61.33	-56.53	-51.17	-45.37	-39.23	-32.87
19.00	1	0.051	-73.22	-73.22	-72.83	-71.78	-69.95	-67.32	-63.92	-59.80	-55.05	-49.77	-44.08
20.00	1	0.051	-70.62	-70.62	-71.03	-71.37	-71.17	-70.21	-68.41	-65.79	-62.40	-58.31	-53.61

MODE KEY: 0=OFF, 1=SOLAR LOOP ON, -1=OUTPUT LOOP ON

TABLE 7

HOTROCKS AIR TEMPERATURE OUTPUT: SINUSOID TEST

TIME	MODE	VEL	INPUT TEMP	TOP	ROCK TEMPERATURE PROFILE							BOTTOM
1.00	1	0.051	-3.96	-4.51	-5.63	-4.75	-1.07	2.46	7.66	13.32	19.25	25.28	31.47
2.00	1	0.051	8.99	8.09	3.93	0.30	-1.46	-1.09	1.15	4.80	9.44	14.71	20.39
3.00	1	0.051	20.25	19.44	15.03	9.95	5.50	2.46	1.17	1.63	3.66	6.97	11.24
4.00	1	0.051	29.13	28.49	24.74	19.92	14.88	10.27	6.60	4.22	3.33	3.91	5.87
5.00	1	0.051	35.06	34.62	31.93	28.15	23.76	19.12	14.63	10.70	7.67	5.79	5.21
6.00	1	0.051	37.67	37.47	36.02	33.67	30.56	26.83	22.73	18.53	14.59	11.21	8.69
7.00	1	0.051	36.79	36.84	36.73	36.00	34.49	32.18	29.19	25.69	21.91	18.14	14.67
8.00	1	0.051	32.49	32.78	34.02	34.96	35.18	34.54	33.06	30.83	27.98	24.72	21.27
9.00	1	0.051	25.03	25.54	28.06	30.61	32.56	33.66	33.87	33.21	31.74	29.60	26.92
10.00	1	0.051	14.88	15.59	19.22	23.23	26.78	29.57	31.48	32.49	32.63	31.95	30.52
11.00	1	0.051	2.67	3.53	8.05	13.26	18.20	22.50	26.00	28.66	30.44	31.36	31.45
12.00	1	0.051	-10.82	-9.87	-4.75	1.34	7.36	12.90	17.78	21.92	25.26	27.77	29.44
13.00	1	0.051	-24.74	-23.75	-18.35	-11.78	-5.06	1.38	7.33	12.69	17.39	21.35	24.53
14.00	1	0.051	-38.23	-37.27	-31.92	-25.27	-18.27	-11.35	-4.70	1.55	7.31	12.48	16.97
15.00	1	0.051	-50.43	-49.56	-44.61	-38.29	-31.46	-24.48	-17.56	-10.80	-4.35	1.71	7.30
16.00	1	0.051	-60.58	-59.85	-55.60	-50.03	-43.79	-37.19	-30.42	-23.60	-16.85	-10.29	-4.02
17.00	1	0.051	-68.04	-67.50	-64.22	-59.73	-54.48	-48.68	-42.49	-36.02	-29.40	-22.75	-16.16
18.00	1	0.051	-72.35	-72.03	-69.93	-66.80	-62.86	-58.23	-53.01	-47.30	-41.23	-34.90	-28.43
19.00	1	0.051	-73.22	-73.14	-72.35	-70.80	-68.42	-65.24	-61.31	-56.73	-51.58	-45.97	-40.01
20.00	1	0.051	-70.62	-70.79	-71.36	-71.46	-70.79	-69.26	-66.88	-63.71	-59.81	-55.27	-50.19

MODE KEY: 0=OFF, 1=SOLAR LOOP ON, -1=OUTPUT LOOP ON

TABLE 8

HOTROCKS ROCK TEMPERATURE OUTPUT: SINUSOID TEST

case. A comparison between the Schumann values and the HOTROCKS values is given in table 9 and the error between the Schumann and HOTROCKS models is graphically shown in figure 9. The results of this test make it quite clear that HOTROCKS tracks the Schumann model very well, at least for the given set of input constants.

The next test performed on HOTROCKS is a comparison between its step reponse output and that of the Alexander-Taft (AT) model. Because the AT model uses only five bed segments (as shown in figure 10) a direct number-by-number comparison of the HOTROCKS and AT outputs is not practical. A graphical comparison is the best method available to illustrate the differences between the two models. Only a limited amount of output data is available for the AT model, and much of it is not directly comparable to HOTROCKS, but a step response and sinusoid input response are available, and are enough to serve to assist in the validation of HOTROCKS.

The AT model uses parameters in the English system of units, but after conversion of the AT constants to the SI system the constants used in this comparison are: ρ_f = 2.851 kg/m^3, ρ_s = 2400. kg/m^3, C_f = 1012 joule/kg•C, C_s = 837. joule/kg•C, f = 0.42, h = 6.076 joule/m^2•sec•C and A* = 72.42 m^{-1}. The bed length is set at 1.829 m and the input air velocity is 0.051 m/sec. Ambient temperature is set at -17.80°C (0°F) and input air temperature is set at 37.78°C (100°F). Notice there are significant differences between these parameters and those used earlier for the HOTROCKS-Schumann comparison. The higher air density indicates that the AT model was simulated for a relatively high positive pressure differential between the bed and ambient, and

TABLE 9

COMPARISON OF HOTROCKS AND SCHUMANN SOLUTIONS

HOTROCKS-Schumann Comparison: Air Temperature

Hour	Bed Segment	TA-HOTROCKS	TA-Schumann	ΔTA
1	1 (top)	70.°C	70.°C	0.°C
1	3	32.14°C	32.14°C	0.°C
1	5	23.90°C	23.90°C	0.°C
1	7	22.33°C	22.33°C	0.°C
2	1	70.°C	70.°C	0.°C
2	3	37.27°C	36.94°C	+0.33°C
2	5	25.81°C	25.69°C	+0.12°C
2	7	22.85°C	22.80°C	+0.05°C
3	1	70.°C	70.°C	0.°C
3	3	41.99°C	41.49°C	+0.5°C
3	5	28.14°C	27.88°C	+0.26°C
3	7	23.62°C	23.52°C	+0.10°C

HOTROCKS-Schumann Comparison: Rock Temperature

Hour	Bed Segment	TS-HOTROCKS	TS-Schumann	ΔTA
1	1	38.20°C	38.20°C	0.°C
1	3	24.66°C	24°66°C	0.°C
1	5	22.42°C	22.42°C	0.°C
1	7	22.06°C	22.06°C	0.°C
2	1	48.93°C	48.93°C	0.°C
2	3	28.17°C	28.05°C	+0.12°C
2	5	23.27°C	23.23°C	+0.04°C
2	7	22.24°C	22.23°C	+0.01°C
3	1	56.04°C	56.04°C	0.°C
3	3	32.13°C	31.88°C	+0.25°C
3	5	24.55°C	24.44°C	+0.11°C
3	7	22.58°C	22.55°C	+0.03°C

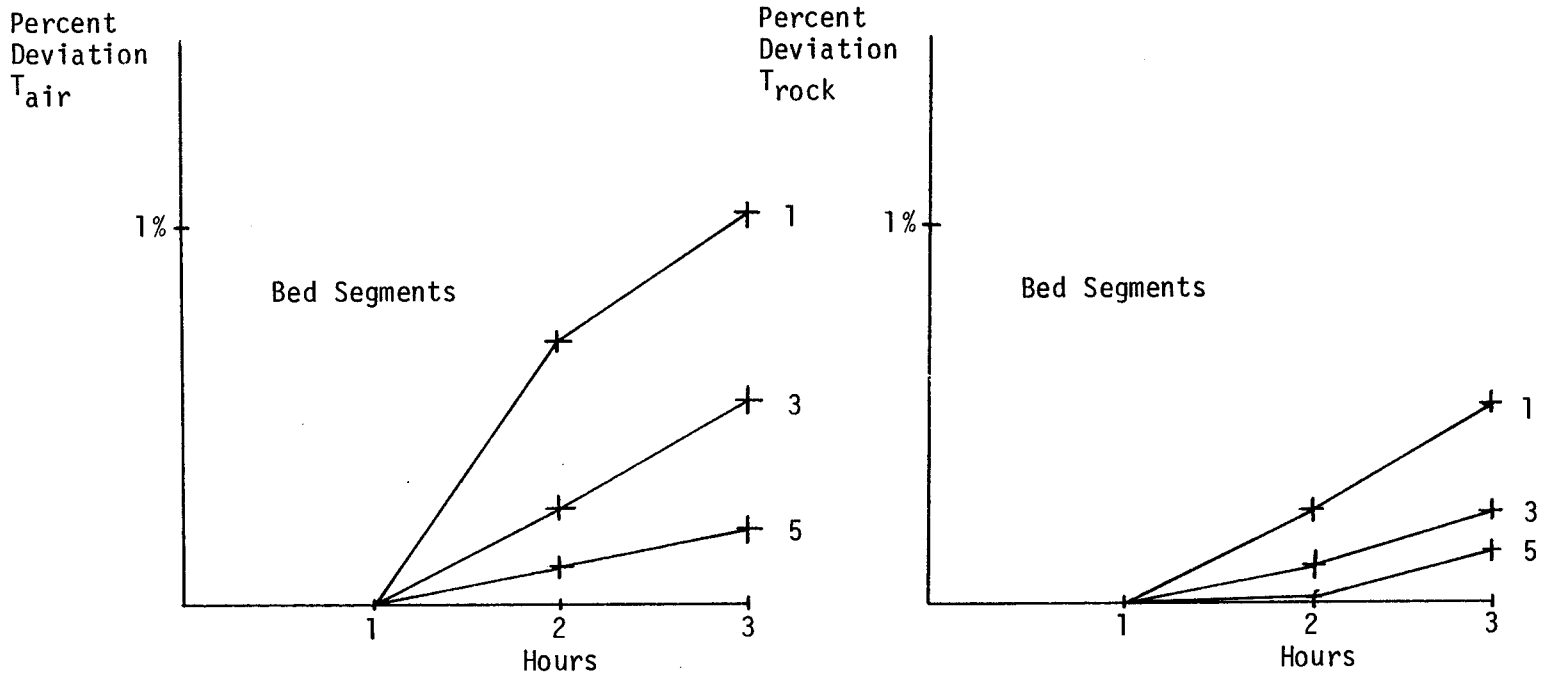


Fig. 9. Difference Between Schumann Output and HOTROCKS Output

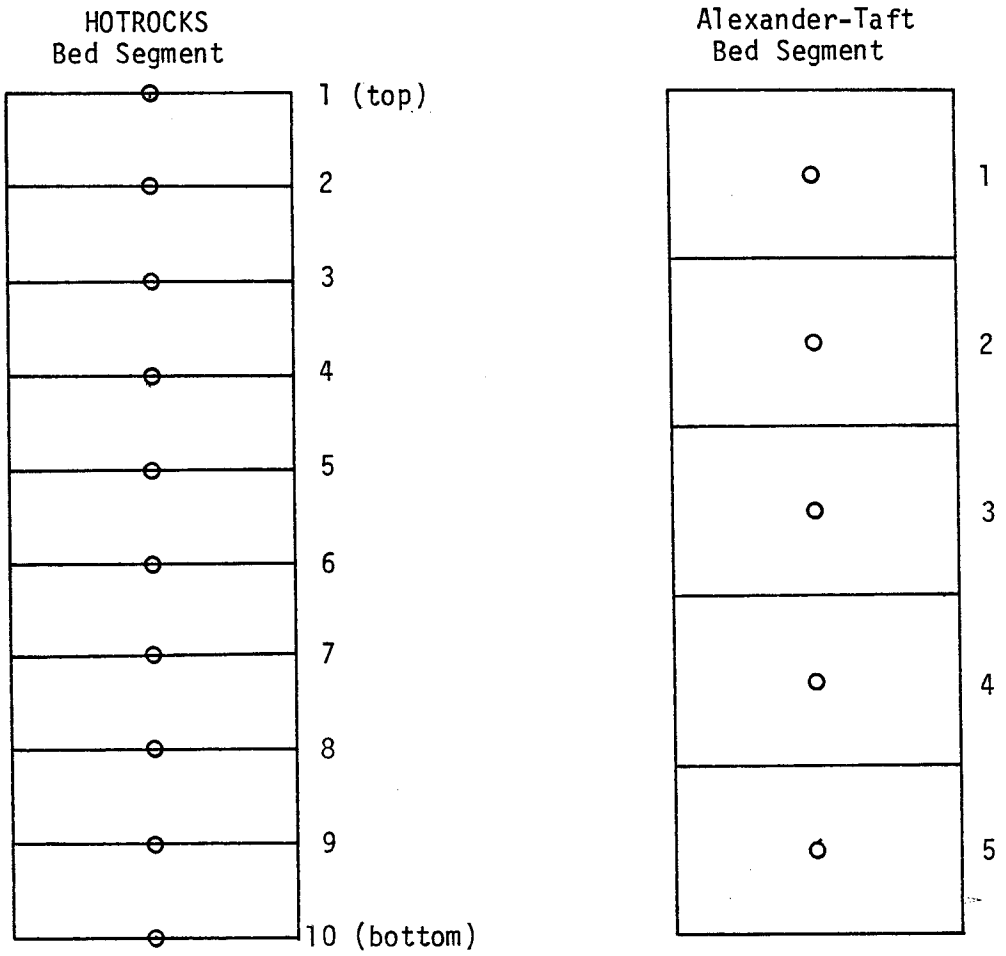


Fig. 10. Comparison of HOTROCKS and AT Model Geometry

the greater A* term implies that the rocks used in the AT bed were of smaller diameter than those used in the earlier HOTROCKS simulation. These differences are useful to test whether HOTROCKS will perform well for widely varying sets of input data. Note that the time increments of calculation are not hours, as with the earlier comparison, but are now time segments of 7100 second duration. The 7100 second time segment was chosen to make HOTROCKS output time compatible with the available AT output data. With a 7100 second time segment, the entire bed is fully charged after approximately ten time segments. A comparison of the first six time segments is all that is necessary to demonstrate the differences between HOTROCKS and AT.

Results of this comparison are illustrated in figures 11 through 18. Figures 11 and 12 show a temperature versus time comparison between two HOTROCKS data points and the AT data points that are physically closest to them. The next six figures are temperature versus bed segment (from top to bottom) maps of both the HOTROCKS and AT models for six successive time segments. For reference, the Schumann model predictions for these input data are also shown. Note that the Schumann and HOTROCKS data compare identically in every case. As shown on the graphs, a temperature difference between the curves of approximately 6 centigrade degrees corresponds to a difference in stored energy of 10 percent, with respect to the total amount of energy that can be stored in the bed.

One can conclude from inspection of this output data that HOTROCKS is also an accurate model for the AT input parameters, for the step response. Note also that the transient response of the

