STATIC AND DYNAMIC STABILITY

OF THIN PLATES USING

FINITE STRIP METHOD

by

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ABSTRACT

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The Finite Strip Method is presented in the thesis for the analysis of two dimensional structures under both static and dynamical loading conditions. The Finite Strip Method can be considered as a special form of the finite element using the displacement (stiffness) approach. The philosophy of the finite strip method is similar to that of the Kantorovich method which is used to reduce a partial differential equation to an ordinary differential equation. The method when applied to most structural problems, yields a narrow banded, small size stiffness and mass matrices, which require a relatively small computational effort. Therefore, this method reduces the analyses of those complex structures to simple, efficient size matrix operations.

In this thesis, a general finite strip theory of structural analysis is presented and the derivation of the stiffness and mass matrices is discussed. General computer programs for a finite strip system are developed to analyze thin plates, folded plate structures and box girders for static and dynamic loading. The theoretical solutions of this method for different structures are compared with other exact and numerical solutions, and in all cases, good agreement in results is observed.

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LIST OF SYMBOLS

SYMBOL	DESCRIPTION
a	Strip length
b	Strip width
$[B]$	Strain matrix
$[C_i]$	Shape functions for a strip
D_x, D_y, D_{xy}	Bending rigidities in Cartesian coordinates
$[D]$	Property matrix
E_x, E_y, E_{xy}	Elastic constants in Cartesian coordinates
e	Eccentricity
$f(x)$	Polynomial functions in transverse (Cartesian) directions
f_i	Nodal parameters of a strip
$\{F\}$	Load vector
G	Shear modulus
$[H]$	Transformation matrix
I	Moment of inertia
I_l	Integrals
$[K]$	General stiffness matrix
$[K_G]$	General geometric stiffness matrix
L_i	Area coordinate
$[M]$	Mass matrix
M_x, M_y, M_{xy}	Bending moments in Cartesian and polar coordinates
$[N_k]$	Shape functions
P	Concentrated load

$\{P\}$	Load vector
q	Uniformly distributed load
$[R]$	Transformation matrix
$[S]$	Stiffness matrix of strip
$[S_G]$	Geometric stiffness matrix of strip
t	Thickness of plates
u, v, w	Displacements in x, y, and z directions, respectively
U	Strain energy
W	Potential energy due to external forces
x, y, z	Rectangular coordinates
X_m, Y_m	Basic functions
$\{X\}$	Redundant force vector
α	Subtended angle
$\epsilon, \epsilon_x, \epsilon_y, \gamma_{xy}$	Strains
$\sigma, \sigma_x, \sigma_y, \tau_{xy}$	Stress
$\{\sigma\}, \{\epsilon\}, \{\tau\}$	Generalized stress and strain vectors
χ_i	Curvature parameters
θ_x, θ_y	Rotations about x and y directions, respectively
μ_m, α_m	Parameters of the basic functions
δ_k	Nodal displacement parameters
$\{\delta\}$	Nodal displacement vector
ϕ	Total potential energy
ν	Poisson's ratio
ω	Natural frequency
ρ	Mass density
λ	Eigenvalues $\omega^2, \frac{1}{p}, \Omega$

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CHAPTER I

INTRODUCTION

1.1 <u>Historical Review</u>

The development of structural mechanics in the area of thin plate theory is classified under the following two broad categories:

I. Classical Methods with subdivisions
   1. Direct Solution of Differential Equation
      (a) Navier
      (b) Levy
   2. Energy Methods
      (a) Rayleigh Ritz
      (b) Galerkin Method
      (c) Kantorovich Method

II. Numerical Methods with subdivisions
   1. Finite Difference Method
   2. Finite Element Method
   3. Finite Strip Method

1.2 <u>Classical Methods</u>

1.2.1 Differential Equation Solutions

Structural analysis commenced with the investigation of static problems in the area of frames and columns. However, the first analytical and experimental studies on thin plates were devoted almost exclusively to free vibration.

The first mathematical approach to the membrane theory was formulated by Euler (1766) who solved the problem of free vibration of rectangular and circular elastic membranes using the analogy of two systems of stretched strings perpendicular to each other. James Bernoulli (1789) extended Euler's analogy to plates by introducing the grid-work analogy. He found only a resemblance between theory and experiment, but no general agreement. Chladini (1802) discovered the various modes of free vibration. Sophie Germain (1816) a French mathematician developed a differential equation for vibration of thin plates using the calculus of variations. Lagrange (1818) corrected Germain's work by adding the strain energy expression of the plate to account for the work done by warping of the middle surface.

The great bridge Engineer, Navier (1820) can be considered as the originator of modern theory of Elasticity. His numerous scientific activities included the solution of various thin plate problems.

The classic theory of Elasticity, which exclusively governed the theory of plates for a long period of time, assumes a linear relationship between stress and strain. The great advantages of this assumption are:

1. Mathematical relation between external and internal forces;
2. The law of superposition.

The fundamental equations of the linearized theory of Elasticity are:

1. Hooke's Law;
2. Stress equilibrium and;
3. Strain (compatibility) Displacement Conditions.

Navier, using the classic theory of Elasticity described the correct differential equation of bending of simply supported rectangular plates. Navier's method is based on the use of trignometric series introduced by Fourier. For simply supported rectangular plates, the solution of the governing fourth order differential equation is reduced to the solution of an algebraic equation by Navier's method.

Poisson (1829) extended the Navier's governing equation of plates to the lateral vibration of circular plates. Kirchoff (1850) developed the extended plate theory to include bending and stretching. A Russian scientist, Krylov, and his student Bovbnov contributed extensively to the theory of thin plates with flexural and extensional rigidities. S. Timoshenko (1930) translated the Russian research in the field of theory of Elasticity. It was after his translation the Western scientists made considerable progress in the area of plates. Among Timoshenko's numerous important contributions we can mention in the area of plates are solution of circular plates considering large deflections and the formulation of Elastic Stability problems.

Foppl (1907), in his book <u>Engineering</u> <u>Mechanics</u>, treated the nonlinear theory of plates. Von Karman developed the final form of differential equations of the large deflection theory. He also investigated the post buckling behavior of plates. Since all classical methods presented by the above authors are based on the theory of Elasticity, an extensive application of higher analysis was necessary. As a result, the methods were highly mathematical, linked with pertinent, geometrical conditions. For the majority of practical problems, a rigorous solution either cannot be found or is of such complicated structure that it can be applied only with great difficulty.

For simply supported rectangular plates, Navier's method yields a mathematical "exact" solution. The convergence of the resulting double Fourier series depends considerably on the continuity of the loading function. Slow convergence is created by discontinuous loading.

The application of Levy's method based on the use of single Fourier series is somewhat more complex, but the solution converges very rapidly. In addition Levy's method is more general. It is used for the case of two opposite edges of the plate simply supported with arbitrary conditions on the opposite two boundaries. The shape of the loading function must be constant at all sections perpendicular to the direction of the simply supported edges.

As mentioned, the number of exact solutions obtainable by the classical methods is severely limited. Many problems of considerable practicable importance cannot be solved by these methods or the solution obtained is too cumbersome. In such cases, the Energy methods are more effective and applicable.

1.2.2 Energy Methods

1.2.2.1 Ritz Method

The Ritz method is used for plates of various shapes, thickness, etc. Since it is essentially an analytical procedure, the accuracy of the solution of plate problems by this and by all energy methods is dependent upon proper and accurate selection of the shape functions. The shape functions transform the potential energy function, which contains a quadratic form of second order derivative terms, into a family of algebraic equations, the number of which depends on the assumed shape function. The Ritz method is recommended when computers are not available, and the solution must be obtained by hand computation. Although the mathematical operations are simple, the method may be quite lengthy. The Ritz Method can be considered as an advantageous method for solving complex boundary conditions. It provides accurate deflections provided proper shape functions are employed.

1.2.2.2 Galerkin Method

The variational method as formulated by Galerkin and Vlasov is an extremely valuable tool for the approximate analysis of various boundary conditions subjected to arbitrary loads. The simplicity of the method, coupled with the high accuracy obtainable, makes it one of the recommended techniques for "hand computations" when computers are not available. The merit of the variational method is that the solution of complex plate problems is reduced to the evaluation of certain definite integrals which in case of need can be evaluated numerically. The accuracy of the method depends considerably on the choice of shape function. The disadvantages of the variational methods are:

1. It requires the knowledge of higher mathematics;
2. Although the computations are simple, it can be quite lengthy;
3. It does not lend itself well to computerization.

1.3 Numerical Methods

1.3.1 Finite Difference Method

For many thin plate problems of practical application, analytic solutions to the governing differential equation can yield approximate results acceptable for most practical purposes. The finite difference method is one of the most general methods in the field of Structural Mechanics.

It can be effectively used to solve a wide variety of plate problems. In applying this method, the derivatives in the differential equation under consideration are replaced by difference quantities at some selected points.

The method is simple and versatile. It is suitable for computer programming. Although the method has been known for a long time, it has gained considerable importance after the development of high speed digital computers. Accuracy of the method is acceptable provided that a relatively fine mesh is used. The accuracy progressively deteriorates beyond a certain mesh width. The method was later improved for accuracy. In the improved finite difference method, special attention has been paid to load representation and the boundary value problems. Duration of improved formulas can be quite involved. Consequently, a simple error distribution technique has been developed for solution of complicated differential equations by the ordinary finite difference method.

The disadvantages of the finite difference methods are:

1. It requires mathematically trained operators.
2. Although it is well suited for computer application, it requires more work to prepare the input data.
3. A concentrated load with a fine grid can create singularity problems.

4. Certain boundary conditions may be difficult to handle.

1.3.2 Finite Element Method

The recently developed finite element methods have proved to be extremely powerful and versatile tools for the analysis of a wide variety of plate and shell problems. The essential feature of the method is the replacement of the continuum by a number of discrete elements connected together at the nodal points, where continuity is expressed. The most general convergence criterion of the finite element solution of plate problem is that the total energy of the substitute system obtained by assembling discrete elements must be equal to that of the original continuum. The most critical and simultaneously the most difficult operation in the finite element method is the generation and evaluation of element stiffness matrices, which are intimately linked to the compatibility of deformations within the element as well as between the adjacent elements. Once the element stiffness coefficients have been determined, the analysis of the structural system follows the familiar procedure of matrix methods for which standard computer programs are available.

The accuracy of the finite element method is influenced by the following parameters:

1. Displacement patterns prescribed for the element.

2. Number of elements.
3. Techniques of load representation.
4. Boundary conditions.
5. Computer program techniques.

Since the achievable results compare very favorably at times with the theoretical solutions, it is conceivable that the finite element method eventually will replace the experimental stress analysis techniques for complex plate problems. At present considerable amount of research time is spent in developing improved shape functions for generation of element stiffness matrices.

The advantages of the finite element method are:

1. The solution is obtained without the use of the governing differential equations.
2. It follows methods familiar to structural engineers.
3. Arbitrary boundary and loading conditions can be handled in the same manner as simple problems.
4. It permits the complete automation of all procedures.
5. It permits the combination of various structural elements such as beams, plates, and shells.
6. It can be extended to cover virtually all fields of continuum mechanics.

The disadvantages of this method are:

1. It requires the use of electronic digital computers of high speed and storage capacity.
2. Preparation of data for each element can be time consuming.
3. Some problems may require special computer programs.
4. It is difficult to ascertain the accuracy of the results when large structural systems are analyzed.

However, due to the limitation of computer size and high cost of the computation time, the cost of accurate solutions can be very high. Generally computers of large capacity (100K-1000K) are required in order to perform the matrix operation involved.

1.3.3 Finite Strip Method

The finite strip method, an extension of the finite element method, is specifically designed to reduce the required computer storage capacity. This efficiency is implemented by a reduction in the number of algebraic equations which must be solved. In turn this reduces the resulting matrix band width, which also directly saves computer time.

In the finite strip method, the continuous structure is divided in one direction only and therefore appears to be a one dimensional problem. However, to account for the second dimension, a basic functional series is assumed in that direction to account for the nature of boundary supports. Such an approach is similar to that of the Kantorovich Method (1950) which modifies the partial differential equations to ordinary differential equations by a separation of variable techniques.

The displacement function must be chosen so as to result in compatible strip deformations. The strain energy in the idealization will represent a lower bound to the strain energy in the actual structure and the finite strip solution can be demonstrated to converge to the true solution as the number of strips increases.

1.4 Purpose of the Thesis

The purpose of the thesis is to investigate the Finite Strip Method (FSM) of Structural Analysis and demonstrate its efficiency for analyzing standard civil engineering structures. This numerical method shows particular promise for structures with cross section geometry that remains constant along the length direction; that is, bridge decks, folded plate roofs and plates.

The thesis presents the solution of the following types of structural problems:

1) Static Analysis of Folded-Roof Structures
2) Static Stability Analysis of Thin Plates
3) Free Vibration Analysis of Thin Plates
4) Dynamic Stability Analysis of Thin Plates

Comparison of analytical results with those of classical methods, experimental methods, and finite elements methods (FEM) is made where possible for the above problems. The advantages of the FSM as compared with the FEM as to Geometric Modeling Time and Data Preparation Time are summarized where possible. Numerical accuracy of the two methods as compared with classical solutions is noted.

The main emphasis of the work is the solution of the Eigenvalue-Eigenvector Problem associated with Problem Types 2), 3) and 4) above. Problem Type 1) above is considered for purposes of background development and understanding of the FSM approach.

1.5 Process of Development

The thesis is divided into five parts.

1) Chapter II presents a procedure using FSM for the analysis of a folded plate structure, described by the equation

$$\left[K\right]\left\{\delta\right\} = \left\{f\right\} \qquad (1\text{-}1)$$

where

$\left[K\right]$ is the bending stiffness matrix

$\left\{\delta\right\}$ is the vector of resulting displacements and

$\left\{f\right\}$ is the vector of applied force

The approach presented herein is simple and straight forward. The element stiffness matrix of a simply supported strip is formulated in a similar manner as that of finite element. The stress matrix is written in terms of nodal displacement parameters. The loadings may be point loads, line loads, or distributed loads acting either on the ridges or directly on the surface of the inclined plates. The whole process of analysis is automated and involves only a small number of elementary matrix operations.

2) Chapter III presents a simple procedure using FSM for the analysis of vibration of a structure which is described by the following Equation

$$\left[K\right]\left\{\delta\right\} = \lambda\left[M\right]\left\{\delta\right\} \qquad (1\text{-}2)$$

where

$[K]$ is the bending stiffness matrix

$\{\delta\}$ is the vector of resulting displacements and

λ is a scalar equal to the square of the natural frequency of free vibration

$[M]$ is the associated mass matrix

A direct solution is uneconomical because matrix $[K]^{-1}[M]$, in general, is not symmetrical although $[K]^{-1}$ and $[M]$ are symmetrical. A "Cholesky" transformation procedure as described in Chapter III is necessary to efficiently solve the equation together with a power iterative technique developed by Anderson (9).

3) Chapter IV presents a simple procedure using FSM for the analysis of the static stability of thin plates with inplane loads which is described by the following Eigenvalue Equation

$$[K]\{\delta\} = +\lambda\,[K_G]\{\delta\} \qquad (1\text{-}3)$$

where

$[K]$ is the bending stiffness matrix

$[K_G]$ is the geometric stiffness matrix

$\{S\}$ is the vector of resulting displacements and scalar

λ is a scalar equal to the Euler Buckling Load

Free vibration and static stability problems share many similar features. Both require the determination of Eigenvalues and Elgenvectors. The solution procedure is similar to that explained in Chapter III.

4) Chapter V presents a procedure using FSM for the analysis of vibration of thin plates with inplane forces which is described by the following characteristic Equation

$$\left[K\right]\left\{\delta\right\} = \lambda\left[M\right]\left\{\delta\right\} + \lambda'\left[K_G\right]\left\{\delta\right\} \tag{1-4}$$

where

$\left[K\right]$ is the bending stiffness matrix

$\left[K_G\right]$ is the geometric stiffness matrix

$\left[M\right]$ is the associated mass matrix

$\left\{\delta\right\}$ is the vector of resulting displacements and scalar

λ is a scalar equal to the square of natural frequency of free vibrations.

λ' is a scalar equal to a fraction of the Euler Buckling Load

The presence of axial loads reduces the value of critical buckling load and this effect is greater on lower frequencies than on the higher frequencies. An incremental technique is developed to determine the reduction of frequency for increase in axial force. The value of zero frequency yields the value of static buckling load.

CHAPTER II

STATIC ANALYSIS

2.1 Chapter Overview

In this chapter the basic theory for the utilization of the FSM in Static Analysis is reviewed. Both the in-plane stiffness matrix and the bending stiffness matrix for a long thin strip are developed, together with the associated consistent load matrix. The comprehensive stiffness matrix formed from the combined in-plane and bending stiffness matrices is formulated. A rotation transformation process is defined to include the condition of an inclined (rotated) plate. The procedure for applying edge boundary conditions is defined and explained. The highly efficient process of simultaneous solution of the resulting Algebraic Equations is summarized. Finally, the numerical problem of a folded-plate structure is investigated in detail utilizing the static computer program of Appendix '2'.

2.2 Inplane Stiffness Formulation

A typical strip is shown in the Figure 1B with nodal lines numbered i and j. Each nodal line has two degrees of freedom, one in x direction, one in y direction, that is,

$$NDF\Big|_{Line} = \begin{matrix} Ui \\ Vi \end{matrix}$$

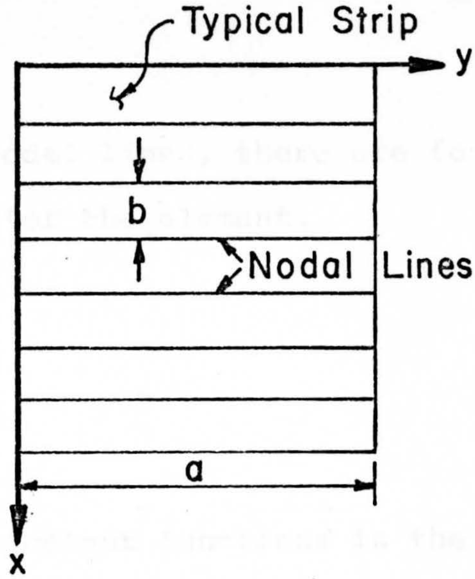

FIGURE 1a. A TYPICAL FLAT THIN PLATE

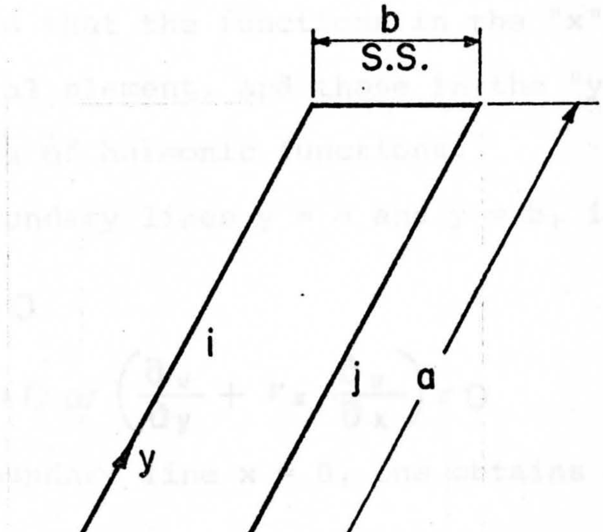

FIGURE 1b. A TYPICAL INPLANE STRIP ELEMENT

Since there are two nodal lines, there are four components of displacement defined for the element,

$$\text{NDF}\Bigg|_{\text{STRIP}} = \begin{Bmatrix} u_i \\ u_j \\ v_i \\ v_j \end{Bmatrix}$$

Assuming linear displacement functions in the x direction as functions of the unknown nodal line displacements, one obtains

$$u = \sum_{m=1}^{r} \left[\left(1 - \frac{x}{b}\right) \left(\frac{x}{b}\right)\right] \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} Y_m(y) \qquad (2\text{-}1)$$

$$v = \sum_{m=1}^{r} \left[\left(1 - \frac{x}{b}\right) \left(\frac{x}{b}\right)\right] \begin{Bmatrix} v_i \\ v_j \end{Bmatrix} Y'_m(y) \qquad (\ )' \equiv \frac{d}{dy}$$

It should be noted that the functions in the "x" variable are those of a uniaxial element, and those in the "y" variable form an infinite series of harmonic functions.

On the boundary lines y = o and y = b, it follows that,

1) $u = 0$

2) $\sigma_y = 0$ or $\left(\frac{\partial v}{\partial y} + \nu_x \frac{\partial u}{\partial x}\right) = 0$

On the boundary line x = 0, one obtains

$u = u_i \qquad v = v_i$

and on the line x = b, it follows that,

$u = u_j \qquad v = v_j$

Equation (2.1) is written in the matrix form as

$$\left\{\tilde{f}\right\} = \left\{\begin{matrix} u \\ v \end{matrix}\right\} = \sum_{m=1}^{r} \begin{bmatrix} \left(1-\frac{x}{b}\right)Y_m & 0 & \left(\frac{x}{b}\right)Y_m & 0 \\ 0 & \left(1-\frac{x}{b}\right)\frac{a}{\mu_m}Y'_m & 0 & \left(\frac{x}{b}\right)\frac{a}{\mu_m}Y'_m \end{bmatrix} \left\{\begin{matrix} u_i \\ v_i \\ u_j \\ v_j \end{matrix}\right\}$$

where $\mu_m = m\pi$

(2-2a)

or in matrix symbolic form as

$$\left\{\tilde{f}\right\} = \sum_{m=1}^{r} \left[N\right]_m \left\{\delta\right\}_m \qquad (2\text{-}2b)$$

2.3 <u>Strain - Displacement - Relationships</u>

The strains for a plane stress problem are composed of two longitudinal strains and a shear strain in the form

$$\left\{\epsilon\right\} = \left\{\begin{matrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{matrix}\right\} = \left\{\begin{matrix} \frac{\delta u}{\delta x} \\ \frac{\delta v}{\delta y} \\ \frac{\delta u}{\delta y} + \frac{\delta v}{\delta x} \end{matrix}\right\} = \sum_{m=1}^{r} \left[B\right]_m \left\{\delta\right\}_m \qquad (2\text{-}3)$$

The strain matrix $\left[B\right]_m$ is obtained by performing the indicated differentiation of the latter definition on Equation (2.2) yielding

$$\left[B\right]_m = \begin{bmatrix} \left(\frac{-1}{b}\right)Y_m & 0 & \left(\frac{1}{b}\right)Y_m & 0 \\ 0 & \left(1-\frac{x}{b}\right)\frac{a}{\mu_m}Y''_m & 0 & \left(\frac{x}{b}\right)\frac{a}{\mu_m}Y''_m \\ \left(1-\frac{x}{b}\right)Y'_m & \left(\frac{-1}{b}\right)\frac{a}{\mu_m}Y'_m & \left(\frac{x}{b}\right)Y'_m & \left(\frac{1}{b}\right)\frac{a}{\mu_m}Y' \end{bmatrix} \qquad (2\text{-}4)$$

2.4 Element Stress

The stresses corresponding to strains ϵ_x, ϵ_y, γ_{xy} for orthotropic materials are defined as σ_x, σ_y, τ_{xy}. They are related to each other by the following linear relationship:

$$\{\sigma\} = [D]\{E\}$$

$$= [D] \sum_{m=1}^{r} [B]_m \{\delta\}_m \qquad (2\text{-}5)$$

in which the matrix is given as

$$[D] = \begin{bmatrix} D_x & D_1 & 0 \\ D_1 & D_y & 0 \\ 0 & 0 & D_{xy} \end{bmatrix} \qquad (2\text{-}6)$$

where D_x, D_y, D_1 and D_{xy} are the orthotropic plate constants defined as

$$\left.\begin{aligned} D_x &= \frac{E_x t^3}{12} \frac{1}{(1-\nu_x \nu_y)} \\ D_y &= \frac{E_y t^3}{12} \frac{1}{(1-\nu_x \nu_y)} \\ D_1 &= \frac{\nu_x E_y t^3}{12(1-\nu_x \nu_y)} = \frac{\nu_y E_x t^3}{12(1-\nu_x \nu_y)} \\ D_{xy} &= \frac{G t^3}{12} \end{aligned}\right\} \qquad (2\text{-}7a)$$

where E_x, E_y, ν_x, ν_y and G are the elastic constants, with the thickness of the platestrip.

For an isotropic plate

$$\left.\begin{aligned} E_x &= E_y = E \\ \nu_x &= \nu_y = \nu \\ G &= \frac{E}{2(1+\nu)} \end{aligned}\right\} \qquad (2\text{-}7b)$$

2.5 Minimization of Total Potential Energy

The strain energy of an elastic body is given as

$$U = \frac{1}{2}\int\left\{\epsilon\right\}^T\left\{\sigma\right\} d(\text{vol.}) \tag{2-8}$$

noting Equations (2.3) and (2.5).

$$\left[\epsilon\right] = \left[B\right]\left\{\delta\right\} , \left\{\sigma\right\} = \left[D\right]\left[B\right]\left\{\delta\right\}$$

Equation (2.8) is rewritten as

$$U = \frac{1}{2}\int\left\{\delta\right\}^T\left[B\right]^T\left[D\right]\left[B\right]\left\{\delta\right\} d(\text{vol.}) \tag{2-9}$$

The potential energy due to external surface loads is given as

$$V = -\int\left\{f\right\}^T\left\{q\right\} d(\text{area}) \tag{2-10}$$

$$\text{BUT } \left\{\tilde{f}\right\} = \left[N\right]\left\{\delta\right\}$$

$$\left\{\tilde{f}\right\}^T = \left\{\delta\right\}^T\left[N\right]^T$$

Therefore, $W = \int\left\{\delta\right\}^T\left[N\right]^T\left\{q\right\} d(\text{area})$

Total potential energy is the sum of the elastic strain energy stored in the body and the potential energy of the loads, hence,

$$\phi = U + V$$

$$= \frac{1}{2}\int\left\{\delta\right\}^T\left[B\right]^T\left[D\right]\left[B\right]\left\{\delta\right\} d(\text{vol.}) - \int\left\{\delta\right\}^T\left[N\right]^T\left\{q\right\} d(\text{area}) \tag{2-11}$$

The principle of minimum total potential energy requires that

$$\frac{\partial \phi}{\partial\left\{\delta\right\}} = 0$$

Thus,

$$\left\{\frac{\partial \phi}{\partial\{\delta\}}\right\} = \int [B]^T [D][B]\{\delta\} d(\text{vol.}) - \int [N]^T \{q\} d(\text{area}) = \{0\}$$

or

$$[S]\{\delta\} - \{F\} = \{0\} \qquad (2\text{-}12a)$$

in which

$$[S] = \int [B]^T [D][B] d(\text{vol.}) \qquad (2\text{-}12b)$$

2.6 In-Plane Stiffness Matrix

Since the thickness of a strip is assumed to be constant, the $[S]$ matrix of Equation (2.12) is structured into a partitioned matrix with individual element matrices given as

$$[S]_{mn} = t \int [B]_m^T [D][B]_n \; d(\text{area}) \qquad (2\text{-}13)$$

$$m = 1,2,3,\ldots\ldots \qquad n = 1,2,3,\ldots\ldots$$

The in-plane stiffness matrix as computed from Equation (2.13) is shown in Table 1 for the case of simply supported boundary conditions, with $Y(y) = \operatorname{Sin}\frac{m\pi y}{a}$ $\qquad m = 1,2,\ldots,r.$

This matrix is a 4x4 square symmetric stiffness matrix. The computer program subroutine FEMP computes the numerical values of this element stiffness matrix.

$$
\begin{bmatrix}
\frac{aE}{2b} + \frac{abk_m^2 G}{6} & & \text{SYMMETRICAL} & \\
\frac{ak_m \nu_x E_2}{4} - \frac{ak_m G}{4} & \frac{abk_m^2 E_2}{6} + \frac{aG}{2b} & & \\
-\frac{aE_1}{2b} + \frac{abk_m^2 G}{12} & -\frac{ak_m \nu_x E_2}{4} - \frac{ak_m G}{4} & \frac{aE_1}{2b} + \frac{abk_m^2 G}{6} & \\
\frac{ak_m \nu_x E_2}{4} + \frac{ak_m G}{4} & \frac{abk_m^2 E_2}{12} - \frac{aG}{2b} & -\frac{ak_m \nu_x E_2}{4} + \frac{ak_m G}{4} & \frac{abk_m^2 E_2}{6} + \frac{aG}{2b}
\end{bmatrix}
$$

$$E_1 = \frac{E_x}{1-\nu_x \nu_y}$$

$$E_2 = \frac{E_y}{1-\nu_x \nu_y}$$

$m = 1,2,\ldots, r.$

TABLE 1 INPLANE STIFFNESS MATRIX OF A SIMPLY SUPPORTED STRIP.

2.7 Bending Stiffness Formulation

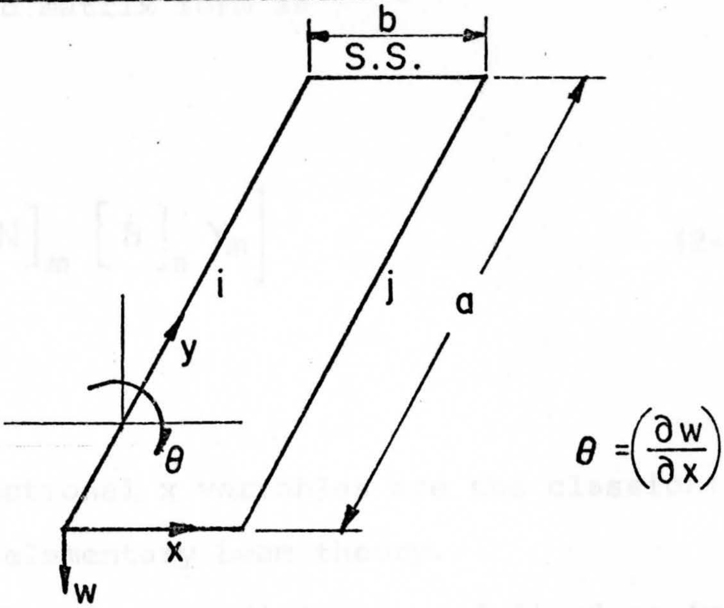

FIGURE 2 TYPICAL BENDING STRIP ELEMENT

A typical bending strip is shown in Figure 2 with nodal lines i and j. Each nodal line has two degrees of freedom, one in the transverse z direction and the other defined as slope θ of a nodal line.

Since there are two nodal lines, there are four degrees of freedom:

$$NDF = \begin{Bmatrix} w_i \\ \theta_i \\ w_j \\ \theta_j \end{Bmatrix}$$

The assumed displacement function is taken as

$$w = \sum_{m=1}^{r} \left[\left[1 - \frac{3x^2}{b^2} - \frac{2x^3}{b^3}\right] \left[x - \frac{2x^2}{b} + \frac{x^3}{b^2}\right] \left[\frac{3x^2}{b^2} - \frac{2x^3}{b^3}\right] \left[\frac{x^3}{b^2} - \frac{x^2}{b}\right] \right] \begin{Bmatrix} w_i \\ \theta_i \\ w_j \\ \theta_j \end{Bmatrix} Y_m \qquad (2\text{-}14a)$$

or in symbolic matrix form as

$$\underset{(1\times 1)}{\left[w\right]} = \sum_{m=1}^{r}\left[\left[N\right]_m \left\{\delta\right\}_n Y_m\right] \qquad (2\text{-}14b)$$

where the functional x variables are the classical shape functions of elementary beam theory.

Edge boundary conditions are defined as follows:

On the lines y = o and y = b:

1) $w = 0$

2) $M_y = 0$ or $D_1 \frac{\partial^2 w}{\partial x^2} + D_y \frac{\partial^2 w}{\partial y^2} = 0$ (2-15)

On the nodal line x = o:

$$w = w_i \qquad \frac{\partial w}{\partial x} = \theta_i$$

On the node line x = b:

$$w = w_j \qquad \frac{\partial w}{\partial x} = \theta_j$$

The strain curvature relationships are defined as

$$\left\{E\right\} = \begin{Bmatrix} -\chi_x \\ -\chi_y \\ 2\chi_{xy} \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w}{\partial x^2} \\ -\frac{\partial^2 w}{\partial y^2} \\ 2\frac{\partial w}{\partial x \partial y} \end{Bmatrix} \qquad (2\text{-}16)$$

which upon by differentiating the Equation 2.14 yields

$$
= \left[\begin{array}{c:c:c:c}
\frac{6}{b^2}\left(1-2\frac{x}{b}\right)Y_m & \frac{2}{b}\left(2-3\frac{x}{b}\right)Y_m & \frac{6}{b^2}\left(-1+2\frac{x}{b}\right)Y_m & \frac{2}{b}\left(-3\frac{x}{b}+1\right)Y_m \\
\hdashline
-\left(1-3\frac{x^2}{b}+2\frac{x^2}{b}\right)Y''_m & -x\left(1-2\frac{x}{b}+\frac{x^2}{b}\right)Y''_m & -\left(3\frac{x^2}{b}-2\frac{x^3}{b}\right)Y''_m & -x\left(\frac{x^2}{b}-\frac{x}{b}\right)Y''_m \\
\hdashline
\frac{2}{b}\left(-6\frac{x}{b}+6\frac{x^2}{b}\right)Y'_m & 2\left(1-4\frac{x}{b}+3\frac{x^2}{b}\right)Y'_m & \frac{2}{b}\left(6\frac{x}{b}-6\frac{x^2}{b}\right)Y'_m & 2\left(3\frac{x^2}{b}-2\frac{x}{b}\right)Y'_m
\end{array}\right]
$$

$m = 1,2,\ldots,r.$

(2-17)

2.8 Bending Moments

The moment curvature relationship for orthotropic materials is given as

$$
\{M\} = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_x & D_1 & 0 \\ D_1 & D_y & 0 \\ 0 & 0 & D_{xy} \end{bmatrix} \begin{Bmatrix} -\frac{\partial^2 w}{\partial x^2} \\ -\frac{\partial^2 w}{\partial y^2} \\ 2\frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \qquad (2\text{-}18)
$$

with

$$D_x = \frac{E_x t^3}{12(1-\nu_x \nu_y)} \qquad D_y = \frac{E_y t^3}{12(1-\nu_x \nu_y)}$$

$$D = \frac{\nu_x E_y t^3}{12(1-\nu_x \nu_y)} \qquad D = \frac{G t^3}{12}$$

In the above equations E_x, E_y, ν_x, ν_y & G are elastic constants, with t the thickness of the platestrip. For an isotropic plate $E_x = E_y = E, \nu_x = \nu_y = \nu, G = \frac{E}{2(1+\nu)}$

2.9 Bending Stiffness Matrix

Similar to the inplane stiffness matrix, using the minimum potential energy principal, the partitioned stiffness matrix element for the plate bending problem becomes

$$S_{mn} = \int [B]_m^T [D][B]_n \, d\,(\text{area}) \qquad (2\text{-}19)$$

The bending stiffness matrix is a (4x4) square symmetric matrix which is computed using Equations (2.17) and (2.19), which after proper integration is shown in Table 2 for the case of simply supported boundary conditions. The subroutine FEMS is written to compute the bending stiffness matrix for a strip element.

2.10 Consistent Load Matrix - Bending

The consistent load matrix is defined as

$$\{F\}_m = \int_0^a \int_0^b \left\{ \begin{matrix} [N_1]_m^T \\ [N_2]_m^T \end{matrix} \right\} \{q\} \, dx\, dy \qquad (2\text{-}20)$$

For a uniformly distributed loadcase, one obtains

$$\{F\}_m = q \int_0^b \left\{ \begin{matrix} [C_1]^T \\ [C_2]^T \end{matrix} \right\} dx \int_0^a Y_m dy$$

$$= q \left[\frac{b}{2} \quad \frac{b^2}{12} \quad \frac{b}{2} \quad \frac{-b^2}{12} \right]^T \int_0^a Y_m dy \qquad (2\text{-}21)$$

$$
\left[\begin{array}{c|c|c|c}
\begin{array}{l}\frac{13ab}{70}k_m^4 D_y + \frac{12a}{5b}k_m^2 D_{xy} \\ + \frac{6a}{5b}k_m^2 D_1 + \frac{6a}{b^2}D_x\end{array} & & & \\
\hline
\begin{array}{l}\frac{3a}{5}k_m^2 D_1 + \frac{a}{5}k_m^2 D_{xy} \\ + \frac{3a}{b^2}D_x + \frac{11ab^2}{420}k_m^4 D_y\end{array} & \begin{array}{l}\frac{ab}{210}k_m^4 D_y + \frac{4ab}{15}k_m^2 D_{xy} \\ + \frac{2ab}{15}k_m^2 D_1 + \frac{2a}{b}D_x\end{array} & & \text{SYMMETRICAL} \\
\hline
\begin{array}{l}\frac{9ab}{140}k_m^4 D_y - \frac{12a}{5b}k_m^2 D_{xy} \\ - \frac{6a}{5b}k_m^2 D_1 - \frac{6a}{b^3}D_x\end{array} & \begin{array}{l}\frac{13ab^2}{840}k_m^4 D_y - \frac{a}{5}k_m^2 D_{xy} \\ - \frac{a}{10}k_m^2 D_1 - \frac{3a}{b^2}D_x\end{array} & \begin{array}{l}\frac{13ab}{70}k_m^4 D_y + \frac{12a}{5b}k_m^2 D_{xy} \\ + \frac{6a}{5b}k_m^2 D_1 + \frac{6a}{b^3}D_x\end{array} & \\
\hline
\begin{array}{l}-\frac{13ab^2}{840}k_m^4 D_y + \frac{a}{5}k_m^2 D_{xy} \\ + \frac{a}{10}k_m^2 D_1 + \frac{3a}{b^2}D_x\end{array} & \begin{array}{l}-\frac{3ab^3}{840}k_m^4 D_y - \frac{ab}{15}k_m^2 D_{xy} \\ - \frac{ab}{30}k_m^2 D_1 + \frac{a}{b}D_x\end{array} & \begin{array}{l}-\frac{11ab^2}{420}k_m^4 D_y - \frac{a}{5}k_m^2 D_{xy} \\ - \frac{3a}{5}k_m^2 D_1 - \frac{3a}{b^2}D_x\end{array} & \begin{array}{l}\frac{ab^3}{210}k_m^4 D_y + \frac{4ab}{15}k_m^2 D_{xy} \\ + \frac{2ab}{15}k_m^2 D_1 + \frac{2a}{b}D_x\end{array}
\end{array}\right]
$$

$k_m = \frac{m\pi}{a}$

$m = 1, 2, \ldots, r.$

TABLE 2 BENDING STIFFNESS MATRIX OF A SIMPLY SUPPORTED STRIP

In the computer program, subroutine QS figures the consistent load matrix.

