Math Methods for Secondary Education Teachers:

Blending New and Old Ideas for Successful Student Learning

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ABSTRACT

This study is an investigation into the mathematics instructional methods of past and present mathematics professors and educators and a blending of these techniques into mathematics instruction in high school mathematics classroom. Selective items chosen for this study were George Polya and his style of instruction, Morris Kline and his views on the American education system, Steven G. Krantz and his views on teaching mathematics, Richard R. Skemp and his assessment of the psychological aspects of learning mathematics, Matt DeLong and Dale Winter and their approach to professional development, and James W. Stigler and James Hiebert and their examination, contrast and comparison of current mathematics instruction in United States, Japan, and Germany. This study also includes discussions on the implementation of various techniques and methods into high school geometry and trigonometry classroom instruction. Specific examples of lessons using some of these techniques are detailed. More successful instructional methods, techniques, and specific lessons time tested by experienced classroom teachers should be passed on to new teacher and fellow instructors through departmental libraries and learning centers.

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Today's mathematics educators are faced with a complex problem of educating students of all academic levels to be functional in the new technology age. No longer is it acceptable to have students fail. Failures are viewed as non-productive members of society. These students do not have the skills that are required to enable the high school graduate to be assimilated into the current work force without additional expense to the employer.

In January, 2002, President George Bush signed the new Elementary and Secondary Educational Act, which made it mandatory that "no child will be left behind." For various reasons, this has become an extremely large task to accomplish. Unto those ends, the Secretary of the United States Department of Education set into motion on March 16, 2005 his Mathematics and Science Initiative, which includes a goal to "develop a research base to improve our knowledge of what boosts student learning in mathematics and science" (U.S. Department of Education, 2005, p.1). This issue has been discussed extensively by mathematics educators for over 60 years. An analysis of the instructional methods of the professors and learned mathematicians of the past 60 years and those of the current wave of mathematics reformers may bring to light some avenues of instruction useful to the elementary and secondary teachers in producing the "math educated" secondary school graduate.

While potential employers of all types examine with much scrutiny the knowledge level of our current graduates, the question arises in my mind as to whether or not the available techniques and procedures for instruction of the mathematics disciplines, if properly utilized, will improve secondary classroom instruction. As an educator, who graduated from college while the lecture method was the dominate form of instruction, I am interested in discovering what has been tested through usage and has proven successful for other mathematics educators. Based on what college students are required to do in our present system to obtain a teaching license, I feel that I was not prepared well enough. Though mathematics teachers often tell the algebra students when working with variables that the students cannot compare "apples and oranges," I feel I must compare my education with the available information to glean a proper perspective of the present level of pedagogy in mathematics instructional methods. My mere one general course each in teaching in secondary schools, teaching mathematics, in educational psychology and in educational philosophy along with one experience in the classroom, student teaching, did not make me aware of all the efforts of mathematics educators to improve mathematics instruction. To me, the efforts of these educational giants did not penetrate the entire college/university system well enough for me to grasp and develop an instructional style that will help me educate all my students to their fullest potential.

For the past five years, I have been compelled to traverse this path as I carry the responsibility of being a member of the Mathematics Curriculum Team for our County Educational Service Center and my local school district. During the summer of 2000, I was a member of the team that wrote several courses of study for the mathematics disciplines at the high school level. Not long afterwards, I voluntarily wrote a variety of Short Cycle Assessments for the eleventh grade courses (Algebra II, Trigonometry, Pre-Calculus and Probability and Statistics) utilizing the Ohio Academic Content Standards. As a member of my district's Mathematics Committee, I have been involved in an in-depth analysis of student achievement, instructional practices and mathematics curriculum for grades K-12. The investigation of instructional practices was based on current research found in journal articles and texts published on these practices. The committee also gathered information on instructional practices in the Buckeye Local School District, evaluated teacher attitudes toward current instructional practices within the district, analyzed student achievement data, and observed classroom instruction at several high performing schools in an effort to make recommendations to improve student achievement and K-12 mathematics curriculum along with viable suggestions for improving instructional practices in the classroom (Williams, Clark, Deak, DeLuca, Drushel, Evans, Falcione, Gawdyda, Hughes, Kimmel, Morris, Obhof, Pasky, Pozum, Robinson, & Scafuro, 2004).

This committee's work is far from done. As the district implements the suggestions, I feel that I must continue researching additional instructional methods and procedures, both from current mathematics reformers and innovative educators of the past, thus creating a pool of knowledge accessible to my peers that would maximize learning in the classroom.

The variety of solutions is as wide as there are people who believe they have discovered a method or procedure that will correct mathematics education. Each person brings to this solution his/her own personal experiences in education from childhood through college. These experiences could have left a positive image of mathematics education or a feeling of doom and gloom. The educator then brings the personal successes and failures into the classroom as an instructor. Some of the solutions meant to set mathematics education on the correct path range from changing one's teaching style by utilizing the knowledge of how the brain encodes new concepts when preparing lesson plans to comprehending the role that the fear factor plays in educating the student. With such research available, I considered my own successes and failures in the classroom and concluded that I would render an analysis of those mathematics educators whose specific procedures and techniques have helped me improve my instructional methods.

Mathematicians and Instructional Innovators

In this paper, I will discuss the instructional method employed by George Polya for finding the solution for problems, comment on Morris Kline's observance of mathematical instruction in American schools in the mid-20th century, summarize the psychological aspects of learning mathematics as revealed by Richard R. Skemp, review the philosophy of mathematics instructional methods by Steven Krantz, and review some of the current reformers who highlight particular aspects of mathematical instruction. I will incorporate classroom examples from a mathematics educator with whom I work and my own personal classroom examples of experience with these instructional methods along with any general comments about my teaching style from my peers, whether in my department or from my district's mathematics committee, concerning the particular items discussed.

George Polya and His Style of Instruction (1887-1985)

An outstanding 20th century mathematics educator, George Polya (1887-1985), determined that his lack of excellence at the Daniel Berzsenyi Gymnasium in Budapest, Hungary was due to the poor teaching he received from two of his three mathematics teachers. This produced in him an indifference toward mathematics. After obtaining his certificate in Latin and Hungarian, Polya wanted to learn about philosophy. Under the direction of his advisor, Polya was to first study physics and mathematics in order to provide himself with a background for philosophy. For the non-mathematics person, there exists within the field of mathematics an area of the philosophy of mathematics and philosophy of logic. This study of logic provides the terminology and the practice of utilizing this type of thought process within the field of mathematics. It lays a foundation for the study of philosophy of any discipline. The philosophy of mathematics dates back to the time of Socrates and Plato. Thus for Polya, one could conclude that the study of mathematics was necessary to provide some experience in an extremely broad area (O'Connor and Robertson, 2002).

Through many successful encounters with prominent mathematicians of the early twentieth century, George Polya began to take seriously the world of mathematics. Thus, the beginning of a life-long career as a mathematics educator had commenced for George Polya (O'Connor and Robertson, 2002).

Personal difficulties with mathematics and recognition of the same for many of his students led Polya to publish his book *How to Solve It* in 1945. His personal method of instruction, a modern heuristic approach, gave a definite instructional tool to mathematics education that differed from the norm of rote memorization and computational drills so prevalent in educational institutions world wide (Polya, 1973).

The general purpose of a heuristic approach to a situation is to focus one's attention on the techniques and processes used in discovering and/or inventing a solution to a problem. This concept was not new or unique to Polya. It has been used by ancient Greek mathematicians/philosophers as well as some Renaissance men. Furthermore, the modern heuristic approach deals with comprehending the procedures used in problem solving, stressing the mental operations that are ordinarily associated with this process. In any heuristic study, no aspect of the problem solving process is ignored or bypassed for any reason, least of all experience. Both personal experiences and the experiences of others in problem solving must be viewed as essential components involved in developing a common methodology especially useful in teaching mathematics (Polya, 1973).

Firmly believing that the primary task of a teacher is to help the student learn, Polya presented his heuristic approach to mathematics teachers in a four phase procedural format. The first phase was to enable the student to understand the problem. To accomplish this phase, the teacher had to make sure that the student could discern what the unknown factor was and whether or not there was sufficient information available to render a possible solution. Often times, it was necessary for the teacher to require that the student draw a diagram or figure that would yield a visual display of the problem. From here the student would need to label and make notations of pertinent conditions (Polya, 1973).

Completing this phase, the teacher would have to help the student devise a plan of action. This plan of action would require that the student find the connection between the collected data and the unknown factor. When this was not readily available to the student, it became the teacher's responsibility to question the student in such as fashion as to lead him/her to the connection or a related problem for which a solution had been rendered. Here is the crux of the task. If the teacher did not furnish the student with sufficient information to discover the connection, the student would be forever lost. Then again, if the teacher furnished the student with too much information, the student would not have anything left to discover. Thus, spoon feeding would result, and the student would be no better off. Discreteness was an essential part of the second phase for the teacher to help instill confidence in the student for establishing a plan of action (Polya, 1973).

The third phase, carrying out the plan of action, would produce a solution in a logical step by step process with each subsequent step being related to the preceding step(s). The teacher had to be assured that the student could clearly see that each step was correct and could prove that it was correct by some relationship, fact, or procedure (Polya, 1973).

Lastly, an examination of the solution was checked. Could the resulting solution be used in solving some of other problems in the lesson? Could this solution be derived differently? Could additional information be obtained from the problem? These were some of the questions that Polya considered essential for the teacher to utilize in recapping the solving process. It does no good for the student to get to the end if he/she has no idea what has been gleaned for future use (Polya, 1973).