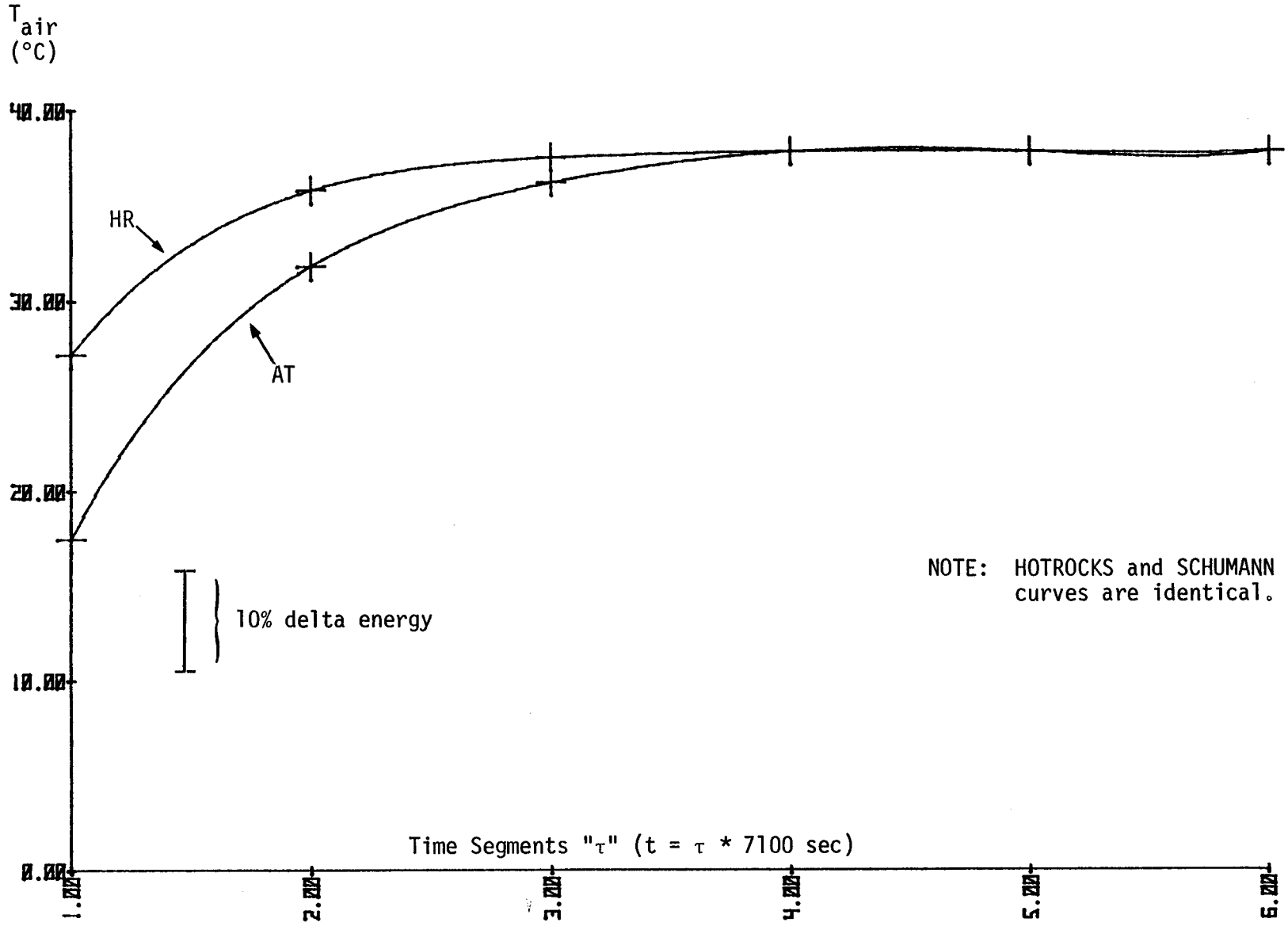


Fig. 11.--HOTROCKS-AT-SCHUMANN Comparison; Step Input, Single Bed Segment, HR 2

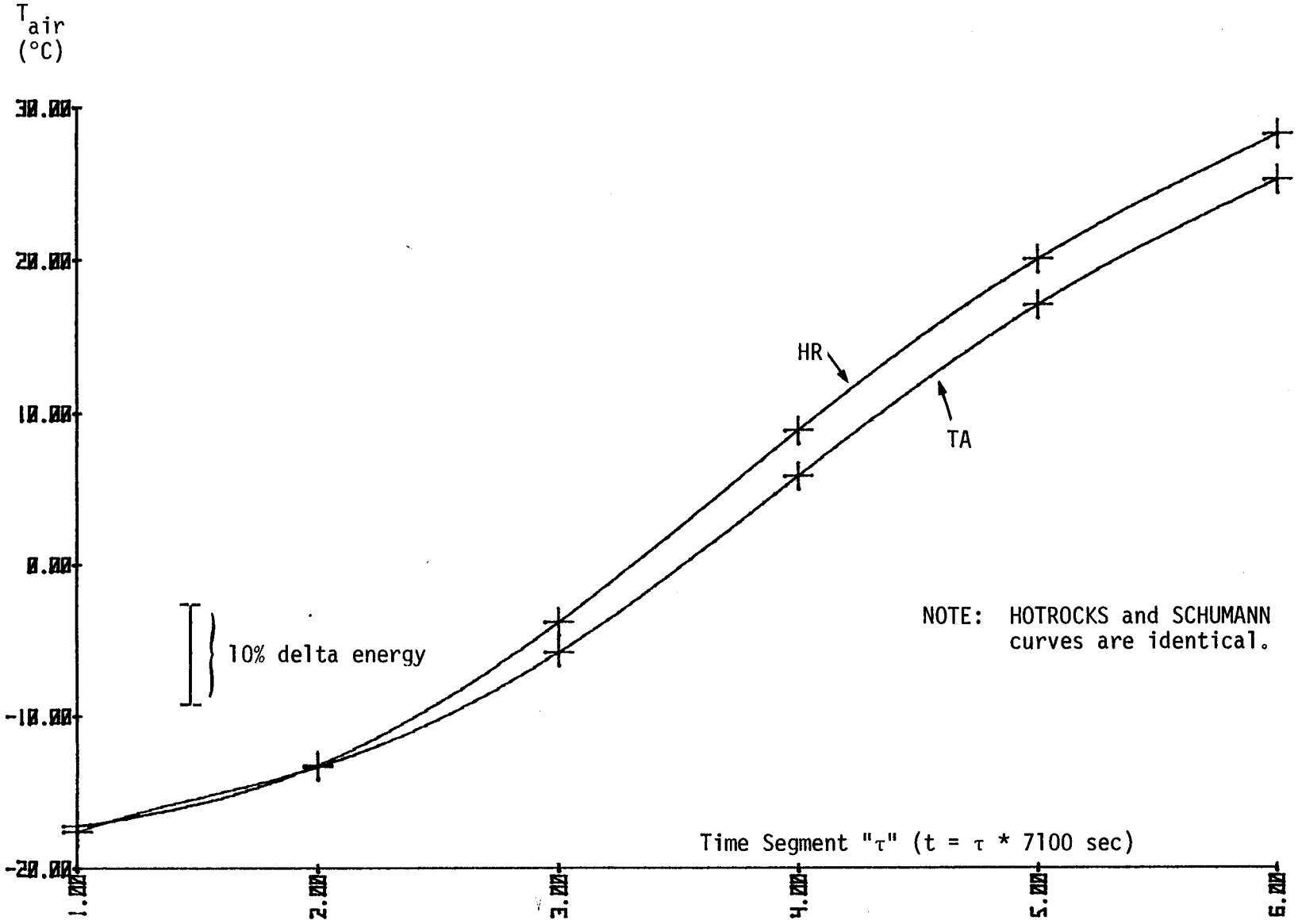


Fig. 12.--HOTROCKS-AT-SCHUMANN Comparison; Step Input, Single Bed Segment, HR 9

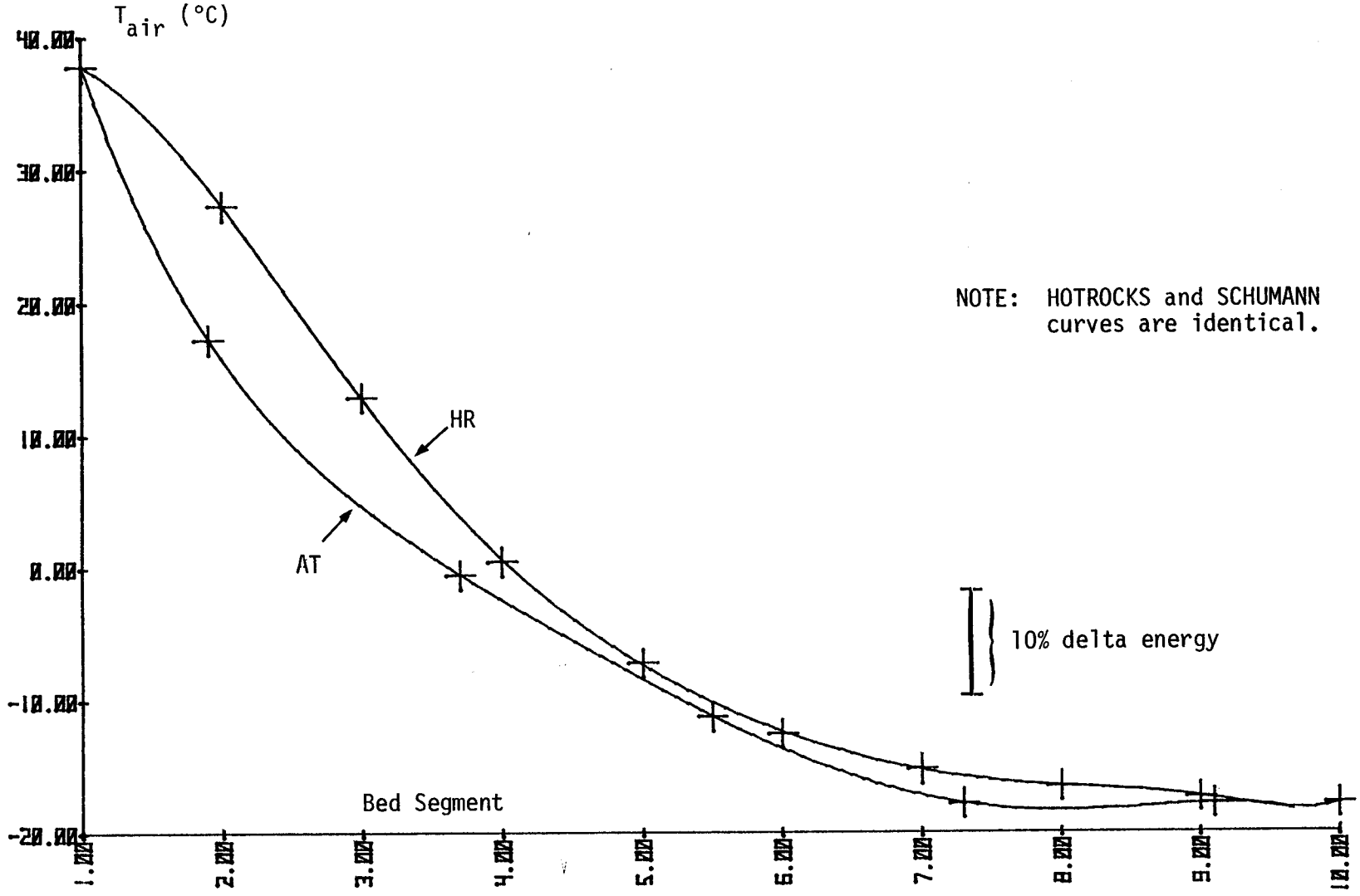


Fig. 13.--HOTROCKS-AT-SCHUMANN Comparison; Step Input, Single Time Segment, #1

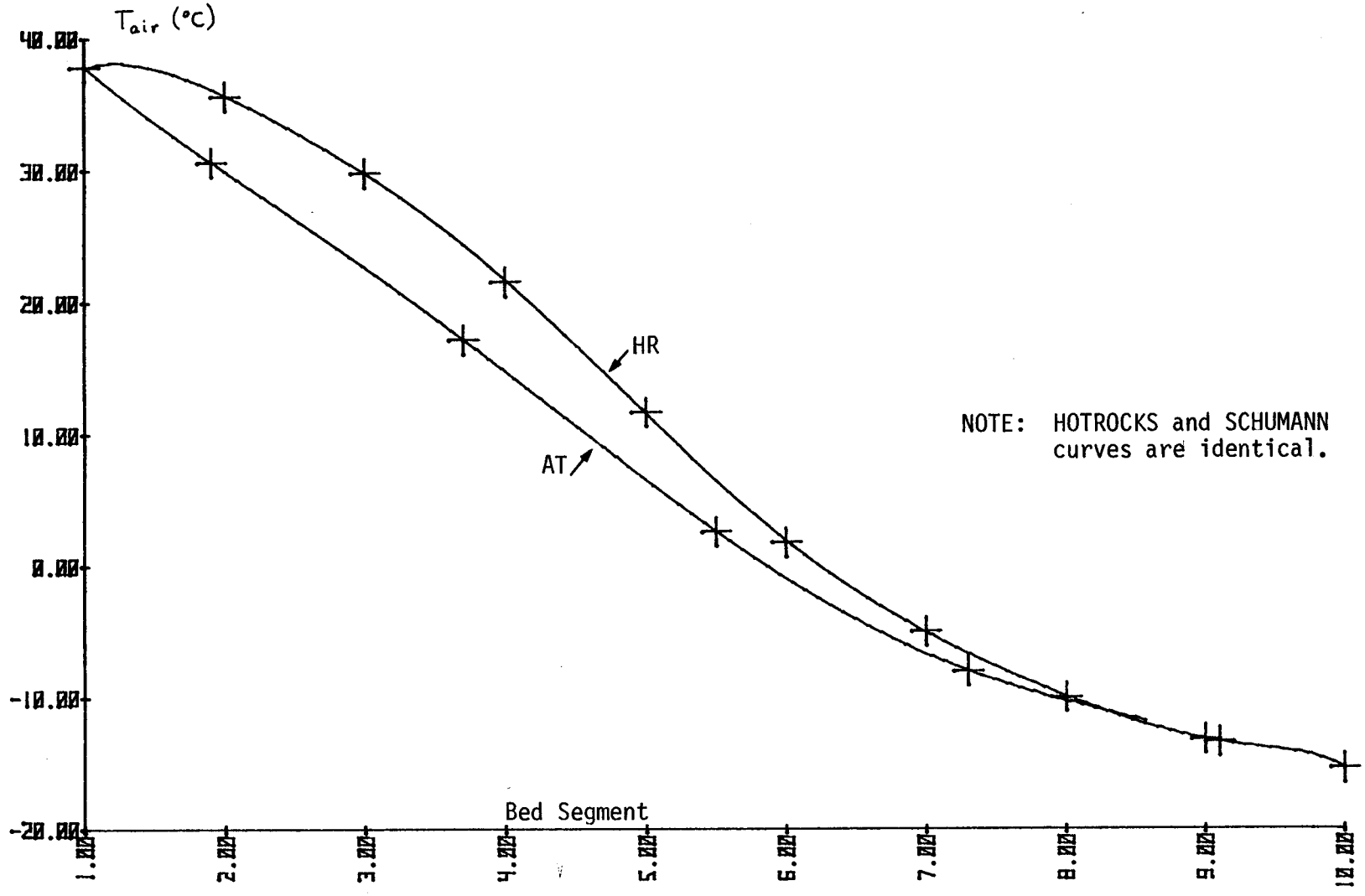


Fig. 14.--HOTROCKS-AT-SCHUMANN Comparison; Step Input, Single Time Segment, #2

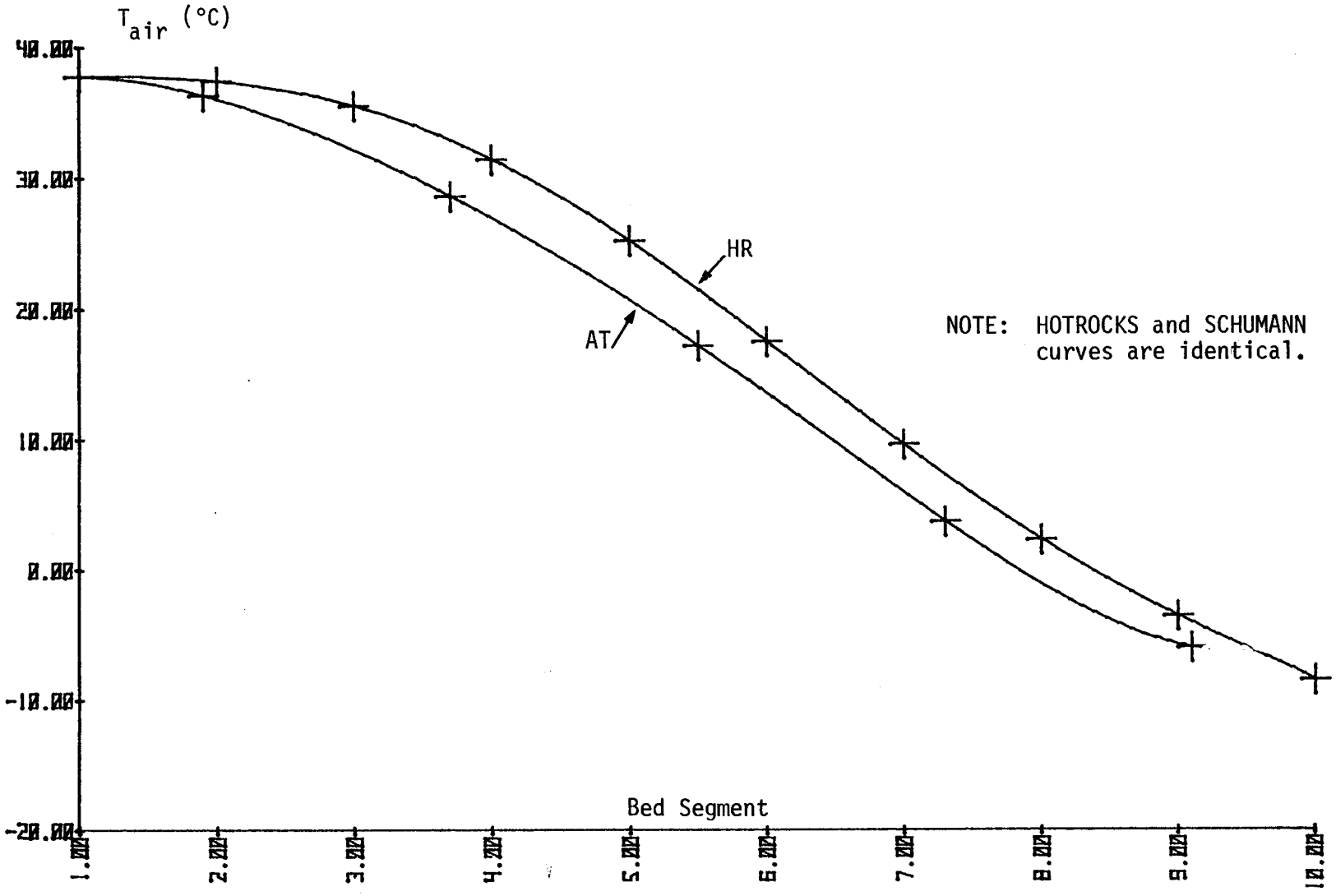


Fig. 15.--HOTROCKS-AT-SCHUMANN Comparison; Step Input, Single Time Segment, #3

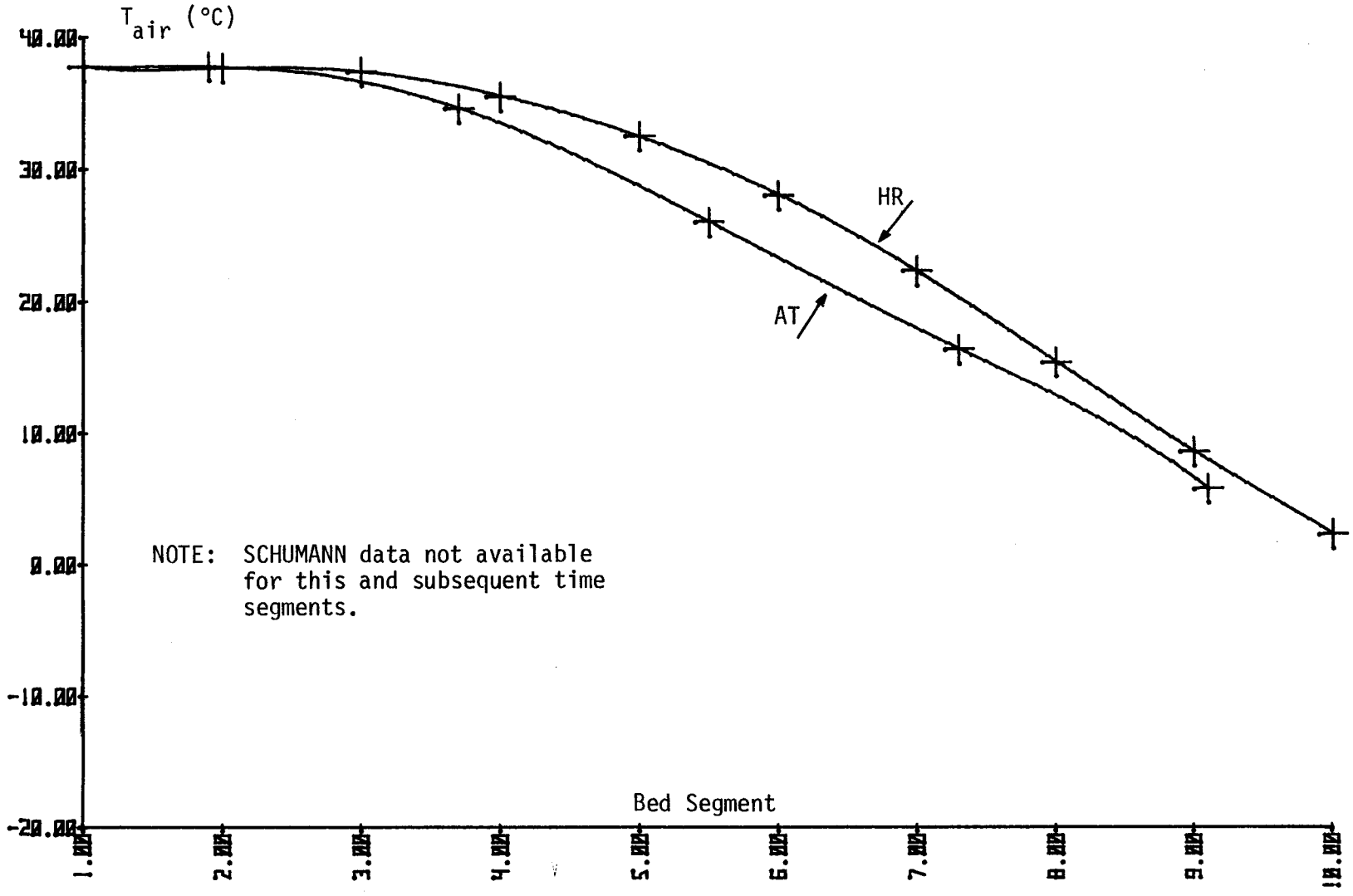


Fig. 16.--HOTROCKS-AT Comparison; Step Input, Single Time Segment, #4

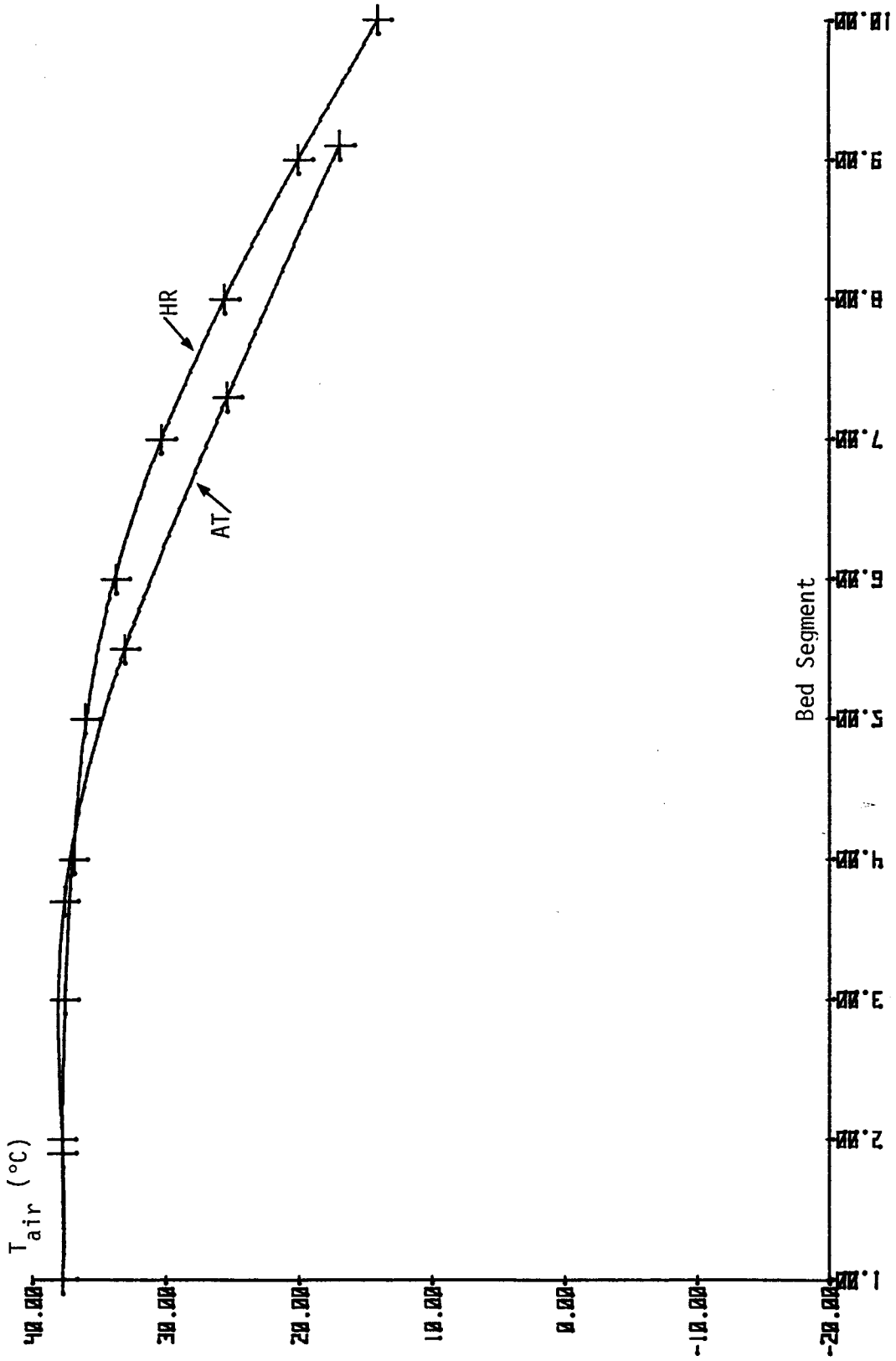


Fig. 17.--HOTROCKS-AT Comparison; Step Input, Single Time Segment, #5

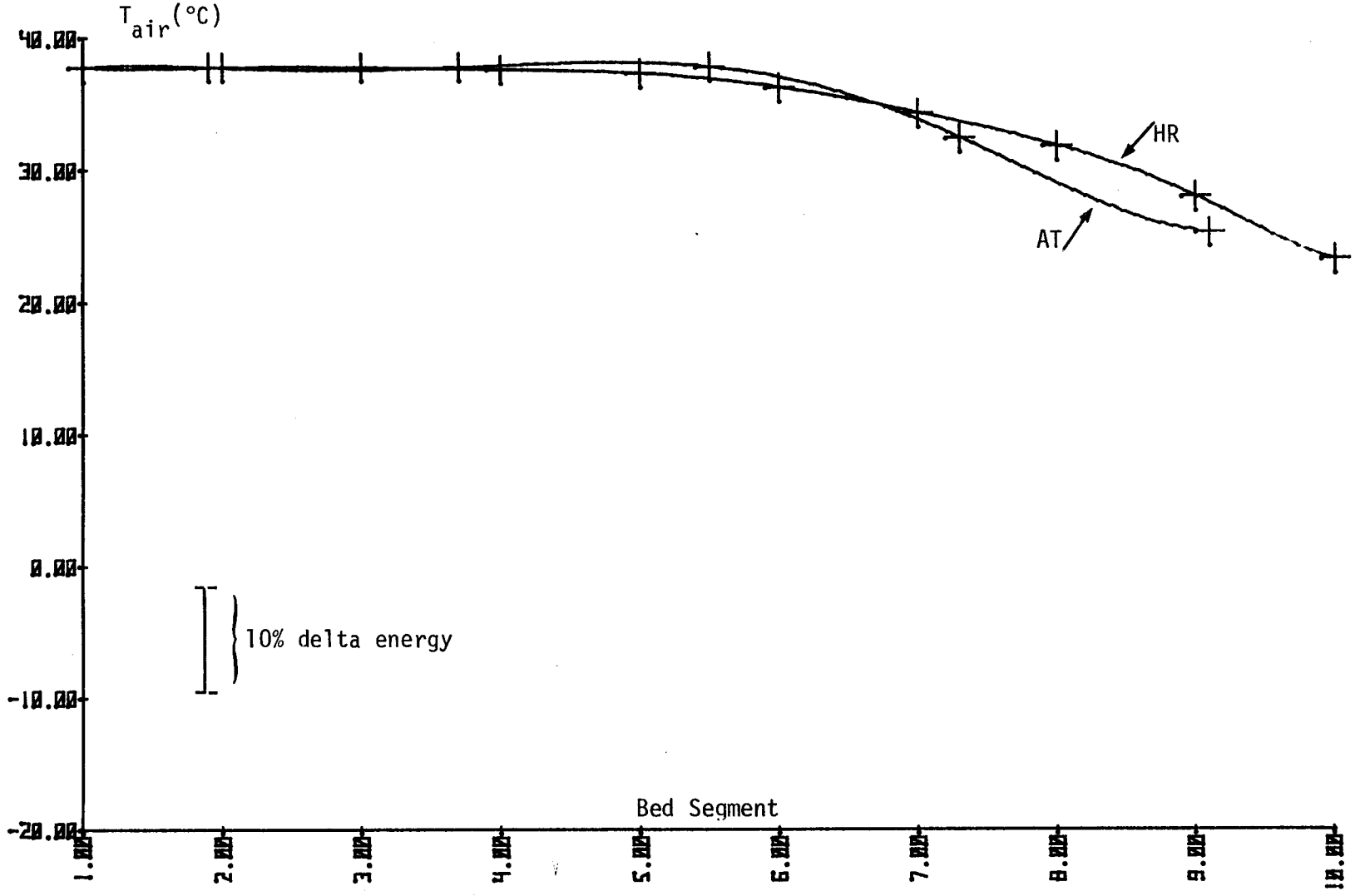


Fig.18.--HOTROCKS-AT Comparison; Step Input, Single Time Segment, #6

HOTROCKS model is better (i.e., the HOTROCKS data compare to Schumann more favorably than AT for the first few time segments) especially for the topmost bed segments. Since these are the highest temperature bed segments during nominal bed operation, HOTROCKS provides a truer model of the high energy bed positions than AT. This may be the result of the fact that HOTROCKS models twice the number of bed segments as does AT. Whether this difference in the models is significant enough to warrant concern is dependent upon how the model is to be used; in no case is the difference much greater than 15 percent and the deviation rapidly converges to near zero for all bed segments as the fully charged bed condition is approached.

The final validiation test to be performed on HOTROCKS is a comparison between its sinusoid input response and the AT sinusoid response. Unfortunately, no other model documented has published results for a sinusoid input, so this is the only comparison possible. Again AT input data is used; the constants are the same as the other AT comparison but the initial bed conditions are different. The bed starts the simulation with a linear temperature gradient, from 43.3°C at the bottom to -12.2°C at the top. These conditions would never occur in the real world but serve as an extreme test of the model.

