Model Following Control
of
DC Servomotor

by

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Abstract

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The linear Model-Following Control is implemented to control Dc Servomotor in the feedforward mode. The model is designed to have desirable characteristics of the actual servomotor for the desired response. The model and the model following control are mathematical models simulated on an analog computer. The control law derived forces the actual servomotor to behave as the model. The simulation results are compared with conventional P+I control to verify the effectiveness of the control law.

ACKNOWLEDGEMENTS

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CHAPTER I

INTRODUCTION

1.1 Background and Objective

Servomechanisms and automatic controls apply to many engineering fields such as electronics, electrical, mechanical, hydraulic, pneumatic, aeronautical, and chemical engineering. Servomotors are the most essential part of a servomechanism. The importance of the dc servomotor in every branch of industry and transport system has enhanced the scope of research in the area of servosystem controls. Different methods of dc servosystem controls have been developed in the past. The basic feedback or feedforward PID compensation techniques led to the development of more advanced techniques. [1] uses adaptive control in feedforward with linear state feedback and [2] discusses computer control methods. [3] uses the difference between the output of the system and the output of the reference model in adaptive mechanism to make the response of the system equal to that of the model. Model-Following Control is one of the advanced methods used in aircraft control and satellite control.

In this thesis, Model-Following Control is considered to control a dc servomotor in the feedforward mode. The aim of this control system design is to make the controlled system follow the model.

1.2 An Overview

This research deals with the derivation of the model, the control law and verification of the control law. The procedure is outlined in four steps:

1. Derive the mathematical Model of the actual servomotor
   - Define the parameters of the servomotor and estimate them.
   - Define the operating range of the servosystem.
   - Simulate the time response of the servomotor and check the time history.
2. Definition of a Model
   - Define the desired model of the servomotor.
   - Define the control objectives, speed or position, etc.
   - Design the control system through feedback or any other technique.
   - Verify performance through simulation.
3. Derive a control law to drive the model of the servomotor
   - Model responses drive the model of the servomotor through Model-Following Control forcing the actual servomotor to follow the model.
   - Verify Model-Following Control through time response simulation by programming on the analog computer.

4. Use the Model and the Model-Following Control to drive the actual servomotor to demonstrate the effectiveness of the control law.

In accordance with this procedure, the control system consists of specifying the desired response of the servomotor in the form of a model. A control law is derived such that the Model-Following Control forces the actual servomotor to follow the model. The advantages of this method are that all the characteristics of the servomotor can be defined in the model. Also, the control law strictly depends upon the servomotor parameters. The control system does not have to be actual hardware; it can be mathematical models simulated on the computer. A general control law for a second order system was derived. The control law was tested on a first order servomotor by analog simulation as well as on an actual modular servosystem unit described in chapter III. Experimental results using PID hardware unit were compared with the simulation results. The results confirmed the validity of the method developed in this thesis.

CHAPTER II

SERVOMECHANISMS

2.1 Definition

A servomechanism is a power amplifying feedback control system in which the controlled variable is mechanical position, or a time derivative of position, such as speed or acceleration.

The input of a servomechanism is usually variable, and the system operates in such a manner that the output closely follows the input signal and its variations. Servomechanisms are used to control physical systems, such as the flight path of an aircraft or a missile, which are automatically controlled to perform specific duties. Servomechanism enables man to control large amounts of power. The control point can be remote from the actual operating point. Efficient operation of many industrial processes, machines, satellite control, radar control, robots, often requires consistent performance; which may not be achievable by human operators. With automatic control the required performance can be achieved accurately and economically. The term Servomechanism, or Servosystem, is universally used to describe Automatic Control Systems. The automatic control in servosystem is actuated by the " error signal," i.e., the difference between the actual output and the desired output.

2.2 Classification of Servomechanisms

The servomechanisms can be classified based on

a. The nature of the input to the control system

b. The operating characteristics of the control system

Consider the first method of classification. In this type the input to the control system is a pre-determined function of time, as in the case of automatic profile milling machine tools. In the second type the input can be a nonpredetermined function of time, as in the case of automatic tracking of an aerial target by radar equipment. In the third type, the input is a constant quantity. This requires the servomechanism to maintain the condition of the controlled quantity at a corresponding constant value irrespective of external disturbances. These types of servomechanisms fall in the special class of automatic regulators such as temperature, or pressure, or voltage regulators.

Consider the second method of classification. In one type the control is a discontinuous function of error, as in the case of non-linear systems. In the other type the control is a continuous function of error, as in the case of linear systems, such as position or speed control servosystems.

In this research a linear servosystem is considered for speed control.

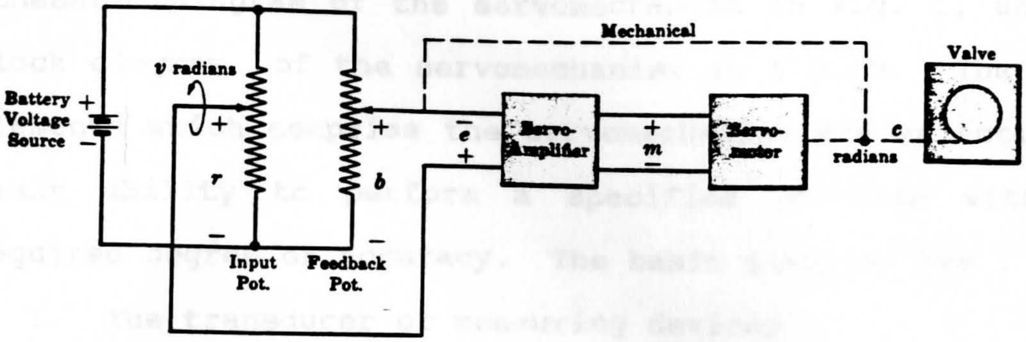


Fig. 1. Schematic diagram of a Servomechanism [15].

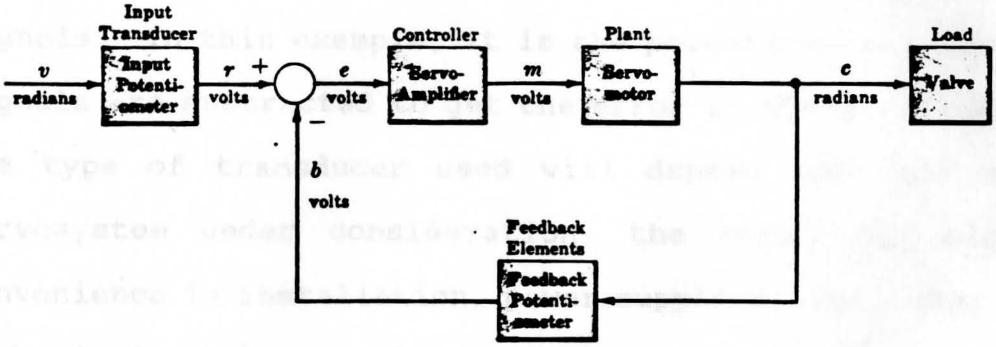


Fig. 2. Block diagram of a Servomechanism [15].

2.3 Concept of servomechanism

As an example of a servomechanism, consider the schematic diagram of the servomechanism in Fig. 1, and the block diagram of the servomechanism in Fig. 2. The basic elements which comprise the servomechanism are selected for their ability to perform a specified function with the required degree of accuracy. The basic elements are :

1. The transducer or measuring devices
2. Amplifiers
3. Actuators or controlling devices.

The first element in the servomechanism control loop is the measuring device through which the required input signal is fed to the system. This signal must be in a form which can be readily compared with the feedback signal from the final element in the loop of the feedback device. Often the same type of device is used to generate the input and the feedback signals. In this example, it is the potentiometer. These two signals are subtracted to get the error in the error detector. The type of transducer used will depend upon the overall servosystem under consideration, the resolution required, convenience in installation, power supplies, etc. They can be mechanical, electrical, hydraulic pneumatic, or some combination of these. In any case, discontinuities of any kind are to be avoided when possible. The most common electrical position measuring device is the potentiometer.

Angular velocity or rotation speed is more easily measured by tacho-generators in feedback line which has an output voltage proportional to speed; this is compared with desired speed represented by an input voltage. The part of the system which compares the input and the output signals and produces an error is commonly termed the 'error channel'. The second element in the loop is the amplifier, which is called the controller. The amplifier in a servosystem multiplies the error by a constant value, without introducing phase shift. This amplifier is sensitive to low-level error signals and provides sufficient output power to drive the controlling device. The overall performance of the servomechanism is dependent upon the characteristics of the servoamplifier, i.e., the gain and frequency response of the amplifier. Since some error detectors produce dc error signals and others produce ac error signals, the amplifier must be suitable for the type of error signal produced. In addition, the controlling device may require ac or dc input, depending upon its design.

The third element in the loop is the servomotor, which is called the actuator or plant. The actuator chosen depends upon the parameter to be controlled. In addition to supplying the necessary mechanical force, the actuator's time response must satisfy the requirements of the application. Usually ac or dc electric motors, solenoids, and electrically operated valves are used as actuators. To transmit the mechanical motion required to position the load, often gear trains, cams and other forms of mechanical coupling are used in conjunction with the actuator. The device is selected carefully for the purpose it has to serve. The choice is determined by the order of accuracy required and the type of input demand. The servomotor which is the actuator in the servomechanism is discussed in the following chapter.

2.4 Advantages of servomechanisms

Some of the functions performed by servomechanisms can be performed by less sophisticated means. But servomechanisms are increasingly used to satisfy the most challenging control requirements because they are

1. accurate due to the feedback principle, which eliminates the effect of normal variations in characteristics of components.
2. automatic due to the correction of errors in the control operation, and fluctuations in characteristics of the controlled process.
3. considerably faster than manual control.
4. reliable as it provides continuous operation with minimum service due to proven long life of the components.
5. flexible because simple re-arrangement of the same basic components permits the solution of a wide

variety of problems.

6. stable as it is not affected by environmental changes.

2.5 Applications of servomechanisms

Servomechanism techniques are used extensively in industry for measuring, recording, and controlling process variables such as temperature, pressure, flow rate, tension, position and speed. For some applications, the equipment records the value of the variable; for other applications, the equipment controls and records. Navigational devices, automatic flight controls, and missile guidance systems frequently employ servomechanism techniques. Also, servomechanisms are frequently employed in analog computers and in analog-to-digital converters.

CHAPTER III

DC SERVOMOTOR ANALYSIS

Since the principles of servomechanism apply to systems in many diverse fields of applications, the term " servomotor" can be used freely to refer to the dc motor used in servomechanism. A servomotor is a special type of motor used in the important application of position and speed control. These motors are designed to have a very high torque-to-inertia ratios with very small rotor inertias. Some dc servomotors have extremely small time constants. Dc servomotors with relatively small power ratings are used in instruments and computer related equipments such as disk drives, tape drives, printers, and word processors. One of the most useful servomotors for small power applications is the separately excited dc motor. It is either designed to be used as an Armature-controlled motor with fixed field or a Field-controlled motor with fixed armature current. The armature-controlled dc motor is often used, as it has convenient characteristics when used in closed loop [14].

3.1 Simplified mathematical model of armature-controlled dc motor with tacho-generator for speed control

The mathematical model of a system is defined as a set of equations used to represent the physical system. No mathematical model of a physical system is exact. Accuracy of the model can be increased by increasing the complexity of the equations, but exactness can never be achieved. The transfer function of a system may be represented by linear time-invariant differential equations. All the characteristics of the equations can be determined from this transfer function. Hence, to the extent that the equations accurately model the physical system, the transfer function yields the characteristics of the physical system.