George Polya (1954) did not cease here. Instead, he continued by writing a two-volume set of books on plausible reasoning. This set of books, entitled *Mathematics and Plausible Reasoning,* was meant for an audience of mathematics students and teachers to enable them to correctly ascertain the thought process necessary to bring them to demonstrative reasoning, the thought process required in doing proofs. Since demonstrative reasoning is precise and exact as established by mathematical procedures and facts, it does not develop new mathematics. Discovering new mathematics is a product of forming conjectures by plausible reasoning. Plausible reasoning is necessary for students to establish the second step in solving proofs. The ability to guess and guess again assists the student in finding a plan of action that will lead him/her to the solution.

Though he wrote these books in the mid-1950s, some ten years later Polya (1962) still passionately felt that "the preparation of high school mathematics teachers is insufficient" (Polya, 1962, p. vii). He stated that all responsible institutions should accept their portions of the blame for this failure. Beyond the fact that the schools of education and departments of mathematics at the colleges and universities should revise the course offerings for teachers of mathematics, Polya held the opinion that this task could not reasonably be accomplished until the controversial issue of what should be taught in high school mathematics programs was resolved. His opinion was stated in print in 1962. Have we listened? Within the last few years, the Ohio Department of Education has revised the "Model" they set forth in 1990 and published an academic content standard book for all K-12 mathematics teachers to utilize in planning and instructing Ohio students. This endeavor has clarified and updated the "what" of K-12 mathematics education. Fortunately for Ohio mathematics teachers, the United States Department of Education considers the instruction of U.S. children in mathematics to be a critical task. At the present time, committees under its direction are investing ways that will enable the students to increase knowledge.

Polya (1962) felt the necessity to provide additional information on his modern heuristic approach to problem solving for potential and current high school mathematics teachers. Polya's (1962; 1965) two-volume set, *Mathematical Discovery*, focuses on understanding, learning and teaching problem solving to potential and current high school mathematics teachers. In developing this set of books, Polya drew heavily from his lectures that he presented to his students. To fully appreciate his efforts, the reader must become a participant, actually doing the problems that he has placed in the chapters. Polya believed that mathematics should be taught, modeled, and imitated. Yet, modeling a problem and imitating what has been done can still prove to be a stumbling block for students and teachers alike. In order to clarify this process, Polya presents "case histories" (Polya, 1962, p. vi) as the models of his problems. These case histories of the solution to the problem include the sequences of essential steps along with the reasons and perspectives that were used to develop these steps. Without analyzing the thought process of the mathematicians who discovered the steps, it becomes difficult to develop a plausible reasoning skill that will lead to demonstrative reasoning and, thus, the solution.

Morris Kline and American Education (1908-1992)

On the other hand, Kline (1973; 1977), a U.S. born professor of mathematics and educator who taught at New York University, had a totally different view of our mathematics educational system than most educators. Unlike the educators and reformers from the mid-twentieth century who firmly believed that the mathematics curriculum should demonstrate a complete understanding of proofs, theorems, and a conglomeration of other operations and processes, Kline (1977) felt that mathematics should be useful to students. He held to the philosophy that our methods of instruction made mathematics "dull, boring, intrinsically meaningless" (p. 165) for students of all ages. Furthermore, Kline believed that the subject matter taught in high schools was fixed because of the expectations university mathematics faculty held concerning high school preparation for college mathematics courses. Kline held the belief that university mathematics faculty were not interested in the usefulness of the knowledge presented to the high school students in algebra, geometry or even trigonometry. The university faculty was simply concerned that high school students should enter college with a mastery of these courses thus eliminating the need for remedial courses in mathematics. However, at the time of the printing of *Why the Professor Can't Teach* at " two-year colleges forty percent of the students taking mathematics were enrolled in remedial courses" (p. 169).

Furthermore, there exists a phenomenon within mathematics education that teachers teach mathematics in the same manner in which they were taught or as presented in a textbook. This instructional practice coupled with the teacher's inability to change the curriculum continues the pattern that results in students' dislike of mathematics. This cyclic effect of the teacher's learning mathematics and subsequently teaching mathematics has no progressive impact on students' mathematics comprehension. Unless new teacher candidates are taught correct and varying instructional methods, students will exhibit the same results when learning mathematics as past generations.

In 1973, Kline stated in his book, *Why Johnny Can't Add,* that before educators can establish a curriculum for elementary and secondary schools, they must consider the objectives of these levels of education. He claims that absolutely no consideration should be given to the preparation of future college students when designing an elementary curriculum as a very small percentage of students from this population will go to college. Though the number of students who matriculate at various institutes of higher learning increases as we examine high school graduates, they, too, are unaware of the nature and importance of mathematics in the everyday operation of the world. Unto those ends, Morris Kline advocates a drastic change in mathematics education from a plug and chug mentality in which processes, proofs and procedures are memorized and regurgitated with no substantial relationship established between mathematics and the world in which we live to a liberal arts education. In a liberal arts education, the high school students should "not only get to know what a subject is about but also what role it plays in our culture and our society" (Kline, p. 145).

Kline (1977) believes that mathematics is fundamental to the student's comprehension of the physical world. Mathematics has always progressed as mankind's desire for knowledge of the physical world has expanded. It has been the tool used by scientists to develop new, creative ideas to solve problems and to advance understanding of nature and its behavior. Mathematics is not an entity apart from development of culture but the tool used to bring about that development in the culture. For example, geometric designs in architecture must be balanced to withstand the stress placed on the structure.

It has been suggested by some educators that mathematics should be combined with the sciences. By doing so, mathematics, which now deals with abstract concepts, would deal with physical problems. Dealing with a hands-on situation or a real life problem provides for the student a tangible item that is easily associated with the mathematics processes. Genuine curiosity or motivation naturally occurs when the student is interested in finding a solution to the real life problem. Along with Kline (1977), The Core-Plus Mathematics Project supports this approach to mathematics instruction. In the preface to the text *Contemporary Mathematics in Context Course 1 Part A,* the mathematics educators state that "through investigations of real-life contexts, students develop a rich understanding of important mathematics that makes sense to them and which, in turn, enables them to make sense out of new situations and problems" (Coxford, Fey, Hirsch, Schoen, Burrill, & Hart,1998, p. x). Furthermore, Kline believed that mathematics achieves a blending when reasoning and concepts are used to obtain results about real world situations. He stated that "mathematicians of the past were inspired by real problems and found the meaning of mathematics through them is beyond dispute" (Kline, 1977, p. 173). For example, functions can be used to represent motion whether on land, in the air, or under the sea. Through real world problems, students realize that functions can also represent the law of gravitation, the mass of the earth, or even grains of sand on a beach.

As Kline (1977) focused on teaching mathematics through real life situations, he made it public that one of the foremost difficulties that any student has is mathematics. These difficulties were reading and interpreting the written word into mathematical expressions. This is called problem solving. If the students can do the algebra and /or the arithmetic, the equation can be solved. Therefore, the problem does not lie with computational mathematics but in reading and formulating the given information into usable mathematical equations. Using the traditional curriculum or even the new math approaches of the 1960s, students had few realistic word problems and little or no opportunity for the type of instruction necessary to practice formulating equations for real life situations. Thus, the connection could not be made and math was never connected to everyday life. With Kline's approach of including application problems in the lessons, the connection can be made. Then students can learn how to use mathematics in their daily lives.

His liberal arts approach also included new pedagogical techniques for high school teachers. In framing the application problem, the teacher could include historical background information and even some biographical information about the men or women involved in creating the formula that eventually rendered the solution. If cleverly done, the student should discover that mathematics is the result of human effort to solve real life problems. Mathematics is an active science that has developed since early man and has progressed to today's level while helping solve many of mankind's issues between nature and humanity (Kline, 1977).

Though Kline (1977) projected that some educators would protest and claim they did teach application problems, these word problems were often devoid of the personal connection to real men and women and their thinking that developed the solution. Furthermore, disregarding the small percentage of students who would actually need the traditional approach to mathematics as a preparation for college, Kline believed that the curriculum design of high school mathematics needed virtually a complete make over. In his own words Kline stated, "The new approach would present what is interesting, enlightening, and culturally significant, with the inclusion of only those concepts and techniques that will serve to further the liberal arts objective" (p. 178). Furthermore, he stated that "the material should be objective-orientated rather than subject-orientated" (p. 178).

In geometry, rigorous proofs would be gone. The teacher could use a cross-disciplinary approach by incorporating geography, studying the construction of transportation systems, developing functions that will describe the path of the planets in our solar system, or studying the refraction of light. Morris Kline (1977) adheres to the philosophy that this approach should continue through the third year of mathematics, which would be trigonometry. Equally important is that the application of real world problems in mathematics is a pedagogical approach. Problems that do not lend themselves to this physical approach should be eliminated from the course of study and perhaps be part of the fourth year of mathematics. In this senior year course, students can be taught the finer points that would be required of them in college mathematics courses.

Though this concept of mathematics education sounds wonderful, it was a different approach than colleges or university were accustomed to. At the time of the publication of his text *Why The Professor Can't Teach*, there existed a great dilemma and a travesty for the education of mathematics teachers. Many colleges and universities were producing teachers whose role models were research professors with little or no ability to actually teach. The mathematics world of the late 1970s had no place for scholars or teachers at the university level. Many educators like Klein (1977) announced a wake up call.

