THE DYNAMIC STABILITY OF RECTANGULAR PLATES

by

Mukundray S. Dalia

Submitted in Partial Fulfillment of the Requirements
for the Degree of
Master of Science in Engineering
in the
Civil Engineering
Program

Adviser ________________ Date 6/10/74

Dean of the Graduate School ________________ Date 6/11/74

YOUNGSTOWN STATE UNIVERSITY
June, 1974

# ABSTRACT


## THE DYNAMIC STABILITY OF RECTANGULAR PLATES

Mukundray S. Dalia
Master of Science in Engineering
Youngstown State University, 1974

The purpose of this thesis is to study the dynamic stability of rectangular plates subjected to transverse bending stress, shear stress and transverse and rotary inertia.

The proper equations of motion are solved using the classical Levy method for a rectangular plate simply supported along two parallel sides. The resulting set of equations are applicable to a particular group of rectangular plate problems subjected to various applied static and dynamic forces with various boundary restraints on the opposite two parallel sides. Thusly, the natural frequency of free vibration and the critical buckling-load are directly attainable.

The natural frequencies and static stability loads are analyzed by resulting equations for the special case of a simply supported rectangular plate on all four edges subjected to biaxial compression forces.

The curves and tables for the natural frequencies and critical buckling-loads are prepared for the simply supported rectangular plate on all four edges with uniaxial compression force acting on it.

# ACKNOWLEDGEMENTS

The author wishes to convey his utmost thanks to his advisor, Dr. Paul X. Bellini, whose time, efforts, guidance and advice contributed directly to the completion and success of this work.

The author also wishes to thank his review committee, Dr. Michael K. Householder and Prof. John F. Ritter, who have taken an interest in this work.

I would also like to thank Mrs. Debbie Corbi who has typed the entire work.

# TABLE OF CONTENTS

# LIST OF SYMBOLS

| SYMBOL | DEFINITION |
|---|---|
| a,b | Rectangular plate dimension along the x and y axis respectively. |
| D | Flexural rigidity of the rectangular plate. |
| E | Modulus of elasticity. |
| G | Shear modulus of elasticity. |
| h | Thickness of the rectangular plate. |
| m,n | Number of modes in the x and y direction respectively. |
| q | Distributed transverse loading. |
| t | Time perameter. |
| u,v,w | Displacement components in direction of x,y, z respectively for any arbitrary point in the plate. |
| x,y,z | Rectangular co-ordinates. |
| $\alpha, \beta, r$ | Lame's coefficient. |
| $\rho$ | Mass density per unit mass. |
| $\nu$ | Poisson's ratio. |
| $\Omega$ | Natural frequency. |
| Nx,Ny,Nxy,Nyx | Normal stress resultants in unit of forces per unit length due to initial stress. |
| Mx,My,Mxy,Myx | Bending stress couples in unit of moment per unit length. |
| Vx, Vy | Shear stress components in the direction of x and y respectively. |

NONDIMENSIONAL SYMBOLIC FORMS:

$\hat{a} = a/h$, $\hat{b} = b/h$, $\hat{x} = x/h$,

$\hat{y} = y/h$, $\hat{t} = t/t_1$, $t_1 = \sqrt{2h/g}$,

$$\hat{G} = \frac{5Gh^3}{6D}, \qquad \hat{m} = \frac{\rho h^5}{D t_1^2}, \qquad \hat{I} = \frac{\hat{m}}{12} = \frac{\rho h^5}{12 D t_1^2},$$

$$\hat{N}_x = -\frac{N_x}{D} h^2, \qquad \hat{N}_y = -\frac{N_y}{D} h^2, \qquad \hat{N}_{xy} = -\frac{N_{xy}}{D} h^2,$$

$$\hat{\omega} = \omega / h, \qquad \hat{\nabla}^2 = \frac{\partial^2}{\partial \hat{x}^2} + \frac{\partial^2}{\partial \hat{y}^2}, \qquad \hat{\Omega} = \Omega t_1,$$

$$\hat{\beta} = \frac{m^2 \cdot \frac{\hat{b}}{\hat{a}} \hat{N}_x + n^2 \frac{\hat{a}}{\hat{b}} \hat{N}_y}{(\hat{N}_{B.A.S.})_{CR}}, \qquad \hat{\alpha} = \frac{m^2 \cdot \frac{\hat{b}}{\hat{a}} \cdot \hat{N}_x + n^2 \cdot \frac{\hat{a}}{\hat{b}} \cdot \hat{N}_y}{(\hat{N}_{B.A.})_{CR}},$$

$$(\hat{N}_{B.S.A.})_{CR} = \left(\frac{b^2}{\pi^2 D}\right) N_x, \qquad \hat{\beta} = \frac{N_x \left(\frac{b^2}{\pi^2 D}\right)}{(\hat{N}_{B.S.A.})_{CR}},$$

$$\hat{\alpha} = \frac{N_x \left(\frac{b^2}{\pi^2 D}\right)}{(\hat{N}_{B.A.})_{CR.}}, \qquad \hat{\Omega}^2_{(B.S.A.TI.)} = \frac{\Omega^2_{(B.S.A.TI.)}}{\pi^4 / \left(\frac{a^2 b^2 \rho h}{D}\right)}$$

$$\hat{\Omega}^2 = \frac{\hat{\Omega}^2_{(B.S.A.TI)}}{\hat{\Omega}^2_{(B.A.TI.)}}, \qquad \hat{\Omega}^2_{(B.A.TI.)} = \frac{\Omega^2_{(B.A.TI)}}{\pi^4 / \left(\frac{a^2 b^2 \rho h}{D}\right)},$$

$$\hat{\Omega}^2_{(B.S.A.TI.RI.)} = \frac{\Omega^2_{(B.S.A.TI.RI.)}}{\pi^4 / \left(\frac{a^2 b^2 \rho h}{D}\right)},$$

$$\hat{N} = \frac{(\hat{N}_{B.S.A.})_{CR}}{(\hat{N}_{B.A.})_{CR}}.$$

## LIST OF FIGURES

# LIST OF TABLES

# CHAPTER I

## INTRODUCTION

Timoshenko and Gere[3] presented the static stability of rectangular plates using the energy method, subjected to transverse bending stress and axial forces. The rectangular plates with two parallel sides simply supported and the other two parallel sides subjected to different boundary conditions are analyzed for the critical buckling-loads.

Wu, C.I. and Vinson[5] have derived the sets of equations for the study of free vibrations of rectangular plates composed of transversely isotropic material, including the effect of transverse shear deformation and rotary inertia. The special case of a simply supported rectangular plates is studied in details. Different values of parameter E/G, the ratio of in plane modulus of elasticity to transverse shear modulus, are considered for transversely isotropic material.

Brunelle[2] derived the equations of motion that include the effect of transverse isotropy, initial stress and initial displacement. These plate equations are suitable for investigating free and forced vibrations, wave propagation and the static stability problem. However, the equations are specialized so that only static stability of a rectangular plates with two parallel sides simply supported and the remaining two sides subjected to a variety of boundary conditions is treated in detail. The effect of transverse isotropy is considered by the parameters S, (ie $S= E/G\ (h/b)^2/(1-\nu^2)$ where E/G is the ratio of in plane modulus of elasticity to transverse shear modulus, h is the thickness of plate, b is the width dimension of plate and $\nu$ is the Poisson's ratio. Considering different values of the parameter S, $0<S<0.1$, the critical buckling loads for the plate subjected to different boundary conditions for the other two parallel sides are studied in detail.

Archer[1] derived the nonlinear shear deformation theory for thin elastic shells, including the effect of shear deformation and transverse and rotary inertia. He presented the fifteen equations for the ten stress resultants and the five displacement functions in orthogonal curvilinear co-ordinates. The equations of motions, specialized for a thin rectangular plate by noting A=B=1 and $1/r_1 = 1/r_2 = 0$, are utilized in this thesis.

The objective of this thesis is to study the dynamic stability of rectangular plates using the equations of motion derived from the reference (1). The natural frequency and the critical buckling-load equations are derived for the rectangular plate simply supported along two parallel sides and the other two parallel sides subjected to different boundary conditions.

From the resulting equations the dynamic stability of a rectangular plate simply supported on all four edges is studied in detail. Setting the natural frequency equal to zero the static critical buckling-load is determined. The effect of transverse shear stress is considered by using the parameter $h^2/ab$, which is related directly to the plate geometry namely the thickness and the length/width dimensions, for the study of natural frequencies and the critical buckling-loads. The range of this parameter is taken as $0.01 < h^2/ab < 0.05$, which is considered sufficient for the theory to be valid (2).

# CHAPTER II

## EQUATIONS OF MOTION

The equations of motion of thin, isotropic, elastic plates subjected to initial stress conditions, as presented in reference (1) are as follows:

$$\frac{D(1-\nu)}{2}\nabla^2\phi + \frac{D(1+\nu)}{2}\frac{\partial^2\phi}{\partial x^2} - \frac{5Gh}{6}\phi - \frac{\rho h^3}{12}\frac{\partial^2\phi}{\partial t^2} + \frac{D(1+\nu)}{2}\frac{\partial^2\psi}{\partial x\partial y} - \frac{5Gh}{6}\frac{\partial w}{\partial x} = 0, \text{---(1)}.$$

$$\frac{D(1+\nu)}{2}\frac{\partial^2\phi}{\partial x\partial y} + \frac{D(1-\nu)}{2}\nabla^2\psi + \frac{D(1+\nu)}{2}\frac{\partial^2\psi}{\partial y^2} - \frac{5Gh}{6}\psi - \frac{\rho h^3}{12}\frac{\partial^2\psi}{\partial t^2} - \frac{5Gh}{6}\frac{\partial w}{\partial y} = 0, \text{---(2)}$$

and

$$-\frac{5Gh}{6}\frac{\partial\phi}{\partial x} - \frac{5Gh}{6}\frac{\partial\psi}{\partial y} - \frac{5Gh}{6}\nabla^2 w - N_x\frac{\partial^2 w}{\partial x^2} - 2N_{xy}\frac{\partial^2 w}{\partial x\partial y} - N_y\frac{\partial^2 w}{\partial y^2}$$

$$+ \rho h\frac{\partial^2 w}{\partial t^2} - q(x,y,t) = 0 . \text{ ----------(3)}$$

The definitions for moment, axial and shear forces for the rectangular plate are as follows:

$$\left.\begin{aligned}
&M_x = D\left(\frac{\partial\phi}{\partial x} + \nu\frac{\partial\psi}{\partial y}\right), \quad M_y = D\left(\frac{\partial\psi}{\partial y} + \nu\frac{\partial\phi}{\partial x}\right),\\
&M_{xy} = M_{yx} = D\left(\frac{1-\nu}{2}\right)\left(\frac{\partial\psi}{\partial x} + \frac{\partial\phi}{\partial y}\right),\\
&N_x = \frac{Eh}{(1-\nu^2)}\left[\left(\frac{\partial u}{\partial x} + \frac{1}{2}\frac{\partial^2 w}{\partial x^2}\right) + \nu\left(\frac{\partial v}{\partial y} + \frac{1}{2}\frac{\partial^2 w}{\partial y^2}\right)\right],\\
&N_y = \frac{Eh}{(1-\nu^2)}\left[\left(\frac{\partial v}{\partial y} + \frac{1}{2}\frac{\partial^2 w}{\partial y^2}\right) + \nu\left(\frac{\partial u}{\partial y} + \frac{1}{2}\frac{\partial^2 w}{\partial x^2}\right)\right],
\end{aligned}\right\} \text{--- (4)}$$

$$N_{xy} = N_{yx} = Gh\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right],$$

$$V_x = \frac{5Gh}{6}\left(\frac{\partial w}{\partial x} + \phi\right), \quad \text{and} \quad V_y = \frac{5Gh}{6}\left(\frac{\partial w}{\partial y} + \psi\right).$$

Boundary conditions for the rectangular plate at edges y=o and y=b are as follows:

(a) Simply supported at y=o and y=b.

W(y)=o, $\phi$(y)=o, and My(y)=o. ----(5)

(b) Fixed at y=o and y=b.

W(y)=o, $\phi$(y)=o, and $\Psi$(y)=o. ----(6)

(c) Free at y=o and y=b.

My(y)=o, Myx(y)=o, and Vy(y)=o. ----(7)

For the free vibration problem q(x,y,t)=o; upon rearranging equations (1)-(3), the following matrix form is obtained:

$$[A]\begin{Bmatrix}\phi(x,y,t)\\ \Psi(x,y,t)\\ w(x,y,t)\end{Bmatrix} = \begin{Bmatrix}o\\ o\\ o\end{Bmatrix}, \text{-------(8)}$$

Where the elements of the symmetric operator matrix [A], defined as $a_{ij}$, are given by the equations

$$a_{11} = \frac{D(1-\nu)}{2}\nabla^2 + \frac{D(1+\nu)}{2}\frac{\partial^2}{\partial x^2} - \frac{5Gh}{6} - \frac{\rho h^3}{12}\frac{\partial^2}{\partial t^2},$$

$$a_{12} = a_{21} = \frac{D(1+\nu)}{2}\frac{\partial^2}{\partial x\partial y},$$

$$a_{13} = a_{31} = -\frac{5Gh}{6}\frac{\partial}{\partial x},$$

$$a_{22} = \frac{D(1-\nu)}{2}\nabla^2 + \frac{D(1+\nu)}{2}\frac{\partial^2}{\partial y^2} - \frac{5Gh}{6} - \frac{\rho h^3}{12}\frac{\partial^2}{\partial t^2}, \quad (9)$$

$$a_{23} = a_{32} = -\frac{5Gh}{6}\frac{\partial}{\partial y}, \text{ and}$$

$$a_{33} = -\frac{5Gh}{6}\nabla^2 - N_x\frac{\partial^2}{\partial x^2} - 2N_{xy}\frac{\partial^2}{\partial x\partial y} - N_y\frac{\partial^2}{\partial y^2} + \rho h\frac{\partial^2}{\partial t^2}.$$

The uncoupled operator for w (x,y,t), $\phi$ (x,y,t) and $\psi$ (x,y,t) is obtained by successive differentiation and combination of the results of equations (1)-(3). The uncoupled operator for w (x,y,t), $\phi$ (x,y,t) and $\psi$ (x,y,t) is given by the determinant of the operator matrix [A] as

$$\left[ D\nabla^4 + \left\{ \left( N_x \frac{\partial^2}{\partial x^2} + 2N_{xy}\frac{\partial^2}{\partial x \partial y} + N_y \frac{\partial^2}{\partial y^2} \right) - \rho h \frac{\partial^2}{\partial t^2} \right\} \left\{ \frac{D\nabla^2}{5Gh/6} - 1 - \frac{\rho h^3}{12}\cdot\frac{1}{5Gh/6}\frac{\partial^2}{\partial t^2} \right\} - \frac{\rho h^3}{12}\nabla^2 \frac{\partial^2}{\partial t^2} \right]$$

$$\left[ \frac{D(1-\nu)}{2}\cdot\frac{1}{5Gh/6}\nabla^2 - 1 - \frac{\rho h^3}{12}\cdot\frac{1}{5Gh/6}\cdot\frac{\partial^2}{\partial t^2} \right]\left( \quad \right) = 0 \quad \text{---(10)}$$

Neglecting the effect of shear and rotary inertia, equation (10) reduces to the classical 4th order equation of motion including the effect of bending, axial forces and transverse inertia.

The uncoupled operator for w(x,y,t) is subsequently to reduce to a 4th order operator rather than the 6th order operator as shown in the equation (10).

Differentiating equation (1) with respect to x, equation (2) with respect to y, adding the results, and introducing the function $\Phi \equiv \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}$, yields

$$D\nabla^2\Phi - \frac{5Gh}{6}\Phi - \frac{\rho h^3}{12}\frac{\partial^2 \Phi}{\partial t^2} - \frac{5Gh}{6}\nabla^2 w = 0 \quad \text{---(11)}$$

substituting $\Phi \equiv \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}$ into equation (3), one obtains

$$-\frac{5Gh}{6}\Phi - \frac{5Gh}{6}\nabla^2 w - N_x\frac{\partial^2 w}{\partial x^2} - 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} - N_y\frac{\partial^2 w}{\partial y^2} + \rho h\frac{\partial^2 w}{\partial t^2} - q(x,y,t) = 0 \quad (12)$$

For the free vibration problem q (x,y,t)=o; upon rearranging equations (11) and (12) the following matrix form is obtained:

$$[B]\begin{Bmatrix} w\ (x,y,t) \\ \Phi\ (x,y,t) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \quad \text{---------------------(13)}$$

where the elements of the nonsymmetrix operator matrix [B], defined as bij, are given by the equations

$$\left.\begin{aligned} b_{11} &= -\frac{5Gh}{6}\nabla^2 - N_x\frac{\partial^2}{\partial x^2} - 2N_{xy}\frac{\partial^2}{\partial x\partial y} - N_y\frac{\partial^2}{\partial y^2} + \rho h\frac{\partial^2}{\partial t^2}, \\ b_{12} &= -\frac{5Gh}{6}, \quad b_{21} = -\frac{5Gh}{6}\nabla^2, \text{ and} \\ b_{22} &= \nabla^2 - \frac{5Gh}{6} - \frac{\rho h^3}{12}\frac{\partial^2}{\partial t^2}. \end{aligned}\right] \text{---(14)}$$

The uncoupled operator for $w(x,y,t)$ [and for the function $\Phi(x,y,t)$] is given by the determinant of operator matrix [B]. The uncoupled equation for $w(x,y,t)$ is expressed in the following 4th order operator form:

$$\left[D\nabla^4 + \left\{\left(N_x\frac{\partial^2}{\partial x^2} + 2N_{xy}\frac{\partial^2}{\partial x\partial y} + N_y\frac{\partial^2}{\partial y^2}\right) - \rho h\frac{\partial^2}{\partial t^2}\right\}\left\{\frac{D\nabla^2}{5Gh/6} - 1 - \frac{\rho h^3}{12}\cdot\frac{1}{5Gh/6}\cdot\frac{\partial^2}{\partial t^2}\right\} - \frac{\rho h^3}{12}\nabla^2\frac{\partial^2}{\partial t^2}\right]\left[w(x,y,t)\right] = 0 \text{ --(15)}$$

Upon defining the following terms,

$$\left.\begin{aligned} &\frac{5Gh^3}{6D} = \hat{G}, \quad \frac{\rho h^5}{Dt_1^2} = \hat{m}, \quad \frac{\rho h^5}{12Dt_1^2} = \frac{\hat{m}}{12} = \hat{I}, \quad -\frac{N_x}{D}h^2 = \hat{N}_x, \\ &-\frac{N_y}{D}h^2 = \hat{N}_y, \quad -\frac{N_{xy}}{D}h^2 = N_{xy}, \quad \frac{x}{h} = \hat{x}; \quad \frac{y}{h} = \hat{y}, \quad \hat{\nabla}^2 = \frac{\partial^2}{\partial\hat{x}^2} + \frac{\partial^2}{\partial\hat{y}^2}, \\ &\frac{t}{t_1} = \hat{t}, \quad t_1 = \sqrt{\frac{2h}{g}}, \text{ and } \frac{\omega}{h} = \hat{\omega}. \end{aligned}\right] \text{--(16)}$$

Equation (10) reduces to the following nondimensional form:

$$\left\{\left[\hat{\nabla}^2 - \left(\frac{\hat{N}_x}{\hat{G}}\frac{\partial^2}{\partial\hat{x}^2} + \frac{2\hat{N}_{xy}}{\hat{G}}\frac{\partial^2}{\partial\hat{x}\partial\hat{y}} + \frac{\hat{N}_y}{\hat{G}}\frac{\partial^2}{\partial\hat{y}^2}\right) - \frac{\hat{m}}{\hat{G}}\frac{\partial^2}{\partial\hat{t}^2}\right]\left[\hat{\nabla}^2 - \hat{G} - \hat{I}\frac{\partial^2}{\partial\hat{t}^2}\right] + \hat{G}\hat{\nabla}^2\right\}\left\{\frac{(1-\nu)}{2}\hat{\nabla}^2 - \hat{G} - \hat{I}\frac{\partial^2}{\partial\hat{t}^2}\right\}\left[\Phi(\hat{x},\hat{y},\hat{t}),\ \Psi(\hat{x},\hat{y},\hat{t})\right] = 0. \quad \text{---(17)}$$

Similarly equation (15) becomes

$$\left\{\left[\hat{\nabla}^2 - \left(\frac{\hat{N}_x}{\hat{G}}\frac{\partial^2}{\partial \hat{x}^2} + \frac{2\hat{N}_{xy}}{\hat{G}}\frac{\partial^2}{\partial \hat{x}\partial \hat{y}} + \frac{\hat{N}_y}{\hat{G}}\frac{\partial^2}{\partial \hat{y}^2}\right) - \frac{\hat{m}}{\hat{G}}\frac{\partial^2}{\partial \hat{t}^2}\right]\left[\hat{\nabla}^2 - \hat{G} - \hat{I}\frac{\partial^2}{\partial \hat{t}^2}\right] + \hat{G}\hat{\nabla}^2\right\}\left[\hat{W}(\hat{x},\hat{y},\hat{t})\right] = 0. \quad ---(18)$$

# CHAPTER III

# ANALYSIS

## 3.1 Free Vibration Of A Rectangular Plate Simply Supported Along Two Parallel Sides:

Using the method of Bernoulli Separation of Variables, and proceeding in the manner as presented by Levy[4], the following non-dimensional displacement and rotations conditions are assumed:

$$\left.\begin{aligned} \hat{\omega}(\hat{x},\hat{y},\hat{t}) &= \hat{W}(\hat{y}) \sin \frac{m\pi\hat{x}}{\hat{a}} e^{i\hat{\Omega}\hat{t}}, \\ \phi(\hat{x},\hat{y},\hat{t}) &= \phi(\hat{y}) \cos \frac{m\pi\hat{x}}{\hat{a}} e^{i\hat{\Omega}\hat{t}}, \text{ and} \\ \psi(\hat{x},\hat{y},\hat{t}) &= \psi(\hat{y}) \sin \frac{m\pi\hat{x}}{\hat{a}} e^{i\hat{\Omega}\hat{t}}, \end{aligned}\right] \text{---(19)}$$

where, $\hat{\Omega} = \Omega t_1$, $\hat{a} = \frac{a}{h}$, and --------------------(20)

m is the number of half waves in the x direction. The assumed solutions hold for the special case of simply supported boundary conditions on the edges $\hat{x}=0$, and $\hat{x}=\frac{a}{h}=\hat{a}$. The boundary conditions on the edges are arbitrary.

Substituting equation (19) into equations (17) and (18) with $\hat{N}_{xy}=0$, yields, respectively

$$\left[A_4 \frac{d^4}{d\hat{y}^4} + A_2 \frac{d^2}{d\hat{y}^2} + A_0\right]\left[C_2 \frac{d^2}{d\hat{y}^2} + C_0\right]\left\{\phi(\hat{y}), \psi(\hat{y})\right\} = 0; \text{-- (21)}$$

and

$$\left[A_4 \frac{d^4}{d\hat{y}^4} + A_2 \frac{d^2}{d\hat{y}^2} + A_0\right]\left\{\hat{W}(\hat{y})\right\} = 0, \quad \text{---- (22)}$$

where

$$\left.\begin{aligned} A_4 &= 1 - \frac{\hat{N}_y}{\hat{G}}, \\ A_2 &= -\left\{\left(\frac{m\pi}{\hat{a}}\right)^2 + \hat{G} - \hat{I}\hat{\Omega}^2\right\}\left\{1 - \frac{\hat{N}_y}{\hat{G}}\right\} - \left(\frac{m\pi}{\hat{a}}\right)^2\left(1 - \frac{\hat{N}_x}{\hat{G}}\right) \\ &\quad + \frac{\hat{m}}{\hat{G}}\hat{\Omega}^2, \\ A_0 &= \left\{\left(\frac{m\pi}{\hat{a}}\right)^4 + \left(\frac{m\pi}{\hat{a}}\right)^2\hat{G} - \left(\frac{m\pi}{\hat{a}}\right)^2\hat{I}\hat{\Omega}^2\right\}\left\{1 - \frac{\hat{N}_x}{\hat{G}}\right\} \\ &\quad + \frac{\hat{I}\hat{m}}{\hat{G}}\hat{\Omega}^4 - \left\{\left(\frac{m\pi}{\hat{a}}\right)^2\frac{\hat{m}}{\hat{G}} + \hat{m}\right\}\hat{\Omega}^2 - \left(\frac{m\pi}{\hat{a}}\right)^2\hat{G}, \text{ and} \end{aligned}\right] \text{----(23)}$$

$$\left.\begin{aligned} C_2 &= (1-\nu)/2, \text{ and} \\ C_0 &= -\frac{(1-\nu)}{2}\left(\frac{m\pi}{\hat{a}}\right)^2 - \hat{G} + \hat{I}\hat{\Omega}^2. \end{aligned}\right] \text{----(24)}$$

Since equations (21) and (22) are linear differential equations with constant coefficients, the solution is assumed in the form:

$$\left.\begin{aligned} \phi(\hat{y}) &= P_1 e^{\lambda\hat{y}}, \\ \psi(\hat{y}) &= P_2 e^{\lambda\hat{y}}, \text{ and} \\ \hat{W}(\hat{y}) &= P_3 e^{\lambda\hat{y}}. \end{aligned}\right] \text{----(25)}$$

The roots of the biquadratic factor in equation (21) and (22) are

$$\left.\begin{aligned} \lambda_{1,2} &= \pm\alpha, \text{ and} \\ \lambda_{3,4} &= \pm i\beta. \end{aligned}\right] \text{--(26)}$$

The roots of the quadratic factor in equation (21) are

$$\lambda_{1,2} = \pm\gamma\ , \qquad \text{----(27)}$$

where $\alpha$, $\beta$ and $\gamma$ are real quantities.

It follows that,

$$\alpha^2 = \frac{-A_2}{2A_4} + \left[\left(\frac{A_2}{2A_4}\right)^2 - \frac{A_0}{A_4}\right]^{1/2},$$

$$\beta^2 = \frac{A_2}{2A_4} + \left[\left(\frac{A_2}{2A_4}\right)^2 - \frac{A_0}{A_4}\right]^{1/2}, \text{ and} \qquad \text{--(28)}$$

$$\gamma^2 = -\frac{C_0}{C_2}.$$

Hence, the solutions of the functions of $\hat{W}, \phi, \psi$ become, respectively,

$$\hat{W}(\hat{y}) = C_1 \operatorname{Sinh} \alpha\hat{y} + C_2 \operatorname{Cosh} \alpha\hat{y} + C_3 \operatorname{Sin} \beta\hat{y} + C_4 \operatorname{Cos} \beta\hat{y},$$

$$\phi(\hat{y}) = C_5 \operatorname{Sinh} \alpha\hat{y} + C_6 \operatorname{Cosh} \alpha\hat{y} + C_7 \operatorname{Sin} \beta\hat{y} + C_8 \operatorname{Cos} \beta\hat{y} + C_9 \operatorname{Sinh} \gamma\hat{y} + C_{10} \operatorname{Cosh} \gamma\hat{y}, \text{ and} \qquad \text{--(29)}$$

$$\psi(\hat{y}) = C_{11} \operatorname{Sinh} \alpha\hat{y} + C_{12} \operatorname{Cosh} \alpha\hat{y} + C_{13} \operatorname{Sin} \beta\hat{y} + C_{14} \operatorname{Cos} \beta\hat{y} + C_{15} \operatorname{Sinh} \gamma\hat{y} + C_{16} \operatorname{Cosh} \gamma\hat{y},$$

where constants $C_1$ through $C_{16}$ are constants of integration.
The above solutions in equation (29) also must satisfy the two parent equations (1) and (2). Thus, the following relationships among the sixteen constants are obtained:

$$C_5 = \delta_1\left(\frac{m\pi}{\hat{a}}\right)C_1, \qquad C_6 = \delta_1\left(\frac{m\pi}{\hat{a}}\right)C_2,$$

$$C_7 = -\delta_2\left(\frac{m\pi}{\hat{a}}\right)C_3, \qquad C_8 = \delta_2\left(\frac{m\pi}{\hat{a}}\right)C_4,$$

$$C_{11} = \delta_1 \alpha C_1, \qquad C_{12} = \delta_1 \alpha C_2, \qquad \text{--(30)}$$

$$C_{13} = -\delta_2 \beta C_4, \qquad C_{14} = \delta_2 \beta C_3,$$

$$
\left.\begin{aligned}
C_{15} &= \left(\frac{m\pi}{\hat{a}}\right)\frac{1}{\gamma}C_{10}\,, \quad \text{and} \\
C_{16} &= \left(\frac{m\pi}{\hat{a}}\right)\frac{1}{\gamma}C_{9}\,,
\end{aligned}\right]
$$

where
$$
\left.\begin{aligned}
\delta_1 &= \hat{G}/[h^2\alpha^2 + \hat{I}\hat{\Omega}^2 - \hat{G} - (m\pi/\hat{a})^2], \\
\delta_2 &= \hat{G}/[h^2\beta^2 + \hat{G} + (m\pi/\hat{a})^2 - \hat{I}\hat{\Omega}^2].
\end{aligned}\right] \quad \text{---(31)}
$$
and

The relationships reduce the number of constants from sixteen to six. Substituting equation (30) into equation (29), one obtains

$$
\begin{Bmatrix} \hat{W} \\ \phi \\ \psi \end{Bmatrix} = [C] \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_9 \\ C_{10} \end{Bmatrix}, \quad \text{---(32)}
$$

where the elements of matrix [C], are given by the equation

$$[C] = \begin{bmatrix} \sinh \alpha\hat{y} & \cosh \alpha\hat{y} & \sin \beta\hat{y} & \cos \beta\hat{y} & 0 & 0 \\ \delta_1 (m\pi/\hat{a}) \cdot \sinh \alpha\hat{y} & \delta_1 (m\pi/\hat{a}) \cdot \cosh \alpha\hat{y} & -\delta_2 (m\pi/\hat{a}) \cdot \sin \beta\hat{y} & -\delta_2 (m\pi/\hat{a}) \cdot \cos \beta\hat{y} & \sinh \gamma\hat{y} & \cosh \gamma\hat{y} \\ \delta_1 \alpha \cosh \alpha\hat{y} & \delta_1 \alpha \sinh \alpha\hat{y} & -\delta_2 \beta \cdot \cos \beta\hat{y} & \delta_2 \beta \cdot \sin \beta\hat{y} & (m\pi/\hat{a}) \cdot \frac{1}{\gamma} \cdot \cosh \gamma\hat{y} & (m\pi/\hat{a}) \cdot \frac{1}{\gamma} \cdot \sinh \gamma\hat{y} \end{bmatrix} . \quad ----(33)$$

Equation (32) is used, along with three appropriate boundary conditions on each edge $\hat{y}=o$ and $\hat{y}=\frac{b}{\bar{h}}=\hat{b}$ to solve the free vibration problem of a biaxially compressed plate subjected to transverse shear stress, bending stress, and transverse and rotary inertia.

3.2 Dynamics Stability Of A Rectangular Plate Simply Supported Along Two Parallel Sides:

Multiplying equations (1) and (2) by $h^2/D$, equation (3) by $h^3/D$ and using equation (16) the following nondimensional form of equations of motion are obtained:

$$\left(\frac{1-\nu}{2}\right)\hat{\nabla}^2\phi + \left(\frac{1-\nu}{2}\right)\frac{\partial^2\phi}{\partial\hat{x}^2} - \hat{G}\phi - \hat{I}\frac{\partial^2\phi}{\partial\hat{t}^2} + \left(\frac{1+\nu}{2}\right)\frac{\partial^2\psi}{\partial\hat{x}\partial\hat{y}} - \hat{G}\frac{\partial\hat{w}}{\partial\hat{x}} = 0, \quad (34)$$

$$\left(\frac{1+\nu}{2}\right)\frac{\partial^2\phi}{\partial\hat{x}\partial\hat{y}} + \left(\frac{1-\nu}{2}\right)\hat{\nabla}^2\psi + \left(\frac{1+\nu}{2}\right)\frac{\partial^2\psi}{\partial\hat{y}^2} - \hat{G}\psi - \hat{I}\frac{\partial^2\psi}{\partial\hat{t}^2} - \hat{G}\frac{\partial\hat{w}}{\partial\hat{y}} = 0, \text{ and} \quad (35)$$

$$-\hat{G}\frac{\partial\phi}{\partial\hat{x}} - \hat{G}\frac{\partial\psi}{\partial\hat{y}} - \hat{G}\hat{\nabla}^2\hat{w} + \hat{N}_x\frac{\partial^2\hat{w}}{\partial\hat{x}^2} + 2\hat{N}_{xy}\frac{\partial^2\hat{w}}{\partial\hat{x}\partial\hat{y}} + \hat{N}_y\frac{\partial^2\hat{w}}{\partial\hat{y}^2} + \hat{m}\frac{\partial^2\hat{w}}{\partial\hat{t}^2} - \frac{h^3}{D}q(x,y,t) = 0. \quad (36)$$

The dynamic stability equations for the free vibration of a simply supported plate at edges $\hat{x}=o$ and $\hat{x}=\frac{a}{\bar{h}}=\hat{a}$, with $\hat{N}x$ and $\hat{N}y$ acting, are obtained by assuming the nondimensional displacement and rotations given in the form:

$$\left.\begin{aligned} \phi &= P_1 e^{\lambda\hat{y}}\cos\left(\frac{m\pi\hat{x}}{\hat{a}}\right)e^{i\hat{\Omega}\hat{t}}, \\ \psi &= P_2 e^{\lambda\hat{y}}\sin\left(\frac{m\pi\hat{x}}{\hat{a}}\right)e^{i\hat{\Omega}\hat{t}}, \text{ and} \\ \hat{W} &= P_3 e^{\lambda\hat{y}}\sin\left(\frac{m\pi\hat{x}}{\hat{a}}\right)e^{i\hat{\Omega}\hat{t}}. \end{aligned}\right] \quad (37)$$

where m is the number of half waves in the x direction. The parameter $\lambda$ is obtained for the given plate by the form of the appropriate boundary conditions on the sides $\hat{y}=o$, and $\hat{y}=\frac{b}{\bar{h}}=\hat{b}$.

Substituting equation (37) into the above equations (34)-(36), upon rearranging terms, the following matrix form is obtained:

$$[D]\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \text{---(38)}$$

where the elements of the matrix [D] defined as dij, are given by the equations

$$\left.\begin{aligned}
d_{11} &= \left(\frac{1-\nu}{2}\right)\left\{\lambda^2-\left(\frac{m\pi}{\hat{a}}\right)^2\right\} - \left(\frac{1+\nu}{2}\right)\left(\frac{m\pi}{\hat{a}}\right)^2 - \hat{G} + \hat{I}\hat{\Omega}^2, \\
d_{12} = -d_{21} &= \left(\frac{1+\nu}{2}\right)\left(\frac{m\pi}{\hat{a}}\right)\lambda, \\
d_{13} = -d_{31} &= \hat{G}\,(m\pi/\hat{a}), \\
d_{22} &= \left(\frac{1-\nu}{2}\right)\left\{\lambda^2-\left(\frac{m\pi}{\hat{a}}\right)^2\right\} + \left(\frac{1+\nu}{2}\right)\lambda^2 - \hat{G} + \hat{I}\hat{\Omega}^2, \\
d_{23} = d_{32} &= -\hat{G}\lambda, \quad \text{and} \\
d_{33} &= -\hat{G}\left\{\lambda^2-\left(\frac{m\pi}{\hat{a}}\right)^2\right\} - \hat{N}_x\left(\frac{m\pi}{\hat{a}}\right)^2 + \hat{N}_y\lambda^2 - \hat{m}\hat{\Omega}^2.
\end{aligned}\right] \quad (39)$$

The uncoupled equation for $\hat{\Omega}$ is obtained by formulating the determinant of operator matrix [D] which yields,

$$[B_4\hat{\Omega}^4 + B_2\hat{\Omega}^2 + B_0][D_2\hat{\Omega}^2 + D_0] = 0, \quad (40)$$

where

$$B_4 = \frac{\hat{m}\hat{I}}{\hat{G}},$$

$$B_2 = -\left[\frac{\hat{I}\pi^2}{(\hat{a}\hat{b})}\left\{m^2\left(\frac{\hat{b}}{\hat{a}}\right)\left(1-\frac{\hat{N}_x}{\hat{G}}\right) - \left(\frac{\lambda\hat{b}}{\pi}\right)^2\left(\frac{\hat{a}}{\hat{b}}\right)\left(1-\frac{\hat{N}_y}{\hat{G}}\right)\right\} + \hat{m}\left(\frac{\pi^2}{\hat{a}\hat{b}\hat{G}}\right)\left\{m^2\left(\frac{\hat{b}}{\hat{a}}\right) - \left(\frac{\lambda\hat{b}}{\pi}\right)^2\left(\frac{\hat{a}}{\hat{b}}\right)\right\} + 1\right)\Bigg],$$

$$B_0 = \frac{\pi^4}{(\hat{a}\hat{b})^2}\left\{m^2\left(\frac{\hat{b}}{\hat{a}}\right)\left(1-\frac{\hat{N}_x}{\hat{G}}\right) - \left(\frac{\lambda\hat{b}}{\pi}\right)^2\left(\frac{\hat{a}}{\hat{b}}\right)\left(1-\frac{\hat{N}_y}{\hat{G}}\right)\right\}$$

$$\left\{m^2\left(\frac{\hat{a}}{\hat{b}}\right) - \left(\frac{\lambda\hat{b}}{\pi}\right)^2\left(\frac{\hat{a}}{\hat{b}}\right)\right\} - \frac{\pi^2}{(\hat{a}\hat{b})}\left\{m^2\left(\frac{\hat{b}}{\hat{a}}\right)\hat{N}_x - \left(\frac{\lambda\hat{b}}{\pi}\right)^2\left(\frac{\hat{a}}{\hat{b}}\right)\hat{N}_y\right\}, \quad \text{---(41)}$$

$$D_2 = \hat{I}, \text{ and}$$

$$D_0 = -\left(\frac{1-\nu}{2}\right)\left(\frac{\pi^2}{\hat{a}\hat{b}}\right)\left\{m^2\left(\frac{\hat{b}}{\hat{a}}\right) - \left(\frac{\lambda\hat{b}}{\pi}\right)^2\left(\frac{\hat{a}}{\hat{b}}\right)\right\} - \hat{G}. \quad \text{---(42)}$$

The natural frequency of free vibration of the plate is obtained from the biquadratic form of equation (40), which is written as

$$B_4\hat{\Omega}^4 + B_2\hat{\Omega}^2 + B_0 = 0 \quad \text{-----------(43)}$$

From the above equation (43), two independent families of frequency curves are obtained:

the higher set is given as

$$\hat{\Omega}_1^2 = \frac{-B_2}{2B_4}\left[1 + \left(1 - \frac{4B_4B_0}{B_2^2}\right)^{1/2}\right], \quad \text{-------(44)}$$

and the lower set becomes

$$\hat{\Omega}_2^2 = \frac{-B_2}{2B_4}\left[1 - \left(1 - \frac{4B_4B_0}{B_2^2}\right)^{1/2}\right]. \quad \text{-------(45)}$$

Substituting $\hat{\Omega} = 0$ into the equation (43), the equation for critical buckling load is obtained, that is,

$$B_o = o \qquad \text{------------------------------------------} (46)$$

Equation (46) yields

$$\left[ m^2 \left(\frac{\hat{b}}{\hat{a}}\right)\hat{N}_x - \left(\frac{\lambda\hat{b}}{\pi}\right)^2 \left(\frac{\hat{a}}{\hat{b}}\right)\hat{N}_y \right]$$

$$= \frac{\left(\frac{\pi^2}{\hat{a}\hat{b}}\right)\left\{m^2\left(\frac{\hat{b}}{\hat{a}}\right) - \left(\frac{\lambda\hat{b}}{\pi}\right)^2\left(\frac{\hat{a}}{\hat{b}}\right)\right\}^2}{\left[1 + \frac{\pi^2}{\hat{a}\hat{b}\hat{G}}\left\{m^2\left(\frac{\hat{b}}{\hat{a}}\right) - \left(\frac{\lambda\hat{b}}{\pi}\right)^2\left(\frac{\hat{a}}{\hat{b}}\right)\right\}\right]} \quad \text{-----} (47)$$

For uniaxial compression case in the $\hat{x}$ direction, that is $\hat{N}y=o$, equation (47) becomes

$$\hat{N}x. = \frac{\frac{\pi^2}{m^2\hat{b}^2}\left\{m^2\cdot\frac{\hat{b}}{\hat{a}} - \left(\frac{\lambda\hat{b}}{\pi}\right)^2\cdot\left(\frac{\hat{a}}{\hat{b}}\right)\right\}^2}{\left[1 + \frac{\pi^2}{\hat{a}\hat{b}\hat{G}}\left\{m^2\cdot\frac{\hat{b}}{\hat{a}} - \left(\frac{\lambda\hat{b}}{\pi}\right)^2\cdot\left(\frac{\hat{a}}{\hat{b}}\right)\right\}\right]} \quad \text{----} (48)$$

For uniaxial compression case in the $\hat{Y}$ direction, that is $\hat{N}x=o$, equation (47) becomes

$$\hat{N}y. = \frac{\frac{\pi^2}{\hat{a}^2(\lambda\hat{b}/\pi)^2}\left\{m^2\left(\frac{\hat{b}}{\hat{a}}\right) - \left(\frac{\lambda\hat{b}}{\pi}\right)^2\cdot\left(\frac{\hat{b}}{\hat{a}}\right)\right\}^2}{\left[1 + \frac{\pi^2}{\hat{a}\hat{b}\hat{G}}\left\{m^2\cdot\frac{\hat{b}}{\hat{a}} - \left(\frac{\lambda\hat{b}}{\pi}\right)^2\cdot\left(\frac{\hat{a}}{\hat{b}}\right)\right\}\right]} \quad \text{---} (49)$$

For the special case of biaxial compression with $\hat{N}x=\hat{N}y=\hat{N}$, equation (47) becomes

$$\hat{N} = \frac{\frac{\pi^2}{\hat{a}\hat{b}}\left\{m^2\left(\frac{\hat{b}}{\hat{a}}\right) - \left(\frac{\lambda\hat{b}}{\pi}\right)^2\left(\frac{\hat{a}}{\hat{b}}\right)\right\}}{\left[1 + \frac{\pi^2}{\hat{a}\hat{b}\hat{G}}\left\{m^2\cdot\frac{\hat{b}}{\hat{a}} - \left(\frac{\lambda\hat{b}}{\pi}\right)^2\cdot\left(\frac{\hat{a}}{\hat{b}}\right)\right\}\right]} \quad \text{----} (50)$$

# CHAPTER IV

## SAMPLE EXAMPLE

Consider a simply supported rectangular plate on all four edges.

### 4.1 Free Vibration Of A Simply Supported Plate.

The following boundary conditions are applied for a simply supported rectangular plate along the lines $\hat{y}$=o and $\hat{y}$=b/h=$\hat{b}$.

$$\left.\begin{array}{l} W(o)=o, \\ W(b)=o, \\ \phi(o)=o, \\ \phi(b)=o, \\ M_y(o)=o, \text{ and} \\ M_y(b)=o. \end{array}\right] \text{-------------- (51)}$$

The last two boundary conditions reduce to the form

$\psi^1(o)=o$, and

$\psi^1(b)=o$, where $(\ )^1$ denotes the derivative with respect to y.

By noting equations (32) and (51), one obtains

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ \mathrm{Sinh}\,\alpha\hat{b} & \mathrm{Cosh}\,\alpha\hat{b} & \mathrm{Sin}\,\beta\hat{b} & \mathrm{Cos}\,\beta\hat{b} & 0 & 0 \\ 0 & \left(\frac{m\pi}{\hat{a}}\right)\delta_1 & 0 & -\left(\frac{m\pi}{\hat{a}}\right)\delta_2 & 0 & 1 \\ (m\pi/\hat{a})\delta_1 \cdot \mathrm{Sinh}\,\alpha\hat{b} & (m\pi/\hat{a})\delta_1 \cdot \mathrm{Cosh}\,\alpha\hat{b} & -(m\pi/\hat{a})\delta_2 \cdot \mathrm{Sin}\,\beta\hat{b} & -(m\pi/\hat{a})\delta_2 \cdot \mathrm{Cos}\,\beta\hat{b} & \mathrm{Sinh}\,\gamma\hat{b} & \mathrm{Cosh}\,\gamma\hat{b} \\ 0 & \delta_1\alpha^2 & 0 & \delta_2\beta^2 & 0 & (m\pi/\hat{a}) \\ \delta_1\alpha^2 \cdot \mathrm{Sinh}\,\alpha\hat{b} & \delta_1\alpha^2 \cdot \mathrm{Cosh}\,\alpha\hat{b} & \delta_2\beta^2 \cdot \mathrm{Sin}\,\beta\hat{b} & \delta_2\beta^2 \cdot \mathrm{cos}\,\beta\hat{b} & (m\pi/\hat{a}) \cdot \mathrm{Sinh}\,\gamma\hat{b} & (m\pi/\hat{a}) \cdot \mathrm{cosh}\,\gamma\hat{b} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_9 \\ C_{10} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \text{---(52a)}$$

or in symbolic matrix form

$$[E]\{C\} = \{o\}. \quad \text{-----------------------------(52b)}$$

For nontrivial solutions of the vactor $\{C\}$, the determinant of [E] must be zero. This conditions yields

$$\left[\delta_1\left\{\alpha^2 - \left(\frac{m\pi}{\hat{a}}\right)^2\right\} - \delta_2\left\{\beta^2 + \left(\frac{m\pi}{\hat{a}}\right)^2\right\}\right]\left[\sinh \gamma\hat{b}\right]$$
$$\left[\sinh(\alpha\hat{b})\right]\left[\sin\beta\hat{b}\right] \quad \text{-------------(53)}$$

Since the first three terms of the equation (53) are always nonzero, it follows that

$$\sin\beta\hat{b} = 0, \quad \text{-------------------(54)}$$

which is satisfied by the conditions

$\beta\hat{b} = n\pi$, where n=1, 2, 3, $\cdots$. (55)

Combining equations (26) and (55), yields

$\lambda_n = i\left(\frac{n\pi}{\hat{b}}\right)$. n= 1,2,3,$\cdots$ . (56)

Combining equations (54) and (52a), one obtains the conditions

$C_1 = C_2 = C_4 = C_9 = C_{10} = o$, and ---------(57a)

letting $C_3 = C_n$, it follows that

$$\hat{W}(\hat{y}) = C_n \sin\frac{n\pi}{\hat{b}}\hat{y},$$

$$\phi(\hat{y}) = C_n\left(-\delta_2\frac{m\pi}{\hat{a}}\right)\sin\frac{n\pi}{\hat{b}}\hat{y}, \text{ and} \quad \text{---(57b.)}$$

$$\psi(\hat{y}) = C_n\left(-\delta_2\frac{m\pi}{\hat{a}}\right)\cos\frac{n\pi}{\hat{b}}\hat{y},$$

where $\delta_2 = \hat{G}\Big/\left[h^2\left(\frac{n\pi}{\hat{b}}\right)^2 + \left(\frac{m\pi}{\hat{a}}\right)^2 + \hat{G} - \hat{I}\hat{\Omega}^2\right]$.

Substituting equation (56) into equation (43), yields

$$B_4\hat{\Omega}^4 + B_2\hat{\Omega}^2 + B_0 = 0, \qquad \text{-------------(58)}$$

where

$$B_4 = \frac{\hat{m}\,\hat{I}}{\hat{G}},$$

$$B_2 = -\left[\hat{I}\left(\frac{\pi^2}{\hat{a}\hat{b}}\right)^2\left\{m^2\left(\frac{\hat{b}}{\hat{a}}\right)\left(1-\frac{\hat{N}_x}{\hat{G}}\right) + n^2\left(\frac{\hat{a}}{\hat{b}}\right)\cdot\left(1-\frac{\hat{N}_y}{\hat{G}}\right)\right\} + \hat{m}\left\{\frac{\pi^2}{\hat{a}\hat{b}\hat{G}}\left(m^2\cdot\frac{\hat{b}}{\hat{a}} + n^2\frac{\hat{a}}{\hat{b}}\right)+1\right\}\right], \text{ and}$$

$$B_0 = \frac{\pi^4}{(\hat{a}\,\hat{b})^2}\left\{m^2\left(\frac{\hat{b}}{\hat{a}}\right)\left(1-\frac{\hat{N}_x}{\hat{G}}\right) + n^2\left(\frac{\hat{a}}{\hat{b}}\right)\left(1-\frac{\hat{N}_y}{\hat{G}}\right)\right\}\left\{m^2\cdot\frac{\hat{b}}{\hat{a}} + n^2\cdot\frac{\hat{a}}{\hat{b}}\right\} - \frac{\pi^2}{\hat{a}\hat{b}}\left\{m^2\cdot\frac{\hat{b}}{\hat{a}}\cdot\hat{N}_x + n^2\cdot\frac{\hat{a}}{\hat{b}}\cdot\hat{N}_y\right\}. \qquad \text{--(59)}$$

The higher set of frequency equations is given as

$$\hat{\Omega}_1^2 = \frac{-B_2}{2B_4}\left[1 + \left(1 - \frac{4B_4B_0}{B_2^2}\right)^{1/2}\right], \qquad \text{-------------(60)}$$

the lower set of frequency equations is given as

$$\hat{\Omega}_2^2 = \frac{-B_2}{2B_4}\left[1 - \left(1 - \frac{4B_4B_0}{B_2^2}\right)^{1/2}\right]. \qquad \text{-------------(61)}$$

Substituting $\hat{\Omega}$=o into the equation (58), the equation for critical buckling load is obtained, that is, for

$$B_0 = 0, \qquad \text{-------------------------------------(62)}$$

equation (62) yields

$$[m^2\cdot\hat{b}/\hat{a}\cdot\hat{N}_x + n^2\cdot\hat{a}/\hat{b}\cdot\hat{N}_y] = \frac{\left(\frac{\pi^2}{\hat{a}\hat{b}}\right)\cdot\left(m^2\cdot\frac{\hat{b}}{\hat{a}} + n^2\cdot\frac{\hat{a}}{\hat{b}}\right)^2}{\left[1 + \frac{\pi^2}{\hat{a}\hat{b}\hat{G}}\left(m^2\cdot\frac{\hat{b}}{\hat{a}} + n^2\cdot\frac{\hat{a}}{\hat{b}}\right)\right]} \qquad \text{.----(63)}$$

For the uniaxial compression case in the $\hat{x}$ direction, that is $\hat{N}_y$=o, equation (63) becomes

$$\hat{N}_x = \frac{\frac{\pi^2}{\hat{b}^2 m^2}\left(m^2\cdot\frac{\hat{b}}{\hat{a}} + n^2\cdot\frac{\hat{a}}{\hat{b}}\right)^2}{\left[1 + \frac{\pi^2}{\hat{a}\hat{b}\hat{G}}\left(m^2\cdot\frac{\hat{b}}{\hat{a}} + n^2\cdot\frac{\hat{a}}{\hat{b}}\right)\right]} \qquad \text{------(64)}$$

$$\hat{N}_y = \frac{\frac{\pi^2}{\hat{a}^2 n^2}\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right)^2}{\left[1 + \frac{\pi^2}{\hat{a}\hat{b}\hat{G}}\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right)\right]} \quad \text{---------}(65)$$

For the special case of biaxial compression with $\hat{N}x = \hat{N}y = \hat{N}$, equation (63) becomes

$$\hat{N} = \frac{\frac{\pi^2}{\hat{a}\hat{b}}\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right)}{\left[1 + \frac{\pi^2}{\hat{a}\hat{b}\hat{G}}\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right)\right]} \quad \text{----------}(66)$$

By substituting equation (55) directly into equation (28), and noting equation (23) equation (59) is produced directly.

4.2 Dynamic Stability

(4.2a) Natural frequency of free vibration including bending stress, shear stress, biaxial forces, transverse and rotary inertia.

The natural frequency as obtained from the equation (60) is written as

$$\hat{\Omega}^2_{(B.S.A.TI.RI.)} = \frac{12\pi^2}{2k\hat{m}}[R]$$

$$\left[1 + \left\{1 - \frac{\frac{4\hat{I}}{\hat{m}\hat{G}}\left[\frac{\pi^4}{(\hat{a}\hat{b})^2}\left(m^2\cdot\frac{\hat{b}}{\hat{a}} + n^2\cdot\frac{\hat{a}}{\hat{b}}\right)^2 - \frac{\pi^2}{\hat{a}\hat{b}}\left\{1+\frac{k}{\hat{a}\hat{b}}\left(m^2\frac{\hat{b}}{\hat{a}}+n^2\frac{\hat{a}}{\hat{b}}\right)\right\}\left\{m^2\frac{\hat{b}}{\hat{a}}\cdot\hat{N}_x + n^2\cdot\frac{\hat{a}}{\hat{b}}\cdot\hat{N}_y\right\}\right]}{[R^2]}\right\}\right] \quad \text{---------------(67)}$$

where

$$k = \pi^2/5(1-\nu),$$

$$R = 1 + \frac{k}{\hat{a}\hat{b}}\left(m^2\cdot\frac{\hat{b}}{\hat{a}} + n^2\cdot\frac{\hat{a}}{\hat{b}}\right) + \frac{\pi^2}{12}\left(\frac{1}{\hat{a}\hat{b}}\right)\left(m^2\frac{\hat{b}}{\hat{a}} + n^2\frac{\hat{a}}{\hat{b}}\right) - \frac{\pi^2}{12}\left(\frac{1}{\hat{a}\hat{b}}\right)\frac{1}{\hat{G}}\left(m^2\cdot\frac{\hat{b}}{\hat{a}}\cdot\hat{N}_x + n^2\cdot\frac{\hat{a}}{\hat{b}}\cdot\hat{N}_y\right).$$

Similarly the second set of the natural frequency as obtained by the equation (61) is written as

$$\hat{\Omega}^2_{(B.S.A.TI.RI.)} = \frac{12\pi^2}{2k\hat{m}}[R]$$

$$\left[1 - \left\{1 - \frac{\frac{4\hat{I}}{\hat{m}\hat{G}}\left[\frac{\pi^4}{(\hat{a}\hat{b})^2}\left(m^2\cdot\frac{\hat{b}}{\hat{a}} + n^2\cdot\frac{\hat{a}}{\hat{b}}\right)^2 - \frac{\pi^2}{\hat{a}\hat{b}}\left\{1+\frac{k}{\hat{a}\hat{b}}\left(m^2\frac{\hat{b}}{\hat{a}}+n^2\frac{\hat{a}}{\hat{b}}\right)\right\}\left\{m^2\cdot\frac{\hat{b}}{\hat{a}}\hat{N}_x + n^2\cdot\frac{\hat{a}}{\hat{b}}\cdot\hat{N}_y\right\}\right]}{[R^2]}\right\}\right] \quad \text{-------------(68)}$$

(4.2b) Natural frequency considering bending stress, shear stress, biaxial forces and transverse inertia.

The natural frequency neglecting rotary inertia, (i.e., $\hat{I}$=o), becomes

$$\hat{\Omega}^2_{(B.S.A.T.I.)} = \frac{\pi^4}{(\hat{a}\hat{b})^2\hat{m}} \left[ \frac{\left(m^2 . \frac{\hat{b}}{\hat{a}} + n^2 . \frac{\hat{a}}{\hat{b}}\right)^2}{1 + \frac{k}{\hat{a}\hat{b}}\left(m^2 . \frac{\hat{b}}{\hat{a}} + n^2 . \frac{\hat{a}}{\hat{b}}\right)} - \left(\frac{\hat{a}\hat{b}}{\pi^2}\right)\left(m^2 . \frac{\hat{b}}{\hat{a}} . \hat{N}_x + n^2 . \frac{\hat{a}}{\hat{b}} . \hat{N}_y\right) \right] \quad \text{-----(69)}$$

(4.2c) Natural frequency considering bending stress, biaxial forces and transverse inertia.

The natural frequency neglecting rotary inertia, (i.e., $\hat{I}$=o) and shear stress, (i.e., $1/\hat{G}$=o), becomes

$$\hat{\Omega}^2_{(B.A.T.I.)} = \frac{\pi^4}{(\hat{a}\hat{b})^2\hat{m}} \left[ \left(m^2 . \frac{\hat{b}}{\hat{a}} + n^2 . \frac{\hat{a}}{\hat{b}}\right)^2 - \left(\frac{\hat{a}\hat{b}}{\pi^2}\right)\left(m^2 . \frac{\hat{b}}{\hat{a}} \hat{N}_x + n^2 . \frac{\hat{a}}{\hat{b}} . \hat{N}_y\right) \right] \quad \text{-----(70)}$$

(4.2d) Natural Frequency Ratio.

A nondimensional ratio of natural frequency including and excluding shear stress for the case when rotary inertia is neglected takes the form.

$$\hat{\hat{\Omega}}^2 = \frac{\hat{\Omega}^2_{(B.S.A.T.I.)}}{\hat{\Omega}^2_{(B.A.T.I.)}} = \frac{1 - \dfrac{\frac{\hat{a}\hat{b}}{\pi^2}\left\{m^2 . \frac{\hat{b}}{\hat{a}} . \hat{N}_x + n^2 . \frac{\hat{a}}{\hat{b}} . \hat{N}_y\right\}\left\{\frac{k}{\hat{a}\hat{b}}\left(m^2 . \frac{\hat{b}}{\hat{a}} + n^2 . \frac{\hat{a}}{\hat{b}}\right)\right\}}{\left\{m^2 . \frac{\hat{b}}{\hat{a}} + n^2 . \frac{\hat{a}}{\hat{b}}\right\}^2 - \left\{\frac{\hat{a}\hat{b}}{\pi^2}\left(m^2 . \frac{\hat{b}}{\hat{a}} . \hat{N}_x + n^2 . \frac{\hat{a}}{\hat{b}} . \hat{N}_y\right)\right\}}}{1 + \frac{k}{\hat{a}\hat{b}}\left(m^2 . \frac{\hat{b}}{\hat{a}} + n^2 . \frac{\hat{a}}{\hat{b}}\right)} \quad \text{--- (71)}$$

4.3 Static Stability Load

The critical buckling load considering bending stress, biaxial forces and shear stress is given in equation (63).

