THE MEASUREMENT AND ANALYSIS OF "DOGBONE" AND LATERAL SPREAD PHENOMENA ON A UNIVERSAL SLABBING MILL

by

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ABSTRACT

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The term "dogbone" is used to describe the shape of the cross section of a steel slab having undergone a reduction in width as a result of a vertical edger draft. Specifically, the deformation of the edges of a slab which is (by definition) of greater magnitude in width than thickness, results in the upsetting of the edges in the vertical direction, such that the slab thickness is no longer uniform. The presence of this additional dogbone thickness causes unequal and often excessive loading of the horizontal rolls in the forward pass portion of the rolling sequence of an uncompensated schedule.

Inherently necessary to the study of slab dogbone is the measurement and analysis of lateral spread, for it is precisely the occurrence of spread, induced by the horizontal mill drafts, which contributes a significant portion to the total dogbone magnitude. In fact, the two effects are mutual in nature inasmuch as dogbone also uniquely contributes to the overall spread during the forward pass thickness reduction.

The study presented herein illustrates a technique of measuring the described effects and analyzing the resultant data for the purpose of developing compensating equations. Furthermore, it is shown how

the resultant model is implemented in an actual on-line computer controlled slabbing mill.

ACKNOWLEDGMENTS

The author acknowledges Mr. David E. Anderson, without whose original conception of dogbone measurement and analysis this paper would not have been possible. Secondly, I wish to thank my adviser, Professor Charles Alexander, for his helpful advice and criticisms. Also, I am extremely grateful to Miss Mary Kennedy for having typed these pages, and to George Reaggle and Dave Creatura for their assistance in producing the illustrations. Finally, I thank my wife, Carolyn, for her patience and encouragement during the writing of this thesis.

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LIST OF SYMBOLS

SYMBOL	DEFINITION
DRAFT	The amount of reduction on the width or thickness of a slab during a given pass
KM	Horizontal mill constant of proportionality
KE	Edger constant of proportionality
DB	Dogbone magnitude, inches
S	Spread magnitude, inches
DE	Edger draft magnitude, inches
DM	Horizontal mill draft magnitude, inches
DF	Dogbone factor
SF	Spread factor
W	Slab width, inches
T	Slab thickness, inches
WF	Edge working factor
i	Pass number
RM	Horizontal Mill screw position, inches
RE	Edger screw position, inches
RAD	Radial arm in torque-force relationship, inches
MILDIA	Horizontal mill roll diameter, inches
α	Bite angle, radians
K	Yield stress, tons in $^{-2}$
P	Specific roll load, tons per inch width
R'	Radius of curvature of elastically deformed roll, inches
s	Normal roll pressure tons in $^{-2}$

SYMBOL	DEFINITION
r	% reduction or draft
h	Entry thickness, inches
y	Slab thickness at plane of intersection, inches
R	Work roll radius, inches

LIST OF FIGURES

CHAPTER I

INTRODUCTION

In this paper is presented a method of analyzing spread and "dogbone" effects for the purpose of developing empirical equations to predict them. Both lateral spread and vertical (dogbone) upset on the edge of a steel slab are problems that are routinely encountered in any slabbing mill rolling process which employs both vertical (edger) and horizontal (mill) work rolls to reduce an ingot to a slab. The cause of the spread and the dogbone or upset edge problem can be shown to be a direct result of the "draft" or reduction of the thickness and width of the slab on the part of the mill and edger rolls respectively. In fact, the major intent of this paper is to describe the dogbone effect as a function of edger draft and spread as a function of mill draft; and secondarily, to present the means by which a unique set of spread and dogbone compensation equations were arrived at and implemented in an on-line control system for a particular slabbing mill.

General Description of Dogbone and Spread

In order to begin to describe the dogbone effect and its significance as a real problem, it is first necessary to briefly explain the practice by which an ingot is reduced to a slab in the so-called reversing slabbing mill process.

Figure 1 illustrates the general dimensions and relative positions of the actual reducing rolls in a typical reversing slabbing mill. As shown, the ingot to be rolled is delivered first on the edger side of the rolling mill. The sequence of reducing passes which follows (as illustrated in Figure 2) terminates with the slab exiting on the mill side.

As shown in Figure 2, the ingot is delivered by table rolls to the vertical edger, then directly into the horizontal mill rolls for width and thickness reduction respectively. (Dimensions shown are approximate.) The steel is then brought back through the rolls for further reduction in the reverse direction (the edgers are required only to hold spread on the reverse pass caused by the two prior sequential horizontal mill drafts). The dogbone phenomenon is specifically the upsetting of the edges of the ingot on the part of the edger rolls as they reduce the width of the ingot such that the cross section of the steel as it exits the edger rolls appears as the shape illustrated in Figure 2; hence, the term dogbone.

It is pointed out here that, except for the first pass, the total dogbone effect as presented to the horizontal rolls is actually the result of two edger drafts; the first being the spread reduction on the reverse pass, and the second being the requested edger draft on the ensuing forward pass. Of course, the significance of the dogbone problem is manifested in the additional load experienced by the horizontal rolls in the process of drafting the thickness dimension, on the forward pass. It is this excessive load which has been observed in uncompensated drafting schedules in the past, but which may be eliminated,or at least anticipated, by the means described in later text.

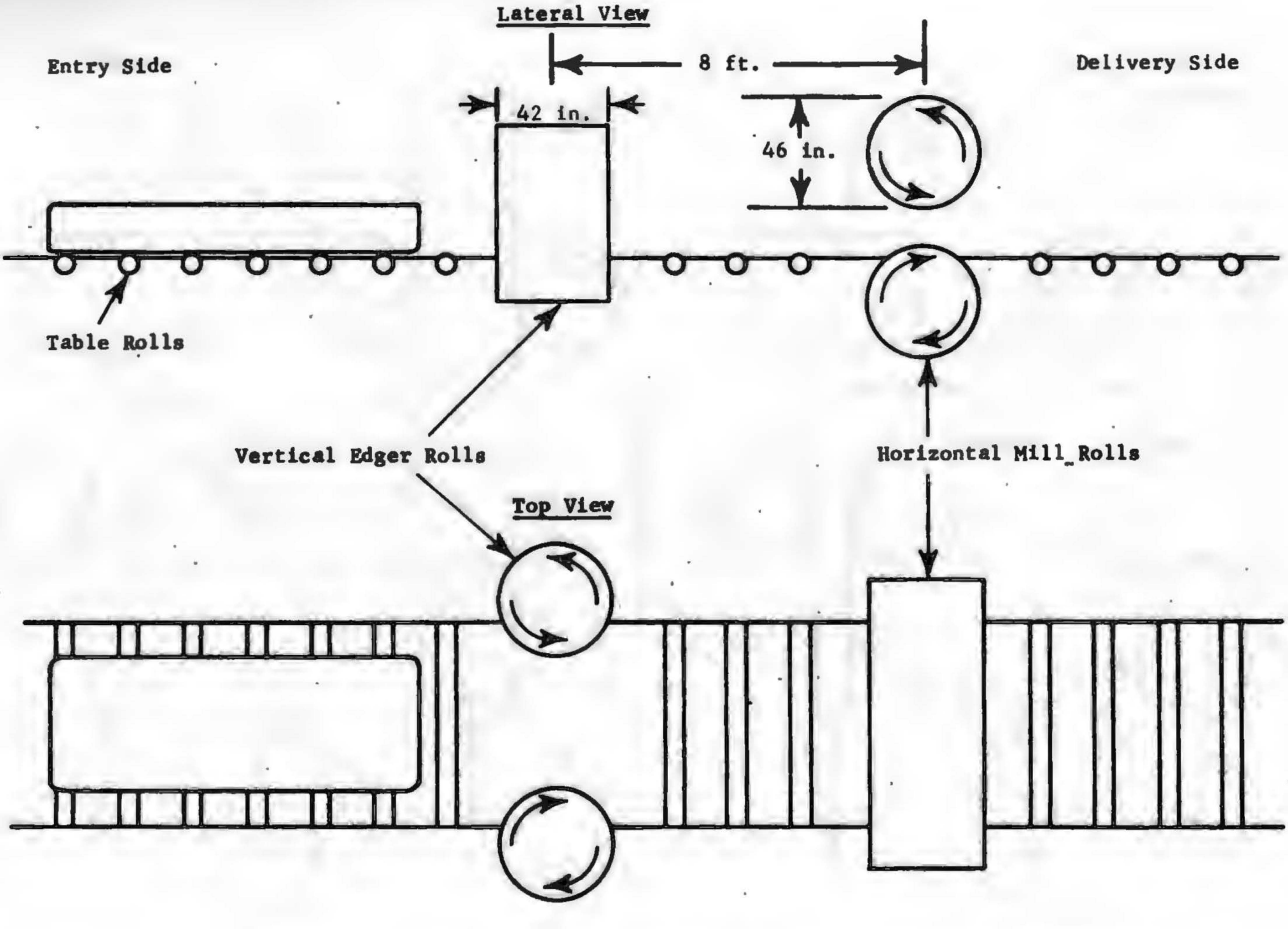

Fig. 1.--General layout of reversing slabbing mill with approximate dimensions.

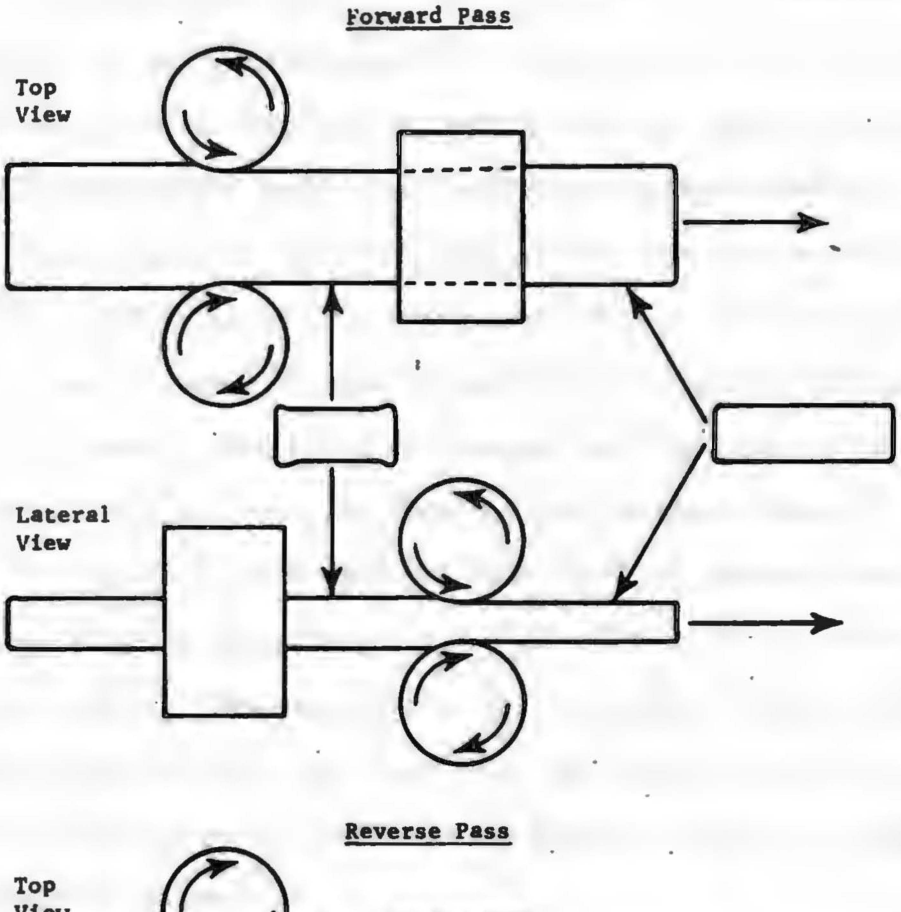

Fig. 2.--Forward and reverse pass sequence showing general slab shape.

The particular mill on which the study herein described was conducted is the No. 2 Slabbing Mill of Youngstown Sheet and Tube Company at Indiana Harbor. Numerous observations of the typical load patterns exhibited throughout portions of various reducing schedules at the mill led to the conclusion that the slabbing mill was consistently more heavily loaded on certain forward passes than on the reverse passes. Such a condition was, of course, contrary to original design intent, in that the process computer which controls the drafting of the mill does so primarily on the basis of constant pass-to-pass torques.

It was suspected that the observed load inequalities could be attributed to the upsetting of the edges of the slab by the draft of the vertical edgers. The suspicion of this so-called dogbone effect was further strengthened by the fact that the largest forward pass loads could be observed specifically during passes in which the edgers were drafting most heavily.

The dogbone study was then initiated with the immediate purpose of reducing excessive forward pass loads in order to reduce the risk of equipment failure and resultant maintenance costs. It was further intended that, by gradually raising the overall level of forward and reverse loads without exceeding the uncompensated peak dogbone loads, a significant increase in rolling rate could be realized.

CHAPTER II

GENERAL APPROACH TO SOLUTION OF DOGBONE PROBLEM

There exist two ways to observe and measure resultant pass-by-pass loads on the vertical and horizontal mills of the system that was studied. The horizontal mill is equipped with a pair of ASEA load cells from which is obtained a voltage output that is proportional to the instantaneous separating force occurring during the drafting of the slab. In addition, both the horizontal and vertical mills may further be monitored by means of available readouts indicating the instantaneous current drawn by the respective motors at any time during the rolling process. Realizing that the mechanical torque on the rolls during reduction of the slab may be directly related to motor current (and furthermore, since the edgers were not equipped with load cells), it was decided that the dogbone and spread characteristics would be measured in terms of horizontal mill and edger motor currents respectively. This was done as follows:

A multichannel strip chart recorder was used to record the edger and horizontal mill motor currents during actual rolling for a wide variety of ingot and slab sizes. Additionally, the load cell output from the horizontal mill was recorded on the same chart paper in order to determine the exact points in time at which the edger and mill were independently loaded. This is essential to eliminate mutual loading effects caused strictly by speed imbalance between the two sets of rolls. The points of interest on the current recordings then represent loads due only to power needed to reduce the given dimension independently by the respective set of rolls.

Figure 3 further clarifies the point at hand. Here is shown a series of typical current and force traces indicating distinct points in time during any given rolling sequence occurring in the middle or later passes of a reducing schedule. (The initial passes are different due to the fact that the ingot is often not long enough at this point in time to occupy the horizontal and vertical rolls simultaneously.) Region (a) - (b) on the forward pass represents independent edger load required strictly to deform the slab for the requested draft. Point (b) indicates the additional current expended by the edger when the horizontal rolls contact the slab (proper threading practice dictates that the edger push the slab slightly on the forward pass). The load cell output is observed as a further verification that the horizontal mill has contacted the slab at point (b). At point (c) the slab has dropped out of the edger (as shown by the edger current) and the mill current level is that which is necessary to independently draft the thickness dimension of the slab for the given pass. The forward pass terminates at point (d). The four points in the reverse pass sequence can be determined similarly.

Mutual loading effects, then, were avoided simply by ensuring that data points were not chosen in regions (b) - (c) or (f) - (g).

A printout from the actual process computer was also utilized which supplied the necessary information regarding pass number and pass-by-pass horizontal and vertical screw positions in inches. In addition to these, the computer supplied as information for each pass a measured

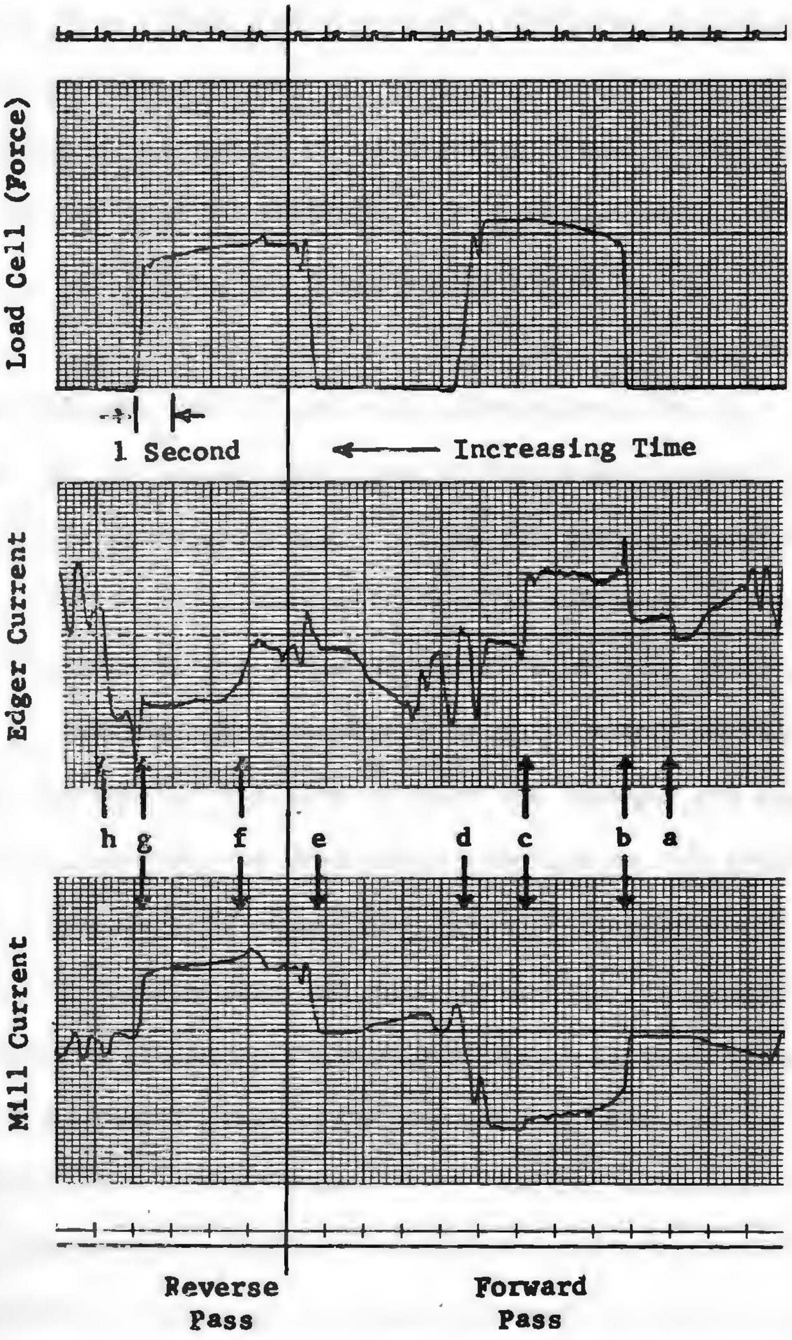

Fig. 3.--Strip chart recording showing forward and reverse pass sequence of events.

torque derived from an average measured separating force from the ASEA load cells (this value is based on the torque-force relationship described in Chapter V) and an "adaptive schedule multiplier" which serves as an indication of the relative hardness of the slab (this adaptive factor will be explained later in the text). Reproductions of the computer printouts for the schedules used in the study are shown in Appendix A.

As one may infer from prior discussion, the true horizontal mill draft on the reverse pass can be determined simply from the difference of the screw settings. Unlike the reverse pass, however, the true effective forward pass draft for the horizontal mill is described by the difference in the screw settings, plus some resultant dogbone effect. In reality, there occurs a certain amount of mill stretch during deformation of the slab on both the forward and reverse pass. This stretch, however, is very small compared to the total draft; consequently, the observed separating force is assumed to be entirely a result of slab deformation and exit dimensions are accepted as equal to the screw positions (plus an approximated lateral spread after completion of a forward pass). The problem is, then, to arrive at some valid description of the forward pass overload (dogbone) as a function of total accumulated edger draft based strictly on the analysis of observed motor currents during both known and effectively unknown drafts.

In order to describe dogbone load as a function of total edger draft, it first becomes necessary to describe lateral spread as a function of the horizontal mill draft. This is simply because the total edger draft immediately prior to a forward pass horizontal mill draft is the summation of the requested forward pass edge reduction plus the immediately prior reverse pass spread which is accumulated as a result of the two earlier horizontal mill drafts.