To these professionals, there was a great need for mathematics scholars who had a wide variety of knowledge of the practical use for mathematics in real life situations. There was also a belief that until the universities recognized the mathematics scholars and teachers as valuable assets to university faculties, our educational system would not improve. Kline (1977) stated that these scholars and teachers needed to acquire training in pedagogical techniques.

At the time that he stated his opinion of mathematics teacher education, Kline (1977) did not stand alone in his assessment of the situation. In the annual report of 1972-73 to the Board of Overseers of Harvard University, President Bok stated that Harvard University had not provided sufficient instruction to its students who were to disseminate knowledge as teachers and educators to America's children. Yet, these same Harvard graduates could obtain a doctorate without ever experiencing the classroom environment, engaging in peer committees at grade level, or being exposed to the inner workings of an academic department as the chairperson. President Bok could not defend this situation. He held to the belief that the preparation of the students' professional lives must be thorough since society was questioning college teaching and the traditional forms of education (Kline, 1977).

For their era, these professional educators were on target. Yet, organizations such as the National Council of Teachers of Mathematics also recognized the dilemma and attempted to provide new ideas and new techniques of teaching U.S.'s mathematics educators through professional publications, seminars and workshops. Along with these efforts from various professional organizations and institutions came a quagmire of problems and political agendas of its members. Kline (1977) insisted that it was time for mathematicians to recognize their inner dependence on research, scholarship and teaching, and to regroup and provide for the education of all students, whether undergraduate or graduate. It was and is important that these undergraduate students learn to teach since poor K-12 educational experiences would hinder the growth of mathematics in the future.

Steven G. Krantz and Teaching Mathematics

Similar remarks came forth from Steven Krantz (1999) in *How To Teach Mathematics (2nd ed.)* that the American educational system K-12 has a twisted, sinister approach to education that stifles the American student's inquisitive nature. Regardless of the grade level of the student in high school or college, the good educator must reawaken the student's natural curiosity and rejuvenate a desire to learn. Over the past 30 years, some of Morris Kline's opinions, along with others, concerning the university faculties' need for effective teachers of mathematics have provoked researchers into taking courses in pedagogical techniques and methods of instruction. No longer is it acceptable to stroll into a class, present the material, and exit stage left. The researcher/educator must employ sound pedagogical practices that cause his/her students to become interested in mathematics. The educator must also arouse the students' natural curiosity and desire to learn, thus making the class thought provoking.

Steven Krantz, a current mathematician, research professor, and mathematics educator at Washington University in St. Louis, Missouri, states that he is a traditionalist when it comes to teaching mathematics, but he sees great value in the reform movement. Though there are no demarcation lines between the two camps in this teaching reform movement, Krantz points out that the reform movement stresses cooperative and group learning along with a discovery approach while utilizing various types of technology in the instruction process. He maintains that the traditionalists want to continue with the lectures and practice exercises, also known as drill work, while insisting that students take the responsibility of their own learning.

Krantz divides a mathematics teacher's performance in a classroom into four major categories: Guiding Principles, Practical Matters, Spiritual Matters, and Difficult Matters. Under these topics, he describes the teacher's appearance and behavior in the classroom, presents his opinion on the structure and dissemination of the lessons, and discusses some of the non-mathematics issues that arise in the classroom. His fourth section deals with the issues that are the most difficult to administer to students. In the first section, Krantz speaks of some guiding points on which the educator needs to focus. The first guiding principle is respect for one's self. Belief in one's ability to be qualified as an instructor, the desire to instruct, and the willingness to come prepared are essential to a successful rapport between teacher and student. Though I found it quite odd, he felt the need to explain that personal hygiene and proper attire convey respect also and should be adhered to at all times. The remainder of his guiding principles stretches across a span from this approach to personal hygiene to the use of computer technology in the classroom.

In his section on practical matters, Krantz (1999) deals with the teacher, as an individual, who is in a position to communicate his/her expertise with other people. He addresses the educator's attitude and professional presentation, communication skills, classroom management issues, along with various aspects of instruction including the tools that may be utilized in accomplishing this task. As we all know, no one can turn us into "super teacher." Yet, Krantz attempts to discuss topics, provide advice, and offer the benefits of his own teaching experiences to assist the teacher in formulating a philosophy and developing a teaching methodology that is effective for him/her in the classroom.

Krantz' opening paragraph in chapter 1 states that "good teaching is a product of preparation, effort, and good attitude on the part of the instructor" (p. 1). Without mastery of these skills of communication, neither student nor teacher will enjoy the educational experience.

While reading Krantz (1999), I focused on specific items in which I was interested. His comments on voice, eye contact, and body language reinforced my own attitude and validated classroom practices that I have developed over the past 30 years. I have found that eye contact is critical in developing a personal involvement with a student in his/her learning. Krantz states that mutual awareness of each other through eye contact demonstrates genuine interest in the activity being done. It sets up an atmosphere for learning. Here, the instructor can draw the students into one's lecture or discussion by making them feel like an active part of the presentation. Using one's body as a tool, the teacher can work the room by moving closer to students who are asking questions, by standing next to them while viewing a problem in discussion on the board, and then by moving back toward the board to point out a particular aspect. Coupled with a voice that contains inflection, interest in the lecture or discussion can be maintained.

Discussion in mathematics is difficult without viewing the interpretation on a screen or chalkboard. Therefore, while the teacher is lecturing he/she should be writing notes on the chalkboard. Using this tool is not as easy as it sounds. Krantz suggests that the chalkboard presentation should be organized. The teacher should write from top to bottom and from left to right. Material should be blocked by dividing the board into sections. For example, the normal high school green board has three individual boards. This is a natural block of space. However, the white erasable boards do not have this natural form. The instructor can naturally regulate the space or can physically draw a line down the center of the board. Krantz stresses emphatically not to erase the board. The students cannot absorb all the material presented as fast as the educator can write it. Therefore, he recommends that the instructor continue writing on board until the entire board is filled. At this point, the instructor should take a break and view what he/she has written. This allows time for the students to take notes on the material, review the material, and formulate questions about the material. Once everything is done concerning the material on the board, then the board may be erased completely. To further help the students absorb the material, Krantz (1999) suggests that the instructor read the notes aloud as they are being written on the board. Krantz states that the educator should "make mathematics happen before their eyes" (p. 41).

There is still another approach that can add information to the student's cache of knowledge. It is the application of the mathematical concepts. This makes the process appear to have usefulness to everyday life and makes mathematics appear as a tool to produce knowledge. The inclusion of application problems prevents the belligerent student, who has no patience with abstract mathematical concepts, from interrupting the class. By presenting these real world problems before someone can say, "When are we ever going to use this stuff?" or similar questions, the instructor defuses a potential adverse situation in class that may make the instructor appear unknowledgeable about his/her area of expertise. These examples need not be detailed. In fact, Krantz states that a brief application problem is best. Otherwise, the instructor will lose the students in the layers of mathematical processes, thus making the example useless.

Besides personal characteristics that affect pedagogical techniques in the classroom, Krantz also views questioning of students and by students a crucial part of instruction. The method that the teacher uses must suit the teacher's personality, teaching style, and work. For those who are wondering what he is talking about, Krantz focuses on his own techniques. He begins by listing topics on the board. He constructs direct questions to a method or part of a procedure that was studied under the particular topic. He hopes that this will cause the student to remember what was taught. He also gives a small quiz once or twice a week over the current material. In his section on exams, besides discussing the typical items of the formulation of question, length of the exam, and the rubrics, Krantz also brings forth information on how to handle questions from the student concerning grading procedures of examination problems. In the section "Difficult Matters", he gives guidelines on dealing with students who fail the exam.

Krantz (1999) believes that the handwritten exam is very good. He explains that the handwritten exam requires the student to hand write each step of the solution. Normally, these questions require a multi-step response. Therefore, he does not particularly care for the multiple choice examination. No partial credit can be given for the student's knowledge in such examinations. Though these exams are easier to grade, they are unfair to the student who might make a minor error in the last step. However, if an instructor can produce a multiple choice exam that can test each step, then Krantz would consider that as an alternative to part of the exam. For those elementary college courses – calculus and below – where computational and procedural processes are critical to advanced courses, the examination should be comprised of straightforward problems that are similar to or reflect the work done in class and on homework assignments. He states that 95% of his exams for those elementary courses are straightforward and 5% of the problems are to discern the bright student in the class.

Krantz (1999) remarked that there are some educators who actually give homework problems on the tests. There are other educators who will announce from the beginning of the course that the examinations will be taken from the homework problems. Thirdly, there are teachers who will give the examination problems at the review session. Krantz does not believe in using these pedagogical techniques because the examination would be seen as a means for testing the student's ability to regurgitate material. It may cause students to memorize the problems and would not be testing for understanding of the material. It may even cause cheating. Though he knows that these styles of examinations give the student little room to complain about the test, he eliminates the adverse situations by not giving the students the exact problems at any time prior to the test. Furthermore, Krantz dislikes the use of examinations to have students reprove theorems and corollaries or to use this opportunity to cover obscure material not already presented in class. For Krantz, the examination is a tool of communication. Since the purpose of the course is to transmit knowledge and information to the student, the exam becomes a tool to test the professor's ability to successfully transmit course information and view areas that he/she may need to stress in the future.