The critical buckling load considering bending stress and biaxial compression neglecting shear stress, (i.e., $1/\hat{G} = o$), into the equation (63), yields

$$(\hat{N}_{B.A.})_{CR} = m^2 \cdot \frac{\hat{b}}{\hat{a}} \cdot \hat{N}_x + n^2 \cdot \frac{\hat{a}}{\hat{b}} \cdot \hat{N}_y$$
$$= \left(\frac{\pi^2}{\hat{a}\hat{b}}\right)\left\{m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right\}^2 . \quad \text{----------------(72)}$$

The critical buckling load ratio is the ratio of the critical buckling load including shear and excluding shear. This yields

$$\hat{\hat{N}} = \frac{(\hat{N}_{B.S.A.})_{CR}}{(\hat{N}_{B.A.})_{CR}}$$
$$= \frac{1}{1 + \left(\frac{k}{\hat{a} \cdot \hat{b}}\right) \cdot \left(m^2 \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right)} . \quad \text{----------------(73)}$$

4.4 Relationship Between Natural Frequency And Critical Buckling Load.

(4.4a) The natural frequency considering bending stress, shear stress, biaxial forces, and transverse and rotary inertia, becomes

$$\hat{\Omega}^2_{(B.S.A.TI.RI.)} = \frac{12\pi^2}{2k\hat{m}} R \cdot \left[ 1 + \left\{ 1 - \frac{(4\hat{I}/\hat{G})\,\hat{\Omega}^2_{(B.S.A.TI.)}}{\hat{N} \cdot R^2} \right\}^{1/2} \right] \text{--(74)}$$

where

$$R = \frac{1}{\hat{N}} + \frac{\pi^2}{12}\left(\frac{1}{\hat{a}\hat{b}}\right)\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right) - \frac{\pi^2}{12}\left(\frac{1}{\hat{a}\hat{b}}\right)\frac{1}{\hat{G}}\left(m^2 \frac{\hat{b}}{\hat{a}} \cdot \hat{N}_x + n^2 \frac{\hat{a}}{\hat{b}} \cdot \hat{N}_y\right)$$

similarly the equation (68) becomes

$$\hat{\Omega}^2_{(B.S.A.TI.RI.)} = \frac{12\pi^2}{2k\hat{m}} \cdot R \cdot \left[ 1 - \left\{ 1 - \frac{(4\hat{I}/\hat{G})\,\hat{\Omega}^2_{(B.S.A.TI.)}}{\hat{N} \cdot R^2} \right\}^{1/2} \right] \text{-------------(75)}$$

(4.4b) Natural frequency considering bending stress, shear stress, biaxial forces and transverse inertia.

Combining the equation (63) and (69), yields

$$\hat{\Omega}^2_{(B.S.A.T.I.)} = \frac{\pi^4}{(\hat{a}\hat{b})^2\hat{m}} \left(\hat{N}_{B.S.A.}\right)_{CR} (1 - \hat{\beta}), \quad \text{-------------(76)}$$

where

$$\hat{\beta} = \frac{m^2 \cdot \frac{\hat{b}}{\hat{a}} \cdot \hat{N}_x + n^2 \cdot \frac{\hat{a}}{\hat{b}} \cdot \hat{N}_y}{(\hat{N}_{B.A.S.})_{CR.}}, \quad 0 < \hat{\beta} < 1. \text{-------------(77)}$$

(4.4c) Natural frequency considering bending stress, biaxial forces and transverse inertia.

Combining the equation (70) and (72), yields

$$\hat{\Omega}^2_{(B.A.TI.)} = \frac{\pi^4}{(\hat{a}\hat{b})^2\hat{m}} \cdot \left(\hat{N}_{B.A.}\right)_{CR} \cdot (1 - \hat{\alpha}) \quad , \text{-------(78)}$$

where $$\hat{\alpha} = \frac{m^2 \cdot \frac{\hat{b}}{\hat{a}} \cdot \hat{N}_x + n^2 \cdot \frac{\hat{a}}{\hat{b}} \cdot \hat{N}_y}{(\hat{N}_{B.A.})_{CR}}, \quad 0 < \hat{\alpha} < 1. \text{-------(79)}$$

(4.4d) Natural frequency ratio.

Combining the equations (76)-(79) and equation (73), yields

$$\hat{\Omega}^2 = \frac{\hat{\Omega}^2(B.S.A.TI.)}{\hat{\Omega}^2(B.A.TI.)} = \frac{1}{1+\hat{\gamma}}, \text{---------(80)}$$

where

$$\hat{\gamma} = \frac{1-\hat{N}}{\hat{N}(1-\hat{\beta})} \text{---------(81)}$$

## SUMMARY SHEET

Natural frequency

$$\hat{\Omega}^2_{(B.S.A.TI.RI.)} = \frac{12\pi^2}{2k\hat{m}}(R)\left[1+\left\{1-\frac{\frac{4\hat{I}}{\hat{m}\hat{G}}\left[\frac{\pi^2}{(\hat{a}\hat{b})^2}\left(m^2\cdot\frac{\hat{b}}{\hat{a}}+n^2\cdot\frac{\hat{a}}{\hat{b}}\right)^2-\frac{\pi^2}{\hat{a}\hat{b}}\left\{1+\frac{k}{\hat{a}\hat{b}}\left(m^2\cdot\frac{\hat{b}}{\hat{a}}+n^2\cdot\frac{\hat{a}}{\hat{b}}\right)\right\}\left\{m^2\cdot\frac{\hat{b}}{\hat{a}}\cdot\hat{N}_x+n^2\cdot\frac{\hat{a}}{\hat{b}}\cdot\hat{N}_y\right\}\right]}{(R^2)}\right\}\right]$$

where

$$R = 1+\frac{k}{\hat{a}\hat{b}}\cdot\left(m^2\cdot\frac{\hat{b}}{\hat{a}}+n^2\cdot\frac{\hat{a}}{\hat{b}}\right)+\frac{\pi^2}{12}\cdot\frac{1}{\hat{a}\hat{b}}\cdot\left(m^2\cdot\frac{\hat{b}}{\hat{a}}+n^2\cdot\frac{\hat{a}}{\hat{b}}\right)-\frac{\pi^2}{12}\left(\frac{1}{\hat{a}\hat{b}}\right)\frac{1}{\hat{G}}\left(m^2\cdot\frac{\hat{b}}{\hat{a}}\cdot\hat{N}_x+n^2\cdot\frac{\hat{a}}{\hat{b}}\cdot\hat{N}_y\right),\quad k=\frac{\pi^2}{5(1-\nu)}\;.$$

The other set of natural frequency.

$$\hat{\Omega}^2_{(B.S.A.TI.RI.)} = \frac{12\pi^2}{2k\hat{m}}(R)\left[1-\left\{1-\frac{\frac{4\hat{I}}{\hat{m}\hat{G}}\left[\frac{\pi^2}{(\hat{a}\hat{b})^2}\left(m^2\cdot\frac{\hat{b}}{\hat{a}}+n^2\cdot\frac{\hat{a}}{\hat{b}}\right)^2-\frac{\pi^2}{\hat{a}\hat{b}}\left\{1+\frac{k}{\hat{a}\hat{b}}\left(m^2\cdot\frac{\hat{b}}{\hat{a}}+n^2\cdot\frac{\hat{a}}{\hat{b}}\right)\right\}\left\{m^2\cdot\frac{\hat{b}}{\hat{a}}\hat{N}_x+n^2\cdot\frac{\hat{a}}{\hat{b}}\cdot\hat{N}_y\right\}\right]}{(R^2)}\right\}\right]$$

Natural frequency neglecting rotary inertia.

$$\hat{\Omega}^2_{(B.S.A.TI.)} = \frac{\pi^4}{(\hat{a}\hat{b})^2\hat{m}}\left[\frac{\left(m^2\cdot\frac{\hat{b}}{\hat{a}}+n^2\cdot\frac{\hat{a}}{\hat{b}}\right)^2}{1+k\cdot\left(\frac{1}{\hat{a}\hat{b}}\right)\left(m^2\cdot\frac{\hat{b}}{\hat{a}}+n^2\cdot\frac{\hat{a}}{\hat{b}}\right)}-\frac{\hat{a}\hat{b}}{\pi^2}\left(m^2\cdot\frac{\hat{b}}{\hat{a}}\cdot\hat{N}_x+n^2\cdot\frac{\hat{a}}{\hat{b}}\cdot\hat{N}_y\right)\right].$$

Natural frequency neglecting rotary inertia and shear stress.

$$\hat{\Omega}^2_{(B.A.TI.)} = \frac{\pi^4}{(\hat{a}\hat{b})^2\hat{m}}\left[\left(m^2\cdot\frac{\hat{b}}{\hat{a}}+n^2\cdot\frac{\hat{a}}{\hat{b}}\right)^2-\left(\frac{\hat{a}\hat{b}}{\pi^2}\right)\left(m^2\cdot\frac{\hat{b}}{\hat{a}}\cdot\hat{N}_x+n^2\cdot\frac{\hat{a}}{\hat{b}}\cdot\hat{N}_y\right)\right].$$

Natural frequency ratio.

$$\hat{\Omega}^2 = \frac{\hat{\Omega}^2_{(B.S.A.TI.)}}{\hat{\Omega}^2_{(B.A.TI)}} = \frac{1 - \dfrac{\frac{\hat{a}\hat{b}}{\pi^2}\left\{m^2 \cdot \frac{\hat{b}}{\hat{a}} \cdot \hat{N}_x + n^2 \cdot \frac{\hat{a}}{\hat{b}} \cdot \hat{N}_y\right\}\left\{\frac{k}{\hat{a}\hat{b}}\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right)\right\}}{\left\{\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \frac{\hat{a}}{\hat{b}}\right)^2 - \frac{\hat{a}\hat{b}}{\pi^2}\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} \hat{N}_x + n^2 \cdot \frac{\hat{a}}{\hat{b}} \cdot \hat{N}_y\right)\right\}}}{1 + \frac{k}{\hat{a}\hat{b}}\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right)}$$

when $\hat{\Omega} = 0$,

Critical buckling load.

$$(\hat{N}_{B.S.A.})_{CR} = m^2 \cdot \frac{\hat{b}}{\hat{a}} \cdot \hat{N}_x + n^2 \cdot \frac{\hat{a}}{\hat{b}} \cdot \hat{N}_y = \frac{\frac{\pi^2}{\hat{a}\hat{b}}\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right)^2}{1 + \frac{k}{\hat{a}\hat{b}}\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right)}$$

Critical buckling load neglecting shear stress.

$$(\hat{N}_{B.A.})_{CR} = m^2 \cdot \frac{\hat{b}}{\hat{a}} \cdot \hat{N}_x + n^2 \cdot \frac{\hat{a}}{\hat{b}} \hat{N}_y = \frac{\pi^2}{\hat{a}\hat{b}}\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right)^2$$

Critical buckling load ratio.

$$\hat{N} = \frac{(\hat{N}_{B.S.A.})_{CR}}{(\hat{N}_{B.A.})_{CR}} = \frac{1}{1 + k\left(\frac{1}{\hat{a}\hat{b}}\right)\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \frac{\hat{a}}{\hat{b}}\right)}$$

Relationship between natural frequency and critical buckling load.

Natural frequency.

$$\hat{\Omega}^2_{(B.S.A.TI.RI.)} = \frac{12\pi^2}{2k\hat{m}}(R)\left[1 + \left\{1 - \frac{(4\hat{I}/\hat{G})\,\hat{\Omega}^2_{(B.S.A.TI.)}}{\hat{N}(R^2)}\right\}^{1/2}\right]$$

where

$$R = \frac{1}{\hat{N}} + \frac{\pi^2}{12}\left(\frac{1}{\hat{a}\hat{b}}\right)\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right) - \frac{\pi^2}{12}\left(\frac{1}{\hat{a}\hat{b}}\right)\frac{1}{\hat{G}}\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} \cdot \hat{N}_x + n^2 \cdot \frac{\hat{a}}{\hat{b}} \cdot \hat{N}_y\right)$$

The other set of natural frequency.

$$\hat{\Omega}^2_{(B.S.A.TI.RI.)} = \frac{12\pi^2}{2k\hat{m}}(R)\left[1 - \left\{1 - \frac{(4\hat{I}/\hat{G})\,\hat{\Omega}^2_{(B.S.A.TI.)}}{\hat{N}\,(R^2)}\right\}^{1/2}\right]$$

Natural frequency neglecting rotary inertia.

$$\hat{\Omega}^2_{(B.S.A.TI.)} = \frac{\pi^4}{(\hat{a}\hat{b})^2\hat{m}}\left(\hat{N}_{B.S.A.}\right)_{CR}(1 - \hat{\beta}),$$

where

$$\hat{\beta} = \frac{m^2 \cdot \frac{\hat{b}}{\hat{a}} \hat{N}_x + n^2 \cdot \frac{\hat{a}}{\hat{b}} \hat{N}_y}{(\hat{N}_{B.S.A.})_{CR}}.$$

Natural frequency neglecting rotary inertia and shear stress.

$$\hat{\Omega}^2_{(B.A.TI.)} = \frac{\pi^4}{(\hat{a}\hat{b})^2\hat{m}}\left(\hat{N}_{B.A.}\right)_{CR}(1 - \hat{\alpha}),$$

where

$$\hat{\alpha} = \frac{m^2 \cdot \frac{\hat{b}}{\hat{a}} \cdot \hat{N}_x + n^2 \cdot \frac{\hat{a}}{\hat{b}} \cdot \hat{N}_y}{(\hat{N}_{B.A.})_{CR}}.$$

Natural frequency ratio.

$$\hat{\hat{\Omega}}^2 = \frac{\hat{\Omega}^2_{(B.S.A.TI.)}}{\hat{\Omega}^2_{(B.A.TI.)}} = \frac{1}{1 + \hat{\hat{\gamma}}}$$

where

$$\hat{\gamma} = \frac{1 - \hat{\hat{N}}}{\hat{\hat{N}}(1 - \hat{\beta})}$$

# CHAPTER V

## GRAPHICAL REPRESENTATION

Using the equations derived in the chapter IV the curves and tables are prepared for the natural frequencies and the critical buckling-loads. The curves and tables are prepared considering the following:

(1) Thin plate theory states that the minimum thickness of the plate is about one tenth of the smaller dimensions of the plate, which implies that $1/\hat{a}\hat{b} = h^2/ab \leq 0.01$. Shear stress effect are considerable as the thickness of the plate increases. The parameter $h^2/ab$, where $0.01 \leq h^2/ab \leq 0.05$ are considered from the reference (2).

(2) The parameter $\hat{\hat{\beta}}$, which is defined as the ratio of actual in plane force to the critical buckling-load including shear, varies over the range $0 \leq \hat{\hat{\beta}} < 1$. The specific values which are considered are $\hat{\hat{\beta}} = 0$, 0.01, 0.25, 0.50, 0.75 and 0.99.

(3) The parameter $\hat{\hat{\alpha}}$, which is defined as the ratio of actual in plane force to the critical buckling-load excluding shear, varies over the range $0 \leq \hat{\hat{\alpha}} < 1$. The specific values which are considered are $\hat{\hat{\alpha}} = 0$, 0.01, 0.25, 0.50, 0.75 and 0.99.

(4) The combinations of modes for the curves of natural frequency and the critical buckling-load, (m=1, 2, 3 and n=1, 2, 3) are used.

(5) The plate dimensions $0 < a/b < 5$, (i.e., length/width) are used.

(6) Poisson's Ratio is assumed as $\nu$=0.30.

(7) For curves and tables the uniaxial force in the x direction, that is $\hat{N}y$=o, is considered.

(8) In general, for all values of $\hat{\beta}$ or $\hat{\alpha}$ and $h^2/ab$ the curve for the first mode, (i.e., m=1, n=1), for the natural frequency and critical buckling-loads are plotted. For all other modes (i.e., m=1, 2, 3 and n=1, 2, 3) the values of natural frequency and critical buckling-loads are given in tabular form.

(9) A summary sheet table for minimum or maximum values of frequency and critical buckling-loads are tabulated. These values are obtained by noting that the exteremum condition requires a/b=m/n.

5.1 Critical Buckling-Load Including Shear vs a/b.

For $\hat{N}y=0$, the equation (63) becomes

$$(\hat{N}_{B.S.A.})_{CR} = \left(\frac{\hat{b}}{\pi}\right)^2 \hat{N}_x = \frac{\frac{1}{m^2}\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right)^2}{1 + \left(\frac{k}{\hat{a}\hat{b}}\right)\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right)} .$$

or

$$(\hat{N}_{B.S.A.})_{CR} = \left(\frac{b^2}{\pi^2 D}\right) N_x = \frac{\frac{1}{m^2}\left(m^2 \cdot \frac{b}{a} + n^2 \cdot \frac{a}{b}\right)^2}{1 + k \cdot \frac{h^2}{ab} \cdot \left(m^2 \frac{b}{a} + n^2 \frac{a}{b}\right)} . \quad \text{----(82)}$$

Using the above equation (82) figures 1A and 1B are drawn and tables 1a through 1d are tabulated.

The figure 1A shows that critical buckling-load decreases as the parameter $h^2/ab$ increases. As the parameter $h^2/ab$ increases from 0.01 to 0.05 the values of minimum critical buckling-load decreases about 17.60%.

5.2 Critical Buckling-Load Excluding Shear vs a/b.

For $\hat{N}y=0$, the equation (73) becomes

$$(\hat{N}_{B.A.})_{CR} = \left(\frac{\hat{b}}{\Pi}\right)^2 \hat{N}_x = \frac{1}{m^2}\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right)^2$$

or

$$(\hat{N}_{B.A.})_{CR} = \left(\frac{b^2}{\Pi^2 D}\right) N_x = \frac{1}{m^2}\left(m^2 \cdot \frac{b}{a} + n^2 \cdot \frac{a}{b}\right)^2 . \qquad \text{----(83)}$$

Using the equation (83) the curve, (#1), is plotted on the figure 1A. and table 1e is tabulated.

Figure 1A shows that the minimum critical buckling-load excluding shear is about 5.34% higher than the minimum critical buckling load including shear, for the parameter $h^2/ab=0.01$ and is about 22% higher for the parameter $h^2/ab=0.05$.

5.3 Critical Buckling-Load Ratio vs a/b.

Combining the equations (82) and (83), yields

$$\hat{\hat{N}} = \frac{(\hat{N}_{B.S.A.})_{CR}}{(\hat{N}_{B.A.})_{CR}} = \frac{1}{1 + \left(\frac{k}{\hat{a}\hat{b}}\right)\left(m^2 \cdot \frac{\hat{b}}{\hat{a}} + n^2 \cdot \frac{\hat{a}}{\hat{b}}\right)}$$

or

$$\hat{\hat{N}} = \frac{(\hat{N}_{B.S.A.})_{CR}}{(\hat{N}_{B.A.})_{CR}} = \frac{1}{1 + k \cdot \frac{h^2}{ab} \cdot \left(m^2 \cdot \frac{b}{a} + n^2 \cdot \frac{a}{b}\right)} \quad \text{------(84)}$$

Using the above equation (84) figures 2A and 2B are drawn and tables 2a through 2d are tabulated.

Figure 2A shows that the critical buckling-load ratio decreases as the parameter $h^2/ab$ increases. As the parameter $h^2/ab$ increases from 0.01 to 0.05 values, the maximum critical buckling-load ratio decreases by about 17.60%.

5.4 Natural Frequency Of Free Vibration Including Shear vs a/b.

Equation (70) becomes

$$\hat{\Omega}^2_{(B.S.A.T.I.)} = \frac{\Omega^2_{(B.S.A.T.I.)}}{\pi^4 / \left(\frac{a^2 b^2 \rho h}{D}\right)} = \frac{\left(m^2 \cdot \frac{b}{a} + n^2 \frac{a}{b}\right)^2 (1-\hat{\hat{\beta}})}{\left[1 + k \cdot \frac{h^2}{ab}\left(m^2 \frac{b}{a} + n^2 \frac{a}{b}\right)\right]} \quad \text{-------(85)}$$

where

$$\hat{\hat{\beta}} = \frac{N_x \left(\frac{b^2}{\pi^2 D}\right)}{(\hat{N}_{B.\hat{S}.A.})_{CR}}, \quad 0 \leq \hat{\hat{\beta}} \leq 1 \quad \text{------(86)}$$

Using the above equations (85) and (86) figures (3) through (7) are plotted and tables (3a) through (3e) to (7a) through (7e) are tabulated.

From the figures (3) through (7) following observations are noted:

(a) Figures (3) through (7) shows that, as the values of $\hat{\hat{\beta}}$ increases the value of natural frequency including shear decreases.

(b) For the case $\hat{\hat{\beta}}$ =0.99, the value of natural frequency approximately zero for $0.5 < a/b < 5.0$.

(c) As the parameter $h^2/ab$ increases from 0.01 to 0.05 for each value of $\hat{\hat{\beta}}$ the minimum natural frequency value decreases about 17% to 18%.

## 5.5 Natural Frequency Excluding Shear vs a/b.

Equation (71), yields

$$\hat{\Omega}^2_{(B.A.TI.)} = \frac{\Omega^2_{(B.A.TI.)}}{\pi^4/\left(\frac{a^2b^2\rho h}{D}\right)} = \left(m^2 \cdot \frac{b}{a} + n^2 \cdot \frac{a}{b}\right)^2 (1-\hat{\alpha}) \text{ , ------(87)}$$

where

$$\hat{\alpha} = \frac{N_x \ (b^2/\pi^2 D)}{(\hat{N}_{B.A.})_{CR.}} \quad , \qquad 0 \le \hat{\alpha} < 1 \text{ . ------(88)}$$

Using the equations (87) and (88) figure (8) is plotted and tables (8a) through (8e) are tabulated.

Figure (8) shows that, as the value of $\hat{\alpha}$ increases the value of natural frequency excluding shear decreases. For the case $\hat{\alpha}$ =0.99, the value of natural frequency approximately zero for 0.5<a/b<5.0.

5.6 Natural Frequency Ratio vs a/b.

Combining the equations (85) through (89), yields

$$\hat{\hat{\Omega}}^2 = \frac{\hat{\hat{\Omega}}^2_{(B.S.A.T.I.)}}{\hat{\hat{\Omega}}^2_{(B.A.T.I.)}} = \frac{1}{1 + \hat{\hat{\gamma}}} , \quad \text{---------(89)}$$

where

$$\hat{\hat{\gamma}} = \frac{1 - \hat{\hat{N}}}{\hat{\hat{N}}(1 - \hat{\hat{\beta}})} . \quad \text{---------(90)}$$

Using the above equations (89) and (90) the figures (9) through (13) are plotted and tables (9a) through (9e) to (13a) through (13e) are tabulated.

From the figures (9) through (13) following observations made:

(a) The natural frequency ratio decreases as the value of $\hat{\hat{\beta}}$ increases.

(b) For the case $\hat{\hat{\beta}} = o$ the values are equal to the values of critical buckling-load ratio.

(c) As the parameter $h^2/ab$ increases from the value 0.01 to 0.05 for $0.01 < \hat{\hat{\beta}} < 0.99$, the maximum value of natural frequency ratio decreases from 18% to 77%.

5.7 Lower Set of Natural Frequency Including Shear and Rotary Inertia vs a/b.

Equation (75) becomes

$$\hat{\Omega}^2_{(B.S.A.RI.TI.)} = \frac{\Omega^2_{(B.S.A.TI.RI.)}}{\pi^4/\left(\frac{a^2b^2\rho h}{D}\right)} = \frac{1}{2\frac{\pi^2}{12}.k}\left(\frac{ab}{h^2}\right)^2(R).$$

$$\left[1-\left\{1-\frac{4.\frac{\pi^2}{12}.k.\left(\frac{h^2}{ab}\right)^2\left(m^2.\frac{b}{a}+n^2.\frac{a}{b}\right)^2(1-\hat{\beta})}{(R)^2}\right\}^{1/2}\right] \text{------(91)}$$

where

$$R = 1+\left(k+\frac{\pi^2}{12}\right).\frac{h^2}{ab}.\left(m^2.\frac{b}{a}+n^2.\frac{a}{b}\right)-\frac{k.\frac{\pi^2}{12}.\hat{\beta}\left(\frac{h^2}{ab}\right)^2\left(m^2.\frac{b}{a}+n^2.\frac{a}{b}\right)^2}{1+k.\frac{h^2}{ab}.\left(m^2.\frac{b}{a}+n^2.\frac{a}{b}\right)}$$

Using the equation (91) figures (14) through (18) are drawn and tables (13a) through (13e) to (18a) through (18d) are tabulated.

Following observations are made:

(a) As the value of $\hat{\beta}$ increases the natural frequency ratio decreases.

(b) As the parameter $h^2/ab$ increases from 0.01 to 0.05, for each $0.01<\hat{\beta}<0.99$, the minimum value of natural frequency decreases to about 20.41%.

5.8 Higher Set of Natural Frequency Including Shear and Rotary Inertia vs a/b.

Equation (74) becomes

$$\hat{\Omega}^2_{(B.S.A.TI.RI.)} = \frac{\Omega^2_{(B.S.A.TI.RI.)}}{\pi^4 \Big/ \left(\frac{a^2 b^2 \rho h}{D}\right)} = \frac{1}{2\frac{\pi^2}{12}\cdot k} \cdot \left(\frac{ab}{h^2}\right)^2 \cdot (R) \cdot \left[1 + \left\{1 - \frac{4 \cdot \frac{\pi^2}{12}\left(\frac{h^2}{ab}\right)^2 (1-\hat{\beta}) \cdot \left(m^2 \cdot \frac{b}{a} + n^2 \frac{a}{b}\right)^2}{(R)^2}\right\}^{\frac{1}{2}}\right] \qquad (92)$$

where

$$R = 1 + \left(k + \frac{\pi^2}{12}\right) \cdot \frac{h^2}{ab} \cdot \left(m^2 \cdot \frac{b}{a} + n^2 \cdot \frac{a}{b}\right) - \frac{k \cdot \frac{\pi^2}{12} \hat{\beta} \cdot \left(\frac{h^2}{ab}\right)^2 \left(m^2 \frac{b}{a} + n^2 \frac{a}{b}\right)^2}{1 + k \cdot \frac{h^2}{ab} \cdot \left(m^2 \frac{b}{a} + n^2 \frac{a}{b}\right)}$$

Using the equation (92) and $\hat{\beta} = 0$, figure (19) is drawn and tables (19a) through (19d) are tabulated.

Figure (19) shows that, as the parameter $h^2/ab$ increases from 0.01 to 0.05 the minimum value of natural frequency decreases by about 94.97%.

## 5.8 Higher Set of Natural Frequency Including Shear and Rotary Inertia vs a/b.

Equation (74) becomes

$$\hat{\Omega}^2_{(B.S.A.TI.RI.)} = \frac{\Omega^2_{(B.S.A.TI.RI.)}}{\pi^4 / \left(\frac{a^2 b^2 \rho h}{D}\right)} = \frac{1}{2\frac{\pi^2}{12}\cdot k}\cdot\left(\frac{ab}{h^2}\right)^2\cdot(R)\cdot$$

$$\left[1+\left\{1-\frac{4\cdot\frac{\pi^2}{12}\left(\frac{h^2}{ab}\right)^2(1-\hat{\beta})\cdot\left(m^2\cdot\frac{b}{a}+n^2\cdot\frac{a}{b}\right)^2}{(R)^2}\right\}^{1/2}\right] \quad \text{-------}(92)$$

where

$$R = 1+\left(k+\frac{\pi^2}{12}\right)\cdot\frac{h^2}{ab}\cdot\left(m^2\cdot\frac{b}{a}+n^2\cdot\frac{a}{b}\right)-\frac{k\cdot\frac{\pi^2}{12}\hat{\beta}\cdot\left(\frac{h^2}{ab}\right)^2\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{1+k\cdot\frac{h^2}{ab}\cdot\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)}$$

Using the equation (92) and $\hat{\beta}=0$, figure (19) is drawn and tables (19a) through (19d) are tabulated.

Figure (19) shows that, as the parameter $h^2/ab$ increases from 0.01 to 0.05 the minimum value of natural frequency decreases by about 94.97%.

# CHAPTER VI

## SUMMARY

### Discussion

Using the equations of motion for the free vibrations of a rectangular plate the uncoupled 6th order operator for the functions $\phi(x,y,t)$ and $\psi(x,y,t)$ and the uncoupled 4th order operator for $w(x,y,t)$ are obtained. Neglecting the effect of shear and rotary inertia the above uncoupled operator reduces to the classical 4th order equation of motion including the effect of bending stress, axial force and transverse inertia.

The sets of natural frequencies are determined from the biquadratic equation for the two parallel sides simply supported and other two parallel sides subjected to different boundary conditions. Setting natural frequency equal to zero the critical buckling loads are also determined, thus the static stability of the rectangular plates are studied from the dynamic stability equations.

For the special case of a simply supported plate on all four edges the natural frequencies are determined. It is seen that the natural frequency excluding rotary inertia, and excluding both shear and rotary inertia yield one set of natural frequency curves, while the natural frequency including rotary inertia and shear have two independent sets of frequencies curves. From numerical analysis it is found that the values of natural frequency excluding rotary inertia, natural frequency excluding both shear and rotary inertia, and the lower set of natural frequency including shear and rotary inertia are dependent on the axial force while the higher set of natural frequency values are affected a negligible amount by the presence of axial force.

## Conclusions

From the curves and tables of the natural frequency and the critical buckling-load the following observations are noted:

(1) For $a/b<0.50$, a large decrease of critical buckling-load occurs for a given change in the parameter $h^2/ab$.

(2) As the parameter $h^2/ab$, which accentuates the effect of shear stress, increases the critical buckling-load decreases. Thus the critical buckling-load excluding shear is higher in value than the value of critical buckling load including shear. The minimum critical buckling load excluding shear is about 5.34% higher than the minimum critical buckling-load including shear for the parameter $h^2/ab=0.01$, and is about 22% higher for the parameter $h^2/ab=0.05$.

(3) Considering the different characteristic mode shapes from the tables, it is seen that when rectangular plate buckles it subdivides approximately into squares.

(4) For $a/b<0.50$, the decrease in the values of natural frequencies including shear is about 97.72% for the parameters $h^2/ab=0.01$ and $\hat{\beta}=0$ and it is 95.60% for the parameters $h^2/ab=0.05$ and $\hat{\beta}=0$. For the parameter $\hat{\beta}=0.25$ the values of natural frequencies including shear decreases by about 97.72% for the parameter $h^2/ab=0.01$ and by 95.60% for the parameter $h^2/ab=0.05$.

(5) As the parameter $h^2/ab$, which accentuates the effect of shear, increases the natural frequency decreases. This means that the natural frequency excluding shear is always higher in value than the value of natural frequency including shear. The minimum value of natural frequency excluding shear is about 5.34% higher than the minimum value of natural frequency including shear for the parameters $h^2/ab=0.01$ and $\hat{\beta}=0$, and is about 22% higher for the parameters $h^2/ab=0.05$ and $\hat{\beta}=0$. For the parameter $\hat{\beta}=0.25$ the minimum value of the natural frequency excluding shear is about 6.99% more than the value of natural frequency including shear for the parameter $h^2/ab=0.01$ and is about 27.32% for the parameter $h^2/ab=0.05$.

(6) As the parameters $\hat{\beta}$ or $\hat{\alpha}$, which accentuate the effect of axial force, increase the natural frequency decreases. Thus, the value of natural frequency excluding axial force is higher than the value of natural frequency including axial force. If the parameter $\hat{\beta}$ increases from 0.0 to 0.25 the minimum value of natural frequency including shear decreases by about 24.99%; when $\hat{\beta}$ increases from 0.25 to 0.50 the minimum value of natural frequency including shear decreases by 33.34%; when $\hat{\beta}$ increases from 0.50 to 0.75 the minimum value of natural frequency including shear decreases by about 49.97%.

(7) Comparing the value of natural frequency excluding rotary inertia with the lower set of natural frequency including rotary inertia shows that the minimum value of natural frequency excluding rotary inertia is about 1.44% higher for the parameter $h^2/ab=0.01$ and is about 4.81% higher for the parameter $h^2/ab=0.05$ for the value $\hat{\beta}=0$.

(8) The ratio of higher set of natural frequencies including shear and rotary inertia to the lower set of natural frequencies including

shear and rotary inertia is about 35:1 for the parameter $h^2/ab=0.01$ and is about 8.86:1 for the parameter $h^2/ab=0.05$.

(9) The effect of the parameter $\hat{\hat{\beta}}$ is negligible on the value of higher set of natural frequencies including shear and rotary inertia.

Brunelle analyzed the rectangular plates subjected to different boundary conditions for evaluating the critical buckling-loads. The combinations of the various boundary conditions considered for two parallel sides are (a) simply supported and free, (b) fixed and free, (c) both simply supported and (d) both fixed. Based on procedure outlined in this thesis, evaluation for the natural frequencies of free vibrations for the above boundary conditions may be carried out for the dynamics analysis of rectangular plates.

# APPENDIX A

## GRAPHICAL RESULTS

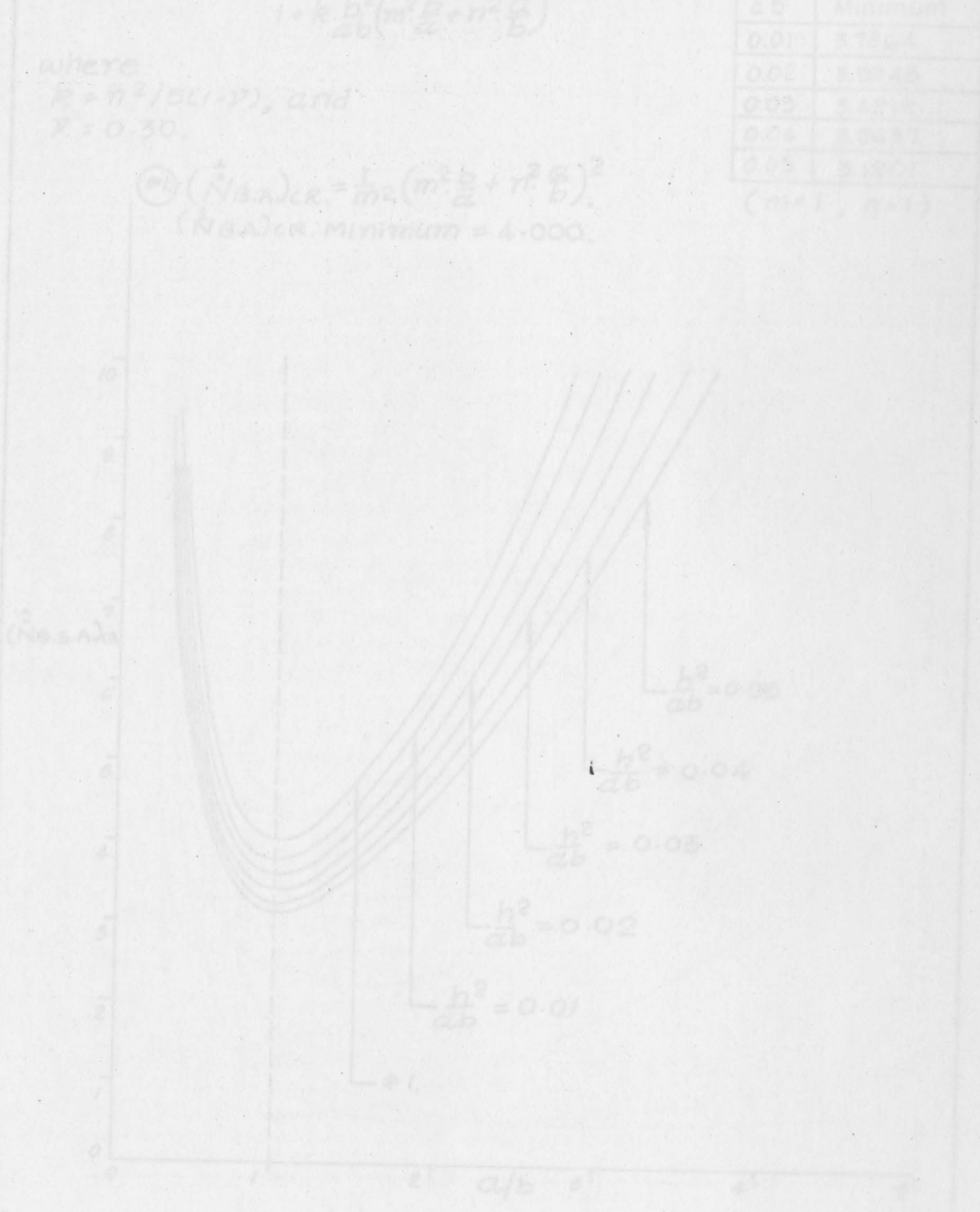



Figure 1-A. Critical Buckling Load Including Shear vs. a/b.

$$\left(\hat{\hat{N}}_{B.S.A.}\right)_{CR.} = \frac{\frac{1}{m^2}\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{1+k\cdot\frac{h^2}{ab}\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)},$$

where
$k = \pi^2/5(1-\nu)$, and
$\nu = 0.30$.

| $\frac{h^2}{ab}$ | $(\hat{\hat{N}}_{B.S.A.})_{CR.}$ Minimum |
|---|---|
| 0.01 | 3.7864 |
| 0.02 | 3.5945 |
| 0.03 | 3.4212 |
| 0.04 | 3.2637 |
| 0.05 | 3.1201 |

(m = 1 , n = 1)

(#1.) $(\hat{\hat{N}}_{B.A.})_{CR.} = \frac{1}{m^2}\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2.$

$(\hat{\hat{N}}_{B.A.})_{CR.}$ Minimum = 4.000.

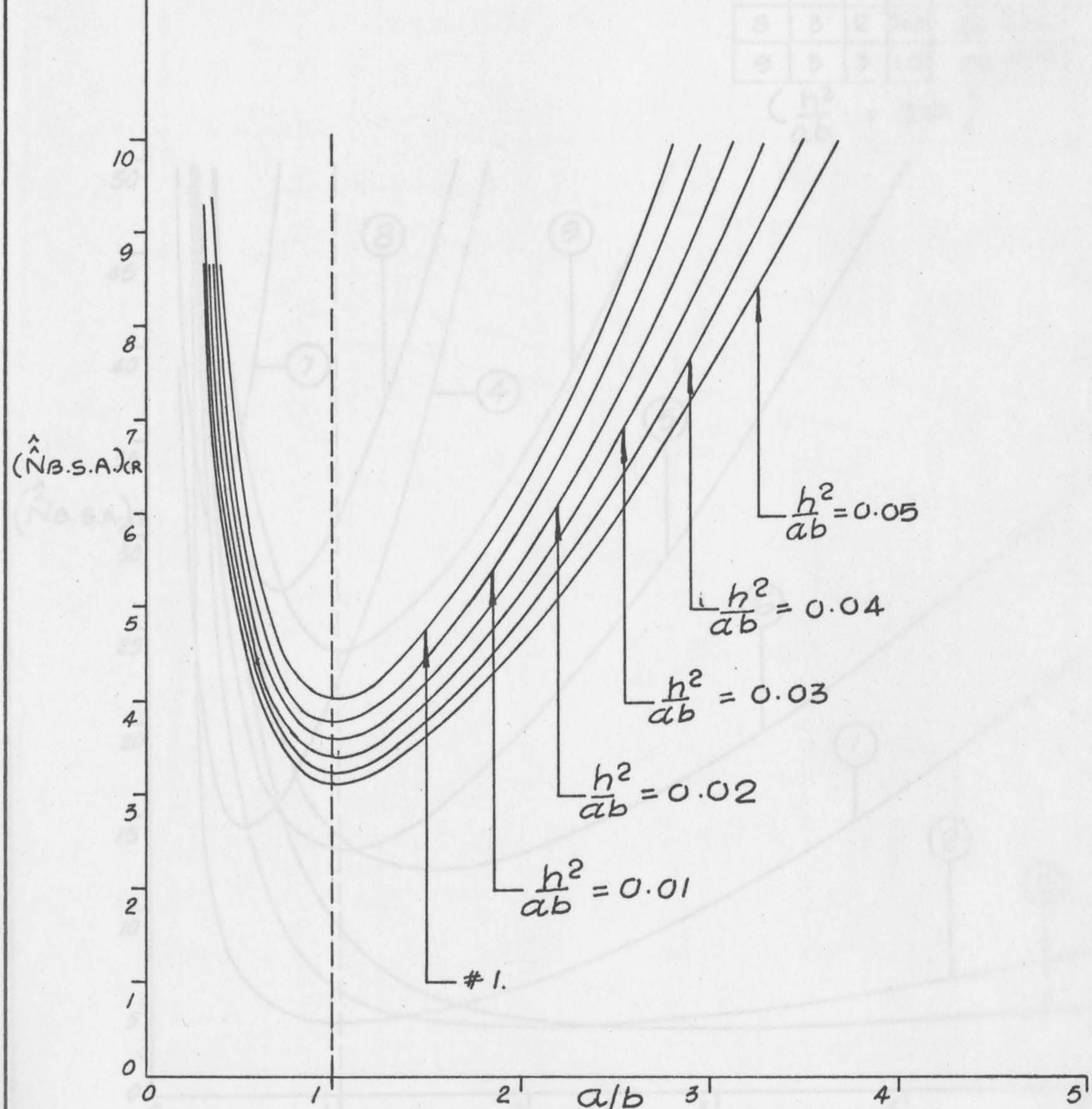



Figure 1-A.Critical Buckling Load Including Shear vs. a/b.

$$\left(\hat{N}_{B.S.A.}\right)_{CR.} = \frac{\frac{1}{m^2}\left(m^2\frac{b}{a} + n^2\frac{a}{b}\right)^2}{1 + k\frac{h^2}{ab}\left(m^2\frac{b}{a} + n^2\frac{a}{b}\right)},$$

where
$k = \pi^2/5(1-\nu)$, and
$\nu = 0.30$.

| # | n | m | $\frac{a}{b}$ | $(\hat{N}_{B.S.A.})_{CR.}$ Minimum |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 3.7864 |
| 2 | 1 | 2 | 2 | 3.5945 |
| 3 | 1 | 3 | 3 | 3.4212 |
| 4 | 2 | 1 | 0.5 | 14.3782 |
| 5 | 2 | 2 | 1.0 | 13.0549 |
| 6 | 2 | 3 | 1.5 | 11.9547 |
| 7 | 3 | 1 | 0.35 | 30.8585 |
| 8 | 3 | 2 | 0.65 | 26.9131 |
| 9 | 3 | 3 | 1.0 | 23.8792 |

$\left(\frac{h^2}{ab} = 0.01\right)$

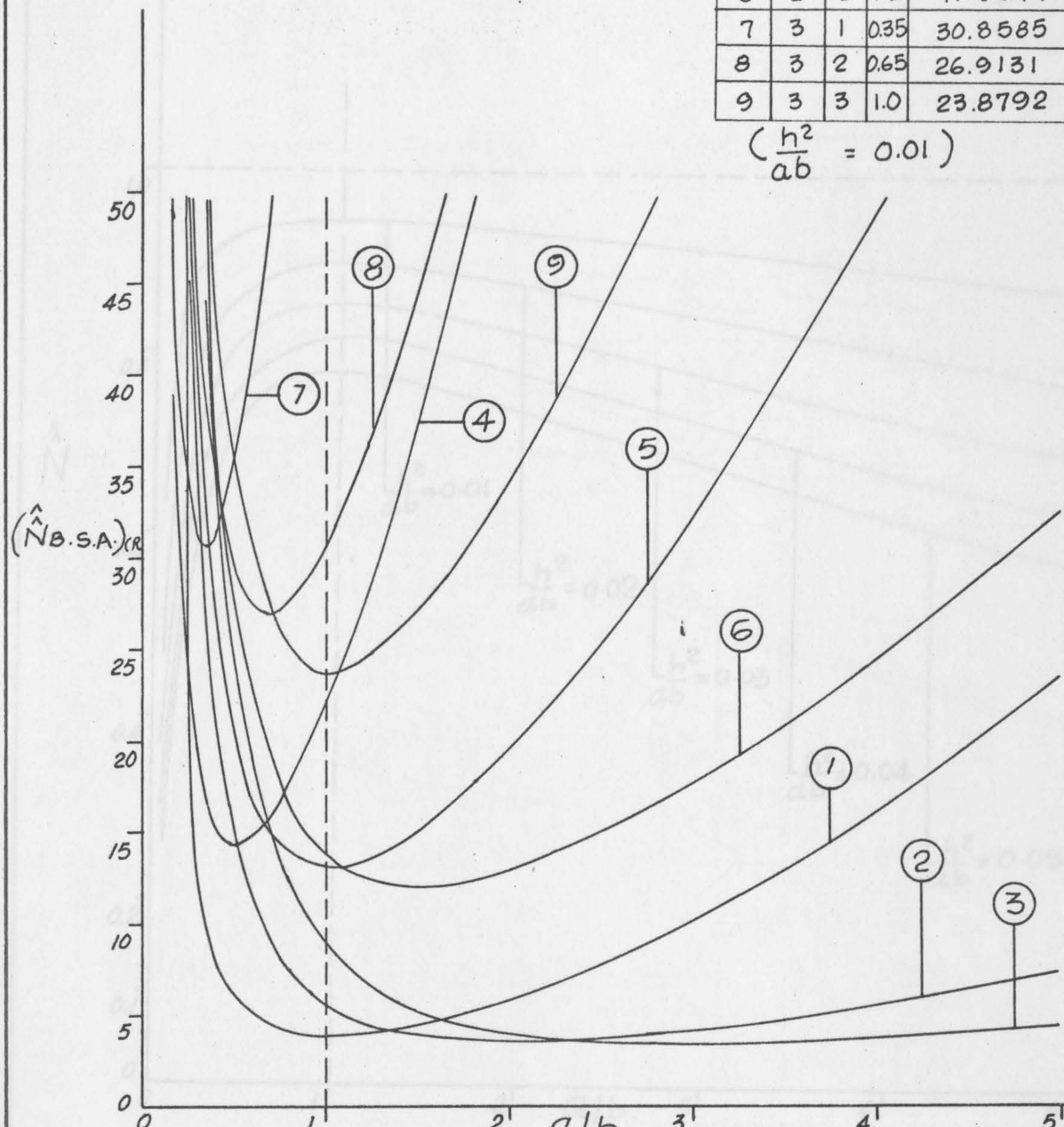



Figure 1-B. Critical Buckling Load Including Shear vs. a/b.

$$\hat{N} = \frac{(\hat{N}_{B.S.A.})_{CR.}}{(\hat{N}_{B.A.})_{CR.}} = \frac{1}{1 + k \cdot \frac{h^2}{ab}\left(m^2 \cdot \frac{b}{a} + n^2 \cdot \frac{a}{b}\right)},$$

where
$k = \pi^2/5(1-\nu)$, and
$\nu = 0.30$.

| $\frac{h^2}{ab}$ | $\hat{N}$ Maximum |
|---|---|
| 0.01 | 0.9466 |
| 0.02 | 0.8986 |
| 0.03 | 0.8553 |
| 0.04 | 0.8159 |
| 0.05 | 0.7800 |

(m = 1, n = 1)

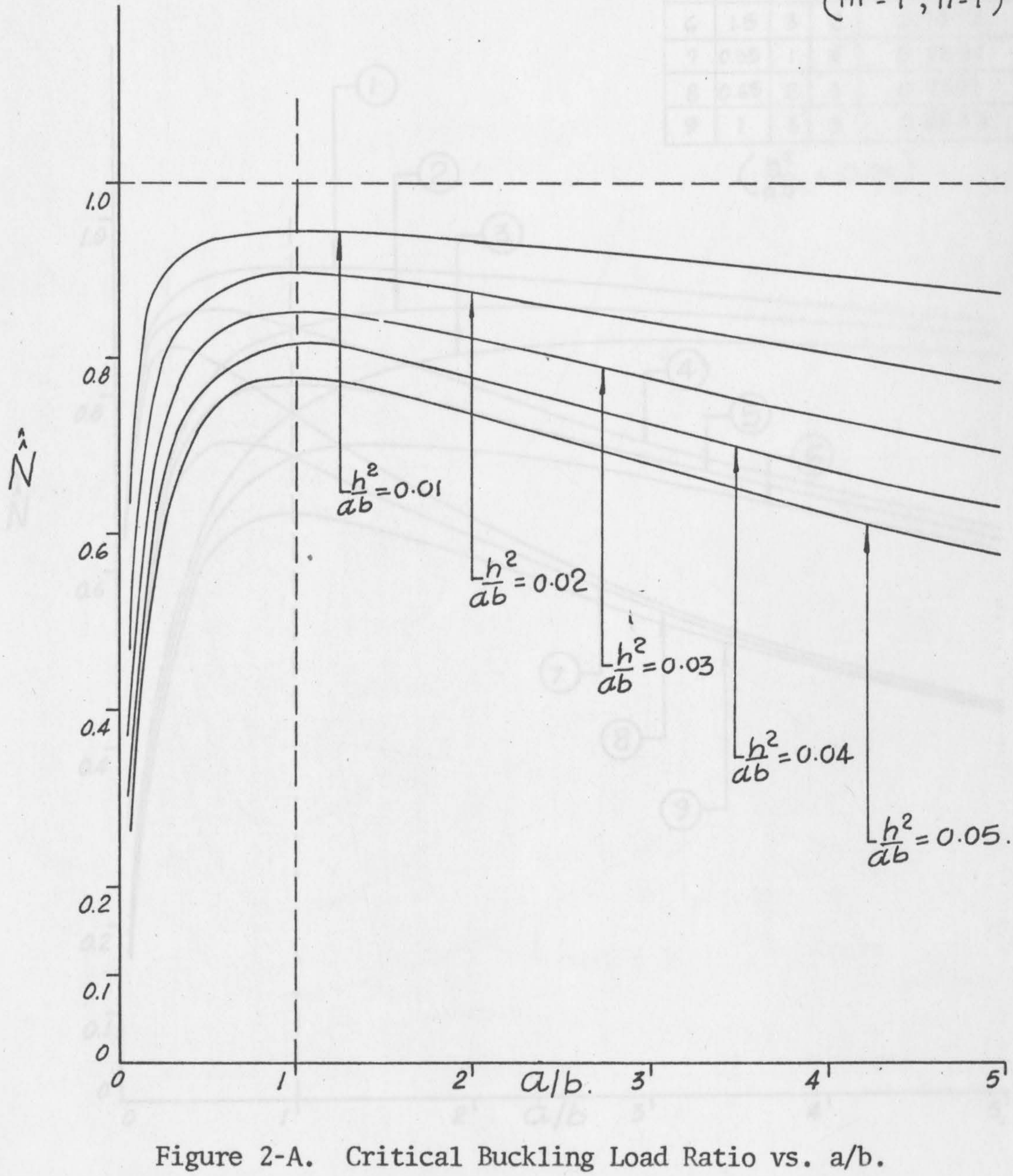



Figure 2-A. Critical Buckling Load Ratio vs. a/b.

$$\hat{\hat{N}} = \frac{(\hat{\hat{N}}_{B.S.A.})_{CR.}}{(\hat{\hat{N}}_{B.A.})_{CR.}} = \frac{1}{1+k\cdot\frac{h^2}{ab}\left(m^2\cdot\frac{b}{a}+n^2\cdot\frac{a}{b}\right)},$$

where
$k = \pi^2/5(1-\nu)$, and
$\nu = 0.30$.

| # | n | m | $\frac{a}{b}$ | $\hat{\hat{N}}$ Maximum |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0.9466 |
| 2 | 2 | 2 | 1 | 0.8986 |
| 3 | 3 | 3 | 1 | 0.8553 |
| 4 | 0.5 | 1 | 2 | 0.8986 |
| 5 | 1.0 | 2 | 2 | 0.8159 |
| 6 | 1.5 | 3 | 2 | 0.7472 |
| 7 | 0.35 | 1 | 3 | 0.8551 |
| 8 | 0.65 | 2 | 3 | 0.7471 |
| 9 | 1 | 3 | 3 | 0.6633 |

$\left(\frac{h^2}{ab} = 0.01\right)$

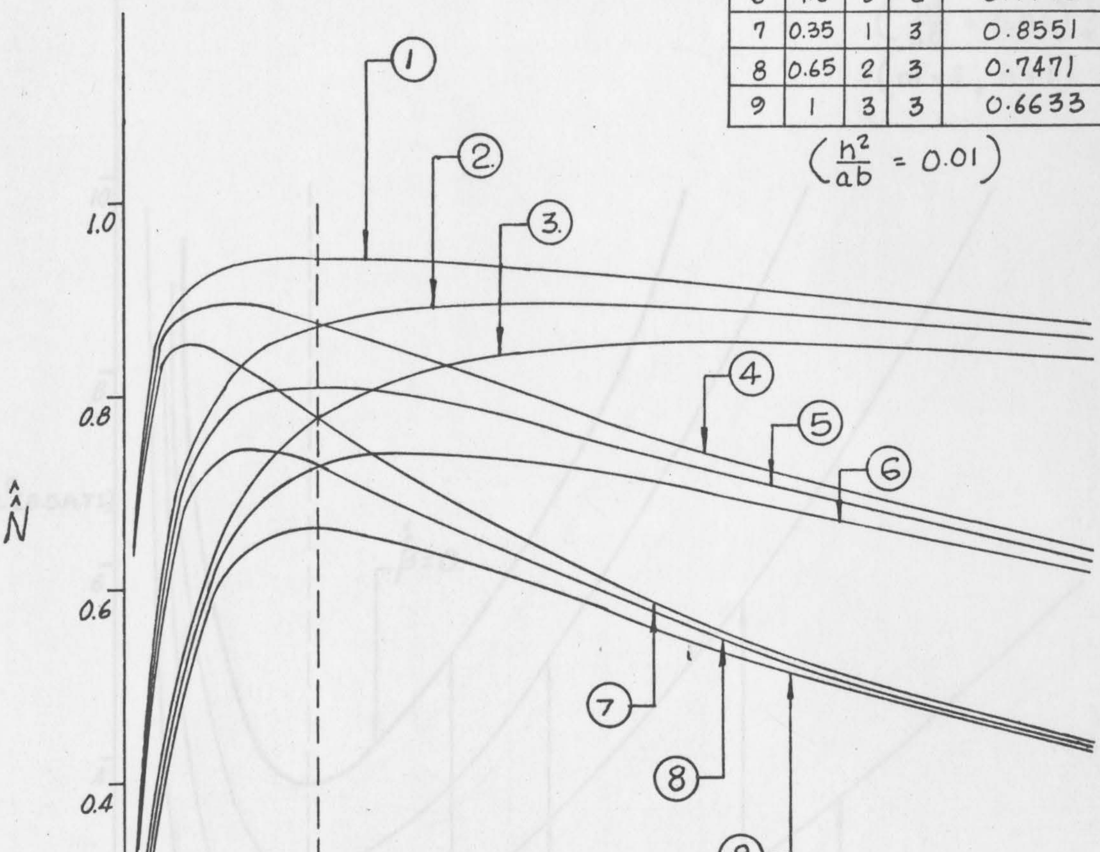



Figure 2-B. Critical Buckling Load Ratio vs. a/b.

$$\hat{\hat{\Omega}}^2_{(B.S.A.T.I.)} = \frac{(1-\hat{\hat{\beta}})\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{1+k\frac{h^2}{ab}\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)},$$

where
$k = \pi^2/5(1-\nu)$, and
$\nu = 0.30$.

| $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2_{(B.S.A.TI)}$ Minimum |
|---|---|
| 0.0 | 3.786 |
| 0.01 | 3.749 |
| 0.25 | 2.840 |
| 0.50 | 1.893 |
| 0.75 | 0.947 |
| 0.99 | 0.038 |

$\left(\frac{h^2}{ab} = 0.01\right)$,
$(m = 1,\ n = 1)$

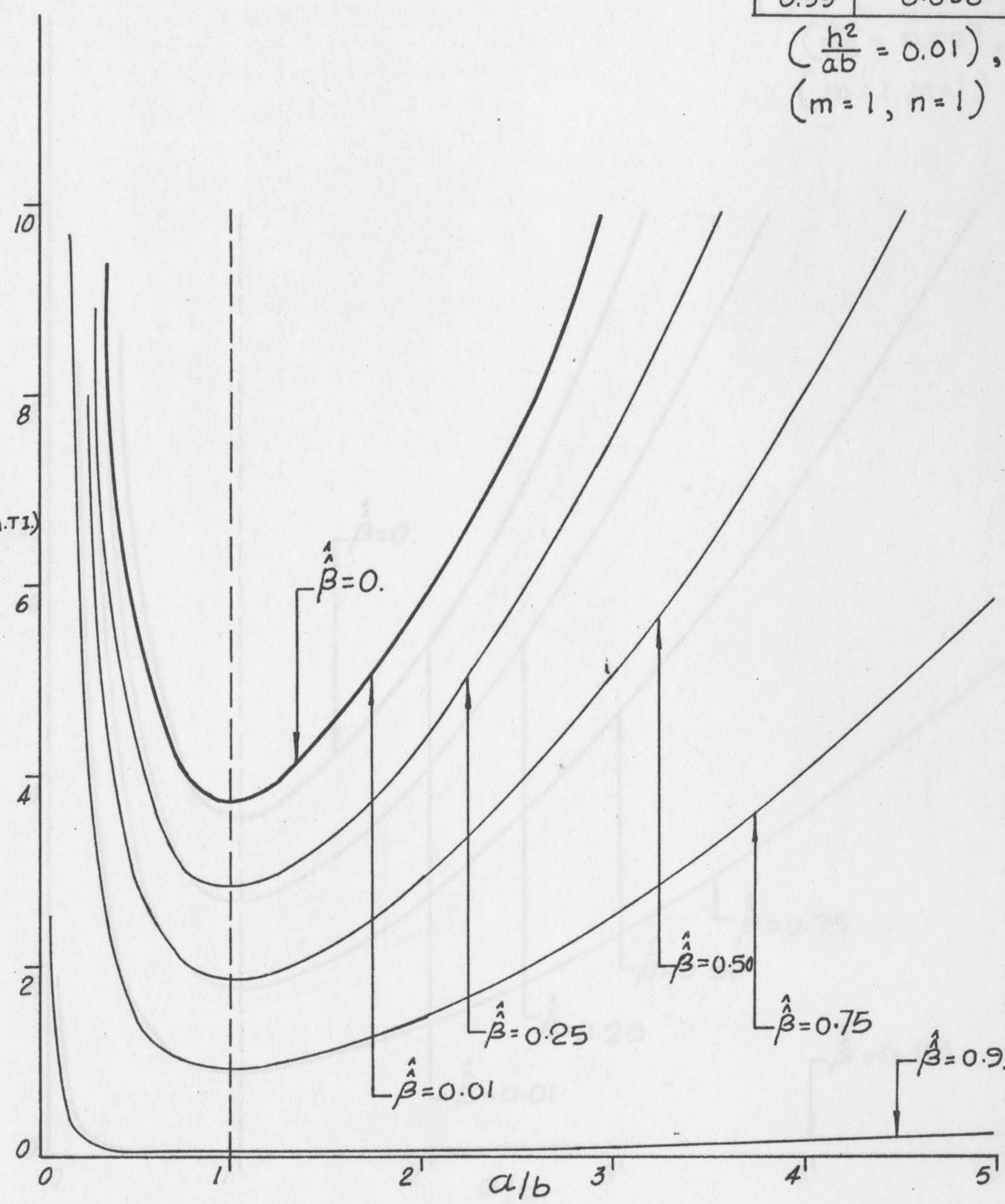



Figure 3. Natural Frequency Including Shear vs. a/b.