A similar approach then is used to analyze the spread phenomenon. In other words, the true edger draft is known on any forward pass as simply a difference in edger screw settings. The reverse pass draft (hence spread), however, can be observed only as a finite current recording. Equations describing spread as a function of horizontal mill draft and dogbone, were developed by observing motor currents for both known and unknown edger drafts.

Application of Data to Fundamental Constraining Equations

The power curve method of drafting a slab (see Chapter V on theory and system description) utilized on the mill in discussion is based on the assumption that the torque per inch width per inch draft for a slab of a particular specification and given thickness is a known constant.

This assumption has been accepted and verified in practice throughout the steel industry and is expressed as follows:

$$\text{TORQUE} = \text{KM} \times (\text{MILL DRAFT}) \times \text{WIDTH} \qquad (1)$$

similarly for the edger:

$$\text{TORQUE} = \text{KE} \times (\text{EDGER DRAFT}) \times \text{THICKNESS} \qquad (2)$$

Thus, in the case of the horizontal mill draft, the constant of proportionality may be determined by observing the horizontal mill motor current (since torque is directly proportional to current) when the mill is independently loaded on the reverse pass; that is, when the slab is in the horizontal rolls but has not yet reached the vertical edgers. It is at this point in the pass sequence when the dogbone influence is absent from the observed load and, therefore, the true draft may be obtained from the difference in screw references from the prior and current passes (this information was conveniently available from the computer log). Furthermore, the entry width may be determined from the edger screw reference plus some estimated spread due to the forward pass horizontal mill draft. These quantities may then be divided into the observed motor torque from the chart recording to render the desired constant KM. Realizing then that this constant must remain fixed throughout the remaining passes (assuming that temperature loss is negligible at least through the immediately ensuing pass) the true forward pass draft is then determined by substituting back into the equation the values of KM, the observed torque when the mill is again independently loaded and the entry width obtained from the vertical edger reference in the computer log. Of course, the difference between the draft indicated by the equation and that obtained from actual screw settings can be considered the effective dogbone in inches.

The effective total accumulated spread (that due to forward and reverse horizontal mill drafts) was measured in the same fashion, solving for KE then substituting in the second equation using reverse pass current and thickness. In this case, the constant of proportionality KE is calculated from forward pass observed torque (when the edger is independently loaded) and difference in edger screw position for indicated true draft. The entry thickness is equal to the horizontal mill position from the prior (reverse) pass. Then, on the following reverse pass the established constant, along with observed torque (again when the edger is independently loaded) and slab entry thickness are substituted into the aforementioned equation to render the true effective draft. This value is then considered to be the total accumulated spread due to the corresponding forward and reverse pass horizontal mill drafts. Such is the case because, unlike the horizontal mill, the vertical edgers are not repositioned in the reverse pass, but are only required to hold induced spread.

A simple computer program was written to accept this interpass data and store the quantitative results obtained over a wide range of slab dimensions. These were later used in a regression analysis program to develop the predictive equations which would subsequently be implemented in the actual process computer.

CHAPTER III

COMPUTER ANALYSIS AND IMPLEMENTATION

Simply having illustrated the means of obtaining quantitative measurements of both dogbone and spread is, of course, of no particular consequence unless the accumulated data can lead further to the development of a scheme to predict these quantities as functions of the known rolling parameters which give rise to them. In order to do this, a few basic assumptions were adopted regarding the proposed predictive equations; namely, that for a given slab width and thickness, dogbone is a function of edger draft and that spread is a function of both horizontal mill draft and dogbone. It was further hypothesized that the terms of the functions could be described as simple product forms as follows:

$$DB|_{W,T} = f(DE) = DF(W,T) \times DE \qquad (3)$$

$$S|_{W,T} = f(DM,DB) = SF(W,T) \times DM + WF \times DB(W,T) \qquad (4)$$

where DB = dogbone magnitude

S = spread magnitude

DE = edger draft magnitude

DM = mill draft magnitude (excluding dogbone)

DF = dogbone factor

SF = spread factor

W = slab width

T = slab thickness

WF = edge working factor

The second term in the spread equation involving "edge working factor," is intuitively explained by the fact that dogbone thickness occurs at the very edges of the slab, thereby contributing strongly to lateral spread.

Given that the edger draft is known on a particular forward pass, then the dogbone magnitude is dependent on the dogbone factor calculated at the particular entry width and thickness DF(W,T). Similarly, given that the dogbone magnitude and mill draft is known on a given pass, then the spread is dependent on the spread factor calculated at the proper entry width and thickness SF(W,T). It is seen now that ultimately the problem at hand is to describe both dogbone factor DF and spread factor SF as functions of slab width and thickness. It is precisely for this purpose that the accumulated torque level data was applied to a computer regression analysis.

The Regression Equations

The general method of applying the accumulated data to the regression program is as follows:

Dogbone Regression

First of all, in order to obtain sufficient data which displayed significant changes in the functions being investigated; namely, dogbone and spread, a select number of the accumulated rolling schedules was chosen in which the vertical edgers were active throughout the majority of passes. These are the schedules represented by the computer logs in Appendix A .

Using the corresponding process computer log and chart recording for each schedule as raw data, the dogbone regression data was developed in a FORTRAN program. The major steps are summarized here:

1. Calculate apparent horizontal mill drafts from the computer log screw settings:

$$DM_i = RM_{i-1} - RM_i \tag{5}$$

where i = pass number

RM = horizontal screw reference

2. Calculate the constant of proportionality for all reverse passes from observed torque (derived from chart recording), prior calculated draft and entry width:

$$KM_i = M_i/(DM_i \times W_i) \tag{6}$$

where M_i = observed torque level

DM_i = calculated horizontal draft

W_i = entry width

i = even pass number

3. Calculate dogbone magnitude for the forward passes:

$$DB_i = M_i/(W_i \times KM_{i+1}) - DM_i \tag{7}$$

where i = odd pass number

4. Calculate forward pass edger drafts from computer log references:

$$DE_i = RE_{i-1} - RE_i \tag{8}$$

where RE = edger screw reference

i = odd pass numbers

5. Calculate the constant of proportionality for forward pass edger drafts from observed edger motor torque, prior calculated drafts, and computer log entry thicknesses:

$$KE_i = E_i/(DE_i \times T_{i-1}) \tag{9}$$

where E_i = observed edger motor torque

DE_i = calculated forward edger draft

T_{i-1} = computer log entry thickness (prior pass horizontal reference)

i = odd pass numbers

6. Calculate reverse pass edger draft, hence lateral spread magnitude:

$$S_i = E_i/(T_i \times KE_{i-1}) \tag{10}$$

where i = even pass number

7. Finally, using the results of steps 3, 4, and 6, calculate a dogbone factor:

$$DF_i = DB_i/(DE_i + S_{i-1}) \tag{11}$$

where i = odd pass numbers

Having executed steps 1-7 for all the listed slabs, the final dogbone factors along with their corresponding entry widths and thicknesses were input to the multiple regression program to obtain the hoped for correlating equation. The results and analysis are reserved for Chapter IV.

Spread Factor Regression Data

Using the intermediate results obtained from the steps outlined in the previous section, lateral spread data was developed for the regression program from the following equation which renders the spread factor for a given spread magnitude and total actual horizontal mill draft.

$$SF_i = S_i/(DM_i + DM_{i-1} + DB_{i-1}) \qquad (12)$$

where S_i = calculated spread magnitude (step 6)

DM_i = calculated reverse pass horizontal mill draft (step 1)

DM_{i-1} = apparent forward pass horizontal mill draft (step 1)

DB_{i-1} = calculated dogbone magnitude (step 3)

i = even pass numbers

The resultant spread factors were then grouped with their respective pass entry data, width, thickness, spread and dogbone magnitude, and total horizontal mill draft (including dogbone magnitude) for input to the multiple regression program. Analysis and results are given in Chapter IV.

Characteristics of Regression Data

Before treating the results of the actual regression analysis, further word is required, at this point, to clarify a number of items regarding the characteristics of the raw data sample applied in the study.

As seen in the computer logs of Appendix A, all listed parameters refer to consecutive passes beginning with pass number 4. The purpose of eliminating the first three passes is simply to avoid the difficulty in attempting to gather reliable predictive data from a portion of the rolling schedule which by its very nature is unpredictable in terms of torque, spread, etc. In fact, the process computer, from which the logs were obtained, does not measure loads for the adaptive function during the first three passes. The reasons for the anomalous behavior of the mill during these early passes include the following: surface scale which causes unpredictably higher loads for a given draft; unsystematic drafting practice on the first pass based on the roller's visual evaluation of the condition of the ingot; uneven overall loads caused by the removal of the taper of the ingot; excessive tail end loads resulting from the presence of abnormal projections (stumps) on some ingots; and finally, insufficient overall slab length which precludes the possibility of obtaining reliable average values of force, torque, etc.

Concerning the overall spread of the raw data in terms of ingot-slab sizes, an attempt was made to cover as wide a range as possible of edger-active schedules; however, the final sample size was dictated additionally by factors which are not pertinent to this thesis. Consequently, the higher width categories (60" - 80") are lacking in data points.

As observed in the computer logs of Appendix A, the measured torque values recorded by the computer do not appear consistent with the constant torque philosophy of the drafting practice in all cases. There are a number of reasons for this. First of all, the alternate odd-even pass variations in recorded torque reflect the dogbone overload on the odd pass draft. Secondly, the adaptive function of the computer dynamically alters the target torque of a schedule by causing reschedules to occur based on a pass-by-pass evaluation of load according to measured force throughout the length of the slab. The percent reduction or increase in target torque is indicated by the value of the adaptive schedule multiplier. A value of 1.0 indicates that no reschedule has yet been requested. An increase in the multiplier corresponds to a reduction in target per unit torque and a decrease corresponds to an increase in target for the remaining passes. Such reschedules according to the described variations of the adaptive multiplier, compensate for the variation in slab hardness due to differences in ingot entry temperature.

Finally, it should be realized that the data sample represents interpass loads for which the gradual change in slab temperature is unaccounted. The method of calculating a new constant KM or KE for each known draft, however, should sufficiently minimize the error resulting from this temperature loss during rolling.

CHAPTER IV

THE REGRESSION ANALYSIS

A line of regression is one which shows "how the mean of the values of one variable associated with a given value of another variable changes with the value of the other variable."[1] Similarly, a surface of regression may be said to describe the change in the mean of a variable according to changes in the values of two other variables. The application of the definition to more than two independent variables is obvious. The regression program applied in this study for the dogbone and spread functions utilizes the "least squares" technique of curve fitting to find the mean response surface describing the given function.

In the regression analysis conducted for dogbone factor, the intent is to determine the variation in dogbone factor DF as a function of slab width W and thickness T. The first set of regression results in Appendix D represents the computer output of various statistical values necessary to evaluate the effectiveness of the data in describing the function DF over the entire data sample; i.e., for slab widths from 27.7 inches up to 78.7 inches.

All of the dogbone factor regressions shown were executed with a total of six input variables as listed on the first page of each

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[1] Acheson J. Duncan, Quality Control and Industrial Statistics (Homewood, Illinois: Richard D. Irwin, Inc., 1959), p. 640.

computer run in the column titled "input." V(1) - V(3) are, of course, the basic data input variables as discussed in Chapter III. The remaining ones are simply variations derived from the thickness variable V(2). To the right of each input variable is listed the minimum, maximum and average values out of the data sample for a given variable. The rightmost column is titled "sigma" which is simply the standard deviation from the mean for each variable.

The simple correlation coefficients are listed next in matrix form. These are the least squares estimates of the coefficients obtained from simple X vs. Y regressions between any two of the input variables with the data standardized; i.e., constrained to having the ranges of both X and Y equal such that the individual units are disregarded. The significance of these so called "r" values is such that they indicate the degree of compliance to a straight line relationship between any two particular variables independent of any other inputs.

The simple correlation coefficients matrix is followed by sets of graphs and corresponding summaries including interaction correlation coefficients, analysis of variance, and the actual regression equation for the particular variables requested. Appendix C in particular contains the results of regression analyses of reverse pass edger draft, hence spread as a function of slab width, thickness, horizontal mill draft and dogbone magnitude.

A total of five "runs" are listed with corresponding graphs. The independent variables for each run are listed along with their respective coefficients in the actual regression equation describing the response surface.

In the case of the edger draft regressions, all the computer runs were performed over the entire range of input data.

The Spread Regression

The computed F ratio for each source of variation indicates the theoretical level of confidence in the judgment that the regression coefficient assigned to the particular variable is not the result of random chance. The actual confidence value in terms of percent is obtained from the standard F charts found in statistics texts. The chart is reproduced in Appendix E, in abridged form, showing pertinent areas of discussion.

As an example, consider computer run No. 2. It is from this particular regression that the best overall results were obtained, based on the fact that the total percent explained variation is the highest (45.71% as shown in the analysis of variance) and the corresponding F ratio for the described response surface is also the highest at a value of 21.1. This F ratio, considering 3 versus 75 degrees of freedom, according to the chart (p. 110, Appendix E), corresponds to a confidence value of better than 99.9%.

The resultant equation obtained from run No. 2 (p. 74, Appendix C) is the following:

$$V(3) = .19 + .52\ V(6) + .30\ V(11) + .003\ V(13) \qquad (12)$$

where

V(3) = reverse pass edger draft (spread)

V(6) = dogbone magnitude

V(11) = V(10) x V(7)

V(13) = V(10) x V(1)

and $V(1) = \text{slab width}$

$V(7) = 1/\sqrt{\text{Thickness}}$

$V(10) = V(6) + V(9) = \text{dogbone magnitude} + \text{horizontal mill draft}$

Using the variable names established in Chapter III and factoring, the equation takes on the final form:

$$S = .19 + .52DB + (.30/\sqrt{T} + .003W) \times DM \tag{13}$$

where $DM = DB + V(9)$

or, in other words, total effective horizontal mill draft including the dogbone contribution.

The quantity in parentheses is recognized from the proposed equation in Chapter III as being the spread factor SF(W,T) and the constant .52 as being the edge working factor WF. The equation indicates further that a bias of .19 exists in the spread magnitude as described by the regression. This should not be interpreted as a strict physical truth that spread is present regardless of the occurrence of dogbone or mill draft. Rather, it simply indicates that the particular sample data as utilized in the regression program shows evidence that a positive bias does exist in the function. This can be explained by a number of reasons, the most likely one being errors in data sampling. Furthermore, the edger motor may indeed experience such an additional load (which appears here as spread) on the reverse pass simply as a consequence of slab misalignment upon exit from the horizontal rolls.

The computer plots of the spread function are shown following each particular analysis of variance. The dependent variable is plotted as a function of each independent variable separately, holding the other two variables at their mean values. The corrective curve is indicated by the letter C and the mean value of the data points for a given value of the abscissa is shown by the letter A.

Accuracy of the Model

The standard error of estimate listed as the last value following the analysis of variance is given as .34.[2] From this it is seen that regardless of the fact that some 46% of the variance is explained by the regression model, the equation for spread only accounts for an improvement of approximately 25% in the error as compared to the uncorrected spread data. This is evidenced by the fact that the original uncorrected standard deviation is given as 0.45.

Although the 25% correction appears small, it is still a significant improvement over the old method of predicting spread. Briefly, this method consists of a spread factor table lookup from which values of factors varying from .2 to .5 are selected depending on the slab cross sectional area. The chosen value is then multiplied by the apparent horizontal mill draft to obtain the spread magnitude.

An analysis of the spread magnitudes predicted by this method was made using the same raw data sample on which the regression was based. The standard deviation was calculated to be 0.49 which is even greater than the standard deviation of the uncorrected raw data.

Therefore, the regression equation for predicting spread is 25% more accurate than the old method considering the overall data range.

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[2] The significance of the .34 standard error of estimate is such that chances are 68% (one standard deviation) that, when the independent variables are at their mean values, the spread magnitude will be within ±.34 of the value predicted by the regression equation.

The Dogbone Regression

The first regression attempted on the dogbone data is summarized in the first set of results in Appendix D. As shown, the full data range of 56 observations was used, the slab widths varying from 27.7 inches to 78.7 inches. It can immediately be recognized that the results are very poor, showing little indication that the variance in dogbone factor is explained by thickness and/or width.

However, if one examines the plot of dogbone factor versus width, corrected for the mean value of thickness, it can be seen that there appears to be a trend in the mean values of dogbone to describe two separate slopes separated by a breakpoint near the width value of 45 inches. Consequently, the regression was repeated within separate width ranges of the data sample. The first sample, as listed, contains 22 observations in the width range from 27.7 inches to 42.4 inches.

The results of the new regression show that 66.4% of the variance in dogbone magnitude is explained by the response surface described by the equation rendered for the given data range. Furthermore, the F ratios for each independent variable and the response surface itself are impressive. The F chart indicates better than 99.9% confidence for both the response surface and the width variable. The confidence value for the slab thickness coefficient is better than 99.5%.

The equation given for dogbone factor in this range of slab widths is as follows (employing previously used variable names):

$$DF = .02 + .02W - .013T \tag{14}$$

The standard error of estimate is seen to be .063 as compared to a standard deviation of .101 for the uncorrected dogbone data. This indicates nearly a 38% improvement in the error by using the regression equation shown in the particular width range of slabs.

The middle width range regression shows a confidence value of better than 95% with 34.47% explained variance. The equation given is as follows:

$$DF = -.33 + .015W - .008T \qquad (15)$$

The standard error of estimate (.051), considering a standard deviation of .057 for the raw dogbone data in this width range, is not very impressive. It is felt that better results could be obtained simply with a larger data sample.

Finally, the last width range shown, from 53.45 inches to 57 inches, illustrates some unusual characteristics of the data sample applied to the regression. Specifically, it is evidenced by the large interaction coefficient of -.9978 that a high degree of correlation exists between the two supposedly independent variables. This means that, at least for this data sample, the two variables should not be treated as independent.

In light of the high interaction factor, it would serve no purpose to discuss the form of the regression equation in this category of widths. The unfortunate circumstance of the width-thickness correlation precludes the possibility of predicting dogbone effects within the particular width range. Nevertheless, for academic purposes, the regression results are presented in Appendix D.

General Comment Concerning the Regression

The general results of the regressions indicate that, although the equations account for fairly high percentages of variance of the functions in most cases, the unexplained error still remains relatively large. In effect, the conclusion must be that although the chosen independent variables account for some of the functions' behavior, there are additional factors involved. The original hypothesis that spread and dogbone may be described by width, thickness and the drafting pattern must further be qualified by a statement regarding characteristics of the steel chemistry. This is significantly illustrated by the dogbone regressions in which there appeared to be categories of behavior.

Although these categories could be selected according to slab width, the real underlying reason for the unique behavior may be that the wider slabs chosen for the data sample were extremely different in terms of the chemistry of the steel.