For the average student, the most important part of the course is the grade. Therefore, a consistent and fair form of grading the course work needs to be conveyed to the student. Krantz (1999) realizes that it is imperative for both the student and the teacher to have this consistent and fair grading system that is defendable to parents, students, and administration. He considers it important to establish this grading system from the outset of the course. Therefore, Krantz strongly recommends that the grading system be part of the syllabus of the course. This syllabus must be given to the student on the first day of class. Whatever value or percentage of the total grade the instructor assigns for homework, quizzes, midterms or unit tests, and the final examination, Krantz states that the teacher should be consistent. One cannot lie or change a grading system during the course. This is unfair and students want to be treated fairly. Treating students fairly does not mean that one needs to be more lenient or give away a grade. Like any teacher, students want to be treated with respect. Therefore, if a question arises about the credit given on a problem, be willing to speak to the student who is complaining. Be prepared to explain the rubrics. Krantz eliminates many student questions by posting the solutions to the examination. If after reviewing the solutions, the student still has a question, one must listen to him/her. If the teacher has made an error in grading and the student merits additional points, the points must be given to the student. If the student does warrant additional credit, it would be advisable to encourage the student to work harder. Since the final course grade is to represent the evaluation of the total work performance of the student or lack thereof, it does seem logical to address a failure after the fact. However, Krantz states that there are a few universities who have a grievance process in place for students who believe that they have received an unfair grade. This would not be as bad as it sounds unless the university never consults with the instructor of the course as to why the grade was given.

In his chapter entitled "Spiritual Matters", the intriguing topic of the *need* for mathematics teachers was a focus. Krantz (1999) freely admits that in the first edition he could not come up with a strong, coherent response. Yet, in the second edition he states some convincing remarks that definitely justify a need for teachers. Mathematics teachers are the primary source of explanation for the procedural steps in the solution of the problem for the students. The teacher sets the pace and helps the students master the processes by making the students adhere to each step with a sufficient amount of practice to imprint the process in the students' memories.

Mathematics teachers are essential in teaching the students to read mathematics symbolism and notation. No one is born with this ability to read these notations and symbols which have been created by mathematicians over the centuries. These notations and symbols are a language unto itself and need to be committed to memory, as any other language, for easy understanding of math equations and inequalities. Furthermore, teachers are needed to formulate a critical thinking process in the mind of the students. The students must be able to evaluate an argument, decide what is true, and then formulate their thoughts about the situation.

The final reason that Krantz (1999) gives for having mathematics teachers is by far the most challenging. Engaging the students in the learning process is a daily challenge for all teachers. It is difficult for the students to focus their total concentration on the subject during class time due to the competition between the teacher voice and the daily noise that society produces. Without total concentration on the subject, it becomes even more difficult to exchange ideas about the process being discussed. Why is it difficult for the students to learn or the teacher to teach? One reason is the daily competition with society as its noise creeps into the classroom. Yet another reason is the passive behavior of the students. Society does not force students to interact with each other on a daily basis. The never ending entertainment by the television or the music on the radio does not require critical thinking skills of communication. Consequently, students are reluctant to engage in discussion about the lesson. Hence, one could summarize that the students who cannot read mathematics or converse about mathematics will not be able to communicate in writing an understandable response to a mathematics problem. This alone justifies the existence of the classroom teacher for the development of communication skills in mathematics.

Much to the dismay of mathematics teachers, there also exists a phenomenon called "math anxiety" that Krantz (1999) briefly discusses (pp. 100-101). This learning disability is defined as the inability of an intelligent person to comprehend mathematics. It produces physical symptoms from a nervous and nauseous stomach to heart palpitations and even paralysis of thought. This fear is real to persons with this disability and should not be disregarded. Because of the psychological effect on the persons who demonstrate math anxiety, the mathematics teacher should recommend that the student get professional assistance in coping with this disability. Whether the instructor believes in existence of this disability or not, he/she should be kind and considerate to the student.

Richard R. Skemp and Psychological Aspects of Learning Mathematics

About the same time that Morris Kline was voicing his ideas on mathematics education in America, Richard R. Skemp (1987) was collecting data for his summary of the psychology of learning mathematics. In Britain in the early 1960s, there was a concentrated effort to improve mathematical education in the school system. Much to the chagrin of the educators and after nearly two decades of hard work to improve mathematical education in schools, a London University research group concluded that students' understanding of mathematics had not increased. This finding was not isolated to British students. It was a global problem. Skemp's curiosity caused him to search for a solution to the problem taking him to such areas as developmental psychology, motivation, human emotions, and human intelligence. There came a point in his search when he realized that behaviorism was not useful in determining why all students could not learn mathematics. Learning mathematics was beyond the scope of comparison with lower level animals. The process of learning required for mathematics was a function of human intelligence. The old IQ based measurement could measure intelligence but provided no information on the development of human intelligence or how human intelligence functions. Skemp held that a new model for human intelligence needed to be defined. This new model considers mathematics as a mental tool, and when properly utilized expands our mental capacity to think at higher levels.

Richard Skemp's (1987) book, *The Psychology of Learning Mathematics,* is not a book about the teaching of mathematics but one of learning mathematics with understanding. I focused on his concepts of the formation of mathematics in the human mind, the development of appropriate schemas, the importance of symbols in the communication of concepts, and the effect of anxiety on student learning. As a high school mathematics teacher, I felt it necessary to understand how the human intelligence assimilated data, made associations, and processed information so as to produce a correlated response.

Skemp claims that human intelligence classifies experiences by similarities. Each day our mental capacities form concepts that allow humans to distinguish new experiences by their similarities and place them in a classification in our intelligence. Closely linked to concepts and concept-formations is the naming of these concepts. This naming permits humans to communicate these concepts to one another and alter these concepts in the course of communication or experiences. Since mathematics is an accumulation of generations of thought processes and generalizations of these processes, the students of mathematics in today's classrooms cannot experience mathematics directly. Their experiences are gleaned from other mathematicians and/or educators and, therefore, are not first hand experiences.

When teaching mathematics, many teachers traditionally begin a new idea with an introduction that defines this idea. They are not alone in this approach. The textbooks usually define the ideas before they present an example. This approach is useless to students without a foundation or already formed concept upon which to build these new ideas. The instructor may understand this approach because the higher-order concepts have already been formed in the teacher's mind. However, the students may or may not have sufficient concepts to process the new ideas. Therefore, the instructor must be able to assist the students in understanding the definition by first presenting a review of previous learned concepts to assure that a foundation can be accessed for retrieval as the new ideas are presented. Next, a presentation of a carefully prepared collection of examples which illustrate the property or properties will guide the students' development of new concepts. This presentation requires a precise understanding of the concepts by the teacher and the skill to communicate these concepts through examples that contain no other properties to clutter the students' view of the process. Hence, a teacher needs to be skilled in the understanding of the development of the mental processes involved in learning mathematics. An educator who is successful at transmitting the new ideas and establishing connections with formed concepts will give students positive experiences and a favorable opinion of mathematics. However, teachers who are unsuccessful at this endeavor will yield students with a dislike or fear of mathematics. At the time Skemp (1987) wrote this book, he believed that there were an extremely large number of students who feared mathematics because of poor teacher instruction.

In order to further understand the learning of mathematics, Skemp (1987) introduces his idea of a schema. This schema is defined as a conceptual structure that weaves together existing knowledge and acts as an instrument for future learning. This schema is also the framework for future understanding. The schemas which are constructed during the early phases of learning a subject can cause an individual to have an easy time advancing in knowledge of that subject or a difficult time assimilating new concepts. There are a few disadvantages to schematic learning. For instance, if a procedure is considered alone without any references to prior knowledge, then schematic learning will possibly take more time to accomplish. In this case, memorization of a rule may be faster at reaching some level of understanding. Yet, this memorization may become burdensome and eventually overwhelming. Therefore, developing a schema, though it may take longer in the beginning, can reduce cognitive strain.

The second disadvantage to schema learning is that it can prove to be an obstruction to learning if the new experience does not fit into any known schema. It will either be not remembered or will be temporally remembered. Lastly, a schema is considered to be an adaptable instrument of knowledge that is utilized in solving new problems and upgrading information. The problem arises when the schema fails to adapt. Assimilation of new experiences cannot be added to the known schema. A reconstruction of the schema must occur or the system breaks down. The schema can be so valued by an individual that the resistance to the change of this schema can be viewed as a threat. Through the centuries as new concepts, such as the Hindu-Arabic system of numerals in Europe, were presented. The resistance to the change was so great that to incorporate the change in the system was illegal (Skemp, 1987).

Since the understanding of a concept is based on an individual's experiences and instruction received, it is subjective by nature. If subjective learning is truly open, reconstruction of schemas can occur. Otherwise, the importance of the schema as a learning tool is rendered inappropriate and/or inefficient in assimilating more complex concepts. Students will resort to the memorization rule if the lack of assimilation persists. This, in turn, will lead to distress for the student. Eventually, it will cause a feeling of frustration and anxiety. Therefore, it is the teacher's responsibility to make sure schematic learning is taking place – that is, understanding when reconstruction of the schema is necessary and when simple assimilation is sufficient (Skemp, 1987).