2.11 Comprehensive Stiffness Matrix

If both inplane stress and bending stress are acting together, both systems of nodal displacements occur simultaneously. The in-plane and bending stiffness matrices are combined in the following form:

$$
= \left[\begin{array}{cccc|cccc}
\overset{p}{k}_{11} & \overset{p}{k}_{12} & \overset{p}{k}_{13} & \overset{p}{k}_{14} & 0 & 0 & 0 & 0 \\
\overset{p}{k}_{21} & \overset{p}{k}_{22} & \overset{p}{k}_{23} & \overset{p}{k}_{24} & 0 & 0 & 0 & 0 \\
\overset{p}{k}_{31} & \overset{p}{k}_{32} & \overset{p}{k}_{33} & \overset{p}{k}_{34} & 0 & 0 & 0 & 0 \\
\overset{p}{k}_{41} & \overset{p}{k}_{42} & \overset{p}{k}_{43} & \overset{p}{k}_{44} & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & \overset{b}{k}_{55} & \overset{b}{k}_{56} & \overset{b}{k}_{57} & \overset{b}{k}_{58} \\
0 & 0 & 0 & 0 & \overset{b}{k}_{65} & \overset{b}{k}_{66} & \overset{b}{k}_{67} & \overset{b}{k}_{68} \\
0 & 0 & 0 & 0 & \overset{b}{k}_{75} & \overset{b}{k}_{76} & \overset{b}{k}_{77} & \overset{b}{k}_{78} \\
0 & 0 & 0 & 0 & \overset{b}{k}_{85} & \overset{b}{k}_{86} & \overset{b}{k}_{87} & \overset{b}{k}_{88}
\end{array} \right]
\left\{ \begin{array}{c} u_1 \\ v_1 \\ u_2 \\ v_2 \\ w_1 \\ \theta_1 \\ w_2 \\ \theta_1 \end{array} \right\}
\tag{2-22}
$$

The form of Equation (2-22) yields an ordering of the displacement components as

$$\begin{bmatrix} u_1 & v_1 & u_2 & v_2 & w_1 & \theta_1 & w_2 & \theta_2 \end{bmatrix}^T$$

For efficiency purposes, the displacement parameters for each nodal line are grouped together in the following form:

$$\left[\, \underline{u_1 \;\; v_1 \;\; w_1 \;\; \theta_1} \;\middle|\; \underline{u_2 \;\; v_2 \;\; w_2 \;\; \theta_2} \,\right]^T$$

Thus, the comprehensive stiffness matrix must be modified to be compatible with the new ordering of the components of the displacement vector as

$$\begin{Bmatrix} U_1 \\ V_1 \\ W_1 \\ M_1 \\ U_2 \\ V_2 \\ W_2 \\ M_2 \end{Bmatrix} = \left[\begin{array}{cc|cc|cc|cc}
k^p_{11} & k^p_{12} & 0 & 0 & k^p_{13} & k^p_{14} & 0 & 0 \\
k^p_{21} & k^p_{22} & 0 & 0 & k^p_{23} & k^p_{24} & 0 & 0 \\
\hline
0 & 0 & k^b_{55} & k^b_{56} & 0 & 0 & k^b_{57} & k^b_{58} \\
0 & 0 & k^b_{65} & k^b_{66} & 0 & 0 & k^b_{67} & k^b_{68} \\
\hline
k^p_{31} & k^p_{32} & 0 & 0 & k^p_{33} & k^p_{34} & 0 & 0 \\
k^p_{41} & k^p_{42} & 0 & 0 & k^p_{43} & k^p_{44} & 0 & 0 \\
\hline
0 & 0 & k^b_{75} & k^b_{76} & 0 & 0 & k^b_{77} & k^b_{78} \\
0 & 0 & k^b_{85} & k^b_{86} & 0 & 0 & k^b_{87} & k^b_{88}
\end{array}\right] \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ \theta_1 \\ u_2 \\ v_2 \\ w_2 \\ \theta_2 \end{Bmatrix}$$

Hence, in the modified comprehensive stiffness matrix, each (4x4) matrix is made up of an appropriate (2x2) bending matrix and a (2x2) inplane matrix in the form

$$\underset{(4\times4)}{\left[S_{mn}\right]_{ij}} = \left[\begin{array}{c|c} \underset{(2\times2)}{[\text{INPLANE}]} & \underset{(2\times2)}{[0]} \\ \hline \underset{(2\times2)}{[0]} & \underset{(2\times2)}{[\text{BENDING}]} \end{array}\right] \qquad (2\text{-}24)$$

In the computer program, subroutine ASSEMBLE is programmed to reassemble the stiffness matrix in the above fashion.

2.12 Transformation of Coordinates

After computing the strip stiffness, before applying the conditions of equilibrium at each nodal line, it is necessary to work in terms of some common system of coordinates.

The global coordinates are defined as x, y and z, and the local (element) coordinates x^1, y^1 and z^1. Applying the rotation transformation law on forces and displacements, it follows that (See Fig. (3)),

$$\left\{F_m\right\} = \left[R\right]\left\{f'_m\right\} \qquad (2\text{-}25a)$$

$$\left\{\delta'_m\right\} = \left[R\right]^T\left\{\delta_m\right\} \qquad (2\text{-}25b)$$

where $[R]$ is the transformation matrix given as

$$\left[R\right] = \left[\begin{array}{c|c} [r] & [0] \\ \hline [0] & [r] \end{array}\right] \qquad (2\text{-}26a)$$

with

$$\left[r\right] = \begin{bmatrix} \text{Cos. } \alpha & 0 & \text{Sin. } \alpha & 0 \\ 0 & 1 & 0 & 0 \\ -\text{Sin. } \alpha & 0 & \text{Cos. } \alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2\text{-}26b)$$

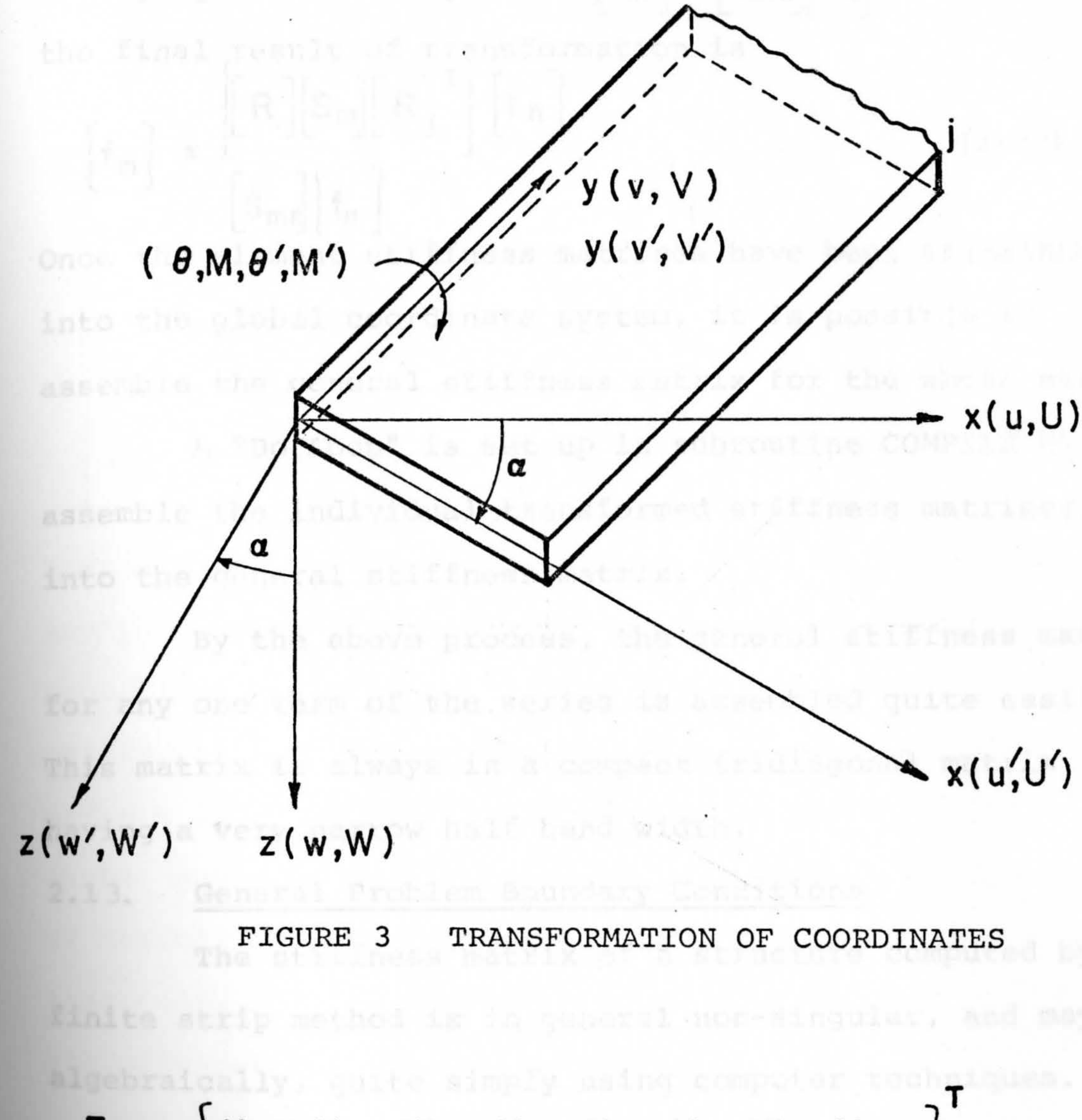

FIGURE 3 TRANSFORMATION OF COORDINATES

$$F_m = \left\{ U_{im} \; V_{im} \; W_{im} \; M_{im} \; U_{jm} \; V_{jm} \; W_{jm} \; M_{jm} \right\}^T$$

$$\delta_m = \left\{ u_{im} \; v_{im} \; w_{im} \; \theta_{im} \; u_{jm} \; v_{jm} \; w_{jm} \; \theta_{jm} \right\}^T$$

The angle α is the angle between the x^1 and x axis.

Noting again that in general $\{f_m\} = [S_{mn}]\{\theta_n\}$

the final result of transformation is

$$\{f_m\} = \frac{\left[[R][S_m][R]^T\right]\{f_n\}}{[S_{mn}]\{f_n\}} \qquad (2\text{-}27)$$

Once the element stiffness matrices have been transformed into the global coordinate system, it is possible to assemble the general stiffness matrix for the whole structure.

A "Do Loop" is set up in subroutine COMPILE to assemble the individual transformed stiffness matrices, into the general stiffness matrix.

By the above process, the general stiffness matrix for any one term of the series is assembled quite easily. This matrix is always in a compact tridiagonal matrix form having a very narrow half bend width.

2.13. General Problem Boundary Conditions

The stiffness matrix of a structure computed by the finite strip method is in general non-singular, and may be solved algebraically, quite simply using computer techniques. Boundary conditions exist along the nodal line interfaces because of node line deformation compatibility. For the case where boundary conditions are prescribed along an edge, the stiffness matrix must be modified in the following manner. Given the stiffness matrix in the form:

$$\begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ k_{31} & k_{32} & \cdots & k_{3n} \\ \vdots & \vdots & & \vdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_n \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_n \end{Bmatrix} \qquad (2\text{-}28)$$

Assume displacement d_3 has a known value β. The process of applying the boundary conditions involves only two simple operations. First, the diagonal term of matrix $[K]$ corresponding to d_3 is multiplied by a very large number, say $n = 10^6$. Secondly, the term P_3 in load vector on the right hand side of the equation is replaced by the product of the new diagonal coefficient with the prescribed value that is $P_3 = (n)\beta$. This process retains the matrix $[K]$ in its original size and arrangement.

The third equation reads

$$k_{31}d_1 + k_{32}d_2 + k_{33}10^{10}d_3 + \ldots\ldots\ldots k_{3n}d_n = k_{33}10^{10}\beta$$

dividing by $k_{33}10^{10}$

$$\frac{k_{31}}{k_{33}10^{10}}d_1 + \frac{k_{32}}{k_{33}10^{10}}d_2 + d_3 + \ldots\ldots\ldots \frac{k_{3n}}{k_{33}10^{10}}d_n = \beta$$

which yields the numerical result $d_3 = \beta$

(2-28b)

2.14 Solution of Simultaneous Equations

A node by node elimination technique is used to solve the simultaneous equations. The stiffness matrix is a banded matrix (matrix coefficients clustered around the diagonal) which reduces the computer time and the storage requirement. In the computer program subroutine SOLVE uses banded matrix solution procedures to solve the simultaneous equations. See Appendix III for interpretation of mathematical procedures.

2.15. Sample Example - Folded Plate Structure

The folded plate structure is supported at the two ends by the stiffened diaphragms as shown in Figure 4a.

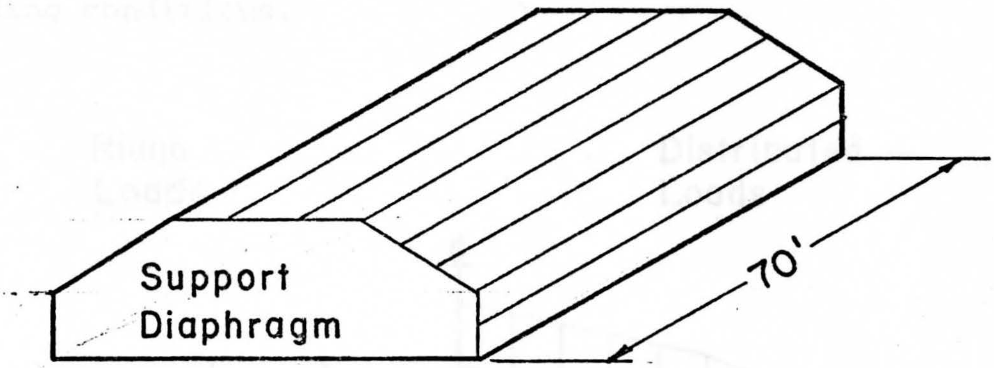

FIGURE 4a FOLDED PLATE ROOF

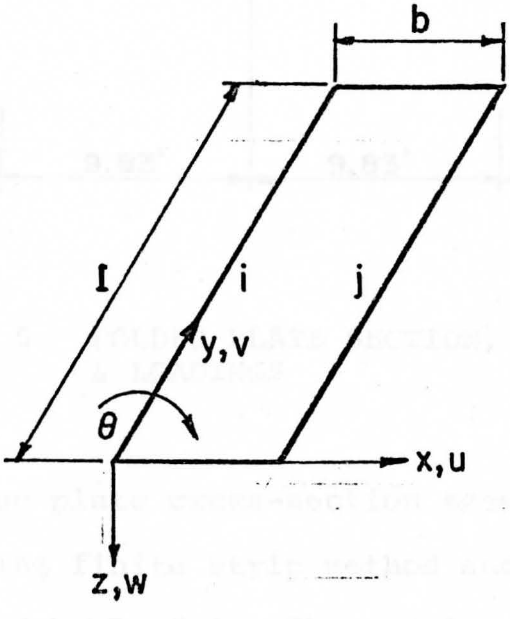

FIGURE 4b A TYPICAL STRIP

A typical strip is shown in Figure 4b where the sides are numbered i and j. The geometric properties of the strip are regarded as constant within the strip and the material is considered isotropic. Each strip is subjected to inplane forces due to plate inclination and transverse bending forces, due to external uniform loading conditions.

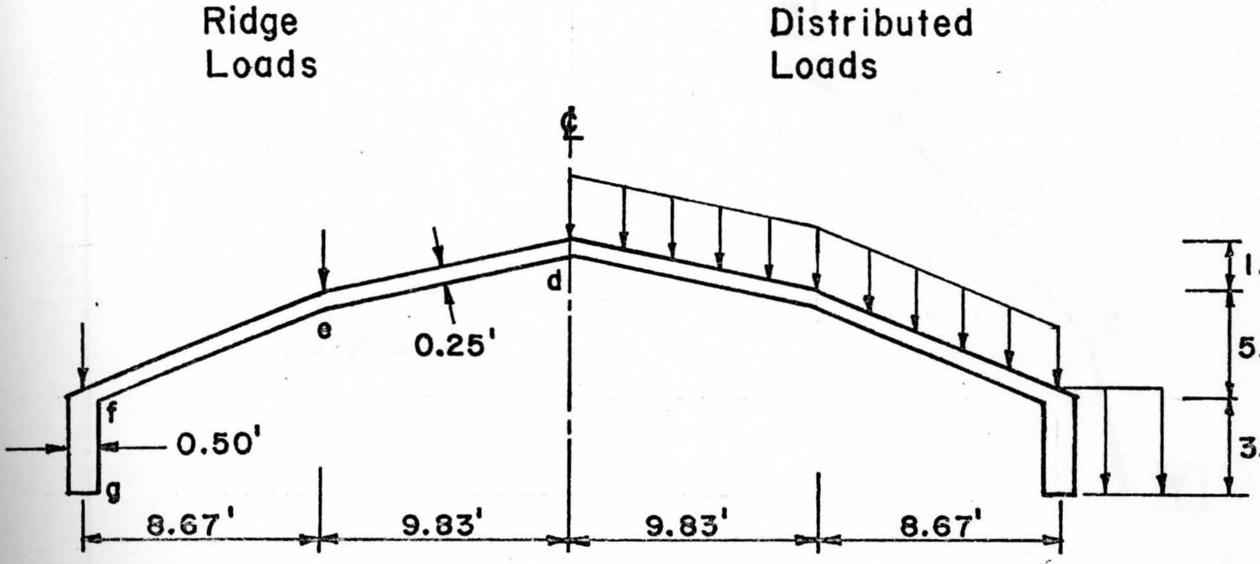

FIGURE 5 FOLDED PLATE SECTION, DIMENSIONS & LOADINGS

A typical folded plate cross-section shown in the Figure 5 is analyzed using finite strip method and compared with the results obtained by De frics-Skene & Scordelocs (17). A span length of 70' is used in order to demonstrate the applicability of the method.

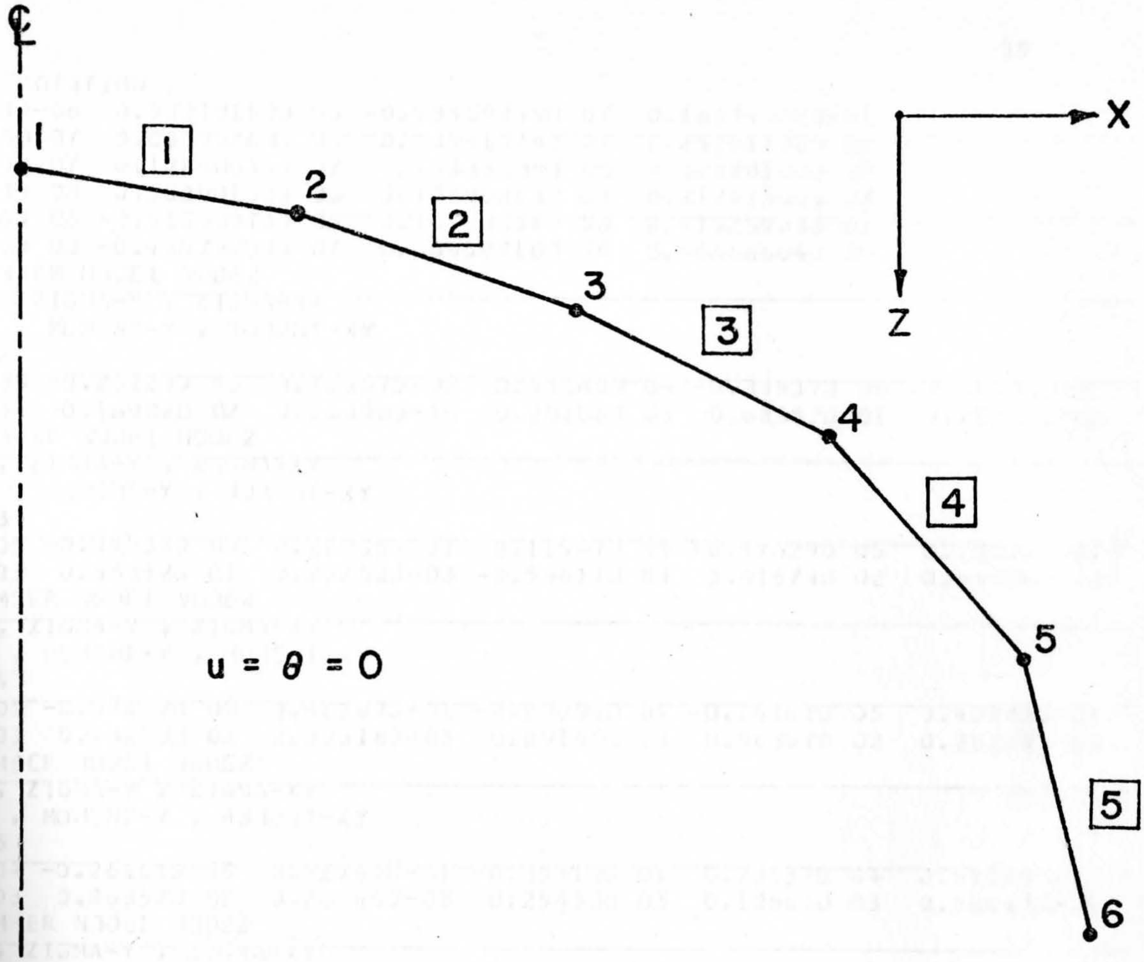

FIGURE 6. GEOMETRY OF FINITE STRIPS

The strips and nodes are numbered as shown in the Figure 6.

The difference of the node numbers between two nodes which are connected with each other determines the band width of the general matrix and therefore kept as small as possible in the numberimg scheme.

The input data for the folded plate structure of Figure 5 with the geometry shown in Figure 6 is tabulated as follows:

```
ECTION ROTATION
70553833D-06   0.58219160D 05 -0.65459849D 07   0.16574922D-04
65225930D 07   0.88099526D 06   0.23541924D 07   0.25261798D 07
03235490D 07   0.12840644D 07   0.15333286D 08   0.32878100D 07
23551230D 08   0.58290103D 05   0.30596818D 08   0.27711356D 07
55457160D 08 -0.51553973D 05   0.36950331D 08   0.57525903D 05
82504150D 08 -0.49675259D 07   0.36959710D 08   0.56466604D 06
MENT NUMBER NODE1 NODE2
IGMA-X , ZIGMA-Y , ZIGMA-XY
OMENT-X , MOMENT-Y , MOMENT-XY
  1   2
13368D 05 -0.26127D 05   0.72557D-02 -0.13368D 05 -0.39537D 05   0.46734D-02
15198D 04 -0.16902D 02   0.10396D-14   0.20200D 03   0.68382D 01   0.25752D-03
MENT NUMBER NODE1 NODE2
IGMA-X , ZIGMA-Y , ZIGMA-XY
OMENT-X , MOMENT-Y , MOMENT-XY
  2   3
11947D 05 -0.39537D 05   0.22905D-01 -0.11947D 05 -0.57629D 05   0.20570D-01
19882D 03   0.68382D 01   0.25752D-03 -0.59611D 03   0.41574D 02   0.33516D-03
MENT NUMBER NODE1 NODE2
IGMA-X , ZIGMA-Y , ZIGMA-XY
OMENT-X , MOMENT-Y , MOMENT-XY
  3   4
85004D 04 -0.57629D 05   0.41387D-01 -0.85004D 04 -0.26161D 05   0.40222D-01
62257D 03   0.40233D 02   0.33516D-03   0.89145D 03   0.86357D 02   0.28249D-03
MENT NUMBER NODE1 NODE2
IGMA-X , ZIGMA-Y , ZIGMA-XY
OMENT-X , MOMENT-Y , MOMENT-XY
  4   5
35713D 04 -0.26161D 05   0.51349D-01 -0.35713D 04   0.23137D 04   0.51150D-01
38839D 03   0.86357D 02   0.28249D-03   0.25433D 03   0.10565D 03   0.58642D-04
MENT NUMBER NODE1 NODE2
IGMA-X , ZIGMA-Y , ZIGMA-XY
OMENT-X , MOMENT-Y , MOMENT-XY
  5   6
39030D 03   0.23137D 04   0.17568D-01 -0.39030D 03   0.22294D 06   0.17522D-01
74208D 02   0.34715D 03   0.46913D-03 -0.64545D 00   0.38292D 03   0.46049D-03
```

TABLE 3a. NODAL COORDINATES

NODE	x(m)	z(m)
1	0.000	0.000
2	4.915	0.875
3	9.830	1.750
4	14.165	4.250
5	18.500	6.750
6	18.500	9.750

TABLE 3b. STRIP PROPERTIES

RIP	LEFT-HAND NODE	RIGHT-HAND NODE	THICKNESS (m)	DISTRIBUTED LOAD (N/m²)
1	1	2	0.25	80
2	2	3	0.25	80
3	3	4	0.25	80
4	4	5	0.25	80
5	5	6	0.50	75

TABLE 3c. PRESCRIBED DISPLACEMENTS

NODE	u	v	w	θ
1	0	1	1	0

TABLE 3d. MATERIAL PROPERTIES & LENGTH OF STRIP

E_x	E_y	v_x	v_y	G	LENGTH (m)
1	1	0	0	0.5	70

TABLE 3 INPUT DATA

STRIP NUMBER	Transverse Stress σ_x	Longitudinal Stress σ_y		Transverse Stress M_x		Longitudinal Stress M_y	
		NODE 1	NODE 2	NODE 1	NODE 2	NODE 1	NODE 2
1	-1.3368	-0.2613	-0.3954	-1.5198	0.2020	-0.1690	0.0684
2	-1.1947	-0.3954	-0.5763	0.1988	-0.0596	0.0684	0.4158
3	-0.8500	-0.5763	-0.2616	-0.0623	0.0892	0.4023	0.8694
4	-0.3751	-0.2616	0.0231	0.0888	0.0254	0.8636	1.0565
5	-0.3903	0.0231	2.2300	0.0074	0.0006	3.4710	3.8292
Multiplier	10^4 K.S.F.	10^5 K.S.F.		10^3 K.S.F./F.		10^2 K.S.F./F.	

TABLE 4

MID-SPAN - STRESSES AND MOMENTS OF FOLDED PLATE ROOF

CHAPTER III

DYNAMIC ANALYSIS

3.1 Chapter Overview

This chapter gives a general review of the formulation procedures necessary to obtain solutions for problems including inertial loads. The consistent mass matrix of the strip element is formed. The equations of free vibration of determined and a modern mathematical approach of equation solution is summarized. The basic steps in the subroutine procedure are detailed and explained.

3.2 Inertial Force Concepts

As shown in Chapter II, any elastic structure subject to static loads reduces to the following matrix equation:

$$\left[K\right]\left\{\delta\right\} = \left\{P\right\} \qquad (3\text{-}1)$$

If dynamic forces are applied to the structure, it is possible to reduce the dynamic problem to a static one by applying D'Alembert's Principle of dynamic equilibrium. Thus, the inertia forces, equal to the product of mass and acceleration, are applied to the structure yielding

$$\left[K\right]\left\{\delta(t)\right\} = -\left[\left[M\right]^{c} + \left[M\right]^{d}\right]\left\{\delta(t)\right\} = -\left[M\right]\left\{\delta(t)\right\} \qquad (3\text{-}2)$$

where

$\left[M\right]^{c}$ is a diagonal matrix of concentrated point masses or line masses, and is simply equal to zero when no such concentrated point masses or line masses are acting on the structure. $\left[M^d\right]$ is a general mass matrix of the structure assembled from individual element consistent mass matrices. For free vibration, the system is vibrating in an harmonic normal mode, thus,

$$\left\{\delta(t)\right\} = \left\{\delta\right\} \text{Sin } \omega t \qquad (3\text{-}3a)$$

$$\left\{\ddot{\delta}(t)\right\} = -\omega^2 \left\{\delta\right\} \text{Sin } \omega t \qquad (3\text{-}3b)$$

where ω is the Natural Frequency of Free Vibration and $(\dot{\ })$ implies derivative with respect to time. Substituting Equation (3.3) into (3.2) yields

$$\left[\left[K\right] - \omega^2 \left[M\right]\right] \left\{\delta\right\} = 0 \qquad (3\text{-}4)$$

$$\left[K\right]\left\{\delta\right\} - \omega^2 \left[M\right]\left\{\delta\right\} = 0$$

Multiplying the Equation (3.4) by $\left[K\right]^{-1}$ assuming $\left|\left[K\right]\right| \neq 0$ gives

$$\left[K\right]^{-1}\left[K\right]\left\{\delta\right\} - \left[K\right]^{-1} \omega^2 \left[M\right]\left\{\delta\right\} = 0$$

Noting

$$\left[K\right]^{-1}\left[K\right] = \left[I\right]$$

it follows that,

$$\left[K\right]^{-1} \omega^2 \left[M\right]\left\{\delta\right\} = \left\{\delta\right\} \qquad (3\text{-}5)$$

or

$$\left[K\right]^{-1}\left[M\right]\left\{\delta\right\} = \frac{1}{\omega^2}\left\{\delta\right\}$$

This is the generalized Eigenvalue problem associated with two symmetric matrices, which may be written in the form

$$[K]^{-1}[M]\{\delta\} = \lambda\{\delta\} \tag{3-6}$$

in which

$$\lambda = \frac{1}{\omega^2} \tag{3-7}$$

3.3 General Stiffness Matrix

Using the procedure described in Chapter II, the element stiffness matrix is computed for a strip and is assembled and compiled to form the general stiffness matrix. This procedure is the exactly same as the procedure described in Chapter II.

3.4 General Mass Matrix of a Strip

Since the mass is distributed throughout the strip, any acceleration induces a distributed loading of magnitude

$$q = -\rho t \frac{\partial^2 w}{\partial t^2} \tag{3-8}$$

Noting the transverse displacement in symbolic form as matrix

$$w = [N]\{\delta\}$$

or in partitioned matrix form as

$$w = \left[[N]_1 [N]_2 \cdots\cdots [N]_r\right] \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_r \end{Bmatrix} \tag{3-9}$$

substituting Equation (3.9) into Equation (3.8) yields

$$q = -\rho t [N]\{\delta(t)\} \tag{3-10}$$

Further substituting of Equation (3.3) in Equation (3.10) gives

$$q = -\rho t \omega^2 [N]\{\delta\} \tag{3-11}$$

By the principal of virtual work, we can obtain equivalent nodal forces for such distributed loads from the relationship

$$\left\{ F \right\} = \int_0^a \int_0^b \left[N \right]^T q \, dx \, dy \qquad (3\text{-}12a)$$

Substituting Equations (3.9) and (3.11) into the latter equation gives

$$\left\{ F \right\} = -\int_0^a \int_0^b \rho t \omega^2 \left[N \right]^T \left[N \right] \left\{ \delta \right\} dx \, dy$$

or in partitioned matrix form

$$\int_0^b \rho t \omega^2 \begin{bmatrix} [N]_1^T [N]_1 & [N]_1^T [N]_2 & \cdots & [N]_1^T [N]_r \\ [N]_2^T [N]_1 & [N]_2^T [N]_2 & \cdots & [N]_2^T [N]_r \\ \vdots & \vdots & & \vdots \\ [N]_r^T [N]_1 & [N]_r^T [N]_2 & \cdots & [N]_r^T [N]_r \end{bmatrix} \left\{ \delta \right\} dx \, dy$$

which is written in the compact matrix form (3-12b)

$$= \omega^2 \left[M \right]^e \left\{ \delta \right\} \qquad (3\text{-}12c)$$

where Matrix $\left[M \right]^e$ is defined as the generalized mass matrix.

3.5 Consistent Mass Matrix of a Plate Strip in Bending

Using the rectangular bending strip discussed in Chapter II, the matrix $\left[N \right]_m$ is defined as

$$= \left[\left(1 - \frac{3x^2}{b^2} + \frac{2x^3}{b^3}\right) \left(x - \frac{2x^2}{b} + \frac{x^3}{b^2}\right) \left(\frac{3x^2}{b^2} - \frac{2x^3}{b^3}\right) \left(\frac{x^3}{b^2} - \frac{x^2}{b}\right) \right] Y_m$$

or $\qquad m = 1, 2, \ldots, r$

$$= \left[C \right] Y_m \qquad (3\text{-}13a)$$

The integrals with respect to the X variable are calculated and shown in the Table 4a on Page 46. The latter equation becomes

$$\left[M\right]^{e}_{mn} = \int_0^a \begin{bmatrix} \frac{13b}{35} & & & \\ \frac{11b^2}{210} & \frac{b^2}{105} & \text{Symmetrical} & \\ \frac{9b}{70} & \frac{13b^2}{420} & \frac{13b}{35} & \\ -\frac{13b^2}{420} & -\frac{3b^2}{420} & -\frac{11b^2}{210} & \frac{b^2}{105} \end{bmatrix} \rho t(y) Y_m Y_n \, dy \qquad (3\text{-}14)$$

TABLE 4a CONSISTENT MASS MATRIX-BENDING

For the special case of simply supported strip with constant thickness, the integrals in the Y variable use the orthogonality property of the deflection functions and reduce to the standard form

$$\int_0^a \rho t Y_m Y_n \, dy = \rho t \int_0^a \sin^2 k_m y \, dy$$

$$= \frac{\rho t a}{2} \qquad \text{for} \quad m = n \qquad (3\text{-}15)$$

$$= 0 \qquad \text{for} \quad m \neq n$$

3.6 Inplane Mass Matrix of a Plate Strip

Using the rectangular inplane plate strip discussed in Chapter II, the matrix $\left[N\right]_m$ is defined as

$$[N]_m = \begin{bmatrix} \left(1-\frac{x}{b}\right)Y_m & 0 & \frac{x}{b}Y_m & 0 \\ 0 & \left(1-\frac{x}{b}\right)\frac{a}{\mu_m}Y'_m & 0 & \left(\frac{x}{b}\right)\frac{a}{\mu_m}Y'_m \end{bmatrix}$$

where $\mu_m = m\pi$ (3-16)

The consistent mass matrix of an inplane strip is

$$[M]^e_{mn} = \int \rho t [N]^T_m [N]_n \, d(\text{area})$$

The mass matrix takes the component form

$$[M]_{mn} = \rho t \begin{bmatrix} \frac{b}{3}I_1 & 0 & \frac{b}{6}I_1 & 0 \\ 0 & \left(\frac{b}{3C_1C_2}\right)I_2 & 0 & \left(\frac{b}{6C_1C_2}\right)I_2 \\ \frac{b}{6}I_1 & 0 & \frac{b}{3}I_1 & 0 \\ 0 & \left(\frac{b}{6C_1C_2}\right) & 0 & \left(\frac{b}{3C_1C_2}\right) \end{bmatrix}$$

where (3-17)

$$I_1 = \int_0^a Y_m Y_n \, dy \quad , \qquad I_2 = \int_0^a Y'_m Y'_n \, dy \quad ,$$

$$I_1 = I_2 = \frac{a}{2} \quad \text{for } m = n \qquad (3\text{-}18)$$

$$= 0 \quad \text{for } m \neq n \quad ,$$

$$C_1 = \frac{\mu_m}{a},$$

$$C_2 = \frac{\mu_n}{a}$$

3.7 Comprehensive Mass Matrix for the Plate Strip

In Chapter II, it was shown that for an inclined plate strip, both the bending and inplane systems of nodal displacements are acting simultaneously, and therefore at each nodal line four components of displacements are present. It was further shown that the components of the comprehensive stiffness matrix $\left[S\right]_{mn}$ are made-up of appropriate elements of the inplane & bending matrices.

For the vibration analysis, the comprehensive mass matrix is made-up in a similar manner as

$$\left[M_{ij}\right]_{mn} = \left[\begin{array}{c|c} \left[M^p_{ij}\right]^e_{mn} & \left[0\right] \\ \hline \left[0\right] & \left[M^b_{ij}\right]^e_{mn} \end{array}\right]$$

Thus, the comprehensive mass matrix is formed the same way as the comprehensive stiffness matrix.

After applying the coordinate transformation law from the element axes to the global coordinate axes, the transformed mass matrices of all the strips are now assembled to form general mass matrix in a similar way as the formation of the general stiffness matrix.

A 'Do-loop' is set up in the subroutine COMPILE to calculate the individual mass matrices, transform the coordinate axes, and the results are assembled together. By the above process, the general mass matrix for any one term of the series is assembled efficiently. This matrix is always a compact tridiagonalized submatrix and has a very narrow half band width.

The boundary conditions are applied to the mass matrix exactly in the same way as to the stiffness matrix.

3.8 Numerical Process of Solution

After developing the general stiffness matrix and general mass matrix, the solution procedures of Equation (3.6) can be worked out as follows:

By Equation (3.6) we have,

$$[K]^{-1}[M]\{\delta\} = \lambda\{\delta\}$$

Although both $[K]$ and $[M]$ are symmetrical, the matrix product $[K]^{-1}[M]$ is in general not symmetrical. Thus, the following transformation is adopted. Assuming $[K]$ is positive definite, it is factorized into upper and lower triangular matrices such that

$$[K] = [L][L]^T \qquad (3\text{-}19)$$

This procedure is known as the Cholesky Transformation Method.

Inverting, we have

$$[K]^{-1} = [L]^{T-1}[L]^{-1} \quad (3\text{-}20)$$

Substituting Equation (3.20) in Equation (3.6) yields

$$[L]^{T-1}[L]^{-1}[M]\{\delta\} = \lambda\{\delta\} \quad (3\text{-}21)$$

Pre-multiply by $[L]$ gives

$$[L]^{-1}[M]\{\delta\} = \lambda\,[L]^{T}\{\delta\} \quad (3\text{-}22)$$

Finally, assuming that

$$\{\delta\} = [L]^{-T}\{\delta^*\} \quad (3\text{-}23)$$

and substituting Equation (3.23) in Equation (3.22), one obtains

$$[L]^{-1}[M][L]^{-T}\{\delta^*\} = \lambda\,[L]^{-T}[L]^{-1\,T}\{\delta^*\} \quad (3\text{-}24a)$$

or

$$[H]\{\delta^*\} = \lambda\,\{\delta^*\} \quad \text{Where } H = [L]^{-1}[M][L]^{-T} \quad (3\text{-}24b)$$

The Eigenvalues of Equation (3.23b) are identical to those of Equation (3.6). The Eigenvectors are different but are related through Equation (3.23).

Subroutine FORM and subroutine POWER are utilized to solve the Eigenvalues. Subroutine FORM formulates the set up of Equation (3.24b). Initially, the stiffness and mass matrices are input into the subroutine as an (n x (n+1)) matrix $[D]$ in the form $\left[[K] \diagdown [M]\right]$ where components of $[M]$ are stored in the upper triangle and the components of $[K]$ in the lower triangle. The subroutine consists of three basic operations;

First, the Cholesky's Method calculates the Matrix $[L]$

Secondly, the $[L]^{-1}$ is calculated and stored in the lower triangle of the $[D]$ Matrix.

Thirdly, the Matrix $[H]$ is calculated and stored in the upper triangle of $[D]$ replacing the $[M]$ Matrix.

Fourthly, the output of subroutine FORM produces a redefined Matrix $[D]$ as

$$[D] = \underbrace{\begin{bmatrix} \ddots & [H] = [L]^{-1}[M][L]^{-T} \\ [L]^{-1} & \ddots \end{bmatrix}}_{[n \times (n+1)]} \qquad (3\text{-}25)$$

Finally, Matrix $[D]$ is input into Subroutine POWER which produces the Eigenvalues and Eigenvectors of the vibration problem defined by the Equation (3.4a).