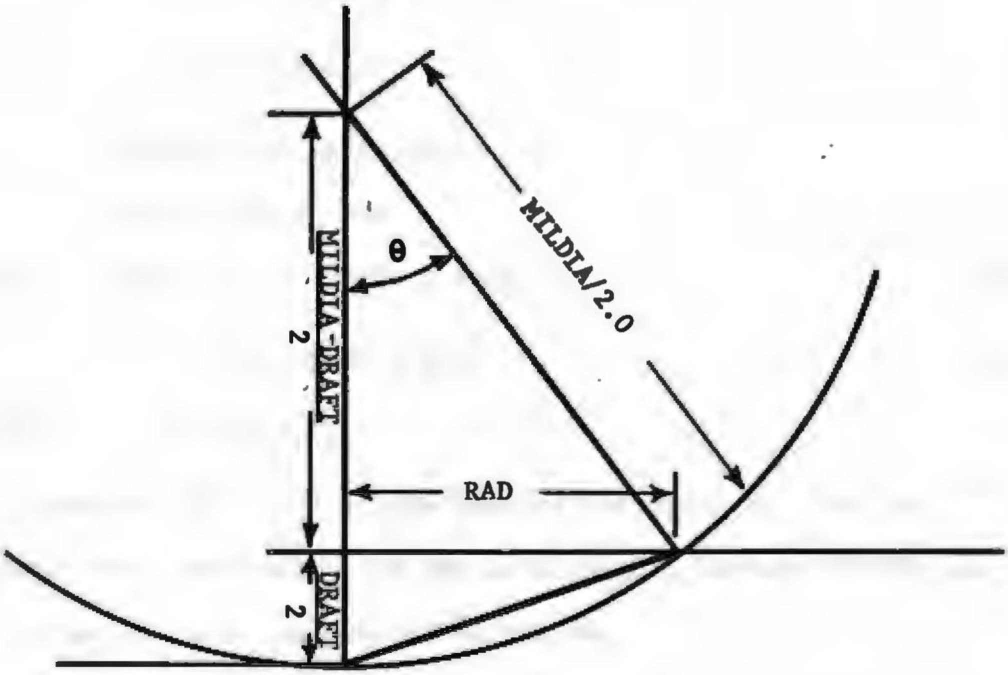
CHAPTER V

PROCESS SYSTEM THEORY

Before describing the way in which the final regression equation for dogbone and spread were implemented on the actual process computer, it is necessary at this point to explain the theory according to which the drafting program calculates reduction schedules. Basically, the computer program which is responsible for generating the pass-by-pass screw position references utilizes the power curve method of determining a systematic drafting schedule prior to the active rolling of the slab. The resultant rolling schedule is one that is primarily torque constrained; that is, upon calculating a particular draft for a given pass, the first criterion in determining the magnitude is the established torque limit. Secondly, a force limit check is made and finally, an absolute draft limit check is applied.

The torque and force algorithms used in the drafting program are based on the following theory:

Referring to Figure 4, given a certain entry thickness for a slab, the rolling force associated with a corresponding draft or reduction in thickness is a function of the contact area projected on the horizontal plane of the slab. The resulting torque on a single roll is proportional to the product of the force and the effective average lever arm (RAD/2). Considering both rolls, the total rolling torque then is proportional to the force times RAD.



MILDIA = Mill Diameter (inches)

DRAFT = Mill Pass Draft (inches)

θ = Bite Angle

RAD = Horizontally projected contact length (inches)

Fig. 4.--Geometry of force-torque relationship during slab deformation.

In summary:

$$\text{FORCE} = K_1 \times \text{CONTACT AREA} \tag{16}$$

$$= K_1 \times \text{WIDTH} \times \text{RAD}$$

where K_1 = a constant

WIDTH = slab entry width

RAD = radial arm

furthermore $$\text{TORQ} = K_2 \times (\text{FORCE} \times \text{RAD}) \tag{17}$$

$$= K \times \text{WIDTH} \times \text{RAD}^2 \tag{18}$$

where $K = K_1 \times K_2$

In order to develop a torque constrained drafting schedule, the computer must obviously know the relationship between torque and draft. To obtain this, we proceed as follows:

Considering the major right triangle in Figure 4 formed by RAD, MILDIA/2, and MILDIA/2 - DRAFT/2 and applying the Pythagorean theorem:

$$\text{RAD}^2 = \left(\frac{\text{MILDIA}}{2}\right)^2 - \left(\frac{\text{MILDIA}}{2} - \frac{\text{DRAFT}}{2}\right)^2 \tag{19}$$

$$\text{RAD}^2 = \frac{\text{MILDIA}^2}{4} - \frac{1}{4}\left(\text{MILDIA}^2 - 2\ (\text{MILDIA} \times \text{DRAFT}) + \text{DRAFT}^2\right) \tag{20}$$

$$\text{RAD}^2 = \frac{\text{MILDIA} \times \text{DRAFT}}{2} - \frac{\text{DRAFT}^2}{4} \tag{21}$$

For MILDIA much greater than DRAFT we may neglect the last term of the above expression. This then yields

$$\text{RAD} = \sqrt{\left(\frac{\text{MILDIA}}{2} \times \text{DRAFT}\right)}$$

Substituting into equation (18)

$$\text{TORQ} = K \times \text{WIDTH} \times \frac{\text{MILDIA}}{2} \times \text{DRAFT} \tag{22}$$

The constant of proportionality K is seen to have the units of force/inch^2, which may be recognized as an effective yield stress. The process computer in the system at hand accommodates variation of the constant K from slab to slab according to current slab thickness and the metallurgical specification (or hardness) of the steel.

The first variation is represented in the computer power curves as a set of four slopes indicating the increasing difficulty in deforming the slab as the thickness decreases (due to a greater percentage draft). The second factor is simply a normalizing constant which multiplies each of the aforementioned slopes to account for variation in spec. from that of mild steel (hence, for mild steel the normalizer is 1.0).

The resulting power curves which show per-unit torque per inch width as a function of slab thickness are shown in Figure 5 with typical numbers used in the computer.

As an example, assume that a slab, possessing specs as shown by the lower curve, is entering a reduction pass with an entry width of 40 inches and thickness of 24 inches, then, according to the power curve for this spec., the computer would calculate the desired draft as follows (for a 200% mill motor load).

$$\Delta y = \frac{2}{40} = .05 \qquad (23)$$

$$\frac{\Delta y}{\Delta x} = \frac{.05}{\Delta x} = \frac{(.47 - .31)}{8} = .02 \qquad (24)$$

$$\Delta x = \text{DRAFT} = \frac{.05}{.02} = 2.5 \text{ inches} \qquad (25)$$

The new entry thickness for the next mill pass then will be 24 - 2.5 or 21.5 inches. This result is stored in the computer for reference in determining the next mill draft. The edger drafting is

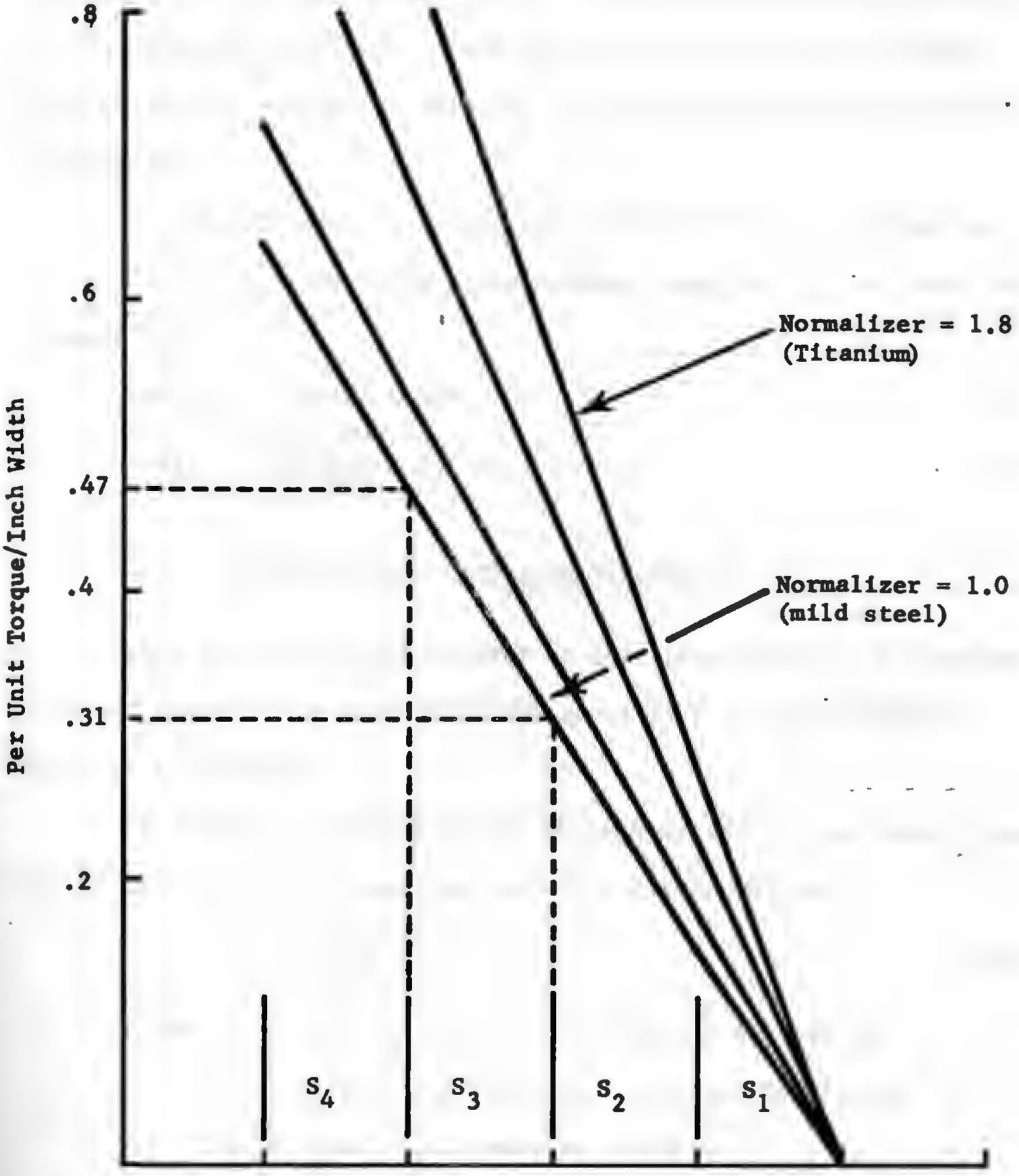


Fig. 5.--Power curves illustrating hardness factors.

calculated in the same fashion and the resultant exit width saved for the next pass calculation. Of course, the power curve for the edger rolls represents a graph of per-unit torque per inch thickness versus entry width.

Having resolved the torque constrained draft, the corresponding roll (per unit) force may be calculated according to the equations repeated here

$$\text{FORCE} = \text{K} \times \text{WIDTH} \times \text{RAD} \tag{26}$$

$$\text{RAD} = \sqrt{\left(\frac{\text{MILDIA}}{2} \times \text{DRAFT}\right)} \tag{27}$$

Support of the Theory in Literature

This result is substantiated in the literature if one considers the exact equation for specific force given by R. B. Sims[3] in the theory of hot rolling:

He states that when the bite angle θ is small, "and where plane deformation occurs, the specific roll load may be written"

$$P = R'\int_0^{\alpha} s \, d\theta \tag{28}$$

where P = specific roll force (tons per inch width)

R' = radius of curvature of elastically deformed roll

s = normal roll pressure (tons/in.^2)

θ = bite angle

α = particular angle in radians

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[3] R. B. Sims, "The Calculation of Roll Force and Torque in Hot Rolling Mills," Research on the Rolling of Strip, A Symposium of Selected Papers (London: Waterlow and Sons, 1958), p. 175.

Which is to say that the specific force is equal to the integral of the normal roll pressure over the arc of contact determined by the bite angle θ.

The integral renders the following expression:

$$P = K\sqrt{R'\delta}\quad Q_p\left(\frac{R'}{h},\ r\right) \qquad (29)$$

where K = yield stress (tons per inch^2)

δ = draft (inches)

and the function Q_p is given as

$$Q_p = \frac{\pi}{2}\sqrt{\frac{1-r}{r}}\tan^{-1}\sqrt{\frac{r}{1-r}} - \frac{\pi}{4} - \sqrt{\frac{1-r}{r}}\sqrt{\frac{R'}{h}}\log_e\frac{Y}{h} + \frac{1}{2}\sqrt{\frac{1-r}{r}}\sqrt{\frac{R'}{h}}\log_e\frac{1}{1-r} \qquad (30)$$

where r = % reduction

h = entry thickness

y = thickness of the slab at the plane of intersection.

The value of R' is calculated by the product of R, the roll radius, and a correction factor accounting for the deformation of the roll. In hot rolling, the correction factor is found to be negligible; therefore, R' may be approximated simply by R, the roll radius. The graph of the function $Q_p\left(\frac{R'}{h},\ r\right)$, shown in Figure 6, shows that indeed, within the range to which the slabbing mill parameters are applicable, the value of Q_p is very close to 1.

Specifically, the relevant area of the graph is bounded by the lower curve ($R'/h = 5$) and the 0.1 - 0.4 abscissa values.

Substituting the result for Q_p back in the original equation for P and recalling that $R' = R$ in this case, renders the equation:

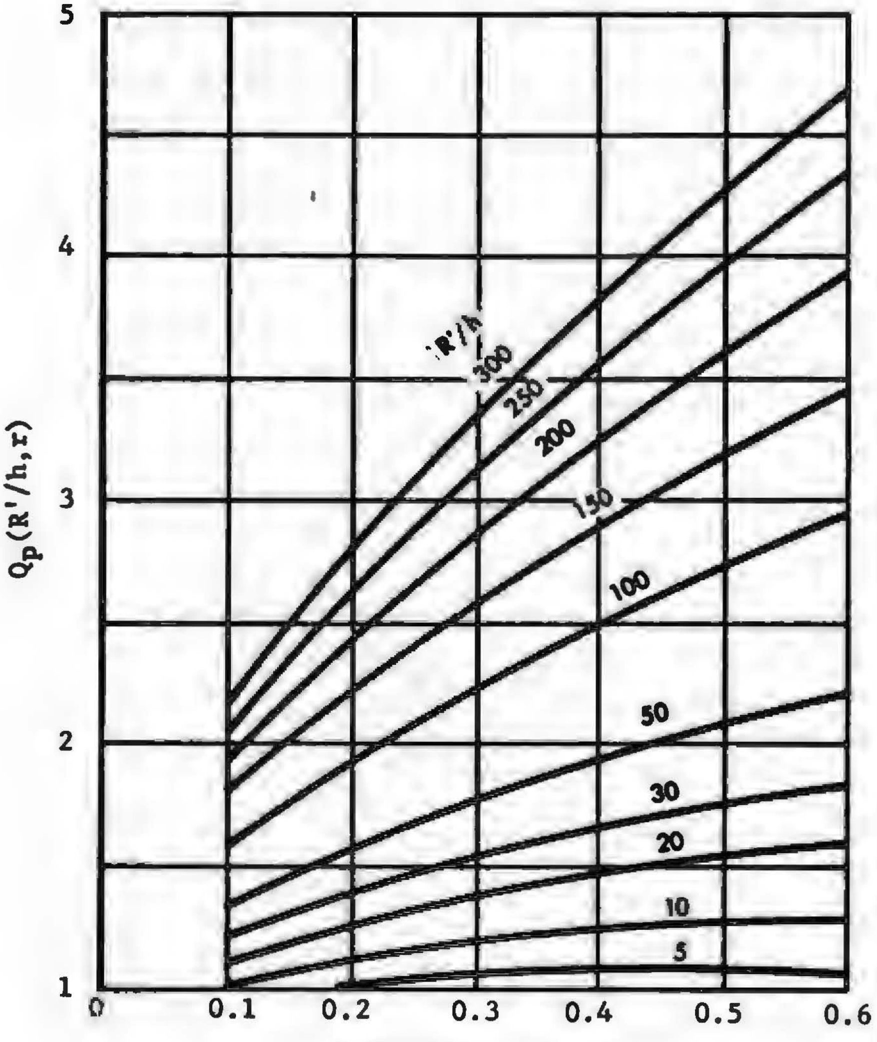

Fig. 6.--Function Q_p showing region applicable to a slabbing mill.[a]

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[a] Sims, p.176

$$P = k \times \sqrt{R\delta} \tag{31}$$

which is the expression for specific force or, in other words, force per inch width; therefore, multiplying by width gives our original result:

$$F = k \times \text{WIDTH} \sqrt{R \times \text{DRAFT}} \tag{32}$$

CHAPTER VI

IMPLEMENTATION OF THE PREDICTIVE EQUATION IN THE PROCESS COMPUTER

Having arrived at a set of equations which are to predict dogbone and spread in an on-line process, the next logical step is to test their accuracy prior to actual implementation, and secondly, to verify that the equations are compatible with the actual on-line program in which they are to be used. This was accomplished by means of two computer programs, the condensed flowcharts of which are shown in Figures 7 and 8.

The first program, referring to the corresponding flowchart, essentially renders a comparison of actual and predicted dogbone and spread loads on a one-to-one rather than a percentage basis. The values for the constants of proportionality KM and KE are computed as averages over the entire reducing schedule for a given slab. These are, in turn, used to calculate interpass edger and mill loads according to the predictive equations in conjunction with actual screw position data from the process computer. The pass-by-pass results may then be compared to the actual strip chart recordings to which the schedules correspond.

Comparative results are listed with the corresponding computer logs in Appendix B.

The purpose of comparing pass-by-pass predicted and actual loads at this point in the project development is simply a safety precaution to determine whether gross errors exist anywhere in the