$$\hat{\Omega}^2_{(B.S.A.TI)} = \frac{(1-\hat{\beta})\left(m^2 \frac{b}{a} + n^2 \frac{a}{b}\right)^2}{1 + k \cdot \frac{h^2}{ab}\left(m^2 \frac{b}{a} + n^2 \frac{a}{b}\right)},$$

where
$k = \pi^2/5(1-\nu)$, and
$\nu = 0.30$.

| $\hat{\beta}$ | $\hat{\Omega}^2_{(B.S.A.TI)}$ Minimum |
|---|---|
| 0.0 | 3.595 |
| 0.01 | 3.559 |
| 0.25 | 2.696 |
| 0.50 | 1.797 |
| 0.75 | 0.899 |
| 0.99 | 0.036 |

$\left(\frac{h^2}{ab} = 0.02\right)$,
$(m = 1, n = 1)$

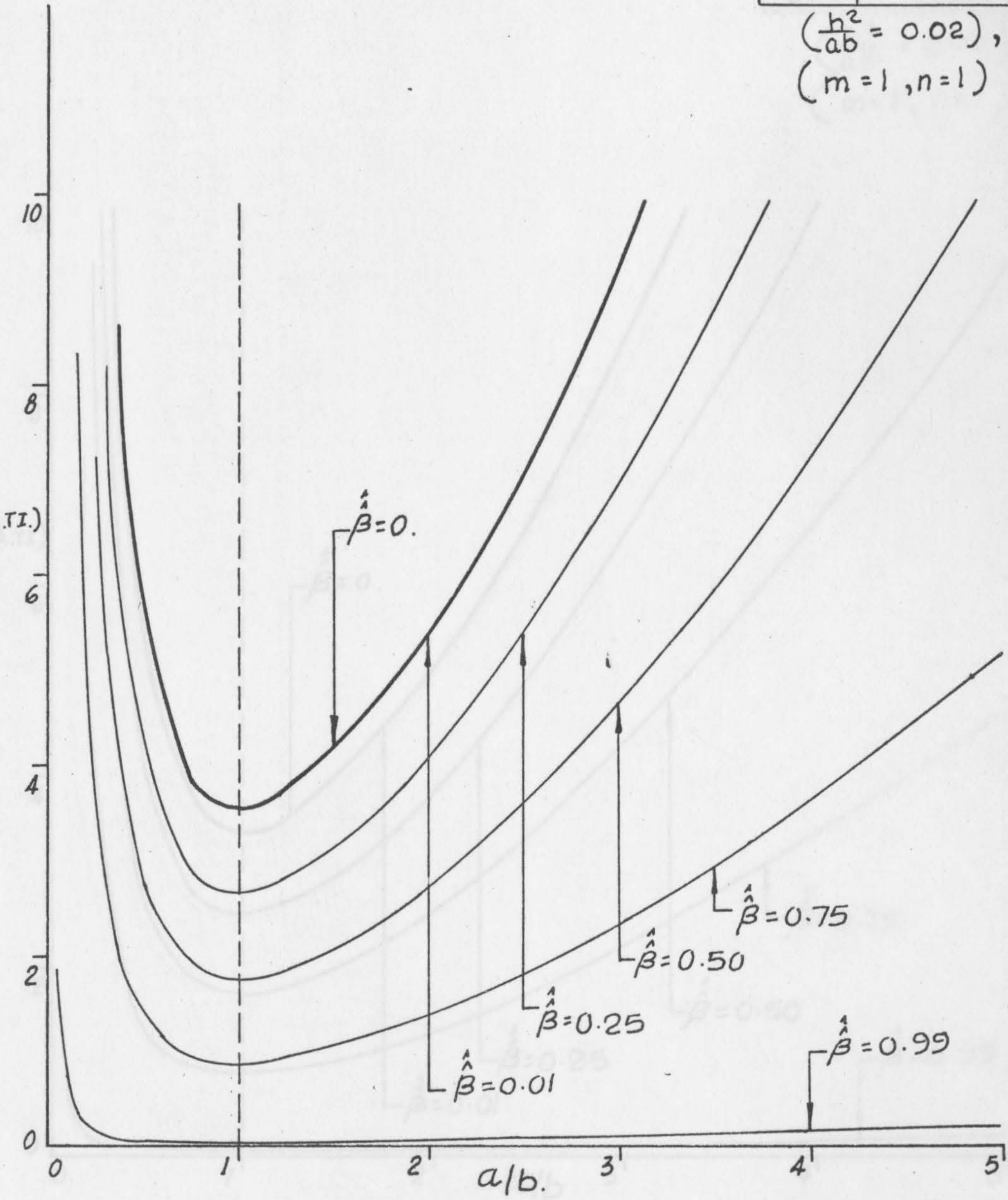



Figure 4. Natural Frequency Including Shear vs. a/b.

$$\hat{\hat{\Omega}}^2_{(B.S.A.T.I.)} = \frac{(1-\hat{\hat{\beta}})\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{1+k\frac{h^2}{ab}\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)},$$

where
$k = \pi^2/5(1-\nu)$, and
$\nu = 0.30$.

| $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2_{(B.S.A.T.I)}$ Minimum |
|---|---|
| 0.0 | 3.421 |
| 0.01 | 3.387 |
| 0.25 | 2.566 |
| 0.50 | 1.711 |
| 0.75 | 0.855 |
| 0.99 | 0.034 |

$\left(\frac{h^2}{ab} = 0.03\right)$,
$(m=1, n=1)$

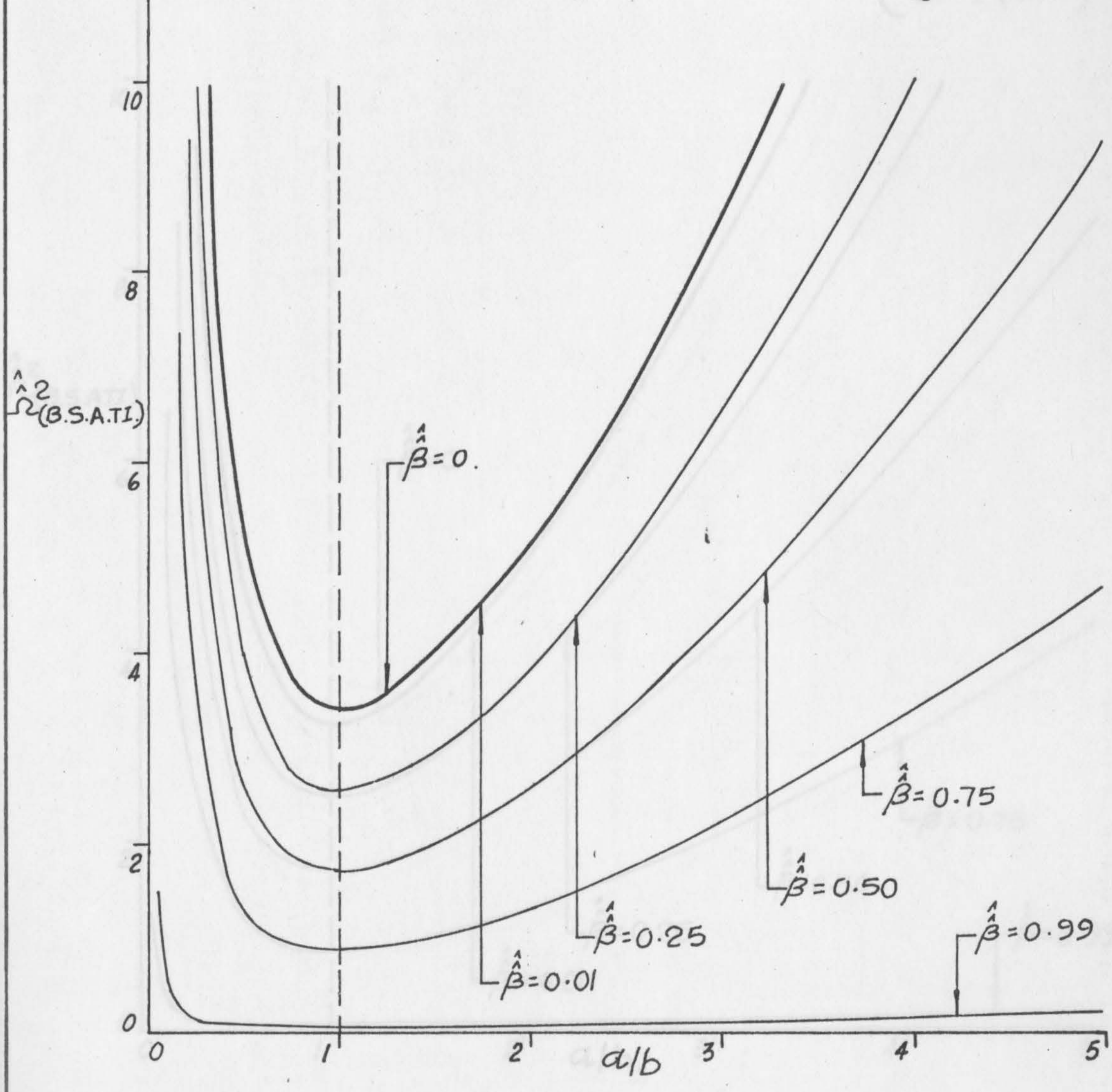



Figure 5. Natural Frequency Including Shear vs. a/b.

$$\hat{\hat{\Omega}}^2_{(B.S.A.T.I.)} = \frac{(1-\hat{\hat{\beta}})\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{1+k\cdot\frac{h^2}{ab}\cdot\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)}$$

where
$k = \pi^2/5(1-\nu)$, and
$\nu = 0.30$.

| $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2$(B.S.A.TI) Minimum |
|---|---|
| 0.0 | 3.264 |
| 0.01 | 3.231 |
| 0.25 | 2.448 |
| 0.50 | 1.632 |
| 0.75 | 0.816 |
| 0.99 | 0.033 |

$\left(\frac{h^2}{ab} = 0.04\right)$,
$(m=1, n=1)$

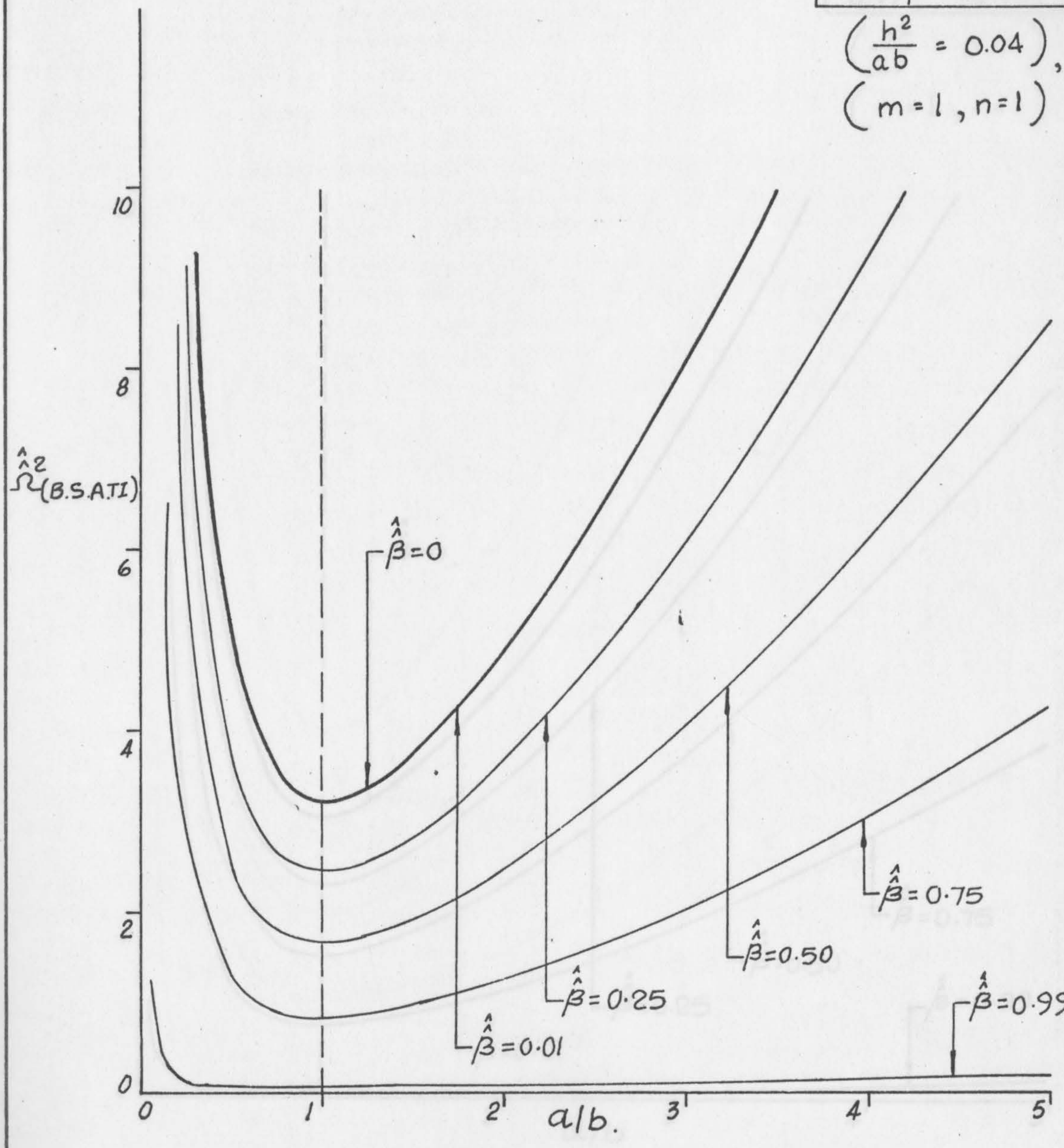



Figure 6. Natural Frequency Including Shear vs. a/b.

$$\hat{\hat{\Omega}}^2_{(B.S.A.T.I.)} = \frac{(1-\hat{\hat{\beta}})\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{1+k\cdot\frac{h^2}{ab}\cdot\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)},$$

where
$k = \pi^2/5(1-\nu)$, and
$\nu = 0.30$.

| $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}$ (B.S.A.T I) Minimum |
|---|---|
| 0.0 | 3.120 |
| 0.01 | 3.089 |
| 0.25 | 2.340 |
| 0.50 | 1.560 |
| 0.75 | 0.780 |
| 0.99 | 0.031 |

$\left(\frac{h^2}{ab} = 0.05\right)$,
$(m=1, n=1)$

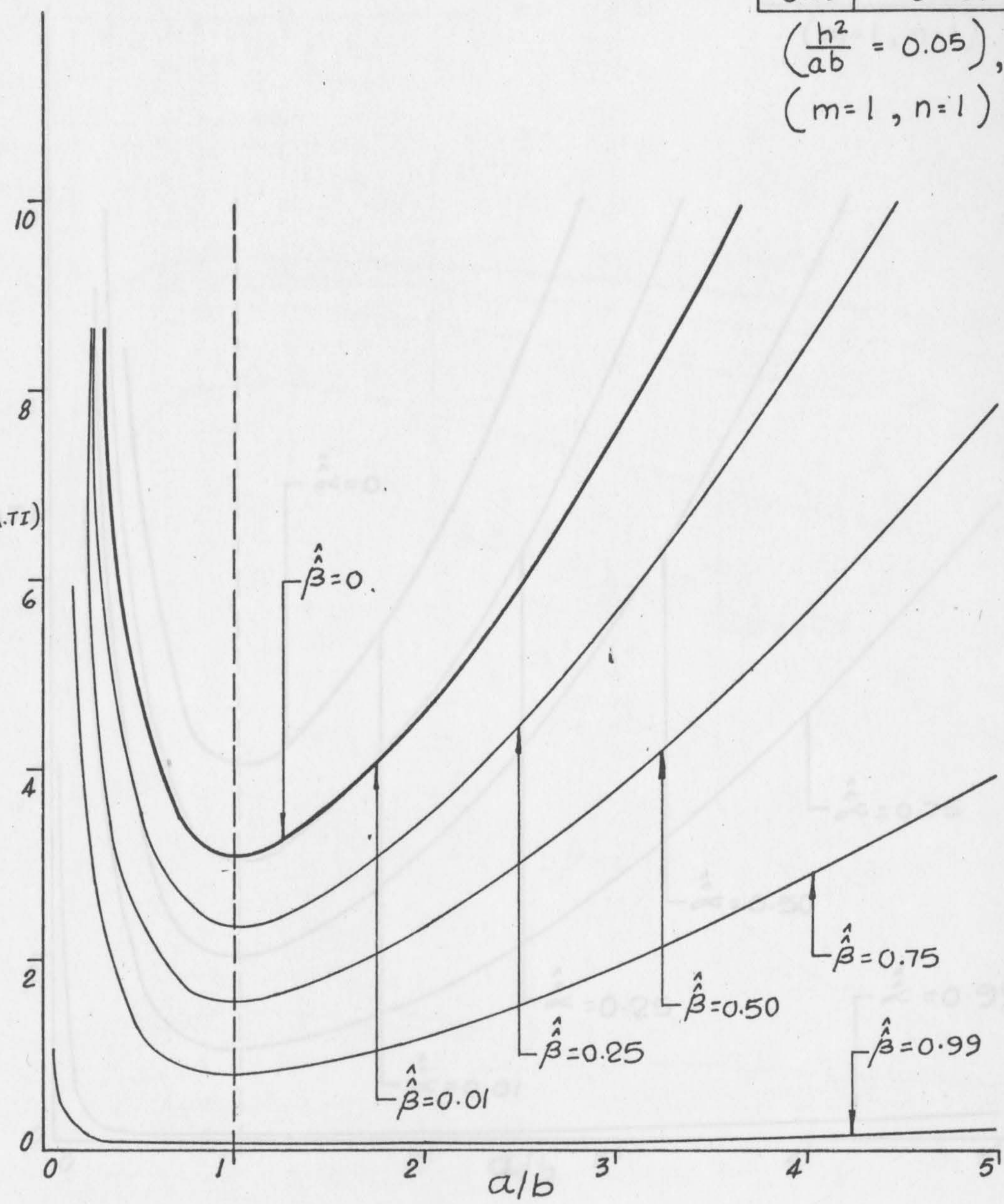



Figure 7. Natural Frequency Including Shear vs. a/b.

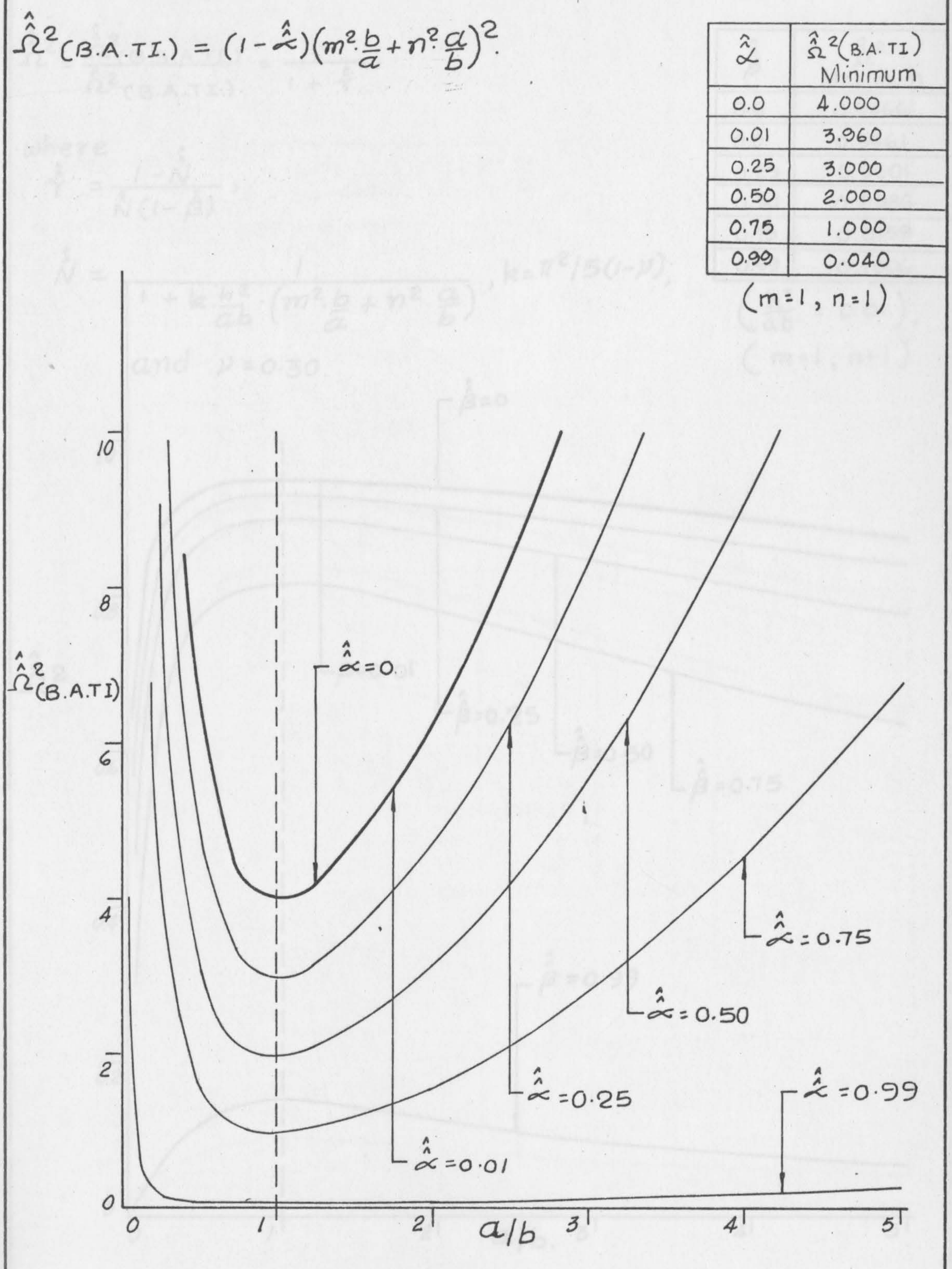



Figure 8. Natural Frequency Excluding Shear vs. a/b.

$$\hat{\hat{\Omega}}^2 = \frac{\hat{\hat{\Omega}}^2_{(B.S.A.T.I.)}}{\hat{\hat{\Omega}}^2_{(B.A.T.I.)}} = \frac{1}{1+\hat{\hat{\gamma}}},$$

where

$$\hat{\hat{\gamma}} = \frac{1-\hat{\hat{N}}}{\hat{\hat{N}}(1-\hat{\hat{\beta}})},$$

$$\hat{\hat{N}} = \frac{1}{1+k\cdot\frac{h^2}{ab}\cdot\left(m^2\cdot\frac{b}{a}+n^2\cdot\frac{a}{b}\right)}, \quad k=\pi^2/5(1-\nu),$$

and $\nu = 0.30$.

| $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2$ Maximum |
|---|---|
| 0.0 | 0.9466 |
| 0.01 | 0.9461 |
| 0.25 | 0.9301 |
| 0.50 | 0.8986 |
| 0.75 | 0.8159 |
| 0.99 | 0.1506 |

$\left(\frac{h^2}{ab} = 0.01\right)$, $(m=1,\ n=1)$

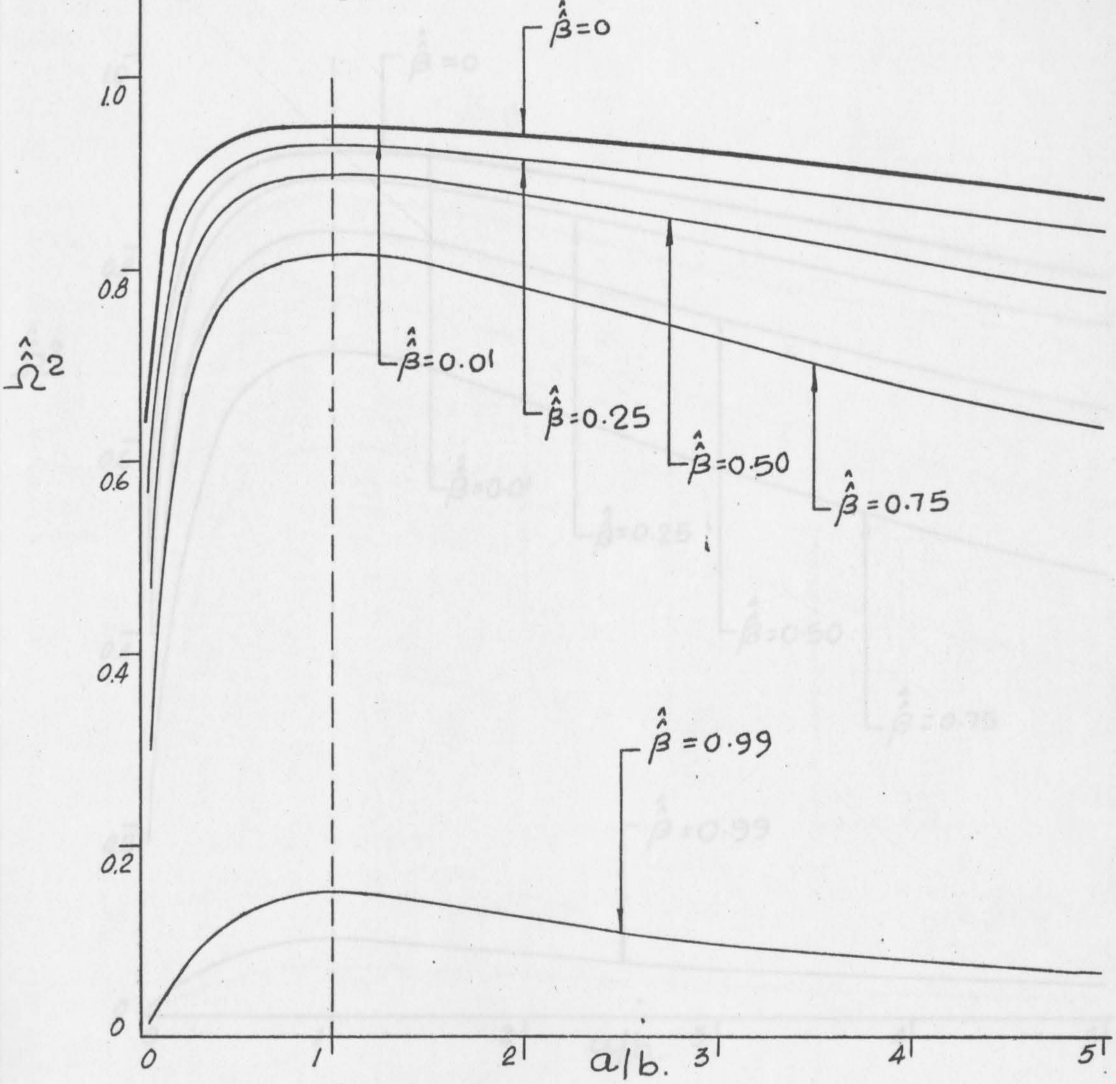



Figure 9. Natural Frequency Ratio vs. a/b.

$$\hat{\hat{\Omega}}^2 = \frac{\hat{\hat{\Omega}}^2_{(B.S.A.T.I.)}}{\hat{\hat{\Omega}}^2_{(B.A.T.I.)}} = \frac{1}{1+\hat{\hat{\gamma}}}, \text{ where}$$

$$\hat{\hat{\gamma}} = \frac{1-\hat{\hat{N}}}{\hat{\hat{N}}(1-\hat{\hat{\beta}})}, \quad \hat{\hat{N}} = \frac{1}{1+k\cdot\frac{h^2}{ab}\left(m^2\cdot\frac{b}{a}+n^2\cdot\frac{a}{b}\right)},$$

$k = \pi^2/5(1-\nu)$, and $\nu = 0.30$.

| $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2$ maximum |
|---|---|
| 0.0 | 0.8986 |
| 0.01 | 0.8977 |
| 0.25 | 0.8693 |
| 0.50 | 0.8159 |
| 0.75 | 0.6891 |
| 0.99 | 0.0814 |

$(h^2/ab = 0.02)$, $(m=1, n=1)$.

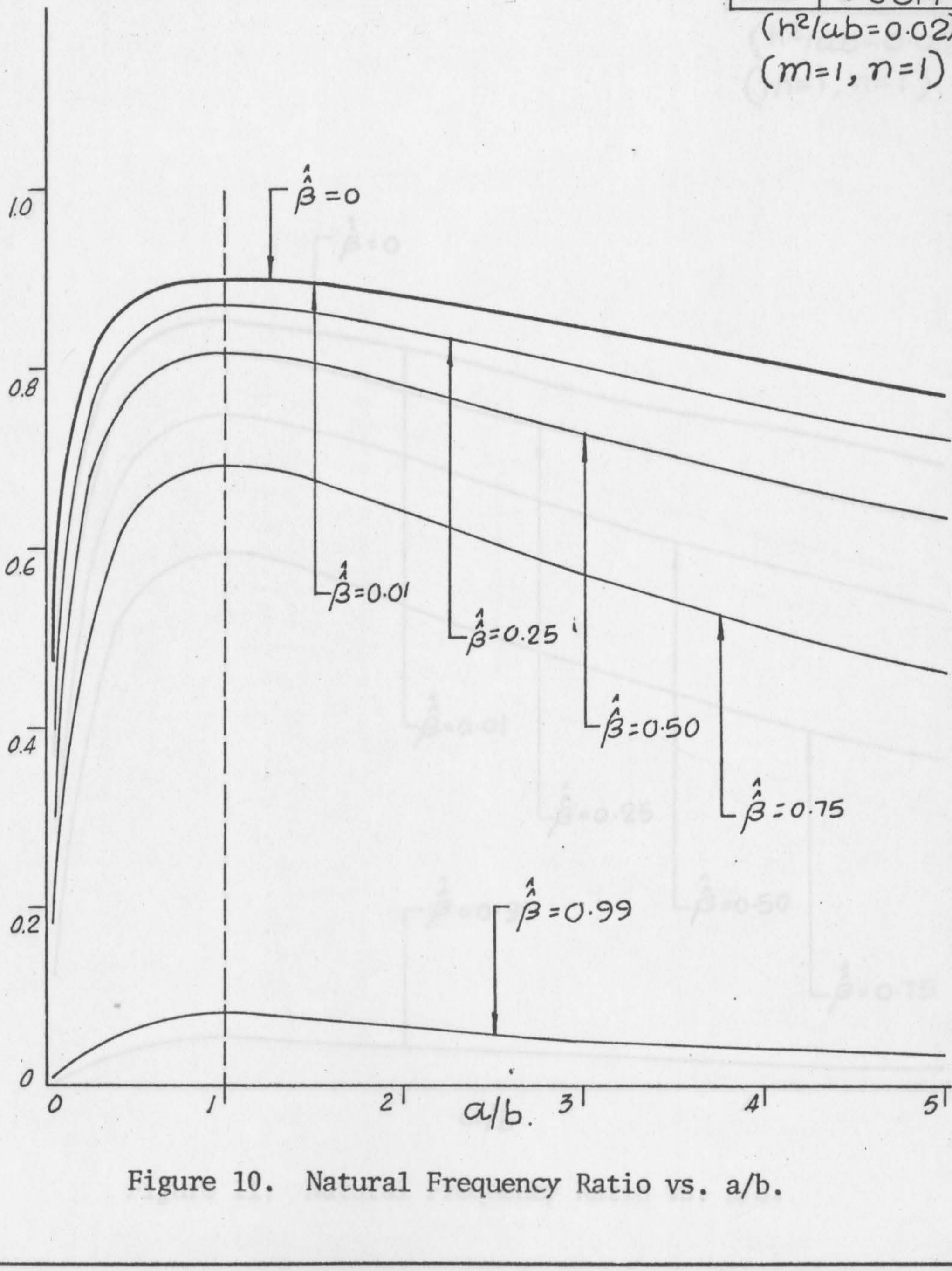



Figure 10. Natural Frequency Ratio vs. a/b.

$$\hat{\hat{\Omega}}^2 = \frac{\hat{\hat{\Omega}}^2(\text{B.S.A.T.I.})}{\hat{\hat{\Omega}}^2(\text{B.A.T.I.})} = \frac{1}{1+\hat{\hat{\gamma}}}, \text{ where}$$

$$\hat{\hat{\gamma}} = \frac{1-\hat{\hat{N}}}{\hat{\hat{N}}(1-\hat{\hat{\beta}})}, \quad \hat{\hat{N}} = \frac{1}{1+k\cdot\frac{h^2}{ab}\left(m^2\cdot\frac{b}{a}+n^2\cdot\frac{a}{b}\right)},$$

$k = \pi^2/5(1-\nu)$, and $\nu = 0.30$.

| $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2$ Maximum |
|---|---|
| 0.0 | 0.8553 |
| 0.01 | 0.8540 |
| 0.25 | 0.8159 |
| 0.50 | 0.7472 |
| 0.75 | 0.5964 |
| 0.99 | 0.0558 |

$(h^2/ab = 0.03)$,
$(m=1, n=1)$.

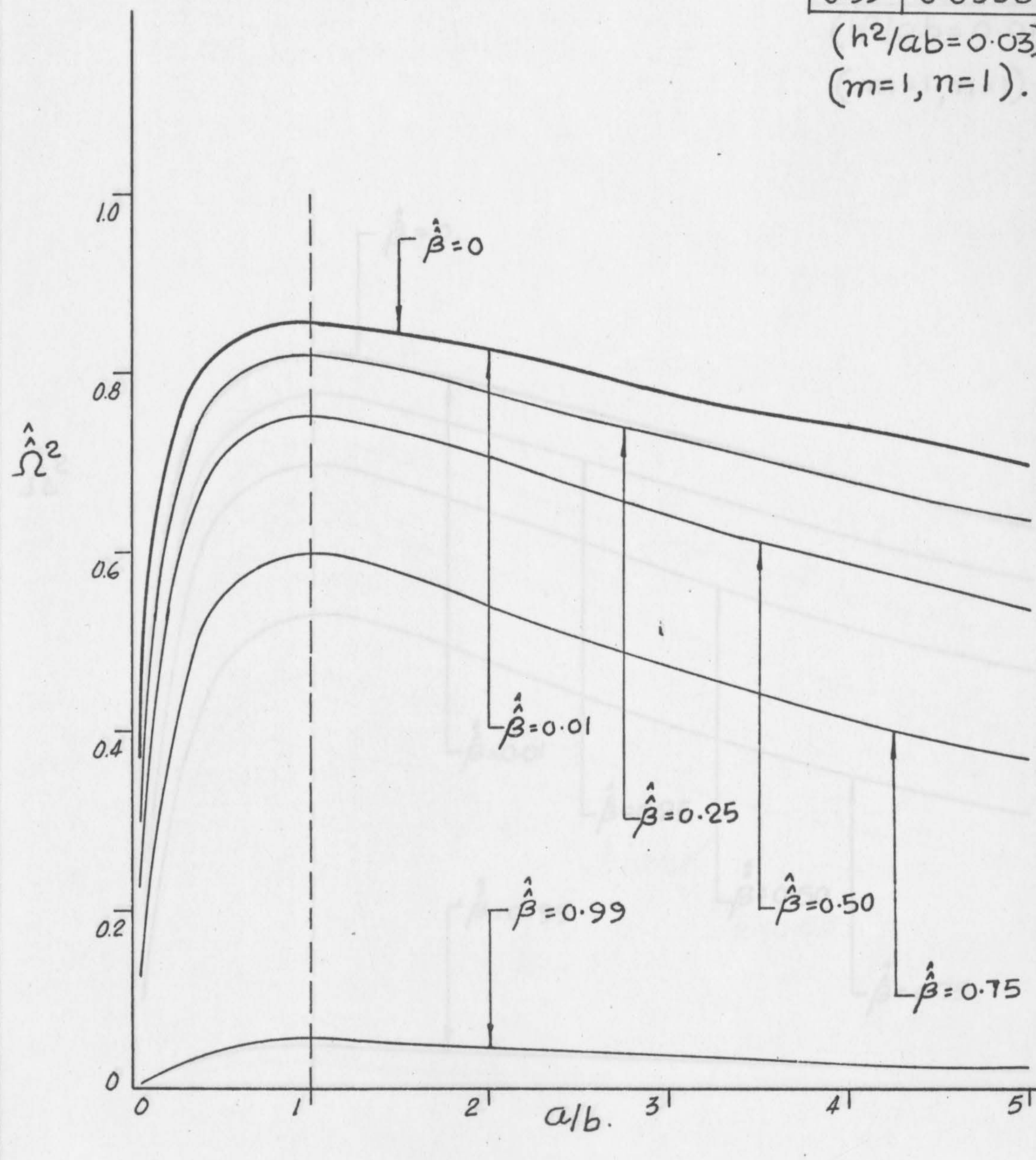



Figure 11. Natural Frequency Ratio vs. a/b.

$$\hat{\hat{\Omega}}^2 = \frac{\hat{\hat{\Omega}}^2_{(B.S.A.T.I.)}}{\hat{\hat{\Omega}}^2_{(B.A.T.I.)}} = \frac{1}{1+\hat{\hat{\gamma}}}, \text{ where}$$

$$\hat{\hat{\gamma}} = \frac{1-\hat{\hat{N}}}{\hat{\hat{N}}(1-\hat{\hat{\beta}})}, \quad \hat{\hat{N}} = \frac{1}{1+k\cdot\frac{h^2}{ab}\cdot\left(m^2\cdot\frac{b}{a}+n^2\cdot\frac{a}{b}\right)},$$

$k = \pi^2/5(1-\nu)$, and $\nu = 0.30$

| $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2$ Maximum |
|---|---|
| 0.0 | 0.8159 |
| 0.01 | 0.8144 |
| 0.25 | 0.7688 |
| 0.50 | 0.6891 |
| 0.75 | 0.5257 |
| 0.99 | 0.0424 |

($h^2/ab = 0.04$),

($m=1, n=1$).

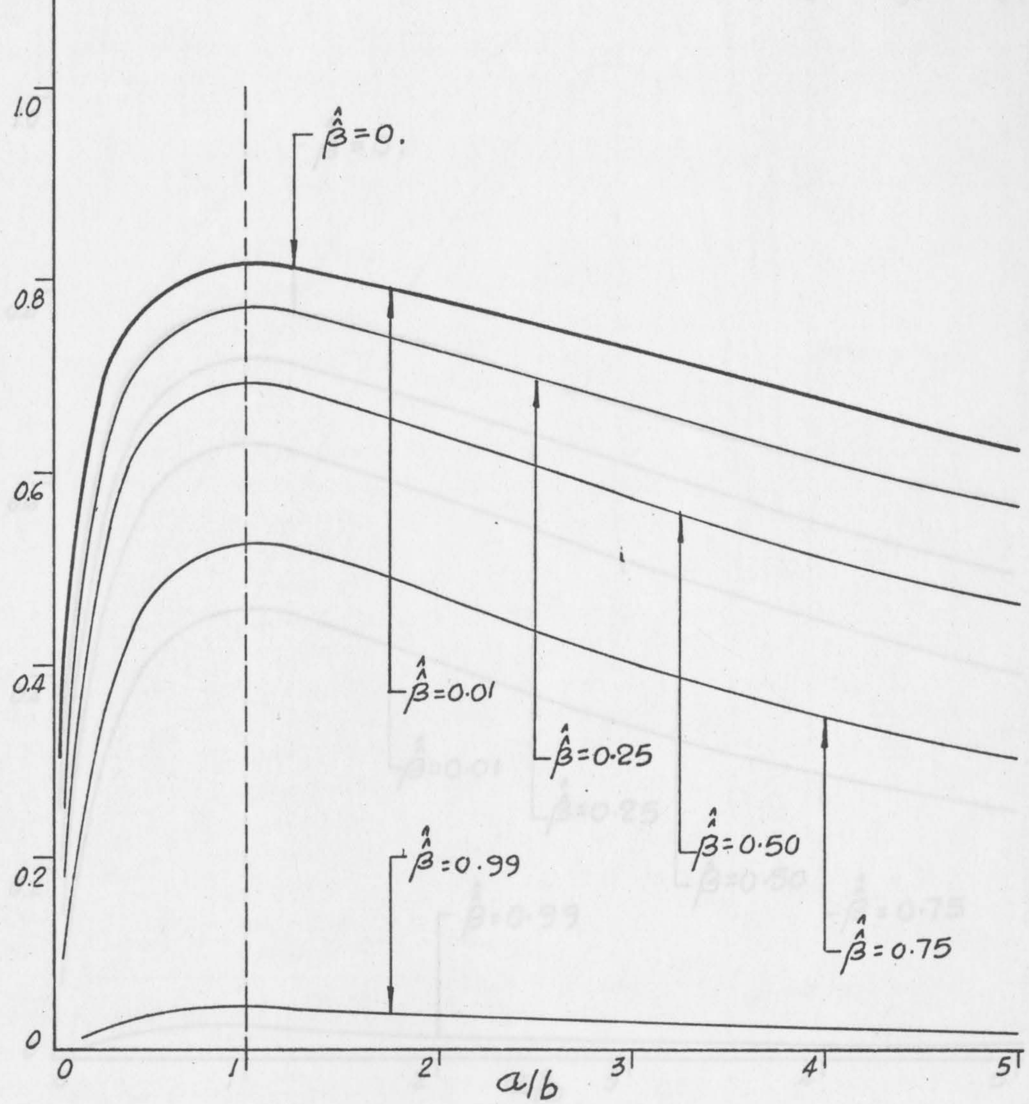



Figure 12. Natural Frequency Ratio vs. a/b.

$$\hat{\hat{\Omega}}^2 = \frac{\hat{\hat{\Omega}}^2(\text{B.S.A.T.I.})}{\hat{\hat{\Omega}}^2(\text{B.A.T.I.})} = \frac{1}{1+\hat{\hat{\gamma}}}, \text{ where}$$

$$\hat{\hat{\gamma}} = \frac{1-\hat{\hat{N}}}{\hat{\hat{N}}(1-\hat{\hat{\beta}})}, \quad \hat{\hat{N}} = \frac{1}{1+k\cdot\frac{h^2}{ab}\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)},$$

$k = \pi^2/5(1-\nu)$, and $\nu = 0.30$.

| $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2$ Maximum |
|---|---|
| 0.0 | 0.7800 |
| 0.01 | 0.7783 |
| 0.25 | 0.7268 |
| 0.50 | 0.6394 |
| 0.75 | 0.4699 |
| 0.99 | 0.0342 |

$(h^2/ab = 0.05)$, $(m=1, n=1)$.

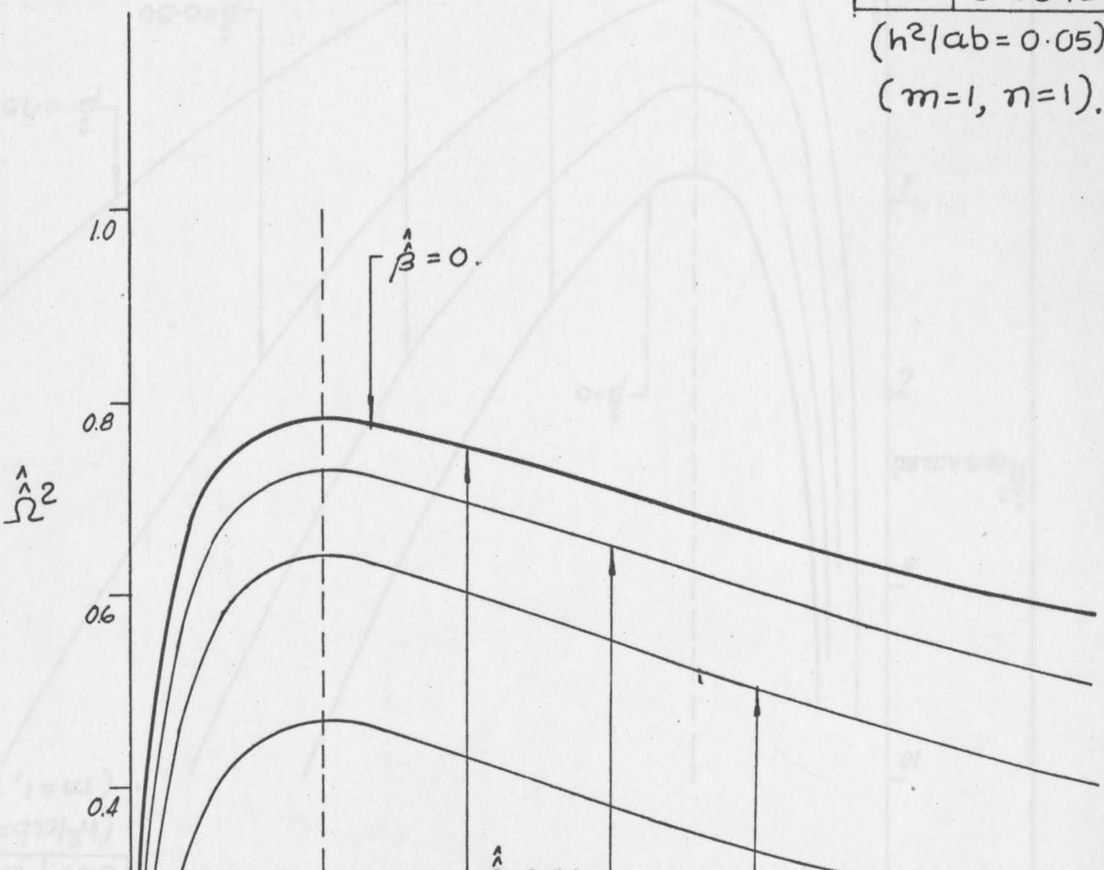



Figure 13. Natural Frequency Ratio vs. a/b.

$$\hat{\hat{\Omega}}^2_{(B.S.A.T.I.R.I.)} = \frac{1}{2\cdot\frac{\pi^2}{12}\cdot k}\left(\frac{ab}{h^2}\right)^2 (R)\left[1-\left\{1-\frac{4\cdot\frac{\pi^2}{12}\cdot k\left(\frac{h^2}{ab}\right)^2\cdot(1-\hat{\hat{\beta}})\left(m^2\cdot\frac{b}{a}+n^2\cdot\frac{a}{b}\right)^2}{(R^2)}\right\}^{1/2}\right],$$

where

$$R = 1+\left(k+\frac{\pi^2}{12}\right)\cdot\left(\frac{h^2}{ab}\right)\left(m^2\cdot\frac{b}{a}+n^2\cdot\frac{a}{b}\right)-\frac{k\cdot\frac{\pi^2}{12}\cdot\left(\frac{h^2}{ab}\right)^2\cdot\hat{\hat{\beta}}\left(m^2\cdot\frac{b}{a}+n^2\cdot\frac{a}{b}\right)^2}{1+k\cdot\frac{h^2}{ab}\left(m^2\cdot\frac{b}{a}+n^2\cdot\frac{a}{b}\right)},$$

$k = \pi^2/5(1-\nu)$ and $\nu = 0.30$.

| $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2_{(B.S.A.T.I.R.I.)}$ Minimum |
|---|---|
| 0.0 | 3.732 |
| 0.01 | 3.694 |
| 0.25 | 2.799 |
| 0.50 | 1.866 |
| 0.75 | 0.933 |
| 0.99 | 0.037 |

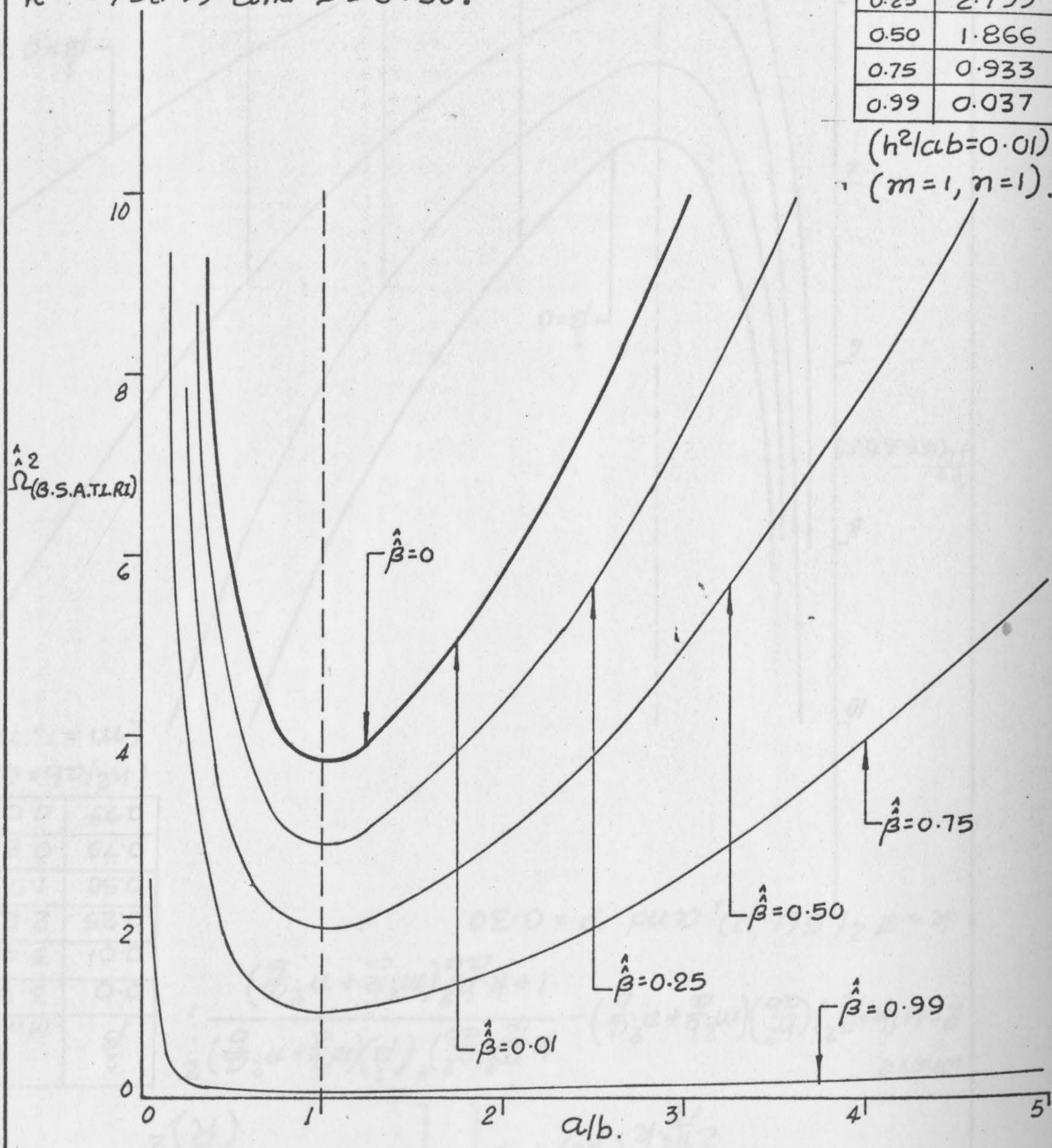



Figure 14. Lower Set of Natural Frequency Including Shear and Rotary Inertia vs. a/b.

$$\hat{\Omega}^2_{(B.S.A.T.I.R.I.)} = \frac{1}{2\cdot\frac{\pi^2}{12}\cdot k}\left(\frac{ab}{h^2}\right)^2(R)\left[1-\left\{1-\frac{4\cdot\frac{\pi^2}{12}k\left(\frac{h^2}{ab}\right)^2(1-\hat{\beta})\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{(R)^2}\right\}^{1/2}\right]$$

where

$$R = 1+\left(k+\frac{\pi^2}{12}\right)\left(\frac{h^2}{ab}\right)\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)-\frac{k\cdot\frac{\pi^2}{12}\left(\frac{h^2}{ab}\right)^2(\hat{\beta})\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{1+k\cdot\frac{h^2}{ab}\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)},$$

$k = \pi^2/5(1-\nu)$, and $\nu = 0.30$.

| $\hat{\beta}$ | $\hat{\Omega}^2_{(B.S.A.T.I.R.I.)}$ Minimum |
|---|---|
| 0.0 | 3.501 |
| 0.01 | 3.466 |
| 0.25 | 2.626 |
| 0.50 | 1.751 |
| 0.75 | 0.875 |
| 0.99 | 0.035 |

$(h^2/ab = 0.02)$, $(m=1, n=1)$.

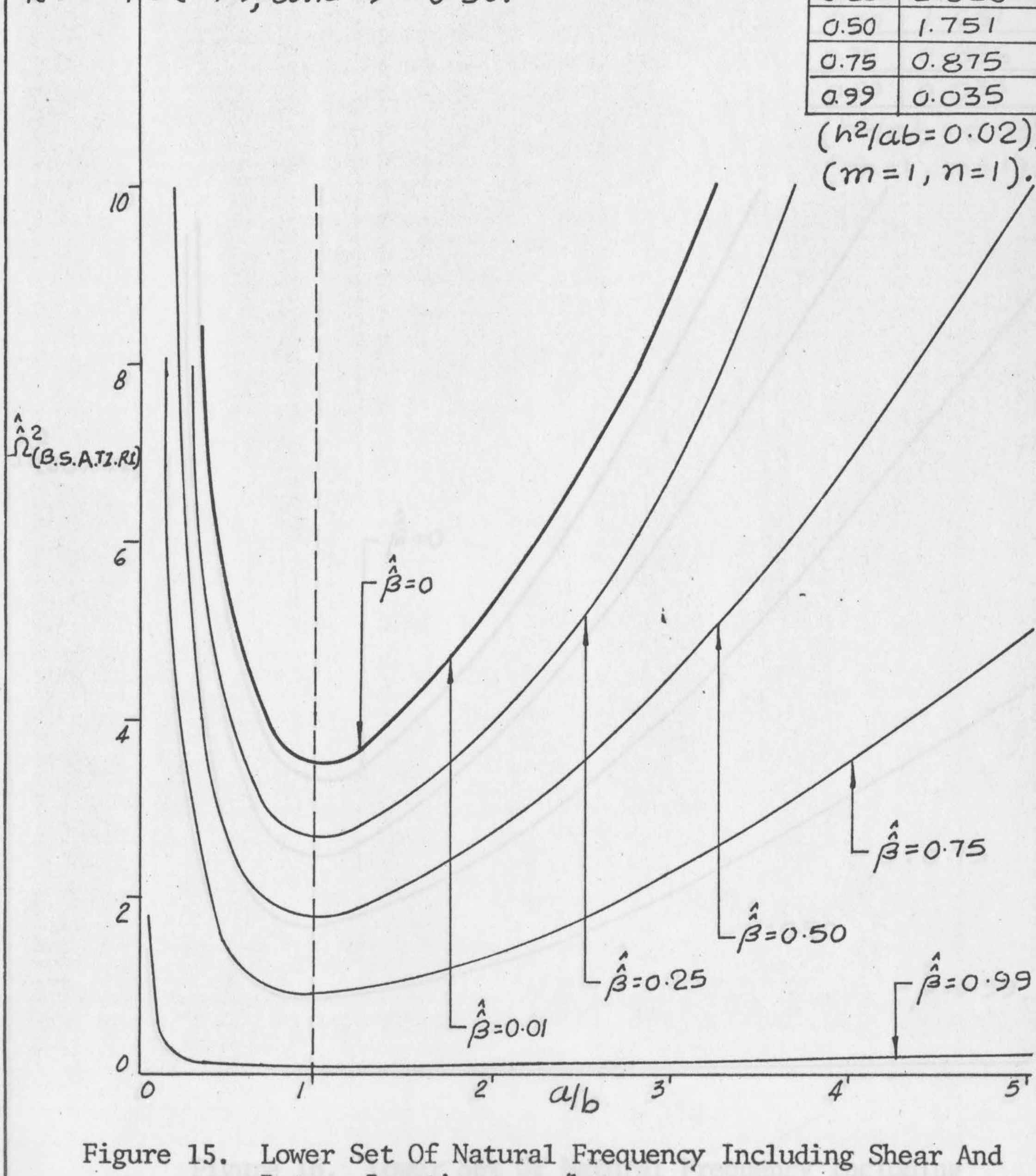



Figure 15. Lower Set Of Natural Frequency Including Shear And Rotary Inertia vs. a/b.

$$\hat{\hat{\Omega}}^2_{(B.S.A.T.I.R.I.)} = \frac{1}{2\cdot\frac{\pi^2}{12}\cdot k}\cdot\left(\frac{ab}{h^2}\right)^2 (R)\left[1-\left\{1-\frac{4\cdot\frac{\pi^2}{12}\cdot k\cdot\left(\frac{h^2}{ab}\right)^2(1-\hat{\hat{\beta}})\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{(R^2)}\right\}^{1/2}\right]$$

where

$$R = 1+\left(k+\frac{\pi^2}{12}\right)\left(\frac{h^2}{ab}\right)\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)-\frac{k\cdot\frac{\pi^2}{12}\left(\frac{h^2}{ab}\right)^2(\hat{\hat{\beta}})\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{1+k\cdot\frac{h^2}{ab}\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)},$$

$k = \pi^2/5(1-\nu)$, and $\nu = 0.30$.

| $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2_{(B.S.A.T.I.R.I.)}$ Minimum |
|---|---|
| 0.0 | 3.301 |
| 0.01 | 3.268 |
| 0.25 | 2.476 |
| 0.50 | 1.651 |
| 0.75 | 0.826 |
| 0.99 | 0.033 |

$(h^2/ab = 0.03)$, $(m=1, n=1)$.