The input forcing function is $T_{in} = 100 \sin (\pi I/25)°F$ where I is a time step equal (for given input constants) to 3590 seconds. Considering time steps for HOTROCKS of $\tau = 7180$ seconds, approximately 20 time steps must elapse before the effect of the initial condition

transient is damped out. This allows the input forcing function to swing from zero through maximum value (100°F or 37.78°C) to minimum value (-100°F or -73.3°C). The resulting 20 time step comparison then will look at three-fourths of one input period.

Figures 19 through 29 detail the differences between the HOTROCKS and AT models for sinusoidal input. Again, figures 19 and 20 are temperature versus time maps of two comparably located bed test points, and the other nine graphs are temperature versus bed position maps for succeeding time segments.

What could be predicted from inspection of the step input comparison is evident from the sinusoid plots. HOTROCKS responds to the initial system transient quicker than AT, but both models' response to the slower input forcing function is approximately identical. This produces a phase shift between the HOTROCKS and AT outputs that is most pronounced in the topmost bed segments (where the HOTROCKS and AT models showed the greatest discrepancy in transient response for the step input) and less obvious in the lower bed segments.

Again, the shift is not very pronounced, amounting to an instantaneous energy stored difference between HOTROCKS and AT of less than 20 percent in almost all cases and a total integrated energy difference over one-half period of less than 10 percent for the topmost segments (determined by graphically summing the area under the two curves in figure 19 over the first one-half period, to include the effect of transients). The bottommost segments compare even more favorably for both instantaneous and integrated energy differences.