3.9 Example Problem

3.9.1 Statement of the Problem

A simply supported square plate (16'-0'x16'-0") of constant thickness is solved for natural frequencies by the finite strip method, using eight equal strips for the plate.

Number of Strips = 8

Number of Nodal Lines = 9

Number of Degrees of Freedom/Nodal Line = 4

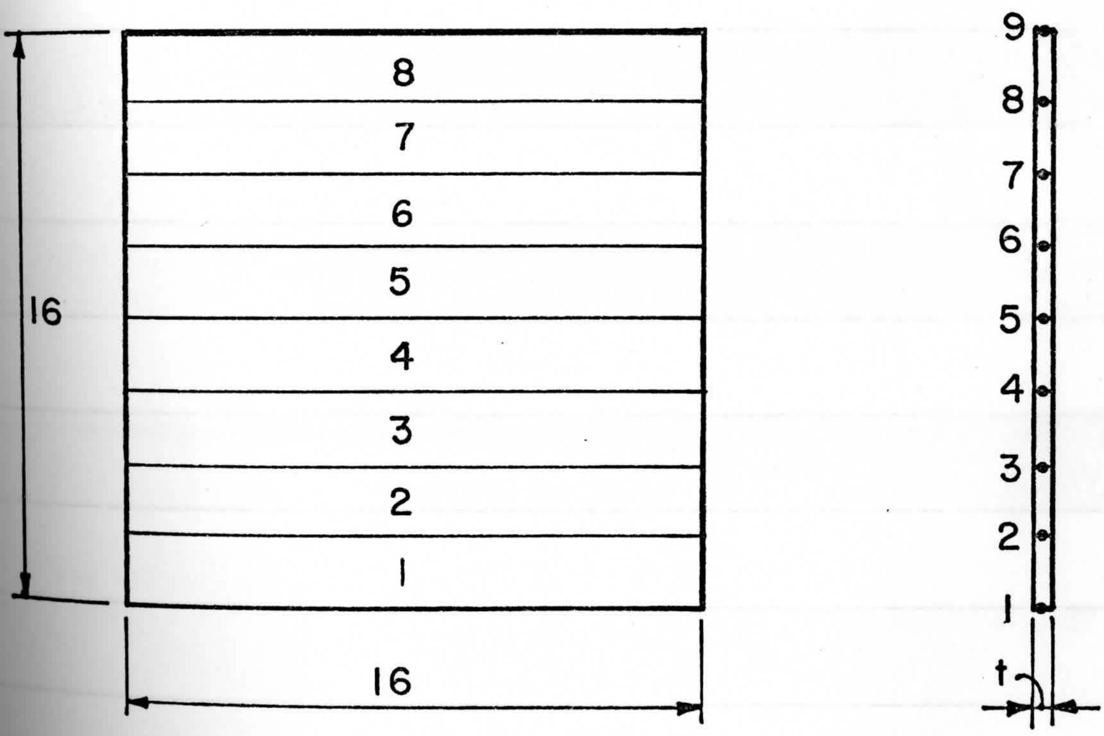

FIGURE 7

PLATE GEOMETRY

```
TE STRIP EIGEN VALUE SYSTEM

AYEN SCHOOL OF ENGR

GSTOWN STATE UNIVERSITY

GSTOWN OHIO

L NUMBER CF PROBLEMS =      3
```

ROBLEM NUMBER = 1

QUARE PLATE DYNAMIC ANALYSIS
INPUT DATA

*** CONTROL PARAMETERS ***

UMBER OF TERMS	3
UMBER OF ELEMENTS	8
UMBER OF NODAL LINES	9
UMBER OF BOUNDARY CONDITIONS	5
UMBER OF DEGREES OF FREEDOM/NODAL LINE	4
OAD TYPE(1=NON SYMM, 2=SYMM)	1
PROB PROBLEM TYPE	1
1=FREE VIBRATION,2=STABILITY, 3=VIB+IN PLANE)	
UMBER OF ITERATIONS	5
ODAL SHAPE OF MODE NUMBER	3
PRINT(1=PRINT DATA,0=NO PRINT)	0

*** PLATE GEOMETRY ***

ENGTH OF PLATE 16.0000

ODE LINE NO	XCOORD	YCOORD
1	0.0	0.0
2	2.0000	0.0
3	4.0000	0.0
4	6.0000	0.0
5	8.0000	0.0
6	10.0000	0.0
7	12.0000	0.0
8	14.0000	0.0
9	16.0000	0.0

STRIP NO	NODE1	NODE2	THICKNESS
1	1	2	2.26803000
2	2	3	2.26803000
3	3	4	2.26803000
4	4	5	2.26803000
5	5	6	2.26803000
6	6	7	2.26803000
7	7	8	2.26803000
8	8	9	2.26803000

*** BOUNDARY CONDITIONS ***

NODE LINE NO	U	V	W	THETA
1	0	0	0	1
3	0	0	1	1
5	0	0	1	1
7	0	0	1	1
9	0	0	0	1

*** MATERIAL PROPERTIES ***

MODULUS OF ELASTICITY 1	1.0000
MODULUS OF ELASTICITY 2	1.0000
POISSONS RATIO X	0.1667
POISSONS RATIO Y	0.1667
MODULUS OF ELASTICITY IN SHEAR G	0.4286
PLATE MASS DENSITY RHO	0.4409

MBER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

DAL LINE NO,	U	V	W	THETA
1	-0.00000000	0.00000000	-0.00000000	-0.51331718
2	-0.00000001	0.00000000	-1.00044969	-0.47424331
3	-0.00000000	0.00000000	-1.84859021	-0.36297025
4	-0.00000001	0.00000000	-2.41529981	-0.19643815
5	-0.00000000	0.00000000	-2.61430180	0.00000001
6	-0.00000001	0.00000000	-2.41529980	0.19643816
7	-0.00000000	0.00000000	-1.84859023	0.36297025
8	-0.00000001	0.00000000	-1.00044971	0.47424332
9	-0.00000000	0.00000000	-0.00000000	0.51331720

REQUENCY=, 0.77565075D-01 RADIANS/SEC
REQUENCY=, 0.12344877D-01 CYCLES/SEC (19.86)

MBER OF ITERATIONS 4

MODAL SHAPE OF MODE NUMBER 2

DAL LINE NO,	U	V	W	THETA
1	0.00000000	-0.00000000	-0.00000000	-0.00345356
2	0.00000000	-0.00000000	-0.00621852	-0.00244203
3	0.00000000	-0.00000000	-0.00879431	0.00000000
4	0.00000000	-0.00000000	-0.00621851	0.00244203
5	0.00000000	-0.00000000	0.00000000	0.00345356
6	0.00000000	-0.00000000	0.00621851	0.00244203
7	0.00000000	-0.00000000	0.00879431	0.00000000
8	0.00000000	-0.00000000	0.00621852	-0.00244203
9	0.00000000	-0.00000000	0.00000000	-0.00345356

REQUENCY=, 0.19353310D 00 RADIANS/SEC
REQUENCY=, 0.30801780D-01 CYCLES/SEC (49.54)

UMBER OF ITERATIONS 3

MODAL SHAPE OF MODE NUMBER 3

ODAL LINE NO,	U	V	W	THETA
1	-0.00000000	0.00000000	-0.00000000	-0.00006570
2	-0.00000000	0.00000000	-0.00010303	-0.00002513
3	-0.00000000	0.00000000	-0.00007883	0.00004646
4	-0.00000000	0.00000000	0.00004270	0.00006057
5	-0.00000000	0.00000000	0.00011147	-0.00000003
6	-0.00000000	0.00000000	0.00004261	-0.00006067
7	-0.00000000	0.00000000	-0.00007883	-0.00004640
8	-0.00000000	0.00000000	-0.00010295	0.00002515
9	-0.00000000	0.00000000	-0.00000000	0.00006553

FREQUENCY=, 0.38676534D 00 RADIANS/SEC
FREQUENCY=, 0.61555675D-01 CYCLES/SEC (99.01)

MODAL SHAPE OF MODE NUMBER 1

NODAL LINE NO,	U	V	W	THETA
1	-0.00000000	0.00000000	-0.00000000	-0.15402630
2	-0.00000001	0.00000001	-0.30019499	-0.14230185
3	-0.00000000	0.00000000	-0.55468831	-0.10891332
4	-0.00000001	0.00000001	-0.72473571	-0.05894368
5	-0.00000000	0.00000000	-0.78444886	-0.00000030
6	-0.00000001	0.00000001	-0.72473677	0.05894322
7	-0.00000000	0.00000000	-0.55468990	0.10891325
8	-0.00000001	0.00000001	-0.30019618	0.14230228
9	-0.00000000	0.00000000	-0.00000000	0.15402699

FREQUENCY=, 0.19350083D 00 RADIANS/SEC
FREQUENCY=, 0.30796643D-01 CYCLES/SEC (49.54)

NUMBER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

NODAL LINE NO,	U	V	W	THETA
1	0.00000000	-0.00000000	0.00000000	-0.00065309
2	0.00000340	-0.00000004	-0.00104216	-0.00026507
3	0.00000000	-0.00000000	-0.00083098	0.00045557
4	0.00000340	-0.00000004	0.00040671	0.00063980
5	0.00000000	-0.00000000	0.00117488	0.00003335
6	0.00000340	-0.00000004	0.00049177	-0.00063960
7	0.00000000	-0.00000000	-0.00083054	-0.00052232
8	0.00000340	-0.00000004	-0.00112697	0.00026437
9	0.00000000	-0.00000000	0.00000000	0.00072436

FREQUENCY=, 0.50415453D 00 RADIANS/SEC
FREQUENCY=, 0.80238763D-01 CYCLES/SEC (129.06)

NUMBER OF ITERATIONS 4

MODAL SHAPE OF MODE NUMBER 3

NODAL LINE NO,	U	V	W	THETA
1	0.00000000	0.00000000	-0.00000000	-0.00004694
2	0.00000000	0.00000000	-0.00008451	-0.00003319
3	0.00000000	0.00000000	-0.00011951	0.00000001
4	0.00000000	0.00000000	-0.00008450	0.00003319
5	0.00000000	0.00000000	-0.00000000	0.00004693
6	0.00000000	0.00000000	0.00008450	0.00003319
7	0.00000000	0.00000000	0.00011951	0.00000001
8	0.00000000	0.00000000	0.00008451	-0.00003319
9	0.00000000	0.00000000	-0.00000000	-0.00004694

FREQUENCY=, 0.31027959D 00 RADIANS/SEC
FREQUENCY=, 0.49382578D-01 CYCLES/SEC (79.43)

MBER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

DAL LINE NO,	U	V	W	THETA
1	-0.00000000	-0.00000000	-0.00000000	-0.13373417
2	-0.00003241	-0.00022809	-0.26055972	-0.12346884
3	-0.00000000	-0.00000000	-0.48132666	-0.09450936
4	-0.00003241	-0.00022809	-0.62888890	-0.05115065
5	-0.00000000	-0.00000000	-0.68051322	0.00021299
6	-0.00003241	-0.00022809	-0.62792373	0.05165207
7	-0.00000000	-0.00000000	-0.47944308	0.09478198
8	-0.00003241	-0.00022809	-0.25878989	0.12296167
9	-0.00000000	-0.00000000	-0.00000000	0.13259154

EQUENCY=, 0.38872506D 00 RADIANS/SEC
EQUENCY=, 0.61867575D-01 CYCLES/SEC (99.51)

MBER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

DAL LINE NO,	U	V	W	THETA
1	0.00000000	0.00000000	0.00000000	0.03640504
2	0.00005171	0.00007328	0.06781126	0.02894494
3	0.00000000	0.00000000	0.10656316	0.00777577
4	0.00005171	0.00007328	0.09299619	-0.02195919
5	0.00000000	0.00000000	0.02182677	-0.04672330
6	0.00005171	0.00007328	-0.07674705	-0.04612562
7	0.00000000	0.00000000	-0.13363124	-0.01065511
8	0.00005171	0.00007328	-0.10925433	0.03913611
9	0.00000000	0.00000000	0.00000000	0.06280139

REQUENCY=, 0.51891994D 00 RADIANS/SEC
REQUENCY=, 0.82588753D-01 CYCLES/SEC (135.84)

MBER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

DAL LINE NO,	U	V	W	THETA
1	-0.00000000	-0.00000000	-0.00000000	0.01669939
2	-0.00000455	-0.00000453	0.02747134	0.00818501
3	-0.00000000	-0.00000000	0.02739859	-0.00801453
4	-0.00000455	-0.00000453	0.00222538	-0.01447080
5	-0.00000000	-0.00000000	-0.01968606	-0.00545116
6	-0.00000455	-0.00000453	-0.01717190	0.00707170
7	-0.00000000	-0.00000000	0.00075735	0.00345559
8	-0.00000455	-0.00000453	0.00919574	-0.00078567
9	-0.00000000	-0.00000000	-0.00000000	-0.00667939

REQUENCY=, 0.66572671D 00 RADIANS/SEC
REQUENCY=, 0.10595380D 00 CYCLES/SEC (170.43)

3.9.3 Comparison of Results

The results of the lowest three modes are summarized in Table 5. The frequencies compare well with the exact value for most of the cases. The results are very accurate considering the fact that only one term in the series is used in the displacement function.

n \ m	1	2	3
1	19.86 (19.73)	49.54 (49.35)	99.51 (98.70)
2	49.54 (49.35)	79.43 (78.96)	132.30 (128.30)
3	99.01 (98.70)	129.06 (128.30)	170.43 (177.65)

TABLE 5. FREQUENCIES FOR A SQUARE PLATE

The above frequencies are compared (See Table 6) with the exact classical solutions (Timoshenko), previous FSM solution, FEM solutions, and experimental solutions (Warburton).

	PROGRAM VALUES	EXACT SOLUTION	Y.K. CHEUNG (REF 4)	FINITE ELEMENT METHOD	WARBURTON (REF 12)
m = 1 n = 1	19.86	19.73	19.74	18.11	19.74
m = 2 n = 1	49.54	49.35	49.35	44.60	49.35
m = 1 n = 3	99.01	98.70	98.64	86.11	98.69
m = 2 n = 2	79.43	78.96	78.95	——	78.95
m = 2 n = 3	129.06	128.30	——	——	——

TABLE 6

COMPARISON OF PLATE FREQUENCIES WITH OTHER METHODS

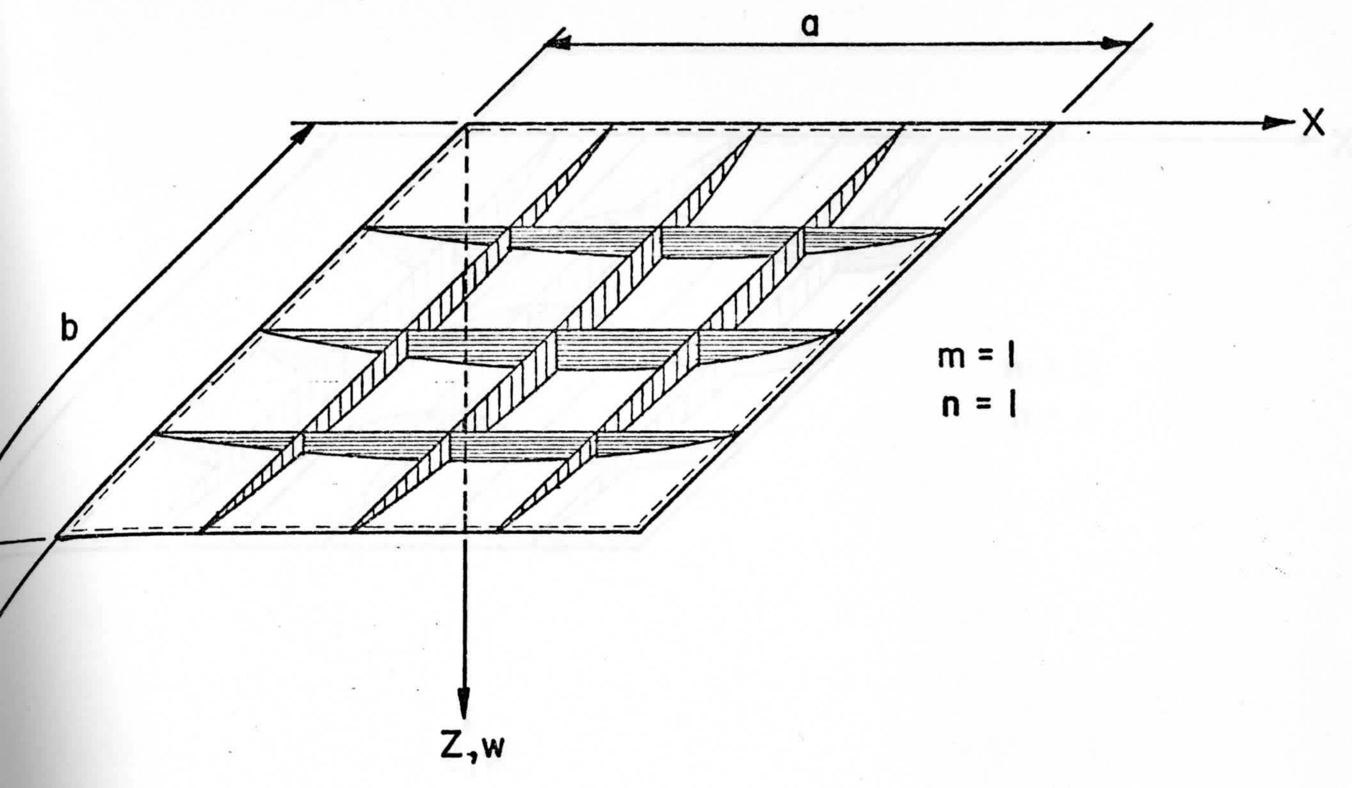

FIGURE 8

FIRST MODE OF TRANSVERSE VIBRATION

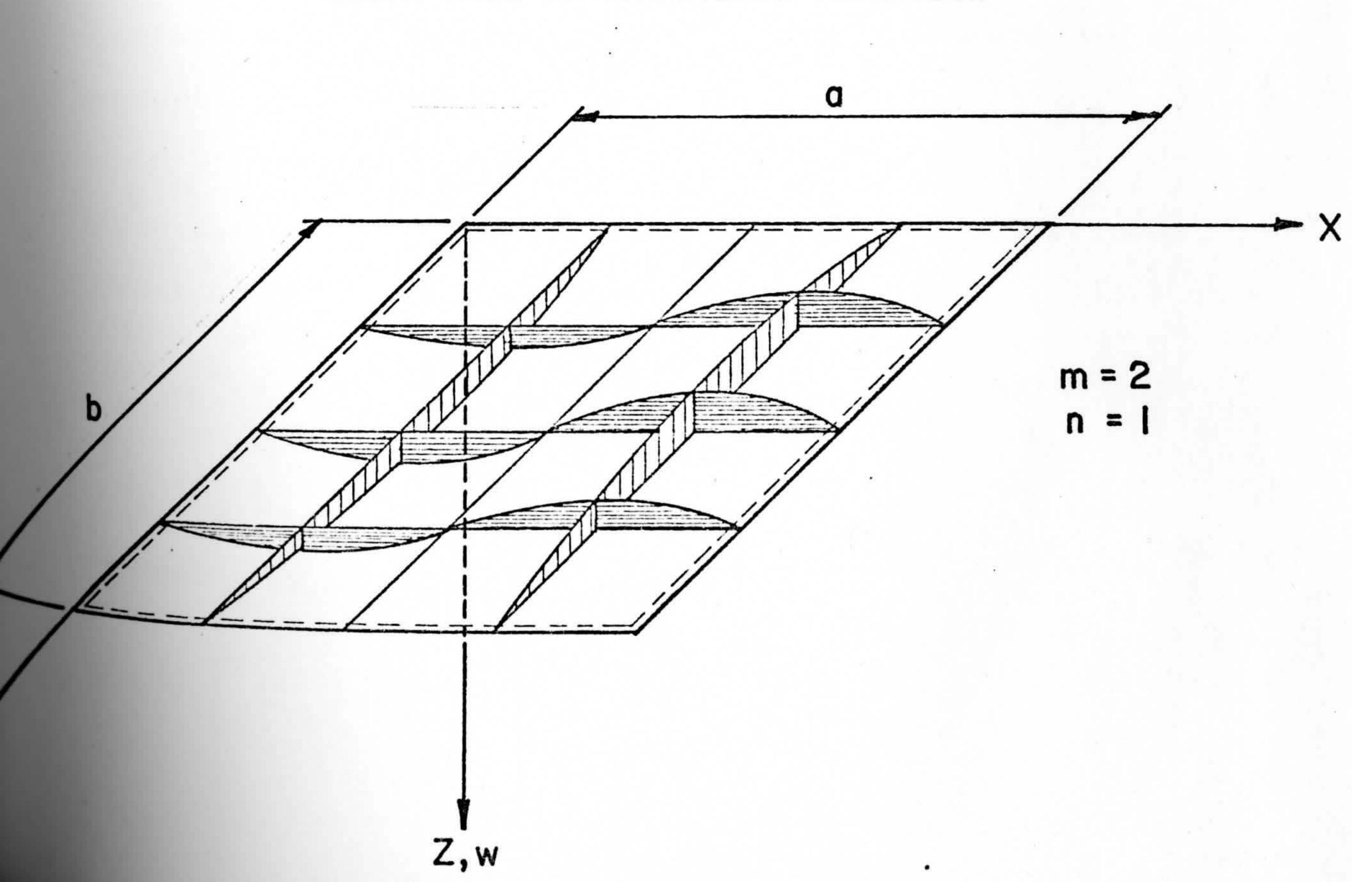

FIGURE 9

SECOND MODE OF TRANSVERSE VIBRATION

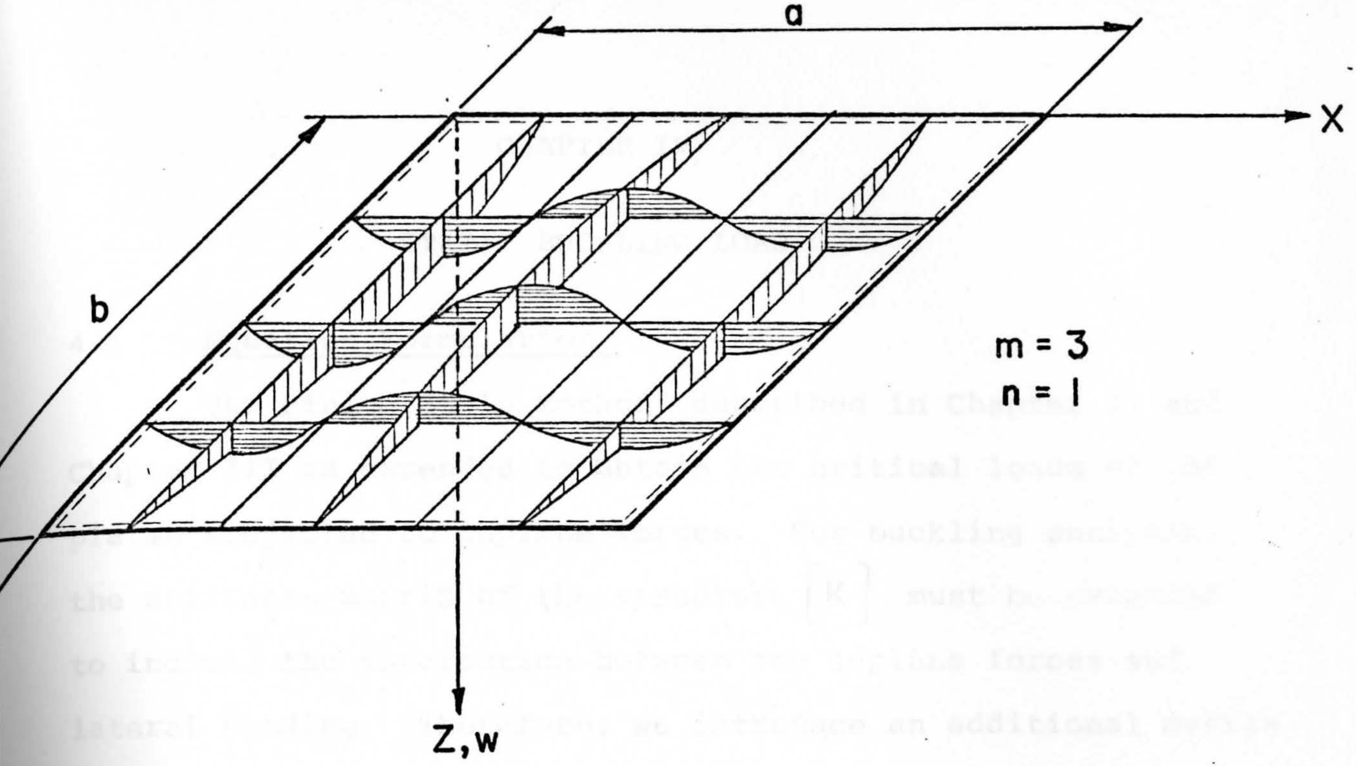

FIGURE 10

THIRD MODE OF TRANSVERSE VIBRATION

CHAPTER IV

STATIC BUCKLING LOAD

4.1 Equation Formulation

The Finite Strip Method, described in Chapter II and Chapter III is extended to obtain the critical loads of thin plates subjected to inplane forces. For buckling analysis, the stiffness matrix of the structure $\left[K\right]$ must be extended to include the interaction between the inplane forces and lateral bending. Therefore, we introduce an additional matrix which is called the Geometric Stiffness Matrix. Since the Matrix $\left[K_G\right]$ is a function of the initial inplane loads, it is also called Initial Stress Matrix. For linear stability, the applied nodal forces are zero and therefore we obtain the homogeneous equation

$$\left[\left[K_E\right] + \lambda \left[K_G\right]\right]\left\{\delta\right\} = \left\{0\right\} \qquad (4\text{-}1)$$

Since the initial compressional stress matrix has a reducing effect on the bending stiffness of the plate, it follows that,

$$\left[\left[K_E\right] - \lambda \left[K_G\right]\right]\left\{\delta\right\} = \left\{0\right\} \qquad (4\text{-}2)$$

or,

$$\left[K_E\right]\left\{\delta\right\} = \lambda \left[K_G\right]\left\{\delta\right\}$$

Pre-multiply by $\left[K_E\right]^{-1}$, gives (4-3)

$$\lambda \left[K_E\right]^{-1}\left[K_G\right]\left\{\delta\right\} = \left[K_E\right]^{-1}\left[K_E\right]\left\{\delta\right\}$$

$$\lambda[K_E]^{-1}[K_G]\{\delta\} = \{\delta\}$$

or finally,

$$[K_E]^{-1}[K_G]\{\delta\} = \frac{1}{\lambda}\{\delta\} \qquad (4\text{-}4)$$

which is a standard form of the Eigenvalue - Eigenvector problem.

To utilize Equation (4.4), one must develop the components of the Geometric Stiffness Matrix. The form of the Elastic Stiffness Matrix is identical to that formed in Chapter II.

4.2 Geometric Stiffness Matrix

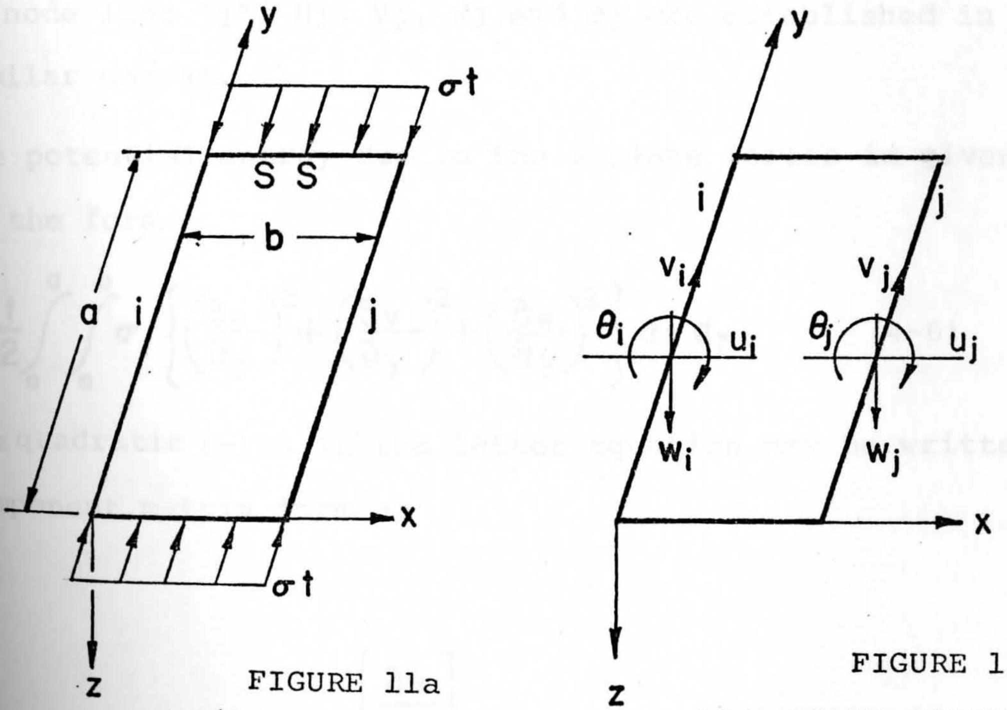

FIGURE 11a
TYPICAL BUCKLING ELEMENT

FIGURE 11b
ASSOCIATED BOUNDARY FORCES

Referring to Figures 11a and 11b, the following displacement functions are assumed:

$$u = \sum_{m=1}^{r}\left[\left(1 - \frac{x}{b}\right) \quad \left(\frac{x}{b}\right)\right]\begin{Bmatrix} u_i \\ u_j \end{Bmatrix} Y_m(y) \qquad (4\text{-}5)$$

$$V = \sum_{m=1}^{r} \left[\left(1 - \frac{x}{b}\right) \quad \left(\frac{x}{b}\right) \right] \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} \frac{a}{\mu_m} Y'_m \qquad \mu_m = m\pi$$

$$w = \sum_{m=1}^{r} \left[\left(1 - \frac{3x^2}{b^2} + \frac{2x^3}{b^3}\right) \quad x\left(1 - \frac{2x}{b} + \frac{x^2}{b^2}\right) \quad \left(\frac{3x^2}{b^2} - \frac{2x^3}{b^3}\right) \quad x\left(\frac{x^2}{b^2} - \frac{x}{b}\right) \right] \begin{Bmatrix} w_i \\ \theta_i \\ w_j \\ \theta_j \end{Bmatrix} Y_m$$

It should be noted that these functions are identical in form to those used in forming the Elastic Stiffness Matrix.

Referring to Figure (11b),

On node line "i", U_i, V_i, W_i and θ_i must be prescribed, either zero, arbitrary, or a definite non-zero number.

On node line "j", Uj, Vj, Wj and θj are established in a similar manner.

The potential energy due to the inplane forces is given in the form

$$\frac{t}{2} \int_0^a \int_0^b \sigma_1 \left\{ \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 \right\} dx\, dy \qquad (4\text{-}6)$$

The quadratic terms in the latter equation may be written in component matrix form as

$$\left[\frac{\partial u}{\partial y} \quad \frac{\partial v}{\partial y} \quad \frac{\partial w}{\partial y} \right] \begin{bmatrix} \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial y} \end{bmatrix} \qquad (4\text{-}7)$$

Noting Equation (4.5), the slopes in the previous equation are related to the nodal displacement parameters of the strip. Assuming

$$\begin{bmatrix} \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial y} \end{bmatrix} = [G]\{\delta\} \qquad (4\text{-}8)$$

the additional potential energy of the whole strip becomes

$$\frac{1}{2}\{\delta\}^T [K_G]\{\delta\} \qquad (4\text{-}9)$$

where

$$[K_G] = t\int_0^a\int_0^b \sigma_1 [G]^T [G] \, dx \, dy \qquad (4\text{-}10)$$

The total potential energy of the strip is the sum of strain energy due to bending, the potential energy due to nodal line forces, and the additional potential energy due to the initial stress. Upon minimization of this potential energy function, one obtains

$$[K_E]\{\delta\} + [K_G]\{\delta\} = \{p\} \qquad (4\text{-}11)$$

The form of the Matrix $[G]$ becomes

$$[G] = \begin{bmatrix} \left(1-\frac{x}{b}\right)Y'_m & 0 & \left(\frac{x}{b}\right)Y'_m & 0 & 0 & 0 & 0 & 0 \\ 0 & \left(1-\frac{x}{b}\right)Y'_m & 0 & \left(\frac{x}{b}\right)Y'_m & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \left(1-\frac{3x^2}{b^2}-\frac{2x^3}{b^3}\right)Y'_m & x\left(1-\frac{2x}{b}-\frac{x^2}{b^2}\right)Y'_m & \left(\frac{3x^2}{b^2}-\frac{2x^3}{b^3}\right)Y'_m & x\left(\frac{x^2}{b^2}-\frac{x}{b}\right)Y'_m \end{bmatrix}$$

(4-12)

Thus, the matrix product $\left[G\right]^T\left[G\right]$ produces an (8x8) matrix. Integrating over the surface area of the plate as defined by Equation (4.10), and assuming simply-supported boundary conditions on edges y = 0 and y = b, yields the Geometric Stiffness Matrix as

$$
= \sigma t \begin{bmatrix}
\frac{b}{3} & & & & & & & \\
0 & \frac{b}{3} & & & & & & \\
\frac{b}{6} & 0 & \frac{b}{3} & & & \text{SYMMETRICAL} & & \\
0 & \frac{b}{6} & 0 & \frac{b}{3} & & & & \\
0 & 0 & 0 & 0 & \frac{13b}{35} & & & \\
0 & 0 & 0 & 0 & \frac{11b^2}{210} & \frac{b^3}{105} & & \\
0 & 0 & 0 & 0 & \frac{9b}{70} & \frac{13b^2}{420} & \frac{13b}{35} & \\
0 & 0 & 0 & 0 & -\frac{13b^2}{420} & -\frac{3b^3}{420} & -\frac{11b^2}{210} & \frac{b^3}{105}
\end{bmatrix} \frac{m\pi^2}{b^2}
$$

$m = 1, 2, \ldots, r$

(4-13)

Components of the displacement vector are

$$\left\{\delta\right\}^T = \left[u_1 \ v_1 \ u_2 \ v_2 \ w_1 \ \theta_1 \ w_2 \ \theta_2\right]^T$$

where the subscripts refer to the node line number. Since it is more efficient to have nodal line parameters for each nodal line grouped together in the form

$$\left[u_1 \ v_1 \ w_1 \ \theta_1 \ \vdots \ u_2 \ v_2 \ w_2 \ \theta_2\right] ,$$

the comprehensive stiffness matrix is modified to be compatible with the new displacement vector, as

$$\left[\begin{array}{cc|cc|cc|cc}
\frac{b}{3} & 0 & 0 & 0 & \frac{b}{6} & 0 & 0 & 0 \\
0 & \frac{b}{3} & 0 & 0 & 0 & \frac{b}{6} & 0 & 0 \\
\hline
0 & 0 & \frac{13b}{35} & \frac{11b^2}{210} & 0 & 0 & \frac{9b}{420} & \frac{-13b^2}{420} \\
0 & 0 & \frac{11b^2}{210} & \frac{b^3}{105} & 0 & 0 & \frac{13b^2}{420} & \frac{-3b^3}{420} \\
\hline
\frac{b}{6} & 0 & 0 & 0 & \frac{b}{3} & 0 & 0 & 0 \\
0 & \frac{b}{6} & 0 & 0 & 0 & \frac{b}{3} & 0 & 0 \\
\hline
0 & 0 & \frac{9b}{70} & \frac{13b^2}{420} & 0 & 0 & \frac{13b}{35} & \frac{-11b^2}{210} \\
0 & 0 & \frac{-13b^2}{420} & \frac{-3b^3}{420} & 0 & 0 & \frac{-11b^2}{210} & \frac{b^3}{105}
\end{array}\right] \frac{m\pi^2}{b^2} \left\{\begin{array}{c} u_1 \\ v_1 \\ \omega_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \omega_2 \\ \theta_2 \end{array}\right\} \qquad (4\text{-}14)$$

4.3 Element Assemblage and Solution

For the case of rotated elements, the transformation law is applied to define stiffness relative to the global system. The transformed stiffness matrices of all strips are combined and assembled to form general geometric stiffness matrix exactly the same way as the general stiffness matrix is formed.

A 'Do loop' is set up in subroutine COMPILE to calculate the individual strip-element stiffness matrices. After coordinate transformation, they are assembled together. The numerical boundary condition technique is applied to the general geometric stiffness matrix exactly in the same way as with the elastic stiffness matrix (see Section (2.12)). After developing the general stiffness matrix and general geometric stiffness matrix, the solution procedures of Equation (4.4) are worked out using the same mathematical process used to solve Equation (3.5), that is,

$$\left[K\right]^{-1}\left[M\right]\left\{\delta\right\} = \lambda\left\{\delta\right\}$$

For stability analysis, the mass matrix is replaced by geometric stiffness matrix, and the numerical solution techniques are identical to each other.

Vibration analysis and stability analysis share many similar mathematical features. Both require the determination of Eigenvalues and Eigenvectors. In vibration analysis, a range of Eigenvalues (natural frequencies) are required. In the stability analysis, the buckling load is computed as the lowest Eigenvalue.

4.4.1 EXAMPLE PROBLEM

The static buckling load of a simply supported 16 x 16 square plate is determined using finite strip method.