All the time that schematic learning is taking place, symbols are playing a critical role in the communication of ideas. Symbols are types of sounds, such as words, and pictorial representation or visual signs that two people hold in common that signify the same mental concept. When these symbols are spoken or viewed, they trigger a projected response within the memory of an individual. It is crucial that the understanding of a common vocabulary word contains the identical meaning to both persons who are attempting to communicate a mental concept to each other. Even people who supposedly speak the same language may have a different mental concept for the same word. In other words, the spoken word is spelled and pronounced the same but carries with it a different denotative and connotative meaning. This makes it imperative that the teacher is able to communicate with the students. The association of symbols with mental concepts makes it possible for students to understand mathematics. An incorrect association can impede learning for students. With this in mind, it becomes necessary for an instructor to be as accurate as possible when communicating to students. Since it is impossible for the instructor to know if the individual student has made the proper connection between the symbol, whether verbal or visual, and the desired mental concept, the instructor must establish some ground rules for conveying the meaning for a symbol. Skemp (1987) has provided three rules for the mathematics teacher to follow. The first rule states that the schema that is being employed is recognized by the student. The second rule makes it mandatory that there be only one idea associated with each symbol in a schema though use of the same symbol in a different context with different meaning is permitted. Lastly, the instructor must not change the schema without informing the student.

If a mathematics teacher can successfully apply these rules in the instruction of students, there still may exist some level of anxiety within the students. This anxiety may have more than one cause but may be utilized as a source of motivation. Though it may still disable some students, it can cause others to employ problem solving techniques and successfully complete the assignment/examination. Yet, for those who cannot accomplish this task, there may exist personal problems that impede the progress and are beyond the scope of mathematics (Skemp, 1987).

In order to deal with anxiety as an impediment to successful instruction, the causes of the anxiety must be made known to the teacher. As Skemp (1987) revealed earlier, poor teacher instruction could cause anxiety or fear of mathematics in students. He also declares that the authoritarian teacher can create an environment in which anxiety may develop if the teacher is critical or ridicules the student for not knowing the material. Although this may occur, the authoritarian teacher may successfully inculcate learning through acceptance of his/her authority by the students. This type of learning, often referred to as rote-learning, can eventually bring about anxiety when higher levels of learning produce a burden on the memory. This cognitive strain could be elevated by another method of instruction – schematic learning.

What about those students whose problem solving techniques, when employed, lead the students to a positive end? Are they interested in doing and understanding mathematics? Skemp (1987) sets forth a question of this nature. In the quest to understand if sufficient methods and techniques are presently valuable to students to successfully learn mathematics, it has been assumed that all persons wish to learn mathematics. This assumption is not true. Not all students are motivated to learn a subject unless they can see that the understanding of the subject matter satisfies some need whether intrinsic or extrinsic. Thus, to assist in achieving the goal of the comprehension of mathematics by all students, instructors must address this issue as well.

For those students who are intrinsically motivated, mathematics is enjoyable and its understanding brings about a satisfaction of success and accomplishment of an intellectual achievement. For those students who are extrinsically motivated, motivation by anxiety and the reward-punishment motivation are the most prevalent types exhibited by students. The desire to please the teacher, a reward-punishment motivation, can be utilized to bring about schematic learning or rote-learning. Motivation by anxiety, normally employed in rote-learning, deals with human emotions which are indeed part of the make up of the classroom environment. Skemp (1987) emphasizes that mainstream psychology separates the cognitive and affective processes in the human experience, thus producing an artificial glimpse of the normal thinking process. Emotions do influence our normal thinking process and, therefore, must be taken into account in the instruction of students.

There are various emotions that affect the learning process of which anxiety is one. Anxiety is an emotion that deals with competence and signals that there is a possible danger in our ability to comprehend the mathematics being taught. In order to keep these emotions in check, the student needs to begin the learning process from a domain or position of security and confidence where competence can be demonstrated. From here the student ventures into the frontier zone, an area of uncertainty just beyond the domain where learning takes place. It is in this area of partial competence that the mixed emotions surface and can cause damage. Therefore, Skemp (1987) believes that it is here that the teacher has the most important function to perform – the function that allows the student to analyze the factors involved and integrate the material into a known schema without permitting the negative emotions to take control.

Skemp (1987) has presented a different approach of understanding how mathematics should be learned and how to avoid the common pitfalls. With this in mind how do present day mathematicians incorporate these ideas into modern teaching methods?

Matt DeLong and Dale Winter and Professional Development

Dr. Matt DeLong of Taylor University and Dr. Dale Winter of the University of Michigan have written a professional development program to assist new teachers, part-time teachers, and teachers who are in a new environment in preparing for instructing the college students who take mathematics courses, especially the introductory level mathematics courses. Since at least four of these courses are taught at the high school level, I thought that it might be interesting to investigate the text that is used to facilitate this professional development program, *Learning to Teach and Teaching to Learn Mathematics* (DeLong & Winter, 2002).

These two individuals incorporated the last 20 years of innovative instructional methods and technological advances into college level mathematics courses, in particular the introductory courses of college algebra, precalculus, and calculus. This program was designed to be used by the faculty of the mathematics departments, who are good teachers, even though they have not had extensive educational work in instructor development. Through this professional development program, the teachers with limited or no experience can draw on the advice and support of the veteran teachers and find suitable training experiences that can assist them in ascertaining the objectives of the course they are teaching (DeLong & Winter, 2002).

This professional development program is one semester in length and provides multiple forums for discussions between new and experienced faculty concerning examinations, instructional practices, and everyday classroom management. Weekly training meetings range in topics from the typical lesson plans and classroom management issues to the more difficult problems of instructor-student situations and student motivation. For the weekly training meetings, the facilitator has been given the description and purpose of the meeting, the goals for the meeting, the instructions in how to prepare for the meeting, the agenda and outline for running the meeting down to the minute intervals for each activity. For each topic of the weekly training meetings, the facilitator is also given a listing of suggested readings which include a brief synopsis of these articles or books. For the Mathematics Association of America's Innovative Teaching Exchange website, DeLong and Winter (2002) indicate which articles pertain to the chapter and also provide a synopsis for these articles.

My curiosity led me first to the topic of student reading of the text. DeLong and Winter (2002) believe that students cannot read a mathematics textbook. Therefore, they cannot complete a homework assignment to read the material before the lecture. All efforts by the students normally prove to be inadequate. For these introductory courses, Delong and Winter state the textbook is a vital part of the course since it provides the basic vocabulary and techniques used in comprehending the lesson. Proper preparation by the students allows the instructor to explore the more difficult material in greater detail. Unto those ends, DeLong and Winter designed a weekly training session with activities that would assist the instructor in developing the skill of reading mathematics.

While all of the chapters are dedicated to weekly training meetings, any one chapter outlines the procedure for that topic in detailed step by step process that the facilitator must do to prepare for the weekly training meeting. Activities, sample hand-out sheets, and suggested readings are provided. For a more experienced educator, this section provides examples and a multitude of internet sites that are excellent sources of pedagogical methods of teaching the skill of reading a mathematics textbook along with ideas for evaluating the students' progress throughout the course.

DeLong and Winter stress that the instructor should first decide what he/she wants the students to glean from reading the text. The instructor should then create a broad list of reasons to present to the students to validate learning how to read mathematics. The instructor needs to relate the course goals and the manner in which the textbook supports the goals. To help the instructor accomplish this task, the authors have given some sample goals and reasons that the textbook supports these goals. DeLong and Winter (2002) also suggest that the instructor should provide the students with a discussion about the difference between reading mathematics and reading the newspaper. There is a list of behaviors provided that can be discussed as methods of reading mathematics such as making marginal notes or making outlines, making a list of questions or solving examples before reading the solutions.

A second topic that captured my attention was "Strategies for Motivating Students," (DeLong & Winter, p. 159). The handout, "Ideas for Motivating Students," (p. 163) provides an explanation of intrinsic motivation and extrinsic motivation. Intrinsic motivation can be defined as motivation that is gleaned through perceiving that the learning activity is valuable in itself. The student who remarks that math interests him/her or that math helps him/her to think clearly is intrinsically motivated. The focus is on math and on educating the students. This proves satisfying to the educator and student alike. Yet, this type of motivation is challenging for the educator. It requires that the instructor knows the student's interests and may require extensive preparations or even special topics.

On the other hand, extrinsic motivation can be caused by the desire to avoid negative consequences, such as losing a scholarship or failure to meet requirements for entrance into another program or college within the university. Unlike intrinsic motivation that requires extensive effort on the part of the teacher, extrinsic motivation requires little preparation by the instructor and does not require that the instructor have any personal knowledge of the students' interests to prepare appropriate lessons (DeLong & Winter, 2002).

DeLong and Winter (2002) continue by relating items of the courses such as grades and subject matter to intrinsic or extrinsic motivation. Grades are extrinsic. As such, an instructor who wishes to motivate his/her students through this avenue needs to view tests and assignments as sources of information that can be used to understand the students' progress. Consideration needs to be given to take-home exams or shorter exams where time is not a factor. This will alleviate math anxiety along with presenting a clearer picture of the student's knowledge.

Subject matter is an intrinsic item. To properly use the subject matter to motivate students, the instructor needs to dedicate a good deal of effort at making the students aware of interesting concepts of mathematics. This can be done by selectively using cue words or phrases that will spark the interest of the students, modeling enthusiastic behavior, and varying the teaching style within the course. Lastly, the educator must plan to arouse the student's natural curiosity by allowing him/her to discover the mathematics or develop a conjecture when applicable. The instructor can create an atmosphere of unexpected surprise (DeLong & Winter, 2002).