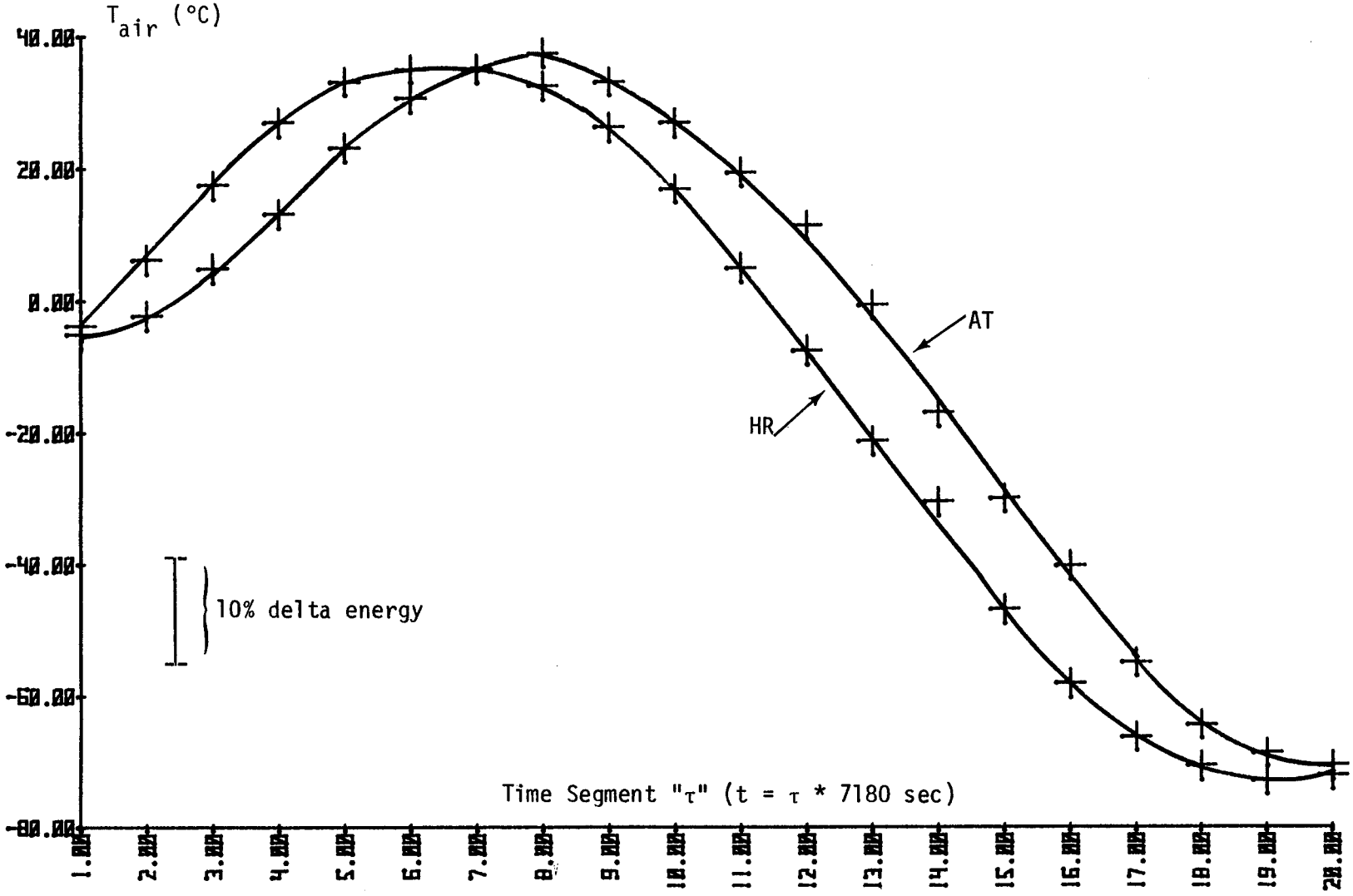


Fig. 19.--HOTROCKS-AT Comparison; Sinusoid Input, Single Bed Segment, HR 2

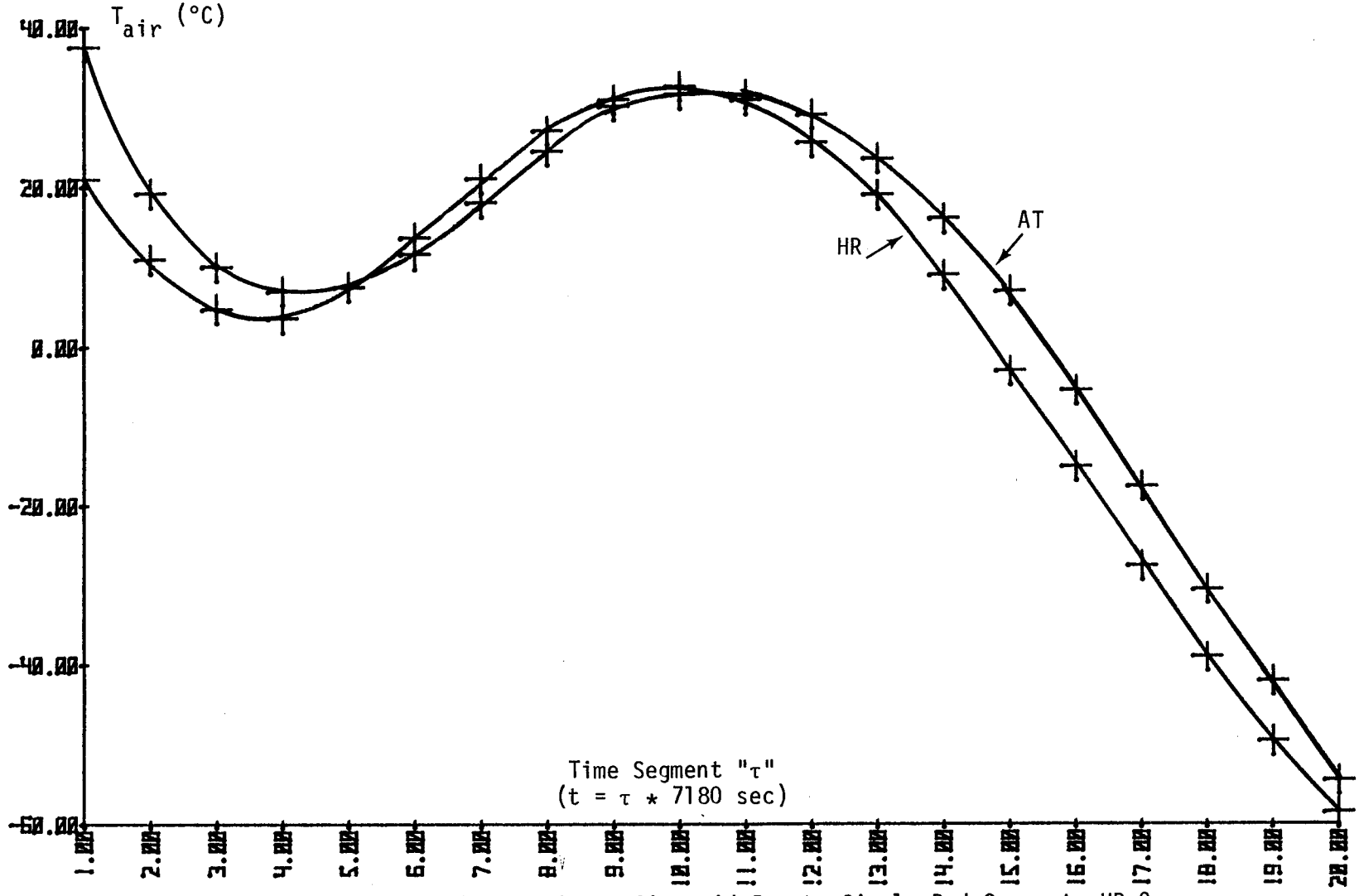


Fig. 20--HOTROCKS-AT Comparison; Sinusoid Input, Single Bed Segment, HR 9

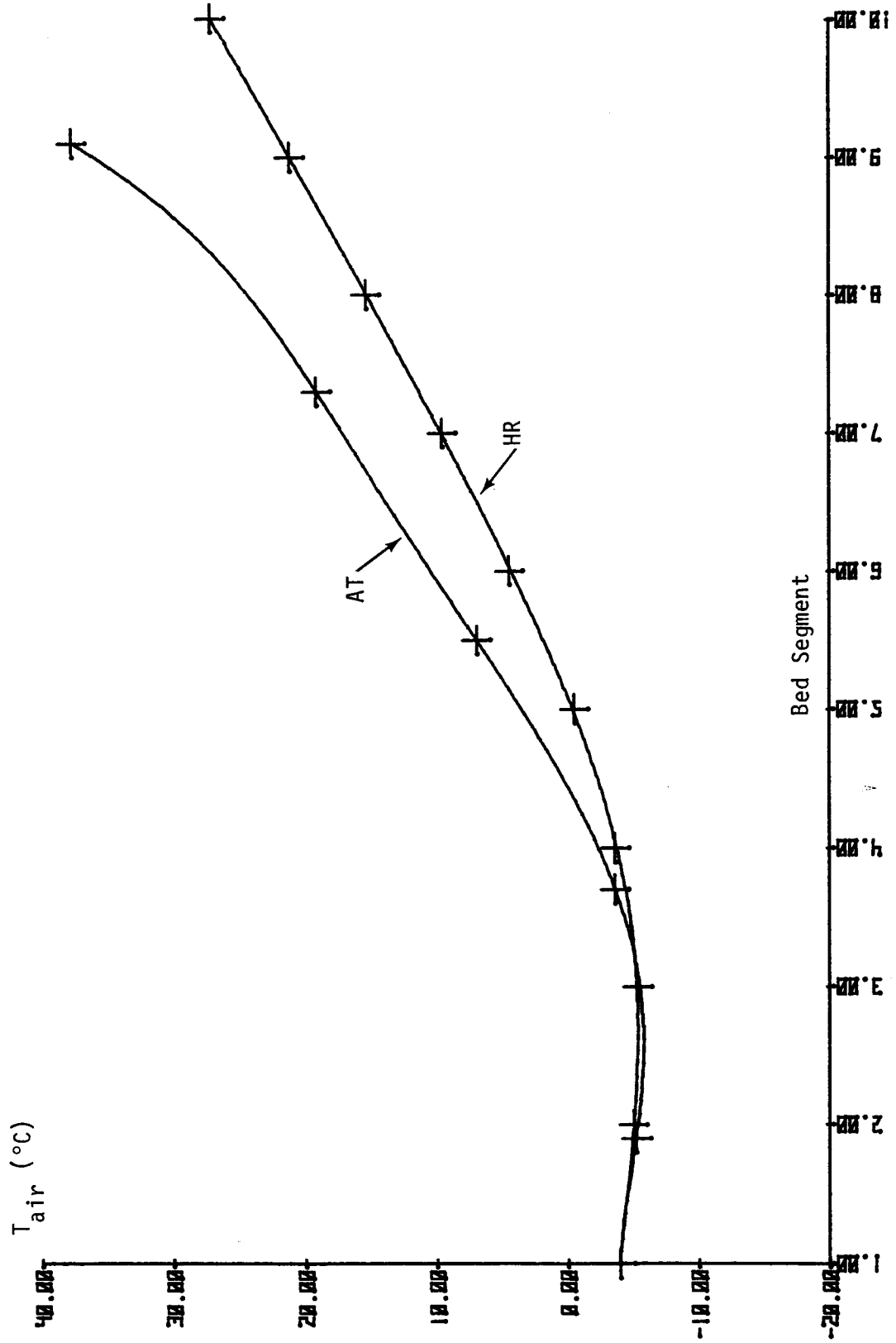


Fig. 21.--HOTROCKS-AT Comparison; Sinusoid Input, Single Bed Segment, #1

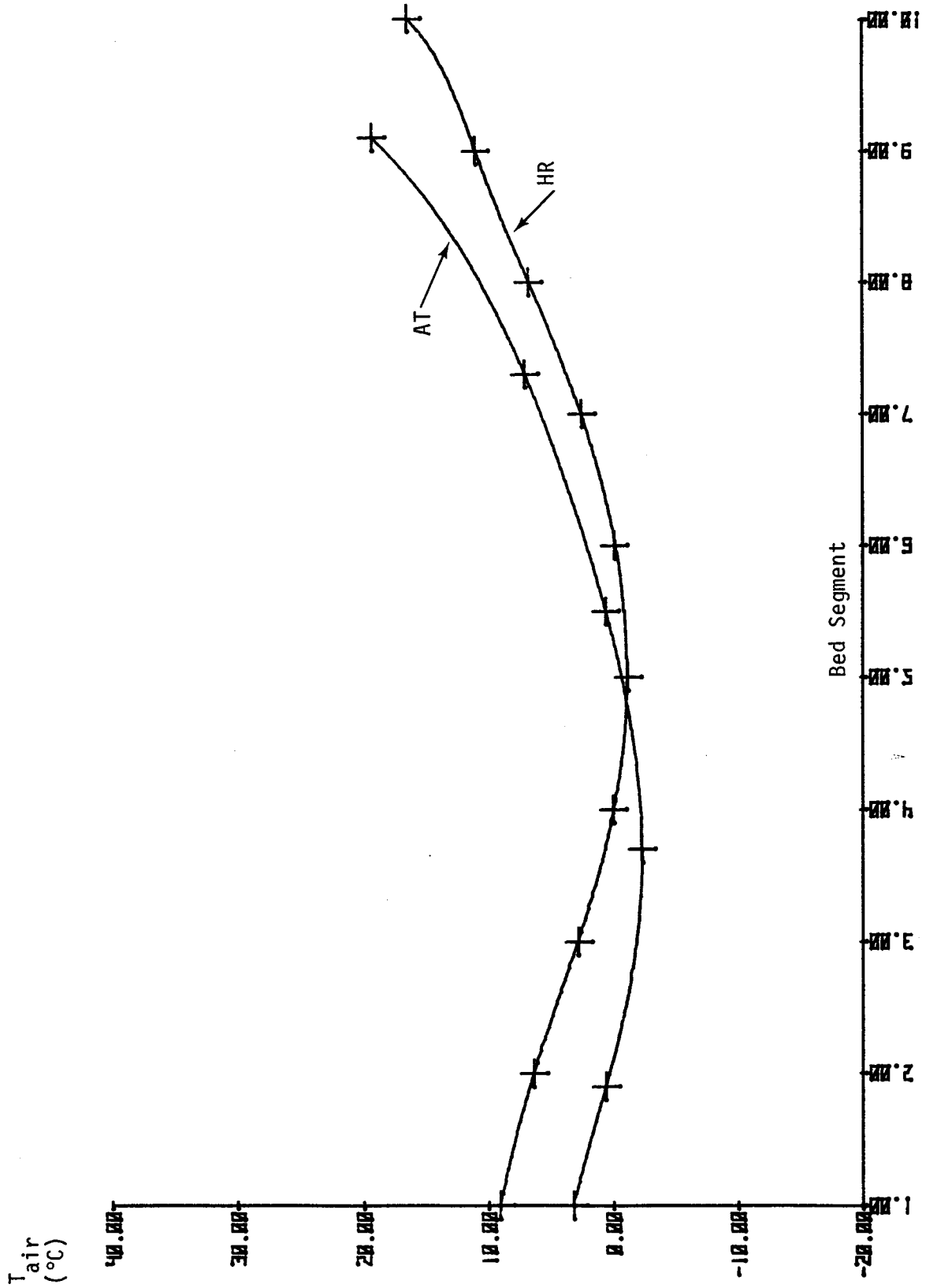


Fig. 22.--HOTROCKS-AT Comparison; Sinusoid Input, Single Time Segment, HR 2

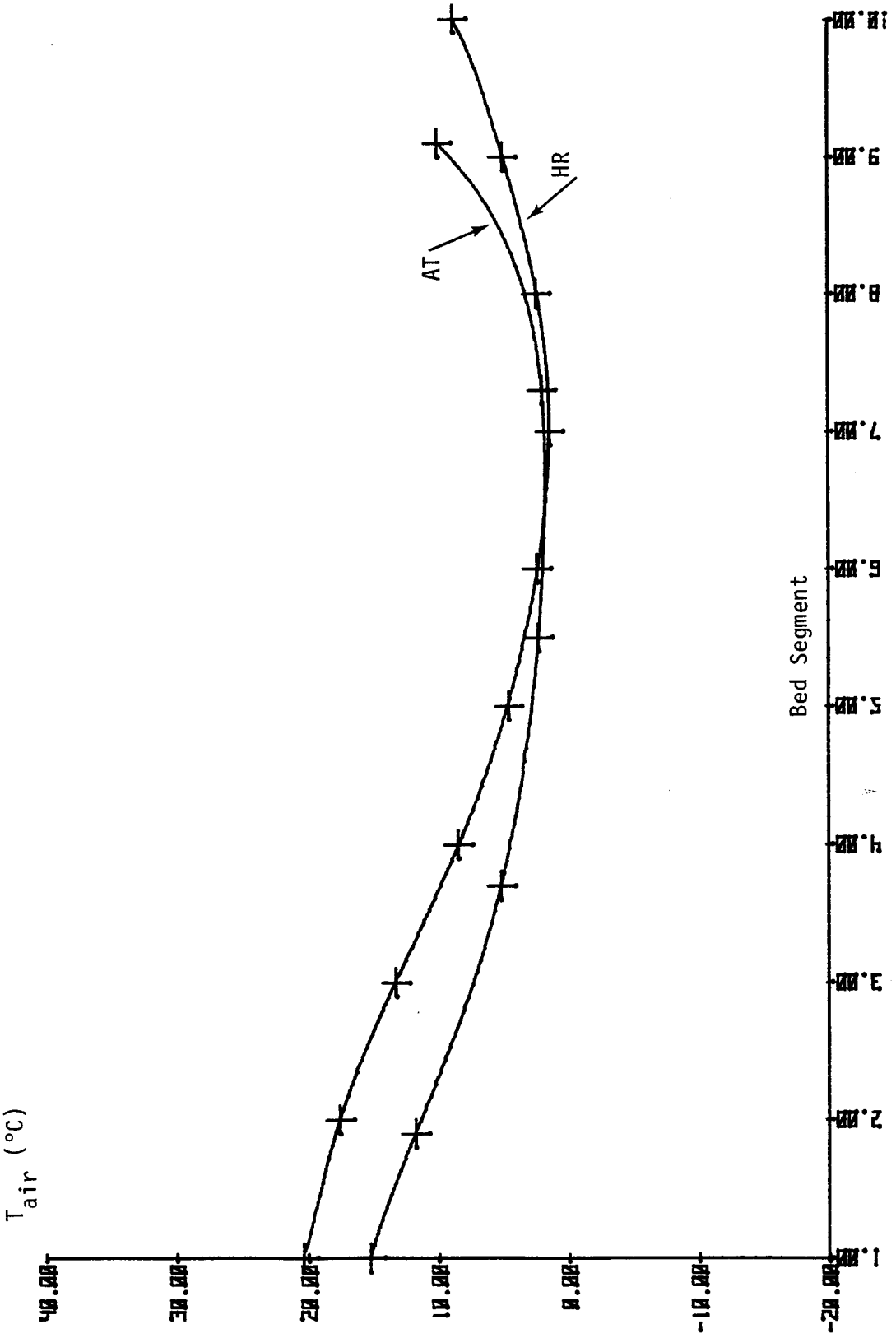


Fig. 23.--HOTROCKS-AT Comparison; Sinusoid Input, Single Time Segment, #3

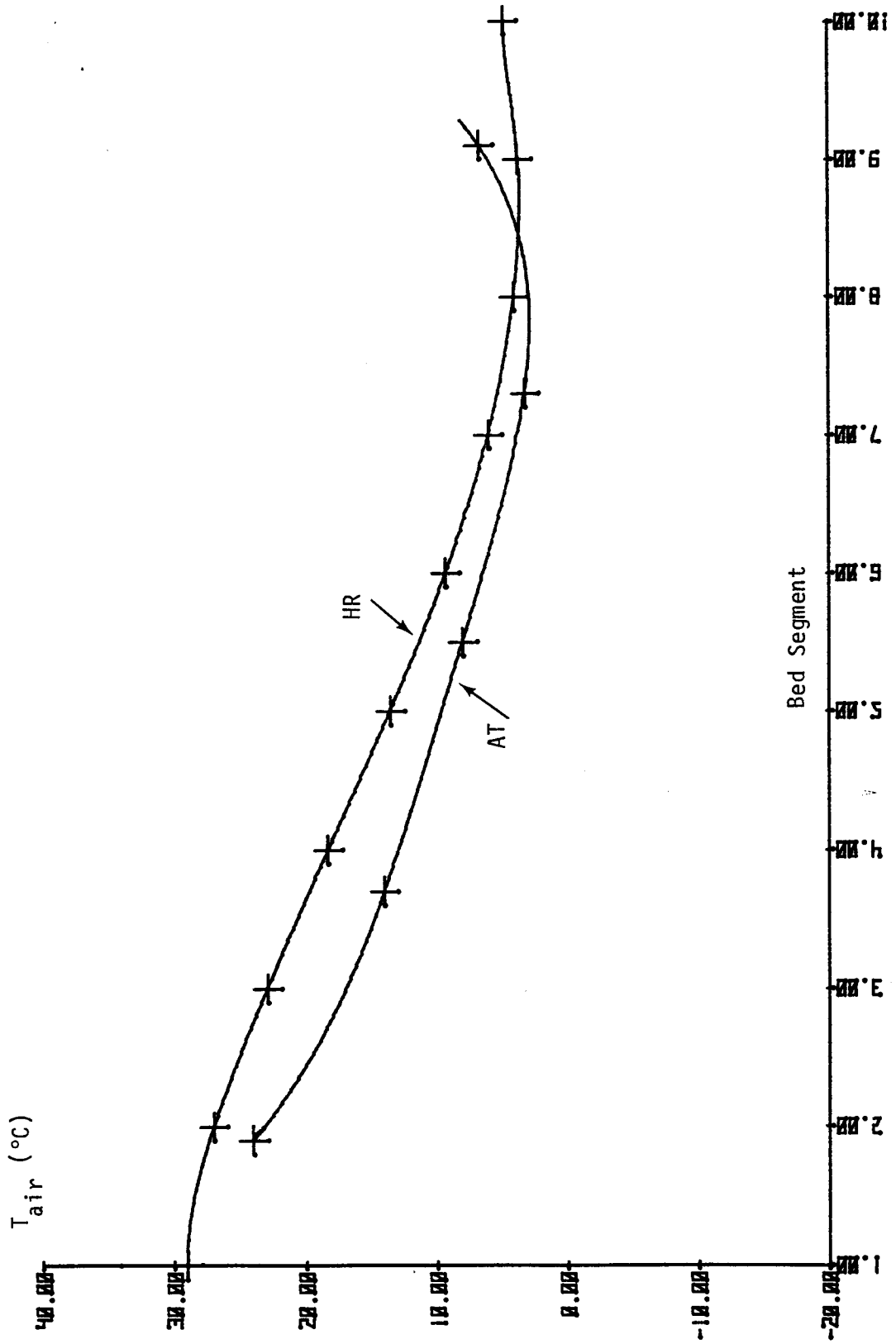


Fig. 24.--HOTROCKS-AT Comparison; Sinusoid Input, Single Time Segment, #4

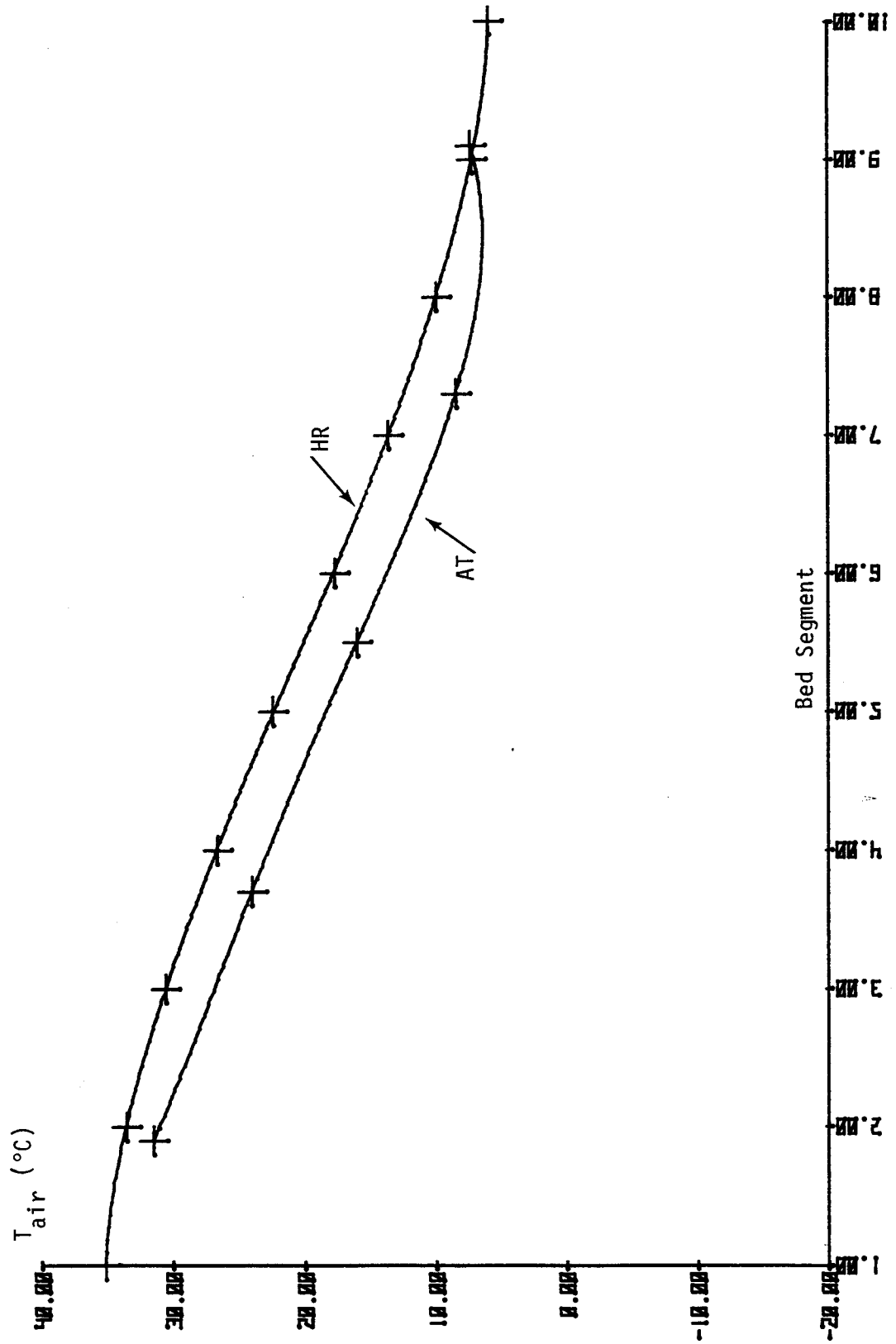


Fig. 25.--HOTROCKS-AT Comparison; Sinusoid Input, Single Time Segment, #5

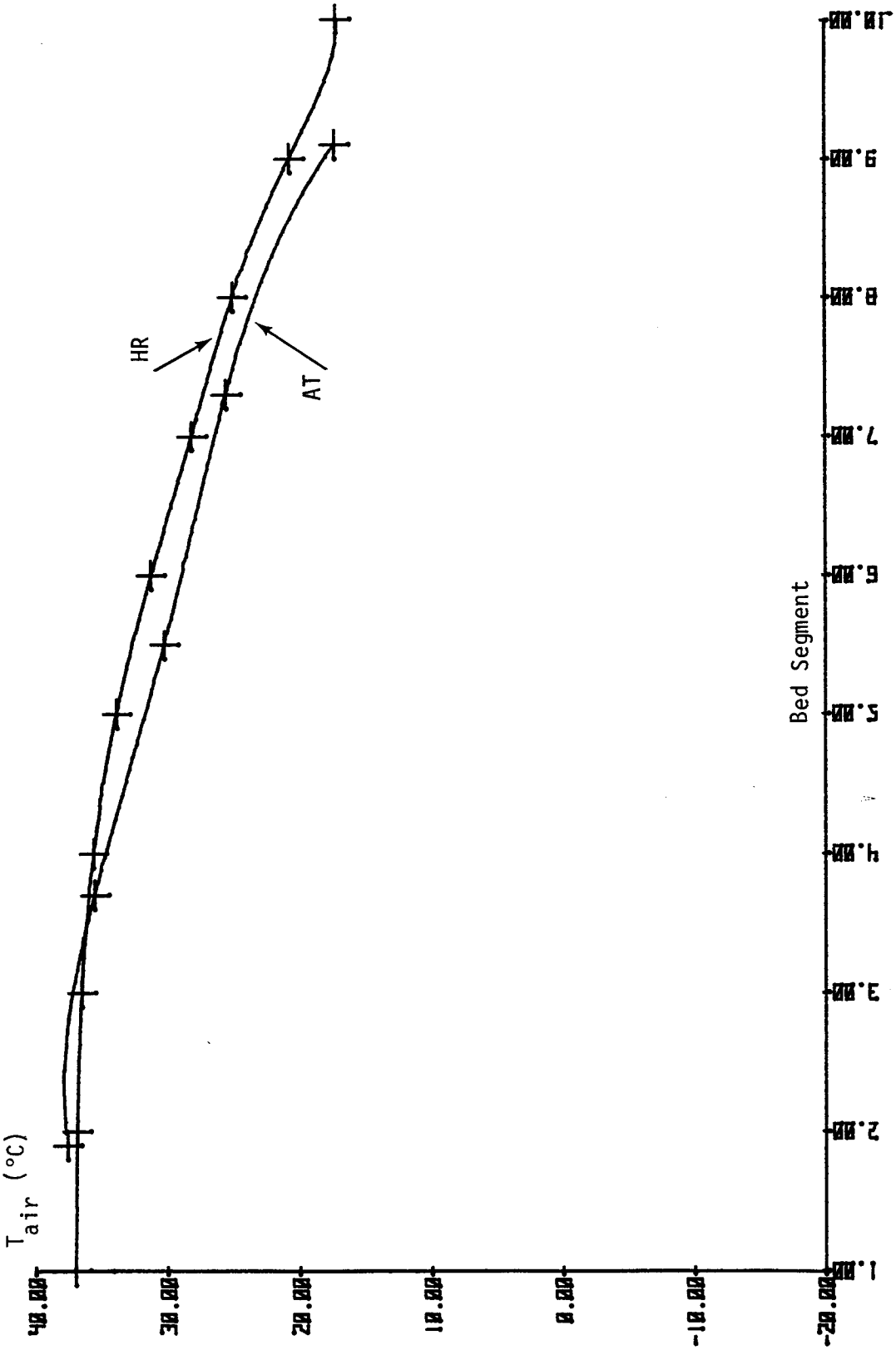


Fig. 26.--HOTROCKS-AT Comparison; Sinusoid Input, Single Time Segment, #7

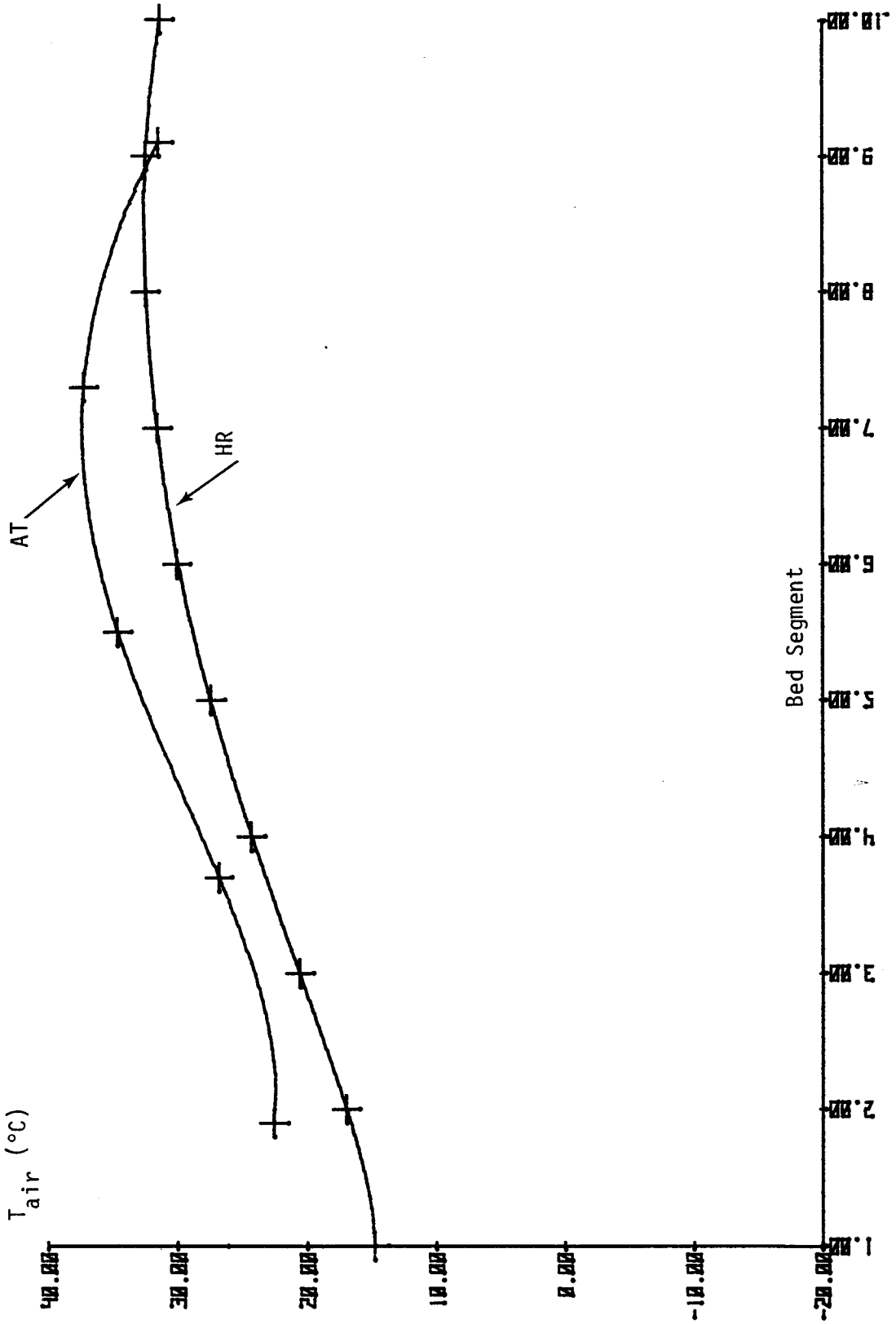


Fig. 27.--HOTROCKS-AT Comparison; Sinusoid Input, Single Time Segment, #10

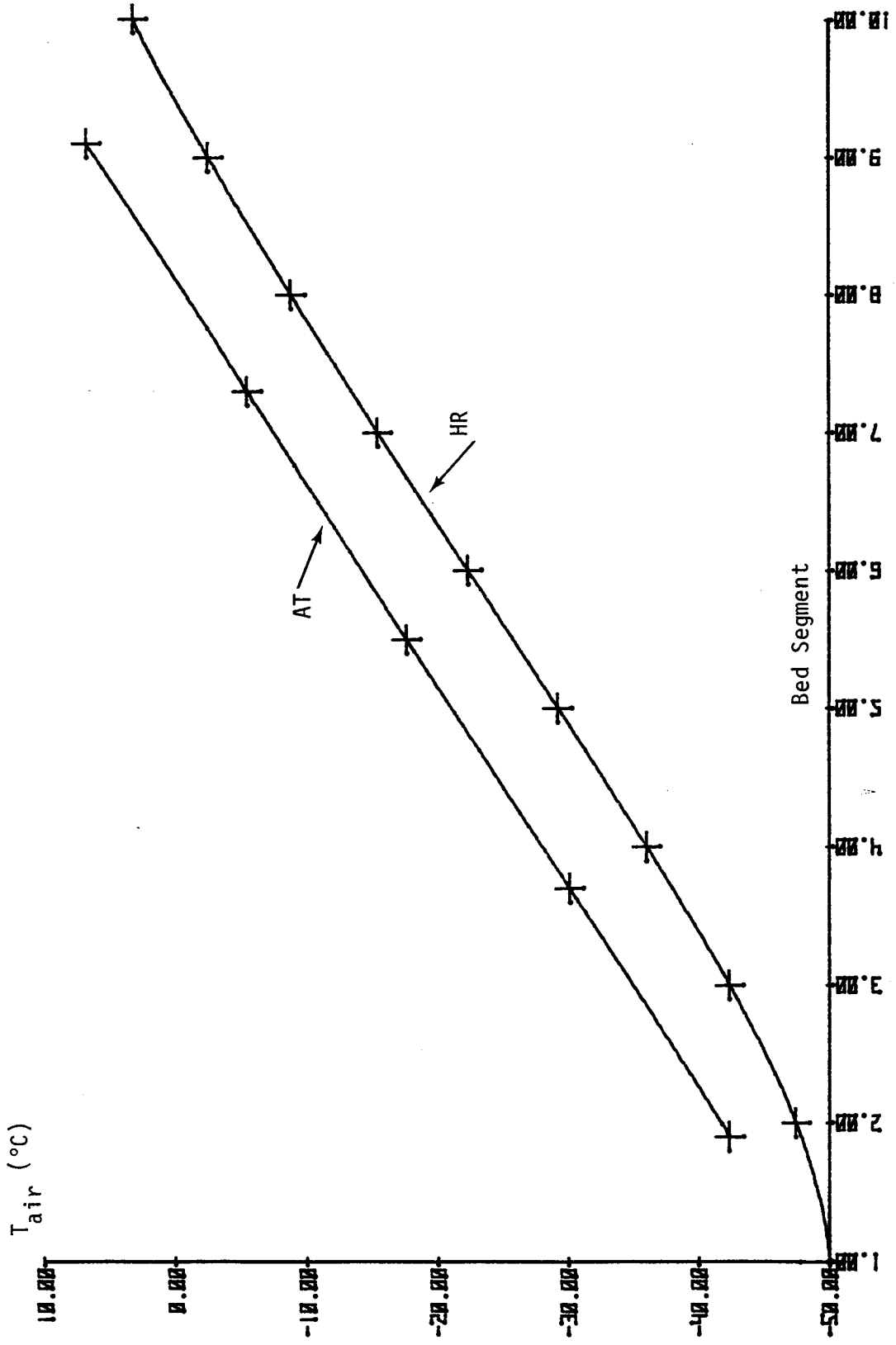


Fig. 28.--HOTROCKS-AT Comparison; Sinusoid Input, Single Time Segment, #15

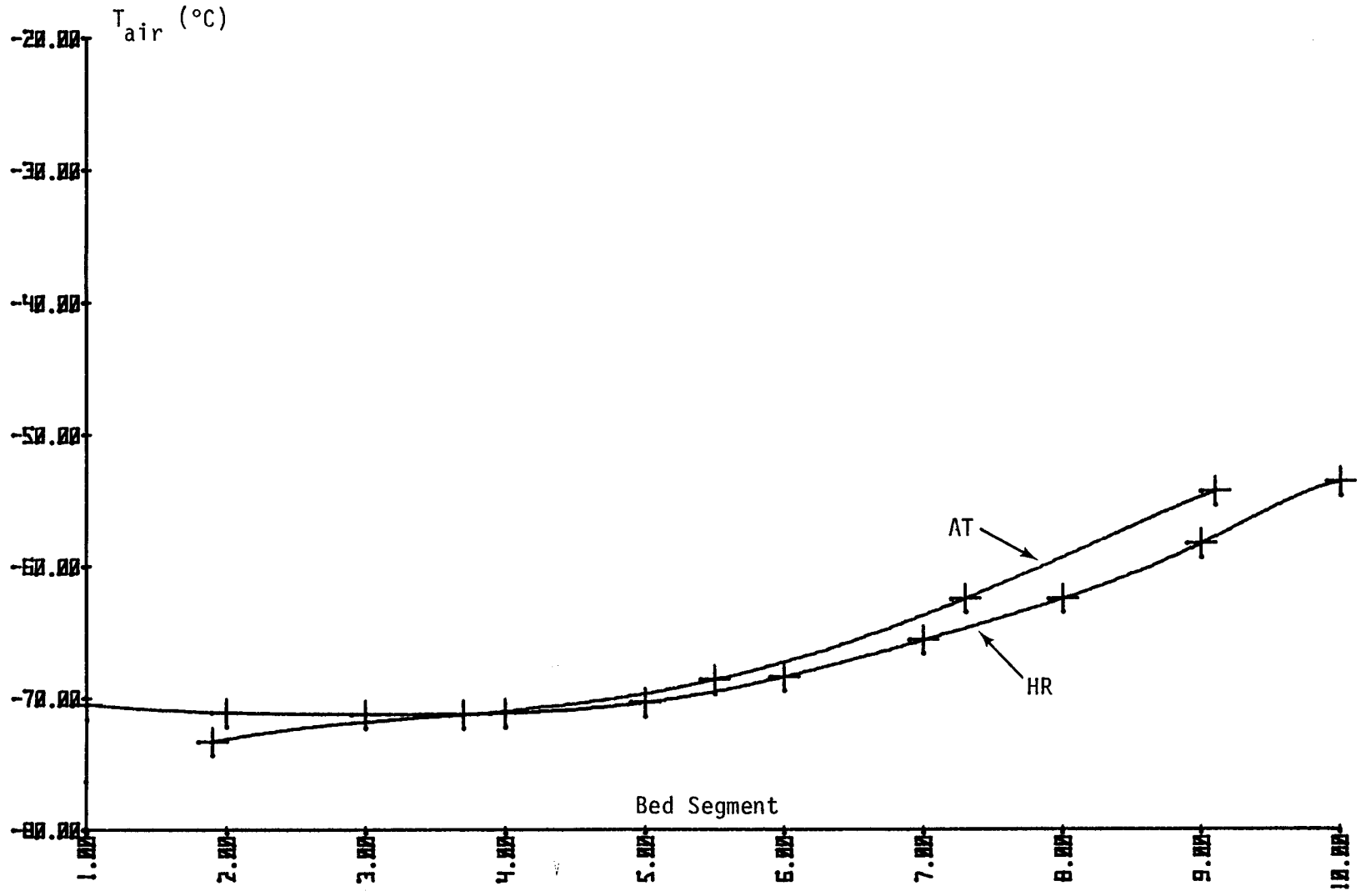


Fig. 29.--HOTROCKS-AT Comparison; Sinusoid Input, Single Time Segment, # 20

As a final note, only the air temperatures have been compared in the AT comparison analysis. This is solely because the AT model only produces an air temperature profile as output, so no rock temperature data were available from AT. However, since air is the energy transport medium in a packed bed system, an air temperature comparison is sufficient to validate the system model.

CHAPTER VI

SUMMARY AND CONCLUSION

It was shown in Chapter I of this paper that a clear need exists for a packed bed thermal energy storage model that is accurate enough to adequately describe transient behavior of a packed bed used for solar energy heat storage, yet uncomplicated enough to be easily used as a system design tool. No other model currently documented in the literature has these two desirable features. It was decided that an expansion of the work conducted by Klinkenberg on an analytical solution to the heat transfer equations developed by Schumann was the most promising approach to build a model that could meet both criteria. The FORTRAN coded packed bed model HOTROCKS was so developed.

After an analysis to ensure that the simplifying assumptions made during the initial phase of model development were valid, HOTROCKS was coded using structured programming techniques. Due to the modular format of the resultant HOTROCKS program, an extensive module-by-module verification procedure was made possible. This modular verification confirmed that the complete program should work satisfactorily for a wide variety of input conditions.

A two-stage validation process was then followed to establish the proper operation of the entire HOTROCKS model. Initially a comparison of HOTROCKS with the step input response Schumann model was made. The HOTROCKS model produced output data virtually identical to

the accepted standard Schumann model for two varied sets of input data; this established a level of confidence in HOTROCKS which was futher enhanced by a favorable comparison of HOTROCKS to the Alexander-Taft numerical packed bed model, for both the step and sinusoidal input cases.

HOTROCKS works. Not only does it produce accurate output, but it also does so quickly. Because the baseline model equations are expressed in terms of easily calculated standard functions (sine, cosine, exponential, etc.) and rapidly converging series instead of difficult to evaluate sets of simultaneous difference equations, HOTROCKS executes quickly. The typical CPU time spent on the generation of one time segment's set of air and rock temperatures, for typical solar packed bed input parameters, is on the order of one to two seconds using a UNIVAC 1110. Also, the HOTROCKS routine is relatively immune to large changes in time segment length. Where numerical difference equation models will need to spend twice as long in brute force number crunching to arrive at a solution for a 2t second time step as opposed to a solution for a 1t second time step, the variable integration step size provision built into HOTROCKS will not require such a large increase in calculation time.

The HOTROCKS model was built as a stand-alone unit, complete with output processing and formatting capabilities, in order to facilitate verification and validation. It can be used as is, to simply predict the behavior of a packed bed, or it can be integrated into a larger, more complete solar energy analysis program which may then be used for computer-aided design and optimization of complete solar energy heat systems.

It has been determined, both from the analysis completed in Chapter II and from data available from the AT model, that for initial development purposes allowance for a convective wall loss term is not necessary as part of HOTROCKS. However, one can envision the case where significant wall losses are a necessary part of a specific packed bed design. Also, many times the design of a packed bed for solar home heating will include within the bed a tank for heating water. Additions such as these to the basic HOTROCKS model will be possible, but are beyond the scope of this presentation. One need only construct a new module to account for the addition, the incorporate that module in the routines that calculate air and rock temperatures. Thus, expansion of the basic model is possible to allow for any off-nominal bed configuration.