Number of Strips = 8

Number of Nodal Lines = 9

Plate Size = 16 -0 x 16 -0

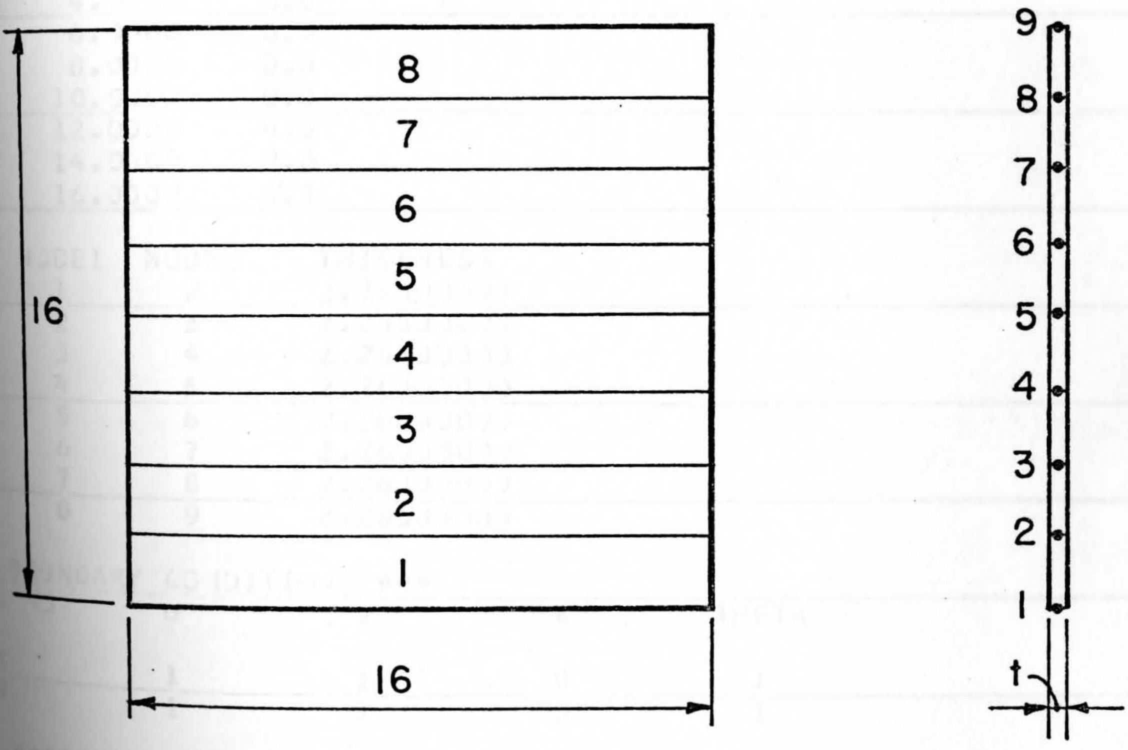

FIGURE 12

PLATE GEOMETRY

NUMBER = 2

LATE STATIC STABILITY
DATA

CONTROL PARAMETERS ***

F TERMS	3
F ELEMENTS	8
F NODAL LINES	9
F BOUNDARY CONDITIONS	2
F DEGREES OF FREEDOM/NODAL LINE	4
E(1=NON SYMM, 2=SYMM)	1
ROBLEM TYPE (VIBRATION, 2=STABILITY, 3=VIB+IN PLANE)	2
F ITERATIONS	5
HAPE OF MODE NUMBER	3
1=PRINT DATA, 0=NO PRINT)	0

PLATE GEOMETRY ***

F PLATE 16.0000

NE NO	XCOORD	YCOORD
1	0.0	0.0
2	2.0000	0.0
3	4.0000	0.0
4	6.0000	0.0
5	8.0000	0.0
6	10.0000	0.0
7	12.0000	0.0
8	14.0000	0.0
9	16.0000	0.0

O	NODE1	NODE2	THICKNESS
	1	2	2.26803000
	2	3	2.26803000
	3	4	2.26803000
	4	5	2.26803000
	5	6	2.26803000
	6	7	2.26803000
	7	8	2.26803000
	8	9	2.26803000

** BOUNDARY CONDITIONS ***

INE NO	U	V	W	THETA
1	1	1	0	1
9	1	1	0	1

** MATERIAL PROPERTIES ***

S OF ELASTICITY 1	1.0000
S OF ELASTICITY 2	1.0000
NS RATIO X	0.1667
NS RATIO Y	0.1667
S OF ELASTICITY IN SHEAR G	0.4286
MASS DENSITY RHO	0.4409

** NUMBER OF PLOT POINTS REQD ***
12

*** RESULTS OF ANALYSIS ***

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

AL LINE NO,	U	V	W	THETA
1	0.00001080	-0.00003088	0.00000000	-0.74290523
2	0.00000644	-0.00003541	-1.44791489	-0.68635575
3	0.00000149	-0.00003534	-2.67540002	-0.52531507
4	-0.00000093	-0.00003241	-3.49558198	-0.28430016
5	-0.00000105	-0.00003073	-3.78359585	-0.00000286
6	-0.00000073	-0.00003178	-3.49559214	0.28429644
7	-0.00000178	-0.00003451	-2.67541376	0.52531536
8	-0.00000472	-0.00003637	-1.44792452	0.68635952
9	-0.00000798	-0.00003504	0.00000000	0.74291066

BILITY FACTOR = , 0.15504955D 00

MBER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

DAL LINE NO,	U	V	W	THETA
1	0.01159080	0.00840142	0.00000000	0.00000001
2	0.01189768	0.00479397	0.00000001	0.00000000
3	0.01187822	0.00254208	0.00000002	0.00000000
4	0.01177965	0.00108707	0.00000003	0.00000000
5	0.01173315	0.00000008	0.00000002	-0.00000001
6	0.01177971	-0.00108696	0.00000000	-0.00000001
7	0.01187829	-0.00254209	-0.00000001	-0.00000000
8	0.01189769	-0.00479405	-0.00000000	0.00000000
9	0.01159078	-0.00840149	0.00000000	0.00000000

ABILITY FACTOR = , 0.53159813D 00

UMBER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

DAL LINE NO,	U	V	W	THETA
1	0.00000004	-0.00000034	-0.00000000	-0.00052478
2	0.00000028	-0.00000012	-0.00094491	-0.00037105
3	0.00000066	-0.00000025	-0.00133629	-0.00000002
4	0.00000066	-0.00000079	-0.00094498	0.00037102
5	0.00000022	-0.00000111	-0.00000012	0.00052478
6	-0.00000030	-0.00000094	0.00094482	0.00037108
7	-0.00000053	-0.00000046	0.00133619	-0.00000002
8	-0.00000055	-0.00000002	0.00094479	-0.00037104
9	-0.00000046	0.00000022	-0.00000000	-0.00052469

ABILITY FACTOR = , 0.97150383D 00

UMBER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

DAL LINE NO,	U	V	W	THETA
1	0.00002132	-0.00010887	0.00000000	-0.32508725
2	-0.00001236	-0.00011739	-0.63359721	-0.30034959
3	-0.00002505	-0.00010238	-1.17074967	-0.22988040
4	-0.00001369	-0.00010046	-1.52966600	-0.12441244
5	0.00000081	-0.00010372	-1.65570575	-0.00000146
6	0.00001442	-0.00010555	-1.52966937	0.12441519
7	0.00002223	-0.00011744	-1.17074659	0.22988309

8	0.00001131	-0.00013402	-0.63359893	0.30034632
9	-0.00001233	-0.00012589	0.00000000	0.32509245

TABILITY FACTOR = , 0.24279279D 00

NUMBER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

NODAL LINE NO,	U	V	W	THETA
1	0.00109368	0.00076613	-0.00000000	0.00000363
2	0.00110730	0.00021682	0.00000707	0.00000323
3	0.00102481	0.00002015	0.00001129	0.00000058
4	0.00095843	-0.00001643	0.00000897	-0.00000271
5	0.00095365	0.00000254	0.00000155	-0.00000443
6	0.00101760	0.00001066	-0.00000734	-0.00000400
7	0.00113156	-0.00005491	-0.00001237	-0.00000060
8	0.00124336	-0.00029830	-0.00000918	0.00000355
9	0.00122901	-0.00092867	-0.00000000	0.00000510

STABILITY FACTOR = , 0.74039544D 00

NUMBER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

NODAL LINE NO,	U	V	W	THETA
1	0.00020678	0.00101462	-0.00000000	-0.00005221
2	0.00031617	0.00087898	-0.00008277	-0.00002139
3	0.00035159	0.00075598	-0.00007004	0.00003188
4	0.00029768	0.00067113	0.00001640	0.00004465
5	0.00019273	0.00065999	0.00007125	0.00000331
6	0.00010743	0.00072305	0.00002793	-0.00004204
7	0.00010483	0.00082411	-0.00005930	-0.00003421
8	0.00019747	0.00087144	-0.00007719	0.00001878
9	0.00031542	0.00074939	-0.00000000	0.00004927

STABILITY FACTOR = , 0.22701110D 01

NUMBER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

NODAL LINE NO,	U	V	W	THETA
1	0.79024488	0.55701650	-0.00000000	0.00855185
2	0.77373944	0.06074265	0.01011164	-0.00167832
3	0.62937165	-0.05299025	-0.01634757	-0.02586532
4	0.51117666	-0.04620370	-0.09124347	-0.04676887
5	0.45677616	-0.00723717	-0.18925086	-0.04677359
6	0.47289210	0.03445319	-0.25995681	-0.01965614
7	0.55738971	0.05161799	-0.25537740	0.02580226
8	0.68046493	-0.03859929	-0.15886146	0.06814346
9	0.69793197	-0.47399868	-0.00000000	0.08527338

STABILITY FACTOR = , 0.63091965D 00

NUMBER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

NODAL LINE NO,	U	V	W	THETA
1	0.01158946	0.00822456	-0.00000000	0.03623054
2	0.01139023	0.00097351	0.07066294	0.03354588
3	0.00935940	-0.00069097	0.13082358	0.02584694
4	0.00767374	-0.00067704	0.17140016	0.01419531
5	0.00678753	-0.00024022	0.18508285	0.00027639
6	0.00677421	0.00026460	0.17239299	-0.01380685
7	0.00766073	0.00050817	0.13222347	-0.02584798
8	0.00910825	-0.00061337	0.07165242	-0.03393403
9	0.00927212	-0.00033679	-0.00000000	-0.03678017

TABILITY FACTOR = , 0.43310645D 00

NUMBER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

NODAL LINE NO,	U	V	W	THETA
1	0.00070828	0.00047217	-0.00000000	-0.00120839
2	0.00067491	0.00001548	-0.00217888	-0.00085921
3	0.00050050	-0.00010668	-0.00310011	-0.00001577
4	0.00032246	-0.00012277	-0.00224077	0.00082949
5	0.00016894	-0.00011422	-0.00011612	0.00118462
6	0.00003519	-0.00010262	0.00202493	0.00084584
7	-0.00008403	-0.00008181	0.00293257	0.00001582
8	-0.00017782	-0.00002641	0.00208694	-0.00081612
9	-0.00020408	0.00011777	-0.00000000	-0.00116095

STABILITY FACTOR = , 0.73777688D 00

4.4.3 The results of the lowest mode of buckling is summarized in Table 7. The buckling load compares well with the exact classical solutions (Timoshenko), FEM solutions.

	F. S. M. PROGRAM VALUES	F. E. M. PROGRAM REF (16)	EXACT SOLUTION REF (2)
$P_{CRITICAL}$	0.1560	0.1515	0.1542
% DIFFERENCE	11.7	17.5	

TABLE 7 COMPARISON OF BUCKLING LOADS

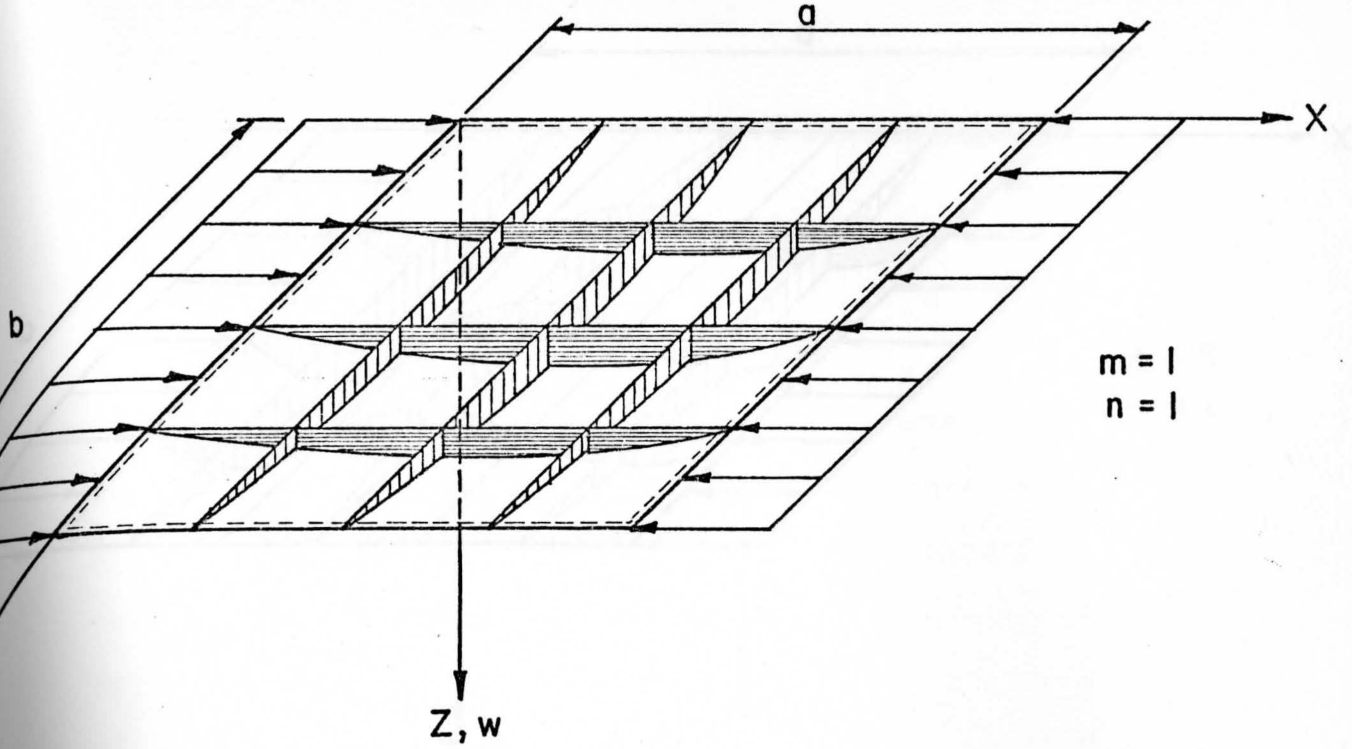

FIGURE 13

LOWEST BUCKLING MODE-SIMPLY SUPPORTED PLATE. NTERM = 1

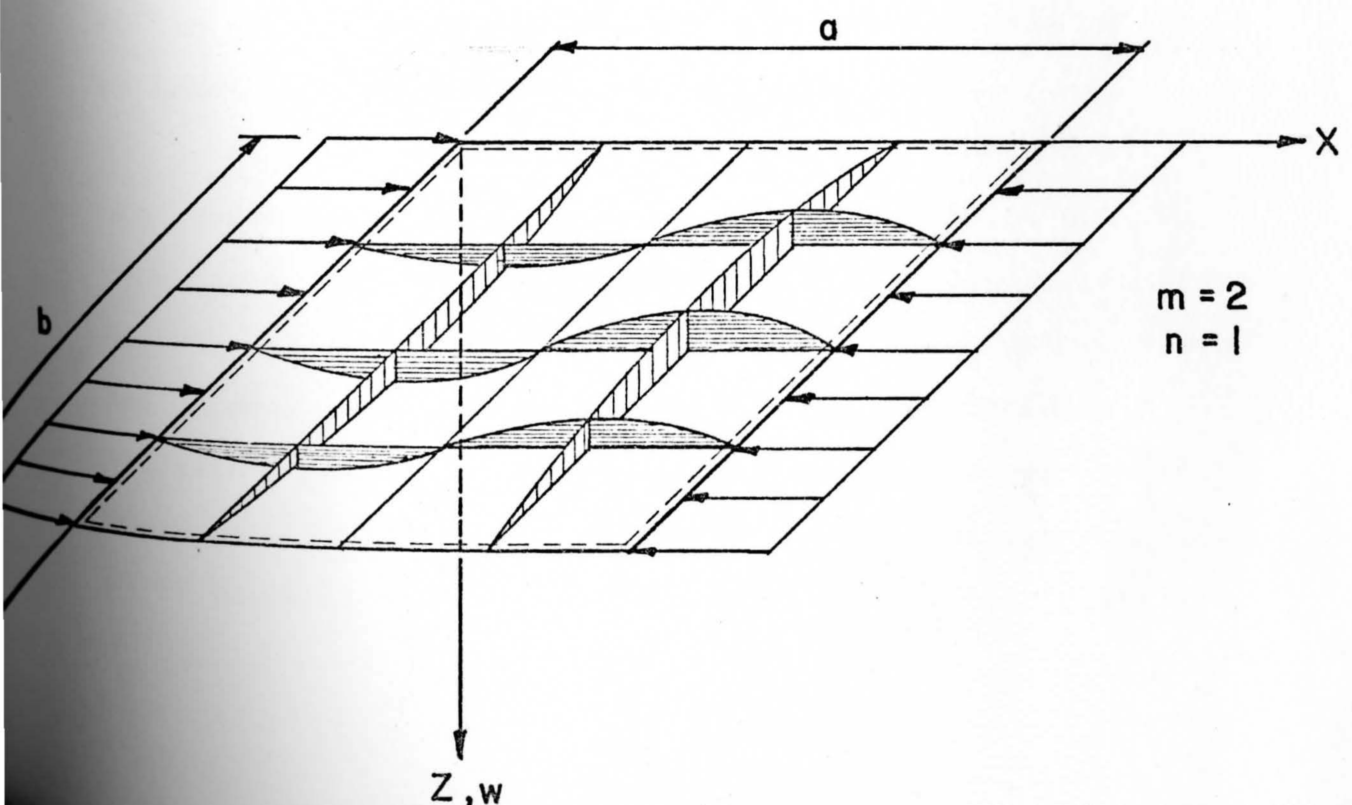

FIGURE 14

SECOND BUCKLING MODE-SIMPLY SUPPORTED PLATE. NTERM = 2

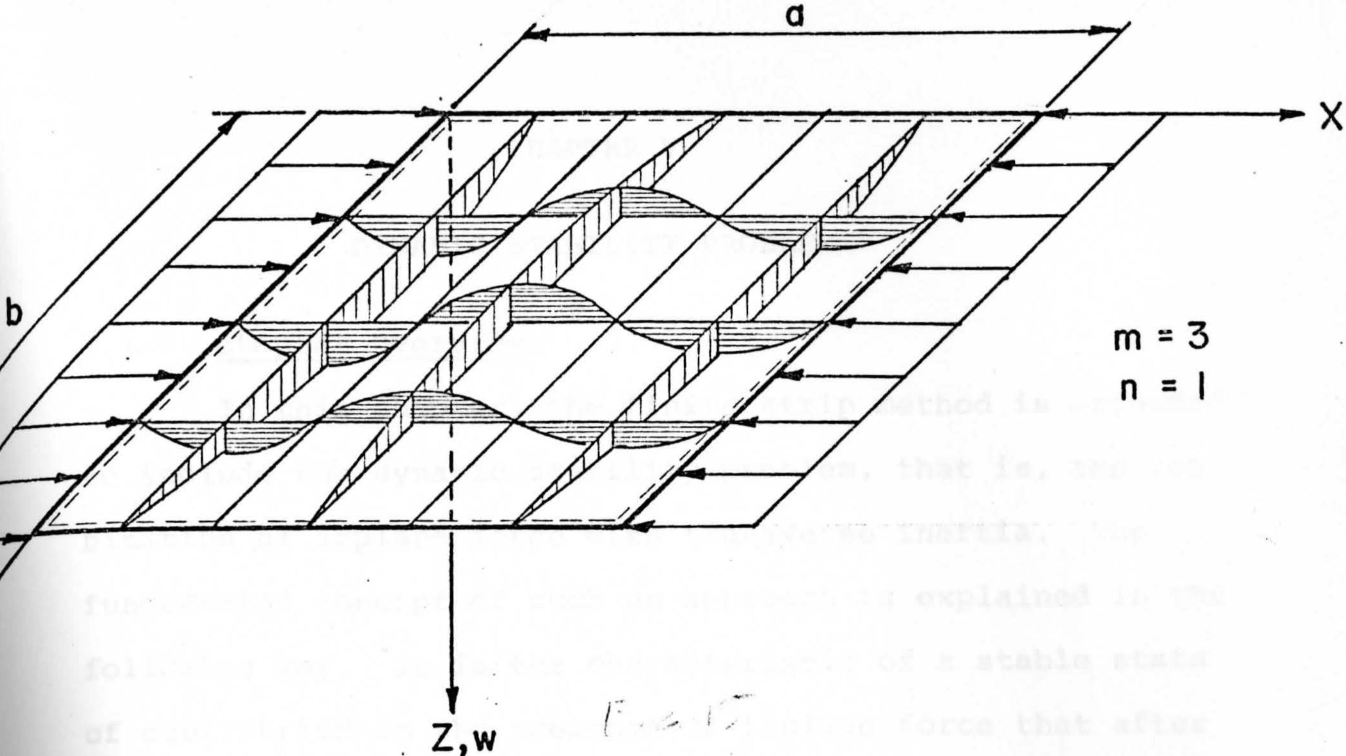

FIGURE 15

THIRD BUCKLING MODE-SIMPLY SUPPORTED PLATE. NTERM = 3

CHAPTER V

DYNAMIC STABILITY PROBLEM

5.1 Chapter Overview

In this chapter, the finite strip method is extended to include the dynamic stability problem, that is, the combination of inplane force with transverse inertia. The fundamental concept of such an approach is explained in the following way. It is the characteristic of a stable state of equilibrium in the presence of inplane force that after the introduction of small oscillations, the system returns to its original undeformed position. If the state of equilibrium is unstable, that is, the inplane force is greater than the critical value, the system does not return to its initial position, and the small disturbance is followed by increasingly large deflections.

Therefore, in setting up the matrix equation of transverse vibration, the effect of inplane force must be considered. The stiffness matrix of the structure [K] must be extended to include the connection between inplane forces and lateral bending. Consequently, one introduces an additional matrix $[K_G]$ to the elastic stiffness matrix [K]. Since matrix $[K_G]$ is a function of the initial inplane loads, it is called the initial stress matrix or geometric stiffness matrix, this terminology referring to its dependency on the geometrical properties of the element.

5.2 Equations of Motion

For large deflections, the equation of motion for a freely vibrating elastic system is obtained by replacing the elastic stiffness matrix [K] with the matrix sum, or

$$\left[[K_E] + [K_G] - \Omega^2[M]\right]\{\delta\} = \{0\} \tag{5-1}$$

The dynamic stability problem may be mathematically reduced to the solution format of either that of the free lateral vibration problem or that of the static stability problem. From Equation (5.1), it follows that, the vibratory frequencies increase with increasing inplane tensile load, and decrease with decreasing compressive inplane load (i.e. axial load is negative). Also, the influence of axial load is greater on the lower frequency than on the higher frequency. Equation (5.1) is modified to reflect a compressive axial force condition, thus,

$$\left[[K_E] - A_N[K_G] - \Omega^2[M]\right]\{\delta\} = \{0\} \tag{5-2}$$

The coefficient A_n preceding the matrix $[K_G]$ is the magnitude of inplane axial load. The negative sign implies that the geometric stiffness matrix has a reducing effect on the bending stiffness matrix of the plate, thus, reducing the overall plate stiffness which inturn decreases plate frequencies. The solution of Equation (5.2) proceeds as the value of A_n increases. For a particular value of A_n, the lowest frequency Ω, reduces to zero. This condition defines the critical buckling load of the plate. For the special case $A_n = 0$, Equation (5.2) reduces to Equation (3.5),

$$\left[[K_E] - \omega^2 [M]\right]\{\delta\} = \{0\} \tag{5-3}$$

which is a standard transverse vibration problem (neglecting axial forces) solved in Chapter III.

When the axial force is sufficient to decrease the first frequency to zero, this condition reflects the static stability problem explained in Chapter IV.

An incremental technique is developed in subroutine EIGEN to calculate the frequencies for different values of A_n where $0 \leq A_n \leq P_{cr}$. Therefore, the solution of Equation (5.2) requires the definition of the general elastic stiffness matrix, the general geometric stiffness matrix and the general mass matrix.

5.3 Matrix Definitions

5.3.1 General Stiffness Matrix

Using the procedure described in Chapter II, the element Bending Stiffness Matrix (See Equation (2.19)) is formulated for a strip and assembled and compiled to form the General Bending Stiffness Matrix. The procedure is similar to that described in Chapter II.

5.3.2 General Mass Matrix

Using the procedure described in Chapter III, the element mass matrix is computed for a strip (See Equation (3-14)) and assembled and compiled to form the General Mass Matrix. The procedure is exactly the same as that described in Chapter III.

5.3.3 General Geometric Stiffness Matrix

Using the procedure described in Chapter IV, the element geometric stiffness matrix is computed for a strip (See Equation (4-10)) and assembled and compiled to form General Geometric Stiffness Matrix. The procedure is similar to that shown in Chapter IV.

5.4 Example Problem

The dynamic buckling load of a simply supported square plate is determined using finite strip method.

Number of Strips = 8

Number of Nodal Lines = 9

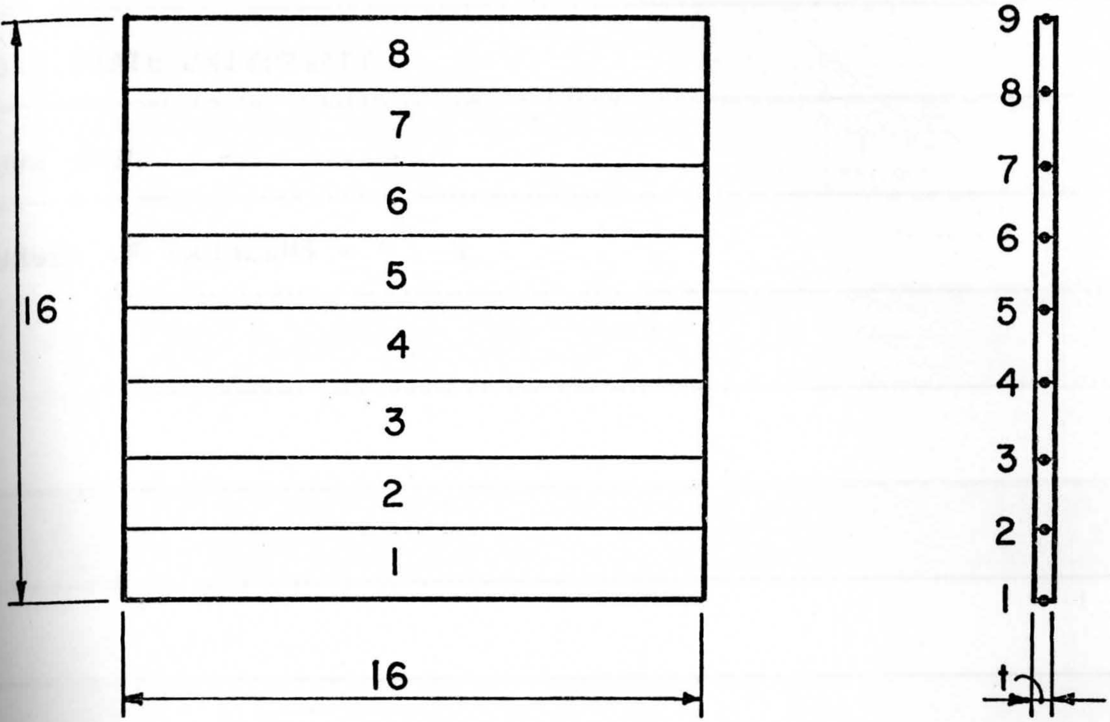

FIGURE 16

PLATE GEOMETRY

STRIP EIGEN VALUE SYSTEM

EN SCHOOL OF ENGR

TOWN STATE UNIVERSITY

TOWN OHIO

NUMBER OF PROBLEMS = 1

EM NUMBER = 1

E PLATE VIBRATION WITH INPLANE FORCES MPROB=3
JT DATA

*** CONTROL PARAMETERS ***

R OF TERMS	3
R OF ELEMENTS	8
R OF NODAL LINES	9
R OF BOUNDARY CONDITIONS	2
R OF DEGREES OF FREEDOM/NODAL LINE	4
TYPE(1=NON SYMM, 2=SYMM)	1
PROBLEM TYPE	3
EE VIBRATION,2=STABILITY, 3=VIB+IN PLANE)	
R OF ITERATIONS	5
SHAPE OF MODE NUMBER	3
T(1=PRINT DATA,0=NO PRINT)	0

*** PLATE GEOMETRY ***

H OF PLATE 16.0000

LINE NO	XCOORD	YCOORD
1	0.0	0.0
2	2.0000	0.0
3	4.0000	0.0
4	6.0000	0.0
5	8.0000	0.0
6	10.0000	0.0
7	12.0000	0.0
8	14.0000	0.0
9	16.0000	0.0

P NO	NODE1	NODE2	THICKNESS
1	1	2	2.26303000
2	2	3	2.26303000
3	3	4	2.26303000
4	4	5	2.26303000
5	5	6	2.26303000
6	6	7	2.26303000
7	7	8	2.26303000
8	8	9	2.26303000

*** BOUNDARY CONDITIONS ***

E LINE NO	U	V	W	THETA
1	1	1	0	1
9	1	1	0	1

*** MATERIAL PROPERTIES ***

ULUS OF ELASTICITY 1	1.0000
ULUS OF ELASTICITY 2	1.0000
SSONS RATIO X	0.1667
SSONS RATIO Y	0.1667
ULUS OF ELASTICITY IN SHEAR G	0.4286
TE MASS DENSITY RHO	0.4409

*** NUMBER OF PLOT POINTS REQD ***

6 9 12

*** RESULTS OF ANALYSIS ***

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	0.00001080	-0.00003088	0.00000000	-0.74290523
2	0.00000644	-0.00003541	-1.44791489	-0.68635575
3	0.00000149	-0.00003534	-2.67540002	-0.52531507
4	-0.00000093	-0.00003241	-3.49558198	-0.28430016
5	-0.00000105	-0.00003073	-3.78359585	-0.00000286
6	-0.00000073	-0.00003178	-3.49559214	0.28429644
7	-0.00000178	-0.00003451	-2.67541376	0.52531536
8	-0.00000472	-0.00003637	-1.44792452	0.68635952
9	-0.00000798	-0.00003504	0.00000000	0.74291066

LITY FACTOR = , 0.15604955D 00

LANE FORCE 0.0

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

L LINE NO,	U	V	W	THETA
1	-0.00001501	-0.00000477	0.00000000	-0.58670550
2	-0.00001405	0.00000017	-1.14348353	-0.54204643
3	-0.00001059	0.00000362	-2.11288615	-0.41486699
4	-0.00000531	0.00000565	-2.76062556	-0.22452703
5	0.00000081	0.00000625	-2.98808637	-0.00000297
6	0.00000679	0.00000542	-2.76063630	0.22452283
7	0.00001175	0.00000322	-2.11290136	0.41486698
8	0.00001493	-0.00000039	-1.14349433	0.54205065
9	0.00001579	-0.00000556	0.00000000	0.58671152

UENCY=, 0.77565075D-01 RADIANS/SEC
UENCY=, 0.12344877D-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

L LINE NO,	U	V	W	THETA
1	0.00163927	0.00104710	0.00000000	0.00000011
2	0.00167291	0.00055602	0.00000018	0.00000005
3	0.00166130	0.00027556	0.00000019	-0.00000004
4	0.00164178	0.00011179	0.00000004	-0.00000008
5	0.00163324	-0.00000013	-0.00000009	-0.00000004
6	0.00164144	-0.00011202	-0.00000011	0.00000002
7	0.00166067	-0.00027567	-0.00000003	0.00000004
8	0.00167209	-0.00055593	0.00000002	0.00000001
9	0.00163838	-0.00104673	0.00000000	-0.00000002

QUENCY=, 0.14954338D 00 RADIANS/SEC
QUENCY=, 0.23800591D-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

AL LINE NO,	U	V	W	THETA
1	-0.00000011	-0.00000005	0.00000000	0.00044034
2	-0.00000011	-0.00000002	0.00079289	0.00031137
3	-0.00000010	-0.00000000	0.00112133	0.00000001
4	-0.00000010	0.00000001	0.00079294	-0.00031135
5	-0.00000009	0.00000001	0.00000011	-0.00044032
6	-0.00000010	0.00000001	-0.00079274	-0.00031136
7	-0.00000010	0.00000002	-0.00112116	-0.00000001
8	-0.00000011	0.00000004	-0.00079279	0.00031133
9	-0.00000011	0.00000008	0.00000000	0.00044029

EQUENCY=, 0.19353310D 00 RADIANS/SEC
EQUENCY=, 0.30801780D-01 CYCLES/SEC

NE FORCE 0.0260

OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	-0.00000653	-0.00000202	-0.00000000	-0.63601857
2	-0.00000611	0.00000013	-1.23959375	-0.58760516
3	-0.00000461	0.00000165	-2.29047271	-0.44973490
4	-0.00000230	0.00000255	-2.99265049	-0.24339629
5	0.00000040	0.00000281	-3.23922555	-0.00000186
6	0.00000305	0.00000245	-2.99265720	0.24339366
7	0.00000524	0.00000146	-2.29048221	0.44973491
8	0.00000665	-0.00000015	-1.23960049	0.58760781
9	0.00000703	-0.00000245	-0.00000000	0.63602232

ENCY=, 0.70806902D-01 RADIANS/SEC
ENCY=, 0.11269279D-01 CYCLES/SEC

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

LINE NO,	U	V	W	THETA
1	0.00113939	0.00073186	0.00000000	0.00000004
2	0.00116305	0.00038994	0.00000007	0.00000002
3	0.00115521	0.00019387	0.00000009	-0.00000001
4	0.00114178	0.00007885	0.00000004	-0.00000003
5	0.00113590	-0.00000010	-0.00000002	-0.00000002
6	0.00114155	-0.00007902	-0.00000005	-0.00000000
7	0.00115478	-0.00019396	-0.00000004	0.00000001
8	0.00116248	-0.00038989	-0.00000002	0.00000001
9	0.00113878	-0.00073161	0.00000000	0.00000001

ENCY=, 0.14588358D 00 RADIANS/SEC
ENCY=, 0.23218115D-01 CYCLES/SEC

R OF ITERATIONS 4

MODAL SHAPE OF MODE NUMBER 3

LINE NO,	U	V	W	THETA
1	-0.00000006	-0.00000005	-0.00000000	0.00031110
2	-0.00000006	-0.00000004	0.00056018	0.00021999
3	-0.00000006	-0.00000003	0.00079223	0.00000001
4	-0.00000007	-0.00000002	0.00056023	-0.00021997
5	-0.00000007	-0.00000001	0.00000007	-0.00031110
6	-0.00000007	-0.00000000	-0.00056011	-0.00021999
7	-0.00000007	0.00000000	-0.00079216	-0.00000001
8	-0.00000006	0.00000001	-0.00056015	0.00021997
9	-0.00000005	0.00000003	-0.00000000	0.00031109

ENCY=, 0.19092497D 00 RADIANS/SEC
ENCY=, 0.30336682D-01 CYCLES/SEC

ANE FORCE 0.0520

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	-0.00000227	-0.00000068	-0.00000000	-0.70149185
2	-0.00000213	0.00000007	-1.36720022	-0.64809423
3	-0.00000161	0.00000061	-2.52625751	-0.49603045
4	-0.00000079	0.00000093	-3.30071615	-0.26845045
5	0.00000017	0.00000102	-3.57267132	-0.00000092
6	0.00000111	0.00000089	-3.30071948	0.26844916
7	0.00000189	0.00000054	-2.52626222	0.49603046
8	0.00000240	-0.00000004	-1.36720355	0.64809554
9	0.00000253	-0.00000087	-0.00000000	0.70149371

ENCY=, 0.63331618D-01 RADIANS/SEC
ENCY=, 0.10079550D-01 CYCLES/SEC

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

LINE NO,	U	V	W	THETA
1	0.00071203	0.00045994	0.00000000	0.00000002
2	0.00072699	0.00024589	0.00000003	0.00000001
3	0.00072225	0.00012265	C.00000004	-0.00000000
4	0.00071396	0.00005001	0.00000002	-0.00000001
5	0.00071031	-0.00000006	-0.00000000	-0.00000001
6	0.00071382	-0.00005012	-0.00000002	-0.00000001
7	C.00072200	-0.00012271	-0.00000003	0.00000000
8	0.00072667	-0.00024587	-0.00000002	0.00000001
9	0.00071168	-0.00045980	0.00000000	0.00000001

UENCY=, 0.14212556D 00 RADIANS/SEC
UENCY=, 0.22620007D-01 CYCLES/SEC

ER OF ITERATIONS 4

MODAL SHAPE OF MODE NUMBER 3

LINE NO,	U	V	W	THETA
1	-0.00000003	-0.00000003	-0.00000000	0.00019518
2	-0.00000003	-0.00000002	0.00035144	0.00013801
3	-0.00000003	-0.00000001	0.00049701	0.00000000
4	-0.00000003	-0.00000001	0.00035145	-0.00013800
5	-0.00000004	-0.00000001	0.00000003	-0.00019517
6	-0.00000004	-0.00000000	-0.00035140	-0.00013801
7	-0.00000003	0.00000000	-0.00049698	-0.00000000
8	-0.00000003	0.00000001	-0.00035142	0.00013800
9	-0.00000003	0.00000002	-0.00000000	0.00019517

UENCY=, 0.18828069D 00 RADIANS/SEC
UENCY=, 0.29965831D-01 CYCLES/SEC

ANE FORCE 0.0730

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	-0.00000056	-0.00000016	-0.00000000	-0.79426060
2	-0.00000052	0.00000003	-1.54800538	-0.73380120
3	-0.00000039	0.00000016	-2.86034127	-0.56162732
4	-0.00000019	0.00000024	-3.73721652	-0.30395075
5	0.00000005	0.00000027	-4.04513467	-0.00000032
6	0.00000029	0.00000023	-3.73721771	0.30395030
7	0.00000049	0.00000014	-2.86034295	0.56162733
8	0.00000062	-0.00000001	-1.54800657	0.73380167
9	0.00000066	-0.00000022	-0.00000000	0.79426127

UENCY=, 0.54346790D-01 RADIANS/SEC
UENCY=, 0.87291462D-02 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

L LINE NO,	U	V	W	THETA
1	0.01951324	0.01267765	0.00000000	0.00000002
2	0.01992860	0.00680165	0.00000003	0.00000001
3	0.01980407	0.00340479	0.00000003	-0.00000001
4	0.01958090	0.00139306	0.00000001	-0.00000001
5	0.01948371	0.00000009	-0.00000001	0.00000000
6	0.01958110	-0.00139290	0.00000001	0.00000001
7	0.01980443	-0.00340469	0.00000004	0.00000001
8	0.01992907	-0.00680167	0.00000003	-0.00000001
9	0.01951373	-0.01267783	0.00000000	-0.00000002

QUENCY=, 0.13826111D 00 RADIANS/SEC
QUENCY=, 0.22004960D-01 CYCLES/SEC

BER OF ITERATIONS 4

MODAL SHAPE OF MODE NUMBER 3

AL LINE NO,	U	V	W	THETA
1	0.00000000	-0.00000000	-0.00000000	-0.00160086
2	0.00000000	-0.00000000	-0.00288252	-0.00113197
3	-0.00000000	-0.00000000	-0.00407649	0.00000000
4	-0.00000000	-0.00000000	-0.00288253	0.00113196
5	-0.00000000	-0.00000000	-0.00000007	0.00160081
6	-0.00000000	-0.00000000	0.00288235	0.00113193
7	-0.00000000	-0.00000000	0.00407628	-0.00000000
8	0.00000000	-0.00000000	0.00288236	-0.00113192
9	0.00000000	-0.00000000	-0.00000000	-0.00160077

EQUENCY=, 0.18559874D 00 RADIANS/SEC
EQUENCY=, 0.29538985D-01 CYCLES/SEC

ANE FORCE 0.10+0

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	-0.00000007	-0.00000002	-0.00000000	-0.94123967
2	-0.00000007	0.00000001	-1.83446596	-0.86959209
3	-0.00000005	0.00000002	-3.38965117	-0.66555700
4	-0.00000002	0.00000003	-4.42879279	-0.36019690
5	0.00000001	0.00000004	-4.79369090	-0.00000006
6	0.00000004	0.00000003	-4.42879303	0.36019682
7	0.00000007	0.00000002	-3.38965152	0.66555702
8	0.00000009	-0.00000000	-1.83446621	0.86959219
9	0.00000009	-0.00000003	-0.00000000	0.94123982