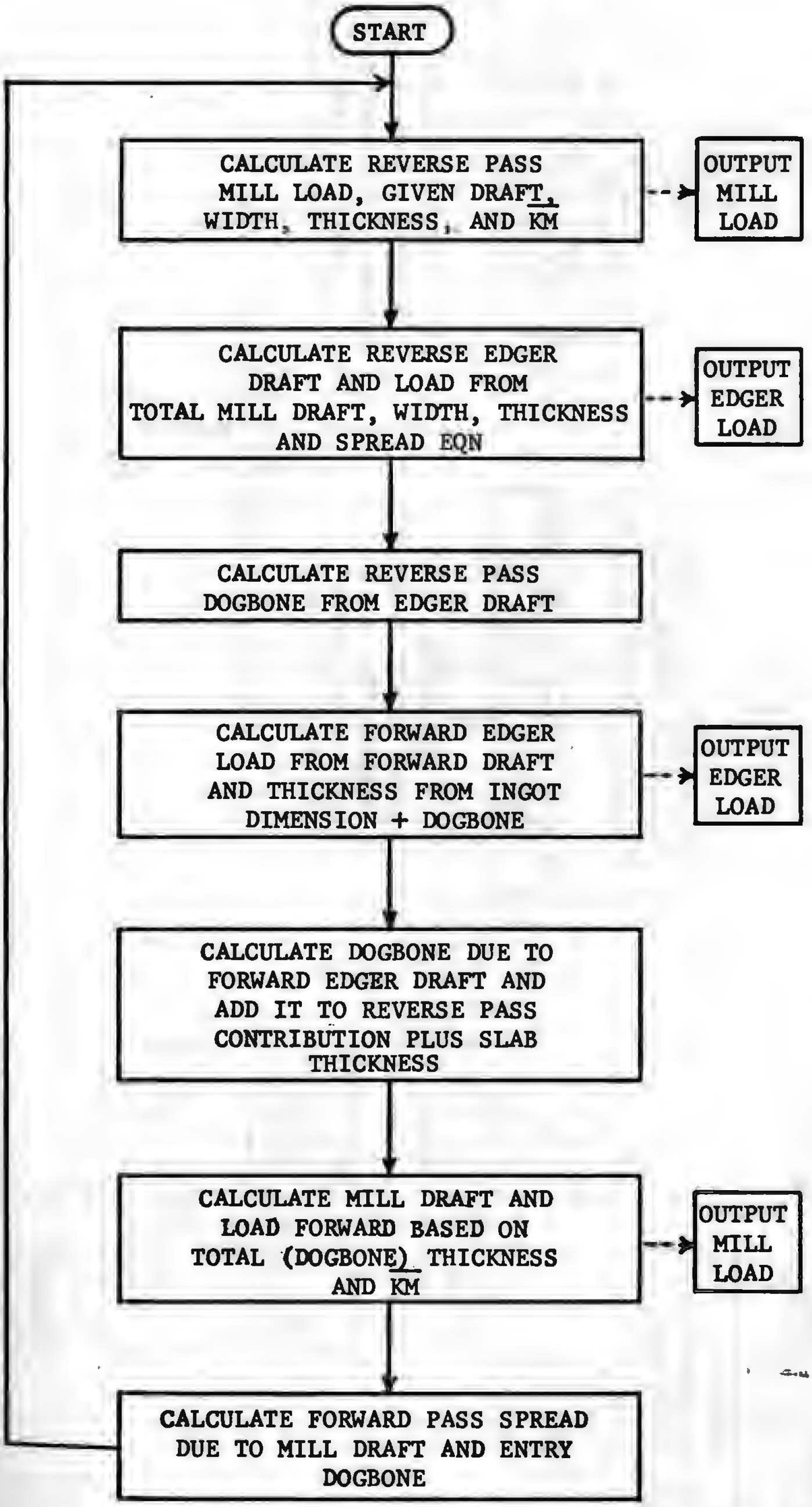

Fig. 7.--Flowchart of Verification Program

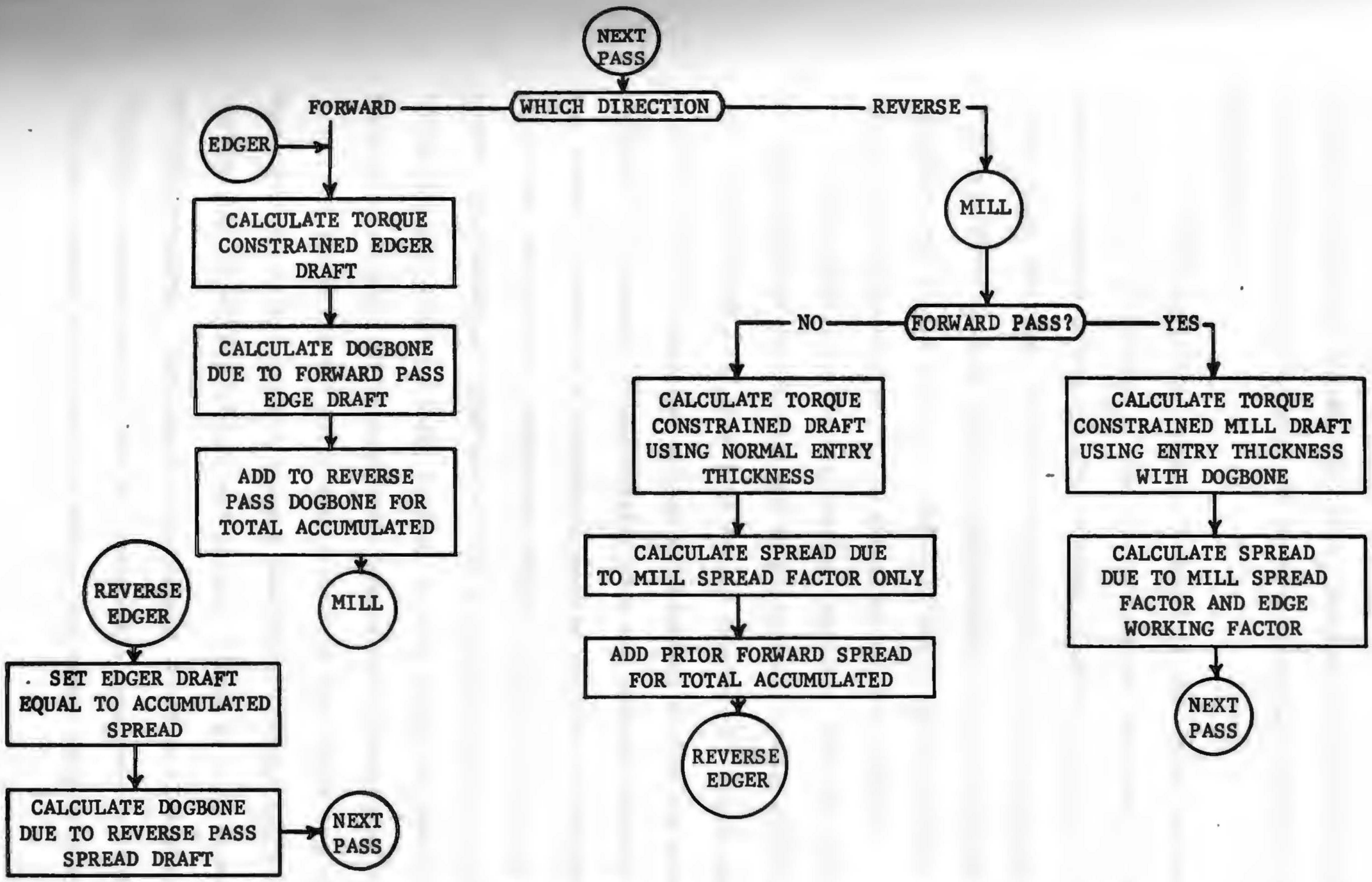

Fig. 8.--Flowchart of Process Computer Schedule Generation Program

overall predictive scheme, such that, the application of the equations within the process program might be of no particular benefit or even detrimental.

Furthermore, the results rendered at this point certainly are not a precise indication of the accuracy of the equations as they would be applied in the on-line process. Specifically, the calculation of the constants of proportionality as an average induces errors in passes whose torques deviate significantly from the average. Such instances may be observed in a number of early passes in which loads are not as predictable as in the remainder of the schedule.

The preliminary results having indicated that the equations at least rendered no gross errors in predicting loading trends on a pass-to-pass basis, the next step was to develop the actual program routines to effectuate the on-line calculations within the process software environment. Compatibility with the process computer program was tested by means of a simulation program (written in FORTRAN) which duplicated the schedule generation program used in the on-line computer. The condensed flowchart of the program is shown in Figure 8, illustrating the point in the logical flow at which the predictive equations are inserted to modify the spread and thickness calculations.

By injecting the predictive equations into their logical areas within the program, it becomes possible to obtain exact schedule results over an entire product mix without having to risk premature on-line debugging. Furthermore, the results obtained from the simulation program verify that all parameters necessary for interpass load calculations are accessible within the normal program flow.

Finally, it offers the advantage of ultimately comparing results from both computers before any on-line tests are attempted.

After the FORTRAN program results were studied, the equivalent machine language coding was developed and installed in the process computer for the purpose of generating drafting schedules off line. Numerous schedules were subsequently produced and compared with those obtained from the FORTRAN program. The calculated torques, references, etc., proved to be identical between the two programs for any given ingot-slab size and specification.

The software modifications as installed in the process computer included two variable gain factors. The first, which varied from 0.0 to 1.0 represents the amount of dogbone compensation desired for a given schedule. A value of 1.0 means that 100% of the calculated dogbone magnitude would be used to compensate each forward pass draft in the schedule. The second factor, which varies from 1.0 to 2.0 represents the requested increase in the overall horizontal mill target load. A value of 1.0 for this factor means that no additional loading of the motors is requested beyond the already established per unit value for any given schedule.

The purpose of the aforementioned gain factors was to facilitate a gradual tuning of the process program in terms of equalizing alternate pass loads and raising overall peak current levels for the purpose of reducing the actual number of passes required for a given slab. Furthermore, the factors permitted an immediate desensitizing of the dogbone compensation equations; hence, a return to the original system equations in the event of unforeseen difficulties at any time during the trial period or thereafter.

Initial On-line Observations

Although, unfortunately, actual on-line trials have not been pursued sufficiently to merit an extensive statistical evaluation of results (due to circumstances beyond the control of the author), a few observations can be made at this point based on the results of a short preliminary trial of the on-line, compensated program.

In general, it was noted that the alternate pass dogbone effects were indeed eliminated in the majority of schedules as regards current, hence torque, overloads; and that except for minor adjustments to the requested target per unit torque on the earlier passes, the overall loads could be increased successfully without exceeding peak loads exhibited by the old uncompensated schedules.

It was interesting to note, however, that in some of the dogbone compensated schedules, particularly for wide slabs, the observed roll force was actually overcompensated, such that, the forward pass horizontal separating forces were lower than those of the reverse passes even though the average forward and reverse pass currents were equal. In fact, the dogbone equation gain factor was reduced from 1.0 to 0.8 before the wide slab schedules indicated equalized alternate pass force readings.

Such overcompensation of the force is not disadvantageous since it is the torque constraint which dictates the drafting pattern on a particular ingot. Furthermore, it is certainly beneficial to be assured of maintaining conservative force levels throughout a schedule which is optimally torque compensated.

The reason for the apparent overcompensation for dogbone in terms of separating force is that the compensating equations were derived from observed current, hence torque, effects, and therefore one would not expect a one-to-one reduction in force attributable to forward pass dogbone magnitude.

CHAPTER VII

SUMMARY AND CONCLUSION

A means of measuring unknown forward pass dogbone loads has been described on the basis of observed loads during known reverse pass horizontal mill drafts. Similarly, it has been illustrated how reverse pass spread may be measured on the basis of observed loading during known forward pass vertical edger drafts.

A general description of the drafting philosophy of the slabbing mill has been given, and it has been pointed out how dogbone compensation may improve the drafting schedules in terms of more equal loading and optimal use of available motor torque.

Simple predictive equations were proposed and a method of applying the accumulated data to a regression program was demonstrated. The results of the spread and dogbone regression were discussed and comments were made regarding the statistical validity and effectiveness of the resultant equations.

The regression results have shown that, at least for this particular effort, the equations based on the present data sample explain a certain percentage of the variance of the given function; however, more is needed to predict them entirely. The study suggests, then, that it is insufficient to specify only slab dimensions and draft magnitudes in order to fully predict corresponding spread and dogbone effects. Nevertheless, actual observations made during the short on-line trial period have indicated that the compensating equations are more effective than one would expect, despite the limited data sample from which the equations were derived.

It is suggested at this point that in order to predict dogbone and spread magnitudes with a high degree of accuracy, it is necessary not only to observe slab geometry and drafting patterns, but also to establish some knowledge of the chemistry of the particular slab. Then perhaps, the resultant data, qualified according to categories of chemistry, would display more definite correlations than were observed in the present study.

APPENDIX A

Process Computer Logs

DESCRIPTION OF CONTENTS

Each table shown here represents the information output by the process computer starting with the fourth pass of the slab through the mill. Each row of data illustrates actual measurements of slab dimensions and average horizontal mill loads as input to the computer during the actual time of the given pass. The width and thickness columns represent the edger and horizontal roll reference position respectively, as preset by the computer prior to initiating the given pass. The Measured Per Unit Torque column is obtained by averaging numerous roll force readings throughout the pass and converting the resultant value to per unit torque according to the force-torque equation derived in Chapter V.

The Adaptive Schedule Multiplier is a unitless number (calculated by the computer) which represents a comparison of measured versus requested per unit torque.

In effect, this number represents the relative difference in hardness of the particular slab primarily due to effects of temperature. A cold slab, for example, would produce the result shown in Table 1; that is, an increase (above 1.0) in the multiplier. This indicates that a reschedule of the remaining passes was made to reduce the absolute torque for these passes. A reschedule is indicated each time the multiplier changes value.

The data shown in these tables were obtained from the slabbing mill process computer during actual mill tests conducted in April 1973.

TABLE 1

PROCESS COMPUTER ENGINEERING LOG FOR SLAB NO. 1
INGOT SIZE: 36 x 62
SLAB SIZE: 8.5 x 46

Pass Count (i)	Width (RE$_i$)	Thickness (RM$_i$)	Measured Per Unit Torque	Adaptive Schedule Multiplier
4	58.60	29.67	2.148	1.052
5	57.00	28.22	2.314	1.052
6	57.00	26.85	2.064	1.052
7	55.45	25.40	2.346	1.052
8	55.45	24.00	1.964	1.052
9	53.65	22.55	2.358	1.052
10	53.65	21.15	1.849	1.052
11	51.65	19.70	2.228	1.052
12	51.65	18.25	1.907	1.052
13	49.65	16.70	2.415	1.052
14	49.65	15.17	1.921	1.052
15	47.65	13.62	2.303	1.052
16	47.65	12.10	1.917	1.052
17	46.65	10.52	2.186	1.052
18	46.65	9.45	1.300	1.052
19	46.30	8.72	0.948	1.052

TABLE 2

PROCESS COMPUTER ENGINEERING LOG FOR SLAB NO. 2
INGOT SIZE: 33 x 84
SLAB SIZE: 7.0 x 76

Pass Count (i)	Width (RE$_i$)	Thickness (RM$_i$)	Measured Per Unit Torque	Adaptive Schedule Multiplier
4	80.40	27.85	1.898	1.000
5	78.70	26.75	2.261	1.000
6	78.70	25.70	1.988	1.000
7	76.90	24.60	2.352	1.000
8	76.90	23.55	2.002	1.000
9	76.65	22.45	2.298	1.000
10	77.04	21.40	1.944	1.000
11	77.44	20.35	2.039	1.000
12	76.65	19.32	1.774	1.000
13	76.65	18.25	2.187	1.000
14	77.03	17.22	1.976	1.000
15	77.44	16.15	1.959	1.000
16	76.65	15.10	1.847	1.000
17	76.65	14.05	1.988	1.000
18	77.02	13.05	1.753	1.000
19	77.42	12.00	1.857	1.000
20	76.65	11.00	1.802	1.000
21	76.65	10.00	1.855	1.000
22	77.04	8.95	1.928	1.000
23	77.37	8.07	1.522	1.000
24	76.65	7.50	1.043	1.000
25	76.45	7.15	0.670	1.000

TABLE 3

PROCESS COMPUTER ENGINEERING LOG FOR SLAB NO. 3

INGOT SIZE: 33 x 73
SLAB SIZE: 7.5 x 66.5

Pass Count (i)	Width (RE_i)	Thickness (RM_i)	Measured Per Unit Torque	Adaptive Schedule Multiplier
4	69.30	27.15	2.260	1.084
5	67.55	26.00	2.314	1.251
6	67.50	24.70	2.557	1.251
7	66.90	23.82	1.897	1.251
8	67.36	22.60	2.140	1.251
9	67.75	21.55	2.031	1.251
10	66.90	20.55	1.823	1.251
11	66.90	19.50	1.872	1.251
12	67.27	18.50	1.662	1.251
13	67.65	17.50	1.815	1.251
14	66.90	16.45	1.817	1.251
15	66.90	15.50	1.761	1.251
16	67.25	14.55	1.509	1.050
17	67.63	13.55	1.722	1.050
18	66.90	12.30	2.135	1.050
19	66.90	11.15	2.004	1.050
20	66.90	10.05	1.888	1.050
21	66.90	9.02	1.791	1.050
22	66.90	8.27	1.263	1.050
23	66.75	7.85	0.776	1.050

TABLE 4

PROCESS COMPUTER ENGINEERING LOG FOR SLAB NO. 4

INGOT SIZE: 36 x 62
SLAB SIZE: 8.5 x 46

Pass Count (i)	Width (RE_i)	Thickness (RM_i)	Measured Per Unit Torque	Adaptive Schedule Multiplier
4	58.65	29.67	2.034	1.000
5	57.15	28.22	2.386	1.130
6	56.90	26.77	2.274	1.130
7	55.35	25.50	2.192	1.130
8	55.35	24.05	2.065	1.130
9	53.65	22.75	2.250	1.130
10	53.65	21.40	1.847	1.130
11	51.75	20.02	2.246	1.130
12	51.75	18.72	1.715	1.130
13	49.70	17.25	2.436	1.130
14	49.70	15.82	1.858	1.130
15	47.70	14.37	2.202	1.130
16	47.70	12.97	1.745	1.130
17	46.65	11.47	2.187	1.130
18	46.65	10.00	1.854	1.130
19	46.00	8.75	1.756	1.130

TABLE 5

PROCESS COMPUTER ENGINEERING LOG FOR SLAB NO. 5
INGOT SIZE: 36 x 62
SLAB SIZE: 8.5 x 46

Pass Count (i)	Width (RE_i)	Thickness (RM_i)	Measured Per Unit Torque	Adaptive Schedule Multiplier
4	58.65	29.67	2.009	1.000
5	57.00	28.22	2.297	1.000
6	57.00	26.77	2.234	1.156
7	55.40	25.52	2.061	1.156
8	55.40	24.15	1.937	1.156
9	53.80	22.85	2.156	1.156
10	53.80	21.55	1.784	1.156
11	51.90	20.22	2.100	1.156
12	51.85	18.92	1.621	1.156
13	49.90	17.52	2.225	1.156
14	49.90	16.12	1.796	1.156
15	47.90	14.67	2.171	1.156
16	47.90	13.32	1.644	1.156
17	46.65	11.85	2.049	1.156
18	46.65	10.42	1.773	1.156
19	46.65	9.62	1.011	1.156
20	46.65	9.07	0.635	1.156
21	46.50	8.75	0.423	1.156

TABLE 6

PROCESS COMPUTER ENGINEERING LOG FOR SLAB NO. 6
INGOT SIZE: 36 x 62
SLAB SIZE: 8.5 x 46

Pass Count (i)	Width (RE_i)	Thickness (RM_i)	Measured Per Unit Torque	Adaptive Schedule Multiplier
4	58.65	29.67	2.142	1.050
5	57.00	28.22	2.369	1.050
6	57.00	26.85	2.056	1.050
7	55.55	25.42	2.379	1.187
8	55.55	24.02	2.064	1.187
9	53.90	22.85	2.028	1.187
10	53.85	21.60	1.719	1.187
11	52.10	20.30	2.122	1.187
12	52.10	19.07	1.570	1.187
13	50.10	17.70	2.240	1.187
14	50.05	16.32	1.835	1.187
15	48.10	14.92	2.225	1.187
16	48.10	13.57	1.687	1.187
17	46.60	12.15	2.148	1.187
18	46.60	10.75	1.777	1.187
19	46.60	9.85	1.251	1.187
20	46.60	9.12	0.840	1.187
21	46.45	8.72	0.535	1.187

TABLE 7

PROCESS COMPUTER ENGINEERING LOG FOR SLAB NO. 7
INGOT SIZE: 36 x 62
SLAB SIZE: 8.5 x 46

Pass Count (i)	Width (RE_i)	Thickness (RM_i)	Measured Per Unit Torque	Adaptive Schedule Multiplier
4	58.55	29.45	2.213	1.063
5	56.90	28.00	2.220	1.063
6	56.90	26.52	2.208	1.063
7	55.20	25.02	2.272	1.200
8	55.20	23.52	1.996	1.200
9	53.45	22.25	1.995	1.200
10	53.45	20.82	1.814	1.200
11	51.45	19.47	2.009	1.200
12	51.45	18.10	1.814	1.200
13	49.45	16.62	2.155	1.200
14	49.45	15.28	1.632	1.200
15	47.45	13.78	2.196	1.200
16	47.45	12.37	1.640	1.200
17	46.65	10.87	2.055	1.200
18	46.60	9.42	1.744	1.200
19	46.35	8.72	0.920	1.200

TABLE 8

PROCESS COMPUTER ENGINEERING LOG FOR SLAB NO. 8
INGOT SIZE: 27 x 49
SLAB SIZE: 11 x 36

Pass Count (i)	Width (RE_i)	Thickness (RM_i)	Measured Per Unit Torque	Adaptive Schedule Multiplier
4	44.95	22.20	1.031	1.053
5	43.00	21.25	0.001	1.053
6	43.00	20.32	0.954	1.053
7	40.90	19.40	0.001	1.053
8	40.90	18.45	1.050	1.053
9	38.90	17.47	0.001	1.053
10	38.90	16.45	1.006	1.053
11	36.90	15.40	0.001	1.053
12	36.90	14.40	0.959	1.053
13	36.55	13.35	1.214	1.212
14	36.55	12.40	0.848	1.212
15	36.10	11.50	1.006	1.212

TABLE 9

PROCESS COMPUTER ENGINEERING LOG FOR SLAB NO. 9
INGOT SIZE: 36 x 48
SLAB SIZE: 7.5 x 40

Pass Count (i)	Width (RE_i)	Thickness (RM_i)	Measured Per Unit Torque	Adaptive Schedule Multiplier
4	43.95	28.40	2.157	1.056
5	42.45	26.57	2.203	1.056
6	42.40	24.72	1.965	1.056
7	40.60	22.85	2.410	1.056
8	40.60	21.00	1.902	1.056
9	40.60	19.12	2.049	1.056
10	40.60	17.30	1.974	1.056
11	40.60	15.50	2.022	1.056
12	40.60	13.78	1.741	1.056
13	40.60	12.10	1.845	1.056
14	40.60	10.37	1.792	1.056
15	40.60	9.30	1.153	1.056
16	40.60	8.47	0.806	1.056
17	40.35	8.02	0.487	1.056