It must be noted that HOTROCKS has been shown to perform in a manner comparable to other models of particle bed heat storage, but has not yet been demonstrated accurate by comparison with real experimental packed bed data. Unfortunately, these data do not exist in the current literature, or such a comparison would have been attempted.

Finally, it should be noted that HOTROCKS, except for the routime OUTPRT, is constructed with ANSI standard FORTRAN and should be compatible with any machine capable of executing a FORTRAN program. During the course of its development, HOTROCKS was successfully brought up and running on an Amdahl 470, an IBM 370, an IBM 360, a Univac 1110 and a PDP-11/45. One need only take care to verify that the integer arithmetic precision in the machine on which one wishes to run HOTROCKS is adequate to handle the computations required.

APPENDIX A

Routine Overview

The HOTROCKS computer model is designed to provide an hour-by-hour simulation of the thermal transfer between air and rock within a packed bed solar thermal energy storage system. The HOTROCKS routine furnishes a map of air and rock temperatures at ten points within the bed, updated every hour. This temperature profile can then be used to determine the amount of thermal energy stored within the bed, and can be used to furnish hourly energy availability information for a real time solar thermal system model.

The routine is based upon Klinkenberg's solutions to the two-phase energy transfer problem first developed by Schumann in 1929. He expresses the temperature within a packed bed (for both air and rock within the bed) as a function of the rate of heat transfer, the position within the bed, and time. The equations for air and rock temperatures are:

$$T_a = \theta_a(Z) - \left\{\theta_a(0)-\theta_s(0)\right\}\left[\frac{\partial F}{\partial Z}\right]_{y,z} - \int_0^z \theta_a'(\alpha)\left[\frac{\partial F}{\partial Z}\right]_{y,(z-\alpha)} d\alpha + \int_0^y \theta_s'(\beta)\left[\frac{\partial F}{\partial Z}\right]_{y-\beta,z} d\beta$$

and

$$T_s = \theta_s(y) + \left\{\theta_a(0)-\theta_s(0)\right\}\left(\frac{\partial F}{\partial Y}\right)_{y,z} + \int_0^z \theta_a'(\alpha)\left(\frac{\partial F}{\partial Y}\right)_{y,(z-\alpha)} d\alpha - \int_0^y \theta_s'(\beta)\left(\frac{\partial F}{\partial Y}\right)_{y-\beta,z} d\beta$$

where

T_a = air temperature

T_s = rock temperature

$\theta_a(z)$ = the inlet air temperature distribution (function of time)

$\theta_s(y)$ = the initial rock temperature distribution (function of position)

F = the heat transfer function

Applicable derivatives of the heat transfer function are developed from Nusselt's series expression for F:

$$F = \sum_{n=0}^{\infty}\left[1-e^{-y}\cdot\sum_{k=0}^{n}\frac{Y^k}{k!}\right]\left[1-e^{-z}\cdot\sum_{k=0}^{n}\frac{Z^k}{k!}\right]$$

Explicitly, those derivatives are:

$$\frac{\partial F}{\partial Y} = \left(e^{-y}\right)\left[\left(1-e^{-z}\right) + \sum_{n=1}^{\infty}\left[1-e^{-z}\sum_{k=0}^{n}\frac{Z^k}{k!}\right]\left(\frac{Y^n}{n!}\right)\right]$$

and

$$\frac{\partial F}{\partial Z} = \left(e^{-z}\right)\left[\left(1-e^{-y}\right) + \sum_{n=1}^{\infty}\left(1-e^{-y}\sum_{k=0}^{n}\frac{y^k}{k!}\right)\left(\frac{z^n}{n!}\right)\right]$$

valid for $y \neq 0$ and $z \neq 0$.

Detailed flowcharts of each of the HOTROCKS routines follow overleaf.

Program Flowchart Descriptors

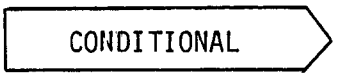
This section presents flowchart symbols with a detailed explanation of the mnemonics necessary to understand the program flows presented in the following appendix.

Four basic symbols are used:

The terminal block designates the beginning or end of a division of the software (i.e., a routine). Within the block is contained either the word BEGIN or END and the software name of the diagrammed routine.

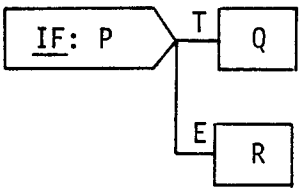
The statement block may contain either a mathematical expression or a logical statement. It may also reference another flow diagram if the software is designed to call another routine at this location. To indicate that another routine is being called, the statement block will contain the software name of the called routine followed by a

brief description of the called routine's function in parentheses.



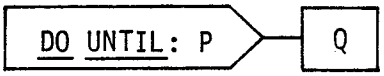
The conditional (i.e., "IF") statement transfers flow to another section of software, provided the condition within the block is met. The symbol is used in two ways dependent upon whether the decision is an IF-THEN-ELSE type or an IF-THEN (no explicit ELSE clause) type:

This case has no explicit ELSE clause. If P is true do Q, then return to the next statement immediately below the decision. NOTE: Q may be more than just a single statement block.

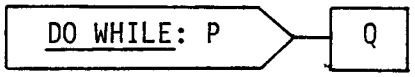


This case has an explicit ELSE clause. If P is true do Q, if P is not true do R, then return to the next statement below the decision.

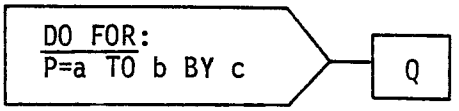
The DO loop is a special kind of conditional statement which transfers flow to another section of software for a prescribed number of iterations or until a particular test condition is met. There are four distinct types of DO statement, described below:



Do Q, then check to see if P is true. Continue to do Q until P is true, then transfer to the next statement below the DO loop block.

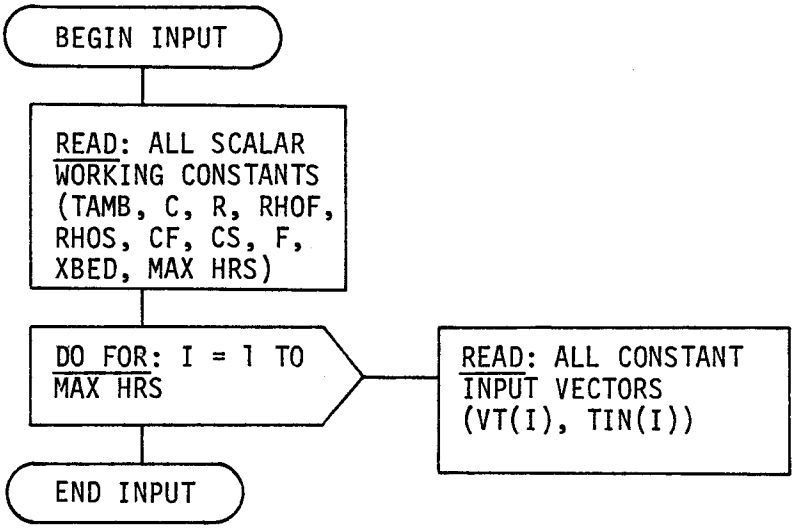


If P is true, do Q. Continue to check P, doing Q as long as P is true; then transfer to the next statement below the decision block.