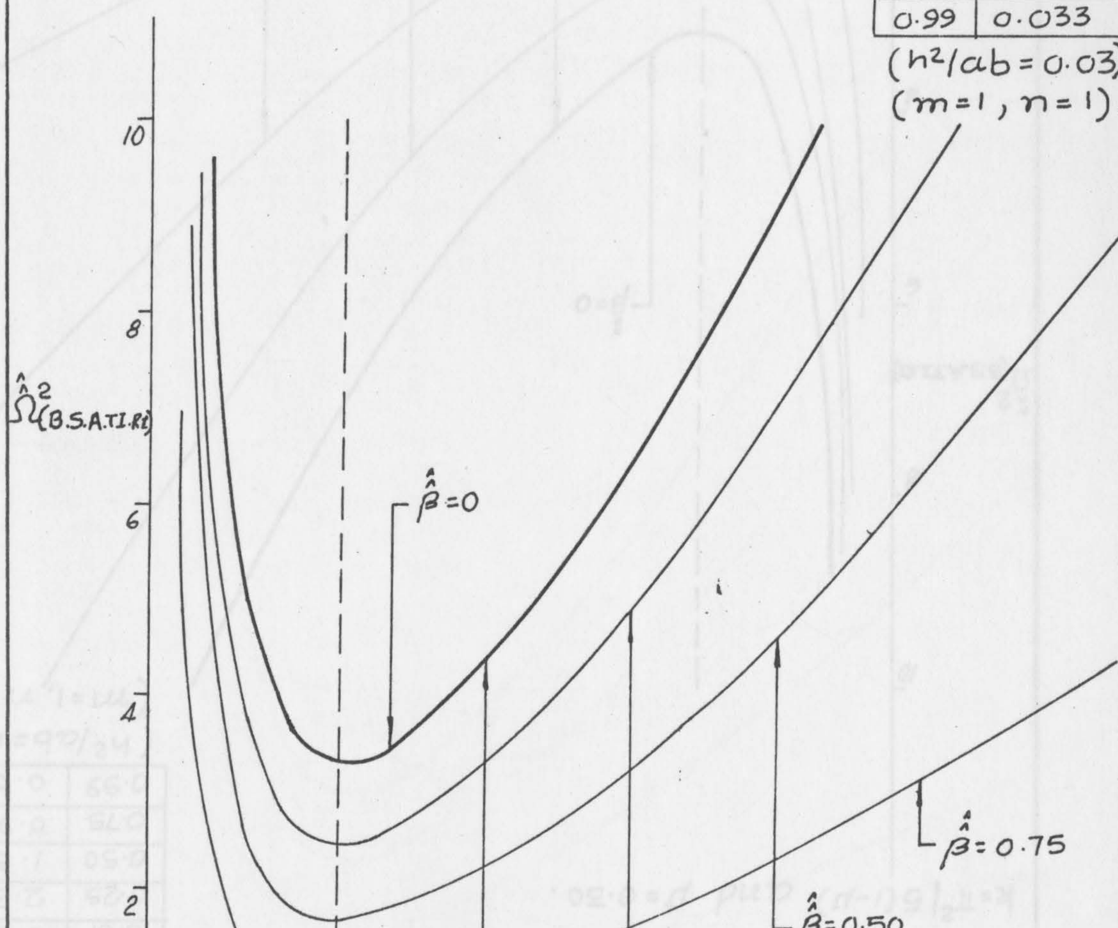



Figure 16. Lower Set Of Natural Frequency Including Shear And Rotary Inertia vs. a/b.

$$\hat{\Omega}^2_{(B.S.A.T.I.R.I.)} = \frac{1}{2\cdot\frac{\pi^2}{12}\cdot k}\left(\frac{ab}{h^2}\right)^2 (R)\left[1 - \left\{1 - \frac{4\cdot\frac{\pi^2}{12}\cdot k\cdot\left(\frac{h^2}{ab}\right)^2 (1-\hat{\hat{\beta}})\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{(R)^2}\right\}^{1/2}\right]$$

where

$$R = 1 + \left(k+\frac{\pi^2}{12}\right)\left(\frac{h^2}{ab}\right)\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right) - \frac{k\frac{\pi^2}{12}\left(\frac{h^2}{12}\right)(\hat{\hat{\beta}})\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{1+k\cdot\frac{h^2}{ab}\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)},$$

$k = \pi^2/5(1-\nu)$ and $\nu = 0.30$.

| $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2_{(B.S.A.T.I.R.I.)}$ Minimum |
|---|---|
| 0.0 | 3.126 |
| 0.01 | 3.094 |
| 0.25 | 2.345 |
| 0.50 | 1.563 |
| 0.75 | 0.782 |
| 0.99 | 0.031 |

$(h^2/ab = 0.04)$

$(m=1, n=1)$.

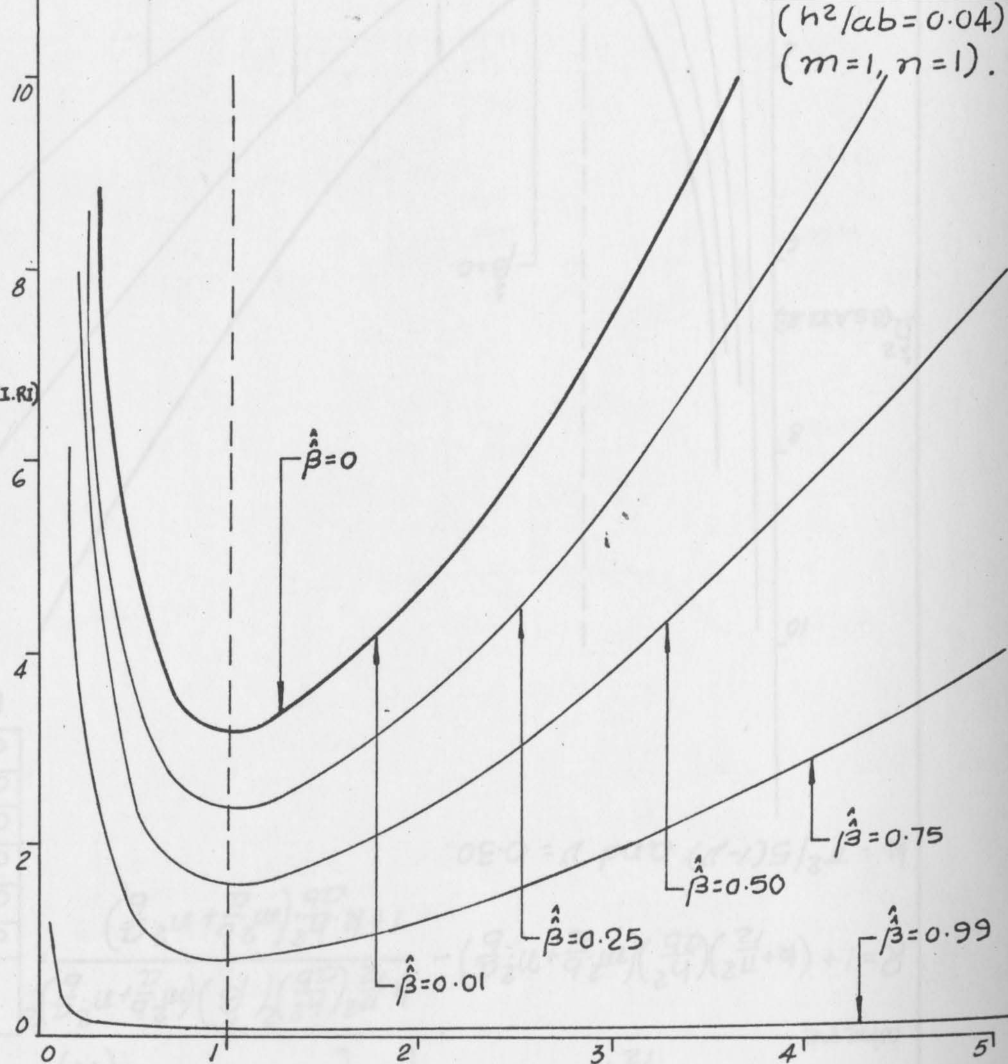



Figure 17. Lower Set Of Natural Frequency Including Shear And Rotary Inertia vs. a/b.

$$\hat{\hat{\Omega}}^2_{(B.S.A.TI.RI.)} = \frac{1.}{2\cdot\frac{\pi^2}{12}\cdot k}\left(\frac{ab}{h^2}\right)^2(R)\left[1-\left\{1-\frac{4\cdot\frac{\pi^2}{12}\cdot k\left(\frac{h^2}{ab}\right)^2(1-\hat{\hat{\beta}})\left(m^2\cdot\frac{b}{a}+n^2\frac{a}{b}\right)^2}{(R)^2}\right\}^{1/2}\right]$$

where

$$R = 1 + \left(k+\frac{\pi^2}{12}\right)\left(\frac{h^2}{ab}\right)\left(m^2\cdot\frac{b}{a}+n^2\cdot\frac{a}{b}\right) - \frac{k\cdot\frac{\pi^2}{12}\left(\frac{h^2}{ab}\right)^2(\hat{\hat{\beta}})\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{1+k\cdot\frac{h^2}{ab}\left(m^2\cdot\frac{b}{a}+n^2\cdot\frac{a}{b}\right)},$$

$k = \pi^2/5(1-\nu)$ and $\nu = 0.30$.

| $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2_{(B.S.A.TI.RI.)}$ Minimum |
|---|---|
| 0.0 | 2.970 |
| 0.01 | 2.940 |
| 0.25 | 2.228 |
| 0.50 | 1.485 |
| 0.75 | 0.743 |
| 0.99 | 0.030 |

$(h^2/ab = 0.05)$,
$(m = 1, n = 1)$.

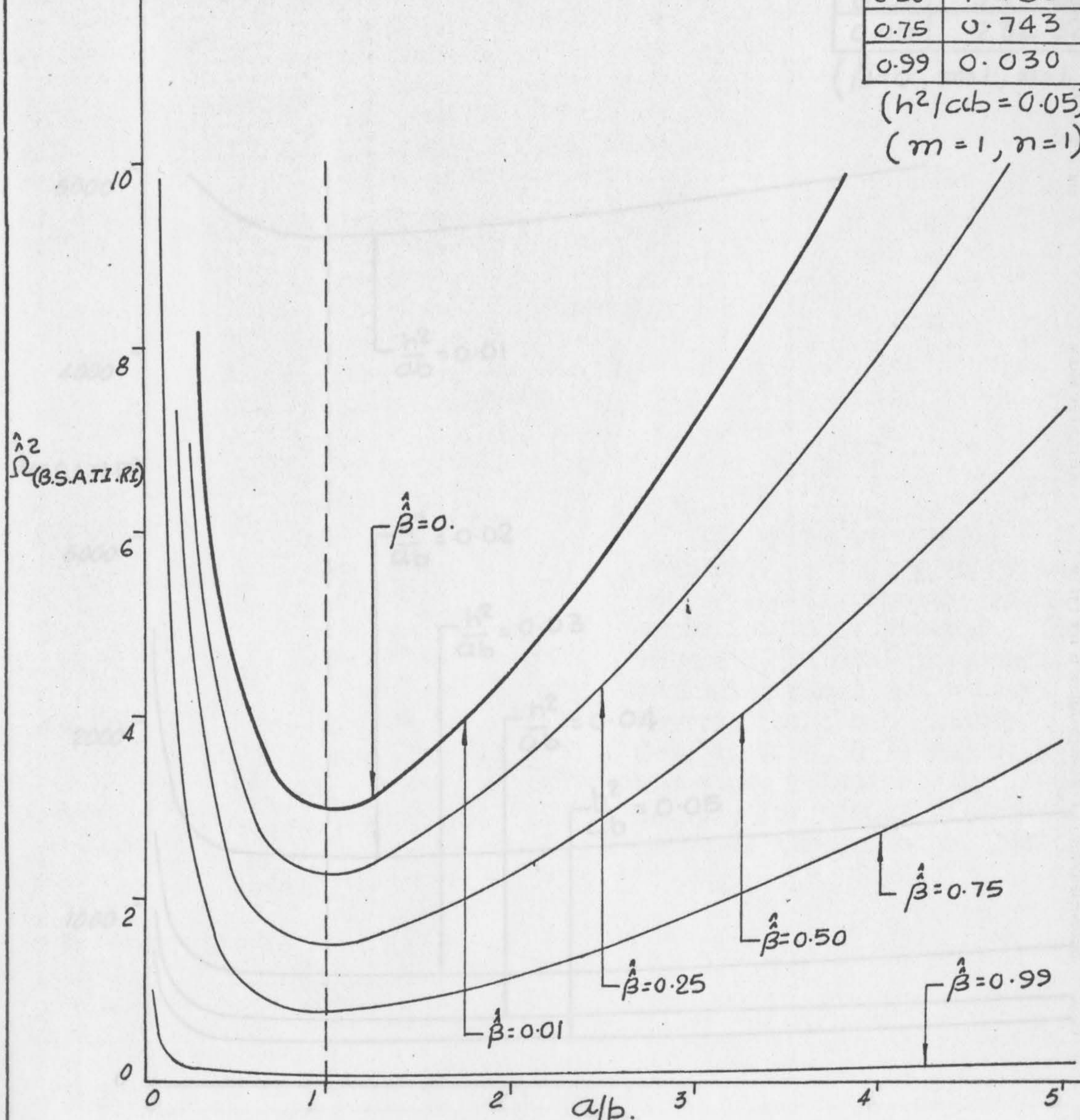



Figure 18. Lower Set Of Natural Frequency Including Shear and Rotary Inertia vs. a/b.

$$\hat{\hat{\Omega}}^2_{(B.S.A.T.I.R.I.)} = \frac{1}{2\cdot\frac{\pi^2}{12}\cdot k}\left(\frac{ab}{h^2}\right)^2 (R)\left[1+\left\{1-\frac{4\cdot\frac{\pi^2}{12}\cdot k\cdot\left(\frac{h^2}{ab}\right)^2(1-\hat{\hat{\beta}})\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{(R^2)}\right\}^{1/2}\right]$$

where

$$R = 1+\left(k+\frac{\pi^2}{12}\right)\frac{h^2}{ab}\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right) - \frac{k\cdot\frac{\pi^2}{12}\left(\frac{h^2}{ab}\right)^2\hat{\hat{\beta}}\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{1+k\cdot\frac{h^2}{ab}\cdot\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)}$$

$k = \pi^2/5(1-\nu)$, and $\nu = 0.30$.

| $h^2/ab$ | $\hat{\hat{\Omega}}^2_{(B.S.A.T.I.R.I.)}$ Minimum |
|---|---|
| 0.01 | 4622.08 |
| 0.02 | 1231.48 |
| 0.03 | 580.48 |
| 0.04 | 344.88 |
| 0.05 | 232.32 |

$(\hat{\hat{\beta}}=0,\ m=1,\ n=1)$.

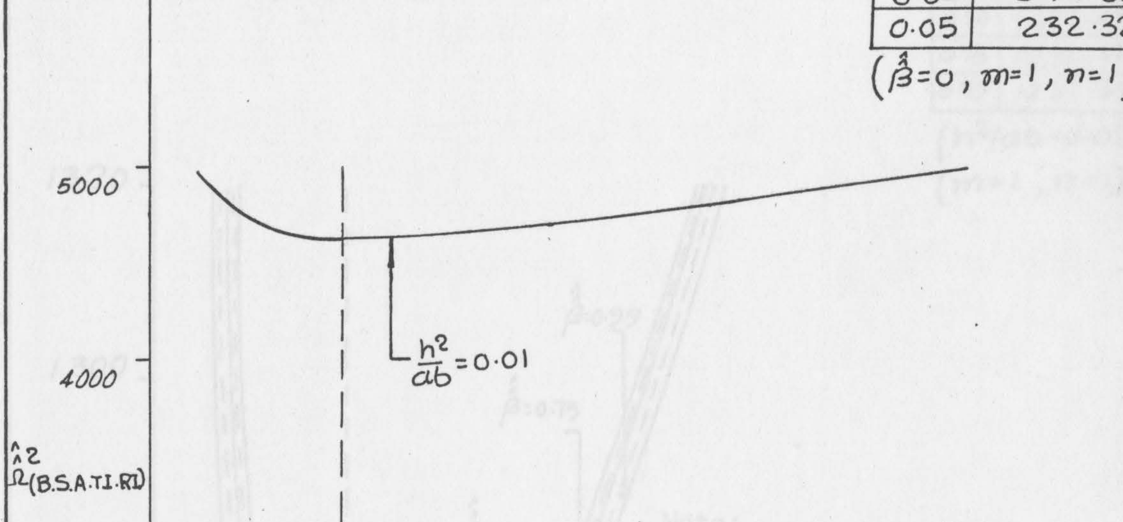



Figure 19. Higher Set Of Natural Frequency Including Shear And Rotary Inertia vs. a/b.

$$\hat{\hat{\Omega}}^2_{(B.S.A.T.I.R.I.)} = \frac{1}{2\cdot\frac{\pi^2}{12}\cdot k}\cdot\left(\frac{ab}{h^2}\right)^2 (R)\cdot\left[1+\left\{1-\frac{4\cdot\frac{\pi^2}{12}\cdot k\cdot\left(\frac{h^2}{ab}\right)^2(1-\hat{\hat{\beta}})\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{(R^2)}\right\}^{1/2}\right],$$

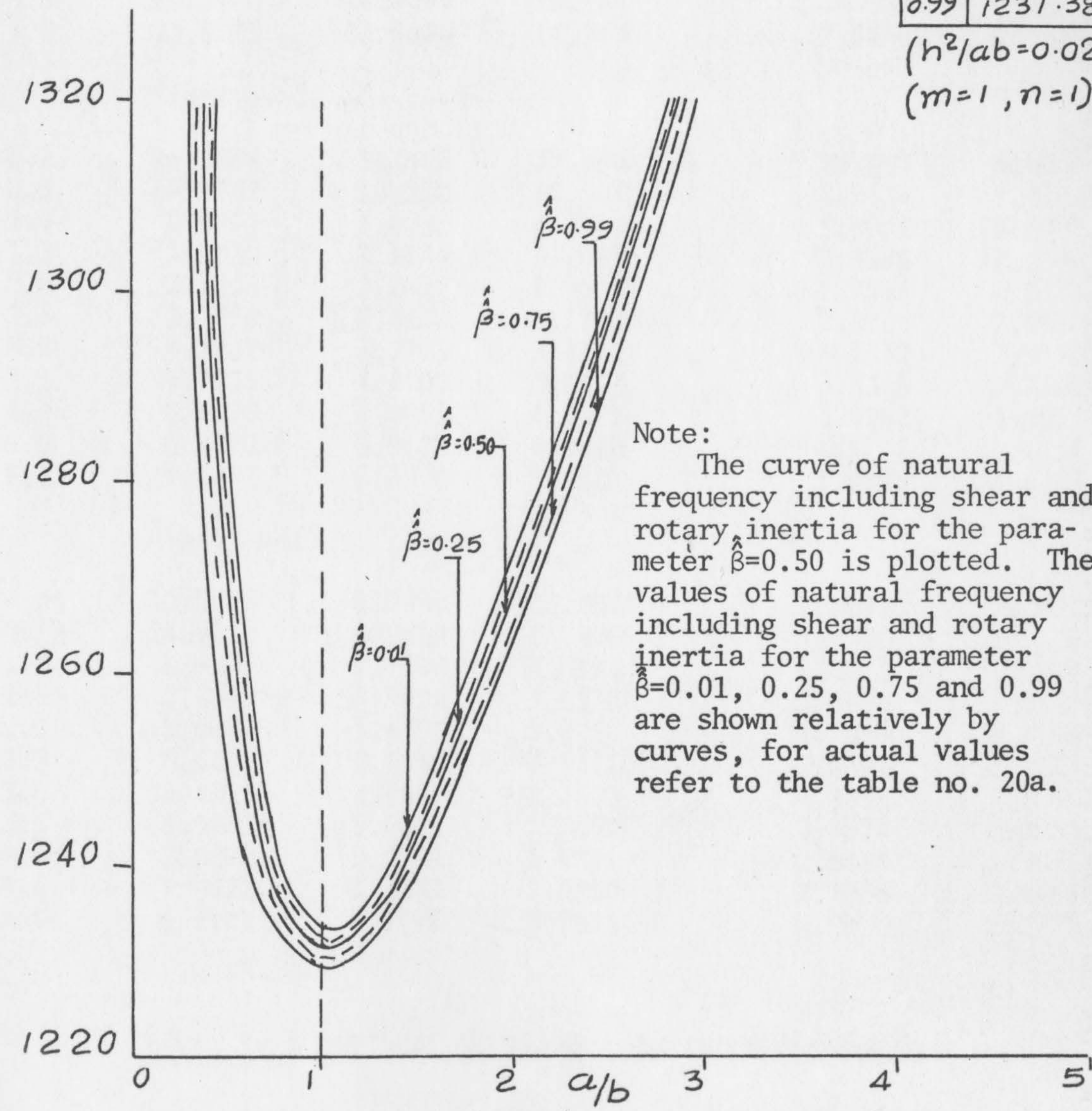
where

$$R = 1+\left(k+\frac{\pi^2}{12}\right)\frac{h^2}{ab}\cdot\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right) - \frac{k\cdot\frac{\pi^2}{12}\cdot\left(\frac{h^2}{ab}\right)^2\cdot\hat{\hat{\beta}}\cdot\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2}{1+k\cdot\frac{h^2}{ab}\cdot\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)},$$

$k = \pi^2/5(1-\nu)$, and $\nu = 0.30$.

| $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2_{(B.S.A.T.I.RI)}$ Minimum |
|---|---|
| 0.01 | 1231.48 |
| 0.25 | 1231.45 |
| 0.50 | 1231.43 |
| 0.75 | 1231.41 |
| 0.99 | 1231.38 |

$(h^2/ab = 0.02)$, $(m=1, n=1)$.




Note:
The curve of natural frequency including shear and rotary inertia for the parameter $\hat{\hat{\beta}}$=0.50 is plotted. The values of natural frequency including shear and rotary inertia for the parameter $\hat{\hat{\beta}}$=0.01, 0.25, 0.75 and 0.99 are shown relatively by curves, for actual values refer to the table no. 20a.

Figure 20. Higher Set Of Natural Frequency Including Shear And Rotary Inertia vs. a/b.

# APPENDIX B.

## TABULAR RESULTS

$(\hat{\hat{N}}_{B.S.A.})_{CR}$

| $a/b$ \ $h^2/ab$ | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|
| | (m=1, n=1) | | | | |
| 0.25 | 16.1295 | 14.570 | 13.286 | 12.210 | 11.295 |
| 0.5 | 5.8384 | 5.4777 | 5.159 | 4.8752 | 4.6211 |
| 1.0 | 3.7864 | 3.5945 | 3.4212 | 3.2637 | 3.1201 |
| 1.5 | 4.4241 | 4.1832 | 3.967 | 3.7725 | 3.5959 |
| 2.0 | 5.8384 | 5.4776 | 5.1589 | 4.8752 | 4.6211 |
| 2.5 | 7.7741 | 7.2278 | 6.753 | 6.3370 | 5.9692 |
| 3.0 | 10.1563 | 9.3527 | 8.6670 | 8.0749 | 7.5586 |
| 3.5 | 12.9490 | 11.8099 | 10.855 | 10.043 | 9.3439 |
| 4.0 | 16.1291 | 14.5699 | 13.286 | 12.2093 | 11.294 |
| 4.5 | 19.6785 | 17.6092 | 15.934 | 14.549 | 13.386 |
| 5.0 | 23.5815 | 20.9078 | 18.779 | 17.0431 | 15.601 |
| | (m=2, n=1) | | | | |
| 0.25 | 45.2708 | 34.446 | 27.800 | 23.303 | 20.058 |
| 0.5 | 14.5702 | 12.210 | 10.507 | 9.2214 | 8.2160 |
| 1.0 | 5.4777 | 4.8752 | 4.3922 | 3.9962 | 3.6657 |
| 1.5 | 3.8839 | 3.5144 | 3.2091 | 2.9526 | 2.7341 |
| 2.0 | 3.5945 | 3.2637 | 2.9887 | 2.7564 | 2.5576 |
| 2.5 | 3.7670 | 3.4132 | 3.1202 | 2.8736 | 2.6630 |
| 3.0 | 4.1832 | 3.7725 | 3.4351 | 3.1532 | 2.9140 |
| 3.5 | 4.7651 | 4.2707 | 3.8692 | 3.5368 | 3.2569 |
| 4.0 | 5.4776 | 4.8752 | 4.3921 | 3.9962 | 3.6657 |
| 4.5 | 6.3022 | 5.5678 | 4.9866 | 4.5153 | 4.1254 |
| 5.0 | 7.2277 | 6.3370 | 5.6417 | 5.0839 | 4.6265 |
| | (m=3, n=1) | | | | |
| 0.25 | 72.2016 | 47.959 | 35.904 | 28.692 | 23.892 |
| 0.50 | 24.9906 | 18.610 | 14.825 | 12.320 | 10.539 |
| 1.0 | 8.6671 | 7.1044 | 6.0191 | 5.2215 | 4.6105 |
| 1.50 | 5.1590 | 4.3922 | 3.8239 | 3.3858 | 3.0377 |
| 2.0 | 3.9673 | 3.4352 | 3.0289 | 2.7086 | 2.4495 |
| 2.5 | 3.5277 | 3.07623 | 2.7271 | 2.4492 | 2.2227 |
| 3.0 | 3.4212 | 2.9887 | 2.6533 | 2.3855 | 2.1669 |
| 3.5 | 3.4971 | 3.0511 | 2.7059 | 2.4310 | 2.2067 |
| 4.0 | 3.6899 | 3.2091 | 2.8391 | 2.5456 | 2.3071 |
| 4.5 | 3.9672 | 3.4351 | 3.0289 | 2.7086 | 2.4495 |
| 5.0 | 4.3111 | 3.7136 | 3.2615 | 2.9076 | 2.6229 |

Table 1a. CRITICAL BUCKLING LOAD INCLUDING SHEAR

$(\hat{\hat{N}}_{B.S.A.})_{CR.}$

| $\frac{a}{b}$ \ $h^2/ab$ | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 21.911 | 19.501 | 17.569 | 15.985 | 14.663 |
| 0.5 | 14.378 | 13.055 | 11.955 | 11.026 | 10.230 |
| 1.0 | 21.911 | 19.501 | 17.569 | 15.985 | 14.663 |
| 1.5 | 37.411 | 32.300 | 28.417 | 25.368 | 22.910 |
| 2.0 | 58.280 | 48.837 | 42.028 | 36.885 | 32.864 |
| 2.5 | 83.632 | 68.173 | 57.537 | 49.772 | 43.854 |
| 3.0 | 112.86 | 89.709 | 74.441 | 63.613 | 55.536 |
| 3.5 | 145.47 | 113.02 | 92.405 | 78.150 | 67.706 |
| 4.0 | 181.08 | 137.78 | 111.20 | 93.210 | 80.232 |
| 4.5 | 219.34 | 163.75 | 130.65 | 108.67 | 93.029 |
| 5.0 | 259.96 | 190.74 | 150.63 | 124.46 | 106.04 |
| | (m=2, n=2) | | | | |
| 0.25 | 48.838 | 36.885 | 29.633 | 24.764 | 21.269 |
| 0.5 | 19.501 | 15.985 | 13.543 | 11.748 | 10.374 |
| 1.0 | 13.055 | 11.026 | 9.5421 | 8.4106 | 7.5189 |
| 1.5 | 15.090 | 12.613 | 10.834 | 9.4954 | 8.4510 |
| 2.0 | 19.501 | 15.985 | 13.543 | 11.748 | 10.374 |
| 2.5 | 25.348 | 20.336 | 16.978 | 14.573 | 12.764 |
| 3.0 | 32.300 | 25.368 | 20.886 | 17.750 | 15.432 |
| 3.5 | 40.172 | 30.920 | 25.131 | 21.169 | 18.285 |
| 4.0 | 48.837 | 36.885 | 29.633 | 24.764 | 21.269 |
| 4.5 | 58.197 | 43.188 | 34.333 | 28.492 | 24.349 |
| 5.0 | 68.172 | 49.772 | 39.193 | 32.323 | 27.502 |
| | (m=3, n=2) | | | | |
| 0.25 | 74.442 | 49.279 | 36.830 | 29.402 | 24.468 |
| 0.5 | 28.418 | 20.886 | 16.510 | 13.650 | 11.635 |
| 1.0 | 13.741 | 10.834 | 8.9428 | 7.6136 | 6.6284 |
| 1.5 | 11.955 | 9.5421 | 7.9398 | 6.798 | 5.9437 |
| 2.0 | 12.836 | 10.183 | 8.4381 | 7.2039 | 6.2847 |
| 2.5 | 14.854 | 11.6303 | 9.5563 | 8.1100 | 7.0440 |
| 3.0 | 17.569 | 13.543 | 11.018 | 9.2869 | 8.0258 |
| 3.5 | 20.795 | 15.772 | 12.7034 | 10.634 | 9.1450 |
| 4.0 | 24.433 | 18.236 | 14.547 | 12.099 | 10.357 |
| 4.5 | 28.417 | 20.886 | 16.510 | 13.650 | 11.635 |
| 5.0 | 32.701 | 23.684 | 18.565 | 15.266 | 12.962 |

<u>Table 1b.</u> CRITICAL BUCKLING LOAD INCLUDING SHEAR

$(\hat{\hat{N}}_{B.S.A.})_{CR.}$

| a/b \ $h^2/ab$ | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|
| | (m=1, n=3) | | | | |
| 0.25 | 33.210 | 28.882 | 25.552 | 22.911 | 20.765 |
| 0.5 | 35.705 | 30.916 | 27.260 | 24.377 | 22.046 |
| 1.0 | 78.004 | 63.939 | 54.172 | 46.993 | 41.495 |
| 1.5 | 143.40 | 111.56 | 91.288 | 77.251 | 66.955 |
| 2.0 | 224.91 | 167.49 | 133.43 | 110.88 | 94.847 |
| 2.5 | 318.64 | 228.85 | 178.53 | 146.36 | 124.01 |
| 3.0 | 421.91 | 293.95 | 225.55 | 182.97 | 153.92 |
| 3.5 | 532.77 | 361.78 | 273.87 | 220.34 | 184.31 |
| 4.0 | 649.80 | 431.62 | 323.13 | 258.22 | 215.03 |
| 4.5 | 771.89 | 503.02 | 373.07 | 296.48 | 245.98 |
| 5.0 | 898.18 | 575.63 | 423.53 | 335.01 | 277.09 |
| | (m=2, n=3) | | | | |
| 0.25 | 54.974 | 41.032 | 32.732 | 27.224 | 23.303 |
| 0.5 | 28.882 | 22.911 | 18.986 | 16.209 | 14.141 |
| 1.0 | 30.92 | 24.377 | 20.121 | 17.131 | 14.914 |
| 1.5 | 44.880 | 34.178 | 27.597 | 23.141 | 19.924 |
| 2.0 | 63.939 | 46.993 | 37.148 | 30.713 | 26.179 |
| 2.5 | 86.450 | 61.547 | 47.783 | 39.049 | 33.015 |
| 3.0 | 111.56 | 77.250 | 59.081 | 47.831 | 40.180 |
| 3.5 | 138.71 | 93.765 | 70.819 | 56.896 | 47.548 |
| 4.0 | 167.49 | 110.88 | 82.866 | 66.154 | 55.052 |
| 4.5 | 197.61 | 128.44 | 95.138 | 75.550 | 62.650 |
| 5.0 | 228.84 | 146.36 | 107.58 | 85.047 | 70.318 |
| | (m=3, n=3) | | | | |
| 0.25 | 78.207 | 51.489 | 38.378 | 30.589 | 25.428 |
| 0.5 | 34.415 | 24.791 | 19.374 | 15.899 | 13.482 |
| 1.0 | 23.879 | 17.865 | 14.270 | 11.880 | 10.176 |
| 1.5 | 27.260 | 20.121 | 15.946 | 13.205 | 11.268 |
| 2.0 | 34.414 | 24.791 | 19.374 | 15.899 | 13.481 |
| 2.5 | 43.600 | 30.619 | 23.594 | 19.191 | 16.173 |
| 3.0 | 54.171 | 37.148 | 28.265 | 22.811 | 19.121 |
| 3.5 | 65.781 | 44.149 | 33.223 | 26.632 | 22.224 |
| 4.0 | 78.206 | 51.488 | 38.377 | 30.588 | 25.428 |
| 4.5 | 91.287 | 59.081 | 43.673 | 34.639 | 28.702 |
| 5.0 | 104.91 | 66.867 | 49.072 | 38.758 | 32.027 |

Table 1c. CRITICAL BUCKLING LOAD INCLUDING SHEAR

| m | n | a/b \ $h^2/ab$. | $(\hat{\hat{N}}$B.A.S.$)$ CR Minimum 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 3.786 | 3.595 | 3.421 | 3.264 | 3.120 |
| 2 | 1 | 2.0 | 3.595 | 3.264 | 2.989 | 2.756 | 2.558 |
| 3 | 1 | 3.0 | 3.421 | 2.989 | 2.653 | 2.386 | 2.167 |
| 1 | 2 | 0.5 | 14.378 | 13.055 | 11.955 | 11.026 | 10.230 |
| 2 | 2 | 1.0 | 13.055 | 11.026 | 9.542 | 8.4106 | 7.519 |
| 3 | 2 | 1.5 | 11.955 | 9.542 | 7.940 | 6.798 | 5.944 |
| 1 | 3 | 0.33 | 30.790 | 26.898 | 23.879 | 21.470 | 19.502 |
| 2 | 3 | 0.67 | 26.898 | 21.470 | 17.865 | 15.296 | 13.373 |
| 3 | 3 | 1.0 | 23.879 | 17.865 | 14.270 | 11.880 | 10.176 |

TABLE 1d. SUMMARY SHEET FOR CRITICAL BUCKLING-LOAD INCLUDING SHEAR.

$(\hat{\hat{N}}_{B.A.})_{CR.}$

| a/b \ m | 1 | 2 | 3 |
|---|---|---|---|
| (n=1) | | | |
| 0.25 | 18.063 | 66.015 | 146.007 |
| 0.5 | 6.250 | 18.063 | 38.028 |
| 1.0 | 4.000 | 6.250 | 11.111 |
| 1.5 | 4.694 | 4.340 | 6.250 |
| 2.0 | 6.250 | 4.000 | 4.695 |
| 2.5 | 8.410 | 4.203 | 4.135 |
| 3.0 | 11.111 | 4.694 | 4.000 |
| 3.5 | 14.331 | 5.389 | 4.096 |
| 4.0 | 18.062 | 6.250 | 4.340 |
| 4.5 | 22.299 | 7.260 | 4.694 |
| 5.0 | 27.039 | 8.410 | 5.138 |
| (n=2) | | | |
| 0.25 | 25.000 | 72.250 | 152.111 |
| 0.5 | 16.000 | 25.000 | 44.444 |
| 1.0 | 25.000 | 16.000 | 18.778 |
| 1.5 | 44.444 | 18.778 | 16.000 |
| 2.0 | 72.249 | 25.000 | 17.361 |
| 2.5 | 108.158 | 33.640 | 20.551 |
| 3.0 | 152.108 | 44.444 | 25.000 |
| 3.5 | 204.077 | 57.325 | 30.512 |
| 4.0 | 264.056 | 72.248 | 37.006 |
| 4.5 | 332.041 | 89.195 | 44.444 |
| 5.0 | 408.029 | 108.157 | 52.803 |
| (n=3) | | | |
| 0.25 | 39.063 | 83.265 | 162.563 |
| 0.5 | 42.250 | 39.063 | 56.250 |
| 1.0 | 100.000 | 42.250 | 36.000 |
| 1.5 | 200.692 | 65.340 | 42.250 |
| 2.0 | 342.244 | 99.999 | 56.249 |
| 2.5 | 524.399 | 145.200 | 75.689 |
| 3.0 | 747.095 | 200.690 | 99.998 |
| 3.5 | 1010.308 | 266.383 | 128.982 |
| 4.0 | 1314.032 | 342.242 | 162.559 |
| 4.5 | 1658.258 | 428.250 | 200.690 |
| 5.0 | 2042.989 | 525.397 | 243.354 |

TABLE 1e. CRITICAL BUCKLING LOAD EXCLUDING SHEAR.

| a/b \ h²/ab ($\hat{N}$) | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|
| (m=1, n=1) | | | | | |
| 0.25 | 0.8930 | 0.8067 | 0.7355 | 0.6760 | 0.6253 |
| 0.5 | 0.9341 | 0.8764 | 0.8254 | 0.7800 | 0.7394 |
| 1.0 | 0.9466 | 0.8986 | 0.8553 | 0.8159 | 0.7800 |
| 1.5 | 0.9424 | 0.8911 | 0.8451 | 0.8036 | 0.7660 |
| 2.0 | 0.9341 | 0.8764 | 0.8254 | 0.7800 | 0.7394 |
| 2.5 | 0.9244 | 0.8594 | 0.8030 | 0.7535 | 0.7098 |
| 3.0 | 0.9141 | 0.8418 | 0.7800 | 0.7268 | 0.6803 |
| 3.5 | 0.9035 | 0.8241 | 0.7574 | 0.7008 | 0.6520 |
| 4.0 | 0.8930 | 0.8067 | 0.7355 | 0.6760 | 0.6523 |
| 4.5 | 0.8825 | 0.7897 | 0.7146 | 0.6525 | 0.6003 |
| 5.0 | 0.8721 | 0.7732 | 0.6945 | 0.6303 | 0.5770 |
| (m=2, n=1) | | | | | |
| 0.25 | 0.6858 | 0.5218 | 0.4211 | 0.3530 | 0.3038 |
| 0.5 | 0.8067 | 0.6760 | 0.5817 | 0.5105 | 0.4549 |
| 1.0 | 0.8764 | 0.7800 | 0.7027 | 0.6394 | 0.5865 |
| 1.5 | 0.8949 | 0.8097 | 0.7394 | 0.6803 | 0.6299 |
| 2.0 | 0.8986 | 0.8159 | 0.7472 | 0.6891 | 0.6394 |
| 2.5 | 0.8964 | 0.8122 | 0.7425 | 0.6838 | 0.6337 |
| 3.0 | 0.8911 | 0.8036 | 0.7318 | 0.6717 | 0.6207 |
| 3.5 | 0.8842 | 0.7925 | 0.7180 | 0.6563 | 0.6044 |
| 4.0 | 0.8764 | 0.7800 | 0.7027 | 0.6394 | 0.5865 |
| 4.5 | 0.8681 | 0.7669 | 0.6869 | 0.6220 | 0.5682 |
| 5.0 | 0.8594 | 0.7535 | 0.6708 | 0.6045 | 0.5501 |
| (m=3, n=1) | | | | | |
| 0.25 | 0.4945 | 0.3285 | 0.2459 | 0.1965 | 0.1636 |
| 0.5 | 0.6572 | 0.4849 | 0.3899 | 0.3240 | 0.2771 |
| 1.0 | 0.7800 | 0.6394 | 0.5417 | 0.4699 | 0.4149 |
| 1.5 | 0.8254 | 0.7027 | 0.6118 | 0.5417 | 0.4860 |
| 2.0 | 0.8451 | 0.7317 | 0.6452 | 0.5770 | 0.5218 |
| 2.5 | 0.8532 | 0.7440 | 0.6596 | 0.5924 | 0.5376 |
| 3.0 | 0.8553 | 0.7472 | 0.6633 | 0.5964 | 0.5417 |
| 3.5 | 0.8538 | 0.7449 | 0.6607 | 0.5935 | 0.5388 |
| 4.0 | 0.8502 | 0.7394 | 0.6541 | 0.5865 | 0.5316 |
| 4.5 | 0.8451 | 0.7318 | 0.6452 | 0.5770 | 0.5218 |
| 5.0 | 0.8391 | 0.7228 | 0.6348 | 0.5659 | 0.5105 |

Table 2a. CRITICAL BUCKLING LOAD RATIO

$\hat{\hat{N}}$

| $\frac{a}{b}$ \ $h^2/ab$ | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|
| (m=1, n=2) | | | | | |
| 0.25 | 0.8764 | 0.7800 | 0.7027 | 0.6394 | 0.5865 |
| 0.5 | 0.8986 | 0.8159 | 0.7472 | 0.6891 | 0.6394 |
| 1.0 | 0.8764 | 0.7800 | 0.7027 | 0.6394 | 0.5865 |
| 1.5 | 0.8418 | 0.7268 | 0.6394 | 0.5708 | 0.5155 |
| 2.0 | 0.8067 | 0.6760 | 0.5817 | 0.5105 | 0.4549 |
| 2.5 | 0.7732 | 0.6303 | 0.5320 | 0.4602 | 0.4055 |
| 3.0 | 0.7420 | 0.5898 | 0.4894 | 0.4182 | 0.3651 |
| 3.5 | 0.7128 | 0.5538 | 0.4528 | 0.3829 | 0.3318 |
| 4.0 | 0.6858 | 0.5218 | 0.4211 | 0.3530 | 0.3038 |
| 4.5 | 0.6606 | 0.4932 | 0.3935 | 0.3273 | 0.2802 |
| 5.0 | 0.6371 | 0.4675 | 0.3692 | 0.3050 | 0.2599 |
| (m=2, n=2) | | | | | |
| 0.25 | 0.6760 | 0.5105 | 0.4101 | 0.3428 | 0.2944 |
| 0.5 | 0.7800 | 0.6394 | 0.5417 | 0.4699 | 0.4149 |
| 1.0 | 0.8159 | 0.6891 | 0.5964 | 0.5257 | 0.4699 |
| 1.5 | 0.8036 | 0.6717 | 0.5770 | 0.5057 | 0.4501 |
| 2.0 | 0.7800 | 0.6394 | 0.5417 | 0.4699 | 0.4149 |
| 2.5 | 0.7535 | 0.6045 | 0.5047 | 0.4332 | 0.3794 |
| 3.0 | 0.7268 | 0.5708 | 0.4699 | 0.3994 | 0.3472 |
| 3.5 | 0.7008 | 0.5394 | 0.4384 | 0.3693 | 0.3190 |
| 4.0 | 0.6760 | 0.5105 | 0.4101 | 0.3428 | 0.2944 |
| 4.5 | 0.6525 | 0.4842 | 0.3849 | 0.3194 | 0.2730 |
| 5.0 | 0.6303 | 0.4602 | 0.3624 | 0.2989 | 0.2543 |
| (m=3, n=2) | | | | | |
| 0.25 | 0.4894 | 0.3240 | 0.2421 | 0.1933 | 0.1609 |
| 0.5 | 0.6394 | 0.4699 | 0.3715 | 0.3071 | 0.2618 |
| 1.0 | 0.7318 | 0.5770 | 0.4762 | 0.4055 | 0.3530 |
| 1.5 | 0.7472 | 0.5964 | 0.4962 | 0.4249 | 0.3715 |
| 2.0 | 0.7394 | 0.5865 | 0.4860 | 0.4149 | 0.3620 |
| 2.5 | 0.7228 | 0.5659 | 0.4650 | 0.3946 | 0.3428 |
| 3.0 | 0.7027 | 0.5417 | 0.4407 | 0.3715 | 0.3210 |
| 3.5 | 0.6815 | 0.5169 | 0.4163 | 0.3485 | 0.2997 |
| 4.0 | 0.6602 | 0.4928 | 0.3931 | 0.3270 | 0.2799 |
| 4.5 | 0.6394 | 0.4699 | 0.3715 | 0.3071 | 0.2618 |
| 5.0 | 0.6193 | 0.4485 | 0.3516 | 0.2891 | 0.2455 |

Table 2b. CRITICAL BUCKLING LOAD RATIO

| $a/b$ \ $h^2/ab$ | $\hat{N}$ 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|
| (m=1, n=3) | | | | | |
| 0.25 | 0.8502 | 0.7394 | 0.6541 | 0.5865 | 0.5316 |
| 0.5 | 0.8451 | 0.7318 | 0.6452 | 0.5770 | 0.5218 |
| 1.0 | 0.7800 | 0.6394 | 0.5417 | 0.4699 | 0.4149 |
| 1.5 | 0.7145 | 0.5559 | 0.4549 | 0.3849 | 0.3336 |
| 2.0 | 0.6572 | 0.4894 | 0.3899 | 0.3240 | 0.2771 |
| 2.5 | 0.6076 | 0.4364 | 0.3405 | 0.2791 | 0.2365 |
| 3.0 | 0.5647 | 0.3935 | 0.3019 | 0.2449 | 0.2060 |
| 3.5 | 0.5273 | 0.3581 | 0.2711 | 0.2181 | 0.1824 |
| 4.0 | 0.4945 | 0.3285 | 0.2459 | 0.1965 | 0.1636 |
| 4.5 | 0.4655 | 0.3033 | 0.2250 | 0.1788 | 0.1483 |
| 5.0 | 0.4396 | 0.2818 | 0.2073 | 0.1640 | 0.1356 |
| (m=2, n=3) | | | | | |
| 0.25 | 0.6602 | 0.4928 | 0.3931 | 0.3270 | 0.2799 |
| 0.5 | 0.7394 | 0.5865 | 0.4860 | 0.4149 | 0.3620 |
| 1.0 | 0.7318 | 0.5770 | 0.4762 | 0.4055 | 0.3530 |
| 1.5 | 0.6869 | 0.5231 | 0.4224 | 0.3542 | 0.3049 |
| 2.0 | 0.6394 | 0.4699 | 0.3715 | 0.3071 | 0.2618 |
| 2.5 | 0.5954 | 0.4239 | 0.3291 | 0.2689 | 0.2274 |
| 3.0 | 0.5559 | 0.3849 | 0.2944 | 0.2383 | 0.2002 |
| 3.5 | 0.5207 | 0.3520 | 0.2659 | 0.2136 | 0.1785 |
| 4.0 | 0.4894 | 0.3240 | 0.2421 | 0.1933 | 0.1609 |
| 4.5 | 0.4614 | 0.2999 | 0.2222 | 0.1764 | 0.1463 |
| 5.0 | 0.4364 | 0.2791 | 0.2051 | 0.1622 | 0.1341 |
| (m=3, n=3) | | | | | |
| 0.25 | 0.4811 | 0.3167 | 0.2361 | 0.1882 | 0.1564 |
| 0.5 | 0.6118 | 0.4407 | 0.3444 | 0.2827 | 0.2397 |
| 1.0 | 0.6633 | 0.4962 | 0.3964 | 0.3300 | 0.2827 |
| 1.5 | 0.6452 | 0.4762 | 0.3774 | 0.3125 | 0.2667 |
| 2.0 | 0.6118 | 0.4407 | 0.3444 | 0.2827 | 0.2397 |
| 2.5 | 0.5760 | 0.4045 | 0.3117 | 0.2536 | 0.2137 |
| 3.0 | 0.5417 | 0.3715 | 0.2827 | 0.2281 | 0.1912 |
| 3.5 | 0.5100 | 0.3423 | 0.2576 | 0.2065 | 0.1723 |
| 4.0 | 0.4811 | 0.3167 | 0.2361 | 0.1882 | 0.1564 |
| 4.5 | 0.4549 | 0.2944 | 0.2176 | 0.1726 | 0.1430 |
| 5.0 | 0.4311 | 0.2748 | 0.2016 | 0.1593 | 0.1316 |

Table 2c. CRITICAL BUCKLING LOAD RATIO

| m | n | a/b \ $h^2/ab$ | $\hat{N}$ Maximum 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 0.9466 | 0.8986 | 0.8553 | 0.8159 | 0.7800 |
| 2 | 1 | 2.0 | 0.8986 | 0.8159 | 0.7472 | 0.6891 | 0.6394 |
| 3 | 1 | 3.0 | 0.8538 | 0.7449 | 0.6607 | 0.5935 | 0.5388 |
| 1 | 2 | 0.5 | 0.8986 | 0.8159 | 0.7472 | 0.6891 | 0.6394 |
| 2 | 2 | 1.0 | 0.8159 | 0.6891 | 0.5964 | 0.5257 | 0.4699 |
| 3 | 2 | 1.5 | 0.7472 | 0.5964 | 0.4962 | 0.4249 | 0.3715 |
| 1 | 3 | 0.33 | 0.8553 | 0.7472 | 0.6633 | 0.5964 | 0.5417 |
| 2 | 3 | 0.67 | 0.7472 | 0.5964 | 0.4962 | 0.4249 | 0.3715 |
| 3 | 3 | 1.0 | 0.6633 | 0.4962 | 0.3964 | 0.3300 | 0.2827 |

TABLE 2d. SUMMARY SHEET FOR CRITICAL BUCKLING-LOAD RATIO.

| a/b \ $\hat{\beta}$ | 0.01 | 0.25 | $\hat{\Omega}^2$ (B.S.A. TI) 0.50 | 0.75 | ($h^2/ab=0.01$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=1) | | | | |
| 0.25 | 15.968 | 12.097 | 8.065 | 4.0324 | 0.1613 |
| 0.5 | 5.7800 | 4.3788 | 2.919 | 1.460 | 0.0584 |
| 1.0 | 3.7486 | 2.8398 | 1.893 | 0.947 | 0.0379 |
| 1.5 | 4.3799 | 3.3181 | 2.212 | 1.106 | 0.0442 |
| 2.0 | 5.7800 | 4.3788 | 2.919 | 1.460 | 0.0584 |
| 2.5 | 7.6964 | 5.8306 | 3.887 | 1.944 | 0.0777 |
| 3.0 | 10.055 | 7.6172 | 5.078 | 2.539 | 0.1016 |
| 3.5 | 12.820 | 9.7118 | 6.475 | 3.237 | 0.1295 |
| 4.0 | 15.968 | 12.097 | 8.065 | 4.032 | 0.1613 |
| 4.5 | 19.482 | 14.759 | 9.839 | 4.920 | 0.1968 |
| 5.0 | 23.346 | 17.686 | 11.791 | 5.895 | 0.2358 |
| | (m=2, n=1) | | | | |
| 0.25 | 179.27 | 135.81 | 90.542 | 45.271 | 1.8108 |
| 0.5 | 57.698 | 43.711 | 29.140 | 14.570 | 0.5828 |
| 1.0 | 21.692 | 16.433 | 10.955 | 5.478 | 0.2191 |
| 1.5 | 15.380 | 11.652 | 7.768 | 3.884 | 0.1554 |
| 2.0 | 14.234 | 10.784 | 7.189 | 3.595 | 0.1438 |
| 2.5 | 14.917 | 11.301 | 7.534 | 3.767 | 0.1507 |
| 3.0 | 16.566 | 12.550 | 8.367 | 4.183 | 0.1673 |
| 3.5 | 18.870 | 14.295 | 9.530 | 4.765 | 0.1906 |
| 4.0 | 21.691 | 16.433 | 10.955 | 5.478 | 0.2191 |
| 4.5 | 24.957 | 18.907 | 12.605 | 6.302 | 0.2521 |
| 5.0 | 28.622 | 21.683 | 14.456 | 7.228 | 0.2891 |
| | (m=3, n=1) | | | | |
| 0.25 | 643.32 | 487.36 | 324.907 | 162.454 | 6.4982 |
| 0.5 | 222.66 | 168.69 | 112.458 | 56.229 | 2.2492 |
| 1.0 | 77.224 | 58.503 | 39.002 | 19.501 | 0.7800 |
| 1.5 | 45.966 | 34.823 | 23.215 | 11.608 | 0.4643 |
| 2.0 | 35.349 | 26.779 | 17.853 | 8.926 | 0.3571 |
| 2.5 | 31.431 | 23.812 | 15.874 | 7.937 | 0.3175 |
| 3.0 | 30.483 | 23.093 | 15.395 | 7.698 | 0.3079 |
| 3.5 | 31.159 | 23.605 | 15.737 | 7.868 | 0.3147 |
| 4.0 | 32.877 | 24.907 | 16.605 | 8.302 | 0.3321 |
| 4.5 | 35.348 | 26.779 | 17.853 | 8.926 | 0.3571 |
| 5.0 | 38.412 | 29.100 | 19.400 | 9.700 | 0.3880 |

TABLE 3a. NATURAL FREQUENCY INCLUDING SHEAR.

| a/b \ $\hat{\hat{\beta}}$ | 0.01 | 0.025 | $\hat{\hat{\Omega}}^2$ (B.S.A. TI) 0.50 | 0.75 | ($h^2/ab=0.01$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 21.692 | 16.433 | 10.955 | 5.478 | 0.2191 |
| 0.5 | 14.234 | 10.784 | 7.189 | 3.595 | 0.1438 |
| 1.0 | 21.692 | 16.433 | 10.955 | 5.478 | 0.2191 |
| 1.5 | 37.037 | 28.058 | 18.706 | 9.353 | 0.3741 |
| 2.0 | 57.697 | 43.710 | 29.140 | 14.570 | 0.5828 |
| 2.5 | 82.795 | 62.724 | 41.816 | 20.908 | 0.8363 |
| 3.0 | 111.73 | 84.643 | 56.429 | 28.215 | 1.1286 |
| 3.5 | 144.02 | 109.11 | 72.737 | 36.369 | 1.4548 |
| 4.0 | 179.27 | 135.81 | 90.540 | 45.270 | 1.8108 |
| 4.5 | 217.14 | 164.50 | 109.67 | 54.834 | 2.1934 |
| 5.0 | 257.36 | 194.97 | 129.978 | 64.989 | 2.5996 |
| | (m=2, n=2) | | | | |
| 0.25 | 193.40 | 146.51 | 97.676 | 48.838 | 1.9535 |
| 0.5 | 77.224 | 58.503 | 39.002 | 19.501 | 0.7800 |
| 1.0 | 51.698 | 39.165 | 26.110 | 13.055 | 0.5222 |
| 1.5 | 59.756 | 45.270 | 30.180 | 15.090 | 0.6036 |
| 2.0 | 77.223 | 58.502 | 39.002 | 19.501 | 0.7800 |
| 2.5 | 100.38 | 76.044 | 50.696 | 25.348 | 1.0139 |
| 3.0 | 127.91 | 96.899 | 64.599 | 32.300 | 1.2920 |
| 3.5 | 159.08 | 120.52 | 80.344 | 40.172 | 1.4769 |
| 4.0 | 193.39 | 146.51 | 97.674 | 48.837 | 1.9535 |
| 4.5 | 230.46 | 174.59 | 116.394 | 58.197 | 2.3279 |
| 5.0 | 269.96 | 204.52 | 136.344 | 68.172 | 2.7269 |
| | (m=3, n=2) | | | | |
| 0.25 | 663.27 | 502.48 | 334.987 | 167.494 | 6.6998 |
| 0.5 | 253.20 | 191.82 | 127.879 | 63.939 | 2.5576 |
| 1.0 | 122.43 | 92.749 | 61.833 | 30.916 | 1.2367 |
| 1.5 | 106.52 | 80.694 | 53.796 | 26.898 | 1.0759 |
| 2.0 | 144.37 | 86.646 | 57.764 | 28.882 | 1.1553 |
| 2.5 | 132.35 | 100.27 | 66.844 | 33.422 | 1.3369 |
| 3.0 | 156.54 | 118.59 | 79.058 | 39.529 | 1.5812 |
| 3.5 | 185.28 | 140.36 | 93.576 | 46.788 | 1.8715 |
| 4.0 | 217.69 | 164.92 | 109.947 | 54.973 | 2.1989 |
| 4.5 | 253.20 | 191.81 | 127.877 | 63.938 | 2.5575 |
| 5.0 | 291.37 | 220.73 | 147.154 | 73.577 | 2.9431 |

TABLE 3b. NATURAL FREQUENCY INCLUDING SHEAR.

| a/b \ $\hat{\beta}$ | 0.01 | 0.25 | $\hat{\Omega}^2$ (B.S.A. TI) 0.50 | 0.75 | ($h^2/ab=0.01$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=3) | | | | |
| 0.25 | 32.878 | 24.907 | 16.605 | 8.3024 | 0.3321 |
| 0.5 | 35.348 | 26.779 | 17.853 | 8.9264 | 0.3571 |
| 1.0 | 77.224 | 58.503 | 39.002 | 19.501 | 0.7800 |
| 1.5 | 141.97 | 107.55 | 71.702 | 35.851 | 1.4341 |
| 2.0 | 222.66 | 168.68 | 112.456 | 56.228 | 2.2491 |
| 2.5 | 315.45 | 238.98 | 159.319 | 79.660 | 3.1864 |
| 3.0 | 417.69 | 316.43 | 210.953 | 105.476 | 4.2191 |
| 3.5 | 527.45 | 399.58 | 266.387 | 133.194 | 5.3278 |
| 4.0 | 643.30 | 487.35 | 324.901 | 162.451 | 6.4981 |
| 4.5 | 764.17 | 578.92 | 385.944 | 192.972 | 7.7189 |
| 5.0 | 889.20 | 673.64 | 449.092 | 224.546 | 8.9819 |
| | (m=2, n=3) | | | | |
| 0.25 | 217.70 | 164.92 | 109.948 | 54.974 | 2.1990 |
| 0.5 | 114.37 | 86.646 | 57.764 | 28.882 | 1.1553 |
| 1.0 | 122.43 | 92.749 | 61.833 | 30.916 | 1.2367 |
| 1.5 | 177.72 | 134.639 | 89.760 | 44.880 | 1.7952 |
| 2.0 | 253.20 | 191.816 | 127.878 | 63.939 | 2.5576 |
| 2.5 | 342.34 | 259.348 | 172.899 | 86.450 | 3.4580 |
| 3.0 | 441.77 | 334.677 | 223.118 | 111.559 | 4.4624 |
| 3.5 | 549.28 | 416.118 | 277.412 | 138.706 | 5.5483 |
| 4.0 | 663.26 | 502.472 | 334.981 | 167.491 | 6.6997 |
| 4.5 | 782.55 | 592.839 | 395.226 | 197.613 | 7.9046 |
| 5.0 | 906.23 | 686.534 | 457.690 | 228.845 | 9.1538 |
| | (m=3, n=3) | | | | |
| 0.25 | 696.83 | 527.899 | 351.933 | 175.966 | 7.0387 |
| 0.5 | 306.63 | 232.899 | 154.866 | 77.433 | 3.0973 |
| 1.0 | 212.76 | 161.185 | 107.457 | 53.728 | 2.1491 |
| 1.5 | 242.89 | 184.005 | 122.670 | 61.335 | 2.4534 |
| 2.0 | 306.63 | 232.297 | 154.865 | 77.432 | 3.0973 |
| 2.5 | 388.47 | 294.299 | 196.199 | 98.100 | 3.9240 |
| 3.0 | 482.67 | 365.657 | 243.771 | 121.886 | 4.8754 |
| 3.5 | 586.11 | 444.024 | 296.016 | 148.008 | 5.9204 |
| 4.0 | 696.82 | 527.891 | 351.927 | 175.964 | 7.0386 |
| 4.5 | 813.37 | 616.189 | 410.793 | 205.396 | 8.2159 |
| 5.0 | 934.73 | 708.128 | 472.085 | 236.043 | 9.4417 |

TABLE 3c. NATURAL FREQUENCY INCLUDING SHEAR.