TABLE 10

PROCESS COMPUTER ENGINEERING LOG FOR SLAB NO. 10
INGOT SIZE: 33 x 40
SLAB SIZE: 7.0 x 32

Pass Count (i)	Width (RE_i)	Thickness (RM_i)	Measured Per Unit Torque	Adaptive Schedule Multiplier
4	36.40	24.32	2.038	1.030
5	34.55	22.05	2.399	1.030
6	34.55	19.90	1.935	1.030
7	32.55	17.52	2.442	1.161
8	32.55	15.50	1.766	1.161
9	32.50	13.37	2.026	1.161
10	32.50	11.37	1.740	1.161
11	32.50	9.57	1.624	1.161
12	32.50	8.25	1.164	1.161
13	32.10	7.45	0.749	1.161

TABLE 11

PROCESS COMPUTER ENGINEERING LOG FOR SLAB NO. 11
INGOT SIZE: 34 x 39
SLAB SIZE: 8.0 x 27.5

Pass Count (i)	Width (RE_i)	Thickness (RM_i)	Measured Per Unit Torque	Adaptive Schedule Multiplier
4	35.45	27.10	1.781	1.174
5	33.85	25.75	0.001	1.174
6	33.95	24.17	1.557	1.174
7	32.10	22.67	0.001	1.174
8	32.10	21.17	1.335	1.174
9	30.10	19.60	1.739	1.383
10	30.10	18.35	1.083	1.383
11	28.40	16.77	1.633	1.383
12	28.40	15.37	1.169	1.383
13	28.40	14.00	1.231	1.383
14	28.40	12.65	1.076	1.383
15	28.40	11.25	1.197	1.383
16	28.40	9.90	1.075	1.383
17	27.75	8.55	1.235	1.383

TABLE 12

PROCESS COMPUTER ENGINEERING LOG FOR SLAB NO. 12
INGOT SIZE: 34 x 39
SLAB SIZE: 8.0 x 28

Pass Count (i)	Width (RE_i)	Thickness (RM_i)	Measured Per Unit Torque	Adaptive Schedule Multiplier
4	35.45	27.07	1.598	1.096
5	33.70	25.55	0.001	1.096
6	33.70	23.92	1.513	1.096
7	31.90	22.27	0.001	1.096
8	31.90	20.72	1.374	1.096
9	29.90	19.02	0.001	1.096
10	29.90	17.30	1.510	1.096
11	28.45	15.50	1.815	1.378
12	28.75	14.20	0.988	1.378
13	29.30	12.72	1.243	1.169
14	28.45	11.15	1.286	1.169
15	28.45	9.90	1.118	1.169
16	28.45	9.02	0.660	1.169
17	28.15	8.55	0.433	1.169

TABLE 13

PROCESS COMPUTER ENGINEERING LOG FOR SLAB NO. 13
INGOT SIZE: 34 x 39
SLAB SIZE: 9.0 x 26

Pass Count (i)	Width (RE_i)	Thickness (RM_i)	Measured Per Unit Torque	Adaptive Schedule Multiplier
4	35.45	27.90	0.001	1.000
5	33.65	26.47	0.001	1.000
6	33.65	25.07	1.419	1.169
7	31.65	23.85	0.001	1.169
8	31.90	22.50	1.331	1.169
9	30.10	21.15	0.001	1.169
10	30.10	19.87	1.116	1.169
11	28.10	18.47	0.001	1.169
12	28.10	17.10	1.210	1.169
13	26.40	15.57	1.499	1.463
14	26.50	14.42	0.859	1.463
15	26.96	13.20	1.029	1.254
16	26.45	11.80	1.088	1.254
17	26.45	10.60	1.043	1.254
18	26.45	9.67	0.718	1.254
19	26.15	9.17	0.443	1.254

TABLE 14

PROCESS COMPUTER ENGINEERING LOG FOR SLAB NO. 14
INGOT SIZE: 34 x 39
SLAB SIZE: 9.0 x 25.5

Pass Count (i)	Width (RE_i)	Thickness (RM_i)	Measured Per Unit Torque	Adaptive Schedule Multiplier
4	35.45	27.90	0.001	1.000
5	33.65	26.47	0.001	1.000
6	33.65	25.05	0.001	1.000
7	31.65	23.55	0.001	1.000
8	31.65	22.12	0.001	1.000
9	29.65	20.55	0.001	1.000
10	29.65	19.00	1.398	1.153
11	27.70	17.60	0.001	1.153
12	27.70	16.07	1.377	1.153
13	26.45	14.62	0.001	1.153
14	26.45	13.17	1.187	1.153
15	26.40	11.67	1.377	1.368
16	26.40	10.40	1.046	1.368
17	25.85	9.20	1.169	1.368

APPENDIX B

Predictive Results Tables

DESCRIPTION OF CONTENTS

The tables contained herein, represent the results of the predicted versus actual percent of rated motor current for both the vertical edger and horizontal mill motors. The "Actual" percent column was obtained from the strip chart recordings made during the rolling of each slab listed. In all cases, an attempt was made to select the "independently loaded" value from the pass sequence as described in Figure 3. The values were then divided by the respective rated motor currents to obtain the percent values shown.

The predicted values were obtained from the drafts predicted by the dogbone and spread equations using the proper corresponding pass entry dimensions to obtain absolute torque and currents.

All the numbers shown in the tables (except pass no.) are in terms of percent, including the standard and mean deviations.

The table and slab numbers correspond to those in Appendix A and represent the same physical slabs.

TABLE 1

PERCENT OF RATED MOTOR CURRENT
SLAB NO: 1
INGOT SIZE: 36 x 62
SLAB SIZE: 8.5 x 46

	Horizontal Mill		Vertical Edger	
Pass No.	Predicted	Actual	Predicted	Actual
2	115	75	34	45
3	139	101	132	127
4	115	131	80	64
5	148	127	132	112
6	109	109	75	67
7	153	150	115	142
8	108	109	70	82
9	169	157	120	131
10	105	97	73	90
11	172	165	117	112
12	105	109	67	97
13	177	180	101	97
14	106	105	59	75
15	174	169	84	75
16	101	105	48	71
17	156	157	34	34
18	-	-	-	-
19	-	-	-	-

	Standard Deviation[a]	Mean Deviation[a]
Horizontal Mill	7.9	3.4
Vertical Edger	17.3	-7.2

[a] Excluding first 4 passes.

TABLE 2

PERCENT OF RATED MOTOR CURRENT
SLAB NO: 2
INGOT SIZE: 33 x 84
SLAB SIZE: 7.0 x 76

	Horizontal Mill		Vertical Edger	
Pass No.	Predicted	Actual	Predicted	Actual
2	118	75	34	30
3	151	112	150	133
4	118	121	85	67
5	163	173	150	150
6	113	135	82	115
7	195	189	147	147
8	110	105	91	116
9	157	165	19	34
10	110	105	34	45
11	125	124	0	0
12	108	97	70	82
13	146	150	0	6
14	108	112	24	0
15	127	133	0	0
16	110	97	56	56
17	147	139	0	4
18	105	90	20	11
19	126	120	0	0
20	105	86	40	34
21	145	131	0	0
22	110	97	14	0
23	111	97	0	0
24	60	90	24	19
25	83	52	5	4

	Standard Deviation[a]	Mean Deviation[a]
Horizontal Mill	14.4	3.6
Vertical Edger	12.7	-2.3

[a] Excluding first 4 passes.

TABLE 3

PERCENT OF RATED MOTOR CURRENT
SLAB NO: 3
INGOT SIZE: 33 x 73
SLAB SIZE: 7.5 x 66.5

	Horizontal Mill		Vertical Edger	
Pass No.	Predicted	Actual	Predicted	Actual
2	121	101	37	91
3	154	146	176	195
4	121	150	92	53
5	166	184	176	176
6	137	128	98	127
7	140	157	55	79
8	127	131	34	75
9	122	120	0	45
10	105	105	86	112
11	143	128	0	30
12	104	96	25	37
13	116	113	0	45
14	110	101	64	82
15	132	120	0	15
16	99	94	19	37
17	116	105	0	67
18	131	150	51	75
19	158	139	0	7
20	114	127	32	37
21	138	113	0	7
22	78	75	22	30
23	76	53	5	7

	Standard Deviation[a]	Mean Deviation[a]
Horizontal Mill	14.1	3.8
Vertical Edger	29.0	-22.5

[a] Excluding first 4 passes.

TABLE 4

PERCENT OF RATED MOTOR CURRENT
SLAB NO: 4
INGOT SIZE: 36 x 62
SLAB SIZE: 8.5 x 46

	Horizontal Mill		Vertical Edger	
Pass No.	Predicted	Actual	Predicted	Actual
2	120	75	38	45
3	144	113	136	157
4	120	129	87	67
5	153	135	136	150
6	120	113	104	86
7	151	143	127	142
8	117	113	75	101
9	161	165	125	127
10	105	98	76	105
11	171	165	124	120
12	98	113	70	105
13	177	187	117	109
14	103	105	65	86
15	173	173	96	82
16	97	97	54	67
17	155	165	42	41

	Standard Deviation[a]	Mean Deviation[a]
Horizontal Mill	9.2	1.0
Vertical Edger	19.1	-8.7

[a] Excluding first 4 passes.

TABLE 5

PERCENT OF RATED MOTOR CURRENT
SLAB NO: 5
INGOT SIZE: 36 x 62
SLAB SIZE: 8.5 x 46

	Horizontal Mill		Vertical Edger	
Pass No.	Predicted	Actual	Predicted	Actual
2	116	67	38	30
3	141	90	150	120
4	116	120	89	52
5	150	120	150	150
6	116	120	85	52
7	140	127	132	112
8	107	109	73	112
9	154	150	119	150
10	99	97	74	101
11	161	165	126	120
12	95	94	74	94
13	166	176	113	112
14	98	97	64	86
15	167	172	99	90
16	91	90	55	67
17	152	165	51	52

	Standard Deviation[a]	Mean Deviation[a]
Horizontal Mill	11.0	1.1
Vertical Edger	22.0	-6.6

---

[a] Excluding first 4 passes.

TABLE 6

PERCENT OF RATED MOTOR CURRENT
SLAB NO: 6
INGOT SIZE: 36 x 62
SLAB SIZE: 8.5 x 46

	Horizontal Mill		Vertical Edger	
Pass No.	Predicted	Actual	Predicted	Actual
2	114	67	39	30
3	138	105	155	150
4	114	120	92	60
5	147	135	155	150
6	108	105	86	45
7	149	150	123	135
8	107	105	78	60
9	143	157	126	124
10	93	90	78	90
11	154	157	120	124
12	88	86	70	97
13	160	180	121	116
14	95	105	69	86
15	161	180	101	97
16	90	90	56	67
17	150	165	64	64

	Standard Deviation[a]	Mean Deviation[a]
Horizontal Mill	11.1	-4.6
Vertical Edger	17.2	-0.6

---

[a] Excluding first 4 passes.

TABLE 7

PERCENT OF RATED MOTOR CURRENT
SLAB NO: 7
INGOT SIZE: 36 x 62
SLAB SIZE: 8.5 x 46

	Horizontal Mill		Vertical Edger	
Pass No.	Predicted	Actual	Predicted	Actual
2	111	71	17	52
3	134	109	69	52
4	111	131	41	22
5	143	131	69	67
6	113	112	39	30
7	167	176	64	75
8	111	112	42	49
9	152	187	58	60
10	102	97	36	49
11	158	187	59	60
12	94	101	33	49
13	164	191	51	45
14	89	86	29	45
15	162	176	43	37
16	90	90	24	34
17	139	154	14	14
18	91	90	16	26
19	76	79	3	7

	Standard Deviation[a]	Mean Deviation[a]
Horizontal Mill	16.0	-8.1
Vertical Edger	9.5	-4.6

[a] Excluding first 4 passes.

TABLE 8

PERCENT OF RATED MOTOR CURRENT
SLAB NO: 8
INGOT SIZE: 27 x 49
SLAB SIZE: 11 x 36

	Horizontal Mill		Vertical Edger	
Pass No.	Predicted	Actual	Predicted	Actual
2	56	50	15	38
3	92	76	128	144
4	56	58	56	58
5	104	101	128	139
6	55	56	56	71
7	103	95	126	135
8	53	65	53	75
9	101	121	109	116
10	55	51	48	71
11	101.	113	97	90
12	51	47	43	64
13	75	88	15	19
14	48	43	23	39
15	63	73	16	14

	Standard Deviation[a]	Mean Deviation[a]
Horizontal Mill	10.3	-4.1
Vertical Edger	14.8	-10.6

[a] Excluding first 4 passes.

TABLE 9

PERCENT OF RATED MOTOR CURRENT
SLAB NO: 9
INGOT SIZE: 36 x 48
SLAB SIZE: 7.5 x 40

	Horizontal Mill		Vertical Edger	
Pass No.	Predicted	Actual	Predicted	Actual
2	107	94	41	79
3	124	135	136	142
4	107	124	90	79
5	131	161	136	135
6	108	105	89	97
7	151	187	142	142
8	104	105	88	105
9	127	139	0	7
10	102	112	53	67
11	117	127	0	0
12	96	90	41	56
13	111	120	0	0
14	97	97	31	45
15	79	75	0	0
16	46	45	16	30
17	42	37	7	11

	Standard Deviation[a]	Mean Deviation[a]
Horizontal Mill	15.0	-6.9
Vertical Edger	10.2	-7.2

[a] Excluding first 4 passes.

TABLE 10

PERCENT OF RATED MOTOR CURRENT
SLAB NO: 10
INGOT SIZE: 33 x 40
SLAB SIZE: 7.0 x 32

	Horizontal Mill		Vertical Edger	
Pass No.	Predicted	Actual	Predicted	Actual
2	105	86	39	19
3	121	127	137	166
4	105	105	84	71
5	144	127	137	142
6	99	101	85	75
7	148	142	121	120
8	88	90	70	56
9	114	116	2	7
10	87	90	36	34
11	94	97	0	7
12	57	52	21	19
13	54	49	10	11

	Standard Deviation[a]	Mean Deviation[a]
Horizontal Mill	7.1	2.1
Vertical Edger	7.2	1.1

[a] Excluding first 4 passes.

TABLE 11

PERCENT OF RATED MOTOR CURRENT
SLAB NO: 11
INGOT SIZE: 34 x 39
SLAB SIZE: 8.0 x 27.5

	Horizontal Mill		Vertical Edger	
Pass No.	Predicted	Actual	Predicted	Actual
2	69	90	28	37
3	84	112	159	187
4	69	90	72	90
5	89	94	159	172
6	80	82	65	60
7	109	105	164	195
8	72	71	82	75
9	112	101	156	142
10	57	56	72	71
11	104	105	115	94

	Standard Deviation[a]	Mean Deviation[a]
Horizontal Mill	5.3	1.2
Vertical Edger	17.3	0.5

[a] Excluding first 4 passes.

TABLE 12

PERCENT OF RATED MOTOR CURRENT
SLAB NO: 12
INGOT SIZE: 34 x 39
SLAB SIZE: 8.0 x 28

	Horizontal Mill		Vertical Edger	
Pass No.	Predicted	Actual	Predicted	Actual
2	74	86	28	45
3	90	97	150	187
4	74	90	69	97
5	95	109	150	142
6	79	79	68	79
7	112	120	136	142
8	71	67	72	75
9	113	112	131	127
10	74	79	66	60
11	108	112	80	82

	Standard Deviation[a]	Mean Deviation[a]
Horizontal Mill	7.2	-3.6
Vertical Edger	6.9	-0.8

[a] Excluding first 4 passes.

TABLE 13

PERCENT OF RATED MOTOR CURRENT
SLAB NO: 13
INGOT SIZE: 34 x 39
SLAB SIZE: 9.0 x 26

	Horizontal Mill		Vertical Edger	
Pass No.	Predicted	Actual	Predicted	Actual
2	77	75	30	37
3	94	90	179	191
4	77	82	77	94
5	100	94	179	184
6	75	74	97	
7	103	105	179	195
8	68	67	59	52
9	101	86	144	154
10	61	60	68	67
11	103	86	142	135
12	61	64	64	64
13	101	97	104	90

	Standard Deviation[a]	Mean Deviation[a]
Horizontal Mill	8.3	4.2
Vertical Edger	12.2	-2.8

[a] Excluding first 4 passes.

TABLE 14

PERCENT OF RATED MOTOR CURRENT
SLAB NO: 14
INGOT SIZE: 34 x 39
SLAB SIZE: 9.0 x 25.5

	Horizontal Mill		Vertical Edger	
Pass No.	Predicted	Actual	Predicted	Actual
2	86	75	31	30
3	105	75	186	191
4	86	97	79	67
5	111	116	186	195
6	85	86	77	105
7	130	112	185	187
8	80	82	87	101
9	130	127	163	161
10	82	79	79	75
11	115	120	137	142
12	75	75	65	60
13	103	105	74	67

	Standard Deviation[a]	Mean Deviation[a]
Horizontal Mill	6.9	1.1
Vertical Edger	12.3	-4.7

[a] Excluding first 4 passes.

APPENDIX C

Computer Output for Spread Regression

DESCRIPTION OF CONTENTS

This Appendix contains the results of five separate regression analyses. In each case, the dependent variable described is that of spread magnitude (referred to as EDGER DRAFT in the actual printouts). The independent variables in each case are DOGBONE MAGNITUDE and various functions of horizontal mill draft, slab width, and thickness.

Computer run No. 2 shows the strongest statistical evidence of correlation between the variables. The other computer runs are shown for academic comparison.

In all cases, the input data is described by the first two pages of the regression results, the first of which lists each variable with its corresponding minimum, maximum, and average values. The second page contains the simple correlation matrix for all the variables.

GENERAL DESCRIPTION OF COMPUTER OUTPUTS

The following description explains the actual computer outputs obtained from the regression program and shown here in Appendix C and Appendix D.

ANALYSIS OF VARIANCE

The first column, labeled SOURCE OF VARIATION, lists the particular response surface and independent variables to which the statistical values in the remaining columns refer. The response surface is simply the net or summed result of all the independent variables.

The column labeled SUM OF SQUARES contains the quantity shown below for the given independent variables and response surface:

$$\Sigma (X - \bar{X})^2$$

The column labeled PCT EXPL contains the percents of explained variance for a given independent variable and the response surface. These are calculated by dividing the sum of squares for a given variable by the sum of squares of the total degrees of freedom.

The values given in the column labeled VARIANCE are simply the squares of the corresponding standard deviations of a given variable.

Finally, the F RATIO is calculated by dividing the variance from the given source of variation by the variance of the residuals.

COMPUTER PLOTS

Each plot shows the dependent variable as a function of one of the independent variables when the other independent variables are at their mean values. Each column of numbers and dashes represents the spread of data points for the given X-value. The summation of these points is shown at the top of the plot. The average value of the Y variable for a given X value is denoted by the letter A. The letter C denotes the point on the correcting curve. An asterisk indicates when the A and C values coincide.

EXPLANATION OF COMPUTER SYMBOLS

SYMBOL	MEANING
SIGMA	Standard deviation of input variables
V(1), V(2), etc.	Variable names of input variables
DD	Total horizontal draft including dogbone for two passes
D	Total horizontal draft excluding dogbone for two passes
D.O.F.	Degrees of Freedom

SOURCE OF REGRESSION EQUATION

The actual coefficients for a given regression equation are found opposite the variable name under the caption "LINEAR SOLUTION FOR (dependent variable)". The column heading for the coefficients is titled "COEFFICIENT" and the independent variable column is titled "FACTOR".

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                       JOB  1  RUN  1
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                 PAGE   1

INPUT CONTAINS 16 VARIABLES WITH   79 OBSERVATIONS.

NOTE  V( 17) IS A CONSTANT.