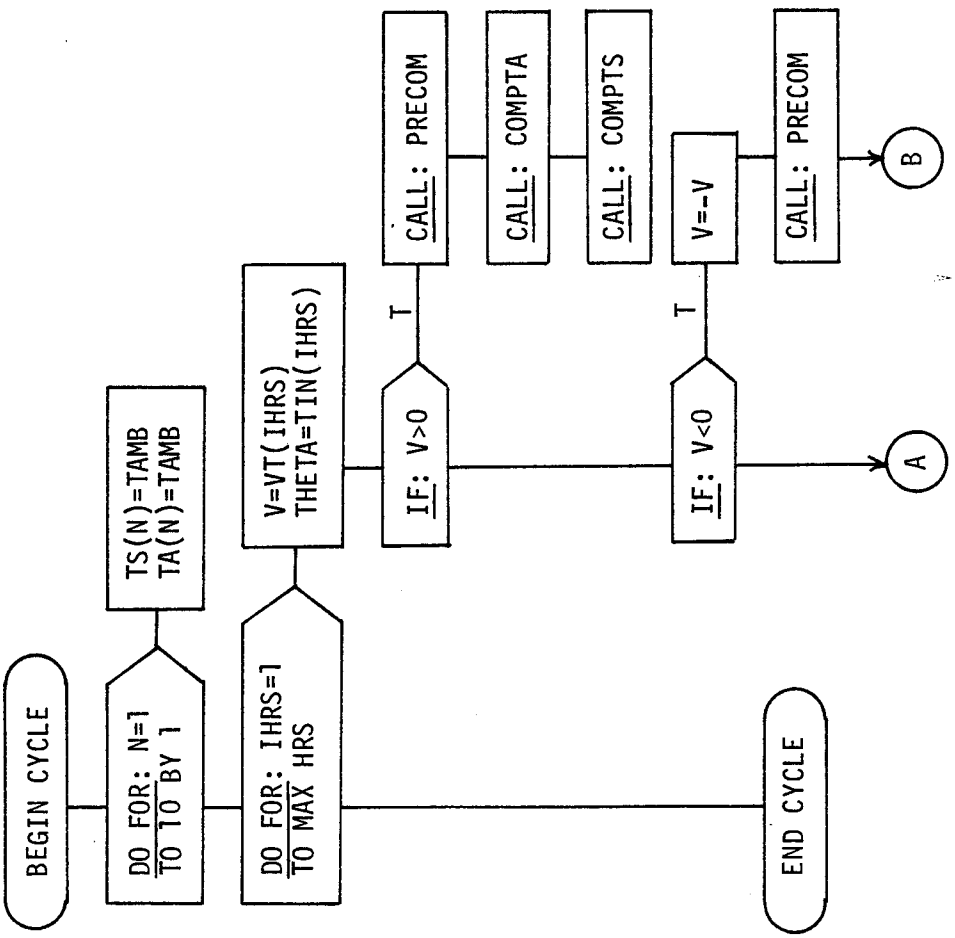


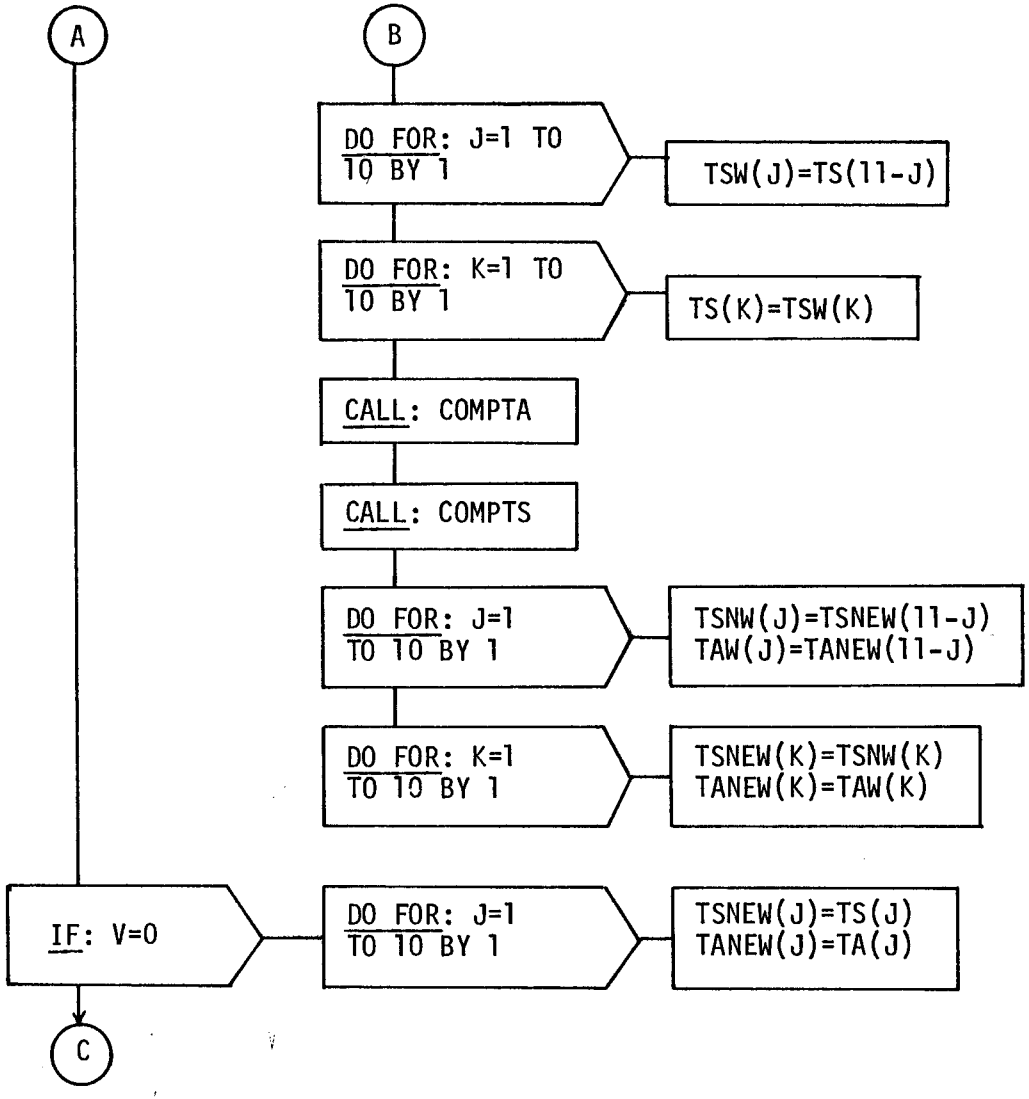
Do Q while P increments from a to b (each increment step = c), then return to the next statement following the decision block.

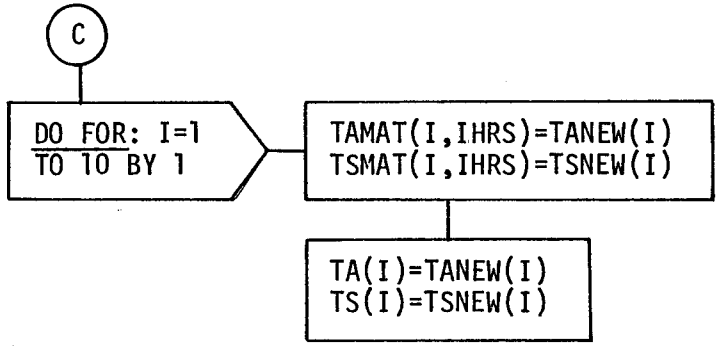
ROUTINE INPUT



ROUTINE CYCLE

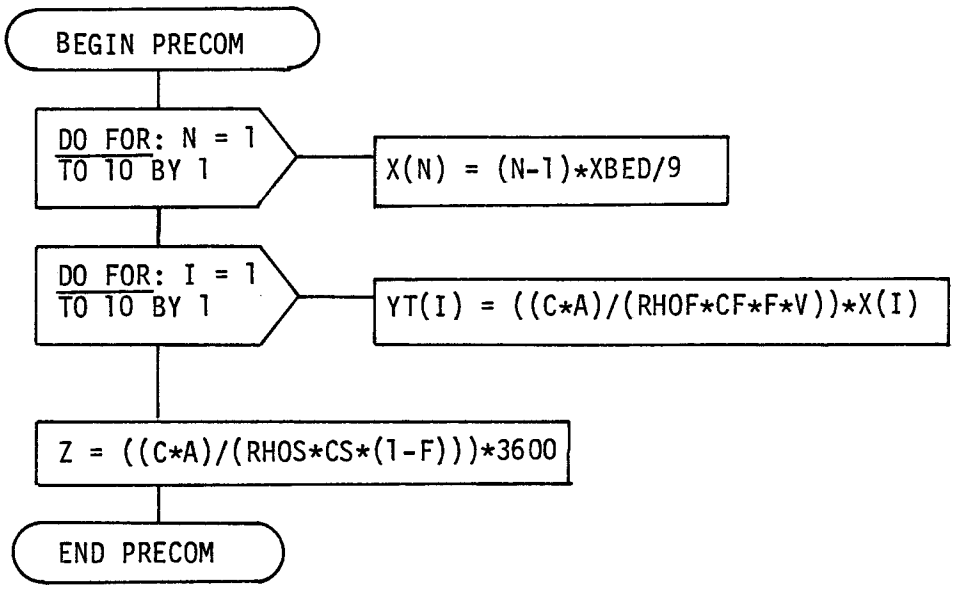



A
B
DO FOR: J=1 TO 10 BY 1
TSW(J)=TS(11-J)
DO FOR: K=1 TO 10 BY 1
TS(K)=TSW(K)
CALL: COMPTA
CALL: COMPTS
DO FOR: J=1 TO 10 BY 1
TSNW(J)=TSNEW(11-J)
TAW(J)=TANEW(11-J)
DO FOR: K=1 TO 10 BY 1
TSNEW(K)=TSNW(K)
TANEW(K)=TAW(K)
IF: V=0
DO FOR: J=1 TO 10 BY 1
TSNEW(J)=TS(J)
TANEW(J)=TA(J)
C


C
DO FOR: I=1
TO 10 BY 1
TAMAT(I,IHRS)=TANEW(I)
TSMAT(I,IHRS)=TSNEW(I)
TA(I)=TANEW(I)
TS(I)=TSNEW(I)

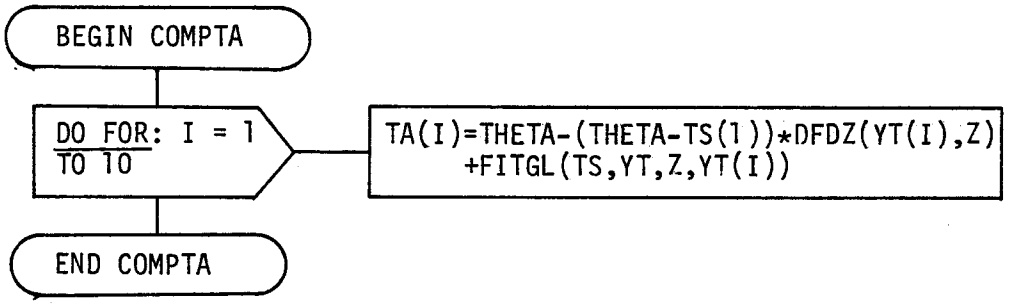
ROUTINE PRECOM

$$Y = \frac{C \cdot A}{\rho_f c_f f} \cdot \frac{x}{v}, \quad Z = \frac{C \cdot A}{\rho_s c_s (1-f)} \cdot t$$



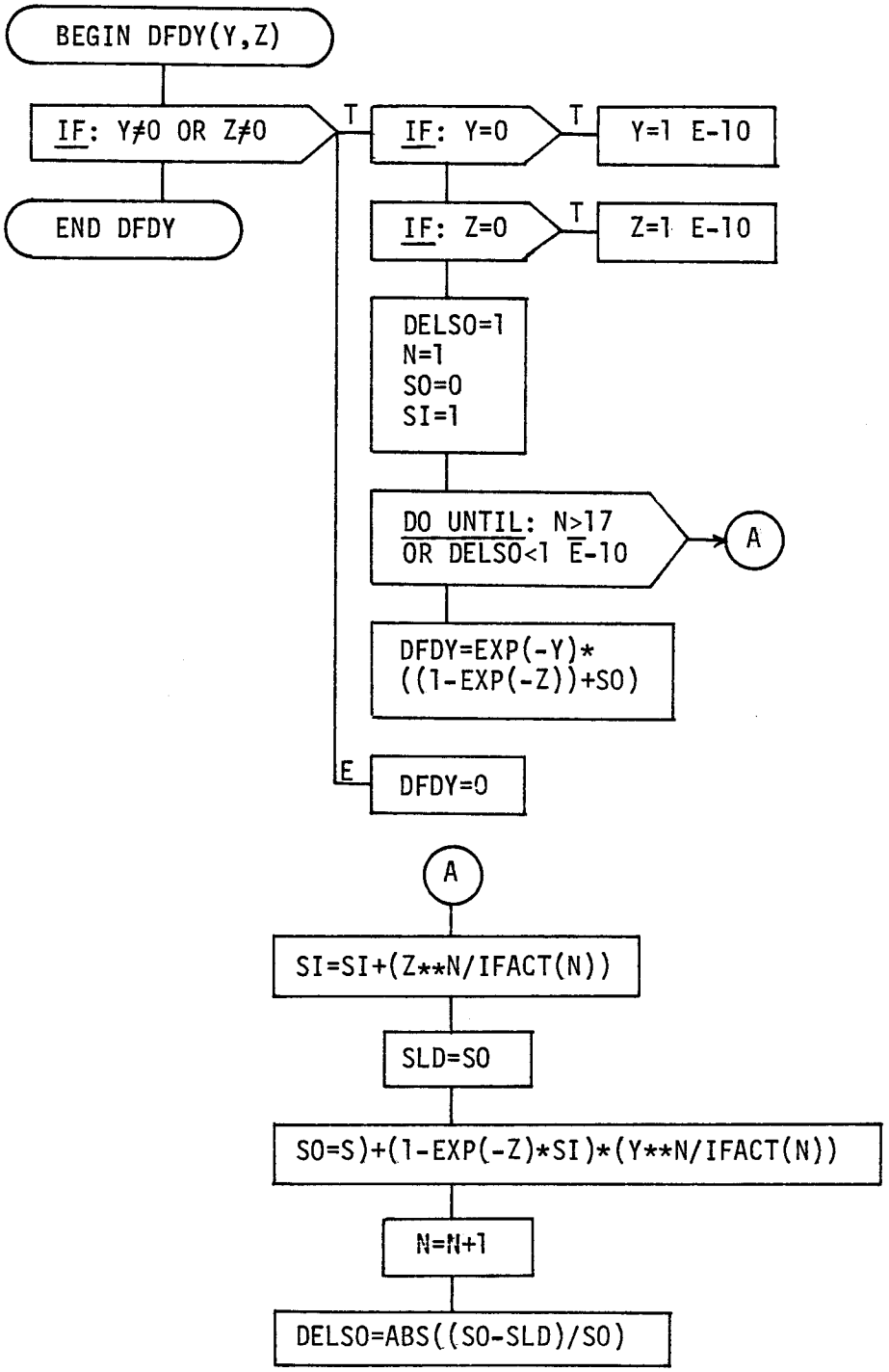
ROUTINE COMPTA/COMPTS

$$T_A(Y,Z) = \Theta_A - [\Theta_A - \Theta_S(0)]\left(\frac{\partial F}{\partial Z}\right)_{y,z} + \int_0^y \Theta_2'(n)\left(\frac{\partial F}{\partial Z}\right)_{y-n,z} dn$$



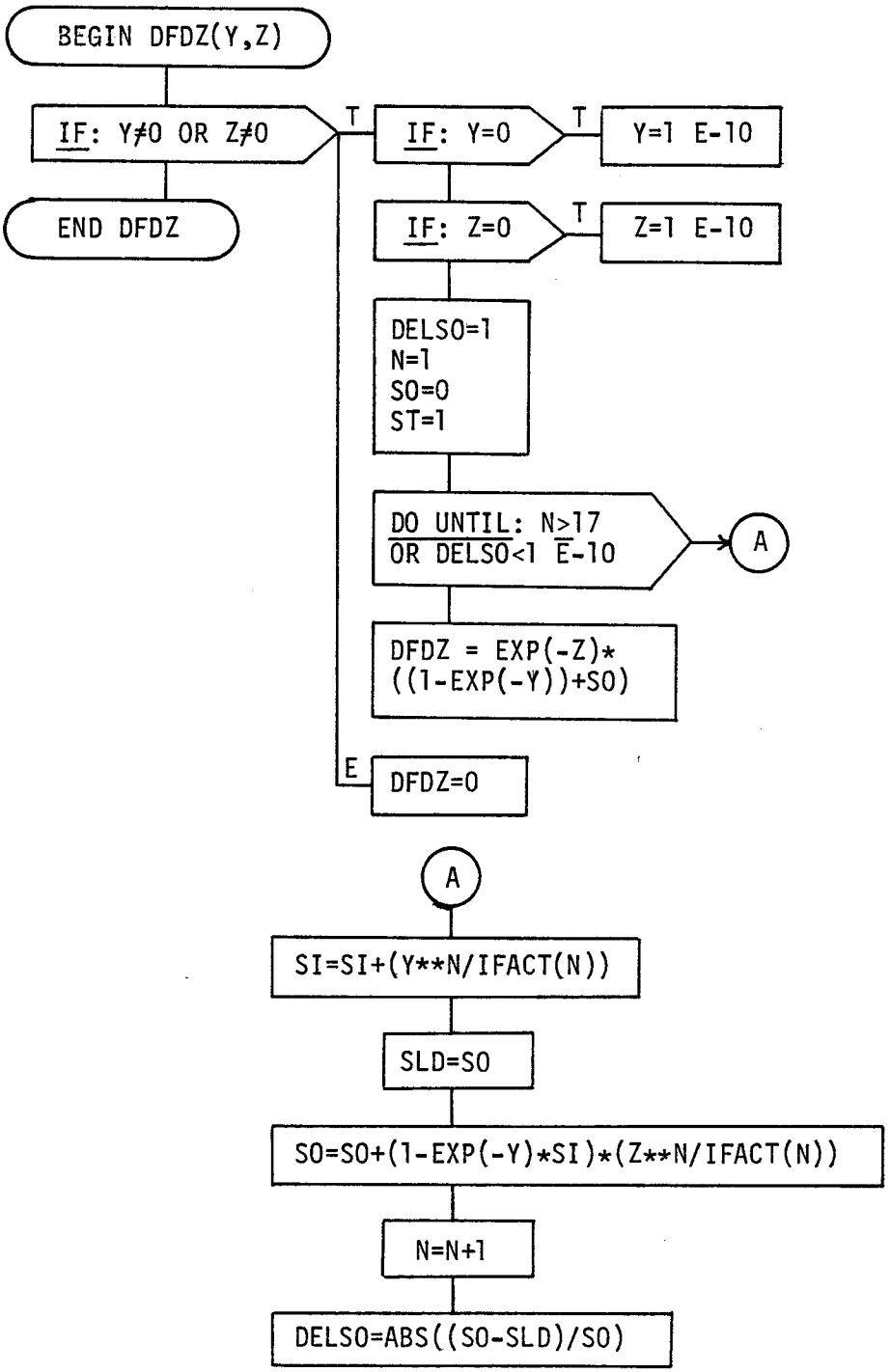
ROUTINE DFDY

$$\frac{\partial F}{\partial Y} = \left(e^{-y}\right)\left[\left(1-e^{-z}\right) + \sum_{n=1}^{\infty}\left(1-e^{-z}\sum_{k=0}^{n}\frac{Z^k}{k!}\right)\left(\frac{Y^n}{n!}\right)\right]$$