JENCY=, 0.44782217D-01 RADIANS/SEC
JENCY=, 0.71273180D-02 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

LINE NO,	U	V	W	THETA
1	0.00945652	0.00617922	0.00000000	0.00000000
2	0.00966023	0.00332656	0.00000000	0.00000000
3	0.00960201	0.00167063	0.00000000	-0.00000000
4	0.00949512	0.00068526	0.00000000	-0.00000000
5	0.00944840	0.00000001	-0.00000000	-0.00000000
6	0.00949514	-0.00068525	0.00000000	0.00000000
7	0.00960204	-0.00167063	0.00000000	0.00000000
8	0.00966027	-0.00332656	0.00000000	-0.00000000
9	0.00945656	-0.00617923	0.00000000	-0.00000000

UENCY=, 0.13428124D 00 RADIANS/SEC
UENCY=, 0.21371543D-01 CYCLES/SEC

ER OF ITERATIONS 4

MODAL SHAPE OF MODE NUMBER 3

LINE NO,	U	V	W	THETA
1	-0.00000000	-0.00000000	-0.00000000	-0.00086626
2	-0.00000000	-0.00000000	-0.00155980	-0.00061254
3	0.00000000	-0.00000000	-0.00220589	-0.00000000
4	0.00000000	-0.00000000	-0.00155980	0.00061253
5	0.00000000	-0.00000000	-0.00000001	0.00086625
6	-0.00000000	-0.00000000	0.00155977	0.00061253
7	-0.00000000	-0.00000000	0.00220585	-0.00000000
8	0.00000000	-0.00000000	0.00155977	-0.00061253
9	0.00000000	-0.00000000	0.00000000	-0.00086624

QUENCY=, 0.18287747D 00 RADIANS/SEC
QUENCY=, 0.29105881D-01 CYCLES/SEC

ANE FORCE 0.1300

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	-0.00000000	-0.00000000	-0.00000000	-1.24030524
2	-0.00000000	0.00000000	-2.41734151	-1.14589263
3	-0.00000000	0.00000000	-4.46666470	-0.87702825
4	-0.00000000	0.00000000	-5.83597868	-0.47464427
5	0.00000000	0.00000000	-6.31681782	0.00000001
6	0.00000000	0.00000000	-5.83597871	0.47464428
7	0.00000000	0.00000000	-4.46666474	0.87702827
8	0.00000000	-0.00000000	-2.41734154	1.14589265
9	0.00000000	-0.00000000	-0.00000000	1.24030527

UENCY=, 0.31665809D-01 RADIANS/SEC
UENCY=, 0.50397749D-02 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

LINE NO,	U	V	W	THETA
1	0.00258066	0.00169610	0.00000000	-0.00000000
2	0.00263693	0.00091624	-0.00000000	-0.00000000
3	0.00262163	0.00046165	-0.00000000	0.00000000
4	0.00259282	0.00018984	0.00000000	0.00000000
5	0.00258019	0.00000000	0.00000000	0.00000000
6	0.00259283	-0.00018984	0.00000000	-0.00000000
7	0.00262163	-0.00046165	-0.00000000	-0.00000000
8	0.00263693	-0.00091624	-0.00000000	0.00000000
9	0.00258066	-0.00169610	0.00000000	0.00000000

UENCY=, 0.13017506D 00 RADIANS/SEC
UENCY=, 0.20718024D-01 CYCLES/SEC

BER OF ITERATIONS 4

MODAL SHAPE OF MODE NUMBER 3

L LINE NO,	U	V	W	THETA
1	-0.00000000	-0.00000000	-0.00000000	-0.00027139
2	-0.00000000	-0.00000000	-0.00048866	-0.00019190
3	0.00000000	-0.00000000	-0.00069107	-0.00000000
4	0.00000000	-0.00000000	-0.00048866	0.00019190
5	0.00000000	-0.00000000	-0.00000000	0.00027139
6	0.00000000	-0.00000000	0.00048866	0.00019190
7	-0.00000000	-0.00000000	0.00069107	0.00000000
8	-0.00000000	-0.00000000	0.00048866	-0.00019190
9	-0.00000000	-0.00000000	0.00000000	-0.00027139

UENCY=, 0.18011509D 00 RADIANS/SEC
UENCY=, 0.28666234D-01 CYCLES/SEC

ANE FORCE 0.1500
A= 36 MAS= 36 GISH= -0.21543474D-07

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NC,	U	V	W	THETA
1	0.00002132	-0.00010887	0.00000000	-0.32508725
2	-0.00001236	-0.00011739	-0.63359721	-0.30034959
3	-0.00002505	-0.00010238	-1.17074967	-0.22988040
4	-0.00001369	-0.00010046	-1.52966600	-0.12441244
5	0.00000081	-0.00010372	-1.65570575	-0.00000146
6	0.00001442	-0.00010555	-1.52966937	0.12441519
7	0.00002223	-0.00011744	-1.17074659	0.22988309
8	0.00001131	-0.00013402	-0.63359893	0.30034682
9	-0.00001233	-0.00012589	0.00000000	0.32509245

ITY FACTOR = , 0.24279279D 00

LANE FORCE 0.0

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

L LINE NO,	U	V	W	THETA
1	0.00000128	0.00000359	0.00000000	-0.22352844
2	0.00000136	0.00000293	-0.43565376	-0.20651358
3	0.00000392	0.00000210	-0.80498482	-0.15806035
4	0.00000341	0.00000003	-1.05177025	-0.08554587
5	-0.00000097	-0.00000079	-1.13843655	-0.00000262
6	-0.00000416	0.00000060	-1.05177720	0.08554593
7	-0.00000334	0.00000235	-0.80498492	0.15806593
8	-0.00000098	0.00000308	-0.43564732	0.20651354
9	-0.00000031	0.00000363	0.00000000	0.22352373

UENCY=, 0.19350083D 00 RADIANS/SEC
UENCY=, 0.30796644D-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

AL LINE NO,	U	V	W	THETA
1	-0.00134956	-0.00073565	-0.00000000	0.00026006
2	-0.00127467	-0.00006479	0.00040714	0.00009862
3	-0.00094992	0.00021604	0.00030900	-0.00018481
4	-0.00049672	0.00033704	-0.00017008	-0.00023720
5	0.00002028	0.00037309	-0.00043553	0.00000251
6	0.00053886	0.00033983	-0.00016408	0.00023665
7	0.00100020	0.00021953	0.00030724	0.00017933
8	0.00133756	-0.00007069	0.00039955	-0.00009783
9	0.00141508	-0.00077316	-0.00000000	-0.00025466

QUENCY=, 0.41015991D 00 RADIANS/SEC
QUENCY=, 0.65279040D-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

AL LINE NO,	U	V	W	THETA
1	0.00008553	0.00005210	0.00000000	0.00000224
2	0.00008547	0.00001220	0.00000409	0.00000165
3	0.00007797	0.00000007	0.00000600	0.00000015
4	0.00007140	-0.00000185	0.00000456	-0.00000154
5	0.00006858	-0.00000042	0.00000035	-0.00000246
6	0.00007020	0.00000108	-0.00000429	-0.00000193
7	0.00007571	-0.00000055	-0.00000650	-0.00000015
8	0.00008245	-0.00001202	-0.00000474	0.00000181
9	0.00008232	-0.00005032	0.00000000	0.00000266

EQUENCY=, 0.34368544D 00 RADIANS/SEC
EQUENCY=, 0.54699288D-01 CYCLES/SEC

NE FORCE 0.0270

OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	-0.00000039	0.00000231	0.00000000	-0.23211675
2	-0.00000052	0.00000249	-0.45239220	-0.21444810
3	0.00000107	0.00000237	-0.83591247	-0.16413202
4	0.00000135	0.00000141	-1.09217465	-0.08882899
5	-0.00000044	0.00000096	-1.18216330	0.00000023
6	-0.00000167	0.00000166	-1.09217244	0.08883139
7	-0.00000082	0.00000251	-0.83590408	0.16413499
8	0.00000072	0.00000272	-0.45238292	0.21444569
9	0.00000117	0.00000245	0.00000000	0.23211107

ENCY=, 0.18243433D 00 RADIANS/SEC
ENCY=, 0.29035354D-01 CYCLES/SEC

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

LINE NO,	U	V	W	THETA
1	0.00040716	0.00025717	-0.00000000	-0.00040467
2	0.00041338	0.00007009	-0.00072786	-0.00028508
3	0.00039523	0.00001891	-0.00102603	0.00000231
4	0.00039243	0.00001645	-0.00072063	0.00028635
5	0.00041645	0.00002767	0.00000555	0.00040141
6	0.00046757	0.00003419	0.00072490	0.00028083
7	0.00053833	0.00001449	0.00101826	-0.00000230
8	0.00060673	-0.00007910	0.00071768	-0.00028260
9	0.00061228	-0.00036840	-0.00000000	-0.00039817

UENCY=, 0.32760438D 00 RADIANS/SEC
UENCY=, 0.52139906D-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

L LINE NO,	U	V	W	THETA
1	-0.00097267	-0.00053063	-0.00000000	0.00021232
2	-0.00091829	-0.00004628	0.00035699	0.00011481
3	-0.00068347	0.00015475	0.00039410	-0.00007690
4	-0.00036379	0.00023749	0.00011632	-0.00017307
5	-0.00000767	0.00026047	-0.00018291	-0.00010333
6	0.00034832	0.00023782	-0.00025647	0.00002602
7	0.00066821	0.00015580	-0.00013531	0.00007577
8	0.00090348	-0.00004345	-0.00001859	0.00003229
9	0.00095835	-0.00052142	-0.00000000	-0.00000353

UENCY=, 0.37868081D 00 RADIANS/SEC
UENCY=, 0.60268981D-01 CYCLES/SEC

ANE FORCE 0.0540

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	-0.00000150	0.00000105	0.00000000	-0.24285566
2	-0.00000163	0.00000180	-0.47332213	-0.22436951
3	-0.00000052	0.00000207	-0.87458548	-0.17172504
4	0.00000017	0.00000172	-1.14270131	-0.09293702
5	-0.00000018	0.00000150	-1.23685012	0.00000129
6	-0.00000030	0.00000182	-1.14269590	0.09293995
7	0.00000061	0.00000215	-0.87457506	0.17172657
8	0.00000173	0.00000201	-0.47331311	0.22436656
9	0.00000205	0.00000120	0.00000000	0.24285043

JENCY=, 0.170651690 00 RADIANS/SEC
JENCY=, 0.271600860-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

L LINE NO,	U	V	W	THETA
1	0.00007152	0.00004562	0.00000000	-0.00039028
2	0.00007191	0.00001225	-0.00070200	-0.00027495
3	0.00006694	0.00000218	-0.00098950	0.00000229
4	0.00006294	0.00000049	-0.00069494	0.00027656
5	0.00006177	0.00000166	0.00000513	0.00038703
6	0.00006462	0.00000304	0.00069897	0.00027105
7	0.00007138	0.00000132	0.00098233	-0.00000203
8	0.00007900	-0.00001013	0.00069253	-0.00027266
9	0.00007932	-0.00004736	0.00000000	-0.00038423

UENCY=, 0.298829430 00 RADIANS/SEC
UENCY=, 0.475602270-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

AL LINE NO,	U	V	W	THETA
1	0.00064240	0.00039906	0.00000000	0.00005200
2	0.00064330	0.00009811	0.00009361	0.00003677
3	0.00058996	0.00000592	0.00013255	0.00000015
4	0.00054426	-0.00000898	0.00009382	-0.00003695
5	0.00052654	0.00000180	-0.00000039	-0.00005226
6	0.00054326	0.00001329	-0.00009401	-0.00003648
7	0.00059061	-0.00000064	-0.00013203	0.00000033
8	0.00064640	-0.00009312	-0.00009307	0.00003661
9	0.00064625	-0.00039592	0.00000000	0.00005167

QUENCY=, 0.330267030 00 RADIANS/SEC
QUENCY=, 0.525636800-01 CYCLES/SEC

NE FORCE 0.0809

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	-0.00000172	0.00000026	0.00000000	-0.25675591
2	-0.00000179	0.00000112	-0.50041346	-0.23721164
3	-0.00000104	0.00000150	-0.92464359	-0.18155385
4	-0.00000031	0.00000143	-1.20810479	-0.09825597
5	-0.00000007	0.00000134	-1.30764180	0.00000124
6	0.00000026	0.00000146	-1.20809953	0.09825826
7	0.00000104	0.00000154	-0.92463538	0.18155456
8	0.00000182	0.00000127	-0.50040691	0.23720931
9	0.00000204	0.00000038	0.00000000	0.25675220

ENCY=, 0.15799276D 00 RADIANS/SEC
ENCY=, 0.25145353D-01 CYCLES/SEC

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

LINE NO,	U	V	W	THETA
1	0.00000869	0.00000661	0.00000000	-0.00029549
2	0.00000880	0.00000258	-0.00053096	-0.00020742
3	0.00000855	0.00000136	-0.00074604	0.00000339
4	0.00000835	0.00000099	-0.00052044	0.00020989
5	0.00000806	0.00000100	0.00000781	0.00029067
6	0.00000825	0.00000125	0.00052648	0.00020143
7	0.00000928	0.00000118	0.00073506	-0.00000315
8	0.00001056	-0.00000029	0.00051655	-0.00020391
9	0.00001076	-0.00000531	0.00000000	-0.00028630

UENCY=, 0.29016029D 00 RADIANS/SEC
UENCY=, 0.46180489D-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

LINE NO,	U	V	W	THETA
1	0.00044305	0.00027707	0.00000000	0.00000342
2	0.00044372	0.00006886	0.00000690	0.00000348
3	0.00040666	0.00000456	0.00001324	0.00000244
4	0.00037479	-0.00000602	0.00001436	-0.00000185
5	0.00036219	0.00000133	0.00000524	-0.00000694
6	0.00037331	0.00000921	-0.00001028	-0.00000753
7	0.00040554	-0.00000053	-0.00002067	-0.00000201
8	0.00044365	-0.00006453	-0.00001668	0.00000586
9	0.00044341	-0.00027284	0.00000000	0.00000965

UENCY=, 0.32424981D 00 RADIANS/SEC
UENCY=, 0.51606009D-01 CYCLES/SEC

ANE FORCE 0.1079

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	-0.00000134	-0.00000008	0.00000000	-0.27557808
2	-0.00000137	0.00000059	-0.53709756	-0.25460104
3	-0.00000091	0.00000090	-0.99242699	-0.19486318
4	-0.00000037	0.00000093	-1.29666816	-0.10545399
5	-0.00000002	0.00000090	-1.40350263	0.00000072
6	0.00000035	0.00000093	-1.29666538	0.10546023
7	0.00000088	0.00000091	-0.99242249	0.19486345
8	0.00000136	0.00000068	-0.53709410	0.25459976
9	0.00000149	0.00000000	0.00000000	0.27557615

ENCY=, 0.14422700D 00 RADIANS/SEC

ENCY=, 0.22954462D-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

LINE NO,	U	V	W	THETA
1	0.00000427	0.00000315	0.00000000	-0.00021348
2	0.00000426	0.00000114	-0.00038394	-0.00015032
3	0.00000396	0.00000050	-0.00054094	0.00000142
4	0.00000366	0.00000030	-0.00037952	0.00015136
5	0.00000333	0.00000030	0.00000329	0.00021147
6	0.00000320	0.00000041	0.00038206	0.00014780
7	0.00000341	0.00000043	0.00053632	-0.00000133
8	0.00000376	-0.00000005	0.00037788	-0.00014884
9	0.00000381	-0.00000181	0.00000000	-0.00020962

UENCY=, 0.28243184D 00 RADIANS/SEC

UENCY=, 0.44950466D-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

LINE NO,	U	V	W	THETA
1	0.00026727	0.00016794	0.00000000	0.00000061
2	0.00026770	0.00004196	0.00000148	0.00000098
3	0.00024517	0.00000278	0.00000391	0.00000128
4	0.00022581	-0.00000369	0.00000540	-0.00000011
5	0.00021828	0.00000077	0.00000279	-0.00000245
6	0.00022509	0.00000546	-0.00000324	-0.00000314
7	0.00024452	-0.00000063	-0.00000786	-0.00000107
8	0.00026738	-0.00003961	-0.00000666	0.00000225
9	0.00026711	-0.00016547	0.00000000	0.00000390

UENCY=, 0.31749490D 00 RADIANS/SEC

UENCY=, 0.50530931D-01 CYCLES/SEC

ANE FORCE 0.1349

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	-0.00000073	-0.00000012	-0.00000000	-0.30258232
2	-0.00000074	0.00000025	-0.58972838	-0.27954969
3	-0.00000052	0.00000042	-1.08967607	-0.21395809
4	-0.00000023	0.00000046	-1.42373039	-0.11579323
5	-0.00000000	0.00000045	-1.54103438	0.00000024
6	0.00000023	0.00000045	-1.42372940	0.11579364
7	0.00000050	0.00000042	-1.08967450	0.21395316
8	0.00000073	0.00000029	-0.58972720	0.27954922
9	0.00000080	-0.00000009	-0.00000000	0.30258168

JENCY=, 0.129000550 00 RADIANS/SEC

JENCY=, 0.205310940-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

LINE NC,	U	V	W	THETA
1	0.00000159	0.00000117	0.00000000	-0.00014730
2	0.00000158	0.00000041	-0.00026508	-0.00010396
3	0.00000145	0.00000017	-0.00037424	0.00000046
4	0.00000132	0.00000010	-0.00026366	0.00010429
5	0.00000121	0.00000011	0.00000107	0.00014666
6	0.00000116	0.00000014	0.00026449	0.00010314
7	0.00000122	0.00000014	0.00037274	-0.00000043
8	0.00000133	-0.00000002	0.00026312	-0.00010348
9	0.00000134	-0.00000064	0.00000000	-0.00014606

QUENCY=, 0.274799510 00 RADIANS/SEC

QUENCY=, 0.437357430-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

AL LINE NO,	U	V	W	THETA
1	0.00013754	0.00008691	0.00000000	-0.00000021
2	0.00013779	0.00002188	-0.00000020	0.00000011
3	0.00012613	0.00000151	0.00000060	0.00000063
4	0.00011613	-0.00000188	0.00000174	0.00000033
5	0.00011227	0.00000042	0.00000142	-0.00000068
6	0.00011582	0.00000281	-0.00000065	-0.00000121
7	0.00012582	-0.00000043	-0.00000261	-0.00000056
8	0.00013754	-0.00002069	-0.00000242	0.00000076
9	0.00013735	-0.00008559	0.00000000	0.00000145

QUENCY=, 0.310585530 00 RADIANS/SEC

QUENCY=, 0.494312700-01 CYCLES/SEC

ANE FORCE 0.1619

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	-0.00000026	-0.00000006	-0.00000000	-0.34448929
2	-0.00000026	0.00000007	-0.67140440	-0.31826665
3	-0.00000019	0.00000013	-1.24059364	-0.24359078
4	-0.00000009	0.00000015	-1.62091374	-0.13183039
5	0.00000000	0.00000015	-1.75446435	0.00000002
6	0.00000009	0.00000015	-1.62091357	0.13183042
7	0.00000018	0.00000013	-1.24059337	0.24359076
8	0.00000026	0.00000008	-0.67140421	0.31826655
9	0.00000028	-0.00000005	-0.00000000	0.34448921

JENCY=, 0.11171775D 00 RADIANS/SEC
JENCY=, 0.17780449D-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

LINE NO,	U	V	W	THETA
1	0.01484751	0.00908159	-0.00000000	-0.00008223
2	0.01465989	0.00191927	-0.00011050	-0.00000567
3	0.01279119	-0.00055698	0.00000933	0.00011577
4	0.01072113	-0.00117236	0.00025446	0.00009671
5	0.00902834	-0.00107721	0.00028547	-0.00007969
6	0.00793546	-0.00079558	-0.00003603	-0.00021412
7	0.00743477	-0.00073018	-0.00041325	-0.00012218
8	0.00749495	-0.00156181	-0.00041730	0.00012309
9	0.00725700	-0.00488216	-0.00000000	0.00025440

UENCY=, 0.30459979D 00 RADIANS/SEC
UENCY=, 0.48478608D-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

L LINE NO,	U	V	W	THETA
1	0.00002190	0.00001389	0.00000000	-0.00204177
2	0.00002195	0.00000351	-0.00367613	-0.00144352
3	0.00002012	0.00000025	-0.00519808	0.00000054
4	0.00001853	-0.00000027	-0.00367440	0.00144397
5	0.00001808	0.00000011	0.00000143	0.00204104
6	0.00001876	0.00000047	0.00367552	0.00144243
7	0.00002043	-0.00000011	0.00519610	-0.00000054
8	0.00002233	-0.00000346	0.00367353	-0.00144238
9	0.00002228	-0.00001402	0.00000000	-0.00204011

QUENCY=, 0.26704615D 00 RADIANS/SEC
QUENCY=, 0.42501756D-01 CYCLES/SEC

ANE FCRCE 0.1833

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	-0.00000005	-0.00000001	-0.00000000	-0.41801795
2	-0.00000005	0.00000001	-0.81471063	-0.38619827
3	-0.00000004	0.00000002	-1.50538899	-0.29558338
4	-0.00000002	0.00000003	-1.96688553	-0.15996360
5	0.00000000	0.00000003	-2.12894157	-0.00000003
6	0.00000002	0.00000003	-1.96688555	0.15996353
7	0.00000003	0.00000002	-1.50538903	0.29558333
8	0.00000005	0.00000001	-0.81471068	0.38619825
9	0.00000005	-0.00000001	-0.00000000	0.41801801

JENCY=, 0.91217152D-01 RADIANS/SEC
JENCY=, 0.14517675D-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

LINE NO,	U	V	W	THETA
1	0.00649451	0.00405461	-0.00000000	-0.00022743
2	0.00645798	0.00093605	-0.00040556	-0.00015532
3	0.00575956	-0.00010799	-0.00055625	0.00001206
4	0.00504539	-0.00033380	-0.00036763	0.00016470
5	0.00455700	-0.00026200	0.00002943	0.00021050
6	0.00437294	-0.00014433	0.00039015	0.00013267
7	0.00447937	-0.00020342	0.00051462	-0.00001248
8	0.00473057	-0.00084712	0.00035114	-0.00014205
9	0.00466543	-0.00303690	-0.00000000	-0.00019271

UENCY=, 0.28815185D 00 RADIANS/SEC
UENCY=, 0.45860836D-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

LINE NO,	U	V	W	THETA
1	-0.00007163	-0.00004511	-0.00000000	-0.00078794
2	-0.00007151	-0.00001087	-0.00141875	-0.00055720
3	-0.00006459	0.00000031	-0.00200653	-0.00000008
4	-0.00005798	0.00000248	-0.00141900	0.00055713
5	-0.00005426	0.00000149	-0.00000019	0.00078305
6	-0.00005416	0.00000023	0.00141885	0.00055735
7	-0.00005732	0.00000145	0.00200681	0.00000008
8	-0.00006171	0.00001036	0.00141911	-0.00055728
9	-0.00006125	0.00003933	0.00000000	-0.00078817

UENCY=, 0.25923150D 00 RADIANS/SEC
UENCY=, 0.41258014D-01 CYCLES/SEC

ANE FORCE 0.2158

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

L LINE NO,	U	V	W	THETA
1	-0.00000000	-J.00J00000	-0.00J00000	-0.58407093
2	-0.00000000	J.00J00000	-1.13834536	-0.53961123
3	-0.00000000	J.00000000	-2.10338801	-0.41300059
4	-0.00000000	J.0J000000	-2.74820891	-0.22351434
5	-0.00000000	J.00J00000	-2.97463992	-0.00000006
6	0.00000000	J.00J00000	-2.74820897	0.22351424
7	0.00000000	J.00J00000	-2.10338811	0.41300052
8	0.00000000	J.00000000	-1.13834547	0.53961122
9	0.00000000	-J.00J00000	-0.00000000	0.58407104

JENCY=, 0.64500253D-01 RADIANS/SEC

JENCY=, 0.10265544D-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

L LINE NO,	U	V	W	THETA
1	-0.00005078	-J.00J02874	0.00000000	-0.00088688
2	-0.00004830	-J.00J00305	-0.00159672	-0.00062691
3	-0.00003677	J.0J000780	-0.00225741	0.00000049
4	-0.00002131	J.00J01213	-0.00159516	0.00062730
5	-0.00000449	J.00J01319	0.00000124	0.00088620
6	0.00001202	J.00J01194	0.00159610	0.00062595
7	0.00002665	J.00J00791	0.00225566	-0.00000053
8	0.00003726	-J.00J00125	0.00159445	-0.00062634
9	0.00003979	-J.00J02169	0.00000000	-0.00088544

UENCY=, 0.25104797D 00 RADIANS/SEC

UENCY=, 0.39955564D-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

L LINE NO,	U	V	W	THETA
1	0.00114939	J.00J73478	0.00000000	-0.00000330
2	0.00115272	J.00018641	-0.00000595	-0.00000234
3	0.00105485	J.00J01072	-0.00000842	-0.00000001
4	0.00097004	-J.00J01942	-0.00000596	0.00000234
5	0.00093851	-J.00J00003	-0.00000000	0.00000331
6	0.00096993	J.00J01937	0.00000596	0.00000234
7	0.00105464	-J.00J01074	0.00000843	0.00000000
8	0.00115243	-J.00018636	0.00000596	-0.00000234
9	0.00114909	-J.00J73459	0.00000000	-0.00000331

QUENCY=, 0.28378828D 00 RADIANS/SEC

QUENCY=, 0.45962126D-01 CYCLES/SEC

ANE FCRCE 0.2428

A= 36 MAS= 36 GISH= -0.56850418D-05

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	0.79024488	0.55701650	-0.00000000	0.00855185
2	0.77373944	0.05074265	0.01011164	-0.00167832
3	0.62937165	-0.05299025	-0.01634757	-0.02586532
4	0.51117666	-0.04620370	-0.09124347	-0.04676887
5	0.45677616	-0.00723717	-0.18925086	-0.04677359
6	0.47289210	0.03445319	-0.25995681	-0.01965614
7	0.55788971	0.05161799	-0.25537740	0.02580226
8	0.63046493	-0.03359929	-0.15886146	0.06814346
9	0.69793197	-0.47399868	-0.00000000	0.08527338

LITY FACTOR = , 0.63091963D 00

ANE FORCE 0.0

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NC,	U	V	W	THETA
1	0.65412613	0.39009904	0.00000000	0.03776575
2	0.63456790	0.02152611	0.06715607	0.02569792
3	0.52494360	-0.03729612	0.09473152	0.00164587
4	0.44429739	-0.02762707	0.07810961	-0.01668208
5	0.40497295	-0.00389859	0.03418714	-0.02589630
6	0.40034612	0.00833735	-0.01908183	-0.02534803
7	0.43021065	0.01873775	-0.05678346	-0.00966168
8	0.48982817	-0.01930150	-0.05023855	0.01632066
9	0.49688771	-0.29463863	0.00000000	0.02978209

ENCY=, 0.530527130 00 RADIANS/SEC
ENCY=, 0.844360950-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

LINE NO,	U	V	W	THETA
1	0.00268568	0.00161090	0.00000000	-0.04486421
2	0.00264684	0.00011574	-0.08737429	-0.04135379
3	0.00229839	-0.00010758	-0.16108021	-0.03138249
4	0.00207601	-0.00008236	-0.20966822	-0.01659203
5	0.00197068	-0.00004099	-0.22587350	0.00054909
6	0.00193993	-0.00000554	-0.20771734	0.01732994
7	0.00199961	0.00001619	-0.15842069	0.03132510
8	0.00217061	-0.00014390	-0.08556490	0.04061464
9	0.00215552	-0.00132764	0.00000000	0.04387400

UENCY=, 0.386531570 00 RADIANS/SEC
UENCY=, 0.615184690-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

L LINE NO,	U	V	W	THETA
1	0.00117429	0.00067258	-0.00000000	-0.00428595
2	0.00110318	-0.00000485	-0.00770808	-0.00301839
3	0.00081741	-0.00013175	-0.01085642	0.00003553
4	0.00055947	-0.00011525	-0.00757397	0.00307818
5	0.00039599	-0.00006180	0.00023355	0.00432622
6	0.00034105	-0.00000592	0.00800826	0.00304550
7	0.00038184	0.00002860	0.01119412	-0.00002775
8	0.00047467	-0.00000750	0.00789388	-0.00310524
9	0.00049812	-0.00028859	-0.00000000	-0.00438195

UENCY=, 0.504420030 00 RADIANS/SEC
UENCY=, 0.802310180-01 CYCLES/SEC

ANE FORCE 0.0523

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

L LINE NO,	U	V	W	THETA
1	0.70809145	0.42742987	0.00000001	0.02326078
2	0.68629601	0.02527185	0.03792993	0.01086589
3	0.56437875	-0.04047037	0.03586202	-0.01294592
4	0.47439759	-0.02940146	-0.00772086	-0.02866093
5	0.43133870	-0.00762301	-0.07047973	-0.03222317
6	0.42824707	0.01241359	-0.12767667	-0.02245864
7	0.46408495	0.02407365	-0.14841468	0.00443412
8	0.53206308	-0.01882398	-0.10325530	0.04019256
9	0.54113423	-0.32103713	0.00000001	0.05758500

UENCY=, 0.46873826D 00 RADIANS/SEC
UENCY=, 0.74602083D-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

L LINE NO,	U	V	W	THETA
1	-0.00094414	-0.00056844	-0.00000000	-0.00555162
2	-0.00091918	-0.00003276	-0.01085984	-0.00518736
3	-0.00076227	0.00006071	-0.02027922	-0.00411941
4	-0.00062793	0.00006327	-0.02691875	-0.00242607
5	-0.00051723	0.00005882	-0.02967236	-0.00027087
6	-0.00041340	0.00005495	-0.02788884	0.00205021
7	-0.00032355	0.00004399	-0.02163252	0.00412994
8	-0.00027280	0.00004285	-0.01180333	0.00556324
9	-0.00024503	0.00016478	-0.00000000	0.00607258

QUENCY=, 0.36236371D 00 RADIANS/SEC
QUENCY=, 0.57672032D-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

AL LINE NO,	U	V	W	THETA
1	-0.00011735	-0.00006819	-0.00000000	0.00018568
2	-C.00011086	0.00000011	0.00033536	0.00013285
3	-0.00008220	0.00001468	0.00048050	0.00000529
4	-0.00005169	0.00001705	0.00035637	-0.00012254
5	-0.00002162	0.00001751	0.00004056	-0.00017669
6	0.00000895	0.00001751	-0.00027982	-0.00012725
7	0.00003966	0.00001516	-0.00041897	-0.00000516
8	0.00006625	0.00000373	-0.00030048	0.00011694
9	0.00007365	-0.00004072	-0.00000000	0.00016744

EQUENCY=, 0.48877880D 00 RADIANS/SEC
EQUENCY=, 0.77791637D-01 CYCLES/SEC

ANE FORCE 0.1032

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	-0.00647666	-0.00261846	0.00000000	-0.16947982
2	-0.00590836	0.00119304	-0.33061987	-0.15700071
3	-0.00407101	0.00215250	-0.61181718	-0.12042348
4	-0.00185219	0.00213838	-0.79933804	-0.06462255
5	-0.00048591	0.00131586	-0.86396996	0.00063237
6	-0.00113133	0.00018548	-0.79753762	0.06482591
7	-0.00372752	-0.00032718	-0.61113166	0.11932357
8	-0.00668591	0.00080355	-0.33156489	0.15678416
9	-0.00735958	0.00538741	0.00000000	0.17032768

JENCY=, 0.33603247D 00 RADIANS/SEC
JENCY=, 0.53431280D-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

L LINE NO,	U	V	W	THETA
1	0.04666363	0.02305359	0.00000000	0.00178928
2	0.04512724	0.00137514	0.00317024	0.00119001
3	0.03695142	-0.00303897	0.00418334	-0.00025486
4	0.03105073	-0.00200602	0.00216625	-0.00167478
5	0.02916686	0.00018617	-0.00188058	-0.00216177
6	0.03152891	0.00238279	-0.00559402	-0.00135096
7	0.03793398	0.00338780	-0.00671745	0.00029322
8	0.04657419	-0.00123649	-0.00449173	0.00183642
9	0.04824783	-0.02385171	0.00000000	0.00245633

QUENCY=, 0.49728546D 00 RADIANS/SEC
QUENCY=, 0.79145515D-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

AL LINE NO,	U	V	W	THETA
1	-0.00000061	0.00002019	-0.00000000	-0.00295480
2	0.00003774	0.00003878	-0.00531447	-0.00208195
3	0.00013639	0.00006385	-0.00749562	0.00001222
4	0.00026466	0.00007202	-0.00528043	0.00208572
5	0.00039275	0.00006911	0.00001066	0.00293158
6	0.00051468	0.00006627	0.00528181	0.00206820
7	0.00064656	0.00005767	0.00746265	-0.00000027
8	0.00078923	-0.00002952	0.00527809	-0.00207201
9	0.00031199	-0.00049505	-0.00000000	-0.00293217

QUENCY=, 0.46737979D 00 RADIANS/SEC
QUENCY=, 0.74385876D-01 CYCLES/SEC

ANE FORCE 0.1577

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	0.00020025	-0.00015064	-0.00000000	-0.17100919
2	0.00021525	-0.00025283	-0.33323195	-0.15790726
3	0.00024524	-0.00031628	-0.61554038	-0.12078947
4	0.00013704	-0.00037808	-0.80418155	-0.06543068
5	-0.00002392	-0.00036778	-0.87056439	-0.00007501
6	-0.00010115	-0.00031114	-0.80440247	0.06540258
7	-0.00006075	-0.00028398	-0.61562511	0.12093538
8	-0.00000524	-0.00033247	-0.33309462	0.15793941
9	-0.00004738	-0.00034534	-0.00000000	0.17088672

JENCY=, 0.307497100 00 RADIANS/SEC
JENCY=, 0.489397300-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

LINE NO,	U	V	W	THETA
1	-0.01702268	-0.01028438	-0.00000000	0.00055548
2	-0.01641957	-0.00045931	0.00100143	0.00039453
3	-0.01328553	0.00125520	0.00142043	0.00000212
4	-0.01087333	0.00094370	0.00100559	-0.00039626
5	-0.00982711	0.00019603	-0.00000798	-0.00056467
6	-0.01021761	-0.00055707	-0.00102591	-0.00039938
7	-0.01196653	-0.00092320	-0.00144617	0.00000155
8	-0.01450095	0.00049685	-0.00102078	0.00040159
9	-0.01495490	0.00908476	-0.00000000	0.00056656

QUENCY=, 0.478501360 00 RADIANS/SEC
QUENCY=, 0.761559300-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

LINE NO,	U	V	W	THETA
1	-0.01813622	-0.01051536	-0.00000000	-0.00565216
2	-0.01683151	0.00039112	-0.01020893	-0.00404095
3	-0.01143070	0.00302171	-0.01456122	-0.00007698
4	-0.00541352	0.00350453	-0.01043046	0.00400723
5	0.00049234	0.00350039	-0.00011130	0.00577241
6	0.00627536	0.00337763	0.01031469	0.00410191
7	0.01200296	0.00284449	0.01463878	0.00000015
8	0.01713138	0.00024625	0.01034091	-0.00406817
9	0.01836234	-0.01070800	-0.00000000	-0.00573908

QUENCY=, 0.475873680 00 RADIANS/SEC
QUENCY=, 0.757377200-01 CYCLES/SEC

ANE FORCE 0.2103

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NC,	U	V	W	THETA
1	0.00004026	-0.00010356	-0.00000000	-0.17776456
2	0.00002969	-0.00012736	-0.34643378	-0.16419315
3	0.00001315	-0.00015701	-0.64005286	-0.12565205
4	-0.00006862	-0.00017846	-0.83627707	-0.06804372
5	-0.00015064	-0.00015084	-0.90528239	-0.00005342
6	-0.00014409	-0.00009571	-0.83643717	0.06802125
7	-0.00003983	-0.00007060	-0.64014577	0.12573369
8	0.00007196	-0.00011369	-0.34639211	0.16422700
9	0.00008564	-0.00019166	-0.00000000	0.17771694

UENCY=, 0.27622664D 00 RADIANS/SEC

UENCY=, 0.43962878D-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

L LINE NC,	U	V	W	THETA
1	0.00589759	0.00360282	-0.00000000	0.00146742
2	0.00569919	0.00018750	0.00263224	0.00102411
3	0.00465569	-0.00038192	0.00368149	-0.00002773
4	0.00395751	-0.00019722	0.00254730	-0.00104272
5	0.00339305	0.00016979	-0.00005826	-0.00142608
6	0.00451693	0.00052963	-0.00259148	-0.00097389
7	0.00577393	0.00066235	-0.00359787	0.00002115
8	0.00731643	-0.00016212	-0.00252332	0.00099748
9	0.00763547	-0.00462379	-0.00000000	0.00139796

QUENCY=, 0.44569389D 00 RADIANS/SEC

QUENCY=, 0.70934454D-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

AL LINE NC,	U	V	W	THETA
1	0.00093295	0.00056915	0.00000000	-0.00114574
2	0.00089951	0.00002671	-0.00206275	-0.00081005
3	0.00072531	-0.00006991	-0.00291692	0.00000012
4	0.00059133	-0.00005186	-0.00206258	0.00080984
5	0.00053567	-0.00000785	-0.00000087	0.00114504
6	0.00056396	0.00003667	0.00206085	0.00080987
7	0.00067094	0.00005734	0.00291535	0.00000029
8	0.00082080	-0.00002690	0.00206176	-0.00080964
9	0.00084840	-0.00051840	0.00000000	-0.00114519

QUENCY=, 0.42956310D 00 RADIANS/SEC

QUENCY=, 0.68367156D-01 CYCLES/SEC

ANE FORCE 0.262)

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

L LINE NO,	U	V	W	THETA
1	0.00001920	-0.00000118	-0.00000000	-0.18541065
2	0.00001088	-0.00001208	-0.36135903	-0.17129349
3	0.00000199	-0.00001747	-0.66769583	-0.13109963
4	-0.00001618	-0.00002102	-0.87239054	-0.07095930
5	-0.00003337	-0.00001440	-0.94429040	-0.00001096
6	-0.00003028	-0.00000162	-0.87242353	0.07095429
7	-0.00000557	0.00000421	-0.66771653	0.13111474
8	0.00001979	-0.00000362	-0.36135622	0.17129904
9	0.00002484	-0.00002006	-0.00000000	0.18540639

UENCY=, 0.24095173D 00 RADIANS/SEC
UENCY=, 0.38348696D-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