INPUT     IDENTITY                       MIN          MAX          AVE           SIGMA

V(  1) = SLAB WIDTH                0.2770E 02   0.7870E 02   0.5075D 02   0.1494D 02

V(  2) = SLAB THICKNESS            0.7500E 01   0.2685E 02   0.1825D 02   0.5231D 01

V(  3) = EDGER DRAFT               0.3800E 00   0.2382E 01   0.1243D 01   0.4495D 00

V(  4) = DRAFT MILL 1              0.5700E 00   0.2150E 01   0.1334D 01   0.2889D 00

V(  5) = DRAFT MILL 2              0.8800E 00   0.2380E 01   0.1361D 01   0.3031D 00

V(  6) = DOGBONE MAGNITUDE         -.2660E 00   0.1491E 01   0.6601D 00   0.3936D 00

                         V( 17)
V(  7) =   1/(V(  2)          )    0.1930E 00   0.3651E 00   0.2430D 00   0.4174D-01
         1/SQRT THICKNESS

V(  8) = 1/LN V(  2)               0.3039E 00   0.4963E 00   0.3552D 00   0.4520D-01
         1/LN THICKNESS

V(  9) =         V(  4)+V(  5)     0.1450E 01   0.4420E 01   0.2695D 01   0.5792D 00
         TOT DRAFT MILL 1 & 2

V( 10) =         V(  6)+V(  9)     0.1184E 01   0.5218E 01   0.3355D 01   0.7868D 00
         DRAFT + DOGBONE

V( 11) =         V( 10).V(  7)     0.4323E 00   0.1359E 01   0.8094D 00   0.2231D 00
         DD * 1/SQRT THK

V( 12) =         V( 10).V(  8)     0.5876E 00   0.1904E 01   0.1185D 01   0.3012D 00
         DD * 1/LN THK

V( 13) =         V( 10).V(  1)     0.8985E 02   0.2261E 03   0.1626D 03   0.3701D 02
         DD * WIDTH

V( 14) =         V(  9).V(  7)     0.4171E 00   0.1225E 01   0.6503D 00   0.1671D 00
         D * 1/SQRT THK

V( 15) =         V(  9).V(  8)     0.6243E 00   0.1699E 01   0.9520D 00   0.2232D 00
         D * 1/LN THK

V( 16) =         V(  9).V(  1)     0.7310E 02   0.1692E 03   0.1316D 03   0.2886D 02
         D * WIDTH

V( 17) =     0.5000000D 00
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                 PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                       JOB - 1  RUN  1
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                 PAGE   2

SIMPLE CORRELATION COEFFICIENTS

        V(  1) V(  2) V(  3) V(  4) V(  5) V(  6) V(  7) V(  8) V(  9) V( 10)

V(  1)  1.0000

V(  2) -0.0322 1.0000

V(  3) -0.1589-0.1193 1.0000

V(  4) -0.5780 0.2472 0.2209 1.0000

V(  5) -0.6030 0.0674 0.1830 0.9140 1.0000

V(  6) -0.4098-0.0431 0.6293 0.2994 0.2541 1.0000

V(  7)  0.0848-0.9659 0.0122-0.2923-0.0914-0.0814 1.0000

V(  8)  0.0928-0.9530-0.0085-0.3015-0.0969-0.1025 0.9989 1.0000

V(  9) -0.6040 0.1586 0.2060 0.9772 0.9793 0.2823-0.1936-0.2011 1.0000

V( 10) -0.6496 0.0952 0.4664 0.8691 0.8480 0.7081-0.1832-0.1993 0.8773 1.0000

V( 11) -0.5512-0.4591 0.4713 0.6399 0.7281 0.6143 0.3843 0.3654 0.7003 0.8227
        1.0000

V( 12) -0.5948-0.3363 0.4801 0.7189 0.7850 0.6535 0.2601 0.2421 0.7695 0.8933
        0.9902 1.0000

V( 13)  0.5181 0.1524 0.4154 0.1920 0.1046 0.3345-0.1856-0.1939 0.1505 0.2781
        0.1706 0.2009 1.0000

V( 14) -0.4892-0.4806 0.2349 0.6935 0.8147 0.2310 0.4552 0.4443 0.7723 0.6841
        0.9044 0.8834 0.0419 1.0000

V( 15) -0.5369-0.3424 0.2288 0.7911 0.8907 0.2448 0.3192 0.3097 0.8608 0.7561
        0.8898 0.8900 0.0665 0.9878 1.0000

V( 16)  0.6884 0.1882 0.0990 0.1551 0.0924-0.1691-0.1568-0.1546 0.1257 0.0080
       -0.0700-0.0525 0.8599 0.0177 0.0458 1.0000
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES              PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                    JOB  1  RUN  1
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                              02/24/75
KLUCHAR                                                               PAGE   3


LINEAR SOLUTION FOR EDGER DRAFT                     VS
FACTOR                        COEFFICIENT  +/-RANGE 95 PCT

CONSTANT                     0.189859D 00
DOGBONE MAGNITUDE            0.736892D 00     0.205072D 00
D * 1/SQRT THK               0.220950D 00     0.476080D 00
D * WIDTH                    0.321907D-02     0.272191D-02

INTERACTION CORRELATION COEFFICIENTS

         B(  6) B( 14) B( 16) B(

B(  6)  1.0000

B( 14) -0.2375 1.0000

B( 16)  0.1780-0.0592 1.0000

ANALYSIS OF VARIANCE IN EDGER DRAFT
SOURCE OF VARIATION D.O.F. PCT EXPL  SUM OF SQUARES          VARIANCE   F RATIO

FIRST ORDER                  3      44.58     0.711525D 01     0.237175D 01 0.201D 02
RESPONSE SURFACE
DOGBONE MAGNITUDE            1      40.61     0.648078D 01     0.648078D 01 0.550D 02
D * 1/SQRT THK               1       1.93     0.307962D 00     0.307962D 00 0.261D 01
D * WIDTH                    1       2.05     0.326506D 00     0.326506D 00 0.277D 01

CALCULATED  RESIDUAL  75                      0.884523D 01     0.117936D 00
OBSERVED    RESIDUAL  75                      0.884523D 01     0.117936D 00

TOTAL                 78                      0.159605D 02     0.204622D 00

EXPECTED MEAN SQUARE                                           0.248969D 01

STANDARD ERROR OF ESTIMATE 0.3434D 00
```

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YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  1  RUN  1
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                 PAGE   4

        0    0    0    0    2    4   16   17   14   16    6    4    0    0    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  30+                                                                          +


    +                                                                          +


  25+                                                                          +
                                                                               C
                                                     1-        1-
    +                                                                     C    +
                                                          1-
                                                                     C
  20+                                                                          +
                                                1-   1-   1-
                                           1-        2-   2A    C
    +                                                2-   1-    A              +
                                      1-             1-    C   1-
                                                     2-   1-   1-
  15+                                      1-   1-    *        1-              +
                                           1-        1-
                                      1-   2-   4C   1-
    +                                 1-   2-   2A   1-                        +
                            1-        1-   1C   2-   1-
                                      3-   2A   1-   2-
  10+                            1-   1-        1-                             +
                                 2-   2*   3-   1-
                             A    A   1-        1-   1-
    +                             C        1-                                  +
                                      2-
                             C   1-        2-
   5+                       1-                                                 +
                        C             2-
                                           1-
    +                                 1-                                       +
                   C

   0+         C                                                                +

         C
    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
            -10        -5         0         5        10        15        20
EDGER DRAFT              VS       DOGBONE MAGNITUDE
V(  3)                             V(  6)
  -1                                 -1
X10                               X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  1 RUN   1
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                 PAGE   5

        0    2    6    7   17   13    7    8    5    4    1    2    2    1    4
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  30+                                                                          +


    +                                                                          +


  25+                                                                          +


    +                                                                          +
                                                               1-

  20+                            1-                                            +
                            1-             1-
                                      1-
    +                                      1-        1-   1A    A              +
                                                                    1-
             1-   1-                             1-
  15+                            1-   2-   1-        1-               A        1-
              A   1-        1-   2-   1-   2-   1-             1-
                  2A   2-   4-   1-    A              A         C    C    C    C
    +                  1C   1C   1C   1C    *    C    C    C        1-   1A    +
         C   1C   1C   1-   3A   1A             1A   1-
                       1A   1-   1-   1-                                      1A
  10+             1-   1-   2-   2-        1-                                 1-
                            1-   1-   1-        1-   1-
                                           1-   1-                            1-
    +                       1-   2-                                            +
                            1-
                            1-
   5+                  1-                  1-                                  +


    +                                                                          +


   0+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
              4         5         6         7         8         9        10
EDGER DRAFT               VS      D * 1/SQRT THK
V(  3)                             V( 14)
 -1                                 -1
X10                               X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                     JOB  1  RUN  1
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                              02/24/75
KLUCHAR                                                               PAGE   6

        1   10    2    7    2    1    6   20   11   11    8    0    0    0    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  30+                                                                          +


    +                                                                          +


  25+                                                                          +


    +                                                                          +
                                                1-

  20+                                      1-   1-                             +
                                                2-

    +                                      2-             1-                   +
                                           2-
                                           1-    A
  15+        1-                            4-   1-        1-              C    C
                  1-            1A   1-    4A   2-   1-   1-    C    C
             1-        1-            2-    1-   1C   1C    *
    +        1-        1-             *    2C   2-        1-                   +
             1-         C    C    C        1-        1-   3-
         C   2*    *        2A       2-                   1-
  10+   1A   1-        1-            1-    2-        2-                        +
                        A                       1-    A
             1-   1-   1-                            3-
    +                  2-                            1-                        +
             1-        1-                            1-
                                                     1-
   5+                                                                          +
             1-

    +                                                                          +


   0+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
              8        10        12        14        16        18        20
EDGER DRAFT               VS      D * WIDTH
V( 3)                              V( 16)
 -1                                  1
X10                               X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES              PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                    JOB  1  RUN  2
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                              02/24/75
KLUCHAR                                                               PAGE   7


LINEAR SOLUTION FOR EDGER DRAFT          VS
FACTOR                     COEFFICIENT  +/-RANGE 95 PCT

CONSTANT                   0.186911D 00
DOGBONE MAGNITUDE          0.523513D 00   0.257758D 00
DD * 1/SQRT THK            0.300975D 00   0.434992D 00
DD * WIDTH                 0.287352D-02   0.219545D-02

INTERACTION CORRELATION COEFFICIENTS

       B(  6) B( 11) B( 13) B(

B(  6)  1.0000

B( 11) -0.6001 1.0000

B( 13) -0.2954 0.0469 1.0000

ANALYSIS OF VARIANCE IN EDGER DRAFT
SOURCE OF VARIATION D.O.F. PCT EXPL  SUM OF SQUARES        VARIANCE   F RATIO

FIRST ORDER            3     45.71     0.729615D 01     0.243205D 01 0.211D 02
RESPONSE SURFACE
DOGBONE MAGNITUDE      1     28.85     0.460417D 01     0.460417D 01 0.399D 02
DD * 1/SQRT THK        1      7.04     0.112355D 01     0.112355D 01 0.973D 01
DD * WIDTH             1      9.83     0.156843D 01     0.156843D 01 0.136D 02

CALCULATED  RESIDUAL  75               0.866433D 01     0.115524D 00
OBSERVED    RESIDUAL  75               0.866433D 01     0.115524D 00

TOTAL                 78               0.159605D 02     0.204622D 00

EXPECTED MEAN SQUARE                                    0.254757D 01

STANDARD ERROR OF ESTIMATE 0.3399D 00
```

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YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  1  RUN  2
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                               02/24/75
KLUCHAR                                                                PAGE   8

        0    0    0    0    2    4   16   17   14   16    6    4    0    0    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  30+                                                                          +


    +                                                                          +


  25+                                                                          +


    +                                                1-                        +
                                                               1-
                                                                               C
  20+                                                     1-                   +
                                           1-   1-                        C
                                                     1-              C
    +                                 1-             2-   2-                   +
                                                     2-   2A    C
                                           1-        1-    C    A
  15+                                                2A        1-              +
                                      1-   1-   1-    C   1-   1-
                            1-        1-   2-   3-   1-        1-
    +                                 1-   2-   3*   1-                        +
                                 1-   3-   2C   2-   1-
                                 1-   1-   1A   1-   2-
  10+                        A   1A   2*   1-   2-   1-                        +
                                  C   1-   2-
                                      1-   1-        1-
    +                       1C                  1-                             +
                        C        1-   1-   1-
                                           1-
   5+              C                  2-                                       +

              C                       1-   1-
    +    C                                                                     +


   0+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
            -10        -5         0         5        10        15        20
EDGER DRAFT              VS       DOGBONE MAGNITUDE
V(  3)                             V(  6)
  -1                                 -1
X10                               X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  1  RUN  2
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                 PAGE   9

        0    0    0    1    6   16   15   13    8    5    9    2    3    1    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  30+                                                                          +


    +                                                                          +


  25+                                                                          +


    +                                                                          +
                                                          1-

  20+                                                                          +
                                 1-   1-         1-
                                                               1-
    +                            1-                       2-        1-         +
                                                                A
                            2-                       1-
  15+                                 1-   2-   2-        2A   1-              +
                            1-   1-   1-   1-        1-                  C    C    C
                             A   4-   2-   3-   1-   1*   1C    C
    +                       1-   1-   1C   1C    *        1-         A         +
                            1C   1*   2-   2A        1-   1-              1A
              C    C   1*   1-   1-   1A        1-                  1-
  10+    C                       2-   1-   2-   1-        1-                   +
                                 1-        1-   2-   1-
                                 1-   1-                            1-
    +                                                                          +
                                 1-   3-
                                      1-
   5+                            1-                                            +
                                           1-

    +                                                                          +


   0+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
              2         4         6         8        10        12        14
EDGER DRAFT               VS     DD * 1/SQRT THK
V(  3)                            V( 11)
 -1                                  -1
X10                               X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES               PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  1  RUN  2
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                 PAGE  10
        0    0    0    0    0   13   10    8   24   19    5    0    0    0    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  30+                                                                          +


    +                                                                          +


  25+                                                                          +


    +                                                                          +

                                                     1-
  20+                                           2-   1-                        +
                                                          1-
                                           1-        1-
    +                                                1-                        C
                                                1-   1-                   C
                                                1-   1-   1-         C
  15+                            1-             2-   3-         C              +
                                           1-   3-   1-   1*
                                 2-   1-   4A   2-   1*
    +                            2-   1-        4*   1-                        +
                                 2-    C   1C   2-   1-   2-
                                 1*   1-        3-
  10+                       C    3-   1-        1-   3-                        +
                        C             2A   1-
                   C                  2-             1-
    +         C                       1-        2-   1-                        +
         C                       1-
                                      1-        1-
   5+                            1-                  1-                        +


    +                                                                          +


   0+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
              0         5        10        15        20        25        30
EDGER DRAFT               VS      DD * WIDTH
V(  3)                             V( 13)
 -1                                  1
X10                               X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                   PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                         JOB  1  RUN  3
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                   02/24/75
KLUCHAR                                                                    PAGE  11

LINEAR SOLUTION FOR EDGER DRAFT           VS
FACTOR                      COEFFICIENT  +/-RANGE 95 PCT

CONSTANT                    0.215943D 00
DOGBONE MAGNITUDE           0.741560D 00    0.206651D 00
D * 1/LN THK                0.121645D 00    0.359535D 00
D * WIDTH                   0.320935D-02    0.273582D-02

INTERACTION CORRELATION COEFFICIENTS

       B(  6) B( 15) B( 16) B(

B(  6)  1.0000

B( 15) -0.2565 1.0000

B( 16)  0.1862-0.0912 1.0000

ANALYSIS OF VARIANCE IN EDGER DRAFT
SOURCE OF VARIATION D.O.F. PCT EXPL  SUM OF SQUARES        VARIANCE   F RATIO

FIRST ORDER              3     44.28   0.706793D 01     0.235598D 01 0.199D 02
RESPONSE SURFACE
DOGBONE MAGNITUDE        1     40.86   0.652184D 01     0.652184D 01 0.550D 02
D * 1/LN THK             1      1.38   0.220574D 00     0.220574D 00 0.186D 01
D * WIDTH                1      2.04   0.325519D 00     0.325519D 00 0.275D 01

CALCULATED  RESIDUAL    75             0.889255D 01     0.118567D 00
OBSERVED    RESIDUAL    75             0.889255D 01     0.118567D 00

TOTAL                   78             0.159605D 02     0.204622D 00

EXPECTED MEAN SQUARE                                    0.247454D 01

STANDARD ERROR OF ESTIMATE 0.3443D 00
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                     JOB  1  RUN  3
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                               02/24/75
KLUCHAR                                                                PAGE  12

        0    0    0    0    2    4   16   17   14   16    6    4    0    0    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  30+                                                                          +


    +                                                                          +


  25+                                                                          +
                                                     1-                        C
                                                               1-
    +                                                                     C    +
                                                          1-
                                                                     C
  20+                                                                          +
                                                1-   2-   1-
                                           1-        1-   2A    C
    +                                                2-   1-    A              +
                                      1-             1-    C
                                                     2-   1-   2-
  15+                                      1-         *        1-              +
                                           1-   1-   1-
                                      1-   2-   3C   1-
    +                                 1-   2-   3A   1-                        +
                            1-        1-   1-   2-   1-
                                      3-   2*   1-   2-
  10+                            2-   1-        1-                             +
                                 1-   2*   3-   1-
                             A    A   1-        1-   1-
    +                             C        1-                                  +
                                      2-
                             C             2-
   5+                       1-   1-   1-                                       +
                        C             1-
                                           1-
    +                                 1-                                       +
                   C

   0+         C                                                                +

         C
    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
            -10        -5         0         5        10        15        20
EDGER DRAFT               VS      DOGBONE MAGNITUDE
V(  3)                             V(  6)
 -1                                 -1
X10                               X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  1  RUN  3
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                 PAGE  13

        0    0    0    2   12   10   24   10    9    4    3    0    3    1    1
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  30+                                                                          +


    +                                                                          +


  25+                                                                          +


    +                                                                          +
                                                     1-

  20+                                 1-                                       +
                                 1-        1-
                                      1-
    +                                      1-        1-                        +
                                                1-        1-
                       1-   1-                  1-
  15+                                 2-   1-   1-    A             1-         +
                        A   2-   1-   3-   3-   1-        1A
                            3-   1-   4-   1A                   C    C    C    C
    +                        C   1C   2C    C    *    C   1C        1A         +
         C    C    C   1C   3A   2A   2A        1-   1-                       1A
                                 1-   1-   1-                            1A
  10+                       2-        2-   1-   1-                             +
                                 1-   2-   1-   1-   1-
                                                2-                  1-
    +                                 2-                                       +
                                 1-   1-
                                 1-
   5+                       1-        1-                                       +


    +                                                                          +


   0+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
              4         6         8        10        12        14        16
EDGER DRAFT               VS      D * 1/LN THK
V(  3)                             V( 15)
 -1                                 -1
X10                               X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES               PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  1  RUN   3
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                 PAGE  14

        1   10    2    7    2    1    6   20   11   11    8    0    0    0    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  30+                                                                          +


    +                                                                          +


  25+                                                                          +


    +                                                                          +
                                                1-

  20+                                      1-   1-                             +
                                                2-

    +                                      3-             1-                   +
                                           1-
                                           1-    A
  15+        1-                            4-   1-        1-              C    C
             1-                  1A   1-   4A   2-   1-   1-     C    C
                  1-   1-             2-   1-   1C    C    *
    +        1-        1-              *   2C   2-   1-   1-                   +
             1-         C    C    C   1-   1-        1-   2-
         C   2*    *        2A        1-                  2-
  10+   1A   1-        1-             1-   2-        2-                        +
                        A                       1-    A
             1-   1-                                 3-
    +                  3-                            1-                        +
             1-        1-                            1-
                                                     1-
   5+                                                                          +
             1-

    +                                                                          +


   0+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
              8        10        12        14        16        18        20
EDGER DRAFT               VS      D * WIDTH
V(  3)                             V( 16)
 -1                                  1
X10                               X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                  PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                        JOB  1  RUN  4
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                  02/24/75
KLUCHAR                                                                   PAGE  15