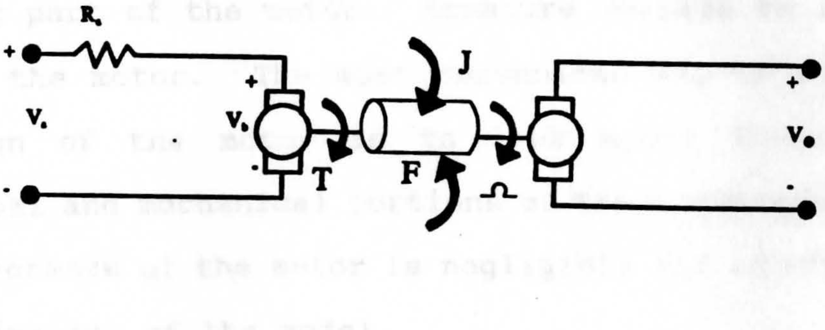


Fig. 3 . Schematic diagram of an armature-controlled dc motor

Parameters of the dc servosystem

T	Motor torque
K_t	Motor-torque constant
K_b	Back emf constant
K_g	Tacho-generator constant
V_a	Armature voltage
V_θ	Tacho-generator output voltage
I_a	Armature current
R_a	Armature resistance
Ω	Speed of the motor
J	Total moment of inertia of the motor and load
F	Total viscous friction

Fig.3. depicts an electromechanical system consisting of an armature-controlled dc motor. The armature is the rotating part of the motor. Armature voltage Va is used to control the motor. The most convenient way to analyze the operation of the motor is to link motor torque between electrical and mechanical portions of the servosystem. Also, the inductance of the motor is negligible and is neglected in the derivation of the model.

In an armature-controlled dc motor, the torque T is proportional to the armature current I_a

$$T = K_t \, I_a \tag{1}$$

where Kt in equation 1 is motor-torque constant, which depends upon the strength of the field and other details of the motor construction.

The motion of a current-carrying conductor in a field produces a voltage in the conductor that opposes the current. This voltage is called back emf. Its magnitude is proportional to the speed and it is given by

$$V_b = K_b \, \Omega \tag{2}$$

where K_b in equation 2 is back emf constant, and Ω is the speed.

The armature current depends upon the difference between the applied voltage V_a and generated back emf V_b

$$I_a = \frac{V_a - K_b \, \Omega}{R_a} \tag{3}$$

The motor torque T, is used up in accelerating the total inertia of the motor and load J, and overcoming the viscous friction torque F, giving

$$T = J \frac{d\Omega}{dt} + F \Omega \qquad (4)$$

These equations may be combined to eliminate the torque and the armature current which gives

$$\frac{J R_a}{F R_a + K_b K_t} \frac{d\Omega}{dt} + \Omega = \frac{K_t}{F R_a + K_b K_t} V_a \qquad (5)$$

Since

$$\Omega = \frac{V_\theta}{K_g} \qquad (6)$$

where K_g in equation 6 is the tacho-generator constant, and V_θ is the output in volts

Replacing Ω, and by application of Laplace transform to equation (5)

$$\frac{J R_a}{F R_a + K_b K_t} S V_\theta (s) + V_\theta (s) = \frac{K_t K_g}{F R_a + K_b K_t} V_a (s) \qquad (7)$$

Therefore the first order transfer function of the system is obtained as

$$\frac{V_\theta (s)}{V_a (s)} = \frac{k}{\tau S + 1} \qquad (8)$$

where

$$\tau = \frac{J\,R_a}{F\,R_a + K_b\,K_t} \tag{9}$$

τ in equation 9 is the motor time constant.

$$k = \frac{K_t\,K_g}{F\,R_a + K_b\,K_t} \tag{10}$$

k in equation 10 is the motor gain constant.

3.2 Modular Servosystem Unit

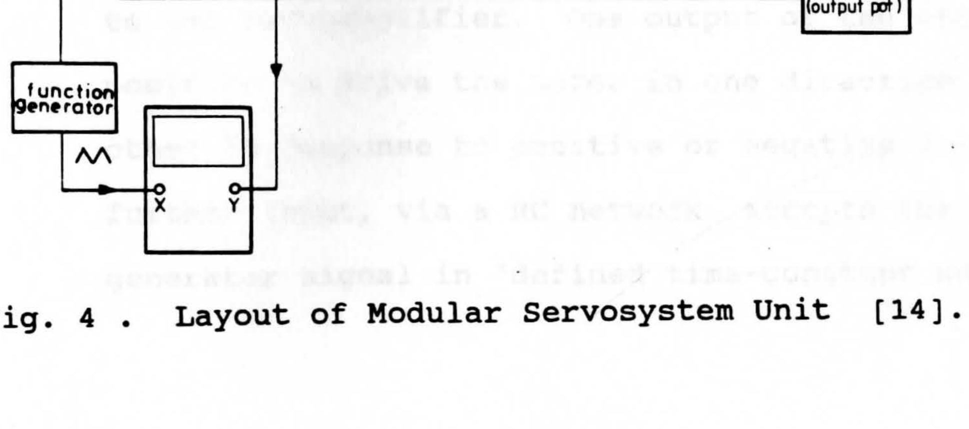


Fig. 4 . Layout of Modular Servosystem Unit [14].

The modular dc servosystem unit shown in Fig. 4 was used for experimental part of this research. The modular dc servo- system essentially consists of the following components:

1. Power supply unit, which supplies a 24 volt dc, 2 amps unregulated and +/-15v dc regulated supply at 100 ma, to the motor through the servo amplifier.
2. Motor unit, which is a dc series-wound split field motor which has an extended shaft, and onto which can be fixed the magnetic brake or inertia disc to vary the load. Integral part of the unit is a dc tacho-generator with output on the top of the unit. The tacho-generator output voltage V_θ, proportional to the speed, can be measured. It also has a low speed shaft driven by a 30:1 reduction gear box coupled to it.
3. Servo amplifier unit which contains power transistors which drive the motor in either direction. The collectors of the power transistors are connected to the ends of the split field winding.
4. Pre-amplifier unit, which provides two output signals to the servoamplifier. One output or the other goes positive to drive the motor in one direction or the other in response to positive or negative input. A further input, via a RC network, accepts the tacho-generator signal in 'defined time-constant mode'. A

toggle switch allows use with either 'defined time-constant mode' or in the 'normal' condition.

5. Operational amplifier unit which comprises a type 741 integrated operational amplifier circuit and associated input and feedback components. This provides three input sockets connected to the summing junction. A rotary switch selects one of the three feedback paths such as
    a. An unit gain,
    b. An unit gain, associated with simple lag of time constant 0.1S
    c. An external network provided by the user.
6. Attenuator unit which contains two variable 10KΩ potentiometers with dials graduated from 0 to 10. The unit can either provide reference voltage when connected to a dc source or be used as a gain control when connected to the output of an amplifier.
7. Load unit which consists of an aluminum disc that can be mounted on the motor shaft with a magnetic brake to produce the loading effect.

8. PID unit which is a Proportional plus Integral plus Derivative control element intended to be used with the Modular Servosystem. The panel layout of the PID unit is shown in Fig. 5. Separate paths for the proportional, integral and derivative signals are

combined in a summing amplifier which has an additional input socket for external signals. The signals in each separate path can be monitored at the socket and can be switched in and out of the combined signal. Gain controls are present in the integral path and derivative path. Since the separate proportional path is not variable, these controls determine the integral and derivative time constants. The output of the summing amplifier is passed through a simple low-pass filter, whose principal function is to limit the frequency range of the differentiator's action, and then to an output amplifier with gain adjustment. All the gain controls have a continuous adjustment range of 10:1, supplemented by 10:1 range multiplier switch, allowing wide variation in setting. The ranges are

Integral action time	0.1s to 1.0s
Derivative action time	1ms to 22ms
	10ms to 220ms
Proportional gain	0.1 to 1
	1 to 10

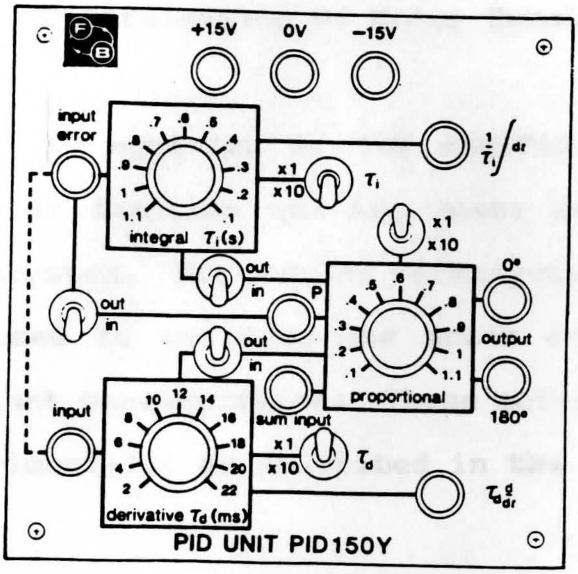


Fig. 5 . Layout of PID unit [14].

3.3 System Identification

The methods are based on calculating the coefficients of the transfer function, or the motor constants, as they are called, from data obtained from measurements on the system input and output. The procedure requires that the order of the transfer function be assumed. The coefficients may be determined from the frequency response, the step response, or the response to a general input. These techniques are considered to be a type of curve fitting, where the assumed transfer function is fitted to the available data to give the "best" fit of the transfer function.

3.4 Determination Of Motor Constants

In equation 8, the coefficients of the first order transfer function are the motor constants, k and τ of the servosystem. The modular servosystem described in section 3.2 was used to estimate the motor constants in defined time-constant configuration. These motor constants were estimated experimentally as described in the next section.

i. Determination Of Motor Gain Constant

The motor gain constant k is equal to the ratio of tacho-generator voltage (V_θ) to the input voltage (V_a) under steady-state conditions. To determine k, input V_a is varied so that V_θ also varies between +/- 12V. Table 1 shows the data for clockwise and counterclockwise rotation of the motor. k was estimated as the slope of the best fit straight line through the origin of data points V_θ vs V_a. From the graph in Fig. 6, the value of k was estimated to be 6.5.

ii. Determination Of Motor Time-Constant

The motor time-constant is estimated by plotting the log of transient response of V_θ vs time. To produce the transient response, a 0.1 HZ, 0.15 V square wave from signal generator was used as the input. The response of V_θ was plotted by using an XY-recorder. From the plot, the differences between the transient response and its steady-state value for an initial time t_0 and then ten equally-spaced times t_1 to t_{10} along the time axis, were measured. Table 2 shows the time, the ratio of each difference to the initial difference, and the log of this ratio. A graph of log of this ratio versus time was plotted. A straight line was fitted through the initial point to the data points. Ideally the graph was a straight line with slope $-1/\tau$. Motor time constant τ was estimated to be 0.26 sec from the graph in Fig. 7.

The estimated values of the motor constants were used in equation 8 of the motor transfer function, to give

$$\frac{V_\theta}{V_a} = \frac{25}{S + 3.85} \qquad (11)$$

This first order transfer function was used for future discussion and was the basis for further experiments.