Student motivation can be affected by math anxiety. DeLong and Winter (2002) provide six conditions in which math anxiety may exist. For these conditions, some solutions that can help alleviate or significantly reduce math anxiety have been suggested as aids to the instructor in coping with this psychological aspect of the fear of mathematics. The student's ability to reach this level of mathematics indicates that he/she has the mental capacity to learn mathematics. Therefore, his/her math anxiety, which is brought about by the student's perception of his/her lack of ability to understand mathematics, is a misconception of the student's true ability. As a means of reducing the fear of forgetting, the instructor can allow students to utilize a "cheat sheet" on tests or to use calculators on an exam. This can be justified since the instructor is testing for understanding and not how well the students memorize.

DeLong and Winter (2002) believe that a teaching portfolio can assist educators in growth and provide documentation of experience. These portfolios can be utilized as an instrument that can unify a professional development program within a department or provide valuable information to current employers for assessment of the educator's abilities. According to the authors, a portfolio is a collection of actual activities that the teacher implements in the classroom. These can range from instructional materials such as the common handouts, lesson plans, or examinations to written reflections on his/her own teaching. It may also include peer evaluations or comments along with reviews from supervisors, videos of classroom teaching, and student evaluations. Furthermore, the educator can include course goals and objectives that can lend insight into his/her pedagogical methods and skills. Often times, materials received from workshops or seminars that have been attended or participated in by the teacher are included as long as the substance has contributed to the professional growth of the instructor or has provided additional pedagogical techniques applicable to his/her academic courses. This portfolio may also assist the teacher in maintaining a record of any innovative practices and results that have been put into action in the classroom. The educator's portfolio may be kept personal or can be made public through community use among educators or as part of a departmental review of general instructional practices.

DeLong and Winter (2002) have complied a plethora of information for the educator of any college course that can be easily adapted to and implemented in upper level high school mathematics courses as these courses are the same as the introductory level courses at the college level, often using college texts as the text of choice. Though I have not discussed all the topics covered in the book, I believe that these assistant professors of mathematics have created a phenomenal professional development program text that can be used as a seminar guide or as a reference for educators truly interested in improving instruction in the classroom.

James W. Stigler and James Hiebert

As the mathematics educators were attempting to provide solutions to the crisis situation of mathematically deficient students graduating from U.S. schools, James W. Stigler, Professor of Psychology at UCLA, approached the American dilemma from a different venue. Under the auspice of the International Association for the Evaluation of Educational Achievement, the Third International Mathematics and Science Study (TIMSS) was conducted under the direction of James W. Stigler. The TIMSS study was an international study that examined student achievement in 40 different countries at five different grade levels. Also involved in the TIMSS study was James Hiebert, Professor of

Education at the University of Delaware. Dr. Hiebert was a consultant to the TIMSS study and assisted Dr. Stigler in interpreting the results of the study and generating recommendations based on the study. Together, they wrote a review that contrasts and compares the teaching practices in Japan and Germany with those in the United States. This review also provides a new perspective on teaching mathematics in America.

Their book, entitled *The Teaching Gap,* is a forum for Stigler and Hiebert (1999) to expound on their findings that teaching is a cultural event. They state that American teachers are educated and are proficient in their knowledge of mathematics. However, American teachers are severely limited in the methods they utilize for classroom instruction. Furthermore, Stigler and Hiebert present the concept that the reform movement in education should refocus its efforts at changing teaching methods and creating a teaching system that will assist educators in continually improving instruction within the classroom.

As educators have discovered over the past few decades, Americans have voiced their concerns for quality educators and have questioned the competence of teachers. In *The Teaching Gap*, Stigler and Hiebert (1999) explain that good teachers using the limited methods available will never be able to elevate student achievement. Mathematics teaching in American schools is centered on isolated procedural skills through repetitive practice which does not promote conceptual understanding.

Through an examination of the video tapes of mathematics classes in Japan, Germany and the United States, Stigler and Hiebert (1999) paint for educators a portrait of education as it looks in each country by charting classroom activities by time and type and by presenting a detailed account of an actual lesson from each country. The TIMSS team of six code developers and many educators analyzed the video tapes of the respective classes in order to create common terminology useful in explaining and interpreting the lessons. In addition to the video tape, the TIMSS team was allowed access to teacher response questionnaires and supplemental materials to help them glean an overall perspective of the education taking place.

Some interesting styles of teaching were presented. For example, German teachers were in charge of the mathematics which was categorized as advanced procedural learning where the execution of the procedures is precise and accurate. This is done by the presentation of a new problem that will generate the day's discussion and work. Students construct a geometric diagram from the oral commands of the teacher. At the direction of the teacher, the students examine the diagram for some commonality or some known fact. Once this is discovered, the teacher will relate information on who first discovered this concept. Here lies the advanced nature of the lesson. The teacher will lead the class through the proof of the law or theorem he/she has presented. This is done by asking students to respond to questions as the teacher develops the proof. Students are never left alone to do it nor are they just given the steps and the reasons. At the end of the lesson, a summary page containing the law or theorem and its history is distributed to the class for designated oral reading. As is familiar in the United States, teachers in Germany assign homework, which will be reviewed during the first ten minutes of the subsequent lesson (Stigler & Hiebert, 1999).

A common lesson in Japan begins with the traditional greeting of bowing. This is followed by a review of the prior lesson for about five minutes. If the prior lesson has an unfinished problem, the students will be given a few minutes to come up with workable solutions. A few of these solutions are briefly discussed and then summarized by the teacher. About 10 minutes into the class, the teacher assigns a task for the day. Students work on this task independently. This is to allow the students to generate their own method of solution. While the students are attempting to complete the task, the teacher is circulating through the room answering questions and making suggestions to assist the students in being successful at accomplishing the task. If success has not been achieved, the students are instructed to get into their groups and choose a problem that all group members agree is challenging and is solvable. Once chosen, the problem from each group is given to the teacher to put on the board. All students will copy the problems and commence solving them. Some work as a group or in pairs, while others work individually in the group setting. The students continue to work in this fashion while the teacher circulates through the room answering the occasional question. Normally after the students have been working for awhile, the teacher will choose students to share their solutions with the class. Students are chosen based on the method they used to obtain the solution. Consequently, the students see a variety of methods used in solving the problem. During the last few minutes of the class, the teacher briefly summarizes the day's lesson on the board. Though this is a brief summary, it is considered a critical process in organizing students' thoughts and understanding. In contrast to American instruction, the chalkboard contains all the information that has been written during the course of the lesson. Japanese teachers believe that the students need to see the entire lesson from beginning to end during the brief summary. They hold that this process assists the students in organizing the lesson in their minds. At this point, all seats are returned to their proper place. Students stand and bow to the teacher (Stigler & Hiebert, 1999).

The preliminary description of the teaching style in the United States is not totally devoid of content as it might first appear. Instruction requires less mathematical reasoning since it is at a lower level. In American classroom situations that were videotaped, the teacher began with warm-up exercises or an activity. After five minutes or so, the homework is checked by oral recitation. At this point, the American teacher usually begins with the definition followed a demonstration of some procedure for solving a problem. The lesson highlighted by Stigler and Hiebert (1999) portrays the educator as passing out a worksheet that illustrates a sample problem at the top of the page. One of the problems on that page was too difficult for the students to work as the required information had not been communicated to the students. The teacher began with a leading question. However, the nature of the question coupled with the tone and inflection in the teacher's voice gave away the answer. The teacher then gave the formula to the students. No thought process was necessary since the teacher did not require the students to generate a formula for the problem. Furthermore, the lesson was interrupted by the teacher making announcements of activities, quizzes and tests prior to reviewing the day's lesson. Though this teacher did not assign homework, it is usually assigned.

As one focuses on the classroom activity, it is easy to view that Germany and the United States follow a similar format with teacher controlled instruction. They deviate during the actual instruction. While the German teacher continues to develop the proof of the law through a question and answer process, the American teacher furnishes the formula, allowing the students to practice their computational skills. By not allowing the students to generate the formula by themselves or with minimal assistance once the key question has been answered, the American students were deprived of actually "doing" mathematics. No analytical thinking took place during the lesson. If the environment had been student controlled, the formula or a variation of the formula would eventually be generated. This formula development method provides for more conceptual learning and elevates the thought process to a higher level. Unfortunately, only nine percent of the American lessons were student-controlled as compared to Germany's 19 percent. Japan had 40 percent of the lessons as student-controlled. These student-controlled lessons are defined as the seatwork time allotted to each class period for students to do work individually or as a group. Stigler and Hiebert (1999) concluded that U.S. students have a marked disadvantage because of the low level of challenging mathematics to which the students are exposed.

Stigler and Hiebert hold the belief that teaching is a cultural activity directed by cultural scripts which are defined as implicitly learned knowledge obtained through observation and/or participation that is stored in one's intellect. These scripts guide one's behavior and forewarn one of an expected response. They state that "teaching, like other cultural activities, is learned through informal participation over a long period of time" (Stigler & Hiebert, 1999, p. 86). Though some may believe that teachers learn to teach in college teacher-preparation programs or that teachers have an innate skill that makes them successful at instructing people, Stigler and Hiebert believe that teaching is a learned process obtain through participating in the education system while growing up.