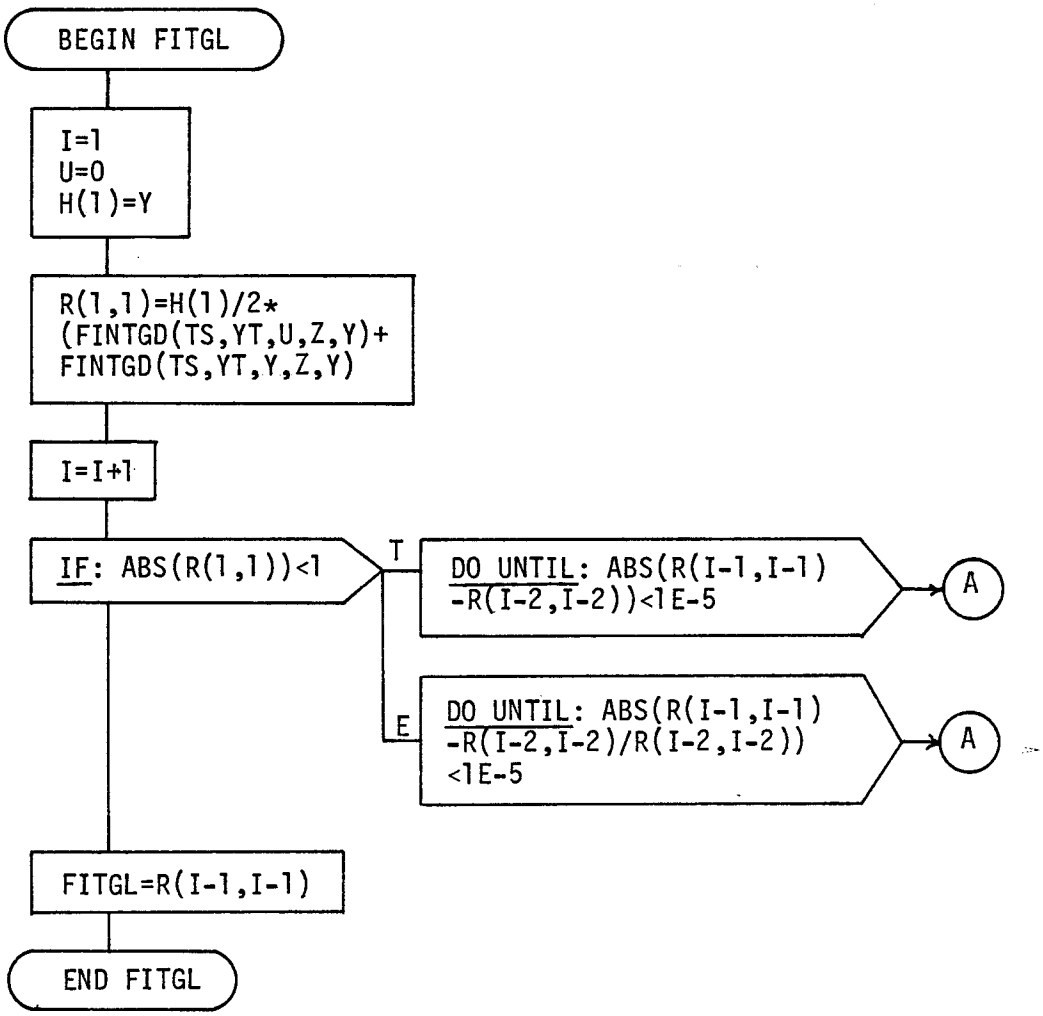


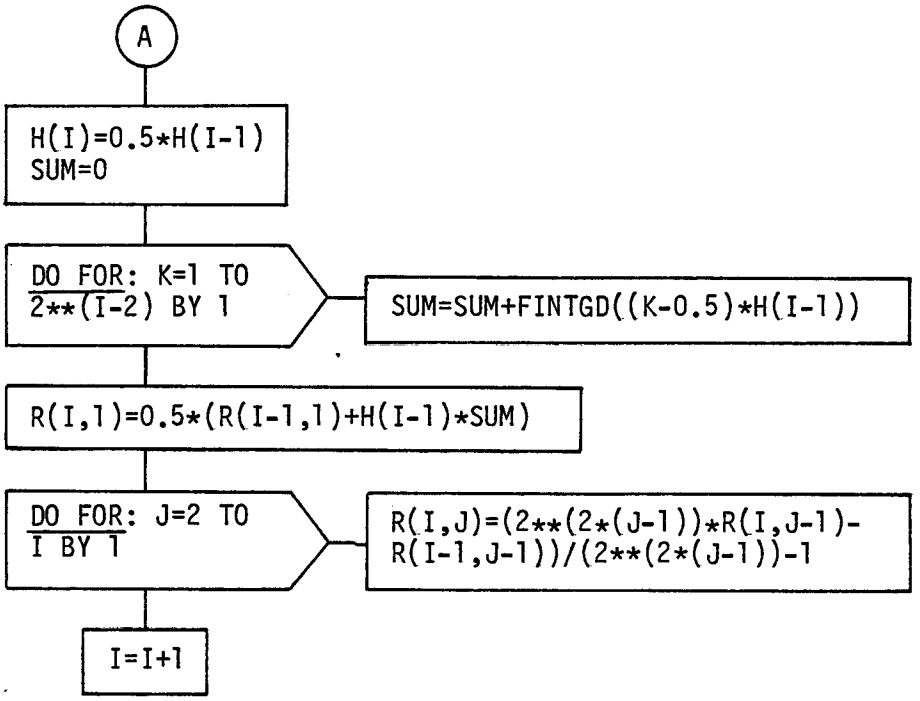
ROUTINE DFDZ

$$\frac{\partial F}{\partial Z} = \left(e^{-z}\right)\left[\left(1-e^{-y}\right) + \sum_{k=0}^{\infty}\left(1-e^{-y}\sum_{k=0}^{n}\frac{Y^k}{K!}\right)\left(\frac{Z^n}{n!}\right)\right]$$

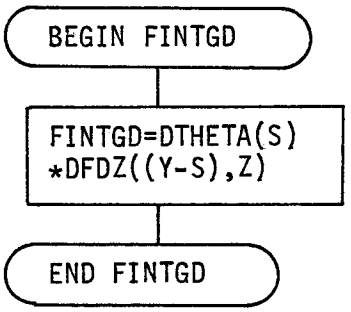


ROUTINE FITGL/SITGL

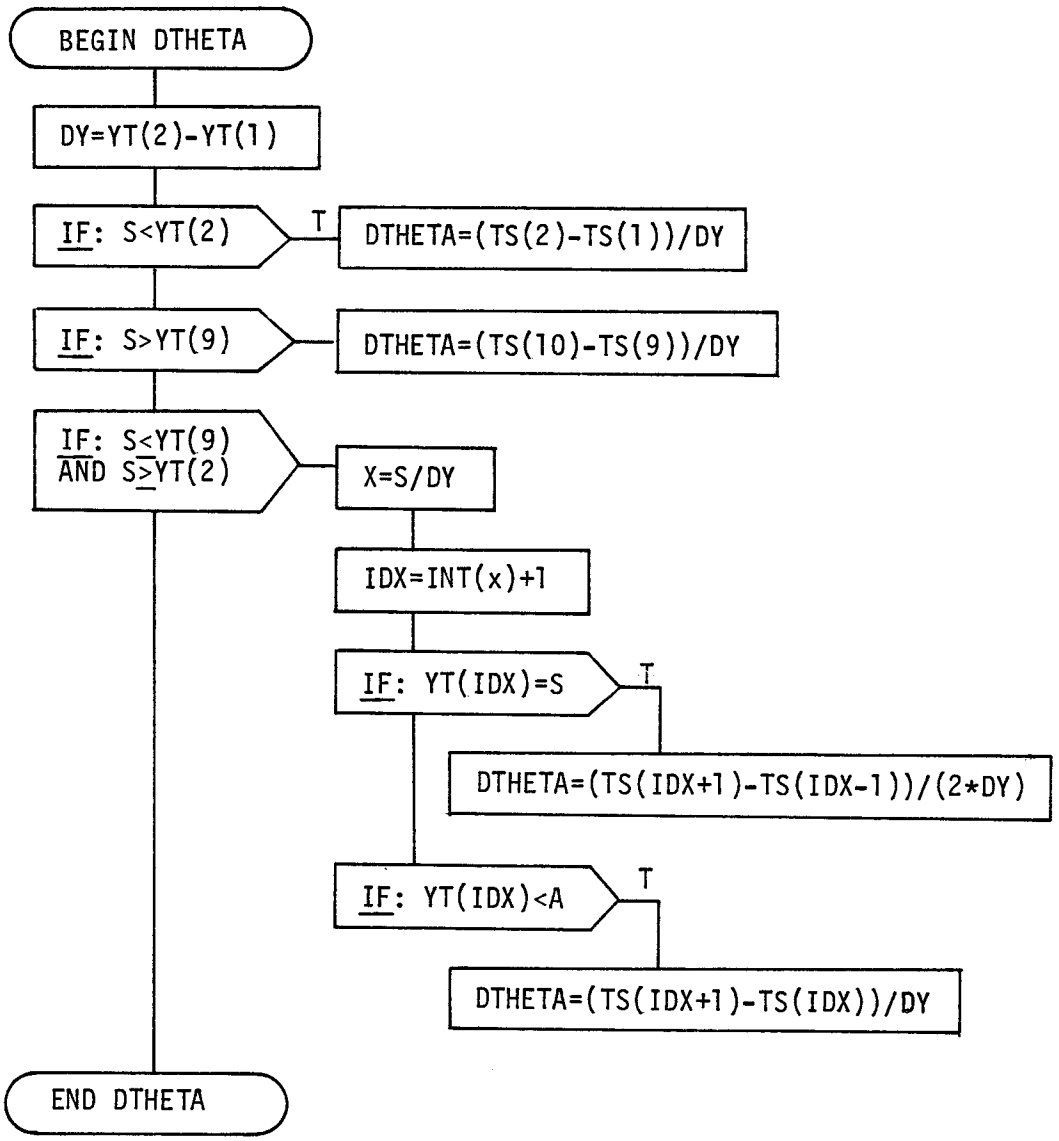



A
H(I)=0.5*H(I-1)
SUM=0
DO FOR: K=1 TO
2**(I-2) BY 1
SUM=SUM+FINTGD((K-0.5)*H(I-1))
R(I,1)=0.5*(R(I-1,1)+H(I-1)*SUM)
DO FOR: J=2 TO
I BY 1
R(I,J)=(2**(2*(J-1))*R(I,J-1)-
R(I-1,J-1))/(2**(2*(J-1))-1
I=I+1

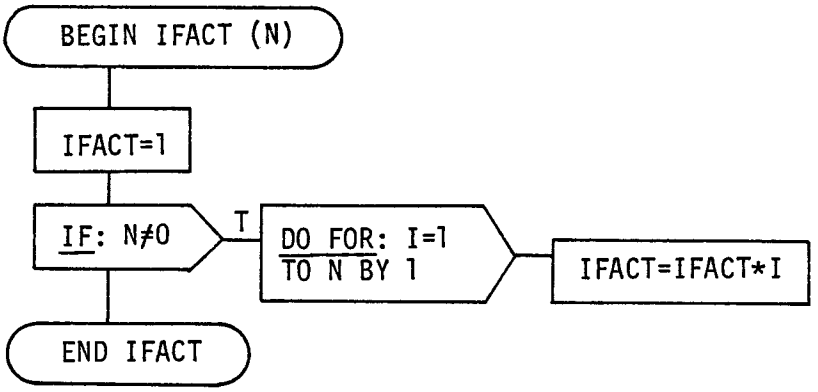
ROUTINE FINTGD/SINTGD



ROUTINE DTHETA



ROUTINE IFACT



APPENDIX B

FORTRAN Listing of HOTROCKS

```
▷LINES:54 FIELDATA
▷@PRT,S SPSDATA.CYCLE
▷FURPUR 27R3A      E35 SL73R1 05/13/81 16:56:15
▷EJ5-N05545*SPSDATA(1).CYCLE
▷      1            SUBROUTINE CYCLE(TAMB,MAXHRS,VT,TIN,C,A,RHOF,RHOS,CF,CS,F,XBE
0,
▷      2           1TAMAT,TSMAT)
▷      3            DIMENSION VT(24),TIN(24),YT(10),TS(10),TA(10),YTW(10),TSW(10)

▷      4            DIMENSION TSNEW(10),TANEW(10),TSNW(10),TAW(10),TAMAT(10,24)

▷      5            DIMENSION TSMAT(10,24)
▷      6            DO 1 N=1,10
▷      7            TS(N)=TAMB
▷      8            TA(N)=TAMB
▷      9          1 CONTINUE
▷     10            DO 2 IHRS=1,MAXHRS
▷     11      C     WRITE(6,99)
▷     12      C  99 FORMAT(10X,'HELLO')
▷     13            V=VT(IHRS)
▷     14            THETA=TIN(IHRS)
▷*OUTPUT INTERRUPT
—
```

```
▷     15              IF(V .LE. 0.) GO TO 3
▷     16              CALL PRECOM(C,A,RHOF,RHOS,CF,CS,F,V,XBED,YT,Z)
▷     17              CALL COMPTA(TS,YT,Z,THETA,TANEW)
▷     18              CALL COMPTS(TS,YT,Z,THETA,TSNEW)
▷     19            3 CONTINUE
▷     20              IF(V .GE. 0.) GO TO 4
▷     21              V=-V
▷     22              CALL PRECOM(C,A,RHOF,RHOS,CF,CS,F,V,XBED,YT,Z)
▷     23              DO 5 J=1,10
▷     24              TSW(J)=TS(11-J)
▷     25            5 CONTINUE
▷     26              DO 6 K=1,10
▷     27              TS(K)=TSW(K)
▷     28            6 CONTINUE
▷     29              CALL COMPTA(TS,YT,Z,THETA,TANEW)
▷     30              CALL COMPTS(TS,YT,Z,THETA,TSNEW)
▷     31              DO 7 J=1,10
▷     32              TSNW(J)=TSNEW(11-J)
▷     33              TAW(J)=TANEW(11-J)
▷     34            7 CONTINUE
▷*OUTPUT INTERRUPT
```

```
▷     35              DO 8 K=1,10
▷     36              TSNEW(K)=TSNW(K)
▷     37              TANEW(K)=TAW(K)
▷     38            8 CONTINUE
▷     39            4 CONTINUE
▷     40              IF(V .NE. 0.) GO TO 9
▷     41              DO 10 J=1,10
▷     42              TSNEW(J)=TS(J)
▷     43              TANEW(J)=TA(J)
▷     44           10 CONTINUE
▷     45            9 CONTINUE
▷     46              DO 11 I=1,10
▷     47              TAMAT(I,IHRS)=TANEW(I)
▷     48              TSMAT(I,IHRS)=TSNEW(I)
▷     49              TA(I)=TANEW(I)
▷     50              TS(I)=TSNEW(I)
▷     51           11 CONTINUE
▷     52            2 CONTINUE
▷     53              RETURN
▷     54              END
▷DATA IGNORED - IN CONTROL MODE
▷
```

```
@PRT,S SPSDATA.PRECOM
EJ5-N05545*SPSDATA(1).PRECOM
      1              SUBROUTINE PRECOM(C,A,RHOF,RHOS,CF,CS,F,V,XBED,YT,Z)
      2              DIMENSION X(10),YT(10)
      3              DO 1 N=1,10
      4              X(N)=(N-1)*XBED/9.
      5            1 CONTINUE
      6              DO 2 I=1,10
      7              YT(I)=X(I)*C*A/(RHOF*CF*F*V)
      8            2 CONTINUE
      9              Z=3600.*C*A/(RHOS*CS*(1.-F))
     10              RETURN
     11              END
```

```
▶
▶EJ5-N05545*SPSDATA(1).COMPTA
▶      1              SUBROUTINE COMPTA(TS,YT,Z,THETA,TANEW)
▶      2              DIMENSION TS(10),YT(10),TANEW(10)
▶      3              DO 1 I=1,10
▶      4              TANEW(I)=THETA-(THETA-TS(1))*DFDZ(YT(I),Z)+FITGL(TS,YT,Z,YT(I
))
▶      5            1 CONTINUE
▶      6              RETURN
▶      7              END
▶
```

```
▶
▶EJ5-N05545*SPSDATA(1).COMPTS
▶      1              SUBROUTINE COMPTS(TS,YT,Z,THETA,TSNEW)
▶      2              DIMENSION TS(10),YT(10),TSNEW(10)
▶      3              DO 1 I=1,10
▶      4              TSNEW(I)=TS(I)+(THETA-TS(1))*DFDY(YT(I),Z)-SITGL(TS,YT,Z,YT(I
))
▶      5            1 CONTINUE
▶      6              RETURN
▶      7              END
▶
```

```
▷@PRT,S SPSDATA.DFDY
▷FURPUR 27R3A      E35 SL73R1 05/13/81 17:12:45
▷EJ5-N05545*SPSDATA(1).DFDY
▷      1              FUNCTION DFDY(Y,Z)
▷      2              DFDY=0.
▷      3              IF(Y .NE. 0.) GO TO 2
▷      4              Y=1.E-10
▷      5            2 CONTINUE
▷      6              IF(Z .NE. 0.) GO TO 3
▷      7              Z=1.E-10
▷      8            3 CONTINUE
▷      9              DELSO=1.
▷     10              N=1
▷     11              SO=0.
▷     12              SI=1.
▷     13            4 CONTINUE
▷     14              SI=SI+(Z**N/IFACT(N))
▷     15              SLD=SO
▷     16              SO=SO+(1.-EXP(-Z)*SI)*(Y**N/IFACT(N))
▷     17              N=N+1
▷     18              DELSO=ABS(SO-SLD)
▷*OUTPUT INTERRUPT
```

```
▷     19              IF(ABS(SO) .GT. 1.) DELSO=DELSO/ABS(SO)
▷     20              IF(DELSO .GT. 1.E-10 .AND. N .LT. 13) GO TO 4
▷     21              DFDY=EXP(-Y)*((1.-EXP(-Z))+SO)
▷     22              RETURN
▷     23              END
▷DATA IGNORED - IN CONTROL MODE
▷
```

```
▷
▷_
```

```
▷@PRT,S SPSDATA.DFDZ
▷FURPUR 27R3A      E35 SL73R1 05/13/81 17:14:41
▷EJ5-N05545*SPSDATA(1).DFDZ
▷      1              FUNCTION DFDZ(Y,Z)
▷      2              DFDZ=0.
▷      3              IF(Y .EQ. 0. .AND. Z .EQ. 0.) GO TO 1
▷      4              IF(Y .NE. 0.) GO TO 2
▷      5              Y=1.E-10
▷      6            2 CONTINUE
▷      7              IF(Z .NE. 0.) GO TO 3
▷      8              Z=1.E-10
▷      9            3 CONTINUE
▷     10              DELSO=1.
▷     11              N=1
▷     12              SO=0.
▷     13              SI=1.
▷     14            4 CONTINUE
▷     15              SI=SI+(Y**N/IFACT(N))
▷     16              SLD=SO
▷     17              SO=SO+(1.-EXP(-Y)*SI)*(Z**N/IFACT(N))
▷     18              N=N+1
▷*OUTPUT INTERRUPT
```

```
▷
▷     19              DELSO=ABS(SO-SLD)
▷     20              IF(ABS(SO) .GT. 1.) DELSO=DELSO/ABS(SO)
▷     21              IF(DELSO .GT. 1.E-10 .AND. N .LT. 13) GO TO 4
▷     22              DFDZ=EXP(-Z)*((1.-EXP(-Y))+SO)
▷     23            1 CONTINUE
▷     24              RETURN
▷     25              END
▷DATA IGNORED - IN CONTROL MODE
▷
```

```
▷FURPUR 27R3A      E35 SL73R1 05/13/81 17:16:38
▷EJ5-N05545*SPSDATA(1).FITGL
▷      1            FUNCTION FITGL(TS,YT,Z,Y)
▷      2            DIMENSION TS(10),YT(10),R(12,12),H(12)
▷      3            I=1
▷      4            U=0.0
▷      5            H(1)=Y
▷      6            R(1,1)=H(1)/2.*(FINTGD(TS,YT,U,Z,Y)+FINTGD(TS,YT,Y,Z,Y))
▷      7            I=I+1
▷      8      C   BEGIN 'DO UNTIL' CONSTRUCT 1
▷      9          1 CONTINUE
▷     10            H(I)=H(I-1)/2.
▷     11            SUM=0.
▷     12            KN=2**(I-2)
▷     13            DO 2 K=1,KN
▷     14            WV=(K-0.5)*H(I-1)
▷     15            SUM=SUM+FINTGD(TS,YT,WV,Z,Y)
▷     16          2 CONTINUE
▷     17            R(I,1)=0.5*(R(I-1,1)+H(I-1)*SUM)
▷     18            DO 3 J=2,I
▷     19            R(I,J)=(2**(2*(J-1))*R(I,J-1)-R(I-1,J-1))/(2**(2*(J-1))-1)
▷*OUTPUT INTERRUPT
```

```
▷
▷     20           3 CONTINUE
▷     21             I=I+1
▷     22             DET=ABS(R(I-1,I-1)-R(I-2,I-2))
▷     23             IF(ABS(R(I-2,I-2)) .GT. 1.) DET=DET/ABS(R(I-2,I-2))
▷     24             IF(DET .GE. 1.E-05 .AND. I .LT. 12) GO TO 1
▷     25       C    END 'DO UNTIL' 1
▷     26             FITGL=R(I-1,I-1)
▷     27             RETURN
▷     28             END
▷DATA IGNORED - IN CONTROL MODE
▷
```

```
▷EJ5-N05545*SPSDATA(1).SITGL
▷      1•            FUNCTION SITGL(TS,YT,Z,Y)
▷      2             DIMENSION TS(10),YT(10),R(12,12),H(12)
▷      3             I=1
▷      4             U=0.0
▷      5             H(1)=Y
▷      6•            R(1,1)=H(1)/2.*(SINTGD(TS,YT,U,Z,Y)+SINTGD(TS,YT,Y,Z,Y))
▷      7             I=I+1
▷      8       C   BEGIN 'DO UNTIL' CONSTRUCT 1
▷      9           1 CONTINUE
▷     10             H(I)=H(I-1)/2.
▷     11             SUM=0.
▷     12             KN=2**(I-2)
▷     13             DO 2 K=1,KN
▷     14             WV=(K-0.5)*H(I-1)
▷     15•            SUM=SUM+SINTGD(TS,YT,WV,Z,Y)
▷     16           2 CONTINUE
▷     17             R(I,1)=0.5*(R(I-1,1)+H(I-1)*SUM)
▷     18             DO 3 J=2,I
▷     19             R(I,J)=(2**(2*(J-1))*R(I,J-1)-R(I-1,J-1))/(2**(2*(J-1))-1)
▷     20           3 CONTINUE
▷*OUTPUT INTERRUPT
```

```
▷
▷     21              I=I+1
▷     22              DET=ABS(R(I-1,I-1)-R(I-2,I-2))
▷     23              IF(ABS(R(I-2,I-2)) .GT. 1.) DET=DET/ABS(R(I-2,I-2))
▷     24              IF(DET .GE. 1.E-05 .AND. I .LT. 12) GO TO 1
▷     25        C   END 'DO UNTIL' 1
▷     26            SITGL=R(I-1,I-1)
▷     27              RETURN
▷     28              END
▷DATA IGNORED - IN CONTROL MODE
▷
```

```
-

▷
▷FURPUR 27R3A      E35 SL73R1 04/16/81 21:10:23
▷EJ5-N05545*SPSDATA(1).FINTGD
▷      1            FUNCTION FINTGD(TS,YT,S,Z,Y)
▷      2            DIMENSION TS(10),YT(10)
▷      3            FINTGD=DTHETA(TS,YT,S)*DFDZ((Y-S),Z)
▷      4            RETURN
▷      5            END
▷
▷EJ5-N05545*SPSDATA(1).SINTGD
▷      1            FUNCTION SINTGD(TS,YT,S,Z,Y)
▷      2            DIMENSION TS(10),YT(10)
▷      3            SINTGD=DTHETA(TS,YT,S)*DFDY((Y-S),Z)
▷      4            RETURN
▷      5            END
▷
```

```
▷EJ5-N05545*TPF$(0).DTHETA
▷      1              FUNCTION DTHETA(TS,YT,S)
▷      2              DIMENSION TS(10),YT(10)
▷      3              DY=YT(2)-YT(1)
▷      4              IF(S .GE. YT(2)) GO TO 1
▷      5              DTHETA=(TS(2)-TS(1))/DY
▷      6            1 CONTINUE
▷      7              IF(S .LE. YT(9)) GO TO 2
▷      8              DTHETA=(TS(10)-TS(9))/DY
▷      9            2 CONTINUE
▷     10              IF(S .GT. YT(9) .OR. S .LT. YT(2)) GO TO 3
▷     11              X=S/DY
▷     12              IDX=INT(X)+1
▷     13              IF(YT(IDX) .NE. S) GO TO 4
▷     14              DTHETA=(TS(IDX+1)-TS(IDX-1))/(2.*DY)
▷     15            4 CONTINUE
▷     16              IF(YT(IDX) .GE. S) GO TO 5
▷     17              DTHETA=(TS(IDX+1)-TS(IDX))/DY
▷     18            5 CONTINUE
▷     19            3 CONTINUE
▷     20              RETURN
▷     21              END
▷
```

```
▷@PRT,S IFACT
▷FURPUR 27R3A       E35 SL73R1 03/10/81 17:40:23
▷EJ5-N05545*TPF$(0).IFACT
▷      1              FUNCTION IFACT(N)
▷      2              IFACT=1
▷      3              IF(N .EQ. 0) GO TO 1
▷      4              DO 2 I=1,N
▷      5              IFACT=IFACT*I
▷      6            2 CONTINUE
▷      7            1 CONTINUE
▷      8              RETURN
▷      9              END
▷
```

```
▶
▶EJ5-N05545*SPSDATA(1).OUTPRT
▶      1              SUBROUTINE OUTPRT(VT,TIN,TAMAT,TSMAT,MAXHRS,C,A,RHOF,RHOS,CF,

▶      2             1CS,F,XBED,TAMB)
▶      3              DIMENSION TAMAT(10,24),TSMAT(10,24),VT(24),TIN(24)
▶      4              WRITE (6,99)
▶      5           99 FORMAT('1',8X,'AIR AND ROCK TEMPERATURE PROFILE FOR EACH HOUR

▶      6             1 PERIOD, ON AN HOURLY BASIS, GIVEN THE LISTED PARAMETERS',///
,
▶      7             29X,'TIME',3X,'MODE',2X,'VEL',2X,'INPUT TEMP',3X,'TOP.........
.
▶      8             3..........AIR TEMPERATURE PROFILE.....................BOTT
0
▶      9             4M',/)
▶     10              DO 1 I=1,MAXHRS
▶     11              MODE=0
▶     12              IF(VT(I) .GT. 0.) MODE=+1
▶     13              IF(VT(I) .LT. 0.) MODE=-1
▶     14              ZW=I
_
```

```
     15          WRITE(6,98) ZW,MODE,VT(I),TIN(I),TAMAT(1,I),TAMAT(2,I),TAMAT

     16         1(3,I),TAMAT(4,I),TAMAT(5,I),TAMAT(6,I),TAMAT(7,I),TAMAT(8,I)

     17         2,TAMAT(9,I),TAMAT(10,I)
     18        98 FORMAT(8X,F5.2,4X,I2,1X,F6.3,2X,F6.2,5X,10(F6.2,2X),/)
     19         1 CONTINUE
     20          WRITE(6,97)
     21        97 FORMAT(25X,'MODE KEY:  0=OFF,  1=SOLAR LOOP ON, -1=OUTPUT
     22         1 LOOP ON',/,'1')
     23          WRITE(6,96)
     24        96 FORMAT(9X,'TIME',3X,'MODE',2X,'VEL',2X,'INPUT TEMP',3X,'TOP

     25         1.....ROCK TEMPERATURE PROFILE.....

     26         2........BOTTOM',/)
     27          DO 2 I=1,MAXHRS
     28          MODE=0
     29          IF(VT(I) .GT. 0.) MODE=1
     30          IF(VT(I) .LT. 0.) MODE=-1
```