$\hat{\Omega}^2$(B.S.A. TI) $(h^2/ab=0.01)$, $(\hat{\beta}=0)$

| a/b \ n | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 16.130 | 21.911 | 33.210 |
| 0.5 | 5.838 | 14.378 | 35.705 |
| 1.0 | 3.786 | 21.911 | 78.004 |
| 1.5 | 4.424 | 37.411 | 143.405 |
| 2.0 | 5.838 | 58.280 | 224.913 |
| 2.5 | 7.774 | 83.632 | 318.638 |
| 3.0 | 10.156 | 112.858 | 421.906 |
| 3.5 | 12.949 | 145.475 | 532.774 |
| 4.0 | 16.129 | 181.080 | 649.803 |
| 4.5 | 19.679 | 219.337 | 771.889 |
| 5.0 | 23.582 | 259.956 | 898.183 |
| (m=2) | | | |
| 0.25 | 181.083 | 195.351 | 219.896 |
| 0.5 | 58.281 | 78.004 | 115.528 |
| 1.0 | 21.911 | 52.220 | 123.666 |
| 1.5 | 15.536 | 60.360 | 179.519 |
| 2.0 | 14.378 | 78.003 | 255.755 |
| 2.5 | 15.068 | 101.392 | 345.798 |
| 3.0 | 16.733 | 129.198 | 446.236 |
| 3.5 | 19.060 | 160.687 | 554.825 |
| 4.0 | 21.910 | 195.348 | 669.962 |
| 4.5 | 25.209 | 232.789 | 790.452 |
| 5.0 | 28.911 | 272.689 | 915.379 |
| (m=3) | | | |
| 0.25 | 649.814 | 669.974 | 703.866 |
| 0.5 | 224.915 | 255.757 | 309.732 |
| 1.0 | 78.004 | 123.666 | 214.913 |
| 1.5 | 46.431 | 107.592 | 245.341 |
| 2.0 | 35.706 | 115.528 | 309.729 |
| 2.5 | 31.749 | 133.688 | 392.398 |
| 3.0 | 30.791 | 158.117 | 487.542 |
| 3.5 | 31.474 | 187.152 | 592.032 |
| 4.0 | 33.209 | 219.893 | 703.854 |
| 4.5 | 35.705 | 255.753 | 821.585 |
| 5.0 | 38.800 | 294.309 | 944.170 |

TABLE 3d. NATURAL FREQUENCY INCLUDING SHEAR.

| | | | $\hat{\Omega}^2$ (B.S.A. TI. RI.) MINIMUM | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | ($h^2/ab=0.01$) | | | | | |
| m | n | $a/b$ \ $\hat{\beta}$ | 0.0 | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
| 1 | 1 | 1.0 | 3.786 | 3.749 | 2.840 | 1.893 | 0.947 | 0.038 |
| 2 | 1 | 2.0 | 14.378 | 14.234 | 10.784 | 7.189 | 3.595 | 0.144 |
| 3 | 1 | 3.0 | 30.791 | 30.483 | 23.093 | 15.395 | 7.698 | 0.308 |
| 1 | 2 | 0.5 | 14.378 | 14.234 | 10.784 | 7.189 | 3.595 | 0.144 |
| 2 | 2 | 1.0 | 52.220 | 51.698 | 39.165 | 26.110 | 13.055 | 0.522 |
| 3 | 2 | 1.5 | 107.592 | 106.516 | 80.694 | 53.796 | 26.898 | 1.076 |
| 1 | 3 | 0.33 | 30.791 | 30.483 | 23.093 | 15.395 | 7.698 | 0.308 |
| 2 | 3 | 0.67 | 107.592 | 106.516 | 80.694 | 53.796 | 26.898 | 1.076 |
| 3 | 3 | 1.0 | 214.913 | 212.764 | 161.185 | 107.457 | 53.728 | 2.149 |

TABLE 3e. SUMMARY SHEET FOR NATURAL FREQUENCY INCLUDING SHEAR.

| a/b \ $\hat{\beta}$ | 0.01 | 0.25 | $\hat{\Omega}^2$ (B.S.A. TI) 0.50 | 0.75 | ($h^2/ab=0.02$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=1) | | | | |
| 0.25 | 14.425 | 10.928 | 7.285 | 3.643 | 0.146 |
| 0.5 | 5.423 | 4.108 | 2.739 | 1.369 | 0.055 |
| 1.0 | 3.559 | 2.696 | 1.797 | 0.899 | 0.036 |
| 1.5 | 4.141 | 3.137 | 2.092 | 1.046 | 0.042 |
| 2.0 | 5.423 | 4.108 | 2.739 | 1.369 | 0.055 |
| 2.5 | 7.156 | 5.421 | 3.614 | 1.807 | 0.072 |
| 3.0 | 9.259 | 7.015 | 4.676 | 2.338 | 0.094 |
| 3.5 | 11.692 | 8.857 | 5.905 | 2.953 | 0.118 |
| 4.0 | 14.424 | 10.927 | 7.285 | 3.643 | 0.146 |
| 4.5 | 17.433 | 14.174 | 8.805 | 4.402 | 0.176 |
| 5.0 | 20.699 | 15.681 | 10.454 | 5.227 | 0.209 |
| | (m=2, n=1) | | | | |
| 0.25 | 136.408 | 103.339 | 68.893 | 34.446 | 1.378 |
| 0.5 | 48.350 | 36.629 | 24.419 | 12.210 | 0.488 |
| 1.0 | 19.306 | 14.626 | 9.751 | 4.875 | 0.195 |
| 1.5 | 13.917 | 10.543 | 7.029 | 3.514 | 0.141 |
| 2.0 | 12.924 | 9.791 | 6.528 | 3.264 | 0.131 |
| 2.5 | 13.516 | 10.240 | 6.827 | 3.413 | 0.137 |
| 3.0 | 14.939 | 11.317 | 7.545 | 3.773 | 0.151 |
| 3.5 | 16.912 | 12.812 | 8.541 | 4.271 | 0.171 |
| 4.0 | 19.306 | 14.626 | 9.750 | 4.875 | 0.195 |
| 4.5 | 22.048 | 16.703 | 11.136 | 5.568 | 0.223 |
| 5.0 | 25.095 | 19.011 | 12.674 | 6.337 | 0.254 |
| | (m=3, n=1) | | | | |
| 0.25 | 427.313 | 323.722 | 215.815 | 107.908 | 4.316 |
| 0.5 | 165.818 | 125.620 | 83.747 | 41.873 | 1.675 |
| 1.0 | 63.300 | 47.955 | 31.970 | 15.985 | 0.639 |
| 1.5 | 39.135 | 29.647 | 19.765 | 9.883 | 0.395 |
| 2.0 | 30.607 | 23.187 | 15.458 | 7.729 | 0.309 |
| 2.5 | 27.409 | 20.764 | 13.843 | 6.921 | 0.277 |
| 3.0 | 26.629 | 20.174 | 13.449 | 6.725 | 0.269 |
| 3.5 | 27.185 | 20.595 | 13.730 | 6.865 | 0.275 |
| 4.0 | 28.593 | 21.661 | 14.441 | 7.221 | 0.289 |
| 4.5 | 30.607 | 23.187 | 15.459 | 7.729 | 0.309 |
| 5.0 | 33.088 | 25.067 | 16.711 | 8.356 | 0.334 |

TABLE 4a. NATURAL FREQUENCY INCLUDING SHEAR.

| a/b \ $\hat{\hat{\beta}}$ | 0.01 | 0.25 | $\hat{\Omega}^2$ (B.S.A. TI) 0.50 | 0.75 | ($h^2/ab$=0.02) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 19.306 | 14.626 | 9.751 | 4.875 | 0.195 |
| 0.5 | 12.924 | 9.791 | 6.528 | 3.264 | 0.131 |
| 1.0 | 19.306 | 14.626 | 9.751 | 4.875 | 0.195 |
| 1.5 | 31.977 | 24.225 | 16.150 | 8.075 | 0.323 |
| 2.0 | 48.349 | 36.628 | 24.419 | 12.209 | 0.488 |
| 2.5 | 67.491 | 51.129 | 34.086 | 17.043 | 0.682 |
| 3.0 | 88.812 | 67.282 | 44.855 | 22.427 | 0.897 |
| 3.5 | 111.890 | 84.765 | 56.510 | 28.255 | 1.130 |
| 4.0 | 136.406 | 103.338 | 68.892 | 34.446 | 1.378 |
| 4.5 | 162.116 | 122.816 | 81.877 | 40.938 | 1.638 |
| 5.0 | 188.830 | 143.053 | 95.369 | 47.684 | 1.907 |
| | (m=2, n=2) | | | | |
| 0.25 | 146.066 | 110.656 | 73.771 | 36.885 | 1.475 |
| 0.5 | 63.300 | 47.955 | 31.970 | 15.985 | 0.639 |
| 1.0 | 43.661 | 33.076 | 22.051 | 11.026 | 0.441 |
| 1.5 | 49.947 | 37.838 | 25.226 | 12.613 | 0.505 |
| 2.0 | 63.300 | 47.954 | 31.970 | 15.985 | 0.639 |
| 2.5 | 80.529 | 61.007 | 40.671 | 20.336 | 0.813 |
| 3.0 | 100.457 | 76.104 | 50.736 | 25.368 | 1.015 |
| 3.5 | 122.442 | 92.759 | 61.839 | 30.920 | 1.237 |
| 4.0 | 146.064 | 110.655 | 73.770 | 36.885 | 1.475 |
| 4.5 | 171.024 | 129.564 | 86.376 | 43.188 | 1.728 |
| 5.0 | 197.096 | 149.315 | 99.544 | 49.772 | 1.991 |
| | (m=3, n=2) | | | | |
| 0.25 | 439.077 | 332.634 | 221.756 | 110.878 | 4.435 |
| 0.5 | 186.094 | 140.980 | 93.987 | 46.993 | 1.880 |
| 1.0 | 96.534 | 73.132 | 48.755 | 24.377 | 0.975 |
| 1.5 | 85.020 | 64.409 | 42.940 | 21.470 | 0.859 |
| 2.0 | 90.727 | 68.733 | 45.822 | 22.911 | 0.916 |
| 2.5 | 103.626 | 78.505 | 52.337 | 26.168 | 1.047 |
| 3.0 | 120.667 | 91.415 | 60.943 | 30.472 | 1.219 |
| 3.5 | 140.527 | 106.460 | 70.973 | 35.587 | 1.420 |
| 4.0 | 162.486 | 123.096 | 82.064 | 41.032 | 1.641 |
| 4.5 | 186.091 | 140.978 | 93.985 | 46.993 | 1.880 |
| 5.0 | 211.027 | 159.869 | 106.579 | 53.290 | 2.132 |

TABLE 4b. NATURAL FREQUENCY INCLUDING SHEAR.

$\hat{\Omega}^2$(B.S.A. TI) ($h^2$/ab=0.02)

| a/b \ $\hat{\beta}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=3) | | | | |
| 0.25 | 28.593 | 21.662 | 14.441 | 7.221 | 0.289 |
| 0.5 | 30.607 | 23.187 | 15.458 | 7.729 | 0.309 |
| 1.0 | 63.300 | 47.955 | 31.970 | 15.985 | 0.639 |
| 1.5 | 110.444 | 83.670 | 55.780 | 27.890 | 1.116 |
| 2.0 | 165.816 | 125.619 | 83.746 | 41.873 | 1.675 |
| 2.5 | 226.557 | 171.634 | 114.423 | 57.211 | 2.289 |
| 3.0 | 291.015 | 220.466 | 146.978 | 73.489 | 2.940 |
| 3.5 | 358.159 | 271.332 | 180.888 | 90.444 | 3.618 |
| 4.0 | 427.306 | 323.717 | 215.811 | 107.906 | 4.316 |
| 4.5 | 497.987 | 377.263 | 251.509 | 125.754 | 5.030 |
| 5.0 | 569.870 | 431.720 | 287.813 | 143.907 | 5.756 |
| | (m=2, n=3) | | | | |
| 0.25 | 162.488 | 123.097 | 82.065 | 41.0324 | 1.641 |
| 0.5 | 90.727 | 68.733 | 45.822 | 22.9109 | 0.916 |
| 1.0 | 96.534 | 73.132 | 48.754 | 24.377 | 0.975 |
| 1.5 | 135.344 | 102.533 | 68.355 | 34.178 | 1.367 |
| 2.0 | 186.092 | 140.979 | 93.986 | 46.993 | 1.880 |
| 2.5 | 243.725 | 184.640 | 123.093 | 61.547 | 2.462 |
| 3.0 | 305.911 | 231.751 | 154.501 | 77.250 | 3.090 |
| 3.5 | 371.309 | 281.294 | 187.530 | 93.765 | 3.751 |
| 4.0 | 439.070 | 332.629 | 221.753 | 110.876 | 4.435 |
| 4.5 | 508.625 | 385.322 | 256.881 | 128.441 | 5.138 |
| 5.0 | 579.574 | 439.071 | 292.714 | 146.357 | 5.854 |
| | (m=3, n=3) | | | | |
| 0.25 | 458.767 | 347.551 | 231.701 | 115.851 | 4.634 |
| 0.5 | 220.889 | 167.340 | 111.560 | 55.780 | 2.231 |
| 1.0 | 159.173 | 120.586 | 80.390 | 40.195 | 1.608 |
| 1.5 | 179.281 | 135.819 | 90.546 | 45.273 | 1.811 |
| 2.0 | 220.887 | 167.339 | 111.559 | 55.780 | 2.231 |
| 2.5 | 272.813 | 206.677 | 137.784 | 68.892 | 2.756 |
| 3.0 | 330.984 | 250.746 | 167.164 | 83.582 | 3.343 |
| 3.5 | 393.365 | 298.003 | 198.669 | 99.335 | 3.973 |
| 4.0 | 458.761 | 347.546 | 231.698 | 115.849 | 4.634 |
| 4.5 | 526.408 | 398.794 | 265.862 | 132.921 | 5.317 |
| 5.0 | 595.782 | 451.350 | 300.900 | 150.450 | 6.018 |

<u>TABLE 4c.</u> NATURAL FREQUENCY INCLUDING SHEAR.

$\hat{\Omega}^2$(B.S.A. TI. RI) ($h^2/ab=0.02$), ($\hat{\beta}=0$)

| b/a \ n | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 14.570 | 19.501 | 28.882 |
| 0.5 | 5.478 | 13.055 | 30.916 |
| 1.0 | 3.595 | 19.501 | 63.939 |
| 1.5 | 4.183 | 32.299 | 111.560 |
| 2.0 | 5.478 | 48.837 | 167.491 |
| 2.5 | 7.228 | 68.173 | 228.845 |
| 3.0 | 9.353 | 89.709 | 293.955 |
| 3.5 | 11.810 | 113.020 | 361.777 |
| 4.0 | 14.570 | 137.784 | 431.622 |
| 4.5 | 17.609 | 163.754 | 503.017 |
| 5.0 | 20.908 | 190.737 | 575.627 |
| (m=2) | | | |
| 0.25 | 137.786 | 147.542 | 164.130 |
| 0.5 | 48.838 | 63.940 | 91.644 |
| 1.0 | 19.501 | 44.102 | 97.509 |
| 1.5 | 14.058 | 50.451 | 136.711 |
| 2.0 | 13.055 | 63.939 | 187.972 |
| 2.5 | 13.653 | 81.343 | 246.187 |
| 3.0 | 15.090 | 101.472 | 309.001 |
| 3.5 | 17.083 | 123.678 | 375.059 |
| 4.0 | 19.501 | 147.540 | 443.505 |
| 4.5 | 22.271 | 172.752 | 513.763 |
| 5.0 | 25.348 | 199.087 | 585.429 |
| (m=3) | | | |
| 0.25 | 431.629 | 443.512 | 463.402 |
| 0.5 | 167.493 | 187.973 | 223.120 |
| 1.0 | 63.940 | 97.509 | 160.781 |
| 1.5 | 39.530 | 85.879 | 181.091 |
| 2.0 | 30.917 | 91.643 | 223.119 |
| 2.5 | 27.686 | 104.673 | 275.569 |
| 3.0 | 26.898 | 121.886 | 334.327 |
| 3.5 | 27.450 | 141.946 | 397.338 |
| 4.0 | 28.882 | 164.128 | 463.395 |
| 4.5 | 30.916 | 187.971 | 531.725 |
| 5.0 | 33.422 | 213.159 | 601.800 |

TABLE 4d. NATURAL FREQUENCY INCLUDING SHEAR.

$\hat{\Omega}^2$(B.S.A. TI) MINIMUM

($h^2/ab=0.02$)

| m | n | $a/b$ \ $\hat{\beta}$ | 0.0 | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 3.595 | 3.559 | 2.696 | 1.797 | 0.899 | 0.036 |
| 2 | 1 | 2.0 | 13.055 | 12.924 | 9.791 | 6.528 | 3.264 | 0.131 |
| 3 | 1 | 3.0 | 26.898 | 26.629 | 20.174 | 13.449 | 6.725 | 0.269 |
| 1 | 2 | 0.5 | 13.055 | 12.924 | 9.791 | 6.528 | 3.264 | 0.131 |
| 2 | 2 | 1.0 | 44.102 | 43.661 | 33.076 | 22.051 | 11.026 | 0.441 |
| 3 | 2 | 1.5 | 85.879 | 85.020 | 64.409 | 42.940 | 21.470 | 0.859 |
| 1 | 3 | 0.33 | 26.898 | 26.629 | 20.174 | 13.449 | 6.725 | 0.269 |
| 2 | 3 | 0.67 | 85.879 | 85.020 | 64.409 | 42.940 | 21.470 | 0.859 |
| 3 | 3 | 1.0 | 160.781 | 159.173 | 120.586 | 80.390 | 40.195 | 1.811 |

TABLE 4e. SUMMARY SHEET FOR NATURAL FREQUENCY INCLUDING SHEAR.

| a/b \ $\hat{\hat{\beta}}$ | 0.01 | 0.25 | $\hat{\hat{\Omega}}^2$ (B.S.A. TI) 0.50 | 0.75 | ($h^2/ab=0.03$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=1) | | | | |
| 0.25 | 13.153 | 9.964 | 6.643 | 3.321 | 0.133 |
| 0.5 | 5.107 | 3.869 | 2.580 | 1.290 | 0.052 |
| 1.0 | 3.387 | 2.566 | 1.711 | 0.855 | 0.034 |
| 1.5 | 3.928 | 2.975 | 1.984 | 0.992 | 0.040 |
| 2.0 | 5.107 | 3.869 | 2.579 | 1.290 | 0.052 |
| 2.5 | 6.686 | 5.065 | 3.377 | 1.688 | 0.070 |
| 3.0 | 8.580 | 6.500 | 4.334 | 2.167 | 0.087 |
| 3.5 | 10.746 | 8.141 | 5.428 | 2.714 | 0.109 |
| 4.0 | 13.153 | 9.964 | 6.643 | 3.321 | 0.133 |
| 4.5 | 15.774 | 11.950 | 7.967 | 3.983 | 0.159 |
| 5.0 | 18.591 | 14.084 | 9.389 | 4.695 | 0.188 |
| | (m=2, n=1) | | | | |
| 0.25 | 110.086 | 82.399 | 55.599 | 27.800 | 1.112 |
| 0.5 | 41.608 | 31.521 | 21.014 | 10.507 | 0.420 |
| 1.0 | 17.393 | 13.177 | 8.784 | 4.392 | 0.176 |
| 1.5 | 12.708 | 9.627 | 6.418 | 3.209 | 0.128 |
| 2.0 | 11.845 | 8.966 | 5.977 | 2.989 | 0.120 |
| 2.5 | 12.356 | 9.361 | 6.241 | 3.120 | 0.125 |
| 3.0 | 13.603 | 10.305 | 6.870 | 3.435 | 0.137 |
| 3.5 | 15.322 | 11.608 | 7.739 | 3.869 | 0.155 |
| 4.0 | 17.393 | 13.176 | 8.784 | 4.392 | 0.176 |
| 4.5 | 19.747 | 14.960 | 9.973 | 4.987 | 0.200 |
| 5.0 | 22.341 | 16.925 | 11.283 | 5.642 | 0.226 |
| | (m=3, n=1) | | | | |
| 0.25 | 319.901 | 242.349 | 161.566 | 80.783 | 3.231 |
| 0.5 | 132.094 | 100.071 | 66.714 | 33.357 | 1.334 |
| 1.0 | 53.630 | 40.629 | 27.086 | 13.543 | 0.542 |
| 1.5 | 34.071 | 25.811 | 17.207 | 8.604 | 0.344 |
| 2.0 | 26.988 | 20.445 | 13.630 | 6.815 | 0.273 |
| 2.5 | 24.299 | 18.408 | 12.272 | 6.136 | 0.245 |
| 3.0 | 23.641 | 17.910 | 11.940 | 5.970 | 0.239 |
| 3.5 | 24.110 | 18.265 | 12.177 | 6.088 | 0.244 |
| 4.0 | 25.297 | 19.164 | 12.776 | 6.388 | 0.256 |
| 4.5 | 26.987 | 20.445 | 13.630 | 6.815 | 0.277 |
| 5.0 | 29.060 | 22.015 | 14.677 | 7.338 | 0.294 |

TABLE 5a. NATURAL FREQUENCY INCLUDING SHEAR.

| $a/b$ \ $\hat{\beta}$ | 0.25 | 0.01 | $\hat{\Omega}^2$ (B.S.A. TI) 0.50 | 0.75 | ($h^2/ab=0.03$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 13.177 | 17.393 | 8.784 | 4.392 | 0.176 |
| 0.5 | 8.966 | 11.835 | 5.977 | 2.989 | 0.120 |
| 1.0 | 13.177 | 17.393 | 8.784 | 4.392 | 0.176 |
| 1.5 | 21.313 | 28.133 | 14.209 | 7.104 | 0.284 |
| 2.0 | 31.521 | 41.608 | 21.014 | 10.507 | 0.420 |
| 2.5 | 43.153 | 56.962 | 28.769 | 14.384 | 0.575 |
| 3.0 | 55.830 | 73.696 | 37.220 | 18.610 | 0.744 |
| 3.5 | 69.304 | 91.481 | 46.202 | 23.101 | 0.924 |
| 4.0 | 83.397 | 110.084 | 55.598 | 27.799 | 1.112 |
| 4.5 | 97.985 | 129.340 | 65.323 | 32.662 | 1.307 |
| 5.0 | 112.97 | 149.123 | 75.315 | 37.657 | 1.506 |
| | (m=2, n=2) | | | | |
| 0.25 | 88.899 | 117.347 | 59.266 | 29.633 | 1.185 |
| 0.5 | 40.629 | 53.630 | 27.086 | 13.543 | 0.542 |
| 1.0 | 28.626 | 37.787 | 19.594 | 9.542 | 0.382 |
| 1.5 | 32.503 | 42.904 | 21.669 | 10.834 | 0.433 |
| 2.0 | 40.629 | 53.630 | 27.086 | 13.543 | 0.542 |
| 2.5 | 50.935 | 67.234 | 33.957 | 16.978 | 0.679 |
| 3.0 | 62.657 | 82.707 | 41.771 | 20.886 | 0.835 |
| 3.5 | 75.394 | 99.520 | 50.263 | 25.131 | 1.005 |
| 4.0 | 88.898 | 117.345 | 59.265 | 29.633 | 1.185 |
| 4.5 | 103.000 | 135.960 | 68.667 | 34.333 | 1.373 |
| 5.0 | 117.580 | 155.205 | 78.386 | 39.193 | 1.568 |
| | (m=3, n=2) | | | | |
| 0.25 | 248.603 | 328.155 | 165.736 | 82.868 | 3.315 |
| 0.5 | 111.444 | 147.106 | 74.296 | 37.148 | 1.486 |
| 1.0 | 60.364 | 79.861 | 40.243 | 20.121 | 0.805 |
| 1.5 | 53.594 | 70.744 | 35.729 | 17.865 | 0.715 |
| 2.0 | 56.957 | 75.183 | 37.971 | 18.986 | 0.759 |
| 2.5 | 64.505 | 85.147 | 43.003 | 21.502 | 0.860 |
| 3.0 | 74.373 | 98.172 | 49.582 | 24.791 | 0.992 |
| 3.5 | 85.748 | 113.187 | 57.165 | 28.583 | 1.143 |
| 4.0 | 98.193 | 129.615 | 65.462 | 32.731 | 1.309 |
| 4.5 | 111.442 | 147.104 | 74.295 | 37.147 | 1.486 |
| 5.0 | 125.316 | 165.417 | 83.544 | 41.772 | 1.671 |

TABLE 5b. NATURAL FREQUENCY INCLUDING SHEAR.

| a/b \ $\hat{\beta}$ | 0.01 | 0.25 | $\hat{\Omega}^2$ (B.S.A. TI) 0.50 | 0.75 | ($h^2/ab=0.03$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=3) | | | | |
| 0.25 | 25.297 | 19.164 | 12.776 | 6.388 | 0.256 |
| 0.5 | 26.988 | 20.445 | 13.630 | 6.815 | 0.273 |
| 1.0 | 53.630 | 40.629 | 27.086 | 13.543 | 0.542 |
| 1.5 | 90.375 | 68.466 | 45.644 | 22.822 | 0.913 |
| 2.0 | 132.093 | 100.070 | 66.714 | 33.357 | 1.334 |
| 2.5 | 176.749 | 133.901 | 89.267 | 44.634 | 1.785 |
| 3.0 | 223.297 | 169.164 | 112.776 | 56.388 | 2.256 |
| 3.5 | 271.136 | 205.406 | 136.937 | 68.469 | 2.739 |
| 4.0 | 319.896 | 242.346 | 161.564 | 80.782 | 3.231 |
| 4.5 | 369.336 | 279.800 | 186.534 | 93.267 | 3.731 |
| 5.0 | 419.293 | 317.646 | 211.764 | 105.882 | 4.235 |
| | (m=2, n=3) | | | | |
| 0.25 | 129.617 | 98.195 | 65.463 | 32.732 | 1.309 |
| 0.5 | 75.184 | 56.957 | 37.972 | 18.986 | 0.759 |
| 1.0 | 79.681 | 60.364 | 40.243 | 20.121 | 0.805 |
| 1.5 | 109.284 | 82.791 | 55.194 | 27.597 | 1.104 |
| 2.0 | 147.104 | 111.443 | 74.295 | 37.148 | 1.486 |
| 2.5 | 189.219 | 143.347 | 95.565 | 47.783 | 1.911 |
| 3.0 | 233.959 | 177.242 | 118.161 | 59.081 | 2.363 |
| 3.5 | 280.444 | 212.452 | 141.638 | 70.819 | 2.833 |
| 4.0 | 328.151 | 248.599 | 165.733 | 82.866 | 3.315 |
| 4.5 | 376.748 | 285.415 | 190.277 | 95.138 | 3.806 |
| 5.0 | 426.016 | 322.740 | 215.160 | 107.580 | 4.303 |
| | (m=3, n=3) | | | | |
| 0.25 | 341.947 | 259.051 | 172.701 | 86.350 | 3.454 |
| 0.5 | 172.619 | 130.772 | 87.181 | 43.591 | 1.744 |
| 1.0 | 127.147 | 96.324 | 64.216 | 32.108 | 1.284 |
| 1.5 | 142.074 | 107.632 | 71.755 | 35.877 | 1.435 |
| 2.0 | 172.618 | 130.771 | 87.181 | 43.590 | 1.745 |
| 2.5 | 210.223 | 159.260 | 106.173 | 53.087 | 2.124 |
| 3.0 | 251.841 | 190.789 | 127.192 | 63.596 | 2.544 |
| 3.5 | 296.017 | 224.256 | 149.504 | 74.752 | 2.990 |
| 4.0 | 341.943 | 259.047 | 172.698 | 86.349 | 3.454 |
| 4.5 | 389.123 | 294.790 | 196.527 | 98.263 | 3.931 |
| 5.0 | 437.235 | 331.238 | 220.826 | 110.413 | 4.417 |

TABLE 5c. NATURAL FREQUENCY INCLUDING SHEAR.

$\hat{\hat{\Omega}}^2$(B.S.A. TI. RI) ($h^2/ab=0.03$), ($\hat{\hat{\beta}}=0$)

| a/b \ n | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 13.286 | 17.569 | 25.552 |
| 0.5 | 5.159 | 11.955 | 27.260 |
| 1.0 | 3.421 | 17.569 | 54.172 |
| 1.5 | 3.967 | 28.417 | 91.288 |
| 2.0 | 5.159 | 42.028 | 133.427 |
| 2.5 | 6.753 | 57.537 | 178.534 |
| 3.0 | 8.667 | 74.441 | 225.552 |
| 3.5 | 10.855 | 92.405 | 273.875 |
| 4.0 | 13.286 | 111.196 | 323.127 |
| 4.5 | 15.934 | 130.646 | 373.067 |
| 5.0 | 18.779 | 150.629 | 423.529 |
| (m=2) | | | |
| 0.25 | 111.198 | 118.532 | 130.926 |
| 0.5 | 42.029 | 54.172 | 75.843 |
| 1.0 | 17.569 | 38.169 | 80.485 |
| 1.5 | 12.837 | 43.337 | 110.388 |
| 2.0 | 11.955 | 54.172 | 148.590 |
| 2.5 | 12.481 | 67.914 | 191.130 |
| 3.0 | 13.741 | 83.543 | 236.323 |
| 3.5 | 15.477 | 100.526 | 283.277 |
| 4.0 | 17.569 | 118.531 | 331.465 |
| 4.5 | 19.947 | 137.333 | 380.554 |
| 5.0 | 22.567 | 156.773 | 430.319 |
| (m=3) | | | |
| 0.25 | 323.132 | 331.470 | 345.401 |
| 0.5 | 133.428 | 148.592 | 174.363 |
| 1.0 | 54.172 | 80.485 | 128.431 |
| 1.5 | 34.415 | 71.458 | 143.510 |
| 2.0 | 27.260 | 75.943 | 174.361 |
| 2.5 | 24.544 | 86.007 | 212.346 |
| 3.0 | 23.879 | 99.164 | 254.385 |
| 3.5 | 24.354 | 114.330 | 299.007 |
| 4.0 | 25.552 | 130.925 | 345.397 |
| 4.5 | 27.260 | 148.590 | 393.053 |
| 5.0 | 29.354 | 167.088 | 441.651 |

TABLE 5d. NATURAL FREQUENCY INCLUDING SHEAR.

$\hat{\Omega}^2$ (B.S.A. TI) MINIMUM

($h^2/ab=0.03$)

| m | n | a/b \ $\hat{\beta}$ | 0.0 | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 3.421 | 3.387 | 2.566 | 1.711 | 0.855 | 0.034 |
| 2 | 1 | 2.0 | 11.955 | 11.835 | 8.966 | 5.977 | 2.989 | 0.120 |
| 3 | 1 | 3.0 | 23.879 | 23.641 | 17.910 | 11.940 | 5.970 | 0.239 |
| 1 | 2 | 0.5 | 11.955 | 11.835 | 8.966 | 5.977 | 2.989 | 0.120 |
| 2 | 2 | 1.0 | 38.169 | 37.787 | 28.626 | 19.084 | 9.542 | 0.382 |
| 3 | 2 | 1.5 | 71.458 | 70.744 | 53.594 | 35.729 | 17.865 | 0.715 |
| 1 | 3 | 0.33 | 23.879 | 23.641 | 17.910 | 11.940 | 5.970 | 0.239 |
| 2 | 3 | 0.67 | 71.458 | 70.744 | 53.594 | 35.729 | 17.865 | 0.715 |
| 3 | 3 | 1.0 | 128.431 | 127.147 | 96.324 | 64.216 | 32.108 | 1.284 |

TABLE 5e. SUMMARY SHEET FOR NATURAL FREQUENCY INCLUDING SHEAR.

| a/b \ $\hat{\beta}$ | 0.01 | 0.25 | $\hat{\Omega}^2$ (B.S.A. TI) 0.50 | 0.75 | ($h^2/ab=0.04$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=1) | | | | |
| 0.25 | 12.087 | 9.157 | 6.105 | 3.052 | 0.122 |
| 0.5 | 4.827 | 3.656 | 2.438 | 1.219 | 0.049 |
| 1.0 | 3.231 | 2.448 | 1.632 | 0.816 | 0.033 |
| 1.5 | 3.735 | 2.829 | 1.886 | 0.943 | 0.038 |
| 2.0 | 4.826 | 3.656 | 2.438 | 1.219 | 0.049 |
| 2.5 | 6.274 | 4.753 | 3.169 | 1.584 | 0.063 |
| 3.0 | 7.994 | 6.056 | 4.037 | 2.019 | 0.081 |
| 3.5 | 9.943 | 7.532 | 5.022 | 2.511 | 0.100 |
| 4.0 | 12.087 | 9.157 | 6.105 | 3.052 | 0.122 |
| 4.5 | 14.404 | 10.912 | 7.275 | 3.637 | 0.146 |
| 5.0 | 16.873 | 12.782 | 8.522 | 4.261 | 0.170 |
| | (m=2, n=1) | | | | |
| 0.25 | 92.279 | 69.909 | 46.606 | 23.303 | 0.932 |
| 0.5 | 36.517 | 27.664 | 18.443 | 9.221 | 0.369 |
| 1.0 | 15.825 | 11.989 | 7.992 | 3.996 | 0.160 |
| 1.5 | 11.692 | 8.858 | 5.905 | 2.953 | 0.118 |
| 2.0 | 10.915 | 8.269 | 5.513 | 2.756 | 0.110 |
| 2.5 | 11.379 | 8.621 | 5.747 | 2.874 | 0.115 |
| 3.0 | 12.487 | 9.460 | 6.306 | 3.153 | 0.126 |
| 3.5 | 14.006 | 10.610 | 7.074 | 3.537 | 0.142 |
| 4.0 | 15.825 | 11.989 | 7.992 | 3.996 | 0.160 |
| 4.5 | 17.881 | 13.546 | 9.031 | 4.515 | 0.181 |
| 5.0 | 20.132 | 15.252 | 10.168 | 5.084 | 0.203 |
| | (m=3, n=1) | | | | |
| 0.25 | 255.642 | 193.668 | 129.112 | 64.556 | 2.582 |
| 0.5 | 109.769 | 83.158 | 55.439 | 27.720 | 1.109 |
| 1.0 | 46.524 | 35.245 | 23.497 | 11.748 | 0.470 |
| 1.5 | 30.167 | 22.854 | 15.236 | 7.618 | 0.305 |
| 2.0 | 24.134 | 18.283 | 12.189 | 6.094 | 0.244 |
| 2.5 | 21.823 | 16.532 | 11.022 | 5.511 | 0.220 |
| 3.0 | 21.255 | 16.102 | 10.735 | 5.367 | 0.215 |
| 3.5 | 21.660 | 16.409 | 10.939 | 5.470 | 0.219 |
| 4.0 | 22.682 | 17.183 | 11.455 | 5.728 | 0.229 |
| 4.5 | 24.133 | 18.283 | 12.189 | 6.094 | 0.244 |
| 5.0 | 25.907 | 19.626 | 13.084 | 6.542 | 0.262 |

TABLE 6a. NATURAL FREQUENCY INCLUDING SHEAR.

| a/b \ $\hat{\beta}$ | 0.01 | 0.25 | $\hat{\Omega}^2$ (B.S.A. TI) 0.50 | 0.75 | ($h^2/ab=0.04$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 15.825 | 11.989 | 7.992 | 3.996 | 0.160 |
| 0.5 | 10.915 | 8.269 | 5.513 | 2.756 | 0.110 |
| 1.0 | 15.825 | 11.989 | 7.992 | 3.996 | 0.160 |
| 1.5 | 25.114 | 19.026 | 12.684 | 6.342 | 0.254 |
| 2.0 | 36.516 | 27.664 | 18.443 | 9.221 | 0.369 |
| 2.5 | 49.274 | 37.329 | 24.886 | 12.443 | 0.498 |
| 3.0 | 62.977 | 47.710 | 31.807 | 15.903 | 0.636 |
| 3.5 | 77.369 | 58.613 | 39.075 | 19.538 | 0.782 |
| 4.0 | 92.278 | 69.908 | 46.605 | 23.303 | 0.932 |
| 4.5 | 107.588 | 81.506 | 54.337 | 27.169 | 1.087 |
| 5.0 | 123.214 | 93.344 | 62.229 | 31.115 | 1.245 |
| | (m=2, n=2) | | | | |
| 0.25 | 98.066 | 74.292 | 49.528 | 24.764 | 0.991 |
| 0.5 | 46.524 | 35.245 | 23.497 | 11.748 | 0.470 |
| 1.0 | 33.306 | 25.232 | 16.821 | 8.411 | 0.336 |
| 1.5 | 37.602 | 28.486 | 18.991 | 9.495 | 0.380 |
| 2.0 | 46.523 | 35.245 | 23.497 | 11.748 | 0.470 |
| 2.5 | 57.707 | 43.718 | 29.145 | 14.573 | 0.583 |
| 3.0 | 70.288 | 53.249 | 35.499 | 17.750 | 0.710 |
| 3.5 | 83.828 | 63.506 | 42.337 | 21.169 | 0.847 |
| 4.0 | 98.064 | 74.291 | 49.527 | 24.764 | 0.991 |
| 4.5 | 112.827 | 85.475 | 56.984 | 28.492 | 1.140 |
| 5.0 | 128.000 | 96.969 | 64.646 | 32.323 | 1.293 |
| | (m=3, n=2) | | | | |
| 0.25 | 261.974 | 198.465 | 132.310 | 66.155 | 2.646 |
| 0.5 | 121.625 | 92.140 | 61.427 | 30.713 | 1.229 |
| 1.0 | 67.837 | 51.392 | 34.261 | 17.131 | 0.685 |
| 1.5 | 60.572 | 45.888 | 30.592 | 15.296 | 0.612 |
| 2.0 | 64.187 | 48.626 | 32.418 | 16.209 | 0.648 |
| 2.5 | 72.260 | 54.743 | 36.495 | 18.248 | 0.730 |
| 3.0 | 82.746 | 62.687 | 41.791 | 20.896 | 0.836 |
| 3.5 | 94.753 | 71.782 | 47.855 | 23.927 | 0.957 |
| 4.0 | 107.806 | 81.671 | 54.448 | 27.224 | 1.089 |
| 4.5 | 121.623 | 92.139 | 61.426 | 30.713 | 1.229 |
| 5.0 | 136.018 | 103.044 | 68.696 | 34.348 | 1.374 |

<u>TABLE 6b</u>. NATURAL FREQUENCY INCLUDING SHEAR.

$\hat{\Omega}^2$(B.S.A. TI) $(h^2/ab=0.04)$

| $a/b$ \ $\hat{\beta}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=3) | | | | |
| 0.25 | 22.682 | 17.183 | 11.456 | 5.728 | 0.229 |
| 0.5 | 24.134 | 18.283 | 12.189 | 6.094 | 0.244 |
| 1.0 | 46.523 | 35.245 | 23.497 | 11.748 | 0.470 |
| 1.5 | 76.478 | 57.938 | 38.625 | 19.313 | 0.773 |
| 2.0 | 109.768 | 83.158 | 55.438 | 27.719 | 1.109 |
| 2.5 | 144.894 | 109.768 | 73.179 | 36.589 | 1.464 |
| 3.0 | 181.145 | 137.231 | 91.487 | 45.744 | 1.830 |
| 3.5 | 218.135 | 165.254 | 110.169 | 55.085 | 2.203 |
| 4.0 | 255.638 | 193.665 | 129.110 | 64.555 | 2.582 |
| 4.5 | 293.510 | 222.356 | 148.238 | 74.119 | 2.965 |
| 5.0 | 331.659 | 251.257 | 167.504 | 83.752 | 3.350 |
| | (m=2, n=3) | | | | |
| 0.25 | 107.807 | 81.672 | 54.448 | 27.224 | 1.089 |
| 0.5 | 64.187 | 48.627 | 32.418 | 16.209 | 0.648 |
| 1.0 | 67.837 | 51.392 | 34.261 | 17.131 | 0.685 |
| 1.5 | 91.639 | 69.424 | 46.282 | 23.141 | 0.926 |
| 2.0 | 121.624 | 92.139 | 61.426 | 30.713 | 1.229 |
| 2.5 | 154.636 | 117.148 | 78.099 | 39.049 | 1.562 |
| 3.0 | 189.409 | 143.492 | 95.661 | 47.831 | 1.913 |
| 3.5 | 225.308 | 170.688 | 113.792 | 56.896 | 2.276 |
| 4.0 | 261.971 | 198.463 | 132.308 | 66.154 | 2.646 |
| 4.5 | 299.177 | 226.649 | 151.100 | 75.550 | 3.022 |
| 5.0 | 336.785 | 255.140 | 170.093 | 85.047 | 3.402 |
| | (m=3, n=3) | | | | |
| 0.25 | 272.546 | 206.474 | 137.650 | 68.825 | 2.753 |
| 0.5 | 141.662 | 107.320 | 71.546 | 35.773 | 1.431 |
| 1.0 | 105.850 | 80.189 | 53.460 | 26.730 | 1.069 |
| 1.5 | 117.657 | 89.134 | 59.423 | 29.711 | 1.189 |
| 2.0 | 141.661 | 107.319 | 71.546 | 35.773 | 1.431 |
| 2.5 | 170.993 | 129.540 | 86.360 | 43.180 | 1.727 |
| 3.0 | 203.243 | 153.972 | 102.648 | 51.324 | 2.053 |
| 3.5 | 237.293 | 179.768 | 119.845 | 59.923 | 2.397 |
| 4.0 | 272.543 | 206.472 | 137.648 | 68.824 | 2.753 |
| 4.5 | 308.632 | 233.813 | 155.875 | 77.938 | 3.118 |
| 5.0 | 345.335 | 261.617 | 174.412 | 87.206 | 3.488 |

<u>TABLE 6c</u>. NATURAL FREQUENCY INCLUDING SHEAR.

$\hat{\hat{\Omega}}^2$ (B.S.A. TI. RI) $(h^2/ab=0.04)$, $(\hat{\hat{\beta}}=0)$

| a/b \ n | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 12.210 | 15.985 | 22.911 |
| 0.5 | 4.875 | 11.026 | 24.377 |
| 1.0 | 3.264 | 15.985 | 46.993 |
| 1.5 | 3.773 | 25.368 | 77.251 |
| 2.0 | 4.875 | 36.885 | 110.877 |
| 2.5 | 6.337 | 49.772 | 146.358 |
| 3.0 | 8.075 | 63.613 | 182.975 |
| 3.5 | 10.043 | 78.150 | 220.338 |
| 4.0 | 12.209 | 93.210 | 258.220 |
| 4.5 | 14.459 | 108.675 | 296.475 |
| 5.0 | 17.043 | 124.458 | 335.009 |
| (m=2) | | | |
| 0.25 | 93.212 | 99.056 | 108.896 |
| 0.5 | 36.886 | 46.993 | 64.835 |
| 1.0 | 15.985 | 33.642 | 68.522 |
| 1.5 | 11.810 | 37.982 | 92.565 |
| 2.0 | 11.026 | 46.993 | 122.852 |
| 2.5 | 11.494 | 58.290 | 156.198 |
| 3.0 | 12.613 | 70.998 | 191.323 |
| 3.5 | 14.147 | 84.675 | 227.584 |
| 4.0 | 15.985 | 99.055 | 264.646 |
| 4.5 | 18.061 | 113.967 | 302.199 |
| 5.0 | 20.336 | 129.293 | 340.187 |
| (m=3) | | | |
| 0.25 | 258.224 | 264.620 | 275.299 |
| 0.5 | 110.878 | 122.853 | 143.093 |
| 1.0 | 46.993 | 68.522 | 106.919 |
| 1.5 | 30.472 | 61.869 | 118.846 |
| 2.0 | 24.377 | 64.835 | 143.092 |
| 2.5 | 22.043 | 72.990 | 172.720 |
| 3.0 | 21.470 | 83.582 | 205.296 |
| 3.5 | 21.879 | 95.711 | 239.690 |
| 4.0 | 22.911 | 108.895 | 275.296 |
| 4.5 | 24.377 | 122.852 | 311.750 |
| 5.0 | 26.168 | 137.392 | 348.823 |

TABLE 6d. NATURAL FREQUENCY INCLUDING SHEAR.

$\hat{\Omega}^2$ (B.S.A. TI) MINIMUM

| m | n | $a/b \setminus \hat{\beta}$ | 0.0 ($h^2/ab=0.04$) | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 3.264 | 3.231 | 2.448 | 1.632 | 0.816 | 0.033 |
| 2 | 1 | 2.0 | 11.026 | 10.915 | 8.269 | 5.513 | 2.756 | 0.110 |
| 3 | 1 | 3.0 | 21.470 | 21.255 | 16.102 | 10.735 | 5.367 | 0.215 |
| 1 | 2 | 0.5 | 11.026 | 10.915 | 8.269 | 5.513 | 2.756 | 0.110 |
| 2 | 2 | 1.0 | 33.462 | 33.306 | 25.232 | 16.821 | 8.411 | 0.336 |
| 3 | 2 | 1.5 | 61.184 | 60.572 | 45.888 | 30.592 | 15.296 | 0.612 |
| 1 | 3 | 0.33 | 21.470 | 21.255 | 16.102 | 10.735 | 5.367 | 0.215 |
| 2 | 3 | 0.67 | 61.184 | 60.572 | 45.888 | 30.592 | 15.296 | 0.612 |
| 3 | 3 | 1.0 | 106.919 | 105.850 | 80.189 | 53.460 | 26.730 | 1.069 |

TABLE 6e. SUMMARY SHEET FOR NATURAL FREQUENCY INCLUDING SHEAR.

| $\frac{a}{b}$ \ $\hat{\beta}$ | 0.01 | 0.25 | $\hat{\Omega}^2$ (B.S.A. TI) 0.50 | 0.75 | ($h^2/ab=0.05$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=1) | | | | |
| 0.25 | 11.182 | 8.471 | 5.647 | 2.824 | 0.114 |
| 0.5 | 4.575 | 3.466 | 2.311 | 1.155 | 0.046 |
| 1.0 | 3.089 | 2.340 | 1.560 | 0.780 | 0.031 |
| 1.5 | 3.560 | 2.697 | 1.798 | 0.899 | 0.036 |
| 2.0 | 4.575 | 3.466 | 2.311 | 1.155 | 0.046 |
| 2.5 | 5.910 | 4.477 | 2.985 | 1.492 | 0.060 |
| 3.0 | 7.483 | 5.669 | 3.779 | 1.890 | 0.076 |
| 3.5 | 9.251 | 7.008 | 4.672 | 2.336 | 0.093 |
| 4.0 | 11.181 | 8.471 | 5.647 | 2.824 | 0.113 |
| 4.5 | 13.252 | 10.040 | 6.693 | 3.347 | 0.134 |
| 5.0 | 15.445 | 11.701 | 7.801 | 3.900 | 0.156 |
| | (m=2, n=1) | | | | |
| 0.25 | 79.431 | 60.175 | 40.117 | 20.058 | 0.802 |
| 0.5 | 32.535 | 24.648 | 16.432 | 8.216 | 0.329 |
| 1.0 | 14.516 | 10.997 | 7.332 | 3.666 | 0.147 |
| 1.5 | 10.827 | 8.202 | 5.468 | 2.734 | 0.109 |
| 2.0 | 10.128 | 7.673 | 5.115 | 2.558 | 0.102 |
| 2.5 | 10.546 | 7.989 | 5.326 | 2.663 | 0.107 |
| 3.0 | 11.540 | 8.742 | 5.828 | 2.914 | 0.117 |
| 3.5 | 12.897 | 9.771 | 6.514 | 3.257 | 0.130 |
| 4.0 | 14.516 | 10.997 | 7.331 | 3.666 | 0.147 |
| 4.5 | 16.337 | 12.376 | 8.251 | 4.125 | 0.165 |
| 5.0 | 18.321 | 13.879 | 9.253 | 4.627 | 0.185 |
| | (m=3, n=1) | | | | |
| 0.25 | 212.880 | 161.273 | 107.515 | 53.758 | 2.150 |
| 0.5 | 93.899 | 71.136 | 47.424 | 23.712 | 0.949 |
| 1.0 | 41.080 | 31.121 | 20.747 | 10.374 | 0.415 |
| 1.5 | 27.066 | 20.505 | 13.670 | 6.835 | 0.273 |
| 2.0 | 21.825 | 16.534 | 11.023 | 5.512 | 0.221 |
| 2.5 | 19.805 | 15.004 | 10.002 | 5.001 | 0.200 |
| 3.0 | 19.307 | 14.627 | 9.751 | 4.876 | 0.195 |
| 3.5 | 19.662 | 14.896 | 9.930 | 4.965 | 0.199 |
| 4.0 | 20.557 | 15.573 | 10.382 | 5.191 | 0.208 |
| 4.5 | 21.825 | 16.534 | 11.023 | 5.511 | 0.221 |
| 5.0 | 23.370 | 17.705 | 11.803 | 5.902 | 0.236 |

TABLE 7a. NATURAL FREQUENCY INCLUDING SHEAR.

| a/b \ $\hat{\beta}$ | 0.01 | 0.25 | $\hat{\Omega}^2$(B.S.A. TI) 0.50 | 0.75 | ($h^2/ab$=0.05) 0.99 |
|---|---|---|---|---|---|
| (m=1, n=2) | | | | | |
| 0.25 | 14.516 | 10.997 | 7.332 | 3.666 | 0.147 |
| 0.5 | 10.128 | 7.673 | 5.115 | 2.558 | 0.102 |
| 1.0 | 14.516 | 10.997 | 7.332 | 3.666 | 0.147 |
| 1.5 | 22.681 | 17.182 | 11.455 | 5.727 | 0.229 |
| 2.0 | 32.535 | 24.648 | 16.432 | 8.216 | 0.329 |
| 2.5 | 43.415 | 32.890 | 21.927 | 10.963 | 0.439 |
| 3.0 | 54.980 | 41.652 | 27.768 | 13.884 | 0.555 |
| 3.5 | 67.028 | 50.779 | 33.853 | 16.926 | 0.677 |
| 4.0 | 79.430 | 60.174 | 40.116 | 20.058 | 0.802 |
| 4.5 | 92.099 | 69.772 | 46.515 | 23.257 | 0.930 |
| 5.0 | 104.975 | 79.526 | 53.018 | 26.509 | 1.050 |
| (m=2, n=2) | | | | | |
| 0.25 | 84.226 | 63.808 | 42.539 | 21.269 | 0.851 |
| 0.5 | 41.080 | 31.121 | 20.747 | 10.374 | 0.415 |
| 1.0 | 29.775 | 22.557 | 15.038 | 7.519 | 0.301 |
| 1.5 | 33.466 | 25.353 | 16.902 | 8.451 | 0.338 |
| 2.0 | 41.079 | 31.121 | 20.747 | 10.374 | 0.415 |
| 2.5 | 50.545 | 38.292 | 25.528 | 12.764 | 0.511 |
| 3.0 | 61.112 | 46.297 | 30.865 | 15.433 | 0.617 |
| 3.5 | 72.410 | 54.856 | 36.571 | 18.285 | 0.731 |
| 4.0 | 84.225 | 63.807 | 42.538 | 21.269 | 0.851 |
| 4.5 | 96.422 | 73.047 | 48.698 | 24.349 | 0.974 |
| 5.0 | 108.909 | 82.507 | 55.005 | 27.502 | 1.100 |
| (m=3, n=2) | | | | | |
| 0.25 | 218.007 | 165.157 | 110.105 | 55.052 | 2.202 |
| 0.5 | 103.668 | 78.536 | 52.357 | 26.179 | 1.047 |
| 1.0 | 59.059 | 44.747 | 29.828 | 14.914 | 0.597 |
| 1.5 | 52.958 | 40.120 | 26.747 | 13.373 | 0.535 |
| 2.0 | 55.997 | 42.422 | 28.281 | 14.141 | 0.566 |
| 2.5 | 62.762 | 47.547 | 31.698 | 15.849 | 0.634 |
| 3.0 | 71.510 | 54.174 | 36.116 | 18.058 | 0.722 |
| 3.5 | 81.482 | 61.729 | 41.152 | 20.576 | 0.823 |
| 4.0 | 92.279 | 69.908 | 46.606 | 23.303 | 0.932 |
| 4.5 | 103.666 | 78.535 | 52.357 | 26.178 | 1.047 |
| 5.0 | 115.493 | 87.494 | 58.330 | 29.165 | 1.167 |

TABLE 7b. NATURAL FREQUENCY INCLUDING SHEAR.

| a/b \ $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2$ (B.S.A. TI) | | | | ($h^2/ab=0.05$) |
|---|---|---|---|---|---|
| | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
| | (m=1, n=3) | | | | |
| 0.25 | 20.557 | 15.573 | 10.382 | 5.919 | 0.208 |
| 0.5 | 21.825 | 16.534 | 11.023 | 5.511 | 0.221 |
| 1.0 | 41.080 | 31.121 | 20.747 | 10.374 | 0.415 |
| 1.5 | 66.286 | 50.216 | 33.478 | 16.739 | 0.670 |
| 2.0 | 93.898 | 71.135 | 47.423 | 23.712 | 0.949 |
| 2.5 | 122.768 | 93.006 | 62.004 | 31.002 | 1.240 |
| 3.0 | 152.380 | 115.439 | 76.959 | 38.480 | 1.539 |
| 3.5 | 182.467 | 138.232 | 92.155 | 46.078 | 1.843 |
| 4.0 | 212.877 | 161.270 | 107.514 | 53.757 | 2.150 |
| 4.5 | 243.516 | 184.482 | 122.988 | 61.494 | 2.460 |
| 5.0 | 274.324 | 207.821 | 138.547 | 69.274 | 2.771 |
| | (m=2, n=3) | | | | |
| 0.25 | 92.280 | 69.909 | 46.606 | 23.303 | 0.932 |
| 0.5 | 55.997 | 42.422 | 28.281 | 14.141 | 0.566 |
| 1.0 | 59.059 | 44.742 | 29.828 | 14.914 | 0.597 |
| 1.5 | 78.900 | 59.773 | 39.848 | 19.924 | 0.797 |
| 2.0 | 103.667 | 78.535 | 52.357 | 26.179 | 1.047 |
| 2.5 | 130.741 | 99.046 | 66.031 | 33.015 | 1.321 |
| 3.0 | 159.112 | 120.539 | 80.360 | 40.180 | 1.607 |
| 3.5 | 188.289 | 142.643 | 95.096 | 47.548 | 1.902 |
| 4.0 | 218.004 | 165.155 | 110.103 | 55.052 | 2.202 |
| 4.5 | 248.095 | 187.951 | 125.301 | 62.650 | 2.506 |
| 5.0 | 278.460 | 210.955 | 140.636 | 70.318 | 2.813 |
| | (m=3, n=3) | | | | |
| 0.25 | 226.563 | 171.639 | 114.426 | 57.213 | 2.289 |
| 0.5 | 120.120 | 91.000 | 60.667 | 30.333 | 1.213 |
| 1.0 | 90.664 | 68.685 | 45.790 | 22.895 | 0.916 |
| 1.5 | 100.402 | 76.062 | 50.708 | 25.354 | 1.014 |
| 2.0 | 120.119 | 90.999 | 60.666 | 30.333 | 1.213 |
| 2.5 | 144.102 | 109.168 | 72.779 | 36.389 | 1.456 |
| 3.0 | 170.366 | 129.066 | 86.044 | 43.022 | 1.721 |
| 3.5 | 198.012 | 150.009 | 100.006 | 50.003 | 2.000 |
| 4.0 | 226.560 | 171.637 | 114.424 | 57.212 | 2.289 |
| 4.5 | 255.734 | 193.738 | 129.159 | 64.579 | 2.583 |
| 5.0 | 285.358 | 216.180 | 144.120 | 72.060 | 2.882 |

TABLE 7c. NATURAL FREQUENCY INCLUDING SHEAR.

$\hat{\hat{\Omega}}^2$(B.S.A. TI. RI) $(h^2/ab=0.05)$, $(\hat{\hat{\beta}}=0)$

| $b/a$ \ $\eta$ | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 11.295 | 14.663 | 20.765 |
| 0.5 | 4.621 | 10.230 | 22.046 |
| 1.0 | 3.120 | 14.663 | 41.495 |
| 1.5 | 3.596 | 22.910 | 66.955 |
| 2.0 | 4.621 | 32.864 | 94.847 |
| 2.5 | 5.969 | 43.854 | 124.008 |
| 3.0 | 7.559 | 55.536 | 153.919 |
| 3.5 | 9.344 | 67.706 | 184.310 |
| 4.0 | 11.294 | 80.232 | 215.027 |
| 4.5 | 13.386 | 93.029 | 245.976 |
| 5.0 | 15.601 | 106.035 | 277.095 |
| (m=2) | | | |
| 0.25 | 80.234 | 85.077 | 93.212 |
| 0.5 | 32.864 | 41.495 | 56.562 |
| 1.0 | 14.663 | 30.076 | 59.655 |
| 1.5 | 10.936 | 33.804 | 79.697 |
| 2.0 | 10.230 | 41.494 | 104.714 |
| 2.5 | 10.652 | 51.055 | 132.061 |
| 3.0 | 11.656 | 61.729 | 160.719 |
| 3.5 | 13.028 | 73.141 | 190.191 |
| 4.0 | 14.663 | 85.076 | 220.206 |
| 4.5 | 16.502 | 97.396 | 250.601 |
| 5.0 | 18.506 | 110.009 | 281.273 |
| (m=3) | | | |
| 0.25 | 215.030 | 220.209 | 228.852 |
| 0.5 | 94.848 | 104.715 | 121.333 |
| 1.0 | 41.495 | 59.656 | 91.579 |
| 1.5 | 27.340 | 53.493 | 101.416 |
| 2.0 | 22.046 | 56.562 | 121.333 |
| 2.5 | 20.005 | 63.396 | 145.557 |
| 3.0 | 19.502 | 72.232 | 172.087 |
| 3.5 | 19.861 | 82.305 | 200.012 |
| 4.0 | 20.764 | 93.211 | 228.849 |
| 4.5 | 22.046 | 104.713 | 258.317 |
| 5.0 | 23.607 | 116.659 | 288.240 |

TABLE 7d. NATURAL FREQUENCY INCLUDING SHEAR.