In the United States, mathematics lessons appear to be a set of procedures for solving problems coupled with an effort at teaching the respective vocabulary so that the concepts and procedures can be understood using the formal terminology. For many American students, mathematics is not interesting. In fact, it is considered boring. Therefore, teachers tend to create an atmosphere at the beginning of the lesson from non-mathematical conversation or by using real-life examples as interest grabbers. However, Japanese teachers believe that mathematics is interesting in itself. They extend this thought to their classrooms by preparing lessons that can be solved several different ways. The students proceed to discover the procedures, thus creating mathematics for themselves. This process is not without the struggling students and does produce mistakes. Yet, Japanese teachers consider this struggling and the mistakes as essential to learning. The contrast between the American teacher and the Japanese teacher lies in the culture which controls education. In brief, the Japanese culture holds classroom instruction in high esteem. Therefore, a classroom lesson takes priority over any other activity. Classroom instruction is never interrupted by the public address system as often occurs in American education. Each lesson is prepared by a group of teachers who select a problem which focuses on the relationship between facts, procedures and concepts and has multiple methods to obtain the solution (Stigler & Hiebert, 1999).

It is here that American education and Japanese education spilt. Japan has a national curriculum. Over the past 50 years, Japanese teachers have painstakingly created a method of developing each lesson that can be used by all teachers in presenting a lesson. These lessons are not created by researchers but by classroom teachers. Classroom teachers in the same building who teach the same subject at the same grade level work in a collaborative effort to produce effective lessons. They are provided with the time to work on the design of the lesson. The lesson study, as it is called, is implemented, tested and improved upon before being shared with other educators to seek their input. This is a continual process for the educator and is considered to be a school-based professional development program. Japanese teachers engage in this process from the time they begin to teach and until they leave the profession. This type of professional development provides a system for mentoring new teachers and provides a built in laboratory for implementing and assessing newly created lessons. These collaborative lessons are based on the philosophy of gradual, but continuous, improvement extended over a long period of time. With Japan's centralized educational system, the division of the content into lessons by grades is easy to accomplish. The lesson study groups can concentrate on the direct improvement of teaching in context (Stigler & Hiebert, 1999).

The United States did have a noted educator in John Dewey. Each educator knows of his laboratory school at the University of Chicago. Here it was difficult to separate the teachers from the researchers since all were involved with learning about teaching in a real classroom environment (Tanner, 1977). After Dewey left Chicago, Charles Judd replaced him. Judd's new method of improvement was to separate the researchers from the teachers with the researchers being moved to university facilities. To this day a large gap exists between researchers and teachers in the education profession. Researchers were assigned to find better ways of teaching in the classroom. The dissemination of this knowledge was done in workshops and in summer classes at the universities. This is still the norm today. Rarely is there any opportunity to implement these skills in a real class to view their effect on education (Stigler & Hiebert, 1999).

Stigler and Hiebert (1999) have proposed a professional development system similar to the Japanese which requires that the classroom teacher must be the focal point of creating a solution. To accomplish their goal, the United States would need to reexamine the educational system from a cultural activity prospective. The cultural activities all have cultural scripts that have been learned by the participants through a life time of living. This would mean that the cultural scripts and cultural activities must be changed. This is not a task for teachers alone. There must be a commitment from the boards of education downward. This would mean that the superintendent must support the efforts of the principal to initiate the lesson study. The second major point is the establishment of standards that contain the goals the district wishes to implement. This can be an easy point to satisfy if the department of education has already created a standards list for each subject and grade level. Lastly, the schools need to be restructured. American schools are not equipped to restructure the teaching system without additional funding. This funding would be for additional teachers since a two hour uninterrupted collaborative time must be set aside each week for the mathematics teachers to get together and create the lessons. This commodity of time must be allotted in the teacher's schedules and take precedence over even student scheduling. How can American education fund this? According to Stigler and Hiebert, this could be funded by redirecting the funds used for reform research.

Looking Ahead to Implementation in the Classroom

The mathematics educators, whose opinions and classroom experiences on mathematical instruction have been examined in this paper, are not the only educators who have delved into the problem of educating today's students. There is still available an enormous amount of information to be examined. Though present day educators have contributed their expertise, I believe that no one educator can assimilate all this knowledge about teaching mathematics. This information offers a resource for educators. However, are there enough resources available concerning the implementation of these ideas and procedures? What are the results of all these suggested techniques and procedures? Have current educators actually employed these procedures in their classrooms?

Teaching Mathematics Through Problem Solving: Grades 6-12 (2003), published by the National Council of Teachers of Mathematics, is a collection of papers intended to assist teachers as they incorporate problem solving activities and approaches in their daily lessons. Sitka, Alaska high school mathematics teachers, Cheryl Bach Hedden and Dan

Langbauer (2003) share their experiences related to implementing a mathematics curriculum that is based on *NCTM Principles and Standards for School Mathematics* (NCTM, 2000). Their article allows other teachers to read about a critical change in instructional practices and classroom procedures that helped to improve students' valuing of active learning and to improve their problem solving skills. The focus of instruction shifted from teacher-centered instruction to a problem-based and learner-centered approach. Their explanation was based on an example of classroom procedures used in developing linear equations (Hedden & Langbauer, 2004).

This proved interesting to me. The shift from teacher-centered approach to the problem-based approach required these teachers to reevaluate their ultimate goals of student achievement – what they wished the students to value. For example, with regard to the quadratic formula, they changed their focus from simply "knowing the equation" to "building the equation" based on the students' problem solving skills with graphs and tables. Yet, knowing that the college mathematics placement tests and college entrance exams focus on symbolic reasoning using decontextualized algebra problems, these teachers used the opening minutes of class to have their students do warm-up problems of this style to prepare the students for these tests. This also helps in classroom management when teachers need to do things, like taking attendance and checking homework (Hedden & Langbauer, 2004).

After reading the works of the mathematicians I have chosen, I began to incorporate into my classroom instruction some of the suggested ideas. Over the last five years as I researched mathematics instruction as a member of the mathematics committee and as a teacher who was integrating new techniques in the classroom, I discovered that I needed to reevaluate what I wanted my students to achieve much as Hedden and

Langbauer (2004) had done. Though I feel that this is an ongoing activity, I am willing to share how I put theory into practice.

Putting Theory into Practice

As I began to think of all the research and methods available for educators to implement in the classroom, I found little in the way of actual implementation to review as to the success or failure of the instructional methods. With this in mind, over the past few years I have attempted to implement some of these techniques to evaluate their success or failure with my students. I realize that my teaching style, classroom environment, and the specific mathematics course being taught had an impact on the usefulness of the implemented method. The educators and mathematicians discussed in section one of this paper are those who influenced my decision to implement some of the methods. I also chose some recent educators (Coxford, et al., 1998; Demana, Waits, Foley, and Kennedy, 2001; Larson, and Hostetler, 1997) whose works provide a more useful structure to assist mathematics teachers in melding certain techniques or methods into one's teaching style.

I will begin with a description of my geometry courses for both the college bound and honors (gifted) students as I taught them after implementing some different techniques while utilizing the Ohio Academic Content Standards for K-12 Mathematics (2002) as the course requirements. I rarely used the textbook that was chosen for this course. The textbook has several deficiencies and was chosen before the Ohio Academic Content Standards for K-12 were published. The format of the textbook was not useful to the design of the presentation/student seatwork that I designed and implemented. Supplementation of the textbook was required because it did not contain sufficient material to meet all the standards. Whenever possible, I used exercise materials from the student's textbook that corresponded with concepts that I was teaching.

In an attempt to successfully cover all of the grade-level indicators of the Ohio Academic Content Standards for mathematics for grades nine and ten for the Honors Geometry ninth grade students, I was compelled to reorganize the order in which I taught the geometry course. Since our high school is on block scheduling, tenth grade students enrolled in geometry would take the Ohio Graduation Test in early March. With geometry courses beginning in late January, these students needed to be exposed to the maximum number of content standards as quickly as possible, thus helping them to test at relatively close to the same level of exposure as the remainder of the Ohio tenth grade students with mathematics courses that are a full year in length. This forced me to begin with a unit on coordinate geometry, which incorporated the algebra necessary to thoroughly cover equations of lines emphasizing a study of lines in general, parallel lines, perpendicular lines, intersecting lines and concurrent lines. This course then progressed into transformational geometry on a coordinate plane. Utilizing line segments and triangles, students were required to reflect these geometric figures over a line in a coordinate plane or locate the image, such as the vertices of a triangle, when given the preimage and the line of reflection. With regard to rotating these figures about the origin, the students were required to first locate the center of rotation. In order to accomplish this, I had to teach the students basic trigonometry and the law of cosines. Hence, my students were well into the course, had acquired a large vocabulary, and had developed a skill at following a long process. This is not the traditional approach to geometry or the order of sequence for most textbooks.

I believe that Stigler and Hiebert (1999) would have approved of the approach that I used to lay the foundation for these academic content standards. I examined the standards to determine which ones would naturally lend themselves to a progression leading to real situations with multiple methods for solutions. As the lessons unfolded, I taught more like the German teacher in the TIMSS study (Stigler and Hiebert, 1999) controlling the development of the concepts through discussion. However, the student-controlled environment time resembled the Japanese approach (Stigler and Hiebert, 1999) to teaching. On these days my students worked in groups to complete an assigned task in which they needed to choose the procedures that they had learned to come to the correct solution. My students found this both challenging and interesting. In this Honors Geometry class, these days became more of a contest among several of the male students to see how each one obtained the solution. Though these students were competitive, they never refused to help others in the class. In my College Preparatory Geometry classes, the groups were slightly competitive but not so much that they would refuse to help each other.