Table 1. To determine dc gain k

V_a V	V_g V	
0.15	- 0.99	Clockwise rotation
0.30	- 1.99	
0.64	- 4.18	
0.77	- 5.04	
1.10	- 7.22	
1.54	- 10.07	
1.82	- 11.92	
- 0.16	0.98	Countercolckwise rotation
- 0.30	1.92	
- 0.65	4.25	
- 0.76	4.97	
- 1.11	7.26	
- 1.51	9.89	
- 1.86	11.94	

Table 2. To determine time constant τ

	Pulse		Square Wave	
Time	V_{ss} - V_θ	ln H	V_θ - V_{ss}	ln L
0.0	15.66	0.00	16.25	0.00
0.1	10.16	- 0.43	11.68	- 0.33
0.2	7.46	- 0.74	8.12	- 0.69
0.3	4.85	- 1.17	5.33	- 1.11
0.4	3.20	- 1.59	3.55	- 1.52
0.5	2.18	- 1.97	2.54	- 1.86
0.6	1.42	- 2.40	1.62	- 2.30
0.7	1.04	- 2.71	1.11	- 2.68
0.8	0.78	- 2.99	0.71	- 3.13
0.9	0.45	- 3.54	0.45	- 3.57
1.0	0.35	- 3.79	0.38	- 3.75

Where H and L are

$$H = \frac{V_{ss} - V_\theta}{V_{ss} - (V_g)_{T=0}} \; ; \quad L = \frac{V_\theta - V_{ss}}{V_\theta - (V_{ss})_{T=0}}$$

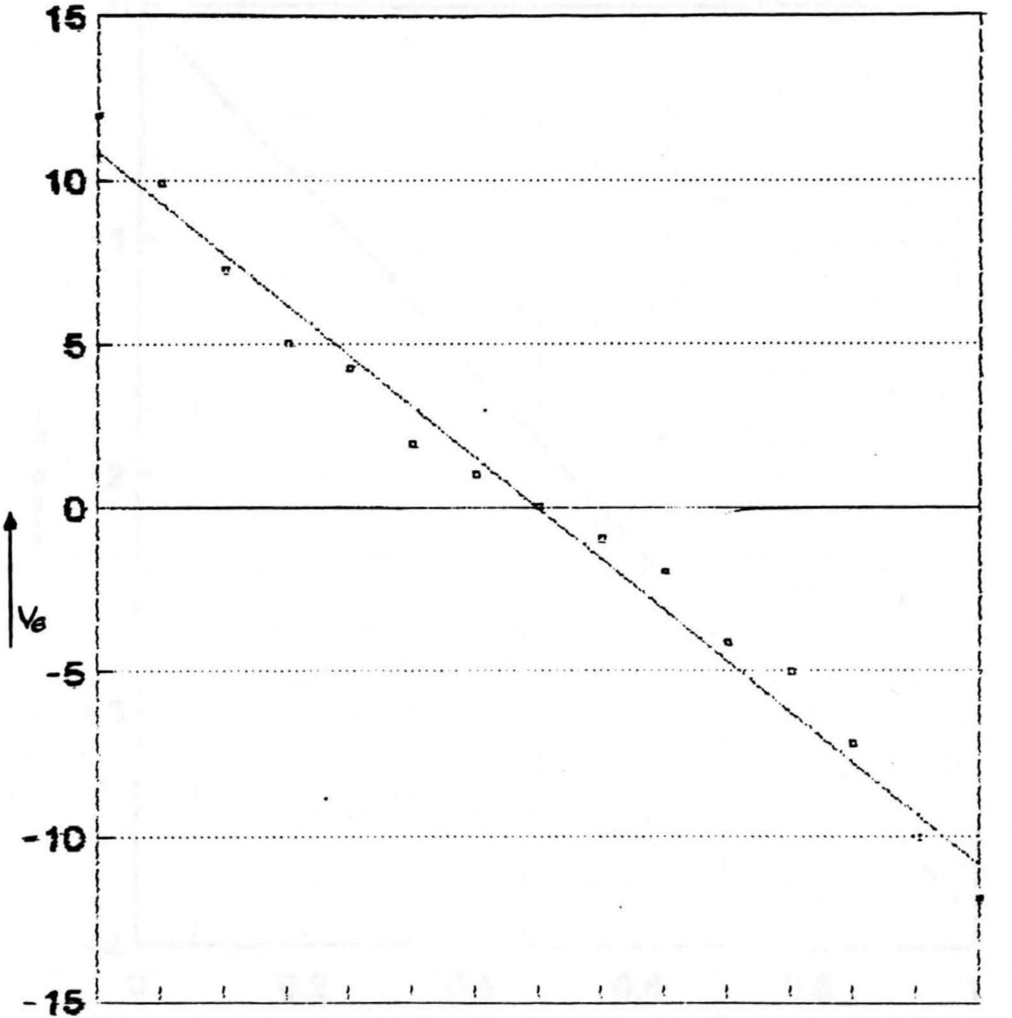


Fig. 6. Determination of Motor gain constant

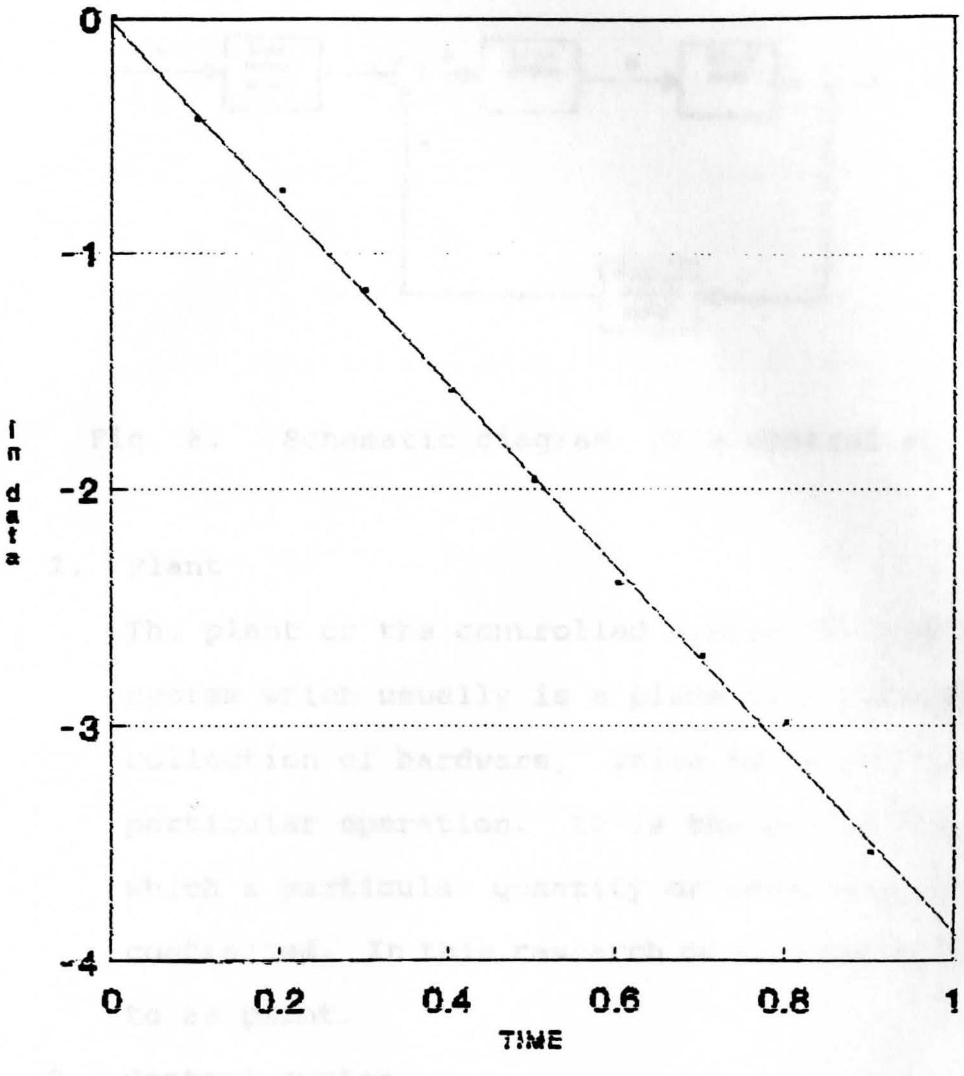


Fig. 7. Determination of Motor time constant

CHAPTER IV

DESIGN OF CONTROL SYSTEM

4.1 Definition of important terms

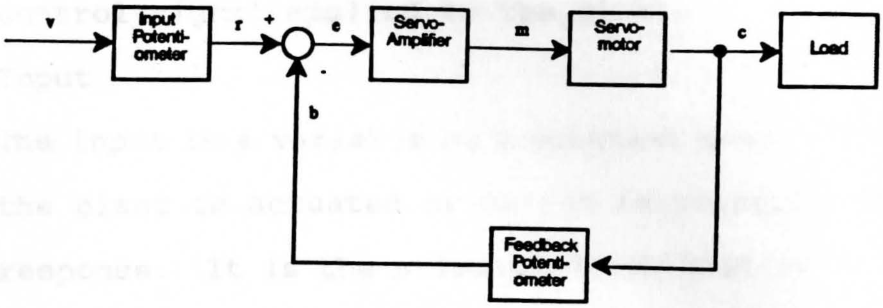


Fig. 8. Schematic diagram of a control system

1. **Plant**

   The plant or the controlled system is a physical system which usually is a piece of equipment or collection of hardware, which is to perform a particular operation. It is the controlled system of which a particular quantity or condition is to be controlled. In this research dc servomotor is referred to as plant.

2. **Control system**

   Control system is an arrangement of physical components such as the controlled system, the controllers, the sensors connected or related in such a manner as to command, direct or regulate itself or another system.

3. Control Element

   Control element also called the controller, consists of the components required to generate the appropriate control signal applied to the plant.

4. Input

   The input is a variable or a constant quantity by which the plant is actuated or driven to produce a specific response. It is the stimulus or excitation applied to the control system from an external energy source.

5. Output

   The output is the actual response obtained from the control system. It is directly related to the plant input . It may or may not be equal to the specified response implied by the input.

6. Error signal or the actuating signal e

   The error signal is the algebraic sum consisting of the input plus or minus the feedback signal.

4.2 Concept of control system

The purpose of the control system usually defines the output and the input. A control system may have more than one input or output. Often all inputs and outputs are well defined by the system description. Mathematical models, in the form of equations, are employed when detailed relationships are required. Most control system may theoretically be characterized by mathematical equations. The solution of these equations represents the system's behavior. The mathematical model of a dc servosystem derived in chapter III is a set of equations used to represent the physical system. Note that no mathematical model of a physical system is exact. The goal of control system design is to improve, or in some cases enable, the performance of a system by the addition of sensors, control processors and actuators to the plant, in order to get the required output response. Control systems are designed for a specific purpose. To enable the control system to perform efficiently it should meet the following requirements:

1. The control system should operate with as little error as possible.
2. The controlled output should follow the changes in the reference input without unduly overshooting or oscillating.
3. The performance of the control system should not be appreciably affected by small changes in certain parameters.
4. The control system should be able to mitigate the effect of undesirable disturbances.

4.3 Classification of control systems

In order for the control system to perform in a specific manner, control equipment is necessary to ensure that a system will operate as expected and do it efficiently. Based on the application of the control system, the requirements vary. In general, the control systems are classified into two general categories, i.e., the open-loop system and the closed-loop system. The distinction is determined by the control action, which is the quantity responsible for activating the system to produce the output.

The closed-loop system is shown in Fig. 8. In this system the control action is dependent on the output. It is also referred to as the feedback control system. In the closed-loop system, the actuating error signal, which is the difference between the input signal and the feedback signal, is the input to the controller. This is done to reduce the error and keep the output of the system at the desired value. Thus this system responds quickly to the demanded changes in the input. The disadvantages are that the system is relatively complex in structure and potentially unstable under fault conditions.

An auto pilot mechanism and the airplane it controls is an example of a closed-loop control system. Its purpose is to maintain a specified airplane heading, despite atmospheric changes. It performs this task by continuously measuring the actual airplane heading, and automatically adjusting the airplane control surfaces(flaps), so as to bring the actual airplane heading into correspondence with the specified heading. The human pilot or operator who presets the autopilot is not part of the control system.