$\hat{\hat{\Omega}}^2$(B.S.A. TI) MINIMUM

($h^2/ab=0.05$)

| m | n | a/b \ $\hat{\beta}$ | 0.0 | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 3.120 | 3.089 | 2.340 | 1.560 | 0.780 | 0.031 |
| 2 | 1 | 2.0 | 10.230 | 10.128 | 7.673 | 5.115 | 2.558 | 0.102 |
| 3 | 1 | 3.0 | 19.502 | 19.307 | 14.627 | 9.751 | 4.876 | 0.195 |
| 1 | 2 | 0.5 | 10.230 | 10.128 | 7.673 | 5.115 | 2.558 | 0.102 |
| 2 | 2 | 1.0 | 30.076 | 29.775 | 22.557 | 15.038 | 7.519 | 0.301 |
| 3 | 2 | 1.5 | 53.493 | 52.958 | 40.120 | 26.747 | 13.373 | 0.535 |
| 1 | 3 | 0.33 | 19.502 | 19.307 | 14.627 | 9.751 | 4.876 | 0.195 |
| 2 | 3 | 0.67 | 53.493 | 52.958 | 40.120 | 26.741 | 13.373 | 0.5349 |
| 3 | 3 | 1.0 | 91.579 | 90.664 | 68.685 | 45.790 | 22.895 | 0.916 |

<u>TABLE 7e.</u> SUMMARY SHEET FOR NATURAL FREQUENCY INCLUDING SHEAR.

| a/b \ $\hat{\hat{\alpha}}$ | $\hat{\hat{\Omega}}^2$(B.A. TI) 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=1) | | | | |
| 0.25 | 17.882 | 13.547 | 9.031 | 4.516 | 0.181 |
| 0.5 | 6.188 | 4.688 | 3.125 | 1.563 | 0.063 |
| 1.0 | 3.960 | 3.000 | 2.000 | 1.000 | 0.040 |
| 1.5 | 4.648 | 3.521 | 2.347 | 1.174 | 0.047 |
| 2.0 | 6.187 | 4.688 | 3.125 | 1.563 | 0.063 |
| 2.5 | 8.326 | 6.307 | 4.205 | 2.103 | 0.084 |
| 3.0 | 11.000 | 8.333 | 5.556 | 2.778 | 0.111 |
| 3.5 | 14.188 | 10.749 | 7.166 | 3.583 | 0.143 |
| 4.0 | 17.882 | 13.547 | 9.031 | 4.516 | 0.181 |
| 4.5 | 22.076 | 16.724 | 11.149 | 5.575 | 0.223 |
| 5.0 | 26.769 | 20.280 | 13.520 | 6.760 | 0.270 |
| | (m=2, n=1) | | | | |
| 0.25 | 261.421 | 198.046 | 132.031 | 66.015 | 2.641 |
| 0.5 | 71.528 | 54.188 | 36.125 | 18.063 | 0.723 |
| 1.0 | 24.750 | 18.750 | 12.500 | 6.250 | 0.250 |
| 1.5 | 17.188 | 13.021 | 8.681 | 4.340 | 0.174 |
| 2.0 | 15.840 | 12.000 | 8.000 | 4.000 | 0.160 |
| 2.5 | 16.642 | 12.607 | 8.405 | 4.203 | 0.168 |
| 3.0 | 18.590 | 14.083 | 9.389 | 4.694 | 0.188 |
| 3.5 | 21.340 | 16.167 | 10.778 | 5.389 | 0.216 |
| 4.0 | 24.750 | 18.750 | 12.500 | 6.250 | 0.250 |
| 4.5 | 28.749 | 21.780 | 14.520 | 7.260 | 0.290 |
| 5.0 | 33.303 | 25.230 | 16.820 | 8.410 | 0.336 |
| | (m=3, n=1) | | | | |
| 0.25 | 1300.922 | 985.547 | 657.031 | 328.516 | 13.141 |
| 0.5 | 338.827 | 256.687 | 171.125 | 85.562 | 3.423 |
| 1.0 | 99.001 | 75.000 | 50.000 | 25.000 | 1.000 |
| 1.5 | 55.688 | 42.188 | 28.125 | 14.063 | 0.563 |
| 2.0 | 41.828 | 31.688 | 21.125 | 10.563 | 0.423 |
| 2.5 | 36.838 | 27.908 | 18.605 | 9.303 | 0.372 |
| 3.0 | 35.640 | 27.000 | 18.000 | 9.000 | 0.360 |
| 3.5 | 36.494 | 27.647 | 18.431 | 9.216 | 0.369 |
| 4.0 | 38.672 | 29.297 | 19.531 | 9.766 | 0.391 |
| 4.5 | 41.827 | 31.687 | 21.125 | 10.562 | 0.423 |
| 5.0 | 45.777 | 34.680 | 23.120 | 11.560 | 0.462 |

TABLE 8a. NATURAL FREQUENCY EXCLUDING SHEAR.

$\hat{\hat{\Omega}}^2$(B.A. TI)

| $a/b$ \ $\hat{\alpha}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 24.750 | 18.750 | 12.500 | 6.250 | 0.250 |
| 0.5 | 15.840 | 12.000 | 8.000 | 4.000 | 0.160 |
| 1.0 | 24.750 | 18.750 | 12.500 | 6.250 | 0.250 |
| 1.5 | 44.000 | 33.333 | 22.222 | 11.111 | 0.444 |
| 2.0 | 71.527 | 54.187 | 36.125 | 18.062 | 0.723 |
| 2.5 | 107.077 | 81.119 | 54.079 | 27.040 | 1.082 |
| 3.0 | 150.587 | 114.081 | 76.054 | 38.027 | 1.521 |
| 3.5 | 202.037 | 153.058 | 102.039 | 51.019 | 2.041 |
| 4.0 | 261.415 | 198.042 | 132.028 | 66.014 | 2.641 |
| 4.5 | 328.720 | 249.030 | 166.020 | 83.010 | 3.320 |
| 5.0 | 403.949 | 306.022 | 204.015 | 102.007 | 4.080 |
| | (m=2, n=2) | | | | |
| 0.25 | 286.109 | 216.750 | 144.500 | 72.250 | 2.890 |
| 0.5 | 99.000 | 75.000 | 50.000 | 25.000 | 1.000 |
| 1.0 | 63.360 | 48.000 | 32.000 | 16.000 | 0.640 |
| 1.5 | 74.360 | 56.333 | 37.555 | 18.778 | 0.751 |
| 2.0 | 98.999 | 74.999 | 50.000 | 25.000 | 1.000 |
| 2.5 | 133.213 | 100.919 | 67.279 | 33.640 | 1.346 |
| 3.0 | 175.997 | 133.331 | 88.887 | 44.444 | 1.778 |
| 3.5 | 227.009 | 171.976 | 114.651 | 57.325 | 2.293 |
| 4.0 | 286.104 | 216.745 | 144.497 | 72.248 | 2.890 |
| 4.5 | 353.213 | 267.586 | 178.391 | 89.195 | 3.568 |
| 5.0 | 428.303 | 324.572 | 216.315 | 108.157 | 4.326 |
| | (m=3, n=2) | | | | |
| 0.25 | 1355.310 | 1026.750 | 684.500 | 342.250 | 13.690 |
| 0.5 | 395.999 | 299.999 | 200.000 | 100.000 | 4.000 |
| 1.0 | 167.310 | 126.750 | 84.500 | 42.240 | 1.690 |
| 1.5 | 142.560 | 108.000 | 72.000 | 36.000 | 1.440 |
| 2.0 | 154.687 | 117.187 | 78.125 | 39.062 | 1.563 |
| 2.5 | 183.109 | 138.719 | 92.479 | 46.240 | 1.850 |
| 3.0 | 222.747 | 168.748 | 112.499 | 56.249 | 2.250 |
| 3.5 | 271.861 | 205.956 | 137.304 | 68.652 | 2.746 |
| 4.0 | 329.726 | 249.792 | 166.528 | 83.264 | 3.331 |
| 4.5 | 395.992 | 299.993 | 199.996 | 99.998 | 4.000 |
| 5.0 | 470.477 | 356.422 | 237.615 | 118.807 | 4.752 |

TABLE 8b. NATURAL FREQUENCY EXCLUDING SHEAR.

| $a/b$ \ $\hat{\alpha}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | | | $\hat{\Omega}^2$(B.A. TI) | | |
| (m=1, n=3) | | | | | |
| 0.25 | 38.672 | 29.297 | 19.531 | 9.766 | 0.391 |
| 0.5 | 41.828 | 31.688 | 21.125 | 10.563 | 0.423 |
| 1.0 | 99.000 | 75.000 | 50.000 | 25.000 | 1.000 |
| 1.5 | 198.685 | 150.519 | 100.346 | 50.173 | 2.007 |
| 2.0 | 338.822 | 256.683 | 171.122 | 85.561 | 3.423 |
| 2.5 | 519.155 | 393.299 | 262.200 | 131.100 | 5.244 |
| 3.0 | 739.624 | 560.321 | 373.547 | 186.774 | 7.471 |
| 3.5 | 1000.204 | 757.731 | 505.154 | 252.577 | 10.103 |
| 4.0 | 1300.891 | 985.523 | 657.016 | 328.508 | 13.140 |
| 4.5 | 1641.675 | 1243.693 | 829.129 | 414.564 | 16.583 |
| 5.0 | 2022.559 | 1532.241 | 1021.494 | 510.747 | 20.430 |
| (m=2, n=3) | | | | | |
| 0.25 | 329.731 | 249.796 | 166.531 | 83.265 | 3.331 |
| 0.5 | 154.688 | 117.188 | 78.125 | 39.063 | 1.563 |
| 1.0 | 167.310 | 126.750 | 84.500 | 42.250 | 1.690 |
| 1.5 | 258.745 | 196.019 | 130.679 | 65.340 | 2.614 |
| 2.0 | 395.994 | 299.996 | 199.997 | 99.999 | 4.000 |
| 2.5 | 574.991 | 435.599 | 290.399 | 145.200 | 5.808 |
| 3.0 | 794.734 | 602.071 | 401.381 | 200.690 | 8.028 |
| 3.5 | 1054.877 | 799.149 | 532.766 | 266.383 | 10.655 |
| 4.0 | 1355.279 | 1026.726 | 684.484 | 342.242 | 13.690 |
| 4.5 | 1695.868 | 1284.749 | 856.499 | 428.250 | 17.130 |
| 5.0 | 2076.612 | 1573.191 | 1048.794 | 524.397 | 20.976 |
| (m=3, n=3) | | | | | |
| 0.25 | 1448.432 | 1097.297 | 731.531 | 365.766 | 14.631 |
| 0.5 | 501.187 | 379.687 | 253.125 | 126.562 | 5.063 |
| 1.0 | 320.759 | 243.000 | 162.000 | 81.000 | 3.240 |
| 1.5 | 376.446 | 285.186 | 190.124 | 95.062 | 3.803 |
| 2.0 | 501.182 | 379.683 | 253.122 | 126.561 | 5.063 |
| 2.5 | 674.387 | 510.899 | 340.599 | 170.300 | 6.812 |
| 3.0 | 890.984 | 674.988 | 449.992 | 224.996 | 9.000 |
| 3.5 | 1149.231 | 870.629 | 580.419 | 290.210 | 11.608 |
| 4.0 | 1448.400 | 1097.273 | 731.515 | 365.758 | 14.630 |
| 4.5 | 1788.146 | 1354.656 | 903.104 | 451.552 | 18.062 |
| 5.0 | 2168.287 | 1642.641 | 1095.094 | 547.547 | 21.902 |

TABLE 8c. NATURAL FREQUENCY EXCLUDING SHEAR.

$\hat{\Omega}^2$( B.A. TI) ($\hat{\alpha}$=0)

| a/b \ n | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 18.063 | 25.000 | 39.063 |
| 0.5 | 6.250 | 16.000 | 42.250 |
| 1.0 | 4.000 | 25.000 | 100.000 |
| 1.5 | 4.694 | 44.444 | 200.692 |
| 2.0 | 6.250 | 72.249 | 342.244 |
| 2.5 | 8.410 | 108.158 | 524.399 |
| 3.0 | 11.111 | 152.108 | 747.095 |
| 3.5 | 14.331 | 204.077 | 1010.308 |
| 4.0 | 18.063 | 264.056 | 1314.032 |
| 4.5 | 22.299 | 332.041 | 1658.258 |
| 5.0 | 27.039 | 408.029 | 2042.989 |
| (m=2) | | | |
| 0.25 | 264.062 | 288.999 | 333.062 |
| 0.5 | 72.250 | 100.000 | 156.250 |
| 1.0 | 25.000 | 64.000 | 169.000 |
| 1.5 | 17.361 | 75.111 | 261.359 |
| 2.0 | 16.000 | 100.000 | 399.995 |
| 2.5 | 16.810 | 134.558 | 580.799 |
| 3.0 | 18.778 | 177.775 | 802.762 |
| 3.5 | 21.556 | 229.302 | 1065.533 |
| 4.0 | 25.000 | 288.994 | 1368.968 |
| 4.5 | 29.040 | 356.781 | 1712.998 |
| 5.0 | 33.639 | 432.629 | 2097.588 |
| (m=3) | | | |
| 0.25 | 1314.063 | 1369.000 | 1463.063 |
| 0.5 | 342.249 | 399.999 | 506.249 |
| 1.0 | 100.000 | 169.000 | 324.000 |
| 1.5 | 56.250 | 144.000 | 380.248 |
| 2.0 | 42.250 | 156.249 | 506.244 |
| 2.5 | 37.210 | 184.958 | 681.199 |
| 3.0 | 36.000 | 224.997 | 899.984 |
| 3.5 | 36.862 | 274.607 | 1160.839 |
| 4.0 | 39.062 | 333.056 | 1463.031 |
| 4.5 | 42.250 | 399.992 | 1806.209 |
| 5.0 | 46.240 | 475.229 | 2190.189 |

TABLE 8d. NATURAL FREQUENCY EXCLUDING SHEAR.

$\hat{\Omega}^2$ (B.A. TI) MINIMUM

| m | n | a/b \ $\hat{\alpha}$ | 0.0 | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 4.000 | 3.960 | 3.000 | 2.000 | 1.000 | 0.040 |
| 2 | 1 | 2.0 | 16.000 | 15.840 | 12.000 | 8.000 | 4.000 | 0.160 |
| 3 | 1 | 3.0 | 36.000 | 35.640 | 27.000 | 18.000 | 9.000 | 0.360 |
| 1 | 2 | 0.50 | 16.000 | 15.840 | 12.000 | 8.000 | 4.000 | 0.160 |
| 2 | 2 | 1.0 | 64.000 | 63.360 | 48.000 | 32.000 | 16.000 | 0.640 |
| 3 | 2 | 1.50 | 144.000 | 142.560 | 108.000 | 72.000 | 36.000 | 1.440 |
| 1 | 3 | 0.33 | 36.000 | 35.640 | 27.000 | 18.000 | 9.000 | 0.360 |
| 2 | 3 | 0.67 | 144.000 | 142.560 | 108.000 | 72.000 | 36.000 | 1.440 |
| 3 | 3 | 1.00 | 324.000 | 320.759 | 243.00 | 162.000 | 81.000 | 3.240 |

TABLE 8e. SUMMARY SHEET FOR NATURAL FREQUENCY EXCLUDING SHEAR.

| $a/b$ \ $\hat{\beta}$ | $\hat{\Omega}^2$ 0.01 | 0.25 | 0.50 | 0.75 | ($h^2/ab=0.01$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 0.8920 | 0.8622 | 0.8067 | 0.6760 | 0.0770 |
| 0.5 | 0.9335 | 0.9141 | 0.8764 | 0.7800 | 0.1242 |
| 1.0 | 0.9461 | 0.9301 | 0.8986 | 0.8159 | 0.1506 |
| 1.5 | 0.9419 | 0.9247 | 0.8911 | 0.8036 | 0.1407 |
| 2.0 | 0.9335 | 0.9141 | 0.8764 | 0.7800 | 0.1242 |
| 2.5 | 0.9237 | 0.9017 | 0.8594 | 0.7535 | 0.1090 |
| 3.0 | 0.9133 | 0.8886 | 0.8418 | 0.7268 | 0.0962 |
| 3.5 | 0.9027 | 0.8754 | 0.8241 | 0.7008 | 0.0857 |
| 4.0 | 0.8920 | 0.8622 | 0.8067 | 0.6760 | 0.0770 |
| 4.5 | 0.8814 | 0.8492 | 0.7897 | 0.6525 | 0.0699 |
| 5.0 | 0.8710 | 0.8365 | 0.7732 | 0.6303 | 0.0638 |
| | (m=2, n=1) | | | | |
| 0.25 | 0.6836 | 0.6207 | 0.5218 | 0.3530 | 0.0214 |
| 0.5 | 0.8051 | 0.7578 | 0.6760 | 0.5105 | 0.0400 |
| 1.0 | 0.8753 | 0.8418 | 0.7800 | 0.6394 | 0.0662 |
| 1.5 | 0.8939 | 0.8646 | 0.8097 | 0.6803 | 0.0784 |
| 2.0 | 0.8977 | 0.8693 | 0.8159 | 0.6891 | 0.0814 |
| 2.5 | 0.8954 | 0.8664 | 0.8122 | 0.6838 | 0.0796 |
| 3.0 | 0.8901 | 0.8599 | 0.8036 | 0.6717 | 0.0756 |
| 3.5 | 0.8832 | 0.8514 | 0.7925 | 0.6563 | 0.0710 |
| 4.0 | 0.8753 | 0.8418 | 0.7800 | 0.6394 | 0.0662 |
| 4.5 | 0.8669 | 0.8315 | 0.7669 | 0.6220 | 0.0617 |
| 5.0 | 0.8582 | 0.8210 | 0.7535 | 0.6045 | 0.0576 |
| | (m=3, n=1) | | | | |
| 0.25 | 0.4920 | 0.4232 | 0.3285 | 0.1965 | 0.0097 |
| 0.5 | 0.6549 | 0.5898 | 0.4894 | 0.3240 | 0.0188 |
| 1.0 | 0.7783 | 0.7268 | 0.6394 | 0.4699 | 0.0342 |
| 1.5 | 0.8240 | 0.7800 | 0.7027 | 0.5417 | 0.0451 |
| 2.0 | 0.8438 | 0.8036 | 0.7318 | 0.5770 | 0.0517 |
| 2.5 | 0.8520 | 0.8134 | 0.7440 | 0.5924 | 0.0549 |
| 3.0 | 0.8540 | 0.8159 | 0.7472 | 0.5964 | 0.0558 |
| 3.5 | 0.8526 | 0.8141 | 0.7449 | 0.5935 | 0.0552 |
| 4.0 | 0.8489 | 0.8097 | 0.7394 | 0.5865 | 0.0537 |
| 4.5 | 0.8438 | 0.8036 | 0.7318 | 0.5770 | 0.0517 |
| 5.0 | 0.8377 | 0.7964 | 0.7228 | 0.5659 | 0.0496 |

TABLE 9a. NATURAL FREQUENCY RATIO

| $a/b$ \ $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2$ 0.01 | 0.25 | 0.50 | 0.75 | ($h^2/ab=0.01$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 0.8763 | 0.8418 | 0.7800 | 0.6394 | 0.0662 |
| 0.5 | 0.8977 | 0.8693 | 0.8159 | 0.6891 | 0.0814 |
| 1.0 | 0.8753 | 0.8418 | 0.7800 | 0.6394 | 0.0662 |
| 1.5 | 0.8404 | 0.7996 | 0.7268 | 0.5708 | 0.0505 |
| 2.0 | 0.8051 | 0.7578 | 0.6760 | 0.5105 | 0.0401 |
| 2.5 | 0.7715 | 0.7189 | 0.6303 | 0.4602 | 0.0330 |
| 3.0 | 0.7400 | 0.6832 | 0.5898 | 0.4182 | 0.0279 |
| 3.5 | 0.7108 | 0.6506 | 0.5538 | 0.3829 | 0.0242 |
| 4.0 | 0.6836 | 0.6207 | 0.5218 | 0.3530 | 0.0214 |
| 4.5 | 0.6583 | 0.5934 | 0.4932 | 0.3273 | 0.0191 |
| 5.0 | 0.6348 | 0.5683 | 0.4675 | 0.3050 | 0.0173 |
| | (m=2, n=2) | | | | |
| 0.25 | 0.6738 | 0.6101 | 0.5105 | 0.3428 | 0.0204 |
| 0.5 | 0.7783 | 0.7268 | 0.6394 | 0.4699 | 0.0342 |
| 1.0 | 0.8144 | 0.7688 | 0.6891 | 0.5257 | 0.0424 |
| 1.5 | 0.8020 | 0.7542 | 0.6717 | 0.5057 | 0.0393 |
| 2.0 | 0.7783 | 0.7268 | 0.6394 | 0.4699 | 0.0342 |
| 2.5 | 0.7516 | 0.6963 | 0.6045 | 0.4332 | 0.0297 |
| 3.0 | 0.7248 | 0.6661 | 0.5708 | 0.3994 | 0.0259 |
| 3.5 | 0.6987 | 0.6372 | 0.5394 | 0.3693 | 0.0229 |
| 4.0 | 0.6738 | 0.6101 | 0.5105 | 0.3428 | 0.0204 |
| 4.5 | 0.6502 | 0.5847 | 0.4842 | 0.3194 | 0.0184 |
| 5.0 | 0.6280 | 0.5612 | 0.4602 | 0.2989 | 0.0168 |
| | (m=3, n=2) | | | | |
| 0.25 | 0.4869 | 0.4182 | 0.3240 | 0.1933 | 0.0095 |
| 0.5 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0174 |
| 1.0 | 0.7298 | 0.6717 | 0.5770 | 0.4055 | 0.0266 |
| 1.5 | 0.7453 | 0.6891 | 0.5964 | 0.4249 | 0.0287 |
| 2.0 | 0.7374 | 0.6803 | 0.5865 | 0.4149 | 0.0276 |
| 2.5 | 0.7208 | 0.6617 | 0.5659 | 0.3946 | 0.0254 |
| 3.0 | 0.7006 | 0.6394 | 0.5417 | 0.3715 | 0.0231 |
| 3.5 | 0.6793 | 0.6161 | 0.5769 | 0.3485 | 0.0210 |
| 4.0 | 0.6580 | 0.5931 | 0.4928 | 0.3270 | 0.0191 |
| 4.5 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0174 |
| 5.0 | 0.6169 | 0.5496 | 0.4485 | 0.2891 | 0.0160 |

TABLE 9b. NATURAL FREQUENCY RATIO.

| $\hat{\beta}$ \ a/b | 0.01 | 0.25 | $\hat{\Omega}^2$ 0.50 | 0.75 | ($h^2/ab=0.01$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=3) | | | | |
| 0.25 | 0.8489 | 0.8097 | 0.7394 | 0.5865 | 0.0537 |
| 0.5 | 0.8438 | 0.8036 | 0.7318 | 0.5770 | 0.0517 |
| 1.0 | 0.7783 | 0.7268 | 0.6394 | 0.4699 | 0.0342 |
| 1.5 | 0.7125 | 0.6525 | 0.5559 | 0.3849 | 0.0244 |
| 2.0 | 0.6549 | 0.5898 | 0.4894 | 0.3240 | 0.0188 |
| 2.5 | 0.6052 | 0.5373 | 0.4364 | 0.2791 | 0.0152 |
| 3.0 | 0.5623 | 0.4932 | 0.3935 | 0.2449 | 0.0128 |
| 3.5 | 0.5248 | 0.4556 | 0.3581 | 0.2181 | 0.0110 |
| 4.0 | 0.4920 | 0.4232 | 0.3285 | 0.1965 | 0.0097 |
| 4.5 | 0.4630 | 0.3951 | 0.3033 | 0.1788 | 0.0086 |
| 5.0 | 0.4372 | 0.3704 | 0.2818 | 0.1640 | 0.0078 |
| | (m=2, n=3) | | | | |
| 0.25 | 0.6580 | 0.5931 | 0.4928 | 0.3270 | 0.0191 |
| 0.5 | 0.7374 | 0.6803 | 0.5865 | 0.4149 | 0.0276 |
| 1.0 | 0.7298 | 0.6717 | 0.5770 | 0.4055 | 0.0266 |
| 1.5 | 0.6847 | 0.6220 | 0.5231 | 0.3542 | 0.0215 |
| 2.0 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0174 |
| 2.5 | 0.5930 | 0.5246 | 0.4239 | 0.2689 | 0.0145 |
| 3.0 | 0.5534 | 0.4892 | 0.3849 | 0.2383 | 0.0124 |
| 3.5 | 0.5182 | 0.4490 | 0.3520 | 0.2136 | 0.0107 |
| 4.0 | 0.4869 | 0.4182 | 0.3240 | 0.1933 | 0.0095 |
| 4.5 | 0.4589 | 0.3912 | 0.2999 | 0.1764 | 0.0085 |
| 5.0 | 0.4339 | 0.3674 | 0.2791 | 0.1622 | 0.0077 |
| | (m=3, n=3) | | | | |
| 0.25 | 0.4786 | 0.4101 | 0.3167 | 0.1882 | 0.0092 |
| 0.5 | 0.6094 | 0.5417 | 0.4407 | 0.2827 | 0.0155 |
| 1.0 | 0.6611 | 0.5964 | 0.4962 | 0.3300 | 0.0193 |
| 1.5 | 0.6429 | 0.5770 | 0.4762 | 0.3125 | 0.0179 |
| 2.0 | 0.6094 | 0.5417 | 0.4407 | 0.2827 | 0.0155 |
| 2.5 | 0.5736 | 0.5047 | 0.4045 | 0.2536 | 0.0134 |
| 3.0 | 0.5392 | 0.4699 | 0.3715 | 0.2281 | 0.0117 |
| 3.5 | 0.5075 | 0.4384 | 0.3423 | 0.2065 | 0.0103 |
| 4.0 | 0.4786 | 0.4101 | 0.3167 | 0.1882 | 0.0092 |
| 4.5 | 0.4524 | 0.3849 | 0.2944 | 0.1726 | 0.0083 |
| 5.0 | 0.4286 | 0.3624 | 0.2748 | 0.1593 | 0.0075 |

TABLE 9c. NATURAL FREQUENCY RATIO.

$\hat{\hat{\Omega}}^2$ $(h^2/ab=0.01)$, $(\hat{\hat{\beta}}=0)$

| $a/b$ \ $n$ | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 0.8930 | 0.8764 | 0.8502 |
| 0.5 | 0.9341 | 0.8966 | 0.8451 |
| 1.0 | 0.9466 | 0.8764 | 0.7800 |
| 1.5 | 0.9424 | 0.8418 | 0.7145 |
| 2.0 | 0.9341 | 0.8067 | 0.6572 |
| 2.5 | 0.9244 | 0.7732 | 0.6076 |
| 3.0 | 0.9141 | 0.7420 | 0.5647 |
| 3.5 | 0.9035 | 0.7128 | 0.5273 |
| 4.0 | 0.8930 | 0.6858 | 0.4945 |
| 4.5 | 0.8825 | 0.6606 | 0.4655 |
| 5.0 | 0.8721 | 0.6371 | 0.4396 |
| (m=2) | | | |
| 0.25 | 0.6858 | 0.6760 | 0.6602 |
| 0.5 | 0.8067 | 0.7800 | 0.7394 |
| 1.0 | 0.8764 | 0.8159 | 0.7318 |
| 1.5 | 0.8949 | 0.8036 | 0.6869 |
| 2.0 | 0.8986 | 0.7800 | 0.6394 |
| 2.5 | 0.8964 | 0.7535 | 0.5954 |
| 3.0 | 0.8911 | 0.7268 | 0.5559 |
| 3.5 | 0.8842 | 0.7008 | 0.5207 |
| 4.0 | 0.8764 | 0.6760 | 0.4894 |
| 4.5 | 0.8681 | 0.6525 | 0.4614 |
| 5.0 | 0.8594 | 0.6303 | 0.4364 |
| (m=3) | | | |
| 0.25 | 0.4945 | 0.4894 | 0.4811 |
| 0.5 | 0.6572 | 0.6394 | 0.6118 |
| 1.0 | 0.7800 | 0.7318 | 0.6633 |
| 1.5 | 0.8254 | 0.7472 | 0.6452 |
| 2.0 | 0.8451 | 0.7394 | 0.6118 |
| 2.5 | 0.8532 | 0.7228 | 0.5760 |
| 3.0 | 0.8553 | 0.7027 | 0.5417 |
| 3.5 | 0.8538 | 0.6815 | 0.5100 |
| 4.0 | 0.8502 | 0.6602 | 0.4811 |
| 4.5 | 0.8451 | 0.6394 | 0.4549 |
| 5.0 | 0.8391 | 0.6193 | 0.4311 |

TABLE 9d. NATURAL FREQUENCY RATIO.

| m | n | $\alpha/b$ \ $\hat{\beta}$ | $(h^2/ab=0.01)$ 0.0 | $\hat{\Omega}^2$ MAXIMUM 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 0.9466 | 0.9461 | 0.9301 | 0.8986 | 0.8159 | 0.1506 |
| 2 | 1 | 2.0 | 0.8986 | 0.8977 | 0.8693 | 0.8159 | 0.6891 | 0.0814 |
| 3 | 1 | 3.0 | 0.8553 | 0.8540 | 0.8159 | 0.7472 | 0.5964 | 0.0558 |
| 1 | 2 | 0.5 | 0.8986 | 0.8977 | 0.8693 | 0.8159 | 0.6891 | 0.0814 |
| 2 | 2 | 1.0 | 0.8159 | 0.8144 | 0.7268 | 0.6891 | 0.5257 | 0.0424 |
| 3 | 2 | 1.5 | 0.7472 | 0.7453 | 0.6891 | 0.5964 | 0.4249 | 0.0287 |
| 1 | 3 | 0.33 | 0.8553 | 0.8540 | 0.8159 | 0.7472 | 0.5964 | 0.0558 |
| 2 | 3 | 0.67 | 0.7472 | 0.7453 | 0.6891 | 0.5964 | 0.4249 | 0.0287 |
| 3 | 3 | 1.0 | 0.6633 | 0.6611 | 0.5964 | 0.4962 | 0.3300 | 0.0193 |

TABLE 9e. SUMMARY SHEET FOR NATURAL FREQUENCY SQUARE RATIO.

| $a/b$ \ $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2$ 0.01 | 0.25 | 0.50 | 0.75 | $(h^2/ab=0.02)$ 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=1) | | | | |
| 0.25 | 0.8051 | 0.7578 | 0.6760 | 0.5105 | 0.0770 |
| 0.5 | 0.8753 | 0.8418 | 0.7800 | 0.6394 | 0.1242 |
| 1.0 | 0.8977 | 0.8693 | 0.8159 | 0.6891 | 0.1506 |
| 1.5 | 0.8901 | 0.8599 | 0.8036 | 0.6717 | 0.1407 |
| 2.0 | 0.8752 | 0.8418 | 0.7800 | 0.6394 | 0.1242 |
| 2.5 | 0.8582 | 0.8210 | 0.7535 | 0.6045 | 0.1090 |
| 3.0 | 0.8404 | 0.7996 | 0.7268 | 0.5708 | 0.0962 |
| 3.5 | 0.8226 | 0.7784 | 0.7008 | 0.5394 | 0.0857 |
| 4.0 | 0.8051 | 0.7578 | 0.6760 | 0.5105 | 0.0770 |
| 4.5 | 0.7880 | 0.7380 | 0.6525 | 0.4842 | 0.0699 |
| 5.0 | 0.7715 | 0.7189 | 0.6303 | 0.4602 | 0.0638 |
| | (m=2, n=1) | | | | |
| 0.25 | 0.5193 | 0.4501 | 0.3530 | 0.2143 | 0.0214 |
| 0.5 | 0.6738 | 0.6101 | 0.5105 | 0.3428 | 0.0400 |
| 1.0 | 0.7783 | 0.7268 | 0.6394 | 0.4699 | 0.0662 |
| 1.5 | 0.8082 | 0.7614 | 0.6803 | 0.5155 | 0.0784 |
| 2.0 | 0.8144 | 0.7688 | 0.6891 | 0.5257 | 0.0814 |
| 2.5 | 0.8107 | 0.7643 | 0.6838 | 0.5195 | 0.0796 |
| 3.0 | 0.8020 | 0.7542 | 0.6717 | 0.5057 | 0.0756 |
| 3.5 | 0.7908 | 0.7412 | 0.6563 | 0.4884 | 0.0710 |
| 4.0 | 0.7783 | 0.7268 | 0.6394 | 0.4699 | 0.0662 |
| 4.5 | 0.7651 | 0.7116 | 0.6220 | 0.4513 | 0.0617 |
| 5.0 | 0.7516 | 0.6963 | 0.6045 | 0.4332 | 0.0576 |
| | (m=3, n=1) | | | | |
| 0.25 | 0.3263 | 0.2684 | 0.1965 | 0.1090 | 0.0097 |
| 0.5 | 0.4869 | 0.4182 | 0.3240 | 0.1933 | 0.0188 |
| 1.0 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0342 |
| 1.5 | 0.7006 | 0.6394 | 0.5417 | 0.3715 | 0.0451 |
| 2.0 | 0.7298 | 0.6717 | 0.5770 | 0.4055 | 0.0517 |
| 2.5 | 0.7421 | 0.6855 | 0.5924 | 0.4209 | 0.0549 |
| 3.0 | 0.7453 | 0.6891 | 0.5964 | 0.4249 | 0.0558 |
| 3.5 | 0.7430 | 0.6866 | 0.5935 | 0.4220 | 0.0552 |
| 4.0 | 0.7374 | 0.6803 | 0.5865 | 0.4149 | 0.0537 |
| 4.5 | 0.7298 | 0.6717 | 0.5770 | 0.4055 | 0.0517 |
| 5.0 | 0.7208 | 0.6617 | 0.5659 | 0.3946 | 0.0496 |

TABLE 10a. NATURAL FREQUENCY RATIO.

| a/b \ $\hat{\beta}$ | $\hat{\Omega}^2$ 0.01 | 0.25 | 0.50 | 0.75 | ($h^2/ab=0.02$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 0.7783 | 0.7268 | 0.6394 | 0.4699 | 0.0662 |
| 0.5 | 0.8144 | 0.7688 | 0.6891 | 0.5257 | 0.0814 |
| 1.0 | 0.7783 | 0.7268 | 0.6394 | 0.4699 | 0.0662 |
| 1.5 | 0.7248 | 0.6661 | 0.5708 | 0.3994 | 0.0505 |
| 2.0 | 0.6738 | 0.6101 | 0.5105 | 0.3428 | 0.0401 |
| 2.5 | 0.6280 | 0.5612 | 0.4602 | 0.2989 | 0.0330 |
| 3.0 | 0.5873 | 0.5188 | 0.4182 | 0.2644 | 0.0279 |
| 3.5 | 0.5513 | 0.4821 | 0.3829 | 0.2368 | 0.0242 |
| 4.0 | 0.5193 | 0.4501 | 0.3530 | 0.2143 | 0.0214 |
| 4.5 | 0.4907 | 0.4219 | 0.3273 | 0.1957 | 0.0191 |
| 5.0 | 0.4650 | 0.3970 | 0.3050 | 0.1800 | 0.0173 |
| | (m=2, n=2) | | | | |
| 0.25 | 0.5080 | 0.4389 | 0.3428 | 0.2068 | 0.0204 |
| 0.5 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0342 |
| 1.0 | 0.6869 | 0.6244 | 0.5257 | 0.3565 | 0.0424 |
| 1.5 | 0.6695 | 0.6054 | 0.5057 | 0.3384 | 0.0393 |
| 2.0 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0342 |
| 2.5 | 0.6021 | 0.5341 | 0.4332 | 0.2765 | 0.0297 |
| 3.0 | 0.5684 | 0.4993 | 0.3994 | 0.2495 | 0.0259 |
| 3.5 | 0.5369 | 0.4676 | 0.3693 | 0.2264 | 0.0229 |
| 4.0 | 0.5080 | 0.4389 | 0.3428 | 0.2068 | 0.0204 |
| 4.5 | 0.4817 | 0.4132 | 0.3194 | 0.1901 | 0.0184 |
| 5.0 | 0.4577 | 0.3900 | 0.2989 | 0.1757 | 0.0168 |
| | (m=3, n=2) | | | | |
| 0.25 | 0.3218 | 0.2644 | 0.1933 | 0.1070 | 0.0095 |
| 0.5 | 0.4674 | 0.3994 | 0.3071 | 0.1814 | 0.0174 |
| 1.0 | 0.5745 | 0.5057 | 0.4055 | 0.2543 | 0.0266 |
| 1.5 | 0.5940 | 0.5257 | 0.4249 | 0.2698 | 0.0287 |
| 2.0 | 0.5841 | 0.5155 | 0.4149 | 0.2618 | 0.0276 |
| 2.5 | 0.5635 | 0.4944 | 0.3946 | 0.2458 | 0.0254 |
| 3.0 | 0.5392 | 0.4699 | 0.3715 | 0.2281 | 0.0231 |
| 3.5 | 0.5144 | 0.4452 | 0.3485 | 0.2110 | 0.0210 |
| 4.0 | 0.4903 | 0.4215 | 0.3270 | 0.1954 | 0.0191 |
| 4.5 | 0.4674 | 0.3994 | 0.3071 | 0.1814 | 0.0174 |
| 5.0 | 0.4461 | 0.3789 | 0.2891 | 0.1690 | 0.0160 |

TABLE 10b. NATURAL FREQUENCY RATIO.

| $a/b$ \ $\hat{\beta}$ | $\hat{\Omega}^2$ 0.01 | 0.25 | 0.50 | 0.75 | ($h^2/ab=0.02$) 0.99 |
|---|---|---|---|---|---|
| (m=1, n=3) | | | | | |
| 0.25 | 0.7374 | 0.6803 | 0.5865 | 0.4149 | 0.0537 |
| 0.5 | 0.7298 | 0.6717 | 0.5770 | 0.4055 | 0.0517 |
| 1.0 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0342 |
| 1.5 | 0.5534 | 0.4842 | 0.3849 | 0.2383 | 0.0244 |
| 2.0 | 0.4869 | 0.4182 | 0.3240 | 0.1933 | 0.0188 |
| 2.5 | 0.4339 | 0.3674 | 0.2791 | 0.1622 | 0.0152 |
| 3.0 | 0.3911 | 0.3273 | 0.2449 | 0.1395 | 0.0128 |
| 3.5 | 0.3558 | 0.2950 | 0.2181 | 0.1224 | 0.0110 |
| 4.0 | 0.3263 | 0.2684 | 0.1965 | 0.1090 | 0.0097 |
| 4.5 | 0.3012 | 0.2462 | 0.1788 | 0.0982 | 0.0083 |
| 5.0 | 0.2797 | 0.2273 | 0.1640 | 0.0893 | 0.0078 |
| (m=2, n=3) | | | | | |
| 0.25 | 0.4903 | 0.4215 | 0.3270 | 0.1954 | 0.0191 |
| 0.5 | 0.5841 | 0.5155 | 0.4149 | 0.2618 | 0.0276 |
| 1.0 | 0.5735 | 0.5057 | 0.4055 | 0.2543 | 0.0266 |
| 1.5 | 0.5206 | 0.4513 | 0.3542 | 0.2152 | 0.0215 |
| 2.0 | 0.4674 | 0.3994 | 0.3071 | 0.1814 | 0.0174 |
| 2.5 | 0.4214 | 0.3556 | 0.2689 | 0.1554 | 0.0145 |
| 3.0 | 0.3825 | 0.3194 | 0.2383 | 0.1353 | 0.0124 |
| 3.5 | 0.3497 | 0.2895 | 0.2136 | 0.1196 | 0.0107 |
| 4.0 | 0.3218 | 0.2644 | 0.1933 | 0.1070 | 0.0095 |
| 4.5 | 0.2978 | 0.2432 | 0.1764 | 0.0967 | 0.0085 |
| 5.0 | 0.2771 | 0.2250 | 0.1622 | 0.0882 | 0.0077 |
| (m=3, n=3) | | | | | |
| 0.25 | 0.3146 | 0.2580 | 0.1882 | 0.1039 | 0.0092 |
| 0.5 | 0.4383 | 0.3715 | 0.2827 | 0.1646 | 0.0155 |
| 1.0 | 0.4937 | 0.4249 | 0.3300 | 0.1976 | 0.0193 |
| 1.5 | 0.4737 | 0.4055 | 0.3125 | 0.1852 | 0.0179 |
| 2.0 | 0.4383 | 0.3715 | 0.2827 | 0.1646 | 0.0155 |
| 2.5 | 0.4021 | 0.3375 | 0.2536 | 0.1452 | 0.0134 |
| 3.0 | 0.3691 | 0.3071 | 0.2281 | 0.1287 | 0.0117 |
| 3.5 | 0.3400 | 0.2807 | 0.2065 | 0.1151 | 0.0103 |
| 4.0 | 0.3146 | 0.2580 | 0.1882 | 0.1039 | 0.0092 |
| 4.5 | 0.2933 | 0.2383 | 0.1726 | 0.0945 | 0.0083 |
| 5.0 | 0.2728 | 0.2213 | 0.1593 | 0.0865 | 0.0075 |

TABLE 10c. NATURAL FREQUENCY RATIO.

$\hat{\hat{\Omega}}^2$ ($h^2/ab=0.02$, $\hat{\hat{\beta}}=0$)

| $\frac{CL}{b}$ \ n | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 0.8067 | 0.7800 | 0.7394 |
| 0.5 | 0.8764 | 0.8159 | 0.7318 |
| 1.0 | 0.8986 | 0.7800 | 0.6394 |
| 1.5 | 0.8911 | 0.7268 | 0.5550 |
| 2.0 | 0.8764 | 0.6760 | 0.4894 |
| 2.5 | 0.8594 | 0.6303 | 0.4364 |
| 3.0 | 0.8418 | 0.5898 | 0.3935 |
| 3.5 | 0.8241 | 0.5538 | 0.3581 |
| 4.0 | 0.8067 | 0.5218 | 0.3285 |
| 4.5 | 0.7897 | 0.4932 | 0.3033 |
| 5.0 | 0.7732 | 0.4675 | 0.2818 |
| (m=2) | | | |
| 0.25 | 0.5218 | 0.5105 | 0.4928 |
| 0.5 | 0.6760 | 0.6394 | 0.5865 |
| 1.0 | 0.7800 | 0.6891 | 0.5770 |
| 1.5 | 0.8097 | 0.6717 | 0.5231 |
| 2.0 | 0.8159 | 0.6394 | 0.4699 |
| 2.5 | 0.8122 | 0.6045 | 0.4239 |
| 3.0 | 0.8036 | 0.5708 | 0.3849 |
| 3.5 | 0.7925 | 0.5394 | 0.3520 |
| 4.0 | 0.7800 | 0.5105 | 0.3240 |
| 4.5 | 0.7669 | 0.4842 | 0.2000 |
| 5.0 | 0.7535 | 0.4602 | 0.2791 |
| (m=3) | | | |
| 0.25 | 0.3285 | 0.3240 | 0.3167 |
| 0.5 | 0.4849 | 0.4699 | 0.4407 |
| 1.0 | 0.6394 | 0.5770 | 0.4962 |
| 1.5 | 0.7027 | 0.5964 | 0.4762 |
| 2.0 | 0.7317 | 0.5865 | 0.4407 |
| 2.5 | 0.7440 | 0.5659 | 0.4045 |
| 3.0 | 0.7472 | 0.5417 | 0.3715 |
| 3.5 | 0.7449 | 0.5169 | 0.3423 |
| 4.0 | 0.7394 | 0.4928 | 0.3167 |
| 4.5 | 0.7319 | 0.4699 | 0.2944 |
| 5.0 | 0.7228 | 0.4485 | 0.2748 |

TABLE 10d. NATURAL FREQUENCY RATIO.

$\hat{\Omega}^2$ MAXIMUM

| m | n | $a/b$ \ $\hat{\beta}$ | 0.0 $(h^2/ab=0.02)$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 0.8986 | 0.8977 | 0.8693 | 0.8159 | 0.6891 | 0.0814 |
| 2 | 1 | 2.0 | 0.8159 | 0.8144 | 0.7688 | 0.6891 | 0.5257 | 0.0424 |
| 3 | 1 | 3.0 | 0.7472 | 0.7453 | 0.6891 | 0.5964 | 0.4249 | 0.0287 |
| 1 | 2 | 0.5 | 0.8159 | 0.8144 | 0.7688 | 0.6891 | 0.5257 | 0.0424 |
| 2 | 2 | 1.0 | 0.6891 | 0.6869 | 0.6244 | 0.5257 | 0.3565 | 0.0217 |
| 3 | 2 | 1.5 | 0.5964 | 0.5940 | 0.5257 | 0.4249 | 0.2698 | 0.0146 |
| 1 | 3 | 0.33 | 0.7472 | 0.7453 | 0.6891 | 0.5964 | 0.4249 | 0.0287 |
| 2 | 3 | 0.67 | 0.5964 | 0.5940 | 0.5257 | 0.4249 | 0.2698 | 0.0146 |
| 3 | 3 | 1.0 | 0.4962 | 0.4937 | 0.4249 | 0.3300 | 0.1976 | 0.0098 |

TABLE 10e. SUMMARY SHEET FOR NATURAL FREQUENCY SQUARE RATIO.

| a/b \ $\hat{\beta}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | | | $\hat{\Omega}^2$ | | ($h^2/ab=0.03$) |
| (m=1, n=1) | | | | | |
| 0.25 | 0.7336 | 0.6760 | 0.5817 | 0.4101 | 0.0271 |
| 0.5 | 0.8240 | 0.7800 | 0.7027 | 0.5417 | 0.0451 |
| 1.0 | 0.8540 | 0.8195 | 0.7472 | 0.5964 | 0.0558 |
| 1.5 | 0.8438 | 0.8036 | 0.7318 | 0.5770 | 0.0517 |
| 2.0 | 0.8240 | 0.7800 | 0.7027 | 0.5417 | 0.0451 |
| 2.5 | 0.8014 | 0.7535 | 0.6708 | 0.5047 | 0.0392 |
| 3.0 | 0.7783 | 0.7268 | 0.6394 | 0.4699 | 0.0342 |
| 3.5 | 0.7556 | 0.7008 | 0.6096 | 0.4384 | 0.0303 |
| 4.0 | 0.7336 | 0.6760 | 0.5817 | 0.4102 | 0.0271 |
| 4.5 | 0.7125 | 0.6525 | 0.5559 | 0.3849 | 0.0244 |
| 5.0 | 0.6924 | 0.6303 | 0.5320 | 0.3624 | 0.0222 |
| (m=2, n=1) | | | | | |
| 0.25 | 0.4187 | 0.3530 | 0.2667 | 0.1539 | 0.0072 |
| 0.5 | 0.5793 | 0.5105 | 0.4101 | 0.2580 | 0.0137 |
| 1.0 | 0.7006 | 0.6394 | 0.5417 | 0.3715 | 0.0231 |
| 1.5 | 0.7374 | 0.6803 | 0.5865 | 0.4149 | 0.0276 |
| 2.0 | 0.7453 | 0.6891 | 0.5964 | 0.4229 | 0.0287 |
| 2.5 | 0.7405 | 0.6838 | 0.5904 | 0.4189 | 0.0280 |
| 3.0 | 0.7298 | 0.6717 | 0.5770 | 0.4055 | 0.0266 |
| 3.5 | 0.7160 | 0.6563 | 0.5601 | 0.3889 | 0.0248 |
| 4.0 | 0.7006 | 0.6394 | 0.5417 | 0.3715 | 0.0231 |
| 4.5 | 0.6847 | 0.6220 | 0.5231 | 0.3542 | 0.0215 |
| 5.0 | 0.6686 | 0.6045 | 0.5047 | 0.3375 | 0.0200 |
| (m=3, n=1) | | | | | |
| 0.25 | 0.2440 | 0.1965 | 0.1402 | 0.0754 | 0.0033 |
| 0.5 | 0.3875 | 0.3240 | 0.2421 | 0.1377 | 0.0063 |
| 1.0 | 0.5392 | 0.4699 | 0.3715 | 0.2281 | 0.0117 |
| 1.5 | 0.6094 | 0.5417 | 0.4407 | 0.2827 | 0.0155 |
| 2.0 | 0.6429 | 0.5770 | 0.4762 | 0.3125 | 0.0179 |
| 2.5 | 0.6574 | 0.5924 | 0.4921 | 0.3264 | 0.0190 |
| 3.0 | 0.6611 | 0.5964 | 0.4962 | 0.3300 | 0.0193 |
| 3.5 | 0.6584 | 0.5935 | 0.4933 | 0.3274 | 0.0191 |
| 4.0 | 0.6519 | 0.5865 | 0.4860 | 0.3210 | 0.0186 |
| 4.5 | 0.6429 | 0.5770 | 0.4762 | 0.3125 | 0.0179 |
| 5.0 | 0.6325 | 0.5659 | 0.4650 | 0.3029 | 0.0171 |

TABLE 11a. NATURAL FREQUENCY RATIO.

| a/b \ $\hat{\hat{\beta}}$ | 0.01 | 0.25 | $\hat{\hat{\Omega}}^2$ 0.50 | 0.75 | ($h^2/ab=0.03$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 0.7006 | 0.6394 | 0.5417 | 0.3715 | 0.0231 |
| 0.5 | 0.7453 | 0.6891 | 0.5964 | 0.4249 | 0.0287 |
| 1.0 | 0.7006 | 0.6394 | 0.5417 | 0.3715 | 0.0231 |
| 1.5 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0175 |
| 2.0 | 0.5793 | 0.5105 | 0.4101 | 0.2580 | 0.0137 |
| 2.5 | 0.5295 | 0.4602 | 0.3264 | 0.2213 | 0.0112 |
| 3.0 | 0.4869 | 0.4182 | 0.3240 | 0.1933 | 0.0095 |
| 3.5 | 0.4503 | 0.3829 | 0.2927 | 0.1714 | 0.0082 |
| 4.0 | 0.4187 | 0.3530 | 0.2667 | 0.1539 | 0.0072 |
| 4.5 | 0.3911 | 0.3273 | 0.2449 | 0.1395 | 0.0064 |
| 5.0 | 0.3668 | 0.3050 | 0.2264 | 0.1276 | 0.0058 |
| | (m=2, n=2) | | | | |
| 0.25 | 0.4077 | 0.3428 | 0.2580 | 0.1481 | 0.0069 |
| 0.5 | 0.5392 | 0.4699 | 0.3715 | 0.2281 | 0.0117 |
| 1.0 | 0.5940 | 0.5257 | 0.4249 | 0.2698 | 0.0146 |
| 1.5 | 0.5745 | 0.5057 | 0.4055 | 0.2543 | 0.0135 |
| 2.0 | 0.5392 | 0.4699 | 0.3715 | 0.2281 | 0.0117 |
| 2.5 | 0.5022 | 0.4332 | 0.3375 | 0.2030 | 0.0101 |
| 3.0 | 0.4674 | 0.3994 | 0.3071 | 0.1814 | 0.0088 |
| 3.5 | 0.4359 | 0.3693 | 0.2807 | 0.1633 | 0.0077 |
| 4.0 | 0.4077 | 0.3428 | 0.2580 | 0.1481 | 0.0069 |
| 4.5 | 0.3825 | 0.3194 | 0.2383 | 0.1353 | 0.0062 |
| 5.0 | 0.3601 | 0.2989 | 0.2213 | 0.1244 | 0.0057 |
| | (m=3, n=2) | | | | |
| 0.25 | 0.2403 | 0.1933 | 0.1377 | 0.0740 | 0.0032 |
| 0.5 | 0.3691 | 0.3071 | 0.2281 | 0.1287 | 0.0059 |
| 1.0 | 0.4737 | 0.4055 | 0.3125 | 0.1852 | 0.0090 |
| 1.5 | 0.4937 | 0.4249 | 0.3300 | 0.1976 | 0.0098 |
| 2.0 | 0.4835 | 0.4149 | 0.3210 | 0.1912 | 0.0094 |
| 2.5 | 0.4625 | 0.3946 | 0.3029 | 0.1785 | 0.0086 |
| 3.0 | 0.4383 | 0.3715 | 0.2827 | 0.1646 | 0.0078 |
| 3.5 | 0.4139 | 0.3485 | 0.2629 | 0.1513 | 0.0071 |
| 4.0 | 0.3907 | 0.3270 | 0.2446 | 0.1394 | 0.0064 |
| 4.5 | 0.3691 | 0.3071 | 0.2281 | 0.1287 | 0.0059 |
| 5.0 | 0.3493 | 0.2891 | 0.2133 | 0.1194 | 0.0054 |

TABLE 11b. NATURAL FREQUENCY RATIO.

$\hat{\hat{\Omega}}^2$ ($h^2/ab=0.03$)

| $a/b$ \ $\hat{\hat{\beta}}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=3) | | | | |
| 0.25 | 0.6519 | 0.5865 | 0.4860 | 0.3210 | 0.0186 |
| 0.5 | 0.6429 | 0.5770 | 0.4762 | 0.3125 | 0.0179 |
| 1.0 | 0.5392 | 0.4699 | 0.3715 | 0.2281 | 0.0117 |
| 1.5 | 0.4524 | 0.3849 | 0.2944 | 0.1726 | 0.0083 |
| 2.0 | 0.3875 | 0.3240 | 0.2421 | 0.1377 | 0.0063 |
| 2.5 | 0.3382 | 0.2791 | 0.2051 | 0.1143 | 0.0051 |
| 3.0 | 0.2998 | 0.2449 | 0.1778 | 0.0976 | 0.0043 |
| 3.5 | 0.2691 | 0.2181 | 0.1568 | 0.0851 | 0.0037 |
| 4.0 | 0.2440 | 0.1965 | 0.1402 | 0.0754 | 0.0033 |
| 4.5 | 0.2232 | 0.1788 | 0.1267 | 0.0677 | 0.0029 |
| 5.0 | 0.2057 | 0.1640 | 0.1156 | 0.0614 | 0.0026 |
| | (m=2, n=3) | | | | |
| 0.25 | 0.3907 | 0.3270 | 0.2546 | 0.1394 | 0.0064 |
| 0.5 | 0.4835 | 0.4149 | 0.3210 | 0.1912 | 0.0094 |
| 1.0 | 0.4737 | 0.4055 | 0.3125 | 0.1852 | 0.0090 |
| 1.5 | 0.4199 | 0.3542 | 0.2677 | 0.1545 | 0.0073 |
| 2.0 | 0.3691 | 0.3071 | 0.2281 | 0.1287 | 0.0059 |
| 2.5 | 0.3269 | 0.2689 | 0.1969 | 0.1092 | 0.0049 |
| 3.0 | 0.2923 | 0.2383 | 0.1726 | 0.0945 | 0.0042 |
| 3.5 | 0.2639 | 0.2136 | 0.1533 | 0.0830 | 0.0036 |
| 4.0 | 0.2403 | 0.1933 | 0.1377 | 0.0740 | 0.0032 |
| 4.5 | 0.2204 | 0.1764 | 0.1250 | 0.0666 | 0.0028 |
| 5.0 | 0.2035 | 0.1622 | 0.1143 | 0.0606 | 0.0026 |
| | (m=3, n=3) | | | | |
| 0.25 | 0.2343 | .1882 | 0.1338 | 0.0717 | 0.0031 |
| 0.5 | 0.3422 | 0.2827 | 0.2080 | 0.1161 | 0.0052 |
| 1.0 | 0.3940 | 0.3300 | 0.2472 | 0.1410 | 0.0065 |
| 1.5 | 0.3751 | 0.3125 | 0.2326 | 0.1316 | 0.0060 |
| 2.0 | 0.3422 | 0.2827 | 0.2080 | 0.1161 | 0.0052 |
| 2.5 | 0.3096 | 0.2536 | 0.1846 | 0.1017 | 0.0045 |
| 3.0 | 0.2806 | 0.2281 | 0.1646 | 0.0897 | 0.0039 |
| 3.5 | 0.2557 | 0.2065 | 0.1478 | 0.0798 | 0.0035 |
| 4.0 | 0.2343 | 0.1882 | 0.1338 | 0.0717 | 0.0031 |
| 4.5 | 0.2159 | 0.1726 | 0.1221 | 0.0650 | 0.0028 |
| 5.0 | 0.2000 | 0.1593 | 0.1121 | 0.0594 | 0.0025 |

TABLE 11c. NATURAL FREQUENCY RATIO.