APPENDIX C

Test Module FORTRAN Listings

PRECOM TEST

This routine verifies that PRECOM is correctly calculating both the bed segment lengths and the 'Y' transformed equivalent values of bed length.

```
CODE        DIMENSION X(10),YT(10)
            DATA K/1./,A/2./,RHOF/3./,RHOS/4./,CF/3./,CS/2./,F/
           2.5/,V/1./,XBED/2./
            CALL PRECOM (K,A,RHOF,RHOS,CF,CS,F,V,XBED,YT,Z)
            DO 1 I=1,10
            WRITE (6,99)I,YT(I)
        99  FORMAT (1X,'I= ',I3,5X,'Y= ',1X,E11.4)
         1  CONTINUE
            WRITE (6,98)Z
        98  FORMAT (1Z,'Z= ',1X,E11.4)
            STOP
            END
```

CYCLE TEST

The following program and its given supporting subroutines will test all the logic paths contained in the routine CYCLE. Unique answers for TA and TS will be supplied for the V > 0, V < 0, and V = 0 cases.

```
      DIMENSION VT(6),TIN(6),TAMAT(10,6)TSMAT(10,6)
      DATA TAMB/20./MAXHRS/6/VT/.5,.5,0.,0,-.5,-.5/TIN/6*70./
      DATA C/1./A/2./RHOF/3./RHOS/4./CF/3./CS/2./F/.5/XBED/2./
      CALL CYCLE (TAMB,MAXHRS,VT,TIN,C,A,RHOF,RHOS,CF,CS,F
     2XBED,TAMAT,TSMAT
      DO 1 I=1,MAXHRS
      DO 2 J=1,10
      WRITE (6,99)I,J,TAMAT(J,I),TSMAT(J,I)
   99 FORMAT (1X,'HR.',1X,I3,2X'BED SEG.',1X,i3,2X,'TA',1X,
     2F7.3,2X,'TS',1X,F7.3)
    2 CONTINUE
    1 CONTINUE
      STOP
      END

      SUBROUTINE PRECOM (C,A,RHOF,RHOS,CF,CS,F,V,XBED,YT,Z)
      DIMENSION YT(10)
      DO 1 I=1,10
      YT(I)=10.*I
    1 CONTINUE
      Z=10.
      RETURN
      END

      SUBROUTINE COMPTA (TS,YT,Z,THETA, TANEW)
      DIMENSION TS(10),YT(10),TANEW(10)
      DO 1 I=1,10
      TANEW(I)=(TS(I)*YT(I)/Z)+THETA
    1 CONTINUE
      RETURN
      END

      SUBROUTINE COMPTS (TS,YT,Z,THETA,TSNEW)
      DIMENSION TS(10),YT(10),TSNEW(10)
      DO 1 I=1,10
      TSNEW(I)=(TS(I)*YT(I))/(2*Z)+THETA
    1 CONTINUE
      RETURN
      END
```

DFDY/DFDZ TEST

This test routine will calculate values of ∂F/∂Y for a range of Y and Z input values, using the function DFDY. Since the analytical form of ∂F/∂Y is so complex, a complete verification of the routine is not possible. Rather this test evaluates DFDY at enough different values of Y and Z to allow an assessment of the reasonableness of the routine DFDY.

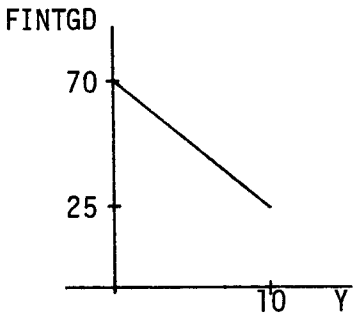
CODE

```
      DO 1 I=1,5
      DO 2 J=1,5
      Y=10.**(J-3)
      Z=10.**(I-3)
      ANS=DFDY(Y,Z)
      WRITE (6,99)Y,Z,ANS
99    FORMAT (5X,F7.3,5X,F7.3,5X,E22.14)
 2    CONTINUE
 1    CONTINUE
      Y=0.
      Z=0.
      ANS=DFDY(Y,Z)
      WRITE (6,99)Y,Z,ANS
      STOP
      END
       .
       .
       .
      (INSERT DYDY,IFACT)
       .
       .
       .
```

NOTE: The same test program may be used for DFDZ, by simply changing DFDY to DFDZ in the above code.

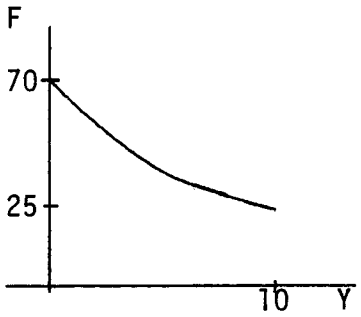
FITGL TEST

The routine FITGL (and its complement for rock temperature, SITGL) is nothing more than an integration routine. As such, it should supply reasonable values when used to integrate several common functions. This routine has FITGL perform the integral of three common functions, a straight line, an exponential and a sinusoid over three different intervals.



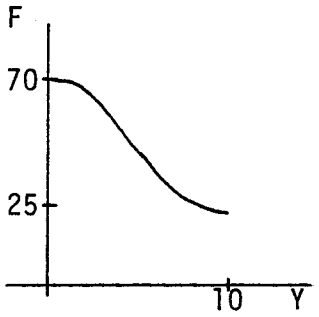
Function A

FINTGD=-4.5Y+70



Function B

$F=71.2e^{-.1Y}-1.2$



Function C

F=22.5 cos .3142Y+47.5

```
CODE          DIMENISON TS(10),YT(10),Y(3)
              DATA TS/10*1./YT/10*1/Y/1.,3.,10./Z/1.0/
              ANS1=FITGL(TS,YT,Z,Y(1))
              ANS2=FITGL(TS,YT,Z,Y(2))
              ANS3=FITGL(TS,YT,Z,Y(3))
              WRITE (6,99)Y(1),ANS1,Y(2),ANS2, Y(3),ANS3
          99  FORMAT (1X,F6.2,5X,E10.3,10X,F6.2,5X,E10.3,10X,F6.2,5X,E10.3)
              STOP
              END
              (INSERT FITGL AND APPROPRIATE FINTGD)

TEST FCN #1   FUNCTION FINTGD(TS,YT,S,Z,Y)
              DIMENSION TS(10),YT(10)
              FINTGD=-4.5*S+70.
              RETURN
              END
```

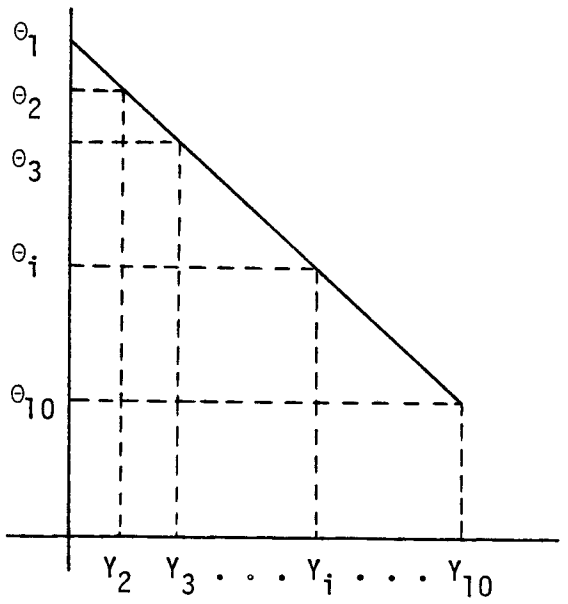
```
TEST FCN #2 FUNCTION FINTGD(TS,YT,S,Z,Y)
            DIMENSION TS(10),YT(10)
            FINTGD=71.2*EXP(-0.1*S)-1.2
            RETURN
            END

TEST FCN #3 FUNCTION FINTGD(TS,YT,S,Z,Y)
            DIMENSION TS(10),YT(10)
            FINTGD=22.5*COS(0.3142*S)+47.5
            RETURN
            END
```

DTHETA TEST

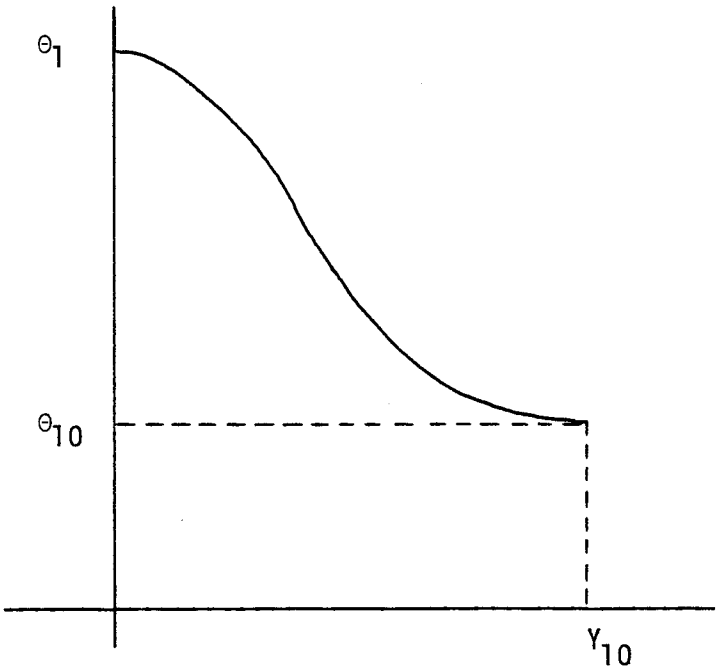
In order to test DTHETA, a routine has been structured to accept as input ten values of T_s assuming a linear temperature distribution and ten values of T_s assuming a cosine function distribution. (see figures). These input data are similar to the data that will actually be supplied to DTHETA within the running of HOTROCKS.

```
CODE          DIMENSION TS(10),YT(10),SOUT(13)
              DATA YT/0.,0.2,0.4,0.6,0.8,1.0,1.2,1.4,1.6,1.8/
FOR LIN →     DATA TS/70.,65.,60.,55.,50.,45.,40.,35.,30.,25./ ←
FOR COS →     DATA TS/70.,68.6,64.7,58.7,51,3,43.5,36.1,30.2,26.3,25./ ←
              DO 1 J=1,19
              S=(0.1,*J)-0.1
              ARG=DTHETA(TS,YT,S)
              WRITE (6,99)S,ARG
           99 FORMAT (5X,'Y= ', F5.2,10X,'DTHETA= ',E12.4)
            1 CONTINUE
              STOP
              END
```



Linear Test
$-25y + 70 = \Theta$

	INPUT	EXPECTED
Y	Θ	Θ'
0	70	-25
0.2	65	-25
0.4	60	-25
0.6	55	-25
0.8	50	-25
1.0	45	-25
1.2	40	-25
1.4	35	-25
1.6	30	-25
1.8	25	-25
Add. Test:		
0.1		-25
1.1		-25
1.7		-25



Cosine Test
22.5 cos/.745y + 47.5 = Θ

	INPUT		EXPECTED
	Y	Θ	Θ′
	0	70.	0
	0.2	68.6	-13.4
	0.4	64.7	-25.2
	0.6	58.8	-34.0
	0.8	51.4	-38.7
	1.0	43.6	-38.7
	1.2	36.3	-34.0
	1.4	30.3	-25.3
	1.6	26.4	-13.5
	1.8	25.	0
Add. Test:			
	0.1		same as Y=0
	1.1		-36.9
	1.7		same as Y=1.8

IFACT TEST

The following code is used as a test program for IFACT. It will output a list of integer numbers corresponding to the factorial of input N, where N shall range from 0 to 20. Values of IFACT for N > 20 will not be needed for HOTROCKS. This test shall also serve to demonstrate the limitations particular computers may have in handling integer arithmetic. Validity of output will be confirmed by comparing to known factorial integers.

```
CODE        DO 1 N=0,20
            IF=IFACT(N)
            WRITE (6,99)N,IF
         99 FORMAT (10X,I2,5X,I30)
          1 CONTINUE
            STOP
            END
             .
             .
             .
            (INSERT FUNCTION IFACT)
             .
             .
             .
```

FOOTNOTES

[1] T. E. W. Schumann, "Heat Transfer: A Liquid Flowing Through a Porous Prism," Journal of the Franklin Institute, CCVIII (September 1929), pp. 405-416.

[2] Neal R. Amundson, "Solid-Fluid Interactions in Fixed and Moving Beds," Industrial and Engineering Chemistry, XLVIII (January 1956), pp. 26-43.

[3] P. J. Hughes, S. A. Klein and D. J. Close, "Packed Bed Thermal Storage Models for Solar Air and Cooling Systems," Transactions of the ASME, Journal of Heat Transfer, XCVIII (May 1976), pp. 336-338.

[4] J. A. Duffie, W. A. Beckman and S. A. Klein, "A Design Procedure for Solar Heating Systems," Solar Energy, XVIII (1976), pp. 113-127.

[5] M. Riaz, "Analytical Solutions for Single and Two Phase Models of Packed Bed Thermal Storage Systems," Transactions of the ASME, Journal of Heat Transfer, XCIX (August 1977), pp. 489-492.

[6] M. Riaz, "Transient Analysis of Packed Bed Thermal Storage Systems," Proceedings of the Annual Meeting-American Section of the International Solar Energy Society, I, Section 17 (June 1977), pp. 17.11-17.17.

[7] S. A. Mumma and W. C. Marvin, "A Method of Simulating the Performance of a Pebble Bed Thermal Energy Storage and Recovery System," ASME Technical Paper No. 76 HT-73, 1976.

[8] John A. Clark, Ronald L. Nabozny and James Heetderks, "Rockbed: A Computer Program for Thermal Storage," Proceedings of the Annual Meeting-American Section of the International Solar Energy Society, I, Section 17 (June 1977), pp. 17.17-17.20.

[9] W. D. Eshleman, C. D. Baird and D. R. Mears, " A Numerical Simulation of Heat Transfer in Rock Beds," Proceedings of the Annual Meeting-American Section of the International Solar Energy Society, I, Section 17 (June 1977), pp. 17.21-17.25.

[10] Jeffrey Taft, David Bailey and Charles Alexander, "A Discrete-Time State Variable Model of Heat Transfer in Rock Storage," (unpublished technical paper, Youngstown State University, 1979).

[11] A. Klinkenberg and A. Harmens, "Unsteady State Heat Transfer in Stationary Packed Beds," Chemical Engineering Science, IX (1960), pp. 260-266.

[12] W. Nusselt, "A New Heat Transfer Formula for Cross-Flow," Technische Mechanik und Thermodynamik, XII (1930).

[13] W. D. Eshleman, et. al., "A Numerical Simulation," p. 17.23.

[14] W. D. Eshleman, et. al., p. 17.22.

[15] W. D. Eshleman, et. al., p. 17.23.

[16] Frank Kreith, Basic Heat Transfer, (New York: Harper and Row, 1980), pp. 438-439.

[17] James R. Welty, Engineering Heat Transfer-SI Version, (New York: John Wiley and Sons, 1978).

[18] R. L. Burden, J. D. Faires and A. C. Reynolds, Numerical Analysis, (Boston: Prindle, Weber and Schmidt, 1978), p. 210.

[19] J. A Duffie and W. A. Beckman, Solar Energy Thermal Processes, (New York: John Wiley and Sons, 1974).

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