The open-loop control system is shown in Fig.9. In this type of control system, the control action is independent of the output. In other words, in an open-loop control system the output is not fed back. Due to the lack of corrective action, it has inaccuracies and is slow in response to demanded changes. It is stable inherently and simple in structure.

An automatic toaster is an example of an open-loop control system because it is controlled by a timer. The time required must be estimated by the user, who is not part of the control system.

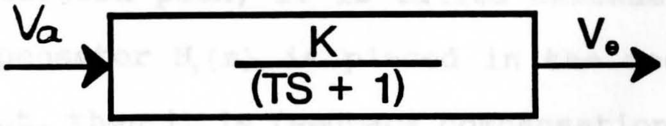


Fig. 9. Block diagram of an open-loop control system

In addition to these two classifications, control systems are categorized in many different ways depending upon the type of control required and the application. Some of the other type of control systems are the linear, non-linear, time invariant, time varying, continuous time, discrete time, etc.

4.4 Compensation

A control system is designed for a specific purpose. If adequate performance cannot be obtained with the control system described above, usually it requires some adjustment so that the various conflicting and demanding specifications may be met. This adjustment is called Compensation. The additional component used with the plant for compensation is called the Compensator. Compensators, like other system components, may be electrical, mechanical, hydraulic, pneumatic, or some other type of device. Electric networks are often used as compensators. The additional compensator $G_c(s)$ is inserted into the system in different ways as shown in Fig. 10. If compensator $G_c(s)$ is inserted into the system in forward path, it is called cascade compensation. If the compensator $H_c(s)$ is placed in the feedback path around the plant, then it is feedback compensation. Some systems involve both cascade and feedback compensation. The compensators are connected with the plant in one of these ways depending upon the need.

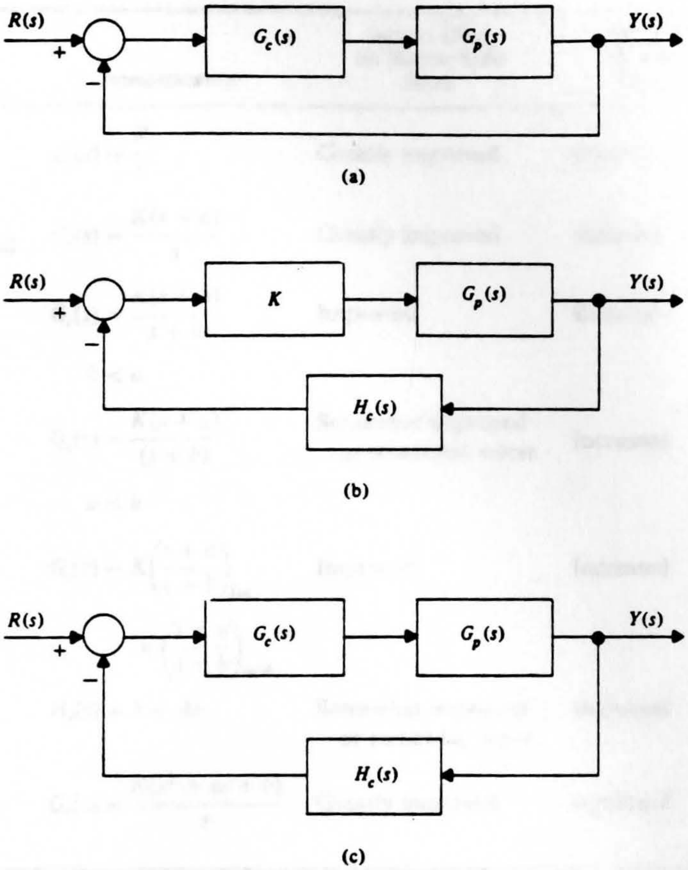
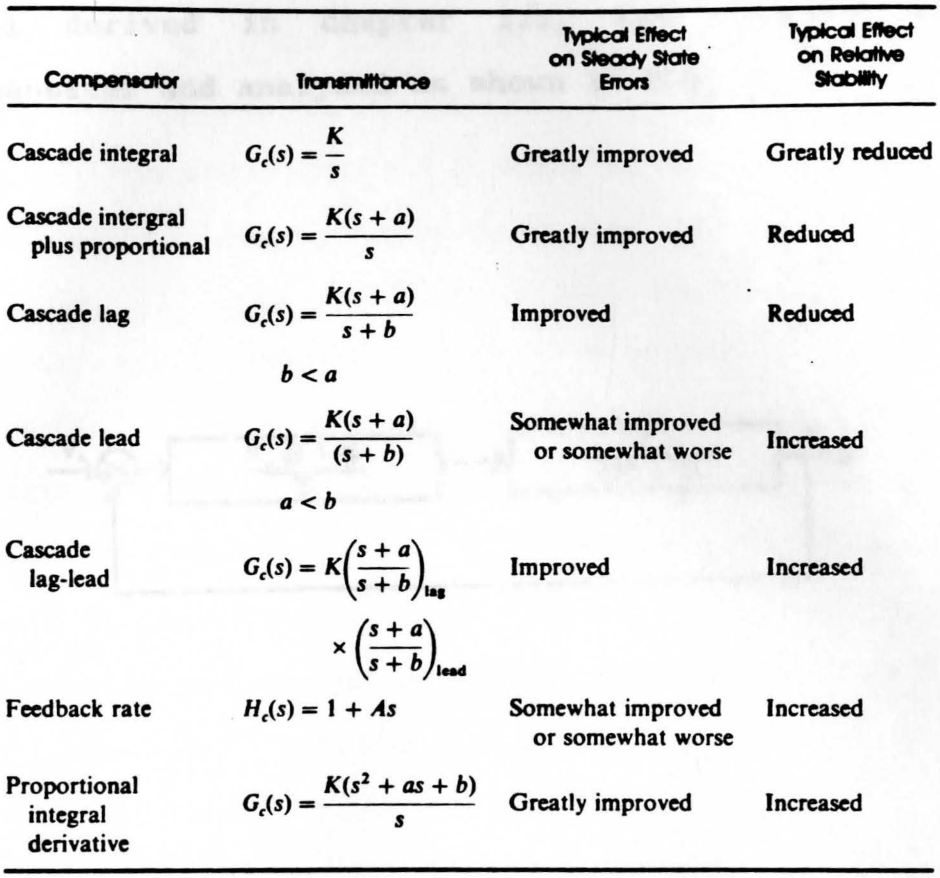


Fig. 10. Compensator configurations. **(a)** Cascade compensated system. **(b)** Feedback compensated system. **(c)** System with feedback and cascade compensation. [10].

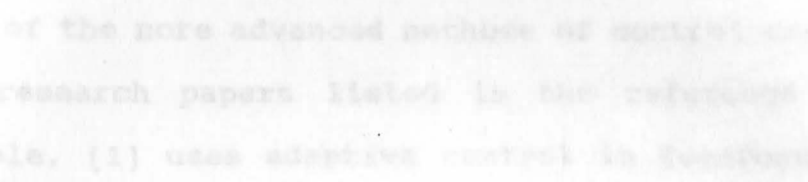
Table 3. Common types of compensators [10].



Compensator	Transmittance	Typical Effect on Steady State Errors	Typical Effect on Relative Stability
Cascade integral	$G_c(s) = \frac{K}{s}$	Greatly improved	Greatly reduced
Cascade intergral plus proportional	$G_c(s) = \frac{K(s+a)}{s}$	Greatly improved	Reduced
Cascade lag	$G_c(s) = \frac{K(s+a)}{s+b}$ $b < a$	Improved	Reduced
Cascade lead	$G_c(s) = \frac{K(s+a)}{(s+b)}$ $a < b$	Somewhat improved or somewhat worse	Increased
Cascade lag-lead	$G_c(s) = K\left(\frac{s+a}{s+b}\right)_{\text{lag}} \times \left(\frac{s+a}{s+b}\right)_{\text{lead}}$	Improved	Increased
Feedback rate	$H_c(s) = 1 + As$	Somewhat improved or somewhat worse	Increased
Proportional integral derivative	$G_c(s) = \frac{K(s^2 + as + b)}{s}$	Greatly improved	Increased

Table 3 shows the different types of compensators and the effect of each compensator on the relative stability and steady state error.

In this research the open-loop plant, dc servomotor model derived in chapter III, was cascaded with P+I compensator and analyzed as shown in Fig. 11.



$$G_m(s) = \frac{K_pKS + aK_pK}{S^2 + (a_0 + K_pK)S + aK_pK}$$

Fig. 11 . Block diagram of servomotor with P+I control

CHAPTER V

DESIGN OF MODEL-FOLLOWING CONTROL SYSTEM

5.1 Overview

In the previous chapter we found the different conventional methods to control a system by using PID unit. Some of the more advanced methods of control are discussed in the research papers listed in the reference section, for example, [1] uses adaptive control in feedforward mode with linear state feedback. [2] discusses computer control methods. [3] uses the difference between outputs of the system and the reference model to drive the system. In this research Model-Following Control is applied to control the speed of the dc servomotor. The control system was developed by specifying the system performance characteristics by means of a model and deriving the control law such that the Model-Following Control forces the servomotor to follow the model. In this control system, a model with desirable characteristics of the plant is defined. A control law is derived such that the plant is made to behave like the model. The control law cancels the plant dynamics and introduces the model dynamics. Therefore the output of the plant is same as the output of the model. Thus the plant is made to behave like the model, just by controlling the model, without incorporating any change in the actual plant.

By this method the plant can be controlled to have desirable characteristics like constant speed, frequency or position.

5.2 Design Of Model-Following Control

Fig. 12. Block diagram showing Model-Following Control

Parameters of the Model-Following Control

Model -- -- Provides any desired characteristics like speed, position of the servosystem

Model-following control - Depends on actual servomotor parameters

V_m -- Input to the model (reference input)

$V_{\theta m}$ -- Command input to model following controller

V_a -- Motor controller which drives actual servomotor towards the model

V_θ -- Servomotor output, speed, in volts

Fig. 12, shows the block diagram of Model-Following Control. It consists of the model, the model-following controller and the plant to be controlled. The model can be anything defined within the plant limitations for the desired response. Therefore, different models can be used for different responses. The Model-Following Controller strictly depends only on the actual servomotor parameters. To test the system with different models, only the model parameters will have to be changed, without changing the parameters of the controller and the actual servosystem. In Model-Following Control method, the control is provided in the feedforward mode. The control system consists of specifying the desired response of the system in the form of a model. The model is designed to have the desirable characteristics of the plant, in this case the actual servomotor. The control law is derived such that the Model-Following Control forces the actual servomotor to follow the model. The advantages of this method are:

1. All the desirable characteristics of the servomotor can be defined in the model.
2. The control law strictly depends only on the servomotor parameters.
3. The model and the Model-Following Control need not be

a hardware, but can be mathematical models simulated on an analog or a digital computer.

4. The control is provided in the feedforward mode eliminating the disadvantages of feedback mode, such as the high loop gain and complexity in the structure of the control system.

5.3 Derivation of the Model-Following Control Law

Controller design or the control law describes the algorithm or the signal processing used by the control processor to generate the actuator signals, from the sensor and command signals it receives. For the derivation of the Model- Following Control Law, the P+I compensation for the first order model of modular servomotor derived in chapter IV was used. The derivation of the Model-Following Control Law for a general second order system follows.