$\hat{\hat{\Omega}}^2$ $(h^2/ab=0.03,\ \hat{\hat{\beta}}=0)$

| a/b \ n | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 0.7355 | 0.7027 | 0.6541 |
| 0.5 | 0.8254 | 0.7472 | 0.6452 |
| 1.0 | 0.8553 | 0.7027 | 0.5417 |
| 1.5 | 0.8451 | 0.6394 | 0.4549 |
| 2.0 | 0.8254 | 0.5817 | 0.3899 |
| 2.5 | 0.8030 | 0.5320 | 0.3405 |
| 3.0 | 0.7800 | 0.4894 | 0.3019 |
| 3.5 | 0.7574 | 0.4528 | 0.2711 |
| 4.0 | 0.7355 | 0.4211 | 0.2459 |
| 4.5 | 0.7146 | 0.3935 | 0.2250 |
| 5.0 | 0.6945 | 0.3692 | 0.2073 |
| (m=2) | | | |
| 0.25 | 0.4211 | 0.4101 | 0.3931 |
| 0.5 | 0.5817 | 0.5417 | 0.4860 |
| 1.0 | 0.7027 | 0.5964 | 0.4762 |
| 1.5 | 0.7394 | 0.5770 | 0.4224 |
| 2.0 | 0.7472 | 0.5417 | 0.3715 |
| 2.5 | 0.7425 | 0.5047 | 0.3291 |
| 3.0 | 0.7318 | 0.4699 | 0.2944 |
| 3.5 | 0.7180 | 0.4384 | 0.2659 |
| 4.0 | 0.7027 | 0.4101 | 0.2421 |
| 4.5 | 0.6869 | 0.3849 | 0.2222 |
| 5.0 | 0.6708 | 0.3624 | 0.2051 |
| (m=3) | | | |
| 0.25 | 0.2459 | 0.2421 | 0.2361 |
| 0.5 | 0.3899 | 0.3715 | 0.3444 |
| 1.0 | 0.5417 | 0.4762 | 0.3964 |
| 1.5 | 0.6118 | 0.4962 | 0.3774 |
| 2.0 | 0.6452 | 0.4860 | 0.3444 |
| 2.5 | 0.6596 | 0.4650 | 0.3117 |
| 3.0 | 0.6633 | 0.4407 | 0.2827 |
| 3.5 | 0.6607 | 0.4163 | 0.2576 |
| 4.0 | 0.6541 | 0.3931 | 0.2361 |
| 4.5 | 0.6452 | 0.3715 | 0.2176 |
| 5.0 | 0.6348 | 0.3516 | 0.2016 |

TABLE 11d. NATURAL FREQUENCY RATIO.

| | | | $\hat{\Omega}^2$ MAXIMUM | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | $(h^2/ab=0.03)$ | | | | | |
| m | n | $a/b$ \ $\hat{\beta}$ | 0.0 | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
| 1 | 1 | 1.0 | 0.8553 | 0.8540 | 0.8159 | 0.7472 | 0.5964 | 0.0558 |
| 2 | 1 | 2.0 | 0.7472 | 0.7453 | 0.6891 | 0.5964 | 0.4249 | 0.0287 |
| 3 | 1 | 3.0 | 0.6633 | 0.6611 | 0.5964 | 0.4962 | 0.3300 | 0.0193 |
| 1 | 2 | 0.5 | 0.7472 | 0.7453 | 0.6891 | 0.5964 | 0.4249 | 0.0287 |
| 2 | 2 | 1.0 | 0.5964 | 0.5940 | 0.5257 | 0.4249 | 0.2698 | 0.0146 |
| 3 | 2 | 1.5 | 0.4962 | 0.4937 | 0.4249 | 0.4055 | 0.1976 | 0.0098 |
| 1 | 3 | 0.33 | 0.6633 | 0.6611 | 0.5964 | 0.4962 | 0.3300 | 0.0193 |
| 2 | 3 | 0.67 | 0.4962 | 0.4937 | 0.4249 | 0.3300 | 0.1976 | 0.0098 |
| 3 | 3 | 1.0 | 0.3964 | 0.3940 | 0.3300 | 0.2472 | 0.1410 | 0.0065 |

TABLE 11e. SUMMARY SHEET FOR NATURAL FREQUENCY SQUARE RATIO.

| $a/b$ \ $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2$ 0.01 | 0.25 | 0.50 | 0.75 | ($h^2/ab=0.04$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=1) | | | | |
| 0.25 | 0.6738 | 0.6101 | 0.5105 | 0.3428 | 0.0204 |
| 0.5 | 0.7783 | 0.7268 | 0.6394 | 0.4699 | 0.0342 |
| 1.0 | 0.8144 | 0.7688 | 0.6891 | 0.5257 | 0.0424 |
| 1.5 | 0.8020 | 0.7542 | 0.6717 | 0.5057 | 0.0393 |
| 2.0 | 0.7783 | 0.7268 | 0.6394 | 0.4699 | 0.0342 |
| 2.5 | 0.7516 | 0.6963 | 0.6045 | 0.4332 | 0.0297 |
| 3.0 | 0.7248 | 0.6661 | 0.5708 | 0.3994 | 0.0259 |
| 3.5 | 0.6987 | 0.6372 | 0.5394 | 0.3693 | 0.0229 |
| 4.0 | 0.6738 | 0.6101 | 0.5105 | 0.3428 | 0.0204 |
| 4.5 | 0.6502 | 0.5847 | 0.4842 | 0.3194 | 0.0184 |
| 5.0 | 0.6280 | 0.5612 | 0.4602 | 0.2989 | 0.0168 |
| | (m=2, n=1) | | | | |
| 0.25 | 0.3507 | 0.2904 | 0.2143 | 0.1200 | 0.0054 |
| 0.5 | 0.5080 | 0.4389 | 0.3428 | 0.2068 | 0.0103 |
| 1.0 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0174 |
| 1.5 | 0.6781 | 0.6148 | 0.5155 | 0.3472 | 0.0208 |
| 2.0 | 0.6869 | 0.6244 | 0.5257 | 0.3565 | 0.0217 |
| 2.5 | 0.6816 | 0.6186 | 0.5195 | 0.3509 | 0.0212 |
| 3.0 | 0.6695 | 0.6054 | 0.5057 | 0.3384 | 0.0200 |
| 3.5 | 0.6540 | 0.5888 | 0.4884 | 0.3231 | 0.0187 |
| 4.0 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0174 |
| 4.5 | 0.6196 | 0.5523 | 0.4513 | 0.2914 | 0.0162 |
| 5.0 | 0.6021 | 0.5341 | 0.4332 | 0.2765 | 0.0151 |
| | (m=3, n=1) | | | | |
| 0.25 | 0.1949 | 0.1550 | 0.1090 | 0.0576 | 0.0024 |
| 0.5 | 0.3218 | 0.2644 | 0.1933 | 0.1070 | 0.0048 |
| 1.0 | 0.4674 | 0.3994 | 0.3071 | 0.1814 | 0.0088 |
| 1.5 | 0.5392 | 0.4699 | 0.3715 | 0.2281 | 0.0117 |
| 2.0 | 0.5745 | 0.5057 | 0.4055 | 0.2543 | 0.0135 |
| 2.5 | 0.5900 | 0.5215 | 0.4209 | 0.2665 | 0.0143 |
| 3.0 | 0.5940 | 0.5257 | 0.4249 | 0.2698 | 0.0146 |
| 3.5 | 0.5911 | 0.5227 | 0.4220 | 0.2674 | 0.0144 |
| 4.0 | 0.5841 | 0.5155 | 0.4149 | 0.2618 | 0.0140 |
| 4.5 | 0.5745 | 0.5057 | 0.4055 | 0.2543 | 0.0135 |
| 5.0 | 0.5635 | 0.4944 | 0.3946 | 0.2458 | 0.0129 |

TABLE 12a. NATURAL FREQUENCY RATIO.

| $\hat{\beta}$ \ a/b | $\hat{\Omega}^2$ 0.01 | 0.25 | 0.50 | 0.75 | ($h^2/ab=0.04$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0174 |
| 0.5 | 0.6869 | 0.6244 | 0.5257 | 0.3565 | 0.0217 |
| 1.0 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0174 |
| 1.5 | 0.5683 | 0.4993 | 0.3994 | 0.2495 | 0.0131 |
| 2.0 | 0.5080 | 0.4389 | 0.3428 | 0.2068 | 0.0103 |
| 2.5 | 0.4577 | 0.3900 | 0.2989 | 0.1757 | 0.0085 |
| 3.0 | 0.4158 | 0.3503 | 0.2644 | 0.1523 | 0.0071 |
| 3.5 | 0.3806 | 0.3176 | 0.2368 | 0.1343 | 0.0062 |
| 4.0 | 0.3507 | 0.2904 | 0.2143 | 0.1200 | 0.0054 |
| 4.5 | 0.3251 | 0.2673 | 0.1957 | 0.1084 | 0.0048 |
| 5.0 | 0.3029 | 0.2477 | 0.1800 | 0.0989 | 0.0044 |
| | (m=2, n=2) | | | | |
| 0.25 | 0.3405 | 0.2812 | 0.2068 | 0.1153 | 0.0052 |
| 0.5 | 0.4674 | 0.3994 | 0.3071 | 0.1814 | 0.0088 |
| 1.0 | 0.5232 | 0.4539 | 0.3565 | 0.2169 | 0.0110 |
| 1.5 | 0.5032 | 0.4341 | 0.3384 | 0.2037 | 0.0101 |
| 2.0 | 0.4674 | 0.3994 | 0.3071 | 0.1814 | 0.0088 |
| 2.5 | 0.4307 | 0.3644 | 0.2765 | 0.1604 | 0.0076 |
| 3.0 | 0.3970 | 0.3328 | 0.2495 | 0.1425 | 0.0066 |
| 3.5 | 0.3669 | 0.3051 | 0.2264 | 0.1277 | 0.0058 |
| 4.0 | 0.3405 | 0.2812 | 0.2068 | 0.1153 | 0.0052 |
| 4.5 | 0.3173 | 0.2604 | 0.1901 | 0.1050 | 0.0047 |
| 5.0 | 0.2968 | 0.2422 | 0.1757 | 0.0963 | 0.0042 |
| | (m=3, n=2) | | | | |
| 0.25 | 0.1917 | 0.1523 | 0.1070 | 0.0565 | 0.0024 |
| 0.5 | 0.3050 | 0.2495 | 0.1814 | 0.0998 | 0.0044 |
| 1.0 | 0.4030 | 0.3384 | 0.2543 | 0.1457 | 0.0068 |
| 1.5 | 0.4224 | 0.3565 | 0.2698 | 0.1559 | 0.0073 |
| 2.0 | 0.4125 | 0.3472 | 0.2618 | 0.1506 | 0.0070 |
| 2.5 | 0.3922 | 0.3284 | 0.2458 | 0.1401 | 0.0065 |
| 3.0 | 0.3691 | 0.3071 | 0.2281 | 0.1287 | 0.0059 |
| 3.5 | 0.3463 | 0.2863 | 0.2110 | 0.1180 | 0.0053 |
| 4.0 | 0.3247 | 0.2670 | 0.1954 | 0.1083 | 0.0048 |
| 4.5 | 0.3050 | 0.2495 | 0.1814 | 0.0998 | 0.0044 |
| 5.0 | 0.2870 | 0.2337 | 0.1690 | 0.0923 | 0.0041 |

<u>TABLE 12b.</u> NATURAL FREQUENCY RATIO.

| $\hat{\hat{\beta}}$ / a/b | $\hat{\hat{\Omega}}^2$ 0.01 | 0.25 | 0.50 | 0.75 | ($h^2/ab=0.04$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=3) | | | | |
| 0.25 | 0.5841 | 0.5155 | 0.4149 | 0.2618 | 0.0140 |
| 0.5 | 0.5745 | 0.5057 | 0.4055 | 0.2543 | 0.0135 |
| 1.0 | 0.4674 | 0.3994 | 0.3071 | 0.1814 | 0.0088 |
| 1.5 | 0.3825 | 0.3194 | 0.2383 | 0.1353 | 0.0062 |
| 2.0 | 0.3218 | 0.2644 | 0.1933 | 0.1070 | 0.0048 |
| 2.5 | 0.2771 | 0.2250 | 0.1622 | 0.0882 | 0.0039 |
| 3.0 | 0.2431 | 0.1957 | 0.1395 | 0.0750 | 0.0032 |
| 3.5 | 0.2164 | 0.1720 | 0.1224 | 0.0652 | 0.0028 |
| 4.0 | 0.1949 | 0.1550 | 0.1090 | 0.0576 | 0.0024 |
| 4.5 | 0.1773 | 0.1404 | 0.0982 | 0.0516 | 0.0022 |
| 5.0 | 0.1626 | 0.1282 | 0.0893 | 0.0467 | 0.0020 |
| | (m=2, n=3) | | | | |
| 0.25 | 0.3247 | 0.2670 | 0.1954 | 0.1083 | 0.0048 |
| 0.5 | 0.4125 | 0.3472 | 0.2618 | 0.1506 | 0.0070 |
| 1.0 | 0.4030 | 0.3384 | 0.2543 | 0.1457 | 0.0068 |
| 1.5 | 0.3519 | 0.2914 | 0.2152 | 0.1206 | 0.0055 |
| 2.0 | 0.3050 | 0.2495 | 0.1814 | 0.0998 | 0.0044 |
| 2.5 | 0.2670 | 0.2162 | 0.1554 | 0.0842 | 0.0037 |
| 3.0 | 0.2365 | 0.1901 | 0.1353 | 0.0726 | 0.0031 |
| 3.5 | 0.2119 | 0.1692 | 0.1196 | 0.0636 | 0.0027 |
| 4.0 | 0.1917 | 0.1523 | 0.1070 | 0.0565 | 0.0024 |
| 4.5 | 0.1750 | 0.1384 | 0.0967 | 0.0508 | 0.0021 |
| 5.0 | 0.1608 | 0.1268 | 0.0882 | 0.0462 | 0.0019 |
| | (m=3, n=3) | | | | |
| 0.25 | 0.1866 | 0.1481 | 0.1039 | 0.0548 | 0.0023 |
| 0.5 | 0.2806 | 0.2281 | 0.1646 | 0.0897 | 0.0039 |
| 1.0 | 0.3278 | 0.2698 | 0.1976 | 0.1096 | 0.0049 |
| 1.5 | 0.3104 | 0.2543 | 0.1852 | 0.1021 | 0.0045 |
| 2.0 | 0.2806 | 0.2281 | 0.1646 | 0.0897 | 0.0039 |
| 2.5 | 0.2517 | 0.2030 | 0.1452 | 0.0783 | 0.0034 |
| 3.0 | 0.2263 | 0.1814 | 0.1287 | 0.0688 | 0.0029 |
| 3.5 | 0.2048 | 0.1633 | 0.1151 | 0.0611 | 0.0026 |
| 4.0 | 0.1866 | 0.1481 | 0.1039 | 0.0548 | 0.0023 |
| 4.5 | 0.1712 | 0.1353 | 0.0945 | 0.0496 | 0.0021 |
| 5.0 | 0.1579 | 0.1244 | 0.0865 | 0.0452 | 0.0019 |

TABLE 12c. NATURAL FREQUENCY RATIO.

$\hat{\hat{\Omega}}^2$ ($h^2/ab=0.04$, $\hat{\hat{\beta}}=0$)

| $\bar{\sigma}$ \ $n$ | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 0.6760 | 0.6394 | 0.5865 |
| 0.5 | 0.7800 | 0.6891 | 0.5770 |
| 1.0 | 0.8159 | 0.6394 | 0.4699 |
| 1.5 | 0.8036 | 0.5708 | 0.3849 |
| 2.0 | 0.7800 | 0.5105 | 0.3240 |
| 2.5 | 0.7535 | 0.4602 | 0.2791 |
| 3.0 | 0.7268 | 0.4182 | 0.2449 |
| 3.5 | 0.7008 | 0.3829 | 0.2181 |
| 4.0 | 0.6760 | 0.3530 | 0.1965 |
| 4.5 | 0.6525 | 0.3273 | 0.1788 |
| 5.0 | 0.6303 | 0.3050 | 0.1640 |
| (m=2) | | | |
| 0.25 | 0.3530 | 0.3428 | 0.3270 |
| 0.5 | 0.5105 | 0.4699 | 0.4149 |
| 1.0 | 0.6394 | 0.5257 | 0.4055 |
| 1.5 | 0.6803 | 0.5057 | 0.3542 |
| 2.0 | 0.6891 | 0.4699 | 0.3071 |
| 2.5 | 0.6838 | 0.4332 | 0.2689 |
| 3.0 | 0.6717 | 0.3994 | 0.2383 |
| 3.5 | 0.6563 | 0.3693 | 0.2136 |
| 4.0 | 0.6394 | 0.3428 | 0.1933 |
| 4.5 | 0.6220 | 0.3194 | 0.1764 |
| 5.0 | 0.6045 | 0.2989 | 0.1622 |
| (m=3) | | | |
| 0.25 | 0.1965 | 0.1933 | 0.1882 |
| 0.5 | 0.3240 | 0.3071 | 0.2827 |
| 1.0 | 0.4699 | 0.4055 | 0.3300 |
| 1.5 | 0.5417 | 0.4249 | 0.3125 |
| 2.0 | 0.5770 | 0.4149 | 0.2827 |
| 2.5 | 0.5924 | 0.3946 | 0.2536 |
| 3.0 | 0.5964 | 0.3715 | 0.2281 |
| 3.5 | 0.5935 | 0.3485 | 0.2065 |
| 4.0 | 0.5865 | 0.3270 | 0.1882 |
| 4.5 | 0.5770 | 0.3071 | 0.1726 |
| 5.0 | 0.5650 | 0.2891 | 0.1593 |

TABLE 12d. NATURAL FREQUENCY RATIO.

$\hat{\Omega}^2$ MAXIMUM

($h^2/ab=0.04$)

| m | n | $a/b$ \ $\hat{\beta}$ | 0.0 | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 0.8159 | 0.8144 | 0.7688 | 0.6891 | 0.5257 | 0.0424 |
| 2 | 1 | 2.0 | 0.6891 | 0.6869 | 0.6244 | 0.5257 | 0.3565 | 0.0217 |
| 3 | 1 | 3.0 | 0.5964 | 0.5940 | 0.5257 | 0.4249 | 0.2698 | 0.0146 |
| 1 | 2 | 0.5 | 0.6891 | 0.6869 | 0.6244 | 0.5257 | 0.3565 | 0.0217 |
| 2 | 2 | 1.0 | 0.5257 | 0.5232 | 0.4539 | 0.3565 | 0.2169 | 0.0110 |
| 3 | 2 | 1.5 | 0.4249 | 0.4224 | 0.3565 | 0.2698 | 0.1559 | 0.0073 |
| 1 | 3 | 0.33 | 0.5964 | 0.5940 | 0.5257 | 0.4249 | 0.2698 | 0.0146 |
| 2 | 3 | 0.67 | 0.4249 | 0.4224 | 0.3565 | 0.2698 | 0.1559 | 0.0073 |
| 3 | 3 | 1.0 | 0.3300 | 0.3278 | 0.2698 | 0.1976 | 0.1096 | 0.0049 |

TABLE 12e. SUMMARY SHEET FOR NATURAL FREQUENCY SQUARE RATIO.

| $a/b$ \ $\hat{\hat{\beta}}$ | 0.01 | 0.25 | $\hat{\hat{\Omega}}^2$ 0.50 | 0.75 | ($h^2/ab=0.05$) 0.99 |
|---|---|---|---|---|---|
| (m=1, n=1) | | | | | |
| 0.25 | 0.6229 | 0.5559 | 0.4549 | 0.2944 | 0.0164 |
| 0.5 | 0.7374 | 0.6803 | 0.5865 | 0.4149 | 0.0276 |
| 1.0 | 0.7783 | 0.7268 | 0.6394 | 0.4699 | 0.0342 |
| 1.5 | 0.7642 | 0.7106 | 0.6207 | 0.4501 | 0.0317 |
| 2.0 | 0.7374 | 0.6803 | 0.5865 | 0.4149 | 0.0276 |
| 2.5 | 0.7077 | 0.6472 | 0.5501 | 0.3794 | 0.0239 |
| 3.0 | 0.6781 | 0.6148 | 0.5155 | 0.3472 | 0.0208 |
| 3.5 | 0.6497 | 0.5842 | 0.4837 | 0.3190 | 0.0193 |
| 4.0 | 0.6229 | 0.5559 | 0.4549 | 0.2944 | 0.0164 |
| 4.5 | 0.5979 | 0.5297 | 0.4289 | 0.2730 | 0.0148 |
| 5.0 | 0.6745 | 0.5057 | 0.4055 | 0.2543 | 0.0135 |
| (m=2, n=1) | | | | | |
| 0.25 | 0.3017 | 0.2466 | 0.1791 | 0.0984 | 0.0043 |
| 0.5 | 0.4524 | 0.3849 | 0.2944 | 0.1726 | 0.0083 |
| 1.0 | 0.5841 | 0.5155 | 0.4149 | 0.2618 | 0.0140 |
| 1.5 | 0.6276 | 0.5608 | 0.4598 | 0.2985 | 0.0167 |
| 2.0 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0174 |
| 2.5 | 0.6313 | 0.5657 | 0.4675 | 0.3019 | 0.0170 |
| 3.0 | 0.6184 | 0.5511 | 0.4501 | 0.2904 | 0.0157 |
| 3.5 | 0.6020 | 0.5340 | 0.4330 | 0.2764 | 0.0150 |
| 4.0 | 0.5841 | 0.5155 | 0.4149 | 0.2618 | 0.0140 |
| 4.5 | 0.5658 | 0.4968 | 0.3969 | 0.2476 | 0.0130 |
| 5.0 | 0.5476 | 0.4784 | 0.3794 | 0.2341 | 0.0121 |
| (m=3, n=1) | | | | | |
| 0.25 | 0.1623 | 0.1280 | 0.0891 | 0.0466 | 0.0020 |
| 0.5 | 0.2751 | 0.2233 | 0.1609 | 0.0875 | 0.0038 |
| 1.0 | 0.4125 | 0.3472 | 0.2618 | 0.1506 | 0.0070 |
| 1.5 | 0.4835 | 0.4149 | 0.3210 | 0.1912 | 0.0094 |
| 2.0 | 0.5193 | 0.4501 | 0.3530 | 0.2143 | 0.0108 |
| 2.5 | 0.5378 | 0.4658 | 0.3676 | 0.2252 | 0.0115 |
| 3.0 | 0.5392 | 0.4699 | 0.3715 | 0.2281 | 0.0117 |
| 3.5 | 0.5363 | 0.4670 | 0.3687 | 0.2260 | 0.0115 |
| 4.0 | 0.5291 | 0.4561 | 0.3620 | 0.2210 | 0.0112 |
| 4.5 | 0.5193 | 0.4501 | 0.3537 | 0.2143 | 0.0108 |
| 5.0 | 0.5080 | 0.4389 | 0.3428 | 0.2068 | 0.0103 |

TABLE 13a. NATURAL FREQUENCY RATIO.

| $a/b$ \ $\hat{\beta}$ | 0.01 | 0.25 | $\hat{\Omega}^2$ 0.50 | 0.75 | ($h^2/ab=0.05$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 0.5841 | 0.5155 | 0.4149 | 0.2618 | 0.0140 |
| 0.5 | 0.6371 | 0.5708 | 0.4699 | 0.3017 | 0.0174 |
| 1.0 | 0.5841 | 0.5155 | 0.4149 | 0.2618 | 0.0140 |
| 1.5 | 0.5130 | 0.4438 | 0.3472 | 0.2101 | 0.0105 |
| 2.0 | 0.4524 | 0.3849 | 0.2944 | 0.1726 | 0.0083 |
| 2.5 | 0.4030 | 0.3383 | 0.2543 | 0.1457 | 0.0068 |
| 3.0 | 0.3628 | 0.3013 | 0.2233 | 0.1257 | 0.0057 |
| 3.5 | 0.3295 | 0.2713 | 0.1989 | 0.1104 | 0.0049 |
| 4.0 | 0.3017 | 0.2466 | 0.1791 | 0.0984 | 0.0043 |
| 4.5 | 0.2782 | 0.2260 | 0.1629 | 0.0887 | 0.0039 |
| 5.0 | 0.2579 | 0.2084 | 0.1493 | 0.0807 | 0.0035 |
| | (m=2, n=2) | | | | |
| 0.25 | 0.2923 | 0.2383 | 0.1726 | 0.0944 | 0.0042 |
| 0.5 | 0.4125 | 0.3472 | 0.2618 | 0.1506 | 0.0070 |
| 1.0 | 0.4674 | 0.2994 | 0.3071 | 0.1814 | 0.0088 |
| 1.5 | 0.4476 | 0.3802 | 0.2904 | 0.1698 | 0.0081 |
| 2.0 | 0.4125 | 0.3472 | 0.2618 | 0.1506 | 0.0070 |
| 2.5 | 0.3771 | 0.3144 | 0.2341 | 0.1326 | 0.0061 |
| 3.0 | 0.3450 | 0.2852 | 0.2101 | 0.1174 | 0.0053 |
| 3.5 | 0.3168 | 0.2600 | 0.1897 | 0.1048 | 0.0047 |
| 4.0 | 0.2923 | 0.2383 | 0.1726 | 0.0945 | 0.0042 |
| 4.5 | 0.2710 | 0.2130 | 0.1581 | 0.0891 | 0.0037 |
| 5.0 | 0.2524 | 0.2037 | 0.1457 | 0.0786 | 0.0034 |
| | (m=3, n=2) | | | | |
| 0.25 | 0.1595 | 0.0756 | 0.0875 | 0.0457 | 0.0019 |
| 0.5 | 0.2598 | 0.1353 | 0.1506 | 0.0814 | 0.0035 |
| 1.0 | 0.3507 | 0.2101 | 0.2143 | 0.1200 | 0.0054 |
| 1.5 | 0.3691 | 0.2419 | 0.2281 | 0.1287 | 0.0059 |
| 2.0 | 0.3597 | 0.2487 | 0.2210 | 0.1242 | 0.0056 |
| 2.5 | 0.3405 | 0.2449 | 0.2068 | 0.1153 | 0.0052 |
| 3.0 | 0.3188 | 0.2348 | 0.1912 | 0.1020 | 0.0047 |
| 3.5 | 0.2976 | 0.2227 | 0.1763 | 0.0967 | 0.0043 |
| 4.0 | 0.2778 | 0.2101 | 0.1627 | 0.0886 | 0.0039 |
| 4.5 | 0.2599 | 0.1979 | 0.1506 | 0.0814 | 0.0035 |
| 5.0 | 0.2436 | 0.1865 | 0.1399 | 0.0752 | 0.0032 |

TABLE 13b. NATURAL FREQUENCY RATIO.

| a/b \ $\hat{\beta}$ | $\hat{\Omega}^2$ 0.01 | 0.25 | 0.50 | 0.75 | ($h^2$/ab=0.05) $\hat{\beta}$ = 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=3) | | | | |
| 0.25 | 0.5291 | 0.4598 | 0.3620 | 0.2210 | 0.0112 |
| 0.5 | 0.5193 | 0.4501 | 0.3520 | 0.2143 | 0.0108 |
| 1.0 | 0.4125 | 0.3472 | 0.2618 | 0.1506 | 0.0070 |
| 1.5 | 0.3314 | 0.2730 | 0.2002 | 0.1112 | 0.0050 |
| 2.0 | 0.2751 | 0.2233 | 0.1609 | 0.0875 | 0.0038 |
| 2.5 | 0.2347 | 0.1885 | 0.1341 | 0.0719 | 0.0031 |
| 3.0 | 0.2044 | 0.1629 | 0.1148 | 0.0649 | 0.0026 |
| 3.5 | 0.1809 | 0.1434 | 0.1004 | 0.0528 | 0.0022 |
| 4.0 | 0.1623 | 0.1280 | 0.0891 | 0.0466 | 0.0020 |
| 4.5 | 0.1471 | 0.1155 | 0.0801 | 0.0417 | 0.0017 |
| 5.0 | 0.1345 | 0.1053 | 0.0727 | 0.0377 | 0.0016 |
| | (m=2, n=3) | | | | |
| 0.25 | 0.2778 | 0.2257 | 0.1627 | 0.0886 | 0.0039 |
| 0.5 | 0.3597 | 0.2985 | 0.2210 | 0.1242 | 0.0056 |
| 1.0 | 0.3507 | 0.2904 | 0.2143 | 0.1200 | 0.0054 |
| 1.5 | 0.3028 | 0.2476 | 0.1799 | 0.0988 | 0.0044 |
| 2.0 | 0.2599 | 0.2101 | 0.1506 | 0.0814 | 0.0035 |
| 2.5 | 0.2256 | 0.1808 | 0.1283 | 0.0685 | 0.0029 |
| 3.0 | 0.1986 | 0.1581 | 0.112 | 0.0589 | 0.0025 |
| 3.5 | 0.1770 | 0.1401 | 0.0980 | 0.0515 | 0.0022 |
| 4.0 | 0.1595 | 0.1257 | 0.0875 | 0.0457 | 0.0019 |
| 4.5 | 0.1450 | 0.1139 | 0.0759 | 0.0411 | 0.0017 |
| 5.0 | 0.1329 | 0.1041 | 0.0719 | 0.0373 | 0.0015 |
| | (m=3, n=3) | | | | |
| 0.25 | 0.1551 | 0.1221 | 0.0848 | 0.0443 | 0.0019 |
| 0.5 | 0.2378 | 0.1912 | 0.1362 | 0.0730 | 0.0031 |
| 1.0 | 0.2806 | 0.2281 | 0.1646 | 0.0897 | 0.0039 |
| 1.5 | 0.2647 | 0.2143 | 0.1539 | 0.0834 | 0.0036 |
| 2.0 | 0.2378 | 0.1912 | 0.1362 | 0.0730 | 0.0031 |
| 2.5 | 0.2120 | 0.1693 | 0.1196 | 0.0636 | 0.0027 |
| 3.0 | 0.1897 | 0.1577 | 0.1057 | 0.0558 | 0.0024 |
| 3.5 | 0.1709 | 0.1350 | 0.0943 | 0.0495 | 0.0021 |
| 4.0 | 0.1551 | 0.1221 | 0.0848 | 0.0443 | 0.0019 |
| 4.5 | 0.1418 | 0.1112 | 0.0770 | 0.0400 | 0.0017 |
| 5.0 | 0.1305 | 0.1021 | 0.0704 | 0.0365 | 0.0015 |

TABLE 13c. NATURAL FREQUENCY RATIO.

$\hat{\hat{\Omega}}^2$ $(h^2/ab=0.05,\ \hat{\hat{\beta}}=0)$

| $a/b$ \ $n$ | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 0.6253 | 0.5865 | 0.5316 |
| 0.5 | 0.7394 | 0.6394 | 0.5218 |
| 1.0 | 0.7800 | 0.5865 | 0.4149 |
| 1.5 | 0.7660 | 0.5155 | 0.3336 |
| 2.0 | 0.7394 | 0.4549 | 0.2771 |
| 2.5 | 0.7098 | 0.4055 | 0.2365 |
| 3.0 | 0.6803 | 0.3651 | 0.2060 |
| 3.5 | 0.6520 | 0.3318 | 0.1824 |
| 4.0 | 0.6523 | 0.3038 | 0.1636 |
| 4.5 | 0.6003 | 0.2802 | 0.1483 |
| 5.0 | 0.5770 | 0.2599 | 0.1356 |
| (m=2) | | | |
| 0.25 | 0.3038 | 0.2944 | 0.2799 |
| 0.5 | 0.4549 | 0.4149 | 0.3620 |
| 1.0 | 0.5865 | 0.4699 | 0.3520 |
| 1.5 | 0.6299 | 0.4501 | 0.3049 |
| 2.0 | 0.6394 | 0.4149 | 0.2618 |
| 2.5 | 0.6337 | 0.3794 | 0.2274 |
| 3.0 | 0.6207 | 0.3472 | 0.2002 |
| 3.5 | 0.6044 | 0.3190 | 0.1785 |
| 4.0 | 0.5865 | 0.2944 | 0.1609 |
| 4.5 | 0.5682 | 0.2730 | 0.1463 |
| 5.0 | 0.5501 | 0.2543 | 0.1341 |
| (m=3) | | | |
| 0.25 | 0.1636 | 0.1609 | 0.1564 |
| 0.5 | 0.2771 | 0.2618 | 0.2397 |
| 1.0 | 0.4149 | 0.3530 | 0.2827 |
| 1.5 | 0.4860 | 0.3715 | 0.2667 |
| 2.0 | 0.5218 | 0.3620 | 0.2397 |
| 2.5 | 0.5376 | 0.3428 | 0.2137 |
| 3.0 | 0.5417 | 0.3210 | 0.1912 |
| 3.5 | 0.5388 | 0.2997 | 0.1723 |
| 4.0 | 0.5316 | 0.2799 | 0.1564 |
| 4.5 | 0.5218 | 0.2618 | 0.1430 |
| 5.0 | 0.5105 | 0.2455 | 0.1316 |

TABLE 13d. NATURAL FREQUENCY RATIO.

| m | n | a/b \ $\hat{\beta}$ | 0.0 ($h^2/ab=0.05$) | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|---|---|---|
| | | | $\hat{\Omega}^2$ MAXIMUM | | | | | |
| 1 | 1 | 1.0 | 0.7800 | 0.7783 | 0.7268 | 0.6394 | 0.4699 | 0.0342 |
| 2 | 1 | 2.0 | 0.6394 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0174 |
| 3 | 1 | 3.0 | 0.5417 | 0.5392 | 0.4699 | 0.3715 | 0.2281 | 0.0117 |
| 1 | 2 | 0.5 | 0.6394 | 0.6371 | 0.5708 | 0.4699 | 0.3071 | 0.0174 |
| 2 | 2 | 1.0 | 0.4699 | 0.4674 | 0.3994 | 0.3071 | 0.1814 | 0.0088 |
| 3 | 2 | 1.5 | 0.3715 | 0.3691 | 0.3071 | 0.2281 | 0.1287 | 0.0059 |
| 1 | 3 | 0.33 | 0.5417 | 0.5392 | 0.4699 | 0.3715 | 0.2281 | 0.0117 |
| 2 | 3 | 0.67 | 0.3715 | 0.3691 | 0.3071 | 0.2281 | 0.1287 | 0.0059 |
| 3 | 3 | 1.0 | 0.2827 | 0.2806 | 0.2281 | 0.1646 | 0.0897 | 0.0039 |

TABLE 13e. SUMMARY SHEET FOR NATURAL FREQUENCY SQUARE RATIO.

$\hat{\hat{\Omega}}^2$ (B.S.A. TI. RI) ($h^2/ab=0.01$)

| $a/b$ \ $\hat{\beta}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=1) | | | | |
| 0.25 | 15.535 | 11.769 | 7.846 | 3.923 | 0.157 |
| 0.5 | 5.678 | 4.302 | 2.868 | 1.434 | 0.057 |
| 1.0 | 3.694 | 2.699 | 1.866 | 0.933 | 0.037 |
| 1.5 | 4.312 | 3.266 | 2.178 | 1.089 | 0.044 |
| 2.0 | 5.678 | 4.302 | 2.868 | 1.434 | 0.057 |
| 2.5 | 7.543 | 5.714 | 3.810 | 1.905 | 0.076 |
| 3.0 | 9.829 | 7.447 | 4.965 | 2.482 | 0.099 |
| 3.5 | 12.501 | 9.471 | 6.314 | 3.157 | 0.126 |
| 4.0 | 15.534 | 11.769 | 7.846 | 3.923 | 0.157 |
| 4.5 | 18.908 | 14.325 | 9.550 | 4.775 | 0.191 |
| 5.0 | 22.608 | 17.128 | 11.419 | 5.710 | 0.228 |
| | (m=2, n=1) | | | | |
| 0.25 | 168.404 | 127.630 | 85.122 | 42.578 | 1.704 |
| 0.5 | 55.165 | 41.796 | 27.867 | 13.935 | 0.558 |
| 1.0 | 21.026 | 15.929 | 10.620 | 5.310 | 0.212 |
| 1.5 | 14.969 | 11.340 | 7.560 | 3.780 | 0.151 |
| 2.0 | 13.866 | 10.504 | 7.003 | 3.502 | 0.140 |
| 2.5 | 14.523 | 11.003 | 7.335 | 3.668 | 0.147 |
| 3.0 | 16.109 | 12.204 | 8.136 | 4.068 | 0.163 |
| 3.5 | 18.322 | 13.880 | 9.254 | 4.627 | 0.185 |
| 4.0 | 21.025 | 15.929 | 10.620 | 5.310 | 0.212 |
| 4.5 | 24.147 | 18.294 | 12.197 | 6.099 | 0.244 |
| 5.0 | 27.644 | 20.943 | 13.963 | 6.982 | 0.279 |
| | (m=3, n=1) | | | | |
| 0.25 | 596.646 | 452.578 | 302.104 | 151.239 | 6.057 |
| 0.5 | 208.523 | 158.051 | 105.422 | 52.738 | 2.111 |
| 1.0 | 73.499 | 55.690 | 37.133 | 18.569 | 0.743 |
| 1.5 | 44.099 | 33.441 | 22.276 | 11.139 | 0.446 |
| 2.0 | 34.042 | 25.791 | 17.195 | 8.598 | 0.344 |
| 2.5 | 30.319 | 22.970 | 15.314 | 7.657 | 0.306 |
| 3.0 | 29.416 | 22.286 | 14.858 | 7.429 | 0.297 |
| 3.5 | 30.059 | 22.774 | 15.183 | 7.592 | 0.304 |
| 4.0 | 31.694 | 24.012 | 16.009 | 8.005 | 0.320 |
| 4.5 | 34.041 | 25.790 | 17.195 | 8.598 | 0.344 |
| 5.0 | 36.948 | 27.993 | 18.663 | 9.332 | 0.373 |

TABLE 14a. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

| $a/b$ \ $\hat{\beta}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | $\hat{\Omega}^2$ (B.S.A. TI. RI) | | | | ($h^2/ab$=0.01) |
| | (m=1, n=2) | | | | |
| 0.25 | 21.026 | 15.929 | 10.620 | 5.310 | 0.212 |
| 0.5 | 13.855 | 10.504 | 7.003 | 3.502 | 0.140 |
| 1.0 | 21.025 | 15.929 | 10.620 | 5.310 | 0.212 |
| 1.5 | 35.644 | 27.005 | 18.004 | 9.003 | 0.360 |
| 2.0 | 55.164 | 41.796 | 27.867 | 13.935 | 0.558 |
| 2.5 | 78.716 | 59.643 | 39.769 | 19.888 | 0.796 |
| 3.0 | 105.721 | 80.111 | 53.421 | 26.717 | 1.069 |
| 3.5 | 135.736 | 102.863 | 68.598 | 34.310 | 1.373 |
| 4.0 | 168.401 | 127.628 | 85.121 | 42.578 | 1.704 |
| 4.5 | 203.421 | 154.182 | 102.840 | 51.445 | 2.059 |
| 5.0 | 240.548 | 182.339 | 121.631 | 60.851 | 2.435 |
| | (m=2, n=2) | | | | |
| 0.25 | 181.473 | 137.539 | 91.734 | 45.887 | 1.836 |
| 0.5 | 73.499 | 55.690 | 37.133 | 18.569 | 0.743 |
| 1.0 | 49.510 | 37.511 | 25.010 | 12.506 | 0.500 |
| 1.5 | 57.102 | 43.264 | 28.846 | 14.425 | 0.577 |
| 2.0 | 73.449 | 55.690 | 37.132 | 18.569 | 0.743 |
| 2.5 | 95.141 | 72.092 | 48.072 | 24.041 | 0.962 |
| 3.0 | 120.772 | 91.519 | 61.030 | 30.524 | 1.221 |
| 3.5 | 149.703 | 113.451 | 75.662 | 37.845 | 1.514 |
| 4.0 | 181.470 | 137.537 | 91.732 | 45.886 | 1.836 |
| 4.5 | 215.722 | 163.510 | 109.065 | 54.561 | 2.184 |
| 5.0 | 252.178 | 191.160 | 127.519 | 63.798 | 2.553 |
| | (m=3, n=2) | | | | |
| 0.25 | 615.106 | 466.593 | 311.468 | 155.931 | 6.245 |
| 0.5 | 236.712 | 179.430 | 119.690 | 59.879 | 2.396 |
| 1.0 | 115.679 | 87.659 | 58.455 | 29.236 | 1.170 |
| 1.5 | 100.864 | 76.430 | 50.965 | 25.488 | 1.020 |
| 2.0 | 108.192 | 81.976 | 54.665 | 27.339 | 1.094 |
| 2.5 | 124.902 | 94.650 | 63.119 | 31.569 | 1.263 |
| 3.0 | 147.344 | 111.663 | 74.469 | 37.248 | 1.490 |
| 3.5 | 173.965 | 131.846 | 87.935 | 43.986 | 1.760 |
| 4.0 | 203.931 | 154.569 | 103.098 | 51.574 | 2.064 |
| 4.5 | 236.709 | 179.427 | 119.688 | 59.878 | 2.396 |
| 5.0 | 271.919 | 206.134 | 137.514 | 68.802 | 2.754 |

TABLE 14b. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\hat{\Omega}}^2$ (B.S.A. TI. RI) $(h^2/ab=0.01)$

| $a/b$ \ $\hat{\hat{\beta}}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=3) | | | | |
| 0.25 | 31.694 | 24.012 | 16.009 | 8.005 | 0.320 |
| 0.5 | 34.042 | 25.790 | 17.195 | 8.598 | 0.344 |
| 1.0 | 73.499 | 55.690 | 37.133 | 18.569 | 0.743 |
| 1.5 | 133.834 | 101.421 | 67.636 | 33.829 | 1.354 |
| 2.0 | 208.521 | 158.049 | 105.420 | 52.737 | 2.111 |
| 2.5 | 294.127 | 222.980 | 148.760 | 73.432 | 2.979 |
| 3.0 | 388.368 | 294.482 | 196.501 | 98.338 | 3.937 |
| 3.5 | 489.607 | 371.319 | 247.819 | 124.043 | 4.967 |
| 4.0 | 596.637 | 452.570 | 302.098 | 151.237 | 6.056 |
| 4.5 | 708.526 | 537.530 | 358.868 | 179.683 | 7.197 |
| 5.0 | 824.553 | 625.647 | 417.757 | 209.196 | 8.380 |
| | (m=2, n=3) | | | | |
| 0.25 | 203.933 | 154.571 | 103.099 | 51.575 | 2.064 |
| 0.5 | 108.182 | 81.977 | 54.665 | 27.339 | 1.094 |
| 1.0 | 115.679 | 87.659 | 58.455 | 29.236 | 1.170 |
| 1.5 | 166.971 | 126.543 | 84.397 | 42.216 | 1.689 |
| 2.0 | 236.710 | 179.428 | 119.688 | 59.878 | 2.396 |
| 2.5 | 318.914 | 241.784 | 161.314 | 80.718 | 3.231 |
| 3.0 | 410.575 | 311.335 | 207.755 | 103.975 | 4.163 |
| 3.5 | 509.759 | 386.615 | 258.036 | 129.161 | 5.172 |
| 4.0 | 615.095 | 466.585 | 311.462 | 155.928 | 6.245 |
| 4.5 | 725.562 | 550.466 | 367.513 | 184.015 | 7.370 |
| 5.0 | 840.373 | 637.663 | 425.788 | 213.221 | 8.541 |
| | (m=3, n=3) | | | | |
| 0.25 | 646.152 | 490.166 | 327.219 | 163.823 | 6.561 |
| 0.5 | 285.998 | 216.813 | 144.643 | 72.371 | 2.897 |
| 1.0 | 199.375 | 151.114 | 100.792 | 50.421 | 2.018 |
| 1.5 | 227.195 | 172.211 | 114.872 | 57.467 | 2.300 |
| 2.0 | 285.996 | 216.812 | 144.642 | 72.370 | 2.897 |
| 2.5 | 361.439 | 274.049 | 182.856 | 91.505 | 3.663 |
| 3.0 | 448.289 | 339.957 | 226.872 | 113.550 | 4.546 |
| 3.5 | 543.776 | 412.439 | 275.288 | 137.804 | 5.518 |
| 4.0 | 646.141 | 490.157 | 327.213 | 163.820 | 6.561 |
| 4.5 | 754.147 | 572.174 | 382.021 | 191.286 | 7.662 |
| 5.0 | 866.874 | 657.792 | 439.241 | 219.964 | 8.811 |

TABLE 14c. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\hat{\Omega}}^2$(B.S.A. TI.) ($h^2/ab=0.01$), ($\hat{\hat{\beta}}=0$)

| a/b \ n | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 15.691 | 21.238 | 32.014 |
| 0.5 | 5.736 | 14.006 | 34.385 |
| 1.0 | 3.732 | 21.238 | 74.241 |
| 1.5 | 4.355 | 36.004 | 135.184 |
| 2.0 | 5.736 | 55.721 | 210.623 |
| 2.5 | 7.619 | 79.510 | 297.090 |
| 3.0 | 9.929 | 106.788 | 392.275 |
| 3.5 | 12.628 | 137.105 | 494.530 |
| 4.0 | 15.691 | 170.099 | 602.631 |
| 4.5 | 19.099 | 205.472 | 715.639 |
| 5.0 | 22.836 | 242.972 | 832.825 |
| (m=2) | | | |
| 0.25 | 170.102 | 183.303 | 205.989 |
| 0.5 | 55.722 | 74.241 | 109.274 |
| 1.0 | 21.238 | 50.010 | 116.856 |
| 1.5 | 15.219 | 57.678 | 168.655 |
| 2.0 | 14.006 | 74.241 | 239.096 |
| 2.5 | 14.670 | 96.102 | 322.125 |
| 3.0 | 16.272 | 121.990 | 414.706 |
| 3.5 | 18.507 | 151.213 | 514.884 |
| 4.0 | 21.238 | 183.300 | 621.274 |
| 4.5 | 24.391 | 217.896 | 732.845 |
| 5.0 | 27.923 | 254.719 | 848.804 |
| (m=3) | | | |
| 0.25 | 602.641 | 621.285 | 652.642 |
| 0.5 | 210.625 | 239.098 | 288.879 |
| 1.0 | 74.241 | 116.846 | 201.385 |
| 1.5 | 44.544 | 101.882 | 229.484 |
| 2.0 | 34.386 | 109.273 | 288.877 |
| 2.5 | 30.625 | 126.162 | 365.077 |
| 3.0 | 29.723 | 148.830 | 452.798 |
| 3.5 | 30.363 | 175.718 | 549.242 |
| 4.0 | 32.014 | 205.986 | 652.631 |
| 4.5 | 34.385 | 239.094 | 761.716 |
| 5.0 | 37.321 | 274.658 | 875.569 |

TABLE 14d. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

| | | | $\hat{\Omega}^2$ (B.S.A. TI. RI) MINIMUM | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $\hat{\beta}$ | ($h^2/ab=0.01$) | | | | | |
| m | n | a/b | 0.0 | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
| 1 | 1 | 1.0 | 3.732 | 3.694 | 2.799 | 1.866 | 0.933 | 0.037 |
| 2 | 1 | 2.0 | 14.006 | 13.866 | 10.504 | 7.003 | 3.502 | 0.140 |
| 3 | 1 | 3.0 | 29.723 | 29.416 | 22.286 | 14.858 | 7.429 | 0.297 |
| 1 | 2 | 0.50 | 14.006 | 13.866 | 10.504 | 7.003 | 3.502 | 0.140 |
| 2 | 2 | 1.0 | 50.010 | 49.510 | 37.511 | 25.010 | 12.506 | 0.500 |
| 3 | 2 | 1.5 | 101.882 | 100.864 | 76.430 | 50.965 | 25.488 | 1.020 |
| 1 | 3 | 0.33 | 29.723 | 29.416 | 22.286 | 14.858 | 7.429 | 0.297 |
| 2 | 3 | 0.67 | 101.882 | 100.864 | 76.430 | 50.965 | 25.488 | 1.020 |
| 3 | 3 | 1.0 | 201.385 | 199.375 | 151.114 | 100.792 | 50.421 | 2.018 |

TABLE 14e. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\Omega}^2$ (B.S.A. TI. RI) ($h^2/ab=0.02$)

| a/b \ $\hat{\beta}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=1) | | | | |
| 0.25 | 13.791 | 10.449 | 6.967 | 3.484 | 0.139 |
| 0.5 | 5.256 | 3.982 | 2.655 | 1.328 | 0.053 |
| 1.0 | 3.466 | 2.626 | 1.751 | 0.875 | 0.035 |
| 1.5 | 4.027 | 3.051 | 2.034 | 1.017 | 0.041 |
| 2.0 | 5.256 | 3.982 | 2.655 | 1.328 | 0.053 |
| 2.5 | 6.911 | 5.236 | 3.491 | 1.745 | 0.070 |
| 3.0 | 8.911 | 6.751 | 4.501 | 2.251 | 0.090 |
| 3.5 | 11.214 | 8.496 | 5.665 | 2.833 | 0.113 |
| 4.0 | 13.791 | 10.449 | 6.967 | 3.484 | 0.139 |
| 4.5 | 16.619 | 12.592 | 8.396 | 4.199 | 0.168 |
| 5.0 | 19.679 | 14.911 | 9.942 | 4.972 | 0.199 |
| | (m=2, n=1) | | | | |
| 0.25 | 126.599 | 96.015 | 64.083 | 32.077 | 1.284 |
| 0.5 | 45.368 | 34.385 | 22.934 | 11.472 | 0.459 |
| 1.0 | 18.375 | 13.923 | 9.283 | 4.642 | 0.186 |
| 1.5 | 13.313 | 10.087 | 6.725 | 3.363 | 0.135 |
| 2.0 | 12.378 | 9.378 | 6.253 | 3.127 | 0.125 |
| 2.5 | 12.936 | 9.801 | 6.535 | 3.268 | 0.131 |
| 3.0 | 14.275 | 10.816 | 7.212 | 3.606 | 0.144 |
| 3.5 | 16.130 | 12.221 | 8.149 | 4.075 | 0.163 |
| 4.0 | 18.375 | 13.992 | 9.283 | 4.642 | 0.186 |
| 4.5 | 20.941 | 15.867 | 10.580 | 5.291 | 0.212 |
| 5.0 | 23.785 | 18.023 | 12.018 | 6.010 | 0.240 |
| | (m=3, n=1) | | | | |
| 0.25 | 398.176 | 302.312 | 201.977 | 101.194 | 4.055 |
| 0.5 | 153.776 | 116.648 | 77.867 | 38.983 | 1.561 |
| 1.0 | 59.178 | 44.858 | 29.923 | 14.970 | 0.599 |
| 1.5 | 36.837 | 27.916 | 18.618 | 9.312 | 0.373 |
| 2.0 | 28.920 | 21.915 | 14.614 | 7.309 | 0.292 |
| 2.5 | 25.943 | 19.658 | 13.109 | 6.556 | 0.262 |
| 3.0 | 25.216 | 19.108 | 12.741 | 6.372 | 0.255 |
| 3.5 | 25.734 | 19.500 | 13.003 | 6.503 | 0.260 |
| 4.0 | 27.045 | 20.494 | 13.666 | 6.835 | 0.273 |
| 4.5 | 28.920 | 21.915 | 14.614 | 7.309 | 0.292 |
| 5.0 | 31.225 | 23.663 | 15.780 | 7.892 | 0.316 |

TABLE 15a. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

| $a/b$ \ $\hat{\beta}$ | $\hat{\Omega}^2$ (B.S.A. TI. RI) 0.01 | 0.25 | 0.50 | 0.75 | ($h^2/ab=0.02$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 18.375 | 13.923 | 9.283 | 4.642 | 0.186 |
| 0.5 | 12.378 | 9.378 | 6.253 | 3.127 | 0.125 |
| 1.0 | 18.375 | 13.923 | 9.283 | 4.642 | 0.186 |
| 1.5 | 30.193 | 22.880 | 15.258 | 7.631 | 0.305 |
| 2.0 | 45.368 | 34.384 | 22.933 | 11.472 | 0.459 |
| 2.5 | 63.045 | 47.790 | 31.880 | 15.590 | 0.638 |
| 3.0 | 82.703 | 62.703 | 41.835 | 20.934 | 0.838 |
| 3.5 | 103.978 | 78.846 | 52.615 | 26.332 | 1.054 |
| 4.0 | 126.596 | 96.014 | 64.082 | 32.076 | 1.284 |
| 4.5 | 150.352 | 114.048 | 76.130 | 38.112 | 1.526 |
| 5.0 | 175.081 | 132.825 | 88.676 | 44.399 | 1.778 |
| | (m=2, n=2) | | | | |
| 0.25 | 135.518 | 102.786 | 68.606 | 34.343 | 1.373 |
| 0.5 | 59.178 | 44.858 | 29.923 | 14.970 | 0.599 |
| 1.0 | 41.030 | 31.095 | 20.739 | 10.373 | 0.415 |
| 1.5 | 46.845 | 35.505 | 23.681 | 11.846 | 0.474 |
| 2.0 | 59.178 | 44.857 | 29.922 | 14.970 | 0.599 |
| 2.5 | 75.068 | 56.910 | 37.968 | 18.998 | 0.760 |
| 3.0 | 93.437 | 70.847 | 47.273 | 23.657 | 0.947 |
| 3.5 | 113.710 | 86.232 | 57.548 | 28.803 | 1.153 |
| 4.0 | 135.516 | 102.784 | 68.605 | 34.342 | 1.375 |
| 4.5 | 158.593 | 120.305 | 80.310 | 40.207 | 1.610 |
| 5.0 | 182.744 | 138.644 | 92.565 | 46.348 | 1.856 |
| | (m=3, n=2) | | | | |
| 0.25 | 409.283 | 310.752 | 207.620 | 104.023 | 4.169 |
| 0.5 | 172.546 | 130.900 | 87.390 | 43.755 | 1.752 |
| 1.0 | 89.821 | 68.103 | 45.441 | 22.740 | 0.910 |
| 1.5 | 79.208 | 60.051 | 40.065 | 20.047 | 0.803 |
| 2.0 | 84.468 | 64.042 | 42.729 | 21.382 | 0.856 |
| 2.5 | 96.359 | 73.064 | 48.754 | 24.399 | 0.977 |
| 3.0 | 112.073 | 84.990 | 56.718 | 28.388 | 1.137 |
| 3.5 | 130.402 | 98.902 | 66.011 | 33.043 | 1.323 |
| 4.0 | 150.694 | 114.308 | 76.303 | 38.199 | 1.530 |
| 4.5 | 172.543 | 130.898 | 87.389 | 43.754 | 1.752 |
| 5.0 | 195.668 | 148.450 | 99.125 | 49.645 | 1.988 |

TABLE 15b. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\hat{\Omega}}^2$ (B.S.A. RI. TI) ($h^2/ab=0.02$)

| $a/b$ \ $\hat{\hat{\beta}}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=3) | | | | |
| 0.25 | 27.046 | 20.494 | 13.666 | 6.835 | 0.273 |
| 0.5 | 28.920 | 21.915 | 14.614 | 7.309 | 0.292 |
| 1.0 | 59.178 | 44.857 | 29.922 | 14.970 | 0.599 |
| 1.5 | 102.645 | 77.834 | 51.939 | 25.994 | 1.041 |
| 2.0 | 153.774 | 116.647 | 77.866 | 38.982 | 1.561 |
| 2.5 | 210.094 | 159.416 | 106.447 | 53.305 | 2.135 |
| 3.0 | 270.165 | 205.048 | 136.949 | 68.595 | 2.748 |
| 3.5 | 333.066 | 252.838 | 168.899 | 84.611 | 3.390 |
| 4.0 | 398.170 | 302.307 | 201.974 | 101.193 | 4.055 |
| 4.5 | 465.029 | 353.113 | 235.942 | 118.222 | 4.738 |
| 5.0 | 533.316 | 405.002 | 270.635 | 135.614 | 5.435 |
| | (m=2, n=3) | | | | |
| 0.25 | 150.696 | 114.309 | 76.304 | 38.180 | 1.530 |
| 0.5 | 84.468 | 64.042 | 42.729 | 21.382 | 0.856 |
| 1.0 | 89.821 | 68.103 | 45.441 | 22.740 | 0.910 |
| 1.5 | 125.616 | 95.269 | 63.585 | 31.827 | 1.274 |
| 2.0 | 172.544 | 130.899 | 87.389 | 43.754 | 1.752 |
| 2.5 | 226.062 | 171.545 | 114.554 | 57.369 | 2.298 |
| 3.0 | 284.091 | 215.628 | 144.022 | 72.140 | 2.890 |
| 3.5 | 345.422 | 262.227 | 175.176 | 87.758 | 3.517 |
| 4.0 | 409.277 | 310.748 | 207.617 | 104.022 | 4.169 |
| 4.5 | 475.117 | 360.778 | 241.067 | 120.791 | 4.841 |
| 5.0 | 542.556 | 412.023 | 275.329 | 137.967 | 5.530 |
| | (m=3, n=3) | | | | |
| 0.25 | 427.894 | 324.894 | 217.075 | 108.763 | 4.359 |
| 0.5 | 204.827 | 155.416 | 103.774 | 51.965 | 2.082 |
| 1.0 | 147.630 | 111.982 | 74.749 | 37.421 | 1.499 |
| 1.5 | 166.235 | 126.108 | 84.188 | 42.150 | 1.688 |
| 2.0 | 204.825 | 155.415 | 103.773 | 51.965 | 2.082 |
| 2.5 | 253.170 | 192.137 | 128.318 | 64.268 | 2.575 |
| 3.0 | 307.569 | 233.466 | 155.948 | 78.118 | 3.130 |
| 3.5 | 366.174 | 277.995 | 185.718 | 93.043 | 3.728 |
| 4.0 | 427.887 | 324.889 | 217.072 | 108.762 | 4.359 |
| 4.5 | 491.994 | 373.603 | 249.642 | 125.090 | 5.013 |
| 5.0 | 557.998 | 423.757 | 283.174 | 141.899 | 5.687 |