LINEAR SOLUTION FOR EDGER DRAFT            VS
FACTOR                       COEFFICIENT  +/-RANGE 95 PCT

CONSTANT                    0.204811D 00
DOGBONE MAGNITUDE           0.534576D 00    0.268077D 00
DD * 1/LN THK               0.189931D 00    0.337016D 00
DD * WIDTH                  0.283304D-02    0.220296D-02

INTERACTION CORRELATION COEFFICIENTS

        B(  6) B( 12) B( 13) B(

B(  6)  1.0000

B( 12) -0.6351 1.0000

B( 13) -0.2740 0.0248 1.0000

ANALYSIS OF VARIANCE IN EDGER DRAFT
SOURCE OF VARIATION D.O.F. PCT EXPL  SUM OF SQUARES        VARIANCE   F RATIO

FIRST ORDER                 3    45.26    0.722293D 01    0.240764D 01 0.207D 02
RESPONSE SURFACE
DOGBONE MAGNITUDE           1    29.46    0.470147D 01    0.470147D 01 0.404D 02
DD * 1/LN THK               1     6.11    0.975125D 00    0.975125D 00 0.837D 01
DD * WIDTH                  1     9.69    0.154634D 01    0.154634D 01 0.133D 02

CALCULATED  RESIDUAL   75                 0.873755D 01    0.116501D 00
OBSERVED    RESIDUAL   75                 0.873755D 01    0.116501D 00

TOTAL                  78                 0.159605D 02    0.204622D 00

EXPECTED MEAN SQUARE                                      0.252414D 01

STANDARD ERROR OF ESTIMATE 0.3413D 00
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  1  RUN  4
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                 PAGE  16

        0    0    0    0    2    4   16   17   14   16    6    4    0    0    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  30+                                                                          +


    +                                                                          +


  25+                                                                          +


    +                                                1-                        +
                                                               1-
                                                                               C
  20+                                                     1-              C    +
                                           1-   1-
                                                     1-              C
    +                                 1-             2-   2A                   +
                                                     2-   2-    C
                                           1-        1-    C    A
  15+                                                2A        1-              +
                                      1-   1-   1-    C   1-   1-
                            1-        1-   3-   3-   1-        1-
    +                                 1-   1-   3*   1-                        +
                                      3-   2C   2-   1-
                                 3-   1-   1A   1-   3-
  10+                        A    A   2*   1-   1-                             +
                                  C   1-   2-   1-
                                           1-   1-   1-
    +                       1C        1-                                       +
                                 1-   1-   1-
                        C                  1-
   5+              C                  2-                                       +

              C                       1-   1-
    +    C                                                                     +


   0+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
            -10        -5         0         5        10        15        20
EDGER DRAFT               VS      DOGBONE MAGNITUDE
V(  3)                             V(  6)
 -1                                 -1
X10                               X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  1  RUN  4
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                               02/24/75
KLUCHAR                                                                PAGE  17

        0    1    2    7    9   13    9   10    4    7    5    6    3    1    2
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  30+                                                                          +


    +                                                                          +


  25+                                                                          +


    +                                                                          +
                                                               1-

  20+                                 1-                                       +
                            1-                  1-
                                                                    1-
    +                  1-                                 1-   1-              +
                                                                    1-
                  1-   1-                        A   1-
  15+              A             1-   1-   1-   2-        2-   1A    A         +
                  1-        1-   1-   1-             1-                        C
                       2-   3-   2-        3-   1-   1-    *    C    C    C
    +                   A   1A   2C    C   1C    C    C        2-              +
              C    C   2C    C   1-   1-   2A         A   1-   1-             1-
         C   1A                  1A   1A             1-                       1A
  10+                       2-   2-        2-        1-   1-        1-         +
                                      1-   1-        2-
                            1-                                           1A
    +                            1-                                            +
                                 2-   1-
                                      1-
   5+                  1-                                                      +
                                      1-

    +                                                                          +


   0+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
              6         8         10        12        14        16        18
EDGER DRAFT               VS      DD * 1/LN THK
V(  3)                             V( 12)
  -1                                -1
X10                               X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  1  RUN  4
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                 PAGE  18

        0    0    0    0    0   13   10    8   24   19    5    0    0    0    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  30+                                                                          +


    +                                                                          +


  25+                                                                          +


    +                                                1-                        +


  20+                                           2-   1-                        +
                                                          1-
                                           1-        1-
    +                                           1-   1-                        +
                                                1-   1-                   C    C
                                                     2-             C
  15+                            1-             2-   2-   1-    C              +
                                           1-   3-   1-   1*
                                 2-   1-   4A   1-   1*
    +                            2-   1-        4*   1-                        +
                                 1-    C   1C   3-   1-   2-
                                 2*             2-
  10+                       C    3-   2-   1-   2-   3-                        +
                        C             2A
                   C                  1-             1-
    +    C    C                       2-        2-   1-                        +
                                 1-   1-
                                                1-   1-
   5+                                                                          +
                                 1-

    +                                                                          +


   0+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
              0         5        10        15        20        25        30
EDGER DRAFT               VS      DD * WIDTH
V(  3)                             V( 13)
  -1                                 1
X10                               X10
```

```
YOUNGSTOWN SHEET AND TUBE CO, TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                     JOB  1  RUN  5
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                               02/24/75
KLUCHAR                                                                PAGE  19


LINEAR SOLUTION FOR EDGER DRAFT          VS
FACTOR                  COEFFICIENT  +/-RANGE 95 PCT

CONSTANT                0.147873D 00
DOGBONE MAGNITUDE       0.767786D 00    0.220534D 00
TOT DRAFT MILL 1 & 2    0.105766D 00    0.171517D 00
SLAB WIDTH              0.598455D-02    0.699402D-02

INTERACTION CORRELATION COEFFICIENTS

        B(  6) B(  9) B(  1) B(

B(  6)  1.0000

B(  9) -0.0479 1.0000

B(  1)  0.3129 0.5580 1.0000

ANALYSIS OF VARIANCE IN EDGER DRAFT
SOURCE OF VARIATION D.O.F. PCT EXPL  SUM OF SQUARES         VARIANCE   F RATIO

FIRST ORDER             3      41.95    0.669608D 01     0.223203D 01 0.181D 02
RESPONSE SURFACE
DOGBONE MAGNITUDE       1      42.31    0.675249D 01     0.675249D 01 0.547D 02
TOT DRAFT MILL 1 & 2    1       2.81    0.448085D 00     0.448085D 00 0.363D 01
SLAB WIDTH              1      -3.16   -0.504494D 00    -0.504494D 00-0.408D 01

CALCULATED  RESIDUAL   75               0.926440D 01     0.123525D 00
OBSERVED    RESIDUAL   75               0.926440D 01     0.123525D 00

TOTAL                  78               0.159605D 02     0.204622D 00

EXPECTED MEAN SQUARE                                     0.235555D 01

STANDARD ERROR OF ESTIMATE 0.3515D 00
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                       JOB  1  RUN  5
SPREAD AND DOGBONE STUDY, NO. 2, SLABBER                               02/24/75
KLUCHAR                                                                PAGE  20

         0    0    0    0    2    4   16   17   14   16    6    4    0    0    0
     +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


     +                                                                          +


   30+                                                                          +


     +                                                                          +


   25+                                                                          C
                                                      1-
                                                                1-
     +                                                     1-              C    +

                                                                      C
   20+                                           1-   1-                        +
                                            1-        1-   2-    C
                                                      1-   1A
     +                                                3-         A              +
                                       1-                  1C
                                                      2-   1-   3-
   15+                                                *                         +
                                            2-   1-   1-
                                       1-   1-   4C   1-
     +                                      2-   2A   1-                        +
                             1-        1-   3-   2-   1-
                                       3-    *   2-   2-
   10+                                 2-   3-                                  +
                                  3-    *   1-   2-
                                   A   3-             1-
     +                        A    C   1-   1-                                  +

                              C   1-   1-   2-
    5+                                                                          +
                             1-        2-
                         C                  1-
     +                                 1-                                       +
                    C

    0+         C                                                                +


     +    C                                                                     +


     +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
             -10        -5         0         5        10        15        20
EDGER DRAFT                VS      DOGBONE MAGNITUDE
V(  3)                             V(  6)
 -1                                 -1
X10                                X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  1  RUN  5
SPREAD AND DOGBONE STUDY, NO. 2, SLABBER                               02/24/75
KLUCHAR                                                                PAGE  21

        0    0    0    1    2   16    5    6   22   16    2    3    3    0    3
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  30+                                                                          +


    +                                                                          +


  25+                                                                          +


    +                                                 1-                       +


  20+                                       1-   1-                            +
                                                      1-
                                       1-
    +                                            2-   2-                       +

                                       1-             1-        1-
  15+                        1-                  4-   2-             1-        +
                             A   1-   1A         4-                         C 1C
                            1-   2-   1-   1-   2A    A    C   1*    C         A
    +                            2-        1*   3C   1C                       1-
                             C   1C    C   1-   2-        1-   1-    A        1-
              C    C    C        4-        1-   1-   3-
  10+    C                        A   1-        1-   1-    A        2-         +
                       1A                            2-
                                 2-                  2-   1-
    +                                            2-                            +
                                 1-
                                 1-
   5+                            1-        1-                                  +
                                 1-

    +                                                                          +


   0+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
             10        15        20        25        30        35        40
EDGER DRAFT                VS     TOT DRAFT MILL 1 & 2
V(  3)                            V(  9)
  -1                                -1
X10                               X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  1  RUN  5
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                 PAGE  22
        0    0    0   12    8    8    3   14   15    0    9    0    9    1    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  30+                                                                          +


    +                                                                          +


  25+                                                                          +


    +                                                                          +
                                           1-

  20+                                      1-   1-                             +
                                           1-             1-

    +                                 1-   2-                             1A    +
                                      1-
                                           1A
  15+                            1-    A   4-             3A         1-         +
                                 2-        2-   1-        3-          C    C    C
                       1-   1-   1-        1-   2-    C   1C    C    1-
    +                  1-              C    C   2C        1-         1-         +
                            3C   1*   1-   1-   1A                   1-
              C    C   1C    A              5-                   1-
  10+    C             3-   2-                  1-                    A         +
                        A   1-                                       1-
                       1-        3-                                  1-
    +                  1-   1-                  2-                             +
                       3-                                            2-

   5+                                                                          +
                       1-

    +                                                                          +


   0+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
              2         3         4         5         6         7         8
EDGER DRAFT               VS      SLAB WIDTH
V(  3)                             V(  1)
 -1                                  1
X10                               X10
```

APPENDIX D

Computer Output for Dogbone Regression

DESCRIPTION OF CONTENTS

This Appendix contains the results of four separate regression analyses. In each case, the dependent variable is DOGBONE FACTOR and the independent variables are slab width and thickness. The first output represents results of the regression conducted over the entire range of input data (56 observations over a slab width range of 27.7 to 78.7 inches). The three remaining computer outputs are results of the regression of the same variables limited to specific ranges of the same input data (width ranges are shown on the first page of each regression).

GENERAL DESCRIPTION OF COMPUTER OUTPUTS

The same general description applies as that given in Appendix C, p. 65.

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                    PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                          JOB  2  RUN  1
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                    02/24/75
KLUCHAR                                                                     PAGE   1

INPUT CONTAINS  6 VARIABLES WITH   56 OBSERVATIONS.

NOTE  V(  7) IS A CONSTANT.

INPUT    IDENTITY                            MIN          MAX          AVE          SIGMA

V(  1) = SLAB WIDTH                     0.2770E 02   0.7870E 02   0.4560D 02   0.1103D 02

V(  2) = SLAB THICKNESS                 0.9420E 01   0.2686E 02   0.1978D 02   0.4477D 01

V(  3) = DOGBONE FACTOR                 0.1460E 00   0.5230E 00   0.3022D 00   0.8812D-01

V(  4) =   1/ V(  2)                    0.3724E-01   0.1062E 00   0.5363D-01   0.1421D-01
         1/THICKNESS

                          V(  7)
V(  5) =   1/(V(  2)             )      0.1930E 00   0.3258E 00   0.2297D 00   0.2911D-01
         1/SQRT THICKNESS

V(  6) = 1/LN V(  2)                    0.3039E 00   0.4459E 00   0.3407D 00   0.3013D-01
         1/LN THICKNESS

V(  7) =     0.5000000D 00

SIMPLE CORRELATION COEFFICIENTS

        V(  1) V(  2) V(  3) V(  4) V(  5) V(  6) V(

V(  1)  1.0000

V(  2)  0.2935 1.0000

V(  3) -0.0934-0.2391 1.0000

V(  4) -0.2019-0.9583 0.2000 1.0000

V(  5) -0.2267-0.9777 0.2137 0.9969 1.0000

V(  6) -0.2166-0.9699 0.2074 0.9990 0.9994 1.0000
```

```
YOUNGSTOWN SHEET AND TUBE CO, TFCHNICAL SERVICES                     PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                           JOB  2  RUN  3
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                    02/24/75
KLUCHAR                                                                     PAGE   6

LINEAR SOLUTION FOR DOGBONE FACTOR        VS
FACTOR                       COEFFICIENT  +/-RANGE 95 PCT

CONSTANT                    0.401609D 00
SLAB WIDTH                 -0.203063D-03   0.222919D-02
SLAB THICKNESS             -0.455824D-02   0.549036D-02

INTERACTION CORRFLATION COEFFICIENTS

       B(  1) B(  2) B(

B(  1)  1.0000

B(  2) -0.2935 1.0000

ANALYSIS OF VARIANCE IN DOGBONE FACTOR
SOURCE OF VARIATION D.O.F. PCT EXPL  SUM OF SQUARES        VARIANCF     F RATIO

FIRST ORDER               2      5.77    0.251063D-01     0.125531D-01 0.162D 01
RESPONSE SURFACE
SLAB WIDTH                1      0.24    0.103175D-02     0.103175D-02 0.133D 00
SLAB THICKNESS            1      5.54    0.240745D-01     0.240745D-01 0.311D 01

CALCULATED  RESIDUAL    53               0.409740D 00     0.773094D-02
OBSERVED    RESIDUAL    53               0.409740D 00     0.773094D-02

TOTAL                   55               0.434846D 00     0.790629D-02

EXPECTED MEAN SQUARE                                      0.202841D-01

STANDARD ERROR OF ESTIMATE 0.8793D-01
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  2  RUN  3
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                 PAGE   7

        0    0    0   10    8    4    3   14   15    0    1    0    0    1    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +

    +                                                                          +

   6+                                                                          +

    +                                                                          +
                                 1-             1-
   5+                            1-                                            +

    +                             A             1-                             +
                            2-   1-

   4+                       1-                                                 +
                                 1-        1-
                                                1-
    +                  1-   1-                  1-                             +
                       1-    A             2-   1-
         C                  1-             1-   3-        1A
   3+         C    C   3C   1C    C    C   1C   1*    C    C    C    C    C    C
                                      1-   2A
                        A                  4-                            1A
    +                  2-   1-             1-   1-                             +
                       1-                       2-
                       1-             1A   1-   1-
   2+                                                                          +
                                           1-   1-
                            1-                  1-
    +                  1-             1-                                       +

   1+                                                                          +

    +                                                                          +

   0+                                                                          +

    +                                                                          +

    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
              2         3         4         5         6         7         8
DOGBONE FACTOR            VS      SLAB WIDTH
V(  3)                             V(  1)
 -1                                  1
X10                               X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  2  RUN  3
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                 PAGE   8

        0    0    0    1    6    7   10   10    6   11    5    0    0    0    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


   6+                                                                          +


    +                                                                          +

                                      1-
   5+                                           1-                             +

                            1-                       1-
    +                            1-                                            +
                                      1-             1-

   4+                                 1-                                       +
         C                            1-
              C             1-   1-                  1-
    +              C    C   1-   1-   1A                                       +
                             *             2-        1-   1-
                            1-   1*   1C
   3+                       1-        1-   1C   2-   1A                        +
                                 1-   1-   1-   1*   3C   1-
                       1A        1-   1-                   C
    +                                 1-   3A        1-         C    C         +
                                 1-        1-              A              C
                            1-                            1-                   C
   2+                                           1-   1-                        +
                                                          1-

    +                                      2-   1-   1-   1-                   +


   1+                                                                          +


    +                                                                          +


   0+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
              5        10        15        20        25        30        35
DOGBONE FACTOR            VS      SLAB THICKNESS
V(  3)                             V(  2)
  -1
X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES             PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                   JOB  2  RUN  1
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                             02/24/75
KLUCHAR                                                              PAGE   1


INPUT CONTAINS  6 VARIABLES WITH   22 OBSERVATIONS.

NOTE  V(  7) IS A CONSTANT.

INPUT     IDENTITY                    MIN          MAX          AVE          SIGMA

V(  1) = SLAB WIDTH               0.2770E 02  0.4240E 02  0.3370D 02  0.4045D 01

V(  2) = SLAB THICKNESS           0.1240E 02  0.2507E 02  0.1961D 02  0.3581D 01

V(  3) = DOGBONE FACTOR           0.1460E 00  0.5230E 00  0.3270D 00  0.1013D 00

V(  4) =   1/ V(  2)              0.3989E-01  0.8065E-01  0.5283D-01  0.1035D-01
         1/THICKNESS

                    V(  7)
V(  5) =   1/(V(  2)          )   0.1997E 00  0.2840E 00  0.2288D 00  0.2199D-01
         1/SQRT THICKNESS

V(  6) = 1/LN V(  2)              0.3104E 00  0.3972E 00  0.3394D 00  0.2237D-01
         1/LN THICKNESS

V(  7) =     0.5000000D 00

SIMPLE CORRELATION COEFFICIENTS

          V(  1) V(  2) V(  3) V(  4) V(  5) V(  6) V(

V(  1)  1.0000

V(  2)  0.0232 1.0000

V(  3)  0.6654-0.4550 1.0000

V(  4)  0.0412-0.9768 0.5018 1.0000

V(  5)  0.0249-0.9874 0.4927 0.9984 1.0000

V(  6)  0.0312-0.9836 0.4962 0.9994 0.9997 1.0000
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES               PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                     JOB  2  RUN  3
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                               02/24/75
KLUCHAR                                                                PAGE   6

LINEAR SOLUTION FOR DOGBONE FACTOR         VS
FACTOR                        COEFFICIENT  +/-RANGE 95 PCT

CONSTANT                     0.174139D-01
SLAB WIDTH                   0.169383D-01    0.666231D-02
SLAB THICKNESS              -0.133158D-01    0.752543D-02

INTERACTION CORRELATION COEFFICIENTS

         B(  1) B(  2) B(

B(  1)  1.0000

B(  2) -0.0232 1.0000

ANALYSIS OF VARIANCE IN DOGBONE FACTOR
SOURCE OF VARIATION D.O.F. PCT EXPL  SUM OF SQUARES        VARIANCE    F RATIO

FIRST ORDER                2     66.41    0.149993D 00     0.749964D-01 0.188D 02
RESPONSE SURFACE
SLAB WIDTH                 1     45.00    0.101626D 00     0.101626D 00 0.255D 02
SLAB THICKNESS             1     21.41    0.483664D-01     0.483664D-01 0.121D 02

CALCULATED   RESIDUAL     19              0.758641D-01     0.399285D-02
OBSERVED     RESIDUAL     19              0.758641D-01     0.399285D-02

TOTAL                     21              0.225857D 00     0.107551D-01

EXPECTED MEAN SQUARE                                       0.789893D-01

STANDARD ERROR OF ESTIMATE 0.6319D-01
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  2  RUN  3
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                               02/24/75
KLUCHAR                                                                 PAGE   7

         0    0    0    0    2    4    8    2    2    3    1    0    0    0    0
     +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


     +                                                                          C


    6+                                                                     C    +

                                                                      C
     +                                                                          +
                                                           1A
                                                                 C
    5+                                                                          +
                                                      1-    C

     +                                                                          +
                                                       C
                                       1-              A
    4+                                           1A   1-                        +
                                                 1C   1-
                                            1-
     +                            1-   1-    C                                  +
                                       1-
                                             A
    3+                       1-   1-   1*                                       +

                                  1*   2-   1-
     +                        A        1-                                       +

                              C
    2+                       1-        1-                                       +
                         C
                                  1-
     +                                                                          +
                    C

    1+         C                                                                +


     +    C                                                                     +


    0+                                                                          +


     +                                                                          +


     +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
              20        25        30        35        40        45        50
DOGBONE FACTOR            VS      SLAB WIDTH
V(  3)                            V(  1)
 -1
X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  2  RUN  3
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                 PAGE   8

        0    0    0    0    1    3    6    5    2    5    0    0    0    0    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


   6+                                                                          +


    +    C                                                                     +

              C
   5+                                                                          +
                   C
                                 1-
    +                   C                                                      +

                            1*    A
   4+                            1-   1-   1-                                  +
                                 1C   1-
                                      1-             1-
    +                                  *                                       +
                                      2-
                                            C   1-   1-
   3+                                 1-             1-                        +
                                           2A    *    A
                                                     1-
    +                                                 C                        +
                                           1-   1-
                                           1-              C
   2+                                                                          +
                                                                C

    +                                                1-              C         +

                                                                          C
   1+                                                                          +
                                                                               C

    +                                                                          +


   0+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
              5        10        15        20        25        30        35
DOGBONE FACTOR            VS      SLAB THICKNESS
V(  3)                             V(  2)
  -1
X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                     PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                            JOB  2  RUN  1
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                      02/24/75
KLUCHAR                                                                   PAGE     1

INPUT CONTAINS  6 VARIABLES WITH   17 OBSERVATIONS.

NOTE  V(  7) IS A CONSTANT.

INPUT     IDENTITY                         MIN         MAX          AVE          SIGMA

V(  1) = SLAB WIDTH                  0.4300E 02  0.5210E 02  0.4917D 02  0.2308D 01

V(  2) = SLAB THICKNESS              0.9420E 01  0.2032E 02  0.1564D 02  0.2936D 01

V(  3) = DOGBONE FACTOR              0.1460E 00  0.3850E 00  0.2896D 00  0.5680D-01

V(  4) =   1/ V(  2)                 0.4921E-01  0.1062E 00  0.6655D-01  0.1425D-01
         1/THICKNESS

                   V(  7)
V(  5) =   1/(V(  2)          )      0.2218E 00  0.3258E 00  0.2566D 00  0.2651D-01
         1/SQRT THICKNESS

V(  6) = 1/LN V(  2)                 0.3320E 00  0.4459E 00  0.3683D 00  0.2868D-01
         1/LN THICKNESS

V(  7) =    0.50000000 00

SIMPLE CORRELATION COEFFICIENTS

          V(  1) V(  2) V(  3) V(  4) V(  5) V(  6) V(

V(  1)  1.0000

V(  2)  0.4062 1.0000

V(  3)  0.4499-0.1520 1.0000

V(  4) -0.4561-0.9724 0.0682 1.0000

V(  5) -0.4485-0.9850 0.0916 0.9981 1.0000

V(  6) -0.4519-0.9787 0.0796 0.9946 0.9994 1.0000
```

```
YOUNGSTOWN SHEET AND TUBE CO, TECHNICAL SERVICES                  PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                        JOB  2  RUN  3
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                 02/24/75
KLUCHAR                                                                  PAGE    6

LINEAR SOLUTION FOR DOGBONE FACTOR         VS
FACTOR                       COEFFICIENT  +/-RANGE 95 PCT

CONSTANT                    -0.332935D 00
SLAB WIDTH                   0.151993D-01   0.116545D-01
SLAB THICKNESS              -0.798579D-02   0.915974D-02

INTERACTION CORRELATION COEFFICIENTS

        B(  1) B(  2) B(

B(  1)  1.0000

B(  2) -0.4062 1.0000

ANALYSIS OF VARIANCE IN DOGBONE FACTOR
SOURCE OF VARIATION D.O.F, PCT EXPL  SUM OF SQUARES          VARIANCE   F RATIO

FIRST ORDER               2      34.47     0.189028D-01     0.945141D-02 0.368D 01
RESPONSE SURFACE
SLAB WIDTH                1      27.78     0.152352D-01     0.152352D-01 0.594D 01
SLAB THICKNESS            1       6.69     0.366761D-02     0.366761D-02 0.143D 01

CALCULATED  RESIDUAL     14                0.359373D-01     0.256695D-02
OBSERVED    RESIDUAL     14                0.359373D-01     0.256695D-02

TOTAL                    16                0.548401D-01     0.342751D-02

EXPECTED MEAN SQUARE                                        0.120184D-01

STANDARD ERROR OF ESTIMATE 0.5067D-01
```