The transfer function of the servomotor in the second order system

is

$$G_p\ (s) = \frac{V_\theta}{V_a} = \frac{K}{s^2 + a_1 s + a_0} \qquad (12)$$

The model is required to provide the desired servomotor characteristics, therefore, the model transfer function is

$$G_m(s) = \frac{V_{\theta m}}{V_m} = \frac{K_m}{s^2 + a_{1m}s + a_{0m}} \tag{13}$$

From equation (12)

$$\ddot{V}_\theta + a_1 \dot{V}_\theta + a_0 V_\theta = V_p K \tag{14}$$

Since the control law is provided by canceling the effect of the servomotor and introducing the effects of the model, from equations (12), (13) and (14)

$$\frac{V_\theta}{V_m} = \frac{K_m}{s^2 + a_{1m}s + a_{0m}} \tag{15}$$

This implies that

$$V_a = \frac{s^2 + a_1 s + a_0}{K} V_{\theta m} \tag{16}$$

Therefore

$$V_\theta = V_{\theta m} \tag{17}$$

And the Model-Following Control Law is

$$V_a = \frac{\ddot{V}_{\theta m}}{K} + \frac{a_1}{K}\dot{V}_{\theta m} + \frac{a_0}{K}V_{\theta m} \tag{18}$$

The above derived Model-Following Control Law can be used for any servosystem application. For the control law to be effective, the dc gain k of the servomotor should be made unity. The output V_a of the Model-Following Control should be modified accordingly before feeding as the input to the plant.

5.4 Model-Following Control Law for the first order system

The first order transfer function for the dc servomotor is

$$G_p(s) = \frac{V_\theta}{V_a} = \frac{K}{S + a_0} \tag{19}$$

The model is required to provide the desired servomotor characteristics, therefore the model transfer function is,

$$G_m(s) = \frac{V_{\theta m}}{V_m} = \frac{K_m}{S + a_{0m}} \tag{20}$$

From (19)

$$\dot{V}_\theta + a_{0m} V_\theta = V_a K \tag{21}$$

Since the control law is provided by canceling the effect of servomotor and introducing the effects of the model, from the equations (19), (20) and (21)

$$\frac{V_\theta}{V_m} = \frac{K_m}{S + a_{0m}} \tag{22}$$

This implies that

$$V_a = \frac{S + a_{0m}}{K} \tag{23}$$

Therefore

$$V_\theta = V_{\theta m} \tag{24}$$

And the Model-Following Control Law is

$$V_a = \frac{\dot{V}_{\theta m}}{K} + \frac{a_0}{K} V_{\theta m} \tag{25}$$

This Model-Following Control Law for the first order system was used for the experimental purposes to test the validity of the Control Law, as explained in the next chapter.

CHAPTER VI

IMPLEMENTATION OF THE MODEL-FOLLOWING CONTROL LAW

6.1 An Overview

The Model-Following Control law for speed control was derived as shown in Chapter V. The next step was to test the validity of this law. The Model-Following Control design was implemented on the analog computer as well as on the actual hardware, which is the modular servosystem unit explained in chapter III. In order to test the derived law, the P+I compensated dc servomotor model, shown in Fig. 11, in Chapter IV, was used. The performance of three different P+I compensator design models used in the control system design were compared. Each P+I compensator model differs from the other only in the values of the parameters, a and Kp the constants. The implementation of the control system design was done in three steps as follows

1. Analog Simulation for the speed control of a dc servomotor.

   In this step the mathematical model of the Model-Following Control system design was programmed on the analog computer. This consisted of the model, the model-following control and the model of the open-loop dc servosystem. The time responses of the analog plant output voltage V_θ, and the output variable of the analog model $V_{\theta m}$ for the three compensator models were plotted using an XY-recorder.

2. Real-time simulation with the actual modular servosystem unit in the control loop

   In this step, only the mathematical models of the model and the Model-Following Control were programmed on the analog computer. In this case the actual modular servosystem unit was connected in the place of the plant and was driven by the output of the Model-Following Control. The time responses of the outputs, which are the tachogenerator output voltage V_θ and the output $V_{\theta m}$ of the three compensator models, were plotted using an XY-recorder.

3. Speed control of dc servomotor with actual PID unit

   In this step the actual modular servosystem was used for speed control along with the PID unit shown in Fig. 5 chapter III, to provide the P+I compensation. The time responses of the tachogenerator voltage $V_{\theta 1}$, which is the output of the plant driven by the PID unit, and $V_{\theta 2}$, which is the output of the plant driven by the Model-Following Control for the three compensator models, were plotted using an XY-recorder.

The three steps mentioned above were implemented for all the three different P+I compensator models. The time responses of the output for each model are shown in the accompanying graphs.

6.2 Experimental procedure

The exact experimental procedure for each one of the three steps mentioned in the overview will now be explained in detail.

1. Analog computer simulation

Generally analog computers consist of an active electrical circuit as the analogous system. They have no moving parts, a high speed of operation, good accuracy, and a high degree of versatility. The active electrical network consists of resistors, capacitors, and operational amplifiers. The forward voltage transfer characteristics of these networks are analogous to the basic linear mathematical operations encountered in the physical system's mathematical model. Therefore the analog computers can be used to simulate mathematical models of a physical system. The input and output voltages of the computer are analogous to the corresponding mathematical variables of the physical system. In certain cases, due to the limitations of the input / output equipment, it becomes necessary to change the scale of the computer variables. The normal procedure for simulating a

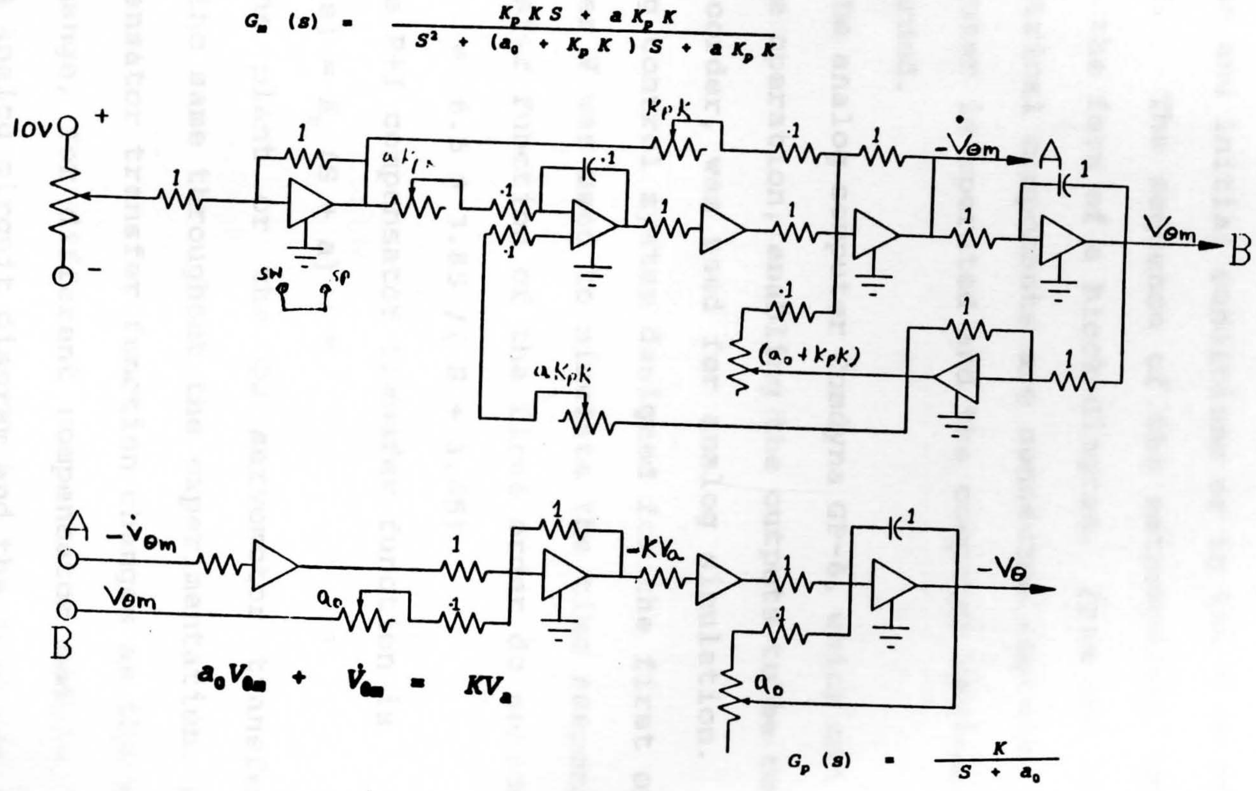


Fig. 13. Analog circuit diagram for step 1

control system involves describing the mathematical model of the system in the form of a set of one or more differential equations and initial conditions or in the form of a transfer function. The sequence of the mathematical operations is shown in the form of a block diagram. From the block diagram the electrical components are connected together or patched. The computer is operated and the computer variables observed and recorded.

The analog computer Comdyna GP-6, which can function in slow time operation, enabling the outputs to be recorded with an XY-recorder, was used for analog simulation. The Model-Following Control system designed for the first order system in Chapter V was used to simulate the time responses. The transfer function of the first order dc servomotor is

$$G_p\ (s) \ = \ 6.5 * 3.85 \ /(\ S + 3.85) \qquad (26)$$

and the P+I compensator transfer function is

$$G_c(s) = K_p\ (S + a)\ /\ S \qquad (27)$$

The plant or the dc servomotor transfer function remains the same throughout the experimentation, whereas the P+I compensator transfer function changes as the value of K_p and a change, for different compensator models. Fig. 13 shows the analog circuit diagram and the transfer function to generate the different models. In each case the values of K_p and a were changed to generate the model.

i. Simulation with P+I compensator1

The transfer function of P+I compensator1 is

$G_{c1}(s) = 0.1*(S+5) / S$ (28)

where $K_p = 0.1$, $a = 5$

The model transfer function is

$$G_{m1}(s) = \frac{2.5S + 12.5}{S^2 + 6.35S + 12.5} \tag{29}$$

The Model-Following Control law equation is

$$KV_a = V_{\theta m} + a\,\dot{V}_{\theta m} \tag{30}$$

The model, Model-Following Control and the open-loop model of the plant, were programmed on comdyna GP-6 as shown in Fig. 13. The time responses of the output of the plant V_θ and output of the model1 $V_{\theta m}$ were plotted.

ii. P+I compensator2

The transfer function of the P+I compensator2 is

$G_{c2}(s) = 0.1(S + 2)/S$ (31)

where $K_p = 0.1$, $a = 4$

The model transfer function is

$$G_{m2}(s) = \frac{2.5S + 5}{S^2 + 6.35S + 5} \tag{32}$$

iii. P+I compensator3

The transfer function of the P+I compensator3 is

$$G_{c3}(s) = 0.3\,(S+5)\,/\,S \tag{33}$$

where $K_p = 0.3$, $a = 5$

The model transfer function is

$$G_{m3}(s) = \frac{7.5S + 37.5}{S^2 + 11.35S + 37.5} \tag{34}$$

The analog time response plots for all the three different compensator models are shown in Fig.14 , Fig.15, and Fig. 16, respectively. In these figures, the set of graphs indicated as graph A shows the analog time responses for the model and the analog plant plotted on the same scale. The set of graphs indicated as graph B shows the analog time responses for the models on a scale double that of the analog plants.