L LINE NO,	U	V	W	THETA
1	0.00132620	0.00082419	0.00000000	0.00072336
2	0.00129046	0.00005465	0.00130192	0.00051087
3	0.00107492	-0.00007269	0.00183924	-0.00000135
4	0.00094051	-0.00003155	0.00129769	-0.00051139
5	0.00094473	0.00004875	-0.00000315	-0.00072138
6	0.00109801	0.00012754	-0.00129992	-0.00050830
7	0.00139169	0.00015429	-0.00183433	0.00000136
8	0.00175038	-0.00004716	-0.00129568	0.00050932
9	0.00182076	-0.00111667	-0.00000000	0.00071940

QUENCY=, 0.42018970D 00 RADIANS/SEC
QUENCY=, 0.66875331D-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

AL LINE NO,	U	V	W	THETA
1	0.00026872	0.00016598	0.00000000	-0.00019637
2	0.00026100	0.00001005	-0.00035352	-0.00013830
3	0.00021691	-0.00001617	-0.00049975	0.00000012
4	0.00018804	-0.00000876	-0.00035328	0.00013872
5	0.00018567	0.00000676	-0.00000015	0.00019616
6	0.00021262	0.00002233	0.00035320	0.00013838
7	0.00026733	0.00002777	0.00049985	0.00000014
8	0.00033473	-0.00001052	0.00035357	-0.00013883
9	0.00034737	-0.00021432	0.00000000	-0.00019639

EQUENCY=, 0.41691736D 00 RADIANS/SEC
EQUENCY=, 0.66354522D-01 CYCLES/SEC

ANE FORCE 0.3155

R OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

LINE NO,	U	V	W	THETA
1	0.00000343	0.00000273	0.00000000	-0.19767657
2	0.00000155	0.00000080	-0.38526743	-0.18262946
3	-0.00000026	0.00000036	-0.71188117	-0.13977829
4	-0.00000246	0.00000011	-0.93011789	-0.07564829
5	-0.00000440	0.00000098	-1.00675436	-0.00000108
6	-0.00000377	0.00000262	-0.93012079	0.07564784
7	-0.00000052	0.00000336	-0.71188263	0.13977960
8	0.00000263	0.00000266	-0.38526720	0.18262966
9	0.00000351	0.00000097	0.00000000	0.19767651

ENCY=, 0.199535350 00 RADIANS/SEC
ENCY=, 0.317570680-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

LINE NO,	U	V	W	THETA
1	0.00076183	0.00047312	-0.00000000	0.00002371
2	0.00073450	0.00002509	0.00004116	0.00001463
3	0.00059375	-0.00005271	0.00005153	-0.00000462
4	0.00049967	-0.00002531	0.00002672	-0.00001311
5	0.00049690	0.00003028	-0.00001090	-0.00001713
6	0.00059609	0.00008481	-0.00003504	-0.00000623
7	0.00078754	0.00010295	-0.00003608	0.00000447
8	0.00101562	-0.00002124	-0.00002098	0.00000970
9	0.00106375	-0.00065368	-0.00000000	0.00001035

UENCY=, 0.412936670 00 RADIANS/SEC
UENCY=, 0.657209760-01 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

LINE NO,	U	V	W	THETA
1	-0.00000032	-0.00000021	0.00000000	0.00000604
2	-0.00000033	-0.00000003	0.00001087	0.00000427
3	-0.00000030	0.00000000	0.00001538	0.00000000
4	-0.00000029	-0.00000000	0.00001088	-0.00000427
5	-0.00000029	-0.00000001	0.00000001	-0.00000604
6	-0.00000031	-0.00000002	-0.00001087	-0.00000428
7	-0.00000035	-0.00000002	-0.00001539	-0.00000000
8	-0.00000041	0.00000002	-0.00001089	0.00000427
9	-0.00000041	0.00000026	0.00000000	0.00000605

UENCY=, 0.379925590 00 RADIANS/SEC
UENCY=, 0.604670930-01 CYCLES/SEC

LANE FORCE 0.3680

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

L LINE NO,	U	V	W	THETA
1	0.00000014	0.00000012	-0.00000000	-0.23274237
2	0.00000007	0.00000004	-0.45360977	-0.21502601
3	-0.00000002	0.00000002	-0.83816158	-0.16457382
4	-0.00000011	0.00000002	-1.09511087	-0.08906678
5	-0.00000018	0.00000005	-1.18533948	-0.00000014
6	-0.00000015	0.00000012	-1.09511089	0.08906655
7	-0.00000002	0.00000015	-0.83816156	0.16457366
8	0.00000011	0.00000012	-0.45360977	0.21502589
9	0.00000015	0.00000005	-0.00000000	0.23274244

QUENCY=, 0.146870630 00 RADIANS/SEC
QUENCY=, 0.233752080-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

AL LINE NO,	U	V	W	THETA
1	0.00010341	0.00006388	-0.00000000	-0.00000246
2	0.00009862	0.00000217	-0.00000448	-0.00000132
3	0.00007680	-0.00000889	-0.00000669	-0.00000029
4	0.00006196	-0.00000384	-0.00000546	0.00000152
5	0.00006289	0.00000636	-0.00000096	0.00000280
6	0.00008229	0.00001639	0.00000472	0.00000257
7	0.00011788	0.00001953	0.00000805	0.00000050
8	0.00015861	-0.00000160	0.00000624	-0.00000226
9	0.00016806	-0.00010313	-0.00000000	-0.00000358

QUENCY=, 0.388870440 00 RADIANS/SEC
QUENCY=, 0.618907110-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

AL LINE NO,	U	V	W	THETA
1	-0.00000041	-0.00000025	0.00000000	-0.00000826
2	-0.00000038	-0.00000000	-0.00001487	-0.00000584
3	-0.00000029	0.00000004	-0.00002103	0.00000000
4	-0.00000023	0.00000001	-0.00001487	0.00000584
5	-0.00000025	-0.00000004	0.00000001	0.00000826
6	-0.00000037	-0.00000010	0.00001487	0.00000584
7	-0.00000058	-0.00000011	0.00002102	-0.00000000
8	-0.00000080	-0.00000000	0.00001486	-0.00000584
9	-0.00000086	0.00000052	0.00000000	-0.00000825

EQUENCY=, 0.355209690 00 RADIANS/SEC
EQUENCY=, 0.565334320-01 CYCLES/SEC

LANE FORCE 0.4206

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 1

L LINE NO,	U	V	W	THETA
1	0.00000000	0.00000000	-0.00000000	-0.57180136
2	0.00000000	0.00000000	-1.11442928	-0.52827630
3	-0.00000000	0.00000000	-2.05919695	-0.40432535
4	-0.00000000	0.00000000	-2.69047058	-0.21881950
5	-0.00000000	0.00000000	-2.91214446	-0.00000036
6	-0.00000000	0.00000000	-2.69047074	0.21881883
7	0.00000000	0.00000000	-2.05919724	0.40432484
8	0.00000000	0.00000000	-1.11442959	0.52827609
9	0.00000000	-0.00000000	-0.00000000	0.57180221

UENCY=, 0.57685382D-01 RADIANS/SEC
UENCY=, 0.91809225D-02 CYCLES/SEC

ER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 2

L LINE NO,	U	V	W	THETA
1	0.00002139	0.00001380	-0.00000000	-0.00013974
2	0.00002096	0.00000119	-0.00025163	-0.00009887
3	0.00001774	-0.00000098	-0.00035605	-0.00000013
4	0.00001589	-0.00000030	-0.00025205	0.00009877
5	0.00001629	0.00000104	-0.00000032	0.00013993
6	0.00001908	0.00000233	0.00025181	0.00009912
7	0.00002413	0.00000267	0.00035651	0.00000014
8	0.00003021	-0.00000110	0.00025223	-0.00009902
9	0.00003130	-0.00001985	-0.00000000	-0.00014012

QUENCY=, 0.33671697D 00 RADIANS/SEC
QUENCY=, 0.53590222D-01 CYCLES/SEC

BER OF ITERATIONS 5

MODAL SHAPE OF MODE NUMBER 3

AL LINE NO,	U	V	W	THETA
1	0.00021271	0.00013546	0.00000000	0.00001389
2	0.00020609	0.00000861	0.00002493	0.00000971
3	0.00016733	-0.00001551	0.00003490	-0.00000025
4	0.00013828	-0.00001111	0.00002415	-0.00000990
5	0.00012829	0.00000013	-0.00000060	-0.00001354
6	0.00013863	0.00001136	-0.00002461	-0.00000925
7	0.00016812	0.00001573	-0.00003405	0.00000025
8	0.00020723	-0.00000860	-0.00002382	0.00000944
9	0.00021393	-0.00013620	0.00000000	0.00001319

EQUENCY=, 0.36604659D 00 RADIANS/SEC
EQUENCY=, 0.58258181D-01 CYCLES/SEC

LANE FCRCE 0.4732
MA= 31 MAS= 31 GLSH= -0.78203476D 01

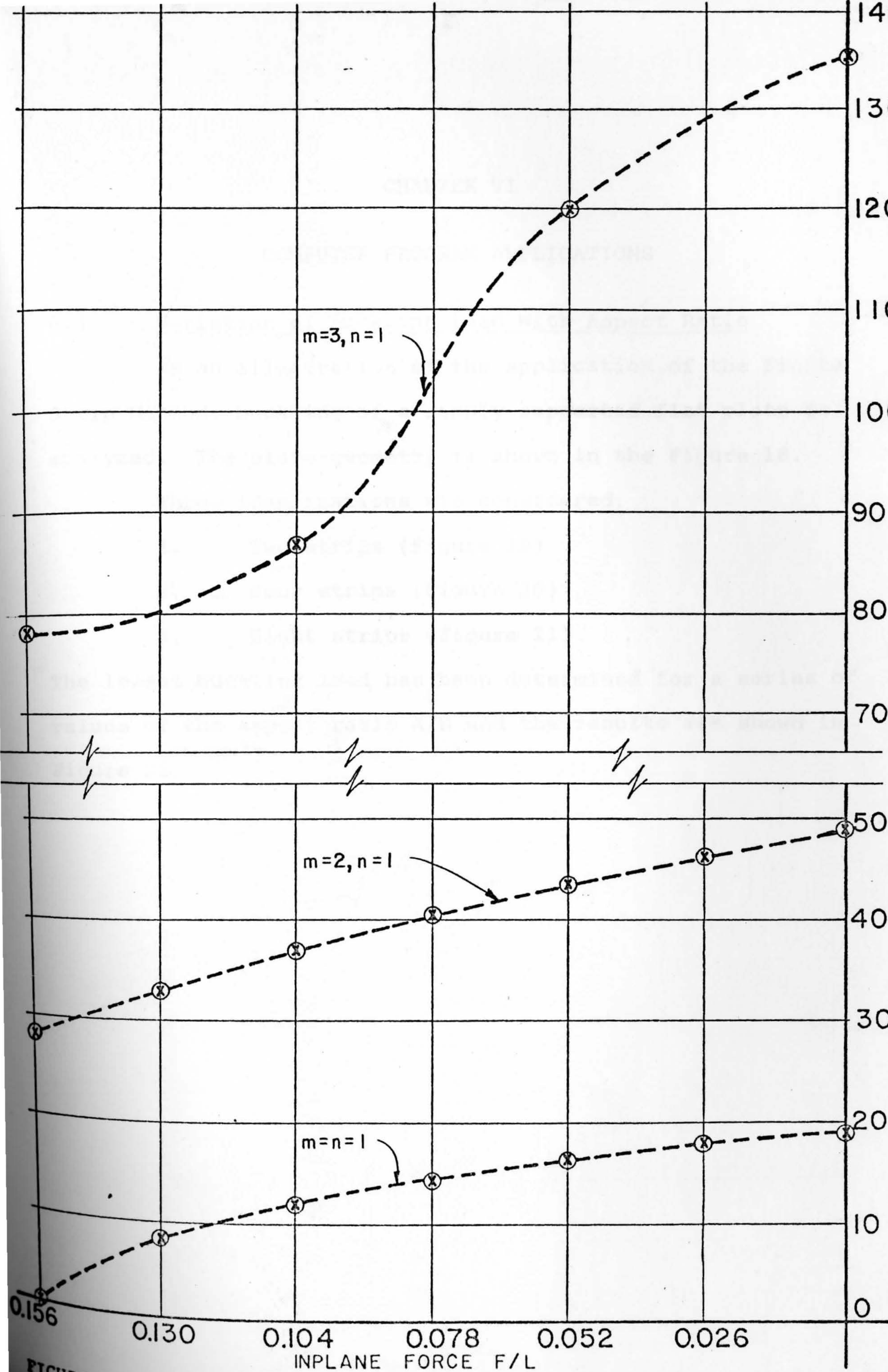

FIGURE 17 PLATE FREQUENCIES FOR VARIOUS VALUES OF INPLANE

CHAPTER VI

COMPUTER PROGRAM APPLICATIONS

6.1 Variation of Buckling Load With Aspect Ratio

As an illustration of the application of the Finite Strip Method, buckling of a simply supported flat plate is analyzed. The plate geometry is shown in the Figure 18.

Three idealizations are considered.

1. Two strips (Figure 19)
2. Four strips (Figure 20)
3. Eight strips (Figure 21)

The lowest buckling load has been determined for a series of values of the aspect ratio A/B and the results are shown in Figure 22.

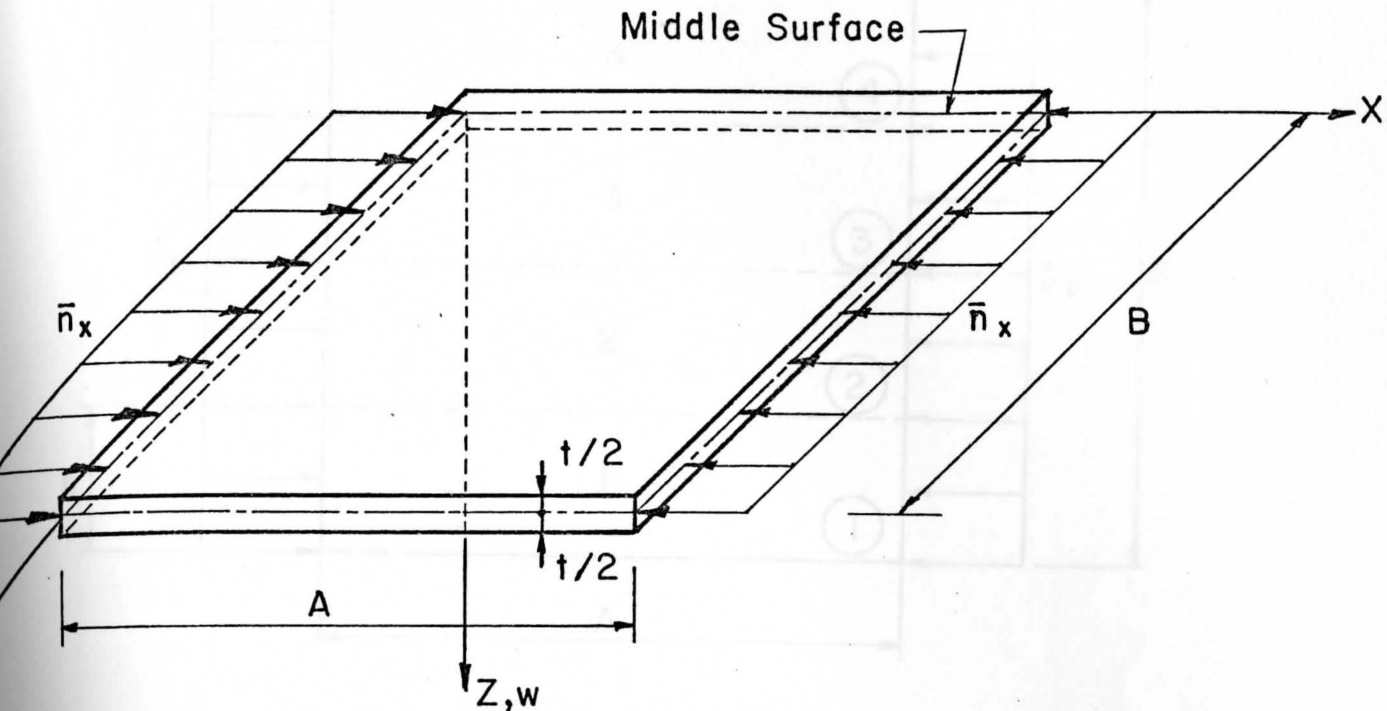

FIGURE 18 SQUARE PLATE WITH INPLANCE FORCES

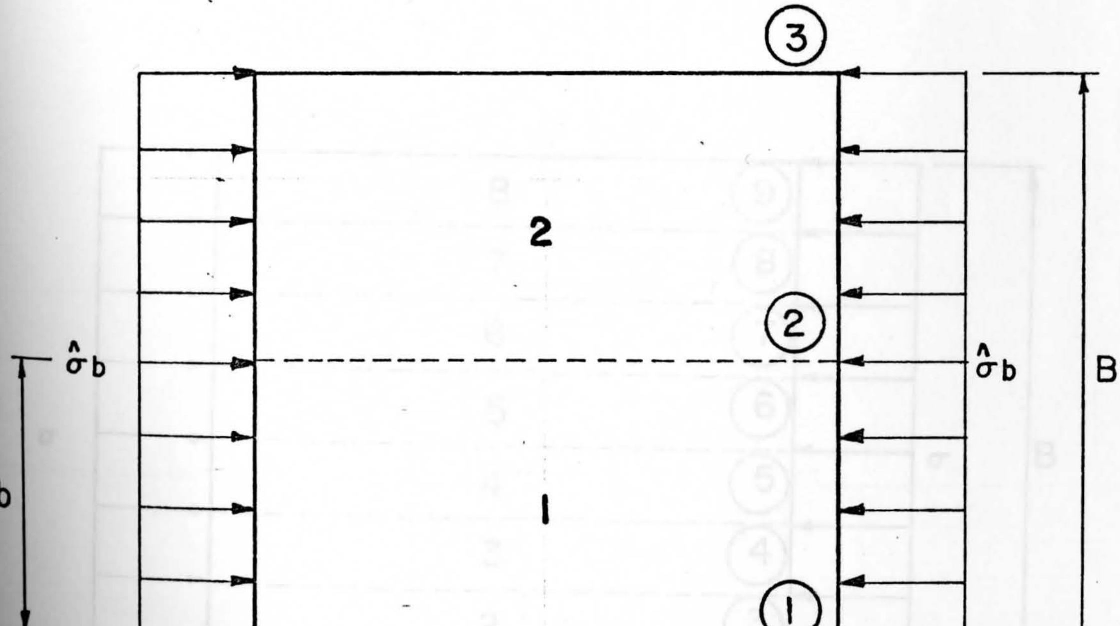

FIGURE 19 SQUARE PLATE TWO STRIPS

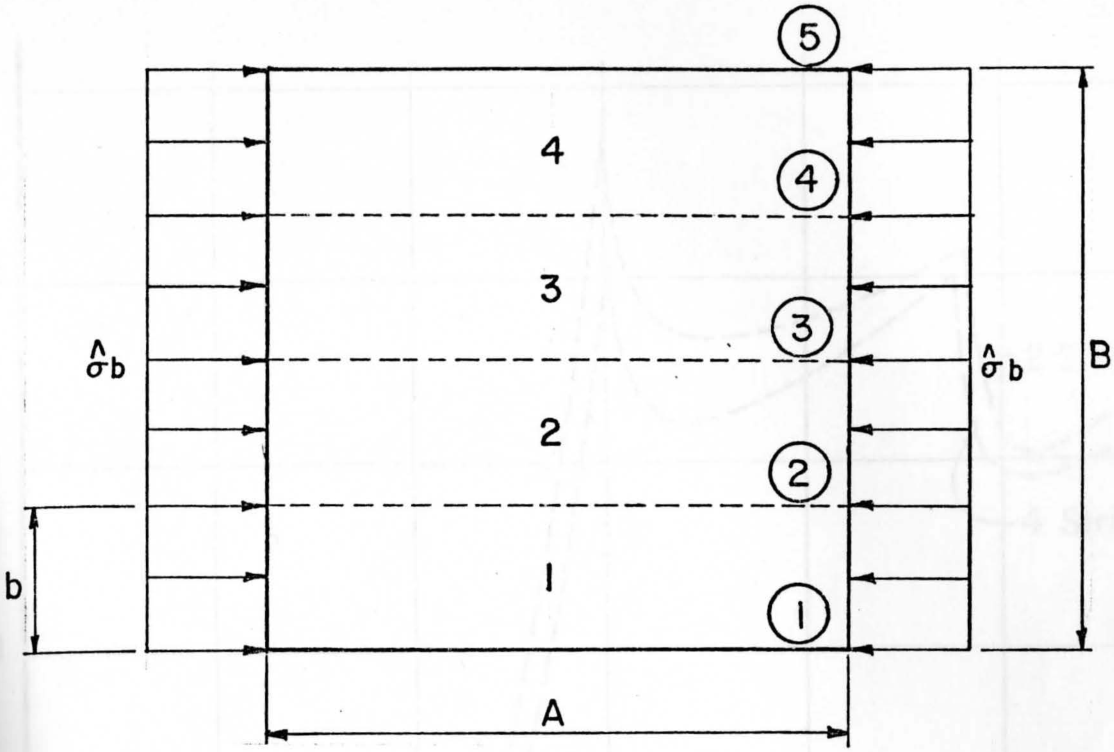

FIGURE 20 SQUARE PLATE FOUR STRIPS

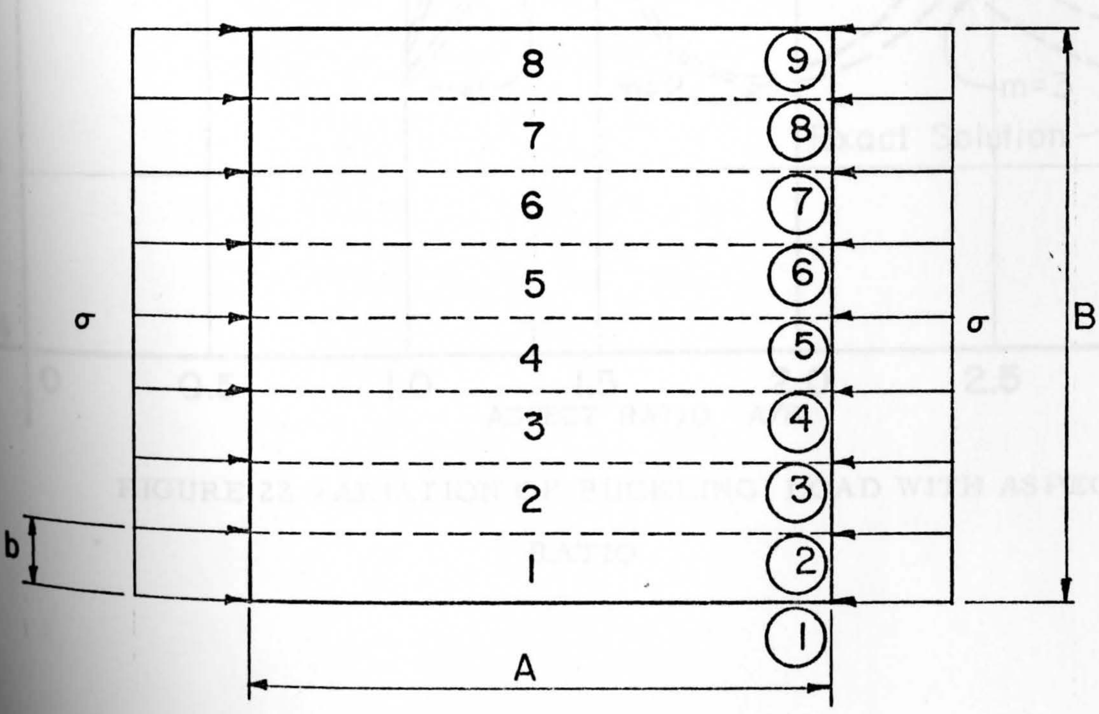

FIGURE 21 SQUARE PLATE EIGHT STRIPS

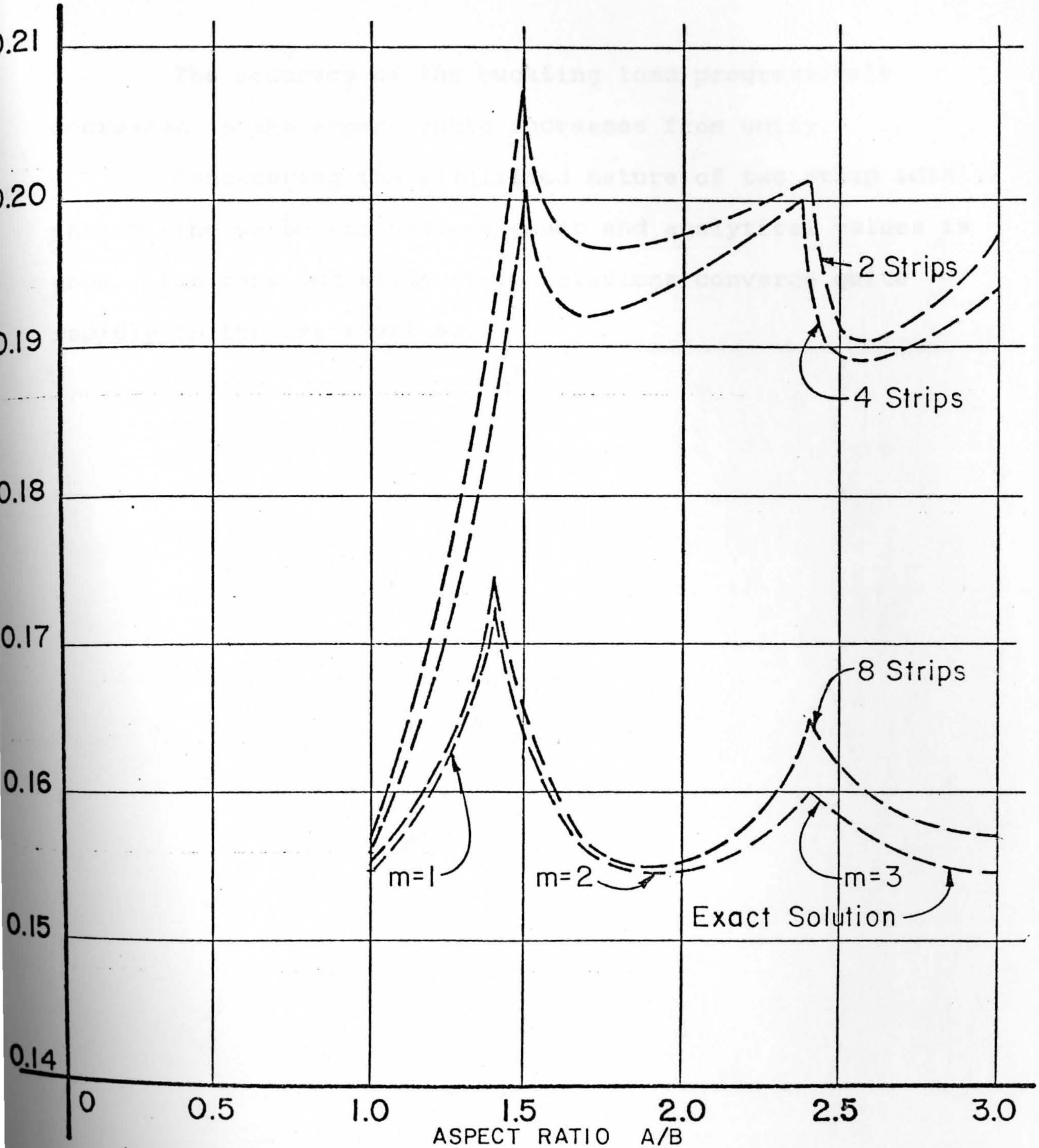

FIGURE 22 VARIATION OF BUCKLING LOAD WITH ASPECT RATIO

The accuracy of the buckling load progressively decreases as the aspect ratio increases from unity.

Considering the simplified nature of two strip idealization, the agreement between exact and analytical values is good. The four and eight strip solutions converge quite rapidly to the exact values.

6.2 Dynamic Analysis of a Slanted Plate

To demonstrate geometric rotation phenomenom of a thin plate by the Finite Strip Method, the natural frequency of a slanted plate is analyzed for various angles of inclination. The plate geometry is as shown in Figure 23. A simply supported plate of 8 strip elements is used in this example problem.

The rate frequencies are determined for different angles of rotation. A graph of angle of rotation versus frequencies is presented in Figure 24.

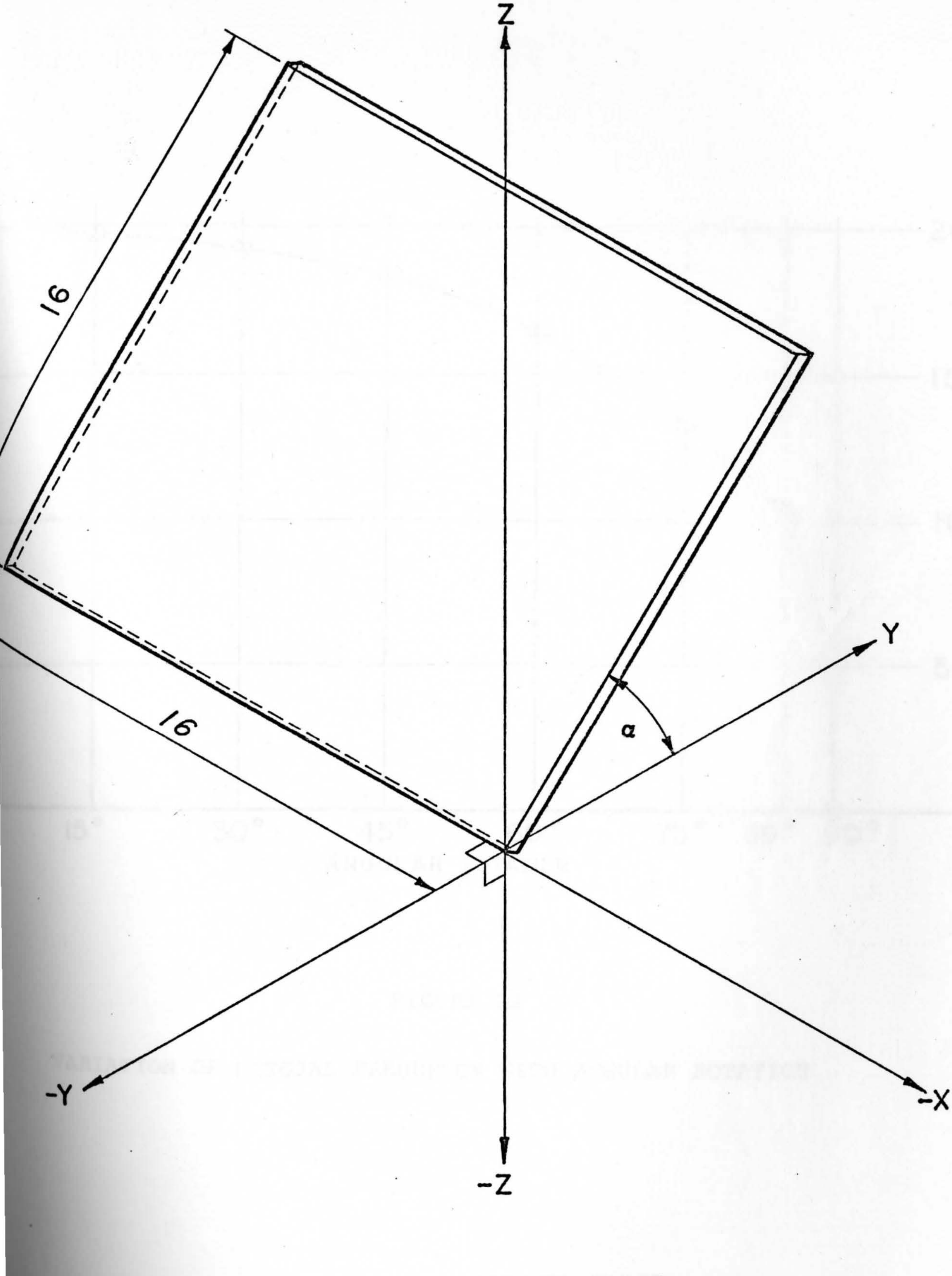

FIGURE 24 SLANTED PLATE

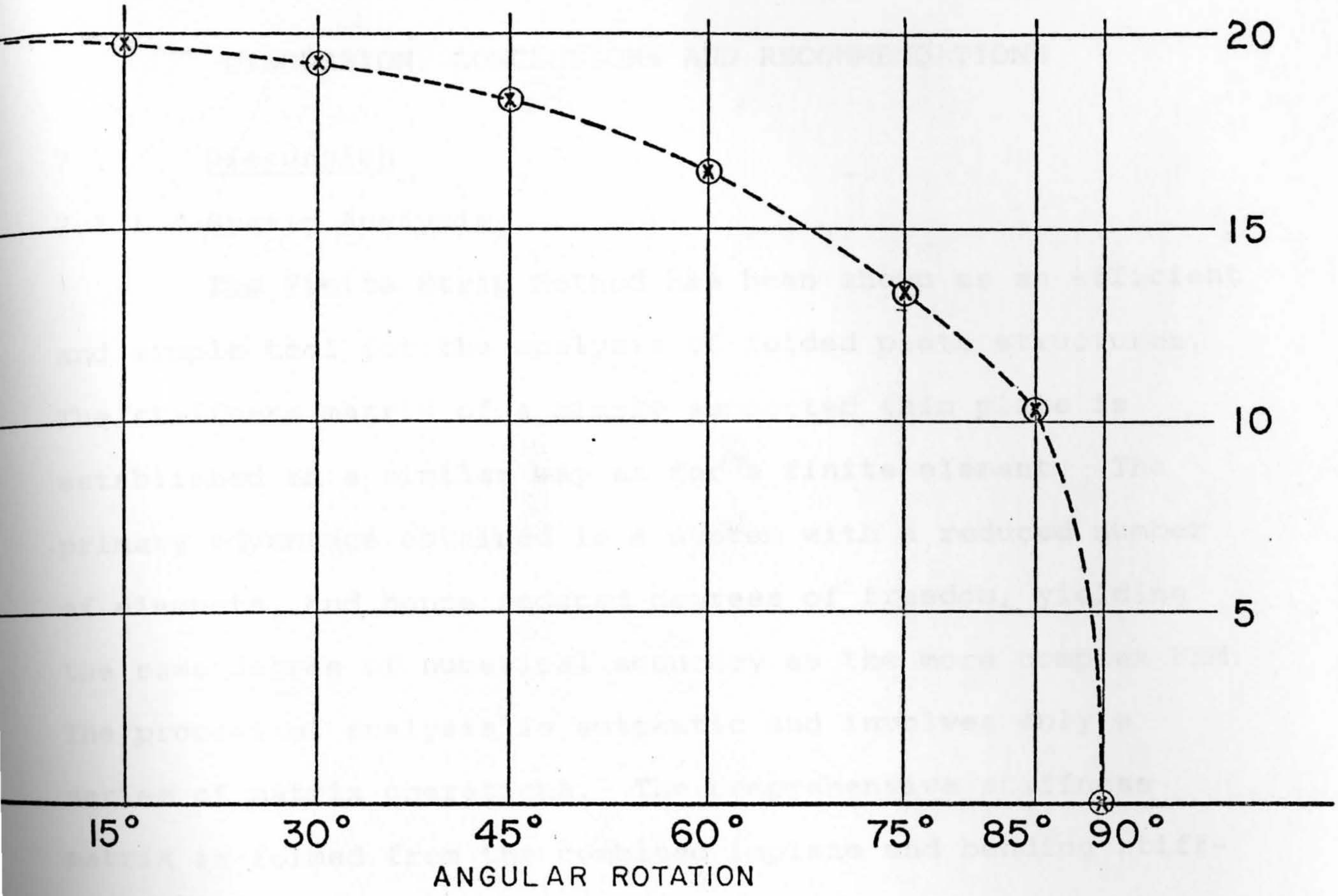

FIGURE 22

VARIATION OF NATURAL FREQUENCY WITH ANGULAR ROTATION

CHAPTER VII

DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

7.1 Discussion

7.1.1 Static Analysis

The Finite Strip Method has been shown as an efficient and simple tool for the analysis of folded plate structures. The stiffness matrix of a simply supported thin plate is established in a similar way as for a finite element. The primary advantage obtained is a system with a reduced number of elements, and hence reduced degrees of freedom, yielding the same degree of numerical accuracy as the more complex FEM. The process of analysis is automatic and involves only a series of matrix operations. The comprehensive stiffness matrix is formed from the combined inplane and bending stiffness matrices. Thus, the analytical method as well as the entire computer process can be applied to the static analysis of flat plates, inclined plates, folded plate structures and box-girder bridges.

7.1.2 Dynamic Analysis

The Finite Strip Method has been shown to be a versatile tool for vibration analysis of rectangular plates. The Mass matrix of a simply supported plate is derived. The required mathematical formulation to reduce the dynamic analysis to a classical eigenvalue-eigenvector problem is discussed. A very efficient power iteration technique is utilized to solve the eigenvalue and eigenvector problem which uses the minimum core storage. Since a finite number of natural frequencies are usually needed, the program is set up to calculate a specified number as determined by the user. Therefore the Finite Strip Method has the advantage of using relatively small matrices, small core storage, and relatively less execution time.

7.1.3 Static Stability

The Finite Strip Method is extended to determine the buckling load of a simply supported thin plate. The mathematical formulation of the geometric stiffness matrix is discussed and presented for the case of simply supported boundary conditions. The solution procedures are the same as that of dynamic analysis. The mass matrix is replaced by geometric stiffness matrix. The condition of an inclined plate is included in both the stiffness matrix and the geometric stiffness matrix. The lowest eigenvalue is the buckling load. Solutions are obtainable for flat plates, inclined plates, folded plates and box girders.

7.1.4 Dynamic Stability

The vibration of simply supported thin plates with inplane forces is discussed. The stiffness matrix of the plate is extended to include the connection between inplane forces and lateral bending. Axial inplane force is applied as a percentage of the lowest critical buckling load. When the axial force is sufficient to decrease the first frequency to zero, this condition defines the static buckling problem. An incremental technique is developed in the program to calculate the frequencies for different values of inplane load. A graph of inplane load versus frequencies is plotted. The program obtains solutions for flat plates, inclined plates, folded plates, and boxed girders.

7.2 Conclusions

It has been demonstrated that the Finite Strip Method is a simple, powerful and efficient tool for the analysis of plate problems including static analysis, dynamic analysis, and static and dynamic stability. This method has the ability to be easily programmed on a relatively small computer (say IBM 1130 with 81K words) which is readily accessible to consulting engineers. The matrices are relatively smaller than that required when solved by the finite element or finite difference methods. This condition is due to the fact that the two dimensional problem (i.e. x,y) is reduced to a one dimensional problem (i.e. y) by the Kantorovich approach.

7.3 Recommendations

It has been shown in our previous discussion that for any one term of the series the accuracy of the results is quite comparable to exact analytical solutions.