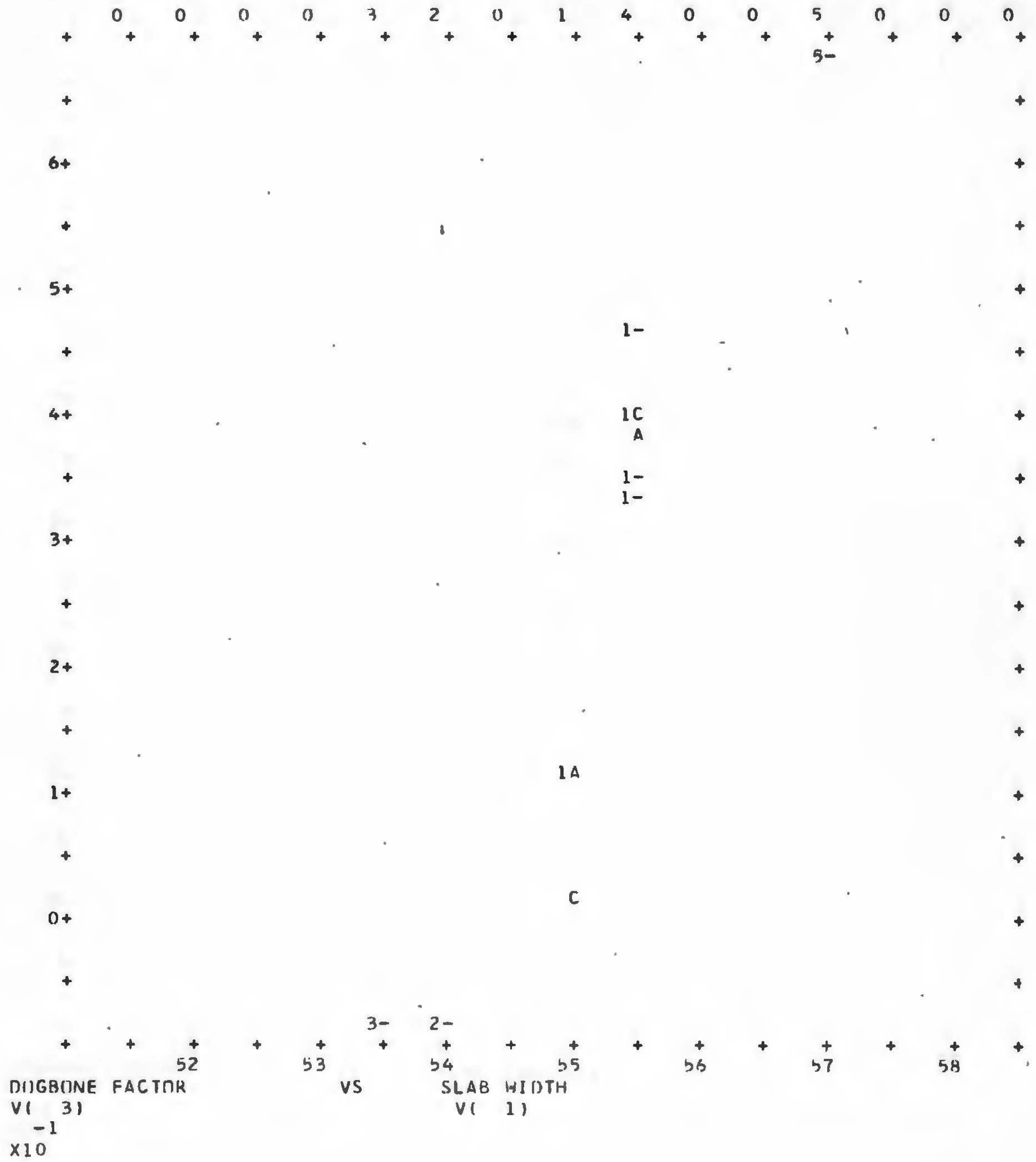
```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                 PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  2  RUN  3
SPREAD AND DOGBONE STUDY, NO. 2, SLABBER                                02/24/75
KLUCHAR                                                                 PAGE   7

        1    0    0    0    2    4    1    4    1    4    0    0    0    0    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  45+                                                                          +


    +                                                                          +

                                                                               C
  40+                                           1A                             +
                                                                          C

    +                                                                C         +
                                                                C
                                           1-
  35+                                                      C                   +
                                 1-                  1-
                                 1-                   C
    +                                                                          +
                                                 C
                                           1-        1A
  30+                                       C        1-                        +
                            1-    A         A        1-
                                 1-   1*
    +                             C                                            +
                                           1-
                             *
  25+                                                                          +
                        C
                                           1-
    +              C                                                           +
                            1-
              C
  20+                            1-                                            +
         C
        1A
    +                                                                          +


  15+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
             44        46        48        50        52        54        56
DOGBONE FACTOR            VS      SLAB WIDTH
V(  3)                             V(  1)
 -2
X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  2  RUN  3
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                02/24/75
KLUCHAR                                                                PAGE    8

        1    0    0    2    2    1    2    3    0    2    3    1    0    0    0
    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +


    +                                                                          +


  45+                                                                          +


    +                                                                          +


  40+                                                                          +


    +                       1-   1A                                            +


  35+                  1-    A                       1-                        +
         C                                 1-
              C
    +              C        1-                                                 +
                        C
        1A                   C
  30+                   A         C         A                                  +
                                       C   1-         A
                                      1-    C
    +                                           C                              +
                                                      C   1C
                                       A                        C
  25+                                      1-                        C         +
                       1-                                  A   1A        C
                                                     1-   2-                   C
    +                                 1-                                       +


  20+                                                                          +


    +                                                                          +


  15+                                                                          +


    +                                                                          +


    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +    +
             10        12        14        16        18        20        22
DOGBONE FACTOR            VS      SLAB THICKNESS
V(  3)                             V(  2)
  -2
X10
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES            PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                  JOB  2  RUN  1
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                            02/24/75
KLUCHAR                                                             PAGE   1

INPUT CONTAINS  6 VARIABLES WITH   15 OBSERVATIONS.

NOTE  V(  7) IS A CONSTANT.

INPUT     IDENTITY                       MIN          MAX          AVE          SIGMA

V(  1) = SLAB WIDTH              0.5345E 02  0.5700E 02  0.5534D 02  0.1344D 01

V(  2) = SLAB THICKNESS          0.2082E 02  0.2685E 02  0.2400D 02  0.2235D 01

V(  3) = DOGBONE FACTOR          0.1520E 00  0.4940E 00  0.2840D 00  0.9299D-01

V(  4) =   1/ V(  2)             0.3724E-01  0.4803E-01  0.4203D-01  0.3935D-02
         1/THICKNESS

                       V(  7)
V(  5) =   1/(V(  2)          )  0.1930E 00  0.2192E 00  0.2048D 00  0.9586D-02
         1/SQRT THICKNESS

V(  6) = 1/LN V(  2)             0.3039E 00  0.3294E 00  0.3154D 00  0.9316D-02
         1/LN THICKNESS

V(  7) =     0.5000000D 00

SIMPLE CORRELATION COEFFICIENTS

        V(  1) V(  2) V(  3) V(  4) V(  5) V(  6) V(

V(  1)  1.0000

V(  2)  0.9978 1.0000

V(  3) -0.1069-0.1546 1.0000

V(  4) -0.9980-0.9977 0.1207 1.0000

V(  5) -0.9984-0.9987 0.1292 0.9999 1.0000

V(  6) -0.9983-0.9984 0.1270 0.9999 1.0000 1.0000
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES              PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                    JOB  2  RUN  3
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                              02/24/75
KLUCHAR                                                               PAGE   6

LINEAR SOLUTION FOR DOGBONE FACTOR        VS
FACTOR                         COEFFICIENT  +/-RANGE 95 PCT

CONSTANT                    -0.3064650 02
SLAB WIDTH                   0.7591770 00     0.4100830 00
SLAB THICKNESS              -0.4618430 00     0.2465320 00

INTERACTION CORRELATION COEFFICIENTS

        B(  1) B(  2) B(

B(  1)  1.0000

B(  2) -0.9978 1.0000

ANALYSIS OF VARIANCE IN DOGBONE FACTOR
SOURCE OF VARIATION D.O.F. PCT EXPL  SUM OF SQUARES          VARIANCE   F RATIO

FIRST ORDER              2      54.44    0.7061900-01     0.3530950-01 0.717D 01
RESPONSE SURFACE
SLAB WIDTH               1    -117.22   -0.1520630 00    -0.1520630 00-0.309D 02
SLAB THICKNESS           1     171.66    0.2226820 00     0.2226820 00 0.452D 02

CALCULATED  RESIDUAL    12               0.5910100-01     0.4925090-02
OBSERVED    RESIDUAL    12               0.5910100-01     0.4925090-02

TOTAL                   14               0.1297200 00     0.9265710-02

EXPECTED MEAN SQUARE                                      0.4023460-01

STANDARD ERROR OF ESTIMATE 0.7018D-01
```

```
YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                 PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                        JOB  2  RUN  3
SPREAD AND DOGBONE STUDY, NO. 2. SLABBER                                 02/24/75
KLUCHAR                                                                  PAGE   7
```



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YOUNGSTOWN SHEET AND TUBE CO. TECHNICAL SERVICES                PROJECT 6400.00
MULTIPLE REGRESSION ANALYSIS                                      JOB  2  RUN  3
SPREAD AND DOGBONE STUDY; NO. 2. SLABBER                               02/24/75
KLUCHAR                                                                PAGE   8
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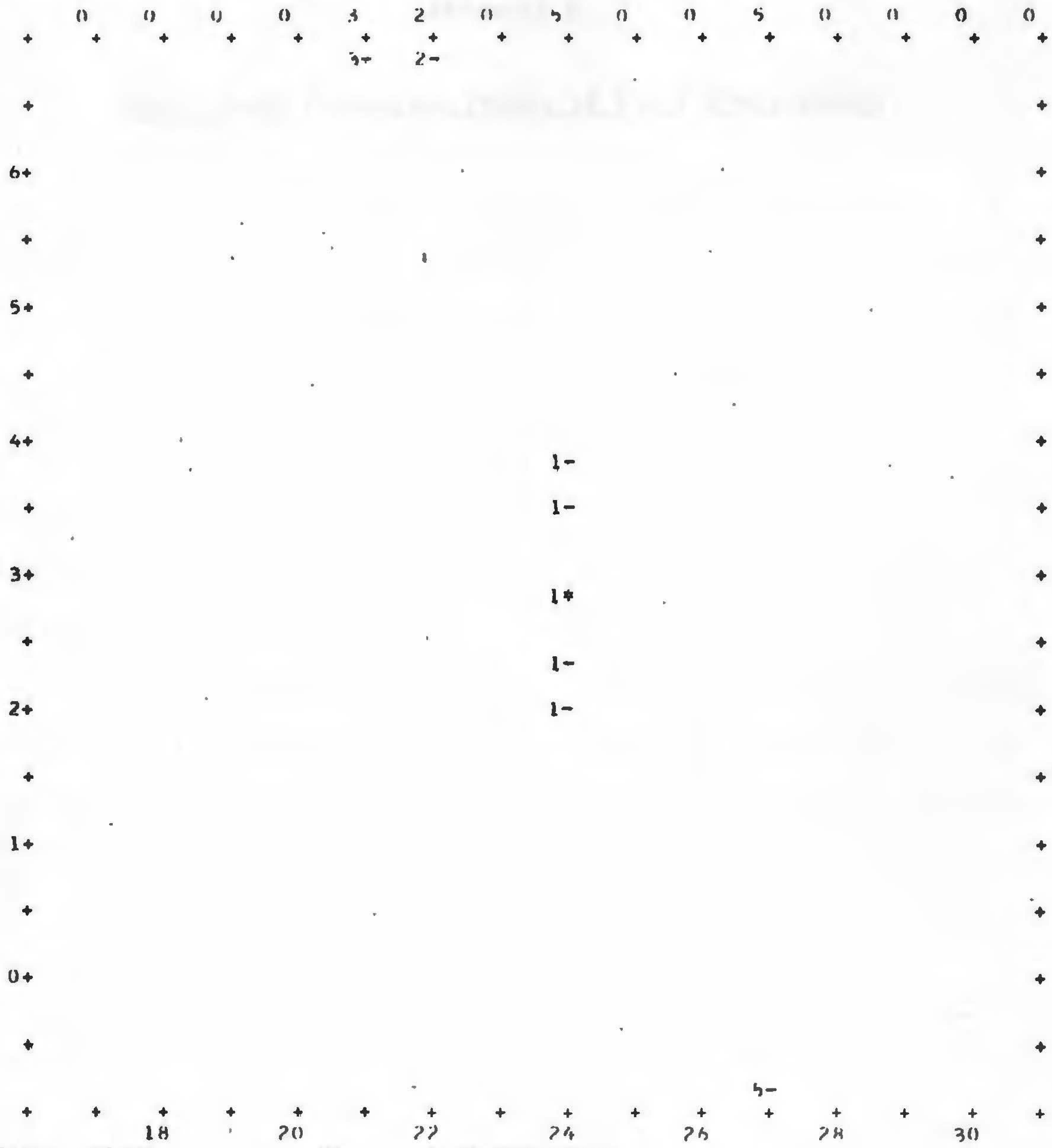

APPENDIX E

Statistical Percentage Points of the F Distribution

DESCRIPTION OF CONTENTS

The table of percentage points is given here as a statistical tool to easily evaluate the calculated F RATIO (the F is in honor of the statistical theorist, R. A. Fisher) as presented in the various regression results. Briefly, this ratio of two variances, when considered with the degrees of freedom of the response surface (denoted by n_1) versus the residuals (denoted by n_2), indicates the measure of likelihood that the particular relationship in question is a result of chance. More specifically, the actual F RATIOS denoted in the regressions are used in conjunction with the P table to indicate the likelihood of chance associated with the relationship described by the particular regression equation.

As an example to illustrate the use of the chart: In a sample of 3 versus 20 degrees of freedom, an F RATIO of 8.10 is shown. This indicates a likelihood of .001 that the given F RATIO may be obtained purely by chance.

PERCENTAGE POINTS OF THE F DISTRIBUTION (ABRIDGED TABLE)

n_2	P	$n_1 = 1$	2	3
12	.500	.484	.735	.835
	.100	3.18	2.81	2.61
	.050	4.75	3.89	3.49
	.025	6.55	5.10	4.47
	.010	9.33	6.93	5.95
	.005	11.8	8.51	7.23
	.001	18.6	13.0	10.8
15	.500	.478	.726	.826
	.100	3.07	2.70	2.49
	.050	4.54	3.68	3.29
	.025	6.20	4.77	4.15
	.010	8.68	6.36	5.42
	.005	10.8	7.70	6.48
	.001	16.6	11.34	9.34
20	.500	.472	.718	.816
	.100	2.97	2.59	2.38
	.050	4.35	3.49	3.10
	.025	5.87	4.46	3.86
	.010	8.10	5.85	4.94
	.005	9.94	6.99	5.82
	.001	14.8	9.95	8.10
60	.500	.461	.701	.798
	.100	2.79	2.39	2.18
	.050	4.00	3.15	2.76
	.025	5.29	3.93	3.34
	.010	7.08	4.98	4.13
	.005	8.49	5.80	4.73
	.001	11.97	7.76	6.17

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