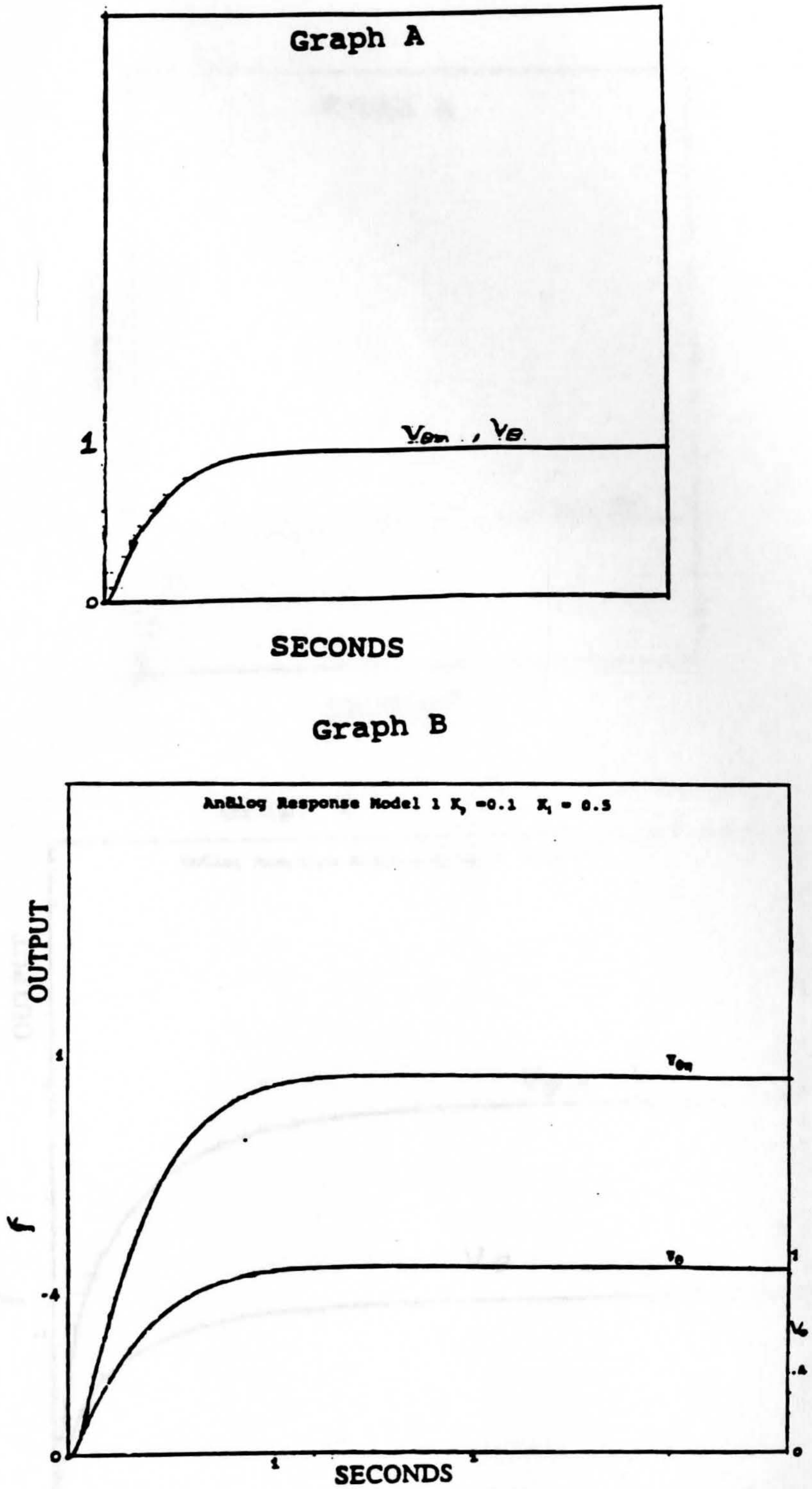
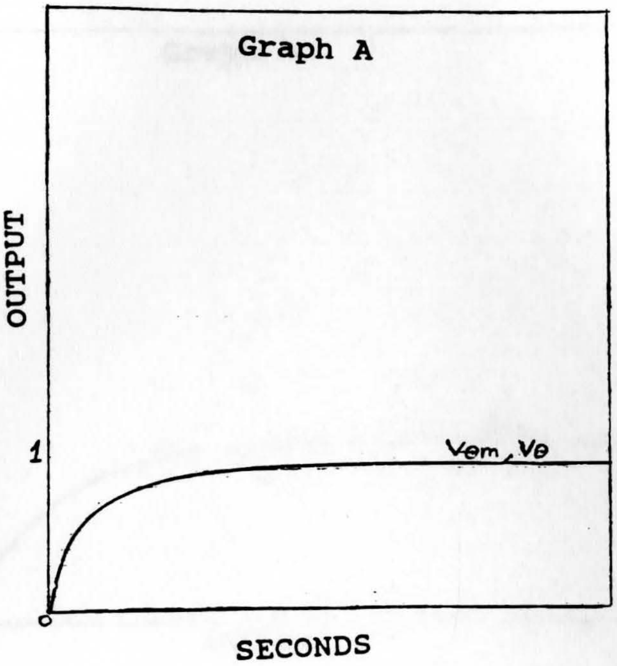


Fig. 14. Analog Response for Model 1



Graph B

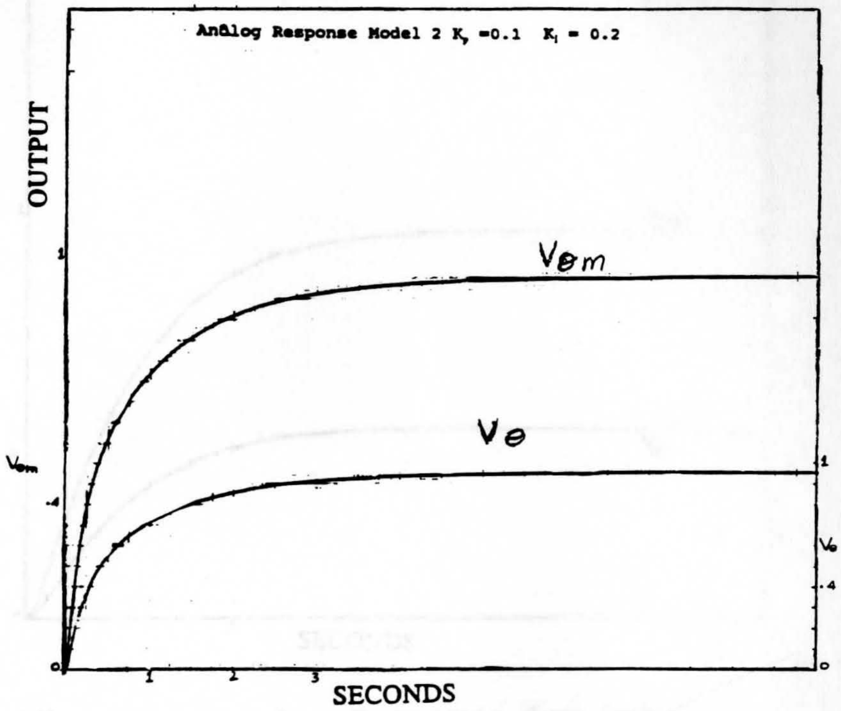


Fig. 15. Analog Response of Model 2

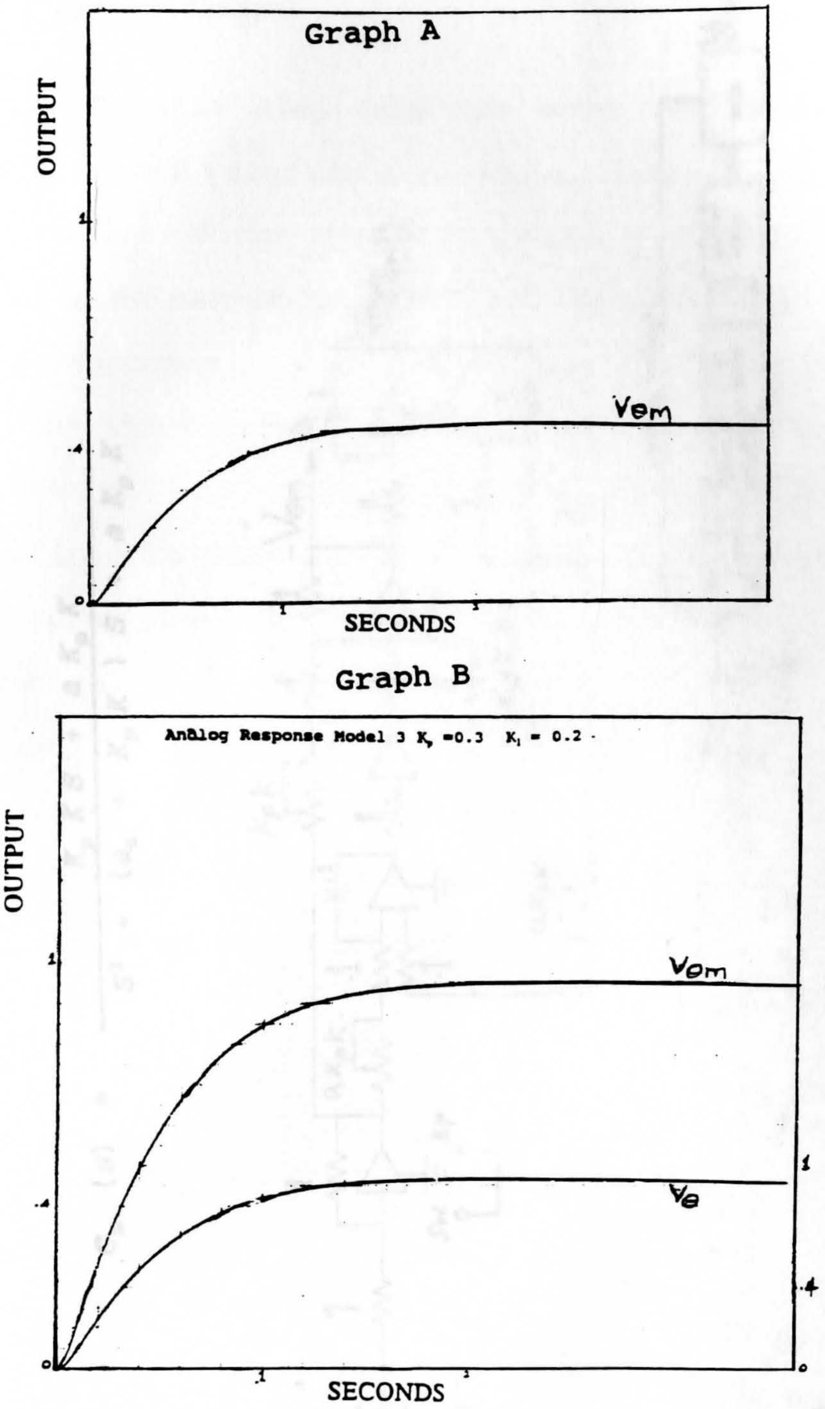
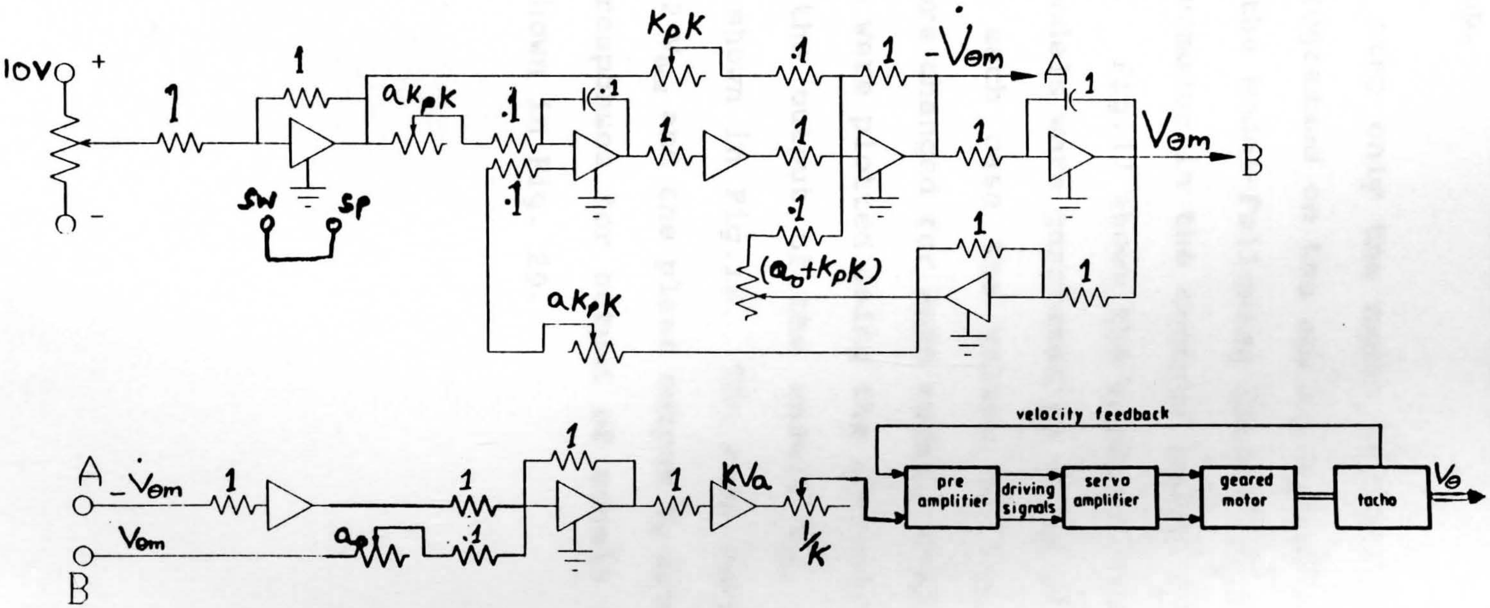