TABLE 15c. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\Omega}^2$(B.S.A. TI. RI) ($h^2/ab=0.02$), ($\hat{\beta}=0$)

| $\frac{a}{b}$ \ $\eta$ | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 13.930 | 18.560 | 27.319 |
| 0.5 | 5.309 | 12.503 | 29.212 |
| 1.0 | 3.501 | 18.560 | 59.774 |
| 1.5 | 4.068 | 30.498 | 103.677 |
| 2.0 | 5.309 | 45.825 | 155.319 |
| 2.5 | 6.981 | 63.680 | 212.202 |
| 3.0 | 9.001 | 83.536 | 272.872 |
| 3.5 | 11.327 | 105.024 | 336.401 |
| 4.0 | 13.930 | 127.869 | 402.154 |
| 4.5 | 16.878 | 151.863 | 469.680 |
| 5.0 | 19.877 | 176.839 | 538.647 |
| (m=2) | | | |
| 0.25 | 127.871 | 136.880 | 152.209 |
| 0.5 | 45.826 | 59.775 | 85.319 |
| 1.0 | 18.560 | 41.444 | 90.725 |
| 1.5 | 13.448 | 47.317 | 126.879 |
| 2.0 | 12.503 | 59.774 | 174.276 |
| 2.5 | 13.066 | 75.824 | 228.330 |
| 3.0 | 14.420 | 94.377 | 286.937 |
| 3.5 | 16.293 | 114.853 | 348.880 |
| 4.0 | 18.560 | 136.878 | 413.372 |
| 4.5 | 21.152 | 160.186 | 479.868 |
| 5.0 | 24.025 | 184.578 | 547.979 |
| (m=3) | | | |
| 0.25 | 402.160 | 413.378 | 432.174 |
| 0.5 | 155.321 | 174.278 | 206.882 |
| 1.0 | 59.775 | 90.725 | 149.114 |
| 1.5 | 37.208 | 80.006 | 167.905 |
| 2.0 | 29.212 | 85.318 | 206.880 |
| 2.5 | 26.204 | 97.328 | 255.707 |
| 3.0 | 25.471 | 113.200 | 310.650 |
| 3.5 | 25.994 | 131.712 | 369.839 |
| 4.0 | 27.318 | 152.208 | 432.168 |
| 4.5 | 29.211 | 174.276 | 496.914 |
| 5.0 | 31.540 | 197.632 | 563.575 |

TABLE 15d. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\hat{\Omega}}^2$ (B.S.A. TI. RI) MINIMUM

$(h^2/ab=0.02)$

| m | n | a/b \ $\hat{\hat{\beta}}$ | 0.0 | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 3.501 | 3.466 | 2.626 | 1.751 | 0.875 | 0.035 |
| 2 | 1 | 2.0 | 12.503 | 12.378 | 9.378 | 6.253 | 3.127 | 0.125 |
| 3 | 1 | 3.0 | 25.471 | 25.216 | 19.108 | 12.741 | 6.372 | 0.255 |
| 1 | 2 | 0.5 | 12.503 | 12.378 | 9.378 | 6.253 | 3.127 | 0.125 |
| 2 | 2 | 1.0 | 41.444 | 41.030 | 31.095 | 20.739 | 10.373 | 0.415 |
| 3 | 2 | 1.5 | 80.006 | 79.208 | 60.051 | 40.065 | 20.047 | 0.803 |
| 1 | 3 | 0.33 | 25.471 | 25.216 | 19.108 | 12.741 | 6.372 | 0.255 |
| 2 | 3 | 0.67 | 80.006 | 79.208 | 60.051 | 40.065 | 20.047 | 0.803 |
| 3 | 3 | 1.0 | 149.114 | 147.630 | 111.982 | 74.749 | 37.421 | 1.499 |

TABLE 15e. SUMMARY SHEET FOR LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

| a/b \ $\hat{\beta}$ | 0.01 | 0.25 | $\hat{\Omega}^2$ (B.S.A. TI. RI) 0.50 | 0.75 | ($h^2/ab=0.03$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=1) | | | | |
| 0.25 | 12.434 | 9.422 | 6.283 | 3.142 | 0.126 |
| 0.5 | 4.900 | 3.712 | 2.475 | 1.238 | 0.050 |
| 1.0 | 3.268 | 2.476 | 1.651 | 0.826 | 0.033 |
| 1.5 | 3.782 | 2.866 | 1.911 | 0.955 | 0.038 |
| 2.0 | 4.900 | 3.712 | 2.475 | 1.238 | 0.050 |
| 2.5 | 6.388 | 4.840 | 3.227 | 1.614 | 0.065 |
| 3.0 | 8.167 | 6.188 | 4.126 | 2.063 | 0.083 |
| 3.5 | 10.192 | 7.723 | 5.150 | 2.575 | 0.103 |
| 4.0 | 12.434 | 9.422 | 6.283 | 3.142 | 0.126 |
| 4.5 | 14.870 | 11.269 | 7.515 | 3.759 | 0.150 |
| 5.0 | 17.482 | 13.249 | 8.836 | 4.420 | 0.177 |
| | (m=2, n=1) | | | | |
| 0.25 | 102.114 | 77.490 | 51.747 | 25.915 | 1.038 |
| 0.5 | 38.726 | 29.362 | 19.591 | 9.803 | 0.392 |
| 1.0 | 16.372 | 12.407 | 8.274 | 4.139 | 0.166 |
| 1.5 | 12.020 | 9.109 | 6.074 | 3.038 | 0.122 |
| 2.0 | 11.207 | 8.492 | 5.663 | 2.832 | 0.113 |
| 2.5 | 11.693 | 8.860 | 5.908 | 2.955 | 0.118 |
| 3.0 | 12.853 | 9.740 | 6.495 | 3.248 | 0.130 |
| 3.5 | 14.451 | 10.951 | 7.303 | 3.653 | 0.146 |
| 4.0 | 16.372 | 12.407 | 8.274 | 4.139 | 0.166 |
| 4.5 | 18.552 | 14.060 | 9.377 | 4.691 | 0.188 |
| 5.0 | 20.952 | 15.880 | 10.592 | 5.298 | 1.212 |
| | (m=3, n=1) | | | | |
| 0.25 | 300.684 | 228.368 | 152.617 | 76.481 | 3.065 |
| 0.5 | 122.647 | 93.088 | 62.174 | 31.142 | 1.248 |
| 1.0 | 49.811 | 37.774 | 25.208 | 12.617 | 0.505 |
| 1.5 | 31.778 | 24.091 | 16.072 | 8.041 | 0.322 |
| 2.0 | 25.224 | 19.135 | 12.764 | 6.385 | 0.256 |
| 2.5 | 22.761 | 17.252 | 11.507 | 5.756 | 0.230 |
| 3.0 | 22.153 | 16.791 | 11.199 | 5.602 | 0.224 |
| 3.5 | 22.587 | 17.119 | 11.419 | 5.712 | 0.229 |
| 4.0 | 23.683 | 17.951 | 11.973 | 5.990 | 0.240 |
| 4.5 | 25.244 | 19.135 | 12.764 | 6.385 | 0.255 |
| 5.0 | 27.157 | 20.585 | 13.732 | 6.870 | 0.275 |

TABLE 16a. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

| $\frac{a}{b}$ \ $\hat{\beta}$ | 0.01 | 0.25 | $\hat{\Omega}^2$(B.S.A. TI. RI) 0.50 | 0.75 | ($h^2/ab=0.03$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 16.372 | 12.407 | 8.274 | 4.139 | 0.166 |
| 0.5 | 11.207 | 8.492 | 5.663 | 2.832 | 0.113 |
| 1.0 | 16.372 | 12.407 | 8.274 | 4.139 | 0.166 |
| 1.5 | 26.301 | 19.937 | 13.299 | 6.653 | 0.266 |
| 2.0 | 38.726 | 29.362 | 19.591 | 9.803 | 0.392 |
| 2.5 | 52.884 | 40.106 | 26.766 | 13.397 | 0.536 |
| 3.0 | 68.344 | 51.843 | 34.607 | 17.325 | 0.694 |
| 3.5 | 84.821 | 64.355 | 42.968 | 21.515 | 0.862 |
| 4.0 | 102.113 | 77.488 | 51.746 | 25.915 | 1.038 |
| 4.5 | 120.073 | 91.132 | 60.866 | 30.486 | 1.221 |
| 5.0 | 138.590 | 105.201 | 70.271 | 35.201 | 1.411 |
| | (m=2, n=2) | | | | |
| 0.25 | 108.879 | 82.629 | 55.182 | 27.637 | 1.107 |
| 0.5 | 49.811 | 37.774 | 25.208 | 12.617 | 0.505 |
| 1.0 | 35.204 | 26.689 | 17.807 | 8.910 | 0.357 |
| 1.5 | 39.920 | 30.268 | 20.196 | 10.106 | 0.405 |
| 2.0 | 49.810 | 37.773 | 25.208 | 12.617 | 0.505 |
| 2.5 | 62.370 | 47.307 | 31.577 | 15.807 | 0.633 |
| 3.0 | 76.686 | 58.177 | 38.840 | 19.446 | 0.779 |
| 3.5 | 92.286 | 70.025 | 46.757 | 23.414 | 0.938 |
| 4.0 | 108.878 | 82.628 | 55.181 | 27.637 | 1.107 |
| 4.5 | 126.262 | 95.834 | 64.010 | 32.062 | 1.285 |
| 5.0 | 144.296 | 109.536 | 73.170 | 36.654 | 1.469 |
| | (m=3, n=2) | | | | |
| 0.25 | 308.601 | 234.383 | 156.638 | 78.496 | 3.146 |
| 0.5 | 136.699 | 103.764 | 69.311 | 34.720 | 1.391 |
| 1.0 | 73.883 | 56.048 | 37.417 | 18.733 | 0.750 |
| 1.5 | 65.614 | 49.770 | 33.222 | 16.631 | 0.666 |
| 2.0 | 69.720 | 52.888 | 35.305 | 17.675 | 0.708 |
| 2.5 | 78.946 | 59.894 | 39.987 | 20.021 | 0.812 |
| 3.0 | 91.033 | 69.073 | 46.121 | 23.096 | 0.925 |
| 3.5 | 105.002 | 79.683 | 53.213 | 26.650 | 1.068 |
| 4.0 | 120.330 | 91.328 | 60.997 | 30.552 | 1.224 |
| 4.5 | 136.697 | 103.762 | 69.310 | 34.719 | 1.391 |
| 5.0 | 153.888 | 116.824 | 78.043 | 39.097 | 1.567 |

TABLE 16b. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\hat{\Omega}}^2$ (B.S.A. TI. RI) $(h^2/ab=0.03)$

| $a/b$ \ $\hat{\beta}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=3) | | | | |
| 0.25 | 23.683 | 17.951 | 11.973 | 5.990 | 0.240 |
| 0.5 | 25.244 | 19.135 | 12.764 | 6.385 | 0.256 |
| 1.0 | 49.811 | 37.774 | 25.208 | 12.617 | 0.505 |
| 1.5 | 83.795 | 63.576 | 42.447 | 21.254 | 0.851 |
| 2.0 | 122.646 | 93.087 | 62.173 | 31.141 | 1.248 |
| 2.5 | 164.552 | 124.927 | 83.460 | 41.813 | 1.676 |
| 3.0 | 208.547 | 158.358 | 105.812 | 53.019 | 2.125 |
| 3.5 | 254.048 | 192.933 | 128.928 | 64.607 | 2.589 |
| 4.0 | 300.679 | 228.365 | 152.615 | 76.480 | 3.065 |
| 4.5 | 348.184 | 264.456 | 176.740 | 88.571 | 3.550 |
| 5.0 | 396.379 | 301.068 | 201.209 | 100.833 | 4.041 |
| | (m=2, n=3) | | | | |
| 0.25 | 120.332 | 91.329 | 60.998 | 30.552 | 1.224 |
| 0.5 | 69.720 | 52.888 | 35.305 | 17.675 | 0.708 |
| 1.0 | 73.883 | 56.048 | 37.417 | 18.773 | 0.750 |
| 1.5 | 101.367 | 76.922 | 51.367 | 25.725 | 1.031 |
| 2.0 | 136.698 | 103.763 | 69.310 | 34.719 | 1.391 |
| 2.5 | 176.308 | 133.860 | 89.433 | 44.807 | 1.796 |
| 3.0 | 218.665 | 166.046 | 110.952 | 55.596 | 2.228 |
| 3.5 | 262.931 | 199.683 | 133.441 | 66.869 | 2.680 |
| 4.0 | 308.596 | 234.380 | 156.635 | 78.495 | 3.146 |
| 4.5 | 355.323 | 269.879 | 180.465 | 90.388 | 3.623 |
| 5.0 | 402.878 | 306.005 | 204.509 | 102.486 | 4.108 |
| | (m=3, n=3) | | | | |
| 0.25 | 321.842 | 244.443 | 163.363 | 81.867 | 3.281 |
| 0.5 | 160.664 | 121.973 | 81.485 | 40.823 | 1.636 |
| 1.0 | 118.025 | 89.576 | 59.826 | 29.965 | 1.201 |
| 1.5 | 131.985 | 100.182 | 66.916 | 33.519 | 1.343 |
| 2.0 | 160.663 | 121.972 | 81.484 | 40.822 | 1.636 |
| 2.5 | 196.160 | 148.945 | 99.519 | 49.864 | 1.998 |
| 3.0 | 235.664 | 178.964 | 119.589 | 59.925 | 2.402 |
| 3.5 | 277.814 | 210.991 | 141.000 | 70.658 | 2.832 |
| 4.0 | 321.837 | 244.440 | 163.360 | 81.866 | 3.281 |
| 4.5 | 367.251 | 278.941 | 186.421 | 93.423 | 3.744 |
| 5.0 | 413.730 | 314.248 | 210.017 | 105.247 | 4.218 |

TABLE 16c. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\Omega}^2$(B.S.A. TI. RI) $(h^2/ab=0.03)$, $(\hat{\beta}=0)$

| a/b \ n | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 12.560 | 16.537 | 23.922 |
| 0.5 | 4.949 | 11.320 | 25.498 |
| 1.0 | 3.301 | 16.537 | 50.312 |
| 1.5 | 3.821 | 26.566 | 84.636 |
| 2.0 | 4.949 | 39.115 | 123.875 |
| 2.5 | 6.452 | 53.416 | 166.199 |
| 3.0 | 8.249 | 69.031 | 210.632 |
| 3.5 | 10.295 | 85.672 | 256.587 |
| 4.0 | 12.560 | 103.137 | 303.683 |
| 4.5 | 15.020 | 121.276 | 351.661 |
| 5.0 | 17.658 | 139.978 | 400.337 |
| (m=2) | | | |
| 0.25 | 103.138 | 109.971 | 121.538 |
| 0.5 | 39.116 | 50.312 | 70.421 |
| 1.0 | 16.537 | 35.558 | 74.625 |
| 1.5 | 12.142 | 40.322 | 102.384 |
| 2.0 | 11.320 | 50.311 | 138.067 |
| 2.5 | 11.811 | 62.997 | 178.072 |
| 3.0 | 12.983 | 77.456 | 220.851 |
| 3.5 | 14.596 | 93.212 | 265.558 |
| 4.0 | 16.537 | 109.969 | 311.679 |
| 4.5 | 18.739 | 127.527 | 358.871 |
| 5.0 | 21.163 | 145.741 | 406.901 |
| (m=3) | | | |
| 0.25 | 303.688 | 311.683 | 325.056 |
| 0.5 | 123.876 | 138.068 | 162.272 |
| 1.0 | 50.312 | 74.625 | 119.207 |
| 1.5 | 32.098 | 66.273 | 133.307 |
| 2.0 | 25.499 | 70.420 | 162.271 |
| 2.5 | 22.991 | 79.739 | 198.122 |
| 3.0 | 22.376 | 91.947 | 238.020 |
| 3.5 | 22.814 | 106.055 | 280.589 |
| 4.0 | 23.921 | 121.536 | 325.052 |
| 4.5 | 25.498 | 138.066 | 370.918 |
| 5.0 | 27.430 | 155.429 | 417.862 |

TABLE 16d. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\hat{\Omega}}^2$ (B.S.A. TI. RI) MINIMUM

$(h^2/ab=0.03)$

| m | n | a/b \ $\hat{\hat{\beta}}$ | 0.0 | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 3.301 | 3.268 | 2.476 | 1.651 | 0.826 | 0.033 |
| 2 | 1 | 2.0 | 11.320 | 11.207 | 8.492 | 5.663 | 2.832 | 0.113 |
| 3 | 1 | 3.0 | 22.376 | 22.153 | 16.791 | 11.199 | 5.602 | 0.224 |
| 1 | 2 | 0.5 | 11.320 | 11.207 | 8.492 | 5.663 | 2.832 | 0.113 |
| 2 | 2 | 1.0 | 35.558 | 35.204 | 26.689 | 17.807 | 8.910 | 0.357 |
| 3 | 2 | 1.5 | 66.273 | 65.614 | 49.770 | 33.222 | 16.631 | 0.666 |
| 1 | 3 | 0.33 | 22.376 | 22.153 | 16.791 | 11.199 | 5.602 | 0.224 |
| 2 | 3 | 0.67 | 66.273 | 65.614 | 49.770 | 33.222 | 16.631 | 0.666 |
| 3 | 3 | 1.0 | 119.207 | 118.025 | 89.576 | 59.826 | 29.965 | 1.201 |

TABLE 16e. SUMMARY SHEET FOR LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

| $\frac{a}{b}$ \ $\hat{\hat{\beta}}$ | 0.01 | 0.25 | $\hat{\hat{\Omega}}^2$ (B.S.A. TI. RI) 0.50 | 0.75 | ($h^2/ab$=0.04) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=1) | | | | |
| 0.25 | 11.342 | 8.596 | 5.733 | 2.868 | 0.115 |
| 0.5 | 4.594 | 3.481 | 2.321 | 1.161 | 0.046 |
| 1.0 | 3.094 | 2.345 | 1.563 | 0.782 | 0.031 |
| 1.5 | 3.569 | 2.704 | 1.803 | 0.902 | 0.036 |
| 2.0 | 4.594 | 3.481 | 2.321 | 1.161 | 0.046 |
| 2.5 | 5.946 | 4.506 | 3.005 | 1.503 | 0.060 |
| 3.0 | 7.5482 | 5.720 | 3.814 | 1.908 | 0.076 |
| 3.5 | 9.356 | 7.091 | 4.729 | 2.365 | 0.095 |
| 4.0 | 11.342 | 8.596 | 5.733 | 2.868 | 0.115 |
| 4.5 | 13.484 | 10.219 | 6.817 | 3.410 | 0.137 |
| 5.0 | 15.761 | 11.948 | 7.970 | 3.987 | 0.160 |
| | (m=2, n=1) | | | | |
| 0.25 | 85.841 | 65.166 | 43.532 | 21.808 | 0.874 |
| 0.5 | 33.880 | 25.697 | 17.152 | 8.568 | 0.344 |
| 1.0 | 14.795 | 11.214 | 7.481 | 3.742 | 0.150 |
| 1.5 | 10.977 | 8.319 | 5.549 | 2.775 | 0.111 |
| 2.0 | 10.258 | 7.774 | 5.185 | 2.593 | 0.104 |
| 2.5 | 10.687 | 8.400 | 5.402 | 2.702 | 0.108 |
| 3.0 | 11.711 | 8.876 | 5.920 | 2.961 | 0.119 |
| 3.5 | 13.115 | 9.941 | 6.631 | 3.317 | 0.133 |
| 4.0 | 14.794 | 11.214 | 7.481 | 3.742 | 0.150 |
| 4.5 | 16.691 | 12.653 | 8.441 | 4.223 | 0.169 |
| 5.0 | 18.767 | 14.228 | 9.492 | 4.749 | 0.190 |
| | (m=3, n=1) | | | | |
| 0.25 | 242.112 | 183.896 | 122.900 | 61.589 | 2.469 |
| 0.5 | 102.321 | 77.688 | 51.905 | 26.006 | 1.042 |
| 1.0 | 43.136 | 32.725 | 21.848 | 10.939 | 0.438 |
| 1.5 | 28.019 | 21.248 | 14.180 | 7.097 | 0.284 |
| 2.0 | 22.455 | 17.026 | 11.360 | 5.685 | 0.228 |
| 2.5 | 20.325 | 15.410 | 10.281 | 5.145 | 0.206 |
| 3.0 | 19.802 | 15.013 | 10.016 | 5.012 | 0.201 |
| 3.5 | 20.175 | 15.296 | 10.205 | 5.107 | 0.204 |
| 4.0 | 21.117 | 16.010 | 10.682 | 5.345 | 0.214 |
| 4.5 | 22.455 | 17.026 | 11.360 | 5.685 | 0.228 |
| 5.0 | 24.090 | 18.266 | 12.188 | 6.100 | 0.224 |

TABLE 17a. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\Omega}^2$ (B.S.A. TI. RI) $(h^2/ab=0.04)$

| $a/b$ \ $\hat{\beta}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 14.795 | 11.214 | 7.481 | 3.742 | 0.150 |
| 0.5 | 10.258 | 7.774 | 5.185 | 2.593 | 0.104 |
| 1.0 | 14.795 | 11.214 | 7.481 | 3.742 | 0.150 |
| 1.5 | 23.359 | 17.712 | 11.818 | 5.914 | 0.237 |
| 2.0 | 33.879 | 25.696 | 17.151 | 8.586 | 0.344 |
| 2.5 | 45.686 | 34.661 | 23.141 | 11.587 | 0.464 |
| 3.0 | 58.421 | 44.333 | 29.606 | 14.827 | 0.594 |
| 3.5 | 71.856 | 54.540 | 36.429 | 18.247 | 0.731 |
| 4.0 | 85.839 | 65.165 | 43.532 | 21.808 | 0.874 |
| 4.5 | 100.261 | 76.123 | 50.859 | 25.481 | 1.021 |
| 5.0 | 115.041 | 87.354 | 58.368 | 29.246 | 1.172 |
| | (m=2, n=2) | | | | |
| 0.25 | 91.284 | 69.302 | 46.298 | 23.195 | 0.930 |
| 0.5 | 43.137 | 32.725 | 21.848 | 10.939 | 0.438 |
| 1.0 | 30.915 | 23.446 | 15.648 | 7.833 | 0.314 |
| 1.5 | 34.882 | 26.458 | 17.660 | 8.840 | 0.354 |
| 2.0 | 43.136 | 32.725 | 21.847 | 10.939 | 0.438 |
| 2.5 | 53.516 | 40.608 | 27.116 | 13.579 | 0.544 |
| 3.0 | 65.238 | 49.513 | 33.068 | 16.562 | 0.664 |
| 3.5 | 77.906 | 59.137 | 39.502 | 19.788 | 0.793 |
| 4.0 | 91.283 | 69.301 | 46.297 | 23.194 | 0.930 |
| 4.5 | 105.211 | 79.884 | 53.373 | 26.742 | 1.072 |
| 5.0 | 119.579 | 90.803 | 60.673 | 30.402 | 1.218 |
| | (m=3, n=2) | | | | |
| 0.25 | 248.249 | 188.557 | 126.015 | 63.150 | 2.531 |
| 0.5 | 113.535 | 86.210 | 57.603 | 28.862 | 1.157 |
| 1.0 | 62.951 | 47.775 | 31.906 | 15.980 | 0.640 |
| 1.5 | 56.182 | 42.633 | 28.469 | 14.257 | 0.571 |
| 2.0 | 59.548 | 45.189 | 30.178 | 15.114 | 0.606 |
| 2.5 | 67.080 | 50.912 | 34.003 | 17.031 | 0.682 |
| 3.0 | 76.893 | 58.367 | 38.987 | 19.530 | 0.783 |
| 3.5 | 88.166 | 66.933 | 44.714 | 22.401 | 0.898 |
| 4.0 | 100.467 | 76.279 | 50.963 | 25.534 | 1.023 |
| 4.5 | 113.534 | 86.209 | 57.602 | 28.862 | 1.157 |
| 5.0 | 127.195 | 96.589 | 64.542 | 32.341 | 1.296 |

TABLE 17b. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\hat{\Omega}}^2$(B.S.A. TI. RI) $(h^2/ab=0.04)$

| $a/b$ \ $\hat{\hat{\beta}}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| (m=1, n=3) | | | | | |
| 0.25 | 21.117 | 16.011 | 10.682 | 5.345 | 0.214 |
| 0.5 | 22.455 | 17.026 | 11.360 | 5.685 | 0.228 |
| 1.0 | 43.136 | 32.725 | 21.848 | 10.939 | 0.438 |
| 1.5 | 71.023 | 53.907 | 36.006 | 18.035 | 0.723 |
| 2.0 | 102.320 | 77.687 | 51.904 | 26.006 | 1.042 |
| 2.5 | 135.639 | 103.006 | 68.832 | 34.492 | 1.382 |
| 3.0 | 170.286 | 129.332 | 86.432 | 43.314 | 1.736 |
| 3.5 | 205.859 | 156.357 | 104.496 | 52.367 | 2.099 |
| 4.0 | 242.109 | 183.893 | 122.899 | 61.588 | 2.468 |
| 4.5 | 278.869 | 211.812 | 141.554 | 70.934 | 2.843 |
| 5.0 | 316.028 | 240.028 | 160.404 | 80.377 | 3.221 |
| (m=2, n=3) | | | | | |
| 0.25 | 100.468 | 76.280 | 50.964 | 25.534 | 1.023 |
| 0.5 | 59.548 | 45.190 | 30.178 | 15.114 | 0.606 |
| 1.0 | 62.951 | 47.775 | 31.906 | 15.980 | 0.640 |
| 1.5 | 85.239 | 64.708 | 43.227 | 21.655 | 0.868 |
| 2.0 | 113.534 | 86.209 | 57.602 | 28.862 | 1.157 |
| 2.5 | 144.926 | 110.063 | 73.550 | 36.857 | 1.477 |
| 3.0 | 178.216 | 135.357 | 90.459 | 45.332 | 1.817 |
| 3.5 | 212.779 | 161.614 | 108.010 | 54.128 | 2.170 |
| 4.0 | 248.245 | 188.554 | 126.013 | 63.149 | 2.531 |
| 4.5 | 284.382 | 215.998 | 144.350 | 72.335 | 2.899 |
| 5.0 | 321.030 | 243.826 | 162.941 | 81.647 | 3.272 |
| (m=3, n=3) | | | | | |
| 0.25 | 258.503 | 196.345 | 131.219 | 65.757 | 2.636 |
| 0.5 | 132.562 | 100.668 | 67.269 | 33.708 | 1.351 |
| 1.0 | 98.621 | 74.877 | 50.025 | 25.063 | 1.004 |
| 1.5 | 109.779 | 83.355 | 55.694 | 27.905 | 1.118 |
| 2.0 | 132.562 | 100.667 | 67.269 | 33.708 | 1.351 |
| 2.5 | 160.560 | 121.942 | 81.492 | 40.837 | 1.637 |
| 3.0 | 191.513 | 145.459 | 97.212 | 48.717 | 1.953 |
| 3.5 | 224.356 | 170.409 | 113.887 | 57.073 | 2.288 |
| 4.0 | 258.500 | 196.342 | 131.217 | 65.756 | 2.635 |
| 4.5 | 293.585 | 222.987 | 149.020 | 74.674 | 2.993 |
| 5.0 | 329.377 | 250.164 | 167.174 | 83.768 | 3.357 |

TABLE 17c. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\hat{\Omega}}^2$(B.S.A. TI. RI) $(h^2/\ ab=0.04)$, $(\hat{\hat{\beta}}=0)$

| a/b \ n | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 11.456 | 14.944 | 21.330 |
| 0.5 | 4.640 | 10.361 | 22.681 |
| 1.0 | 3.126 | 14.944 | 43.570 |
| 1.5 | 3.605 | 23.594 | 71.735 |
| 2.0 | 4.640 | 34.220 | 103.343 |
| 2.5 | 6.006 | 46.145 | 136.995 |
| 3.0 | 7.624 | 59.006 | 171.987 |
| 3.5 | 9.451 | 72.576 | 207.915 |
| 4.0 | 11.456 | 86.699 | 244.526 |
| 4.5 | 13.619 | 101.264 | 281.654 |
| 5.0 | 15.920 | 116.191 | 319.184 |
| (m=2) | | | |
| 0.25 | 86.700 | 92.198 | 101.474 |
| 0.5 | 34.220 | 43.570 | 60.145 |
| 1.0 | 14.944 | 31.226 | 63.582 |
| 1.5 | 11.087 | 35.233 | 86.092 |
| 2.0 | 10.361 | 43.569 | 114.670 |
| 2.5 | 10.795 | 54.053 | 146.375 |
| 3.0 | 11.829 | 65.892 | 179.996 |
| 3.5 | 13.247 | 78.687 | 214.904 |
| 4.0 | 14.943 | 92.196 | 250.724 |
| 4.5 | 16.859 | 106.263 | 287.221 |
| 5.0 | 18.956 | 120.775 | 324.236 |
| (m=3) | | | |
| 0.25 | 244.530 | 250.728 | 261.085 |
| 0.5 | 103.344 | 114.671 | 133.887 |
| 1.0 | 43.570 | 63.582 | 99.608 |
| 1.5 | 28.300 | 56.745 | 110.877 |
| 2.0 | 22.681 | 60.145 | 133.887 |
| 2.5 | 20.530 | 67.752 | 162.164 |
| 3.0 | 20.001 | 77.663 | 193.426 |
| 3.5 | 20.378 | 89.049 | 226.596 |
| 4.0 | 21.330 | 101.472 | 261.081 |
| 4.5 | 22.681 | 114.669 | 296.517 |
| 5.0 | 24.332 | 128.466 | 332.667 |

TABLE 17d. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

| | | | $\hat{\Omega}^2$ (B.S.A. TI. RI) MINIMUM ($h^2/ab=0.04$) | | | | | |
|---|---|---|---|---|---|---|---|---|
| m | n | a/b \ $\hat{\beta}$ | 0.0 | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
| 1 | 1 | 1.0 | 3.126 | 3.094 | 2.345 | 1.563 | 0.782 | 0.031 |
| 2 | 1 | 2.0 | 10.361 | 10.258 | 7.774 | 5.185 | 2.593 | 0.104 |
| 3 | 1 | 3.0 | 20.001 | 19.802 | 15.013 | 10.016 | 5.012 | 0.201 |
| 1 | 2 | 0.5 | 10.361 | 10.258 | 7.774 | 5.185 | 2.593 | 0.104 |
| 2 | 2 | 1.0 | 31.226 | 30.915 | 23.446 | 15.648 | 7.833 | 0.314 |
| 3 | 2 | 1.5 | 56.745 | 56.182 | 42.633 | 28.469 | 14.257 | 0.571 |
| 1 | 3 | 0.33 | 20.001 | 19.802 | 15.013 | 10.016 | 5.012 | 0.201 |
| 2 | 3 | 0.67 | 56.745 | 56.182 | 42.633 | 28.469 | 14.257 | 0.571 |
| 3 | 3 | 1.0 | 99.608 | 98.621 | 74.877 | 50.025 | 25.063 | 1.004 |

TABLE 17e. SUMMARY SHEET FOR LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

| $\frac{a}{b}$ \ $\hat{\beta}$ | 0.01 | 0.25 | $\hat{\Omega}^2$(B.S.A. TI. RI) 0.50 | 0.75 | ($h^2/ab=0.05$) 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=1) | | | | |
| 0.25 | 10.441 | 7.914 | 5.280 | 2.642 | 0.106 |
| 0.5 | 4.327 | 3.279 | 2.187 | 1.094 | 0.044 |
| 1.0 | 2.940 | 2.228 | 1.485 | 0.743 | 0.030 |
| 1.5 | 3.381 | 2.562 | 1.708 | 0.854 | 0.034 |
| 2.0 | 4.327 | 3.279 | 2.187 | 1.094 | 0.044 |
| 2.5 | 5.567 | 4.219 | 2.814 | 1.407 | 0.056 |
| 3.0 | 7.025 | 5.324 | 3.551 | 1.776 | 0.071 |
| 3.5 | 8.659 | 6.563 | 4.378 | 2.190 | 0.088 |
| 4.0 | 10.440 | 7.914 | 5.280 | 2.641 | 0.106 |
| 4.5 | 12.350 | 9.363 | 6.247 | 3.126 | 0.125 |
| 5.0 | 14.371 | 10.896 | 7.270 | 3.638 | 0.146 |
| | (m=2, n=1) | | | | |
| 0.25 | 74.171 | 56.320 | 37.632 | 18.856 | 0.756 |
| 0.5 | 30.166 | 22.887 | 15.281 | 7.652 | 0.307 |
| 1.0 | 13.515 | 10.247 | 6.837 | 3.421 | 0.137 |
| 1.5 | 10.113 | 7.666 | 5.114 | 2.559 | 0.102 |
| 2.0 | 9.469 | 7.177 | 4.488 | 2.395 | 0.096 |
| 2.5 | 9.854 | 7.470 | 4.983 | 2.493 | 0.100 |
| 3.0 | 10.771 | 8.165 | 5.447 | 2.725 | 0.109 |
| 3.5 | 12.023 | 9.115 | 6.081 | 3.043 | 0.122 |
| 4.0 | 13.515 | 10.247 | 6.837 | 3.421 | 0.137 |
| 4.5 | 15.193 | 11.520 | 7.687 | 3.847 | 0.154 |
| 5.0 | 17.022 | 12.908 | 8.614 | 4.311 | 0.173 |
| | (m=3, n=1) | | | | |
| 0.25 | 202.861 | 154.076 | 102.965 | 51.594 | 2.068 |
| 0.5 | 87.921 | 66.769 | 44.618 | 22.358 | 0.896 |
| 1.0 | 38.111 | 28.921 | 19.314 | 9.673 | 0.388 |
| 1.5 | 25.099 | 19.040 | 12.710 | 6.363 | 0.255 |
| 2.0 | 20.256 | 15.363 | 10.253 | 5.132 | 0.206 |
| 2.5 | 18.391 | 13.947 | 9.308 | 4.659 | 0.187 |
| 3.0 | 17.932 | 13.599 | 9.075 | 4.542 | 0.182 |
| 3.5 | 18.259 | 13.847 | 9.241 | 4.625 | 0.185 |
| 4.0 | 19.085 | 14.474 | 9.660 | 4.835 | 0.194 |
| 4.5 | 20.256 | 15.362 | 10.253 | 5.132 | 0.206 |
| 5.0 | 21.683 | 16.446 | 10.977 | 5.495 | 0.220 |

TABLE 18a. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\hat{\Omega}}^2$ (B.S.A. TI. RI) ($h^2/ab=0.05$)

| $\frac{a}{b}$ \ $\hat{\hat{\beta}}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| | (m=1, n=2) | | | | |
| 0.25 | 13.515 | 10.247 | 6.837 | 3.421 | 0.137 |
| 0.5 | 9.469 | 7.177 | 4.788 | 2.395 | 0.096 |
| 1.0 | 13.515 | 10.247 | 6.837 | 3.421 | 0.137 |
| 1.5 | 21.046 | 15.962 | 10.654 | 5.333 | 0.214 |
| 2.0 | 30.166 | 22.887 | 15.281 | 7.652 | 0.307 |
| 2.5 | 40.288 | 30.575 | 20.420 | 10.227 | 0.410 |
| 3.0 | 51.108 | 38.796 | 25.915 | 12.982 | 0.520 |
| 3.5 | 62.442 | 47.408 | 31.673 | 15.869 | 0.636 |
| 4.0 | 74.170 | 56.319 | 37.631 | 18.856 | 0.756 |
| 4.5 | 86.206 | 65.466 | 43.746 | 21.921 | 0.879 |
| 5.0 | 98.491 | 74.800 | 49.987 | 25.050 | 1.004 |
| | (m=2, n=2) | | | | |
| 0.25 | 78.720 | 59.777 | 39.943 | 20.015 | 0.802 |
| 0.5 | 38.111 | 28.921 | 19.314 | 9.673 | 0.388 |
| 1.0 | 27.607 | 20.944 | 13.982 | 7.001 | 0.280 |
| 1.5 | 31.030 | 23.543 | 15.719 | 7.871 | 0.315 |
| 2.0 | 38.110 | 28.921 | 19.314 | 9.673 | 0.388 |
| 2.5 | 46.951 | 35.638 | 23.804 | 11.924 | 0.478 |
| 3.0 | 56.869 | 43.173 | 28.842 | 14.449 | 0.579 |
| 3.5 | 67.524 | 51.270 | 34.255 | 17.163 | 0.688 |
| 4.0 | 78.719 | 59.777 | 39.943 | 20.014 | 0.802 |
| 4.5 | 90.325 | 68.596 | 45.839 | 22.970 | 0.921 |
| 5.0 | 102.255 | 77.660 | 51.899 | 26.008 | 1.042 |
| | (m=3, n=2) | | | | |
| 0.25 | 207.865 | 157.875 | 105.503 | 52.865 | 2.119 |
| 0.5 | 97.242 | 73.851 | 49.353 | 24.731 | 0.991 |
| 1.0 | 54.938 | 41.706 | 27.861 | 13.957 | 0.559 |
| 1.5 | 49.212 | 37.355 | 24.952 | 12.499 | 0.501 |
| 2.0 | 52.062 | 39.520 | 26.400 | 13.225 | 0.530 |
| 2.5 | 58.422 | 44.353 | 29.630 | 14.845 | 0.595 |
| 3.0 | 66.674 | 50.623 | 33.823 | 16.946 | 0.679 |
| 3.5 | 76.115 | 57.798 | 38.620 | 19.351 | 0.776 |
| 4.0 | 86.378 | 65.596 | 43.823 | 21.965 | 0.880 |
| 4.5 | 97.241 | 73.850 | 49.352 | 24.731 | 0.991 |
| 5.0 | 108.561 | 82.452 | 55.102 | 27.613 | 1.107 |

TABLE 18b. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\hat{\Omega}}^2$(B.S.A. TI. RI) ($h^2/ab=0.05$)

| $\frac{a}{b}$ \ $\hat{\hat{\beta}}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| (m=1, n=3) | | | | | |
| 0.25 | 19.085 | 14.474 | 9.660 | 4.835 | 0.194 |
| 0.5 | 20.256 | 15.363 | 10.253 | 5.132 | 0.206 |
| 1.0 | 38.111 | 28.921 | 19.314 | 9.673 | 0.388 |
| 1.5 | 61.742 | 46.876 | 31.317 | 15.690 | 0.629 |
| 2.0 | 87.920 | 66.768 | 44.617 | 22.358 | 0.896 |
| 2.5 | 115.543 | 87.756 | 58.648 | 29.391 | 1.178 |
| 3.0 | 144.083 | 109.438 | 73.139 | 36.653 | 1.469 |
| 3.5 | 173.247 | 131.588 | 87.941 | 44.069 | 1.766 |
| 4.0 | 202.858 | 154.074 | 102.963 | 51.594 | 2.068 |
| 4.5 | 232.801 | 176.806 | 118.147 | 59.198 | 2.372 |
| 5.0 | 262.997 | 199.728 | 133.455 | 66.863 | 2.679 |
| (m=2, n=3) | | | | | |
| 0.25 | 86.379 | 65.597 | 43.834 | 21.965 | 0.880 |
| 0.5 | 52.062 | 39.521 | 26.400 | 13.225 | 0.530 |
| 1.0 | 54.938 | 41.706 | 27.861 | 13.957 | 0.559 |
| 1.5 | 73.667 | 55.938 | 37.376 | 18.728 | 0.751 |
| 2.0 | 97.241 | 73.851 | 49.352 | 24.731 | 0.991 |
| 2.5 | 123.209 | 93.580 | 62.541 | 31.342 | 1.256 |
| 3.0 | 150.595 | 114.384 | 76.445 | 38.309 | 1.535 |
| 3.5 | 178.907 | 135.887 | 90.813 | 45.507 | 1.824 |
| 4.0 | 207.862 | 157.873 | 105.501 | 52.865 | 2.119 |
| 4.5 | 237.284 | 180.210 | 120.420 | 60.337 | 2.418 |
| 5.0 | 267.058 | 202.809 | 135.513 | 67.894 | 2.721 |
| (m=3, n=3) | | | | | |
| 0.25 | 216.221 | 164.220 | 109.740 | 54.988 | 2.204 |
| 0.5 | 113.000 | 85.825 | 57.357 | 28.743 | 1.152 |
| 1.0 | 84.840 | 64.427 | 43.052 | 21.573 | 0.865 |
| 1.5 | 94.122 | 71.481 | 47.768 | 23.937 | 0.959 |
| 2.0 | 113.000 | 85.824 | 57.356 | 28.743 | 1.152 |
| 2.5 | 136.086 | 103.363 | 69.080 | 34.618 | 1.388 |
| 3.0 | 161.500 | 122.667 | 81.980 | 41.082 | 1.647 |
| 3.5 | 188.369 | 143.072 | 95.613 | 47.912 | 1.920 |
| 4.0 | 216.219 | 164.218 | 109.739 | 54.987 | 2.204 |
| 4.5 | 244.767 | 185.890 | 124.214 | 62.236 | 2.494 |
| 5.0 | 273.831 | 207.950 | 138.945 | 69.612 | 2.789 |

TABLE 18c. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\hat{\Omega}}^2$(B.S.A. TI. RI) ($h^2/ab=0.05$), ($\hat{\hat{\beta}}=0$)

| $a/b$ \ $\hat{\hat{\beta}}$ | 1 | 2 | 3 |
|---|---|---|---|
| (m=1) | | | |
| 0.25 | 10.546 | 13.651 | 19.277 |
| 0.5 | 4.371 | 9.564 | 20.459 |
| 1.0 | 2.970 | 13.651 | 38.493 |
| 1.5 | 3.415 | 21.257 | 62.360 |
| 2.0 | 4.371 | 30.469 | 88.799 |
| 2.5 | 5.624 | 40.692 | 116.697 |
| 3.0 | 7.096 | 51.620 | 145.522 |
| 3.5 | 8.746 | 63.067 | 174.977 |
| 4.0 | 10.546 | 74.911 | 204.884 |
| 4.5 | 12.474 | 87.068 | 235.126 |
| 5.0 | 14.516 | 99.475 | 265.625 |
| (m=2) | | | |
| 0.25 | 74.913 | 79.507 | 87.242 |
| 0.5 | 30.469 | 38.493 | 52.583 |
| 1.0 | 13.651 | 27.885 | 55.488 |
| 1.5 | 10.215 | 31.341 | 74.404 |
| 2.0 | 9.564 | 38.492 | 98.213 |
| 2.5 | 9.953 | 47.421 | 124.440 |
| 3.0 | 10.879 | 57.438 | 152.099 |
| 3.5 | 12.144 | 68.200 | 180.694 |
| 4.0 | 13.651 | 79.506 | 209.938 |
| 4.5 | 15.346 | 91.228 | 239.655 |
| 5.0 | 17.194 | 103.276 | 269.726 |
| (m=3) | | | |
| 0.25 | 204.887 | 209.941 | 218.381 |
| 0.5 | 88.800 | 98.213 | 114.129 |
| 1.0 | 38.493 | 55.488 | 85.688 |
| 1.5 | 25.352 | 49.705 | 95.063 |
| 2.0 | 20.460 | 52.583 | 114.128 |
| 2.5 | 18.576 | 59.006 | 137.445 |
| 3.0 | 18.112 | 67.341 | 163.113 |
| 3.5 | 18.443 | 76.877 | 190.840 |
| 4.0 | 19.277 | 87.241 | 218.378 |
| 4.5 | 20.459 | 98.212 | 247.212 |
| 5.0 | 21.901 | 109.646 | 276.567 |

TABLE 18d. LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

| m | n | a/b \ $\hat{\hat{\beta}}$ | $\hat{\hat{\Omega}}^2$ (B.S.A. TI. RI) MINIMUM ($h^2/ab=0.05$) 0.0 | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 2.970 | 2.940 | 2.228 | 1.485 | 0.743 | 0.030 |
| 2 | 1 | 2.0 | 9.954 | 9.469 | 7.177 | 4.788 | 2.395 | 0.096 |
| 3 | 1 | 3.0 | 18.112 | 17.932 | 13.599 | 9.075 | 4.542 | 0.182 |
| 1 | 2 | 0.50 | 9.954 | 9.469 | 7.177 | 4.788 | 2.395 | 0.096 |
| 2 | 2 | 1.0 | 27.885 | 27.607 | 20.944 | 13.982 | 7.001 | 0.280 |
| 3 | 2 | 1.50 | 49.705 | 49.212 | 37.355 | 24.952 | 12.499 | 0.501 |
| 1 | 3 | 0.33 | 18.112 | 17.932 | 13.599 | 9.075 | 4.542 | 0.182 |
| 2 | 3 | 0.67 | 49.705 | 49.212 | 37.355 | 24.952 | 12.499 | 0.501 |
| 3 | 3 | 1.0 | 85.688 | 84.840 | 64.427 | 43.052 | 21.573 | 0.865 |

TABLE 18e. SUMMARY SHEET FOR LOWER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

| a/b \ $\hat{\beta}$ | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|
| | | | $\hat{\Omega}^2$ (B.S.A. TI. RI) | | |
| | (m=1, n=1) | | | | |
| 0.25 | 4963.48 | 1397.73 | 689.01 | 424.89 | 295.41 |
| 0.5 | 4698.59 | 1268.93 | 605.00 | 363.00 | 246.62 |
| 1.0 | 4622.08 | 1231.48 | 580.48 | 344.88 | 232.32 |
| 1.5 | 4647.63 | 1244.00 | 588.68 | 350.95 | 237.11 |
| 2.0 | 4698.59 | 1268.93 | 605.00 | 363.00 | 246.62 |
| 2.5 | 4759.53 | 1298.67 | 624.44 | 377.34 | 257.93 |
| 3.0 | 4825.27 | 1330.67 | 645.33 | 392.73 | 270.07 |
| 3.5 | 4893.62 | 1363.87 | 666.96 | 408.67 | 282.63 |
| 4.0 | 4963.47 | 1397.72 | 689.01 | 424.89 | 295.41 |
| 4.5 | 5034.22 | 1431.95 | 711.26 | 441.27 | 308.32 |
| 5.0 | 5105.52 | 1466.37 | 733.64 | 457.72 | 321.28 |
| | (m=2, n=1) | | | | |
| 0.25 | 6693.66 | 2226.08 | 1226.63 | 820.80 | 607.97 |
| 0.5 | 5590.91 | 1699.56 | 884.94 | 568.99 | 408.98 |
| 1.0 | 5075.71 | 1451.99 | 724.29 | 450.85 | 315.87 |
| 1.5 | 4950.96 | 1391.67 | 685.06 | 421.99 | 293.13 |
| 2.0 | 4925.90 | 1379.52 | 677.16 | 416.17 | 288.54 |
| 2.5 | 4940.94 | 1386.81 | 681.90 | 419.66 | 291.30 |
| 3.0 | 4975.98 | 1403.78 | 692.94 | 427.79 | 297.70 |
| 3.5 | 5022.36 | 1426.21 | 707.53 | 438.52 | 306.16 |
| 4.0 | 5075.71 | 1451.99 | 724.29 | 450.85 | 315.87 |
| 4.5 | 5133.63 | 1479.93 | 742.44 | 464.20 | 326.39 |
| 5.0 | 5194.66 | 1509.34 | 761.54 | 478.25 | 337.45 |
| | (m=3, n=1) | | | | |
| 0.25 | 9402.12 | 3522.29 | 2073.08 | 1448.22 | 1106.19 |
| 0.5 | 7006.50 | 2375.31 | 1323.68 | 892.49 | 664.75 |
| 1.0 | 5807.97 | 1803.40 | 952.27 | 618.54 | 448.08 |
| 1.5 | 5445.04 | 1629.66 | 839.61 | 535.65 | 382.69 |
| 2.0 | 5298.15 | 1559.13 | 793.86 | 502.01 | 356.17 |
| 2.5 | 5239.09 | 1530.72 | 775.42 | 488.45 | 345.49 |
| 3.0 | 5224.29 | 1523.61 | 770.80 | 485.05 | 342.82 |
| 3.5 | 5234.86 | 1528.69 | 774.10 | 487.48 | 344.73 |
| 4.0 | 5261.25 | 1541.39 | 782.34 | 493.54 | 349.50 |
| 4.5 | 5298.14 | 1559.13 | 793.85 | 502.00 | 356.17 |
| 5.0 | 5342.32 | 1580.35 | 807.63 | 512.13 | 364.15 |

TABLE 19a. HIGHER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

| a/b \ $\hat{\hat{\beta}}$ | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|
| | | | $\hat{\hat{\Omega}}^2$ (B.S.A. TI. RI) | | |
| | (m=1, n=2) | | | | |
| 0.25 | 5075.71 | 1451.99 | 724.29 | 450.85 | 315.87 |
| 0.5 | 4925.90 | 1379.52 | 677.16 | 416.16 | 288.54 |
| 1.0 | 5075.71 | 1451.99 | 724.29 | 450.85 | 315.87 |
| 1.5 | 5322.70 | 1570.93 | 801.51 | 507.64 | 360.61 |
| 2.0 | 5590.90 | 1699.56 | 884.93 | 568.99 | 408.98 |
| 2.5 | 5865.50 | 1830.90 | 970.10 | 631.66 | 458.44 |
| 3.0 | 6141.85 | 1962.86 | 1055.69 | 694.71 | 508.23 |
| 3.5 | 6418.14 | 2094.68 | 1141.25 | 757.79 | 558.11 |
| 4.0 | 6693.63 | 2226.07 | 1226.62 | 820.79 | 607.96 |
| 4.5 | 6967.99 | 2356.94 | 1311.72 | 883.66 | 657.75 |
| 5.0 | 7241.10 | 2487.27 | 1396.56 | 946.38 | 707.47 |
| | (m=2, n=2) | | | | |
| 0.25 | 6798.24 | 2275.97 | 1259.06 | 844.74 | 626.93 |
| 0.5 | 5807.96 | 1803.40 | 952.27 | 618.54 | 448.08 |
| 1.0 | 5518.10 | 1664.68 | 862.32 | 552.36 | 395.86 |
| 1.5 | 5615.13 | 1711.16 | 892.46 | 574.52 | 413.35 |
| 2.0 | 5807.96 | 1803.40 | 952.26 | 618.53 | 448.07 |
| 2.5 | 6037.37 | 1912.98 | 1023.34 | 670.87 | 489.40 |
| 3.0 | 6283.70 | 2030.54 | 1099.62 | 727.08 | 533.83 |
| 3.5 | 6538.65 | 2152.15 | 1178.59 | 785.33 | 579.90 |
| 4.0 | 6798.22 | 2275.96 | 1259.05 | 844.74 | 626.93 |
| 4.5 | 7060.27 | 2400.97 | 1340.37 | 904.83 | 674.53 |
| 5.0 | 7323.59 | 2526.65 | 1422.20 | 965.35 | 722.51 |
| | (m=3, n=2) | | | | |
| 0.25 | 9501.26 | 3569.97 | 2104.34 | 1471.46 | 1124.69 |
| 0.5 | 7213.61 | 2474.15 | 1388.01 | 940.06 | 702.45 |
| 1.0 | 6236.51 | 2008.02 | 1085.00 | 716.31 | 525.31 |
| 1.5 | 6094.42 | 1940.22 | 1041.00 | 683.89 | 499.68 |
| 2.0 | 6165.55 | 1974.17 | 1063.03 | 700.12 | 512.51 |
| 2.5 | 6321.41 | 2048.53 | 1111.30 | 735.70 | 540.64 |
| 3.0 | 6518.61 | 2142.59 | 1172.38 | 780.75 | 576.27 |
| 3.5 | 6738.50 | 2247.47 | 1240.53 | 831.06 | 616.10 |
| 4.0 | 6971.86 | 2358.78 | 1312.92 | 884.54 | 658.45 |
| 4.5 | 7213.57 | 2474.13 | 1388.00 | 940.05 | 702.48 |
| 5.0 | 7460.70 | 2592.12 | 1464.87 | 996.93 | 747.55 |

TABLE 19b. HIGHER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

| a/b \ h²/ab | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|
| | | | $\hat{\Omega}^2$(B.S.A. TI. RI) | | |
| (m=1, n=3) | | | | | |
| 0.25 | 5261.25 | 1541.39 | 782.34 | 493.54 | 349.50 |
| 0.5 | 5298.14 | 1559.13 | 793.85 | 502.01 | 356.17 |
| 1.0 | 5807.96 | 1803.41 | 952.27 | 618.54 | 488.08 |
| 1.5 | 6401.38 | 2086.68 | 1136.06 | 753.96 | 555.08 |
| 2.0 | 7006.48 | 2375.31 | 1323.67 | 892.49 | 664.75 |
| 2.5 | 7611.02 | 2663.93 | 1511.68 | 1031.59 | 775.05 |
| 3.0 | 8212.07 | 2951.38 | 1699.33 | 1170.56 | 885.48 |
| 3.5 | 8809.05 | 3237.47 | 1886.45 | 1309.54 | 995.87 |
| 4.0 | 9402.06 | 3522.26 | 2073.06 | 1448.20 | 1106.18 |
| 4.5 | 9991.40 | 3805.91 | 2259.19 | 1586.66 | 1216.41 |
| 5.0 | 10577.44 | 4088.56 | 2444.93 | 1724.94 | 1326.55 |
| (m=2, n=3) | | | | | |
| 0.25 | 6971.87 | 2358.79 | 1312.93 | 884.55 | 658.46 |
| 0.5 | 6165.55 | 1974.17 | 1063.03 | 700.12 | 512.51 |
| 1.0 | 6236.50 | 2008.02 | 1085.00 | 716.31 | 525.31 |
| 1.5 | 6682.01 | 2220.53 | 1223.01 | 818.13 | 605.86 |
| 2.0 | 7213.59 | 2474.14 | 1388.00 | 940.06 | 702.45 |
| 2.5 | 7774.45 | 2742.03 | 1562.63 | 1069.32 | 805.00 |
| 3.0 | 8346.70 | 3015.84 | 1741.46 | 1201.91 | 910.31 |
| 3.5 | 8923.31 | 3292.29 | 1922.35 | 1336.20 | 1017.07 |
| 4.0 | 9501.21 | 2569.94 | 2104.32 | 1471.46 | 1124.68 |
| 4.5 | 10078.89 | 3848.08 | 2286.89 | 1607.27 | 1232.82 |
| 5.0 | 10655.69 | 4126.34 | 2469.77 | 1743.44 | 1341.30 |
| (m=3, n=3) | | | | | |
| 0.25 | 9666.21 | 3649.33 | 2156.41 | 1510.18 | 1155.52 |
| 0.5 | 7556.45 | 2637.85 | 1494.68 | 1019.00 | 765.06 |
| 1.0 | 6937.21 | 2342.26 | 1302.17 | 876.60 | 652.16 |
| 1.5 | 7144.68 | 2441.25 | 1366.59 | 924.22 | 689.90 |
| 2.0 | 7556.43 | 2637.85 | 1494.67 | 1018.99 | 765.06 |
| 2.5 | 8045.59 | 2871.70 | 1647.28 | 1132.06 | 854.82 |
| 3.0 | 8570.36 | 3123.00 | 1811.54 | 1253.92 | 951.65 |
| 3.5 | 9113.32 | 3383.52 | 1982.11 | 1380.60 | 1052.39 |
| 4.0 | 9666.16 | 3649.30 | 2156.39 | 1510.17 | 1155.51 |
| 4.5 | 10224.52 | 3918.28 | 2333.01 | 1641.60 | 1260.16 |
| 5.0 | 10785.98 | 4189.27 | 2511.16 | 1774.28 | 1365.87 |

TABLE 19c. HIGHER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\hat{\Omega}}$ (B.S.A. TI. RI) MINIMUM ($\hat{\beta}=0$)

| m | n | $\frac{a}{b}$ \ $\frac{h^2}{ab}$ | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 4622.08 | 1231.48 | 580.58 | 344.88 | 232.32 |
| 2 | 1 | 2.0 | 4925.90 | 1379.52 | 677.16 | 416.17 | 288.54 |
| 3 | 1 | 3.0 | 5224.29 | 1523.61 | 770.80 | 485.05 | 342.82 |
| 1 | 2 | 0.5 | 4925.90 | 1379.52 | 677.16 | 416.17 | 288.54 |
| 2 | 2 | 1.0 | 5518.10 | 1664.68 | 862.32 | 552.36 | 395.86 |
| 3 | 2 | 1.5 | 6094.42 | 1940.22 | 1041.00 | 683.89 | 499.68 |
| 1 | 3 | 0.33 | 5224.29 | 1523.61 | 770.80 | 485.05 | 342.82 |
| 2 | 3 | 0.67 | 6094.42 | 1940.22 | 1041.00 | 683.89 | 499.68 |
| 3 | 3 | 1.0 | 6937.21 | 2342.26 | 1302.17 | 876.60 | 652.16 |

TABLE 19d. SUMMARY SHEET FOR HIGHER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA.

$\hat{\hat{\Omega}}^2$ (B.S.A. TI. RI.)

($m=1$, $n=1$, $h^2/ab=0.02$)

| $a/b$ \ $\hat{\hat{\beta}}$ | 0.01 | 0.25 | 0.50 | 0.75 | 0.99 |
|---|---|---|---|---|---|
| 0.25 | 1397.72 | 1397.56 | 1397.40 | 1397.25 | 1397.09 |
| 0.5 | 1268.93 | 1268.89 | 1268.84 | 1268.80 | 1268.76 |
| 1.0 | 1231.48 | 1231.45 | 1231.43 | 1231.41 | 1231.38 |
| 1.5 | 1243.99 | 1243.97 | 1243.94 | 1243.91 | 1243.88 |
| 2.0 | 1268.93 | 1268.89 | 1268.84 | 1268.80 | 1268.76 |
| 2.5 | 1298.66 | 1298.60 | 1298.54 | 1298.48 | 1298.42 |
| 3.0 | 1330.67 | 1330.58 | 1330.50 | 1330.41 | 1330.32 |
| 3.5 | 1363.86 | 1363.75 | 1363.63 | 1363.51 | 1363.39 |
| 4.0 | 1397.72 | 1397.56 | 1397.40 | 1397.24 | 1397.09 |
| 4.5 | 1431.94 | 1431.74 | 1431.53 | 1431.33 | 1431.13 |
| 5.0 | 1466.36 | 1466.11 | 1465.84 | 1465.60 | 1465.35 |

TABLE 20a. HIGHER SET OF NATURAL FREQUENCY INCLUDING SHEAR AND ROTARY INERTIA

# REFERENCES


1. Archer, R.R., A Nonlinear Shear Deformation Theory For Thin Elastic Shells, Technical report No. EM68-1, April 1968.

2. Brunelle, E.J., Buckling of Transversely Isotropic Mindlin Plates, AIAA Journal, Vol. 9, No. 6, June, 1971.

3. Timoshenko, S.P. and Gere, J.M., Theory of Elastic Stability, McGraw-Hill, New York, 1961.

4. Timoshenko, S.P., Woinowsky-Krieger, S., Theory of Plates and Shell, McGraw-Hill, New York, 1959.

5. Wy, C.I. and Vinson, J.R., Free Vibrations of Plates and Beams of Pyrolytic Graphite Type Materials, AIAA Journal, Vol. 8, No. 2, February, 1970.