It is the observation of this author that future work on this problem should investigate the following:

1. The stiffness matrix can be modified to include additional terms in the series terms of the displacemen function.
2. The problem can be extended to include other boundary conditions: fixed, free.
3. The method can be extended to curved strips, cylindrical shell strip, skew plates, continuous plates and mixed strips, by modifying terms in the stiffness mass, and geometric stiffness matrix.
4. The program as currently written applies a prescribed boundary condition, by multiplying the corresponding diagonal terms by large number (i.e. 10). This, in turn, makes the required displacement zero. However, the original size of the matrix is maintained throughout the operation. An investigation of the reduction in size of the [H] matrix to reflect the boundary condition (similar to Finite Element Method) should be of interest.

APPENDIX "1"

User Instructions for the "FINSTRIP" Program

Card Type 1

1 card with NPROB Format (I5)

col. 1 - 5 Number of problems to be solved.

Card Type 2

1 card with TITLE Format (12A6)

col. 1 - 72 Title of the problem.

Card Type 3

1 card with Control parameters Format (10I4)

col. 1 - 4 NTERM, number of terms in series

col. 5 - 8 NELEM, number of elements

col. 9 - 12 NP, number of nodal lines

col. 13 - 16 NBOUN, number of boundary conditions

col. 17 - 20 NDF, number of degrees of freedom/nodal line

col. 21 - 24 NJ, load type (1 = non symm., 2 = symm.)

col. 25 - 28 NPROB, problem type

1 - Vibration problem - (Dynamic Analysis)

2 - Stability problem - (Static Stability)

3 - Vibration with inplane forms - (Dynamic Stability)

col. 28 - 32 NIT, number of iterations

col. 32 - 36 MODE, modal shape of mode number col. 36 - 40 IPRINT, print option

1 - Print data

2 - No data print

Card Type 4

Plate Geometry

Length of plate. Format (F10.4)

Card Type 5

1 card per nodal line Format (2F10.4)

col. 1 to 10 - X coordinate

col. 11 to 20 - Y coordinate

Card Type 6

1 card per strip element Format (3I4, 1F16.8)

col. 1 to 4 - Strip number

col. 5 to 8 - Node i

col. 9 to 12 - Node j

col. 12 to 28 - Thickness of strip

Card Type 7

1 card per boundary nodal line Format (5I4)

col. 1 to 4 - Boundary nodal line

Numbers giving boundary condition for each degree of freedom of that boundary line.

0 - Indicates degree of freedom fixed.

1 - Indicates degree of freedom free.

col. 4 to 8 for u

col. 9 to 12 for v

col. 12 to 16 for w

col. 16 to 28 for θ

Card Type 8

1 card for material properties Format (7F10.4)

col. 1 to 10 - E1, Modulus of Elasticity X

col. 11 to 20 - E2, Modulus of Elasticity Y

col. 21 to 30 - Px, Poisson's ratio x

col. 31 to 40 - Py, Poisson's ratio y

col. 41 to 50 - G, Modulus of Elasticity in shear

col. 51 to 60 - RHO, Plate mass density

Card Type 9

1 card with number of plot points Format (I5)

(Only for problems of dynamic stability)

(Blank card for MPROB = 1 and MPROB = 2)

col. 1 to 5 - Number of plot points required for NTERM = 1

col. 6 to 10 - Number of plot points required for NTERM = 2 and so on.

Listings of Routines

Listings of Master segment, GDATA, INITIL, COMPLE, BOUN, PRINTR, and EIGEN are given.

The listings of the Master segment indicates how all the other subprograms are called into use;

GDATA - Gathers data.

INITIL - Initializes the SM, SG, ST matrices.

COMPLE - Compiles stiffness, geometric stiffness, and mass matrices.

IPRINT - Can print all the intermediate matrices. (If required)

EIGEN - Solution of Eigen value - Eigen vector problem associated with dynamic analysis. The routine is programmed to analyze free vibration, static stability, and dynamic stability problems.

Output

A data list together with the output is given for the example problem of a simply supported, square plate (lower order strip) shown in Figure 7, Page 52.

```
MASTER
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION S(4,4),C(4,4),D(4,4),E(4,4),F(4,4),PM(4,4),BM(4,4),
PG(4,4),BS(4,4),ST(37,37),SG(37,37),SM(37,37)
COMMON/CONTRL/NTERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
COMMON/NPI/A,B,BB,BN1,BN2,BNII,CO,R(4,4)
COMMON/NODEN/X(50),Z(50),T(50),NOD(50,2)
COMMON/BOUND/NF(8),NB(8,4)
COMMON/GLOBL/ST,SM,SG
COMMON NEQ,NE,H,E1,E2,IPN(10),BUCP

LYSIS OF VIBRATION,STABILITY, & VIBRATION WITH INPLANE FORCES OF PLATES

MASTER ROUTINE READS IN ALL OTHER ROUTINES


VARIABLE                    DEFINITION

X(50),Z(50)                 X AND Z COORDINATES OF NODAL POINTS
T(50)                       THICKNESS OF STRIP
ST(37,37)                   STRUCTURE STIFFNESS MATRIX
SG(37,37)                   GEOMETRIC STIFFNESS MATRIX
SM(37,37)                   STRUCTURE MASS MATRIX
NF(8)                       RESTRAINED BOUNDARY NODE NUMBERS
NB(8,4)                     BOUNDARY CONDITION TYPE FOR U,V,W AND THETA
                            (0 = FIXED, 1 = FREE)
NOD(50,2)                   NODE NUMBERS OF ALL THE STRIPS
NTERM                       NUMBER OF TERMS
NELEM                       NUMBER OF ELEMENTS
NP                          NUMBER OF NODAL LINES
NBOUN                       NUMBER OF RESTRAINED BOUNDARY POINTS
NDF                         NUMBER OF DEGREES OF FREEDOM PER NODAL LINE
NIT                         NUMBER OF ITERATIONS
MPROB                       VIB=1, STAB=2, VIB+INPLANE= 3
NI                          LOAD TYPE ( 1=NON-SYMMETRICAL , 2=SYMMETRICAL)
                            OR LINE LOADS

  WRITE(6,111)
  WRITE(6,112)
  WRITE(6,113)
  WRITE(6,114)
  READ(5,1) NPROB
  WRITE(6,8)NPROB
8 FORMAT(// ' TOTAL NUMBER OF PROBLEMS = 'I5)
  DO 400 NPR=1,NPROB
  WRITE(6,5)NPR
5 FORMAT(1H1 'PROBLEM NUMBER = 'I5)
  CALL G DATA
  DO 600 II=1,NTERM,NI
  CALL INITIL
  CALL COMPLE
  CALL BOUN
  CALL PRINTR
```

```
CALL EIGEN
CONTINUE
CONTINUE
GO TO 200
FORMAT(915)
FORMAT( '1FINITE STRIP EIGEN VALUE SYSTEM')
FORMAT('-WM RAYEN SCHOOL OF ENGR')
FORMAT('-YOUNGSTOWN STATE UNIVERSITY')
FORMAT('-YOUNGSTOWN OHIO')
STOP
END
```

```
      SUBROUTINE GDATA
      IMPLICIT REAL * 8(A-H,C-Z)
      DIMENSION S(4,4),C(4,4),D(4,4),E(4,4),F(4,4),PM(4,4),BM(4,4),
     1PG(4,4),SS(4,4),ST(37,37),SG(37,37),SM(37,37)
      REAL*8 TITLE(12)
      COMMON/CONTRL/NTERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
      COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
      COMMON/NPI/A,B,BB,BN1,BN2,BNII,CO,R(4,4)
      COMMON/NODEN/X(50),Z(50),T(50),NOD(50,2)
      COMMON/BOUND/NF(8),NB(8,4)
      COMMON/GLOBL/ST,SM,SG
      COMMON NEW,NE,H,E1,E2,IPN(10),BUCP
C
C     SUBROUTINE GDATA GATHERS DATA
C
      READ (5,7)  TITLE
      WRITE(6,8)  TITLE
      WRITE (6,4)
      READ (5,11) NTERM,NELEM,NP,NBOUN,NDF,NI,MPROB,NIT,MODE,IPRINT
      WRITE(6,12) NTERM,NELEM,NP,NBOUN,NDF,NI,MPROB,NIT,MODE,IPRINT
      READ  (5,35) A
      WRITE (6,36) A
C
C     READ X AND Z COORDINATES OF ALL POINTS
C
      WRITE(6,37)
      DO 2 I=1,NP
      READ (5,35) X(I),Z(I)
    2 WRITE(6,33)I,X(I),Z(I)
C
C     READ NUMBER NODAL NUMBERS THICKNESS
C
      WRITE(6,44)
      DO 109 I=1,NELEM
      READ (5,45) NUM,(NOD(I,J),J=1,2),T(I)
  109 WRITE(6,43) NUM,(NOD(I,J),J=1,2),T(I)
C
C     READ BOUNDARY CONDITIONS 0=FIXED 1=FREE
C
      WRITE(6,46)
      READ (5,19)  (NF(I),(NB(I,J),J=1,4),I=1,NBOUN)
      WRITE(6,18)  (NF(I),(NB(I,J),J=1,4),I=1,NBOUN)
C
C     READ YOUNG'S MODULUS E1 E2 POISSON'S RATIO PX PY SHEAR MODULUS G
C
      READ (5,35)E1,E2,PX,PY,G,RHO
      WRITE(6,50) E1,E2,PX,PY,G,RHO
C
C     READ NUMBER OF PLOTS REQUIRED FOR VIB +INPLANE CASE
C
      READ(5,40)(IPN(I),I=1,NTERM)
      WRITE(6,55)
      WRITE(6,40)(IPN(I),I=1,NTERM)
      EX = E1/(1.-PX*PY)
```

```
EY= E2/(1.-PX*PY)
WRITE(6,60)
FORMAT( '    INPUT  DATA    ')
FORMAT(12A6)
FORMAT(1H0,12A6)
FORMAT(12I4)
FORMAT(/6X,'*** CONTROL PARAMETERS *** ',
X       /1X,'NUMBER OF TERMS                                     ',I4,
X       /1X,'NUMBER OF ELEMENTS                                  ',I4,
X       /1X,'NUMBER OF NODAL LINES                               ',I4,
X       /1X,'NUMBER OF BOUNDARY CONDITIONS                       ',I4,
X       /1X,'NUMBER OF DEGREES OF FREEDOM/NODAL LINE             ',I4,
X       /1X,'LOAD TYPE(1=NON SYMM, 2=SYMM)                       ',I4,
X       /1X,'MPROB PROBLEM TYPE                                  ',I4,
X       /1X,'(1=FREE VIBRATION,2=STABILITY, 3=VIB+IN PLANE)',
X       /1X,'NUMBER OF ITERATIONS                                ',I4,
X       /1X,'MODAL SHAPE OF MODE NUMBER                          ',I4,
X       /1X,'IPRINT(1=PRINT DATA,0=NO PRINT)                     ',I4,
X       /1X)
 FORMAT(5I10)
 FORMAT(5I4)
 FORMAT(7F10.4)
 FORMAT(/6X,'***     PLATE GEOMETRY   ***',
X       /1X,'LENGTH OF PLATE                                     ',F10.4)
 FORMAT(/1X,'NODE LINE NO  XCCORD    YCOORD')
 FORMAT(I10,2F10.4)
 FORMAT(10I5)
 FORMAT(/1X,'STRIP NO  NODE1  NODE2    THICKNESS')
 FORMAT(3I4,2F16.8)
 FORMAT(/6X,'*** BOUNDARY CONDITIONS ***   ',
X       /1X,'NODE LINE NO       U          V          W         THETA',
X       /1X)
3 FORMAT(3I7,2F16.8)
0 FORMAT(/6X,'*** MATERIAL PROPERTIES ***   ',
X       /1X,'MODULUS OF ELASTICITY 1                             ',F10.4
X       /1X,'MODULUS OF ELASTICITY 2                             ',F10.4
X       /1X,'POISSONS RATIO X                                    ',F10.4
X       /1X,'POISSONS RATIO Y                                    ',F10.4
X       /1X,'MODULUS OF ELASTICITY IN SHEAR G                    ',F10.4
X       /1X,'PLATE MASS DENSITY RHO                              ',F10.4
X       /1X)
5 FORMAT(/6X'*** NUMBER OF PLOT POINTS REQD *** ')
0 FORMAT (1H1 '       *** RESULTS OF ANALYSIS *** ')
  RETURN
  END
```

```
SUBROUTINE INITIL
IMPLICIT REAL * 8(A-H,O-Z)
DIMENSION S(4,4),C(4,4),D(4,4),E(4,4),F(4,4),PM(4,4),BM(4,4),
PG(4,4),BG(4,4),ST(37,37),SG(37,37),SM(37,37)
COMMON/CONTRL/NTERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
COMMON/NPI/A,B,BB,BN1,BN2,BNII,CO,R(4,4)
COMMON/NODEN/X(50),Z(50),T(50),NOD(50,2)
COMMON/BOUND/NF(8),NB(8,4)
COMMON/GLOBL/ST,SM,SG
COMMON NEQ,NE,H,E1,E2

SUBROUTINE INITIL INITIALIZES SM,SG,ST MATRICES

NE=0
NEQ = NDF*NP
DO  8 I=1,NEQ
DO 8 J=1,NEQ
SM(I,J)=0
SG(I,J)=0.
ST(I,J)=0.
BNII = 3.14159*II
CO = BNII*BNII
BN1 = BNII/A
BN2 = BN1*BN1
RETURN
END
```

```
      SUBROUTINE COMPLE
      IMPLICIT REAL * 8(A-H,O-Z)
      DIMENSION S(4,4),C(4,4),D(4,4),E(4,4),F(4,4),PM(4,4),BM(4,4),
     1PG(4,4),BG(4,4),ST(37,37),SG(37,37),SM(37,37)
      COMMON/CONTRL/NTERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
      COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
      COMMON/NP1/A,B,BB,BN1,BN2,BNII,CO,R(4,4)
      COMMON/NODEN/X(50),Z(50),T(50),NOD(50,2)
      COMMON/BOUND/NF(8),NB(8,4)
      COMMON/GLOBL/ST,SM,SG
      COMMON NEQ,NE,H,E1,E2

      SUBROUTINE COMPLE COMPILES STIFFNESS,GEOMETRIC,MASS MATRICES

      DO 70 LK=1,NP
      IF(IPRINT.EQ.1)
     1WRITE(6,150) LK
      IF (LK .GT. 1) GO TO 9
    1 NE = NE +1
      IF ( NE - NELEM) 9,9,92
    9 IF ( NOD(NE,1) - LK) 92,3,92
    3 N1 = NOD(NE,1)
      N2 = NOD(NE,2)
      XP = X(N2) - X(N1)
      ZP = Z(N2) - Z(N1)
      H = T(NE)
      B = DSQRT(XP*XP + ZP*ZP)
      BB = B*B
      DX=E1*H**3/(12.*(1.-PX*PY))
      DY=E2*DX/E1
      DXY=G*DX/E1
      D1=PX*E2*DX/E1
      HXY=D1+4*DXY
      IF(IPRINT.EQ.1)
     1WRITE(6,152)
      CALL       TRAN(  XP,ZP)
      IF(IPRINT.EQ.1)
     1WRITE(6,155)((R(I,J),J=1,4),I=1,4)
  155 FORMAT(4E15.7)
      CALL ELASTK(E,F)
      IF(MPROB.NE.1)CALL KGEOMT(PG,BG)
      IF(MPROB.NE.2)CALL MASS(PM,BM)
      CALL ASMBLE(ST,E,F)
      IF(MPROB.NE.1)CALL ASMBLE(SG,PG,BG)
      IF(MPROB.NE.2)CALL ASMBLE(SM,PM,BM)
      GO TO 1
   92 CONTINUE
   70 CONTINUE
  150 FORMAT(' LK= ',I2//)
  152 FORMAT('0CALL TRAN R MATRIX'/)
      RETURN
      END
```

```
      SUBROUTINE ASMBLE(ST,E,F)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION S(4,4),C(4,4),D(4,4),E(4,4),F(4,4),PM(4,4),BM(4,4),
     1PG(4,4),BG(4,4),ST(37,37),SG(37,37),SM(37,37)
      COMMON/CONTRL/NTERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
      COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
      COMMON/NPI/A,B,BB,BN1,BN2,BNII,CO,R(4,4)
      COMMON/NODEN/X(50),Z(50),T(50),NOD(50,2)
      COMMON NEJ,NE,H,E1,E2
      DO 80 LL=1,2
      J1 = 2*LL -1
      J2 = J1 + 1
      J = NDF*(NOD(NE,LL) - 1)
      DO 80 KK=1,2
      DO 81 K=1,NDF
      DO 81 L=1,NDF
      D(K,L) =0.
      I1 = 2*KK-1
      I2 = I1 + 1
      I = NDF*(NOD(NE,KK) - 1)
      D(1,1) = E(I1,J1)
      D(2,1) = E(I2,J1)
      D(1,2) = E(I1,J2)
      D(2,2) = E(I2,J2)
      D(3,3) = F(I1,J1)
      D(4,3) = F(I2,J1)
      D(3,4) = F(I1,J2)
      D(4,4) = F(I2,J2)
      CALL M3TM(D,R,C,4,4,4)
      CALL M3TTM(R,C,D,4,4,4)
      IF(IPRINT.EQ.1)
     1WRITE(6,188) LL,KK
      IF(IPRINT.EQ.1)
     1WRITE(6,190)
      IF(IPRINT.EQ.1)
     1WRITE(6,155)((D(IPRT,JPRT),JPRT=1,4),IPRT=1,4)
      DO 5 NJ = 1,NDF
      JN = J + NJ
      DO 5 MI = 1,NDF
      IM = MI + I
    5 ST(IM,JN) = ST(IM,JN) + D(MI,NJ)
   80 CONTINUE
  188 FORMAT('0LL='I2,'KK='I2)
  190 FORMAT('TRANSFORMED INDIVIDUAL STIFFNESS'/)
  155 FORMAT(4E16.7)
      RETURN
      END
```

```
      SUBROUTINE ELASTK(E,F)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION E(4,4),F(4,4)
      IF(IPRINT.EQ.1)
     1WRITE(6,180)
C
C     FORMS INPLANE STIFFNESS MATRIX
C
      CALL FEMP(E)
      IF(IPRINT.EQ.1)
     1WRITE(6,155)((E(I,J),J=1,4),I=1,4)
      IF(IPRINT.EQ.1)
     1WRITE(6,170)
C
C     FORMS BENDING STIFFNESS MATRIX
C
      CALL FEMS(F)
      IF(IPRINT.EQ.1)
     1WRITE(6,155)((F(I,J),J=1,4),I=1,4)
  170 FORMAT('0 ALL FEMS BENDING STIFFNESS MATRIX'/)
  180 FORMAT('0 ALL FEMP PLANE STIFFNESS MATRIX'/)
  155 FORMAT(4E16.7)
      RETURN
      END
```

```
      SUBROUTINE FEMP(E)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION E(4,4)
      COMMON/CONTRL/NTERM,NELEM,NP,NBCUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
      COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
      COMMON/NPI/A,B,BB,BN1,BN2,BNII,CO,R(4,4)
      COMMON NEQ,NE,H
C
C     IN-PLANE STIFFNESS MATRIX E(4,4) OF STRIP
C
      E(1,1) = EX*.5*A/B  +B*BN2*A*G/6.
      E(2,1) = .25*BN1  *A*PX*EY - .25*BN1 *A*G
      E(3,1) = -.5*A*EX/B + B*A*BN2*G/12.
      E(4,1) = .25*A*BN1 *PX*EY + .25*A*BN1 *G
      E(2,2) =A*B*BN2*EY/6.   +  .5*A*G/B
      E(3,2) = -.25*A*BN1 *PX*EY - .25*A*BN1 *G
      E(4,2) =A*B*BN2*EY/12. - .5*A*G/B
      E(3,3) = .5*A*EX/B +A*B*BN2*G/6.
      E(4,3) = -.25 *A*BN1 *PX*EY + .25*A*BN1 *G
      E(4,4) =A*B*EY*BN2/6.+ .5*A*G/B
      DO 20 I=1,4
      DO 20 J=1,I
      E(I,J)=E(I,J)*H
   20 E(J,I) = E(I,J)
      RETURN
      END
```

```
      SUBROUTINE FEMS(F)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION D(4,4),E(4,4),C(4,4),F(4,4)
      COMMON/CONTRL/NTERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
      COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
      COMMON/NPI/A,B,BB,BN1,BN2,BNII,CO,R(4,4)
C
C     BENDING STIFFNESS MATRIX F(4,4) OF STRIP
C
      AA = A*A
      BBBB= BB*BB
      COCO = CO*CO
      BBB = B*BB
      D(1,1) = COCO*DY*B*.5/AA
      D(2,1) =    COCO*BB*DY*.25/AA
      D(3,1) = -CO*B*D1    +   .166667*COCO*BBB*DY/AA
      D(4,1) = -1.5*CO*BB*D1 +  .125*COCO*BBBB*DY/AA
      D(2,2) =    .166667*COCO*BBB*DY/AA  +  2.*CO*B*DXY
      D(3,2) =-CO*BB*D1*.5+.125*COCO*BBBB*DY/AA   +   2.*CO*BB*DXY
      D(4,2) = -CO*BBB*D1   +  .1*COCO*BBBB*B*DY/AA + 2.*CO*BBB*DXY
      D(3,3) =  2.*AA*B*DX -.6666667*CO*BBB*D1   +
     1 .1*COCO*BBBB*B     *DY/AA    + 2.66667*CO*BBB*DXY
      D(4,3) = 3.*AA*BB*DX    -CO*BBBB*D1   +   .0833333*COCO*BBBB*BB*DY/AA
     1 +   3.*CO*BBBB*DXY
      D(4,4) = 4.*BBB*DX*AA    - 1.2*CO*BBBB*B*D1    + .0714286*COCO*BBB
     1*BBBB*DY/AA+ 3.6*CO*BB*BBB*DXY
      DO 5 I=1,4
      DO 5 J=1,I
      D(I,J) = D(I,J)/A
    5 D(J,I) = D(I,J)
      CALL EM(B,BB,BBB,E)
      CALL MBTM(D,E,C,4,4,4)
      CALL MBTTM(E,C,F,4,4,4)
      RETURN
      END
```

```
      SUBROUTINE MASS(PM,BM)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION PM(4,4),BM(4,4)
C
C     FORMS INPLANE MASS MATRIX
C
      CALL FSIM(PM)
C
C  FORMS BENDING MASS MATRIX
C
      CALL FSBM(BM)
      RETURN
      END
```

```
      SUBROUTINE FSIM(F)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION F(4,4)
      COMMON/CONTRL/NTERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
      COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
      COMMON/NP1/A,B,BB,BN1,BN2,BNII,CD,R(4,4)
      COMMON NEQ,NE,H,E1,E2
C
C     IN PLANE MASS MATRIX
C
      C1=BN1
      C2=BN1
      F(1,1)=B/3.
      F(2,1)=0.
      F(3,1)=B/6.
      F(4,1)=0.
      F(2,2)=B/3*C1*C2
      F(3,2)=0.
      F(4,2)=B/6*C1*C2
      F(3,3)=B/3.
      F(4,3)=0.
      F(4,4)=B/3*C1*C2
      H1=RHO*H*A/2.
      DO 20 I=1,4
      DO 20 J=1,I
      F(I,J)=F(I,J)*H1
   20 F(J,I)=F(I,J)
      RETURN
      END
```

```
SUBROUTINE FSBM(E)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION E(4,4)
COMMON/CONTRL/NTERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
COMMON/NP1/A,B,BB,BN1,BN2,BN11,CO,R(4,4)
COMMON NEQ,NE,H,E1,E2

BENDING MASS MATRIX OF A STRIP

E(1,1)=13.*B/35.
E(2,1)=11.*B*B/210.
E(3,1)=9.*B/70.
E(4,1)=-13*B*B/420.
E(2,2)=B*B*B/105.
E(3,2)=13*B*B/420.
E(4,2)=-3*B*B*B/420.
E(3,3)=13*B/35.
E(4,3)=-11*B*B/210.
E(4,4)=B*B*B/105.
H1=RHO*H*A/2.
DO 20 I=1,4
DO 20 J=1,I
E(I,J)=E(I,J)*H1
E(J,I)=E(I,J)
RETURN
END
```

```
      SUBROUTINE KGEOMT(PG,BG)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION PG(4,4),BG(4,4)
C
C     FORMS INPLANE GEOMETRIC MATRIX
C
      CALL FSIG(PG)
C
C     FORMS BENDING GEOMETRIC MATRIX
C
      CALL FSBG(BG)
      RETURN
      END
```

```
      SUBROUTINE FSIG(F)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION F(4,4)
      COMMON/CONTRL/NTERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
      COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
      COMMON/NPI/A,B,BB,BN1,BN2,BNII,CO,R(4,4)
C
C     PLANE GEOMETRIC MMATRIX
C
      C1=BN1
      C2=BN1
      F(1,1)=B/3.
      F(2,1)=0.
      F(3,1)=B/6.
      F(4,1)=0.
      F(2,2)=B/3
      F(3,2)=0.
      F(4,2)=B/6
      F(3,3)=B/3.
      F(4,3)=0.
      F(4,4)=B/3
      H1=C1*C2*A/2
      DO 20 I=1,4
      DO 20 J=1,I
      F(I,J)=F(I,J)*H1
      F(J,I)=F(I,J)
      RETURN
      END
```

```
SUBROUTINE FSBG(E)
IMPLICIT REAL * 8(A-H,O-Z)
DIMENSION E(4,4)
COMMON/CONTRL/ITERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
COMMON/NPI/A,B,BB,BN1,BN2,BNII,CO,R(4,4)

ING GEOMETRIC MATRIX

C1=BN1
C2=BN1
E(1,1)=13.*B/35.
E(2,1)=11.*B*B/210.
E(3,1)=9.*B/70.
E(4,1)=-13*B*B/420.
E(2,2)=B*B*B/105.
E(3,2)=13*B*B/420.
E(4,2)=-3*B*B*B/420.
E(3,3)=13*B/35.
E(4,3)=-11*B*B/210.
E(4,4)=B*B*B/105.
H1=C1*C2*A/2
DO 20 I=1,4
DO 20 J=1,I
E(I,J)=E(I,J)*H1
E(J,I)=E(I,J)
RETURN
END
```

```
SUBROUTINE EM(B,BB,BBB,E)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION E(4,4)
E(1,1) = 1.
E(2,1) = 0.
E(3,1) = -3./BB
E(4,1) = 2./BBB
E(1,2) = 0.
E(2,2) = 1.
E(3,2) = -2./B
E(4,2) = 1./BB
E(1,3) = 0.
E(2,3) = 0.
E(3,3) = 3./BB
E(4,3) = -2./BBB
E(1,4) = 0.
E(2,4) = 0.
E(3,4) = -1./B
E(4,4) = 1./BB
RETURN
END
```

```
SUBROUTINE TRAN(XP,ZP)
IMPLICIT REAL*8(A-H,O-Z)
COMMON/CONTRL/ITERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
COMMON/NPI/A,B,BB,BN1,BN2,BNII,CO,R(4,4)

TRANSFORMATION MATRIX R(4,4) OF STRIP

DO 1 I=1,4
DO 1 J=1,4
R(I,J)=0.
S=ZP/B
C=XP/B
R(1,1)=C
R(2,2)=1.
R(3,1)=-S
R(1,3)=S
R(3,3)=C
R(4,4)=1.
RETURN
END
```

```
      SUBROUTINE MBTM(D,B,DB,L,M,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION D(4,4),B(4,4),DB(4,4)
C     MATRIX MULTIPLICATION DB(L,N) = D(L,M) X B(M,N)
      DO 110 J=1,N
      DO 110 I=1,L
      DB(I,J)=0.
      DO 110 K=1,M
  110 DB(I,J)=DB(I,J)+D(I,K)*B(K,J)
      RETURN
      END
```

```
      SUBROUTINE FORM(D,IVALD)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION D(37,37)
      COMMON/CONTRL/NTERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
      COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
      COMMON/NPI/A,B,BB,BN1,BN2,BNII,CO,R(4,4)
      COMMON/NODEN/X(50),Z(50),T(50),NOD(50,2)
      DIMENSION V1(37),V2(37)
      PRE-EIGENVALUE CHOLESKY REDUCTIONS
      NEQ=NDF*NP
      NFREE=NEQ
      ND=NFREE
      ND1=ND+1
      DO 2 MA=1,ND
      MA1=MA+1
      DO 2 MAS=MA,ND
      MAS1=MAS+1
      GISH=D(MAS,MA)
      MASH=1
    3 IF(MA-MASH)5,5,4
    4 GISH=GISH-D(MA,MASH)*D(MAS,MASH)
      MASH=MASH+1
      GO TO 3
    5 IF(MAS-MA)6,6,7
    6 IF(GISH)221,222,222
  221 WRITE(3,401) MA,MAS,GISH
  401 FORMAT(5X,4H MA=,I3,5X,5H MAS=,I3,5X,6H GISH=,E16.8)
      IVALD=1
      RETURN
  222 DIAG2=DSQRT(GISH)
    7 D(MAS,MA)=GISH/DIAG2
    2 CONTINUE
      TRIANGLE    INVERSION OF LOWER L  (L*LT=K)
      DO 10 K=1,NFREE
   10 D(K,K)=1./D(K,K)
      DO 9  I=2,NFREE
      I1=I-1
      GISH=D(I,I)
      DO 9  K=1,I1
      GAS=0.
      DO 8  J=K,I1
      GASH=D(J,K)*D(I,J)
    8 GAS=GAS-GISH*GASH
    9 D(I,K)=GAS
      TRIPLE MULTIPLICATION OF L*MASS*LT
      I=NFREE
   12 DO 11 J=1,I
      GASH=0
      L=1
      LD=1
      M=J+1
      MD=0
      DO 13 K=1,I
      GASH=GASH+D(I,K)*D(L,M)
```

1/2

```
L=L+LD
IF(L-M)13,113,113
LD=0
MD=1
L=L-1
M=M+MD
V2(J)=GASH
DO 32 J=1,I
GASH=0
DO 33 K=1,J
GASH=GASH+D(J,K)*V2(K)
D(J,I+1)=GASH
I=I-1
IF(I)129,129,12
CONTINUE
RETURN
END
```

```
SUBROUTINE POWER(D,IVALD)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION D(37,37)
DIMENSION V1(37),V2(37),V3(37),V4(37)
COMMON/CONTRL/NTERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODC,IPRINT
COMMON NEQ,NE,H,E1,E2,IPN(10),BUCP
NFREE=NEQ
MODE=MODC
MOD=MODE+1
ND=NFREE
ND1=ND+1
DO 116 I=1,ND
V4(I)=1.
RP=0.0
PUT 1.0 IN V1 FROM 1 TO ND AND ITERATE
DO 114 I=1,ND
V1(I)=V4(I)
PUT LARGE NUMBERS IN V3
DO 115 I=1,ND
V3(I)=10.**12
NUMIT=1
DEN=0.
R=0.
CRITN=0.
ALAM2=0.
ALAM3=0.
DO 15 I=1,ND
I1=I+1
GASH=0.
DO 18 J=1,I
GASH=GASH+V1(J)*D(J,I1)
CONTINUE
IF(I-ND)16,19,19
DO 20 J=I1,ND
GASH=GASH+V1(J)*D(I,J+1)
CONTINUE
V2(I)=GASH
ALAM2=ALAM2+GASH*GASH
ALAM3=ALAM3+V1(I)*V2(I)
CONTINUE
ALAMB=ALAM2/ALAM3
DO 21 I=1,ND
V2(I)=V2(I)/ALAMB
CONTINUE
DO 142 I=1,ND
CD=V1(I)-V2(I)
DEL=V3(I)-CD
V3(I)=CD
DEN=DEN+DEL**2
R=R+CD*DEL
CRITN=CRITN+CD*CD
2 CONTINUE
R=RP+(RP-1.)*R/DEN
RP=R
```

```
  IF(NUMIT-1)37,37,38
7 DO 58 I=1,ND
8 V1(I)=V2(I)
  NUMIT=NUMIT+1
  GO TO 29
8 DO 143 I=1,ND
  V1(I)=V2(I)+R*V3(I)
  IF(CRITN.GT.0.001)V4(I)=V1(I)
3 CONTINUE
  Z=1./10.**12
  IF(CRITN.GT.Z)GO TO 149
  GO TO 22
9 IF(NUMIT-NIT)25,22,22
5 NUMIT=NUMIT+1
  GO TO 29
2 CONTINUE
  WRITE(6,39)NUMIT
9 FORMAT(/2X, 'NUMBER OF ITERATIONS ' 5I4)
0 MUKTIPLY (L-1)T*(Y)
  DO 23 J=1,NFREE
  GASH=0.
  DO 24 I=J,NFREE
4 GASH=GASH+V1(I)*D(I,J)
3 V2(J)=GASH
  NMOD=MOD-MODE
  WRITE(6,105) NMOD
05 FORMAT(22X,'MODAL SHAPE OF MODE NUMBER',I2)
  WRITE(6,102)
02 FORMAT(/2X,'NODAL LINE NO,'5X,'U',14X,'V',14X,'W',14X,'THETA'/)
  DO 203 I=1,NP
  J1=(I-1)*NDF+1
  J2=J1+3
  WRITE(6,106)I,(V2(J),J=J1,J2)
  FORMAT(I10,4F15.8)
31 FORMAT(1H,3E16.8)
  OMEGA=1./ALAMB
  IF (MPROB-2) 206,207,206
06 IF(OMEGA)209,209,208
09 IVALD=1
  RETURN
08 IF(IVALD.EQ.2)GO TO 207
  OMEGA=DSQRT(OMEGA)
  WRITE(6,101) OMEGA
  OMEGA=OMEGA/6.28318
  WRITE(6,103) OMEGA
  GO TO 202
207 WRITE(6,100) OMEGA
  IF(NMOD.EQ.1)BUCP=OMEGA
  IF(MPROB.EQ.3)RETURN
202 CONTINUE
100 FORMAT(21H STABILITY FACTOR = ,E16.8)
101 FORMAT(13H  FREQUENCY=,E16.8,12H RADIANS/SEC)
103 FORMAT(13H  FREQUENCY=,E16.8,12H  CYCLES/SEC)
  CHANGING TO NEXT MODE
```

```
 SUM=0.
 DO 32 I=1,ND
SUM=SUM+V1(I)*V1(I)
DO 30 I=1,ND
DO 30 J=I,ND
J1=J+1
D(I,J1)=D(I,J1)-ALAMB*V1(I)*V1(J)/SUM
MODE=MODE-1
IF (MODE)31,31,34
CONTINUE
RETURN
END
```

```
      SUBROUTINE BOUN
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ST(37,37),SG(37,37),SM(37,37)
      COMMON/CONTRL/NTERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
      COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
      COMMON/NPI/A,B,BB,BN1,BN2,BNII,CO,R(4,4)
      COMMON/NODEN/X(50),Z(50),T(50),NOD(50,2)
      COMMON/BOUND/NF(8),NB(8,4)
      COMMON/GLOBL/ST,SM,SG

      INTRODUCTION OF BOUNDARY CONDITIONS

      DO 230 I=1,NBOUN
      K=(NF(I)-1)*NDF
      DO 230 J=1,NDF
      L=J+K
      IF(NB(I,J)) 230,345,230
    5 ST(L,L)=ST(L,L)*.1E+12
      SM(L,L)=SM(L,L)*.1E+12
      SG(L,L)=SG(L,L)*.1E+12
    0 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE EIGEN
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ST(37,37),SG(37,37),SM(37,37),SGSCL(37,37),STMOD(37,37)
      COMMON/CONTRL/NTERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
      COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
      COMMON/NPI/A,B,BB,BN1,BN2,BNII,CO,R(4,4)
      COMMON/NODEN/X(50),Z(50),T(50),NOD(50,2)
      COMMON/GLOBL/ST,SM,SG
      COMMON NEQ,NE,H,E1,E2,IPN(10),BUCP
      IF(MPROB-2) 1000,2000,2000
      DO 310 I=1,NEQ
      DO 310 J=1,NEQ
      ST(I,J+1)=SM(I,J)
      CALL FORM(ST,IVALD)
      IVALD=0
      CALL POWER (ST,IVALD)
      GO TO 9999
      DO 320 I=1,NEQ
      DO 320 J=1,NEQ
      STMOD(I,J+1)=SG(I,J)
      DO 325 I = 1,NEQ
      DO 325 J =1,NEQ
      STMOD(J,I)=ST(J,I)
      CALL FORM(STMOD,IVALD)
      IVALD=2
      CALL POWER(STMOD,IVALD)
      IF(MPROB.EQ.2) GO TO 9999
      CONTINUE
      AN=0
      IVALD=0
      SGINC=BUCP/IPN(II)          6 WRITE
1     DO 330 I=1,NEQ
      DO 330 J=1,NEQ
0     SGSCL(I,J+1)=SM(I,J)
      DO 340 I=1,NEQ
      DO 340 J=1,NEQ
0     SGSCL(J,I)=ST(J,I)-AN*SG(J,I)
      WRITE(6,50)AN
0     FORMAT(/'L IN PLANE FORCE    ',F10.4)
      CALL FORM(SGSCL,IVALD)
      IF(IVALD.EQ.1)RETURN
      CALL POWER(SGSCL,IVALD)
      IF(IVALD.EQ.1) GO TO 4000
      AN=AN+SGINC
      GO TO 3001
0     WRITE(6,350)
0     FORMAT('0 ERROR MESSAGE'/)
9     RETURN
      END
```

```
SUBROUTINE PRINTR
IMPLICIT REAL * 8(A-H,O-Z)
DIMENSION S(4,4),C(4,4),D(4,4),E(4,4),F(4,4),PM(4,4),BM(4,4),
PG(4,4),BG(4,4),ST(37,37),SG(37,37),SM(37,37)
COMMON/CONTRL/NTERM,NELEM,NP,NBOUN,NDF,NI,II,MPROB,NIT,MODE,IPRINT
COMMON/ORTHO/EX,EY,PX,PY,DX,DY,D1,DXY,G,RHO
COMMON/NPI/A,B,BB,BN1,BN2,BNII,CO,R(4,4)
COMMON/NODEN/X(50),Z(50),T(50),NOD(50,2)
COMMON/GLOBL/ST,SM,SG
IF(IPRINT.NE.1)GO TO 310
WRITE (6,250)
FORMAT('0 ST MATRIX'/)
NM=NP-1
DO 50 IX=1,NM
IN=IX*NDF-3
IK=IN+7
WRITE(6,300)((ST(I,J),J=IN,IK),I=IN,IK)
WRITE (6,260)
FORMAT('0 SM MATRIX'/)
DO 60 IX=1,NM
IN=IX*NDF-3
IK=IN+7
WRITE(6,300)((SM(I,J),J=IN,IK),I=IN,IK)
WRITE(6,270)
FORMAT('0 SG MATRIX')
DO 70 IX=1,NM
IN=IX*NDF-3
IK=IN+7
WRITE(6,300)((SG(I,J),J=IN,IK),I=IN,IK)
FORMAT(8E13.5)
CONTINUE
RETURN
END
```