Fig. 16. Analog Response for Model 3

$$G_m\ (s)\ =\ \frac{K_p\,K\,S\ +\ a\,K_p\,K}{S^2\ +\ (a_0\ +\ K_p\,K\ )\ S\ +\ a\,K_p\,K}$$



$$a_0 V_{\theta m}\ +\ \dot{V}_{\theta m}\ =\ K V_a$$

$$G_p\ (s)\ =\ \frac{K}{S\ +\ a_0}$$

Fig. 17. Circuit diagram for step 2

2. Simulation with the actual servomotor in the control system loop.

In this step only the model and the Model-Following Control were programmed on the analog computer comdyna GP-6. The output of the Model-Following Control drives the actual modular dc servomotor in the control system loop to get the time responses. Fig.17 shows the schematic diagram for step 2. The three models were generated by changing the values of K_p and a. In each case the values of the potentiometer coefficients were changed for each model. Step responses for the three cases were plotted using the XY-recorder. The step responses for the output of the model1 $V_{\theta m1}$ and the plant output V_θ, are shown in Fig.18. The step responses for the output of model2 $V_{\theta m2}$ and the plant output V_θ are shown in Fig. 19. The step responses for output of model3 $V_{\theta m}$ and plant output V_θ are shown in Fig. 20.

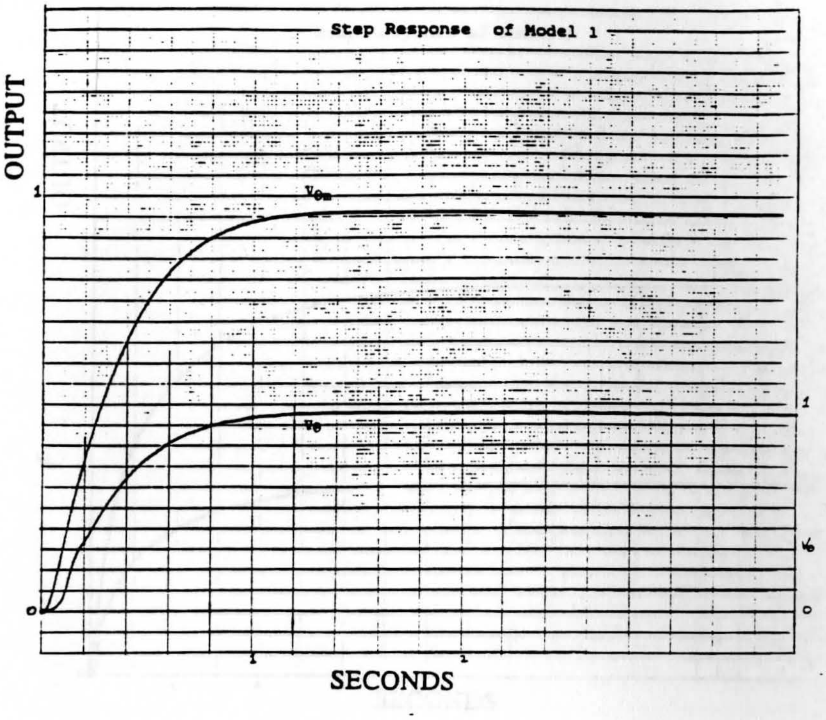


Fig. 18. Step Response for Model 1

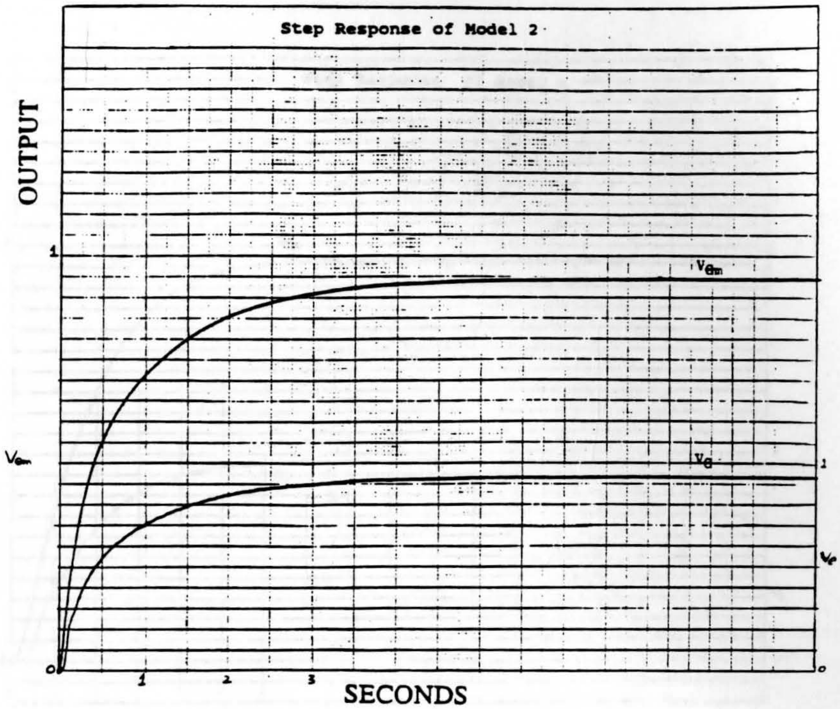


Fig. 19. Step Response for Model 2

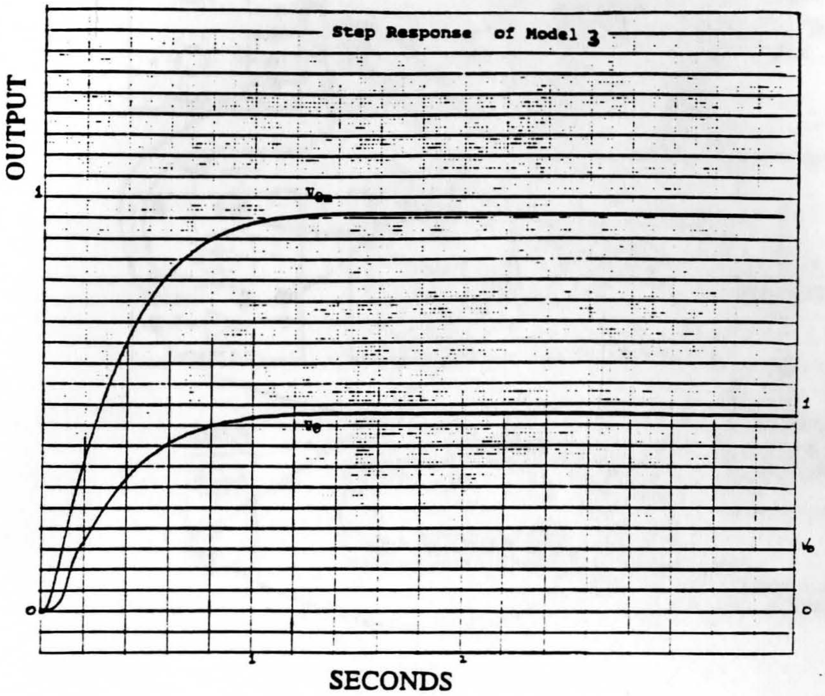


Fig. 20. Step Response for Model 3

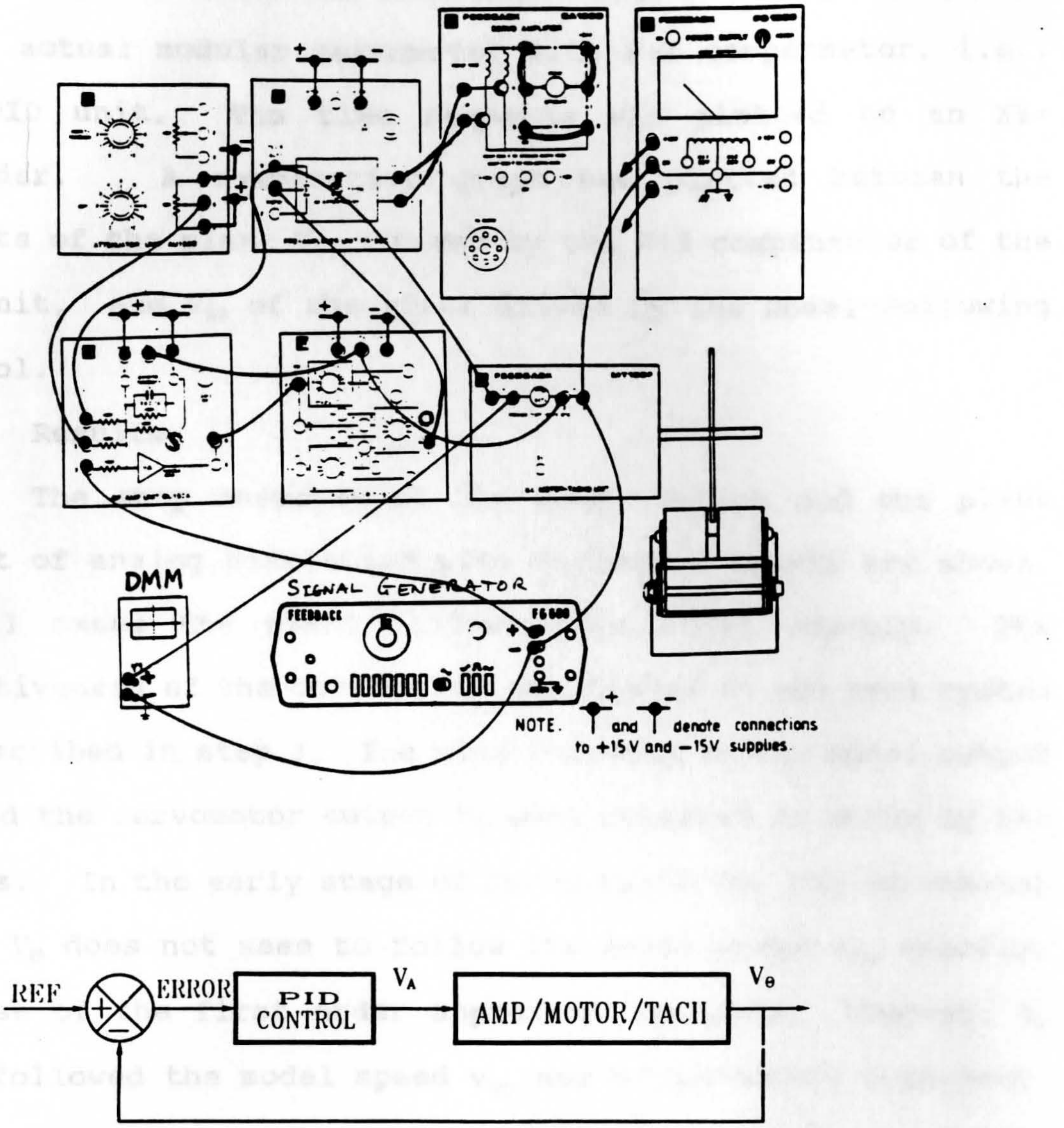


Fig. 21. Circuit diagram for step 3

3. Speed control of dc servomotor with the actual PID unit

Fig. 21 shows the schematic diagram for speed control of an actual modular servomotor with P+I compensator, i.e., the PID unit. The time response was plotted on an XY-recorder. A comparative graph was plotted between the outputs of the plant $V_{\theta 1}$, driven by the P+I compensator of the PID unit, and $V_{\theta 2}$ of the plant driven by the Model-Following Control.