APPENDIX "2"

```
MASTER
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION X(50),Z(50),T(50),FORC(50),NX(20),NY(20),NL(20),FP(20),
1QS(4),QN(2),S(4,4),C(4,4),D(4,4),E(4,4),F(4,4),QP(4)
COMMON P(30),ST(30,30),DIS(200),R(4,4),Q(30,4),NF(8),NB(8,4)
COMMON A,B3,EX,EY,PX,PY,G,DX,DY,D1,DXY,NOD(50,2),UM(600,3),DN(9)

ANALYSIS OF FOLDED PLATES AND BRIDGES

VARIABLE              DEFINITION

X(50),Z(50)           X AND Z COORDINATES OF NODAL POINTS
T(50)                 THICKNESS OF STRIP
FORC(50)              DISTRIBUTED VERTICAL LOAD ACTING ON STRIP
QN(1)                 LOAD COEFFICIENT FOR SINE SERIES VARIATION
QN(2)                 LOAD COEFFICIENT FOR COSINE SERIES VARIATION
ST(30,30)             FORWARD ELIMINATION WORKING AREA FOR STIFFNESS
P(30)                 FORWARD ELIMINATION WORKING AREA FOR LOAD
DIS(200)              DISPLACEMENT PARAMETER ARRAY
NF(8)                 RESTRAINED BOUNDARY NODE NUMBERS
NB(8,4)               BOUNDARY CONDITION TYPE FOR U,V,W AND THETA
                      (0 = FIXED, 1 = FREE)
NOD(50,2)             NODE NUMBERS OF ALL THE STRIPS
NX(20)                NODE NUMBERS AT WHICH CONCENTRATED LOADS ACT
NY(20)                CORRESPONDING Y-POSITION OF CONCENTRATED LOAD
NL(20)                LOAD DEFINITION (1=FOR X-LOAD,Z-LOAD,M-LOAD,
                      2=Y-LOAD)
FP(20)                MAGNITUDE OF CONCENTRATED LOAD
UM(600,3)             STRESS ARRAY
NTERM                 NUMBER OF TERMS
NELEM                 NUMBER OF ELEMENTS
NP                    NUMBER OF NODAL LINES
NBOUN                 NUMBER OF RESTRAINED BOUNDARY POINTS
NMOM                  NUMBER OF POINT ALONG A NODAL LINE FOR OUTPUT-
                      ING STRESSES AND MOMENTS
NDF                   NUMBER OF DEGREES OF FREEDOM PER NODAL LINE
NBAND                 MAXIMUM HALF BANDWIDTH
NI                    LOAD TYPE ( 1=NON-SYMMETRICAL , 2=SYMMETRICAL)
                      OR LINE LOADS
NCON                  NUMBER OF NODAL LINES WITH CONCENTRATED LOADS

REWIND 2
WRITE (6,4)
4 FORMAT( '   INPUT  DATA   ')
READ (5,11) NTERM,NELEM,NP,NBOUN,NMOM,NDF,NBAND,NI,NCON
WRITE(6,11) NTERM,NELEM,NP,NBOUN,NMOM,NDF,NBAND,NI,NCON
READ  (5,33) A, (DN(I),I=1,NMOM)
WRITE (6,33) A, (DN(I),I=1,NMOM)
K =4*NMOM*NELEM
NDF1=NDF + 1
NSIZ= NBAND - NDF
NEQ = NDF*NP
```

```
   NA  =  2*NDF
   NCOLN  =  1
   DO 123 J=1,3
   DO 123 I=1,K
 3 CM(I,J) =0

   READ X AND Z COORDINATES OF ALL POINTS

   DO 2 I=1,NP
   READ (5,35) X(I),Z(I)
 2 WRITE(6,35) X(I),Z(I)

   READ NUMBER NODAL NUMBERS THICKNESS VERTICAL DISTRIBUTED LOAD

   DO 109 I=1,NELEM
   READ (5,45) NUM,(NOD(I,J),J=1,2),T(I),FORC(I)
09 WRITE(6,45) NUM,(NOD(I,J),J=1,2),T(I),FORC(I)

   READ BOUNDARY CONDITIONS 0=FIXED 1=FREE

   READ (5,19)  (NF(I),(NB(I,J),J=1,4),I=1,NBOUN)
   WRITE(6,19)  (NF(I),(NB(I,J),J=1,4),I=1,NBOUN)
19 FORMAT(5I4)

   READ CONCENTRATED OR LINE LOAD DATA

   DO 222 I=1,NCON
   READ (5,45)     NX(I),NY(I),NL(I),FP(I)
222 WRITE(6,45)    NX(I),NY(I),NL(I),FP(I)
 11 FORMAT(9I4)
 35 FORMAT( 7F10.4)
 45 FORMAT(3I4,2F16.8)

   READ YOUNG'S MODULUS E1 E2 POISSON'S RATIO PX PY SHEAR MODULUS G

   READ (5,35)E1,E2,PX,PY,G
   WRITE(6,35) E1,E2,PX,PY,G
   EX = E1/(1.-PX*PY)
   EY= E2/(1.-PX*PY)
   DO 600 II=1,NTERM,NI
   CALL INIT(NBAND,NCOLN,NDF)
   NE=0
   REWIND 4
   DO 12 I=1,NEQ
12 DIS(I   ) =  0.
   DO  8 I=1,NBAND
   P(I)=0.
   DO 8 J=1,NBAND
 8 ST(I,J)=0.

   READ FOURIER LOAD COEFFICIENTS

   READ (5,35) QN(1),  QN(2)
   WRITE(6,35) QN(1),QN(2)
```

```
  BNII = 3.14159*II
  CO = BNII*BNII
  BN1 = BNII/A
  BN2 = BN1*BN1
  DO 70 LK=1,NP
  WRITE(6,150) LK
  IF (LK .GT. 1) GO TO 9
1 NE = NE +1
  IF ( NE - NELEM) 9,9,92
9 IF ( NOD(NE,1) - LK) 92,3,92
3 N1 = NOD(NE,1)
  N2 = NOD(NE,2)
  XP = X(N2) - X(N1)
  ZP = Z(N2) - Z(N1)
  H = T(NE)
  B = DSQRT(XP*XP + ZP*ZP)
  BB = B*B
  DX=E1*H**3/(12.*(1.-PX*PY))
  DY=E2*DX/E1
  DXY=G*DX/E1
  D1=PX*E2*DX/E1
  HXY=D1+4*DXY
  WRITE(6,152)
  CALL       TRAN(  XP,ZP)
  WRITE(6,155)((R(I,J),J=1,4),I=1,4)
  WRITE ( 4 )((R(I,J),I=1,4),J=1,4)
  WRITE(6,160)
  CALL       MOMP(              D,E,F)
  WRITE(6,155)((D(I,J),J=1,4),I=1,3)
  WRITE(6,165)
  CALL       MOMS(D,E,F)
  WRITE(6,155)((F(I,J),J=1,4),I=1,3)
  WRITE(6,170)
  CALL FEMS(CO,A,B,BB,DX,DY,D1,DXY,D,E,F)
  WRITE(6,155)((F(I,J),J=1,4),I=1,4)
  WRITE(6,175)
  CALL LOADS(QN,QS,NE,FCRC,B,BB,R)
  WRITE(6,155)(QS(I),I=1,4)
  WRITE(6,180)
  CALL       FEMP(              E,H,CO)
  WRITE(6,155)((E(I,J),J=1,4),I=1,4)
  WRITE(6,185)
  CALL LOADP(QN,QP,NE,FCRC,B,R)
  WRITE(6,155)(QP(I),I=1,4)
  DO 80 LL=1,2
  J1 = 2*LL -1
  J2 = J1 + 1
  J = NDF*(NOD(NE,LL) - LK)
  S(1,1)=QP(J1)*A
  S(2,1)=QP(J2)*A
  S(3,1)=QS(J1)*A
  S(4,1)=QS(J2)*A
  CALL MBTTM(R,S,C,4,4,1)
  DO 15 JJ=1,NDF
```

```
      JN = J + NJ
15    P(JN)  =  P(JN)  + C(NJ,1)
      DO 80 KK=1,2
      DO 81 K=1,NDF
      DO 81 L=1,NDF
81    D(K,L) =0.
      I1 = 2*KK-1
      I2 = I1 + 1
      I = NDF*(NOD(NE,KK) - LK)
      D(1,1) = E(I1,J1)
      D(2,1) = E(I2,J1)
      D(1,2) = E(I1,J2)
      D(2,2) = E(I2,J2)
      D(3,3) = F(I1,J1)
      D(4,3) = F(I2,J1)
      D(3,4) = F(I1,J2)
      D(4,4) = F(I2,J2)
      CALL MBTM(D,R,C,4,4,4)
      CALL MBTTM(R,C,D,4,4,4)
      WRITE(6,183) LL,KK
      WRITE(6,190)
      WRITE(6,155)((D(IPRT,JPRT),JPRT=1,4),IPRT=1,4)
      DO 5 NJ = 1,NDF
      JN = J + NJ
      DO 5 MI = 1,NDF
      IM = MI + I
 5    ST(IM,JN) = ST(IM,JN) + D(MI,NJ)
80    CONTINUE
      WRITE (6,250)
250   FORMAT('0 ST MATRIX'/)
      WRITE(6,275)((ST(I,J),J=1,8),I=1,8)
275   FORMAT(8E13.5)
      GO TO 1
92    DO 67 I=1,NCON
      IF (LK - NX(I)) 67,54,67
54    J = NL(I)
      IF (N7(I)) 75,75,76
75    IF ( J .EQ. 2) P(J) = P(J) + FP(I)*QN(2)*A
      IF ( J.NE. 2) P(J) = P(J) + FP(I)*QN(1)*A
      GO TO 67
76    P(J)=P(J)+FP(I)*QN(1)
67    CONTINUE
      CALL BOUN(LK,NBOUN)
      CALL SOLVE
70    CONTINUE
      REWIND 4
      CALL BSUB
      CALL MOM(D,E,F,C,S)
600   CONTINUE
      K = 4*NMOM
      DO 133 LL = 1,NELEM
      N1 = (LL - 1)*K + 1
      N2 = LL*K
      WRITE(6,27)
```

```
7 FORMAT(27H ELEMENT NUMBER NODE1 NODE2)
  WRITE(6,28)
3 FORMAT(  '    ZIGMA-X , ZIGMA-Y , ZIGMA-XY    '  )
  WRITE (6,29)
) FORMAT(  '    MOMENT-X , MOMENT-Y , MOMENT-XY   '   )
3 WRITE (6,17) LL,NOD(LL,1),NOD(LL,2),((OM(I,J),J=1,3),I=N1,N2)
7 FORMAT(3I4/(6E13.5))
) FORMAT('LK=',I2//)
2 FORMAT('0ALL TRAN R MATRIX'/)
5 FORMAT(4E13.7)
) FORMAT('0 ALL MOMP STRESS MATRIX'/)
5 FORMAT('0 ALL MOMS MOMENT MATRIX'/)
) FORMAT('0 ALL FEMS BENDING STIFFNESS MATRIX'/)
5 FORMAT('0 ALL LOADS QS'/)
) FORMAT('0 ALL FEMP PLANE STIFFNESS MATRIX'/)
5 FORMAT('0 ALL LOADP QP'/)
3 FORMAT('0LL='I2,'KK='I2)
) FORMAT('TRANSFORMED INDIVIDUAL STIFFNESS'/)
  STOP
  END
```

```
      SUBROUTINE FEMP(E,H,CO)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION E(4,4)
      COMMON P(30),ST(30,30),DIS(200),R(4,4),Q(30,4),NF(8),NB(8,4)
      COMMON NTERM,NELEM,NP,NMDM,NDF,NDF1,NBAND,NSIZ,II,NI,BNI1,BN1,BN2,B
      COMMON A,BB,EX,EY,PX,PY,G,DX,DY,D1,DXY,NOD(50,2),UM(600,3),DN(9)

      IN-PLANE STIFFNESS MATRIX E(4,4) OF STRIP

      E(1,1) = EX*.5*A/B   +B*BN2*A*G/6.
      E(2,1) = .25*BN1  *A*PX*EY - .25*BN1 *A*G
      E(3,1) = -.5*A*EX/B + B*A*BN2*G/12.
      E(4,1) = .25*A*BN1 *PX*EY + .25*A*BN1 *G
      E(2,2) =A*B*BN2*EY/6.    +   .5*A*G/B
      E(3,2) = -.25*A*BN1 *PX*EY - .25*A*BN1 *G
      E(4,2) =A*B*BN2*EY/12. - .5*A*G/B
      E(3,3) = .5*A*EX/B +A*B*BN2*G/6.
      E(4,3) = -.25 *A*BN1 *PX*EY + .25*A*BN1 *G
      E(4,4) =A*B*EY*BN2/6.+ .5*A*G/B
      DO 20 I=1,4
      DO 20 J=1,I
      E(I,J)=E(I,J)*H
   20 E(J,I) = E(I,J)
      RETURN
      END
```

```
      SUBROUTINE FEMS(CO,A,B,BB,DX,DY,D1,DXY,D,E,F)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION D(4,4),E(4,4),C(4,4),F(4,4)

      BENDING STIFFNESS MATRIX F(4,4) OF STRIP

      AA = A*A
      BBBB= BB*BB
      COCO = CO*CO
      BBB = B*BB
      D(1,1) = COCO*DY*B*.5/AA
      D(2,1) =    COCO*BB*DY*.25/AA
      D(3,1) = -CO*B*D1    +   .166667*COCO*BBB*DY/AA
      D(4,1) = -1.5*CO*BB*D1 +  .125*COCO*BBBB*DY/AA
      D(2,2) =      .166667*COCO*BBB*DY/AA   +   2.*CO*B*DXY
      D(3,2) =-CO*BB*D1*.5+.125*COCO*BBBB*DY/AA   +   2.*CO*BB*DXY
      D(4,2) = -CO*BBB*D1   +   .1*COCO*BBBB*B*DY/AA + 2.*CO*BBB*DXY
      D(3,3) =   2.*AA*B*DX -.6666667*CO*BBB*D1   +
     1 .1*COCO*BBBB*B     *DY/AA     + 2.66667*CO*BBB*DXY
      D(4,3) = 3.*AA*BB*DX    -CO*BBBB*D1   +   .0833333*COCO*BBBB*BB*DY/AA
     1 +    3.*CO*BBBB*DXY
      D(4,4) = 6.*BBB*DX*AA     - 1.2*CO*BBBB*B*D1   + .0714286*COCO*BBB
     1*BBBB*DY/AA+ 3.6*CO*BB*BBB*DXY
      DO 5 I=1,4
      DO 5 J=1,I
      D(I,J) = D(I,J)/A
    5 D(J,I) = D(I,J)
      CALL EM(B,BB,BBB,E)
      CALL MBTM(D,E,C,4,4,4)
      CALL MBTTM(E,C,F,4,4,4)
      RETURN
      END
```

```
      SUBROUTINE MOMP( D,E,F)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION D(4,4),E(4,4),F(4,4)
      COMMON P(30),ST(30,30),DIS(200),R(4,4),J(30,4),NF(8),NB(8,4)
      COMMON NTERM,NELEM,NP,NMOM,NDF,NDF1,NBAND,NSIZ,I1,N1,BNI1,BN1,BN2,B
      COMMON A,BB,EX,EY,PX,PY,G,DX,DY,D1,DXY,NOD(50,2),OM(600,3),DN(9)

      IN-PLANE STRESS MATRIX D(4,4) OF STRIP

      DO 83 INDEX=1,2
      IF (INDEX-1) 91,91,92
 1    X = 0
      GO TO 93
 2    X = B
 3    DO 82 M = 1, NMOM
      Z=DN(M)/A
      H1=DSIN(BNI1*Z)
      H2=DCOS(BNI1*Z)
      D(1,1)=-H1*EX/B
      D(2,1)=-H1*PX*EY/B
      D(3,1)=(1.-X/B)*BN1*H2*G
      D(1,2)=-(1.-X/B)*BN1*H1*PX*EY
      D(2,2)=-(1.-X/B)*BN1*H1*EY
      D(3,2)=-H2*G/B
      D(1,3)=H1*EX/B
      D(2,3)=H1*PX*EY/B
      D(3,3)=X*BN1*H2*G/B
      D(1,4)=-X*BN1*H1*PX*EY/B
      D(2,4)=-X*BN1*H1*EY/B
      D(3,4)=H2*G/B
      WRITE(4) ((D(I,J),I=1,3),J=1,4)
 32   CONTINUE
 33   CONTINUE
      RETURN
      END
```

```
      SUBROUJTINE MOMS(D,E,F)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION D(4,4),E(4,4),F(4,4)
      COMMON P(30),ST(30,30),DIS(200),R(4,4),Q(30,4),NF(8),NB(8,4)
      COMMON NTERM,NELEM,NP,NMOM,NDF,NDF1,NBAND,NSIZ,I1,NI,BNI1,BN1,BN2,B
      COMMON A,BB,EX,EY,PX,PY,G,DX,DY,D1,DXY,NOD(50,2),UM(500,3),DN(9)

      MOMENT MATRIX F(4,4) OF STRIP

      CO=BN2
      DO83 INDEX=1,2
      IF(INDEX-1) 91,91,92
   91 X=0.
      GO TO 93
   92 X=B
   93 DO 82 M=1,NMOM
      Z=DN(M)/A
      H1=DSIN(BNI1*Z)
      H2=DCOS(BNI1*Z)
      H3=-H1
      D(1,1)=-CO*H3*D1
      D(2,1)=-CO*H3*DY
      D(3,1)=0
      D(1,2)=-CO*H3*D1*X
      D(2,2)=-CO*X*H3*DY
      D(3,2)=2.*BN1*H2*DXY
      D(1,3)=-2.*H1*DX-CO*X*X*H3*D1
      D(2,3)=-2.*H1*D1-CO*X*X*H3*DY
      D(3,3)=4.*BN1*H2*DXY*X
      D(1,4)=-6.*X*H1*DX-CO*X*X*H3*D1*X
      D(2,4)=-6.*X*H1*D1-CO*X*X*X*H3*DY
      D(3,4)=6.*BN1*X*X*H2*DXY
      BBB=BB*B
      CALL EM(B,BB,BBB,E)
      CALL MUTM(D,E,F,3,4,4)
   82 WRITE (4   ) ((F(I,J),I=1,3),J=1,4)
   83 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TRAN(XP,ZP)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON P(30),SI(30,30),DIS(200),R(4,4),Q(30,4),NF(8),NB(8,4)
      COMMON NTERM,NELEM,NP,NNUM,NDF,NDF1,NBAND,NSIZ,II,NI,BNII,BNI,BN2,B
      COMMON A,BB,EX,EY,PX,PY,G,DX,DY,D1,DXY,NOD(50,2),OM(800,3),DN(9)

      TRANSFORMATION MATRIX R(4,4) OF STRIP

      DO 1 I=1,4
      DO 1 J=1,4
    1 R(I,J)=0.
      S=ZP/B
      C=XP/B
      R(1,1)=C
      R(2,2)=1.
      R(3,1)=-S
      R(1,3)=S
      R(3,3)=C
      R(4,4)=1.
      RETURN
      END
```

```
      SUBROUTINE LOADP(QN,QP,LK,FORC,B,R)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION QN(2),QP(4),FORC(99),R(4,4)

      IN-PLANE NODAL FORCES QP(4) DUE TO DISTRIBUTED LOADS

      FOR=FORC(LK)*R(1,3)
      QP(1)=QN(1)*FOR*B*.5
      QP(2)=0.
      QP(3)=QP(1)
      QP(4)=0.
      RETURN
      END
```

```
      SUBROUTINE LOADS(QN,QS,LK,FORC,B,BB,R)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION QN(2),QS(4),FORC(99),R(4,4)

      OUT-OF-PLANE NODAL FORCES QS(4) DUR TO DISTRIBUTED LOADS

      FOR=FORC(LK)*R(1,1)
      DO 26 KK=1,2
      I = 2*(KK-1)
      QS(I+1)=QN(1)*FOR*B*.5
      GO TO (24,25),KK
   24 QS(I+2)=QN(1)*BB*FOR/12.
      GO TO 26
   25 QS(I+2)=-QN(1)*BB*FOR/12.
   26 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BOUN(  LK,NBOUN)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON P(30),ST(30,30),DIS(200),R(4,4),Q(30,4),NF(8),NB(8,4)
      COMMON NTERM,NELEM,NP,NMOM,NDF,NDF1,NBAND,NSIZ,II,NI,BNII,BN1,BN2,B
      COMMON A,B3,EX,EY,PX,PY,G,DX,DY,D1,DXY,NOD(50,2),JM(600,3),DN(9)

      INTRODUCTION OF BOUNDARY CONDITIONS

      DO 230 I=1,NBOUN
      IF(NF(I)-LK) 78,79,78
   79 DO 230 J=1,NDF
      IF(NB(I,J)) 230,345,230
  345 ST(J,J)=ST(J,J)*.1E+12
   78 CONTINUE
  230 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SOLVE
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON P(30),ST(30,30),DIS(200),R(4,4),Q(30,4),NF(8),NB(8,4)
      COMMON ITERM,NELEM,NP,NMUM,NDF,NDF1,NBAND,NSIZ,I1,N1,BNI1,BN1,BN2,B
      COMMON A,BB,EX,EY,PX,PY,G,DX,DY,D1,DXY,NOD(50,2),JM(600,3),BN(9)

      FORWARD ELIMINATION

      WRITE (6,250)
  50  FORMAT('0 MODIFIED STIFFNESS MATRIX,'/)
      WRITE(6,210)((ST(I,J),J=1,8),P(I),I=1,NDF)
  10  FORMAT(9E13.5)
      CALL MATINV(ST,NDF)
      CALL STORE(ST,P,NDF)
      DO 111 J=1,NDF
      DO 111 I=1,NSIZ
      L=I+NDF
      Q(I,J)=0.
      DO 111 K=1,NDF
  11  Q(I,J)=Q(I,J)+ST(K,L)*ST(K,J)
      DO 112 I=NDF1,NBAND
      L=I-NDF
      DO 112 K=1,NDF
      DO 113 J=NDF1,NBAND
 113  ST(I,J)=ST(I,J)-Q(L,K)*ST(K,J)
 112  P(I      )=P(I      ) - Q(L,K)*P(K      )
      DO 114 I=1,NSIZ
      K=I+NDF
      P(I      ) = P(K      )
      P(K      ) = 0.
      DO 114 J=1,NSIZ
      L = J+NDF
      ST(I,J)=ST(K,L)
      ST(I,L)=0.
      ST(K,J)=0.
 114  ST(K,L)=0.
      RETURN
      END
```

```
      SUBROUTINE BSUB
      IMPLICIT REAL*8(A-H,O-Z)
      COMMONP(30),ST(30,30),DIS(200),R(4,4),Q(30,4),NF(8),NB(8,4)
      COMMONITEM,NELEM,NP,NMOM,NDF,NDF1,NBAND,NSIZ,I1,NI,BNI1,BN1,BN2,B
      COMMON A,BB,EX,EY,PX,PY,G,DX,DY,D1,DXY,NOD(50,2),DM(600,3),DN(9)
C     BACK-SUBSTITUTION
      DO 30IP=1,NP
      M=NP-IP
      CALL RDBK(ST,P,NDF)
      DO 1 I=1,NDF
      DO 1 J=NDF1,NBAND
    1 P(I   ) = P(I   ) - ST(I,J) * P(J   )
      DO 2 I=1,NDF
      Q(I,1)=0.
      DO 2 J=1,NDF
    2 Q(I, 1) = Q(I, 1)+ST(I,J)*P(J)
      DO 3 I=1,NDF
      P(I)=Q(I,1)
      J = NDF*M + I
    3 DIS(J)=Q(I,1)
      DO 114 I=1,NSIZ
      L = NBAND - I + 1
      K = L - NDF
   14 P(L)=P(K)
   30 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MOM(D,E,F,C,S)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION D(4,4),E(4,4),F(4,4),C(4,4),S(4,4)
      COMMON P(30),ST(30,30),DIS(200),R(4,4),Q(30,4),NF(3),NB(8,4)
      COMMON INTERM,NELEM,NP,NMOM,NDF,NDF1,NBAND,NSIZ,II,NI,BNII,BN1,BN2,B
      COMMON A,BB,EX,EY,PX,PY,G,DX,DY,D1,DXY,NOD(50,2),OM(600,3),DN(9)

      COMPUTATION OF STRESSES (D=FC),MOMENTS(D=FS) AT SPECIFIED POINTS
      ALONG NODAL LINES AND DISPLACEMENTS DIS(200) AT CENTRE POINTS

      NUM=0
      WRITE(6,10)II
   10 FORMAT(I4)
      DO 81 LK=1,NELEM
      READ (4  ) ((R(I,J),I=1,4),J=1,4)
      DO 24 LL=1,2
      I = NDF*(NOD(LK,LL)-1)
      DO 23 J=1,NDF
      K=I+J
   23 D(J,1)=DIS(K)
      CALL MBTM(R,D,E,4,4,1)
      C(2*LL-1,1) = E(1,1)
      C(2*LL,1)   = E(2,1)
      S(2*LL-1,1) = E(3,1)
      S(2*LL,1)   =   E(4,1)
   24 CONTINUE
      DO 25 LL=1,2
      DO 25 M=1,     NMOM
      READ (4    ) ((F(I,J),I=1,3),J=1,4)
      CALL MBTM(F,C,D,3,4,1)
      NUM = NUM + 1
      DO 3 I=1,3
    3 OM (NUM,I) = OM(NUM,I) + D(I,1)
   25     CONTINUE
      DO 26 LL=1,2
      DO 26 M=1,    NMOM
      READ (4    ) ((F(I,J),I=1,3),J=1,4)
      CALL MBTM(F,S,D,3,4,1)
      NUM = NUM + 1
      DO 4 I=1,3
    4 OM (NUM,I) = OM(NUM,I) + D(I,1)
   26     CONTINUE
   81 CONTINUE
      NP2 = NDF*NP
      DO 7 I =1,NP2,NDF
      DIS(I    ) = DIS(I     )*DSIN(BNII*.5)
      DIS(I+2) = DIS(I+2)*DSIN(BNII*.5)
    7 DIS(I+3) = DIS(I+3)*DSIN(BNII*.5)
      WRITE(6,15)
   15 FORMAT(11H DEFLECTION,9H ROTATION)
      WRITE (6,5)   (DIS(I   ),I=1,NP2)
    5 FORMAT(4E16.8)
      RETURN
      END
```

```
      SUBROUTINE EM(B,BB,BBB,E)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION E(4,4)
      E(1,1) = 1.
      E(2,1) = 0.
      E(3,1) = -3./BB
      E(4,1) = 2./BBB
      E(1,2) = 0.
      E(2,2) = 1.
      E(3,2) = -2./B
      E(4,2) = 1./BB
      E(1,3) = 0.
      E(2,3) = 0.
      E(3,3) = 3./BB
      E(4,3) = -2./BBB
      E(1,4) = 0.
      E(2,4) = 0.
      E(3,4) = -1./B
      E(4,4) = 1./BB
      RETURN
      END
```

```
      SUBROUTINE MATIN(ST,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ST(30,30)

      MATRIX INVERSION

      DO 19 I=1,N
      Z=ST(I,I)
      ST(I,I)=1.
      DO 60 J=1,N
   60 ST(I,J)=ST(I,J)/Z
      DO 19 K=1,N
      IF(K-I)3,19,3
    3 Z=ST(K,I)
      ST(K,I)=0
      DO 4 J=1,N
    4 ST(K,J)=ST(K,J)-Z*ST(I,J)
   19 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MBTM(D,B,DB,L,M,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION D(4,4),B(4,4),DB(4,4)

      MATRIX MULTIPLICATION DB(L,N) = D(L,M) X B(M,N)

      DO 110 J=1,N
      DO 110 I=1,L
      DB(I,J)=0.
      DO 110 K=1,M
  110 DB(I,J)=DB(I,J)+D(I,K)*B(K,J)
      RETURN
      END
```

```
      SUBROUTINE MBTTM(D,B,DB,L,M,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION D(4,4),B(4,4),DB(4,4)

      MATRIX TRANSPOSE MULTIPLICATION DB(L,N) = D(M,L) X B(M,N)

      DO 110 J=1,N
      DO 110 I=1,L
      DB(I,J)=0.
      DO 110 K=1,M
  110 DB(I,J)=DB(I,J)+D(K,I)*B(K,J)
      RETURN
      END
```

```
      SUBROUTINE INIT ( NBAND, NCOLN,NDF )
C     THIS SUBROUTINE MUST BE CALLED ONCE BEFORE SUBROUTINES
C     STORE AND RDBK ARE CALLED IN ORDER TO INITIALISE THE BLOCK
C     CONTROL COUNTERS.
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/BUFDA/NBD,NCOL,IS,NA,LRECL,NREC,L,IDUMMY,X(2000)
      NA = 2000
      IS = 1
      NBD = NBAND
      NCOL = NCOLN
      LRECL =(NBD + NCOL )*NDF
      NREC=0
      IF (LRECL-NA)1,1,2
    1 RETURN
    2 WRITE (6,4) LRECL,NA
    4 FORMAT ('0LOGICAL RECORD LENGTH OF  ',I6,'EXCEEDS BUFFER SET AT',
     1  I6  )
      STOP
      END
```

```
SUBROUTINE STORE (ST,P,NDF)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION ST(30,30),P(30)
COMMON/BUFDA/NBD,NCOL,IS,NA,LRECL,NREC,L,IDUMMY,X(2000)

STORE FORWARD ELIMINATION RESULTS IN BUFFER AREA AND WRITE ON TAPE
IN A BLOCK OF 2000 WORDS TO SAVE TRANSFER TIME

TEST IF ROOM IN CURRENT BUFFER
IF (IS + LRECL-NA)5,5,50
ROOM IN BUFFER
DO 10 I=1,NBD
DO 10 J=1,NDF
X(IS)=ST(J,I)
IS=IS+1
DO 15 I=1,NDF
X(IS)=P(I)
IS=IS+1
RETURN
NO ROOM LEFT IN BUFFER
L=IS-1
WRITE(2)(X(J),J=1,L)
IS=1
NREC=NREC+1
GO TO 5
END
```

```
      SUBROUTINE RDBK(ST,P,NDF)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ST(30,30),P(30)
      COMMON/BUFDA/NBD,NCOL,IS,NA,LRECL,NREC,L,IDUMMY,X(2000)

      READ FORWARD ELIMINATION RESULTS FOR BACK-SUBSTITION

      TEST IF NEXT RECORD IN BUFFER
      IS=IS-LRECL
      IF(IS-1) 40,12,12
      RECORD IS IN BUFFER
   12 DO 11 I=1, NBD
      DO 11 J=1,NDF
      ST(J,I)=X(IS)
      IS=IS+1
      DO 15 I=1,NDF
      P(I)=X(IS)
      IS=IS+1
      IS=IS-LRECL
      RETURN
      LAST BLOCK WRITTEN MUST BE READ
      IF(NREC)100,100,41
      NREC=NREC-1
      BACKSPACE 2
      READ(2)(X(J),J=1,L)
      BACKSPACE 2
      IS=L+1
      GO TO 10
      ILLOGICAL ERROR
      WRITE (6,101)
      FORMAT ('0 ATTEMPT TO READ BACK TOO MANY RECORDS.')
      STOP
      END
```

APPENDIX "3"

APPENDIX "3"

Subroutine Solve

In the stiff matrix shown in Figure A-1 each stepped area represents the equations associated with one node. The first set of nodal equations can be divided into a diagonal matrix $[K_{11}]$ and an off diagonal submatrix $[K_{1T}] = [[K_{12}][K_{13}] \cdots\cdots [K_{1n}]]$. If the nodal displacement parameters $\{\delta_1\}$ are to be eliminated, then the area ABCD will be affected and we could have the modified matrices.

$$[K_{TT}]^* = [K_{TT}] - [K_{1T}]^T [K_{11}]^{-1} [K_{1T}],$$

$$\{P_T\}^* = \{P_T\} - [K_{1T}]^T [K_{11}]^{-1} \{P_1\},$$

where

$$[K_{TT}] = \begin{bmatrix} [K_{22}] & \cdots\cdots & [K_{2n}] \\ \vdots & & \vdots \\ [K_{2n}] & \cdots\cdots & [K_{nn}] \end{bmatrix}$$

and

$$\{P_T\} = \begin{Bmatrix} P_2 \\ \vdots \\ P_n \end{Bmatrix}$$

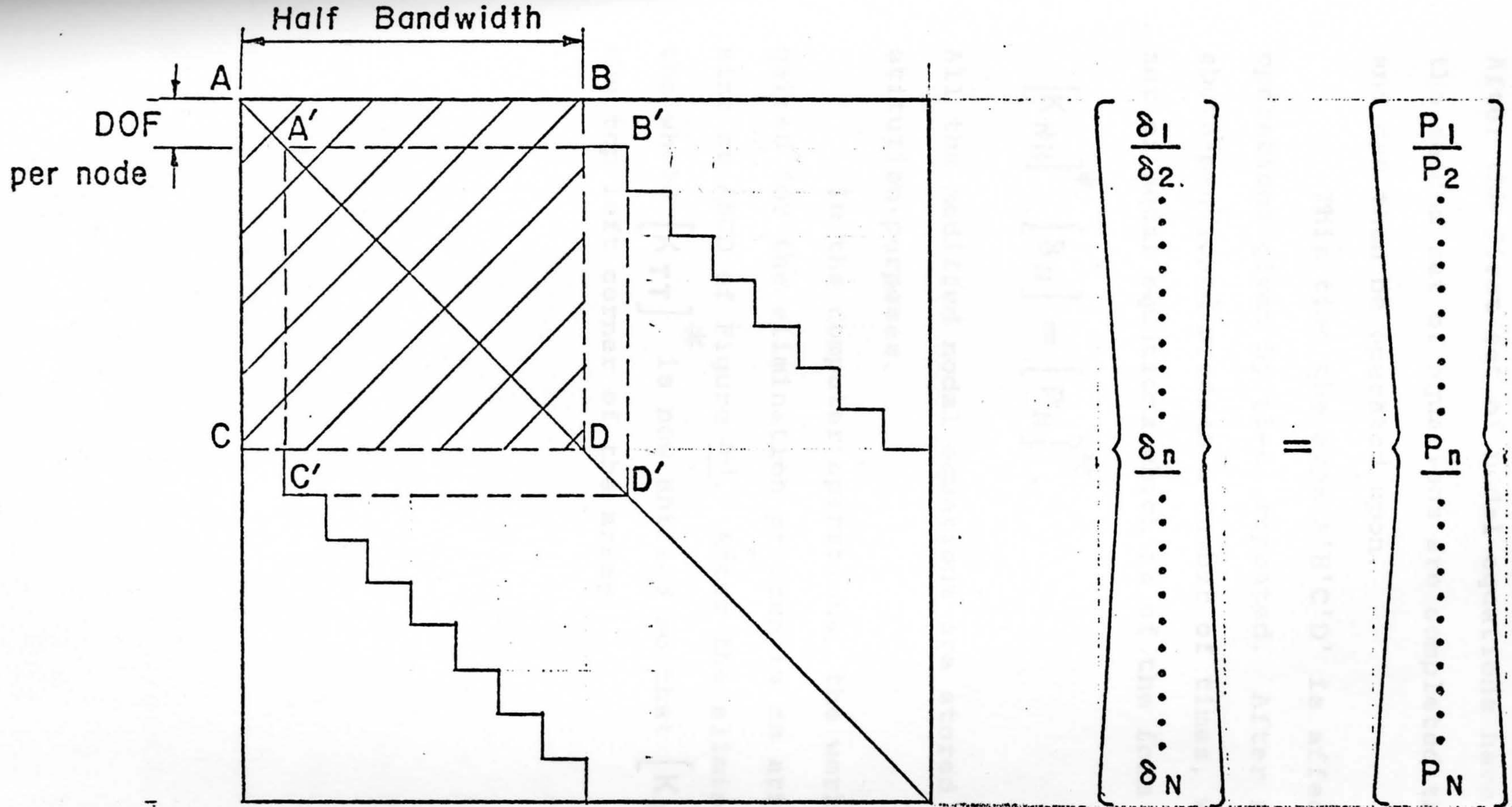

FIGURE "A-1" A NODE BY NODE ELIMINATION SCHEME

After the first set of nodal equations have been eliminated, the second set of equations are completed through assembly and can then be operated upon.

This time the area A'B'C'D' is affected and the operations given by (1-4) repeated. After repeating the above-mentioned process a number of times, we reach the last set of nodal equations which is of the form

$$[K_{NN}]^{*} \{\delta_N\} = \{P_N\}^{*}.$$

All the modified nodal equations are stored for basic substitution purposes.

In the computer operations, the working store required for the elimination process is an array of the same size as ABCD of Figure A-1. After the elimination of $[K_{11}]$ the whole $[K_{TT}]^{*}$ is now shifted so that $[K_{22}]^{*}$ now occupies the top left corner of the array.

BIBLIOGRAPHY

1. Timoshenko, S.P., and Woinowsky - Krieger, S. Theory of Plates and Shells, 2nd ed., McGraw-Hill, N.Y. (1959).

2. Timoshenko, S.P., and Gene, J.M. Theory of Elastic Stability, 2nd ed., McGraw-Hill, N.Y. (1961).

3. Zienkiewicz, O.C., and Cheung, Y.K. Finite Element Method in Structural and Continuum Mechanics, McGraw-Hill (1967).

4. Cheung, Y.K. Finite Strip Method in Structural Analysis, 1st ed., Pergamon Press (1976).

5. Szilard, R. Theory and Analysis of Plates, Prentice Hall.

6. Przemieniecki, J.S. Theory of Matrix Structural Analysis, McGraw-Hill (1968).

7. Kantorovich, L.V., and Krylov, V.I. Approximate Methods of Higher Analysis, Interscience Publishers, N.Y. (1958).

8. Timoshenko, S.P., and Goodier, J.N. Theory of Elasticity, 2nd ed., McGraw-Hill (1951).

9. Anderson, R.G. A Finite Element Eigenvalue Solution System, Ph.D. Thesis, University of Wales, Swansea (1968).

10. Cheung, M.S. Finite Strip Analysis of Structures, Ph.D. Thesis, University of Calgary, Canada (1971).

11. Yoshida, Koichiro. Buckling Analysis of Plate Structures by Strip Elements., Proc. Jap. Soc. of Naval Architects (1971).

12. Warburton, G.B. The Vibration of Rectangular Plates.

13. Bishop, R.E.D., et al. The Matrix Analysis of Vibration, Cambridge University Press (1965).

14. Simitses, G.J. An Introduction to the Elastic Stability of Structures, Prentice Hall.

15. Clough, R.W., and Penzren, J. Dynamics of Structures, McGraw-Hill, N.Y. (1975).

BIBLIOGRAPHY
(Continued)

16. Kapur, K.K., and Hartz, B.J., "Stability of Thin Plates Using Finite Element Method." Proc. ASCE, 92, J. Eng. Mech. Div. Em 2 (April 1966).

17. Scordelis, A.C., and A. De-Fries-Skene. Direct Stiffness Solution for Folded Plates, Am. Soc. Civ. Engrs. 90, St, (1964).