6.3 Results

The step response of the model output and the plant output of analog simulation with different models are shown. In all cases the plant followed the model exactly. The effectiveness of the control law was tested on the real system as described in step 3. The step response of the model output $V_{\theta m}$ and the servomotor output V_θ were compared as shown by the graphs. In the early stage of speed build-up, the servomotor speed V_θ does not seem to follow the model speed $V_{\theta m}$ exactly, because of the first order approximation used. However, V_θ soon followed the model speed $v_{\theta m}$ and satisfactory transient response was obtained. The output of the plant driven by the Model-Following Control was compared with that of the output of the plant driven by the conventional PID unit as shown in Fig. 22, Fig. 23, and Fig. 24. It was seen that the servomotor output $V_{\theta 2}$ due to Model-Following Control had a faster response than $V_{\theta 1}$ due to the PID control.

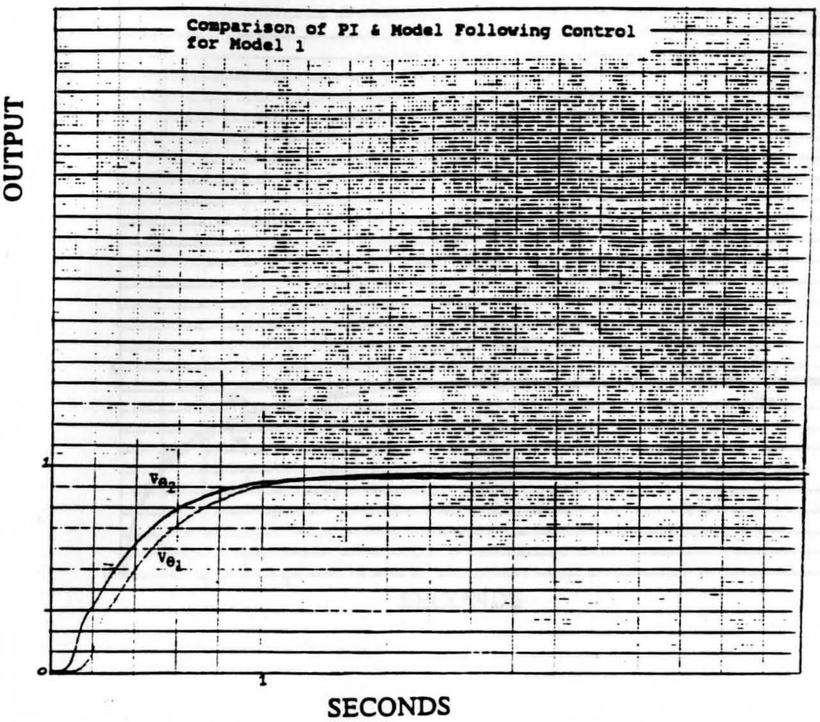


Fig. 22. Comparison of P+I and Model-Following Control for Model 1 .

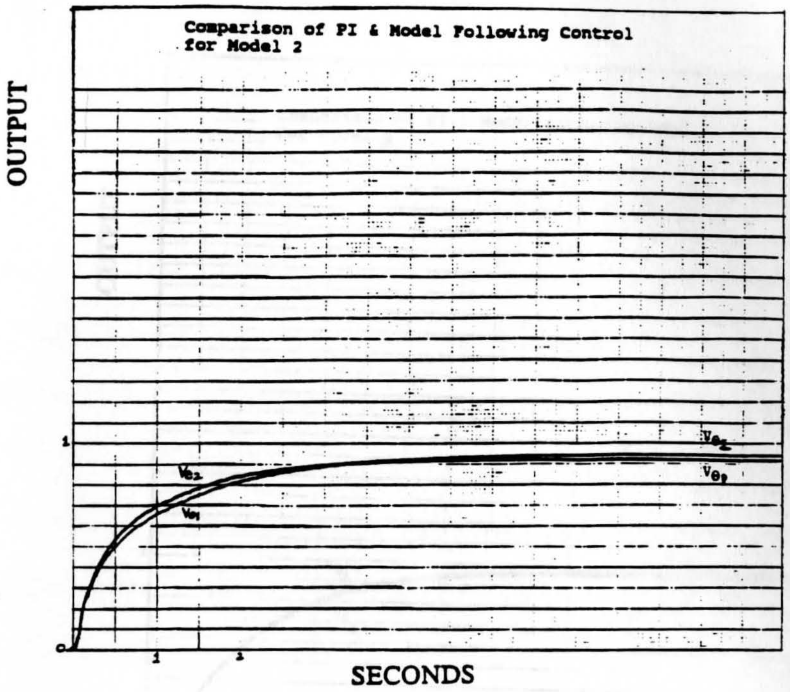


Fig. 23. Comparison of P+I and Model-Following Control for Model 2 .

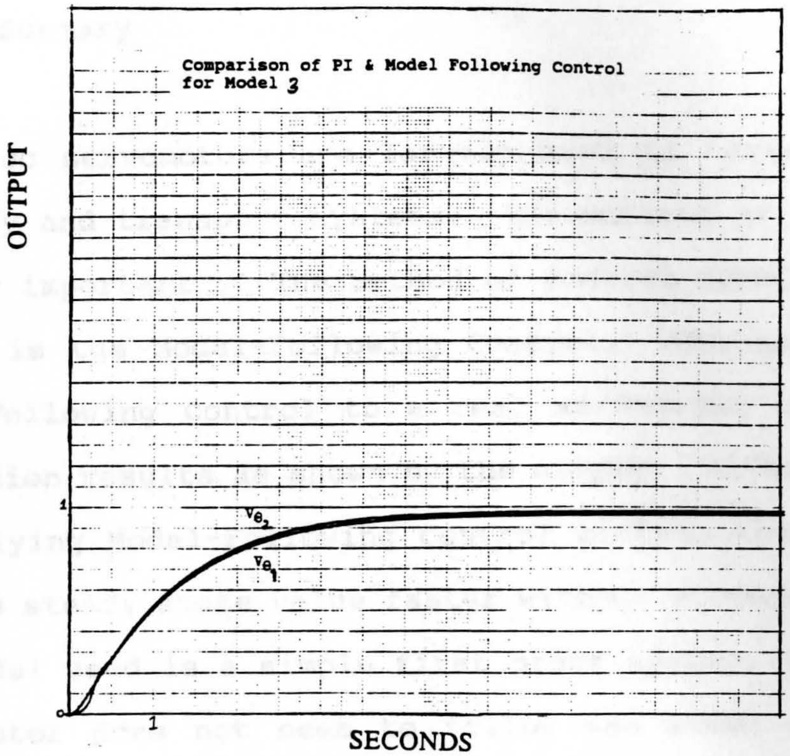


Fig. 24. Comparison of P+I and Model-Following Control for Model 3

CHAPTER VII

CONCLUSION

7.1 Summary

Dc servomotors are largely used in every branch of industry and transport systems. The control of a servomotor is very important . The method of control developed in this thesis is the Model-Following Control. The application of Model-Following Control to a real servomotor confirmed the simulation results as shown by the graphs. It has shown that by applying Model-Following Control method, the motor speed reaches steady state value faster without a overshoot. Since the model used is a simple first order system, initially the servomotor does not seem to follow the model exactly. If complex and higher order models are used, the results may be improved. The advantage of the Model Following Control method is that a system can be made to behave like the model with the desired response. Thus the hardware controller can be replaced by the mathematical model in the control system loop. The Model-Following Control depends upon the plant parameters. Thus the plant can be made to behave as the desired model by changing the model alone for the desired plant characteristics, without changing the plant or the Model-Following Control.

7.4 Future recommendations

The Model-Following Control method developed in this thesis could be extended to position control of dc servomotor, or for any other type of control of a system. If the disturbance is measurable, this method could be used to model the system with disturbance and control it. This method could be used for more complex and higher order systems.Also digital simulation could be used in place of analog simulation. As an extension of this thesis, the method developed could be tested on a second order system and also used to model non-linear systems.

LIST OF REFERENCES

[1] Kelly R., " A linear-state feedback plus adaptive feedforward control for dc servomotors," IEEE Trans. Ind. Elect.,Vol. IE-34, no. 2, pp 153-157, May 1987.

[2] Jing-Ping, J. and Marleau, R. S. ," Digitally controlled dc drive motors," IEEE Trans. Ind. App., Vol. IA-18, no. 6, pp 728-735, November/December 1982

[3] Butler, h., Honderd, G., and Amerogen, J., "Model reference adaptive control of a direct-drive dc motor," IEEE Control System Magazine, Vol. 9, no.1, pp 80-84, 1989.

[4] Astrom, Karl J., and Witten Mark, Bojn., Computer Controlled Systems, New Jersey: prentice-hall, 1984

[5] Bollinger, John G., and Duffie, Neil A., Computer Control of Machines and processes. New York: Addison Wesley publishing Company, 1988.

[6] Borrie, John A., Modern Control Systems a manual of Design Methods, New Jersey: Prentice-Hall International, 1986.

[7] Burdett, G.A.T, Automatic Control Hand Book, Great Britain: Adlard and Sons, Ltd, 1962.

[8] Friedland, Bernard, control System Design, New York: Mc Graw-Hill, 1986.

[9] Frankline, Gene F., and Powel, David J., control of Dynamic Systems. New York: Addison-Wesley publishing

Company, 1980

[10] Hosttetter, Gene H., Savant, Clement J, and Stefani Raymond T,. Design of Feedback Control Systems. New York: Holt, Rinehart and Winston Inc., 1988.

[11] Sinha, Naresh K, Control Systems. New York: Holt, Rinehart, and Winston Inc., 1988.

[12] Ogata, K., Modern Control Engineering. Second Edition, New Jersey: Englewood Cliffs, 1990.

[13] The GP-6 Analog Computer Operators and Maimtaince Manual. Illinois: Comdyna Inc.

[14] The Modular Servo System MS150 MkII Book 1,2,3,4,5,6. Feedback technical information, Crowborough: Feedback Instruments Ltd.

[15] Di Stefano III, Stubberud, and Williams. Feedback and Control Systems, Schaum's outline series, New York: McGraw-Hill Book Company., 1967.

[16] Jalali J, and Chitra, K. S, "Deriving Model Following Control Law to Control Dc Servomotor," Modelling and Simulation proceeding, Vol no. 23, pp 1897-1904, University of Pittsburgh, Pittsburgh, 1992.