As the Japanese teacher did, I normally chose three different students to put their correct solutions on the board based on the approach chosen by each student. Since I was circulating the room and observing each group's work, I was able to make these choices keeping in mind not to choose the same students. Once each chosen student had explained his/her method of acquiring a correct solution, those students who had had difficulty in developing a process would request a list of procedural steps that would assure them of obtaining a correct solution. This list of procedures was then generated by the students under my guidance as a summary of the lesson. At this point, additional problems were assigned for homework for everyone to practice this new skill. The students in the Honors Geometry class always did every homework problem assigned. The students in my College Preparatory Geometry classes had an extremely high percentage of students who did homework. However, there were times when these students did not always completely understand all the problems. They were challenged by the problems and needed more reassurance during class instruction and practice to feel secure enough to tackle the homework.

Furthermore, I was amazed at their eagerness to show off what they had done and their desire to correctly execute a problem. To my surprise, the board work became neater as the semester progressed. These students began to take pride in their displays of knowledge. In the past, I rarely had students who wanted to use graph paper for accuracy. Eventually, I had to make it mandatory to use graph paper forcing my students to see a correct visualization of the solution. Now I need to keep a constant stack of graph paper on top of the front file cabinet as my students believe that they need to know exactly what they have obtained for a solution. On occasion, a few of these students observed that the solution obtained on their assigned problem was incorrect as its graph did not reflect the required image. Thus, they knew that they needed to find the error in their work. In the three classes in which I implemented this process, I observed an increased eagerness from students to learn. At this time I reserve my judgment as to whether it is an intrinsic motivation, such as the desire to learn for the sake of learning, or an extrinsic motivation, such as passing the Ohio Graduation Test, that caused the increased eagerness to learn.

I believe that Stigler and Hiebert (1999) have made some excellent observations about American classroom instruction. Though neither Germany nor Japan outwardly define the terminology as an assignment, American teachers do. Since my students are often unfamiliar with the precise mathematical terminology necessary to grasp a concept, I believe that knowledge of the precise mathematical terminology is integral to the continuation of studying mathematics. In the past, I assumed my students would automatically learn the terminology associated with each lesson as I did. However, as the years passed fewer students voluntarily did so. Thus, I was forced to require the students to define and be tested on these words to accomplish this task. Until my students demonstrate a higher level of maturity toward mathematics education, I will continue to require them to define and be tested on their understanding of these words. Since it takes valuable time in class to explain each word relevant to the material, I prefer to give my students a vocabulary list for each unit prior to the beginning of the new unit. A new vocabulary list is given to each student the day of the test for the previous list. Though they begin immediately to work on these vocabulary words after they have turned in the test for the previous list, they finish the task at home. My students may use their own textbook glossary, the previous textbook stored on the classroom bookshelf, a mathematics dictionary that I provide, or use the internet at www.dictionary.com searching under the mathematics meaning. No one source has all the vocabulary words as they pertain to the concept being learned. The students are required to define the words prior to the lecture on the material. This encourages the students to be prepared for the presentation since I properly use these words in the class discussion. For those students who may have difficulty in locating a definition or two, I encourage them to consult with each other first before coming to me. In doing this, I try to establish a rapport between the students as I use partners and learning groups in class.

As the course of study progressed, the students' knowledge base had expanded through an increased vocabulary and the ability to utilize their learned processes to formulate procedures in the development of a solution. As I soon discovered, this increased ability helped my students to assimilate the unit on reasoning, logic and proofs more quickly. As part of my preliminary study of this unit, I began, as usual, with logic and reasoning. This time I added a lecture on congruency and the congruency postulates of SSS (side-side-side), SAS (side-angle-side) and ASA (angle-side-angle) along with the theorem of AAS (angle-angle-side). At this point, I did not prove the theorem AAS (angle-angle-side). I had the students accept it. Later in the chapter, I returned to the theorem AAS and had the students prove it as a homework exercise.

My first attempt at a proof was the first principle of distance: *If a point is on the perpendicular bisector of a given line segment, then the point is equidistant from the endpoints of the given line segment.* Using Polya's (1973) process without first explaining the process to the class, I reviewed with the students which part of the statement was the "given" information and what was to be proved. Trying to encourage my students to become actively involved, I asked for a volunteer to assist me in drawing the "if" statement. Colored chalk was used to help the students distinguish between the parts of the diagram under construction. A yard stick and a protractor were utilized by the volunteer student to insure a proficient skill level at using these mathematical tools. The volunteer student drew the given line and its perpendicular in yellow, the right angle mark was drawn in blue and the floating point on the perpendicular bisector and the endpoints of the given line segment were drawn in light red. At my insistence, the student made a dotted line of light red from the floating point to each of the endpoints. This was accompanied by the student remarking that this looked like a couple of triangles.

After admitting that there was a resemblance to a couple of triangles, I asked a general question about what the class recognized in the picture. Responses included that the intersection is the midpoint of the line segment. I wrote point E is the midpoint of $\overline{AB}$ on the chalkboard. Next, I heard a remark that the two segments are equal. I wrote $AE = BE$ on the chalkboard. This was followed by a comment about right angles at point E. Again, I wrote that ∠AEP = 90° and ∠ BEP = 90°. The class and I barely had mentioned that the perpendicular bisector had a segment, $\overline{EP}$, that was shared by both triangles. All of a sudden, there came from the back of the room a rather louder than normal comment about the dotted lines being equal. "YES!" was my response.

I then told them that this must be written down formally so others could read it and know what we had done. Once all the steps were put in order and everyone became aware of the shared line as the reflexive property of equality, we declared that we used the SAS congruence postulate.

At this point, my students were sure that they could not do this on their own. I started to tell them about Polya (1973) and his experience in school with math. They seemed to relate well to this story. Once I was finished with this biographical approach, I wrote the four steps on the board. Much to my surprise, a young lady said that we had not gathered any new information other than the sides were congruent.

My response was to make another list. We succeeded in noting that the outside triangle was an isosceles triangle. We looked to see that the base angles were congruent, the vertex angle was bisected into two equal angles, the segment, $\overline{EP}$, of the perpendicular bisector was an altitude and a median. One question lead to another and the students discovered that the point of concurrency of the concurrent lines of the altitudes, medians, angle bisectors, and perpendicular bisectors all had formal names. They are, in corresponding order: orthocenter, centroid, incenter and circumcenter.

I proceeded to do two more problems with their help. Then, one of my students insisted on a process that he could follow to actually write out the proof in a logical sequence. Rephrasing Polya's (1973) four step process and adding four more steps each under Polya's second and third step provided my students with a tangible list for order in writing the actual proof. Under my direction, I managed to get the students to rephrase Polya's four steps. Some of the students were not confident enough that they would be able to come up with the items they needed. Unto those ends, we developed four more steps each under gathering information and logical sequencing. Doing this activity together provided a type of ownership of the process which in turn helped the students remember the steps. Since my students like catchy names for titles they decided on…

POLYA'S PROCESS FOR PROOFS

1. Draw a diagram of the given information.
2. Gather all the information available from the known facts and the diagram.
   a. If it exists, find a common or shared line – Reflexive Property of Equality.
   b. If it exists, find a pair of vertical angles.
   c. If given parallel lines, are there alternate interior angles or corresponding angles in the diagram?
   d. If given perpendicular lines, are there right angles?
3. Write the proof in logical sequence:
   a. Restate the given.
   b. Translate the given into math equations or usable notation.

c. Select items from the gathered information in step 2 that leads to the necessary conclusion.

d. State the conclusion. This is the "to prove" statement.

4. Examine the answer for additional information that is evident that could possibly be used later. These are often called corollaries.

I no longer see the same fear in the faces of my students when they do proofs as I have seen in the past. They can do the easy ones on their own with little difficulty. Some still have a feeling that they are not sure of themselves, but their work progressed well. This is the first time that I have been able to teach proofs in such a short time. I tried this method with three separate classes. One class consisted of all gifted freshman students. The other two classes consisted of sophomores, juniors and a few seniors. While the upper classmen were somewhat hesitant, the freshmen were extremely confident. Of course, we had practice problems. These classes were given a problem to prove with each homework assignment as we continued with a study of polygons and circles.

As I review the entire unit in relationship to the entire course, I can see that a second factor contributed to the ease of this process. This second factor is the order in which the course was taught. By the time my students were introduced to proofs, they had a larger knowledge base and had worked many problems that required knowledge of sequenced steps. Further, I believe that the biographical material on Polya's life added interest to the process. Though my presentation was more like a point of interest, Kline (1973) believed that mathematics is "truly liberal arts education wherein students not only get to know what a subject is about but also what role it plays in our culture and our society" (p. 145). Furthermore, Kline stated that "Mathematics is the key to our understanding of the physical world; it has given us power over nature; and it has given man the conviction that he can continue to fathom the secrets of nature" (p. 145). I find that integrating biographical and historical facts gives a personal touch to the rigid classroom environment. It also gave the students some time to step back from a formal presentation. Since I have my students write the process as a journal entry after we have spent time doing proofs, I am confident that they will retain at least a skeleton process in their mind for future reference. Furthermore, the encounter with proofs in the beginning of a geometry course sours students' taste for the subject and puts up yet another barrier to learning. The effect on students may come from the teacher himself/herself as well as a lack of understanding of the process as Dr. Richard Skemp (1987) suggests.

In addition to implementing Polya's approach to proofs and the restructuring of the material to be covered in the course, I decided to do the presentation of the material differently. I have always used the overhead projector to present my material since I wanted to accommodate many of my visual learners with correct diagrams in color and to be able to see the faces of my students as I discussed the new material. This helped me focus attention on those that appeared to be lost. However, after reading about the Japanese usage of the chalkboard as a tool that contained all the written aspects of the classroom lesson from beginning of the lesson to end of the lesson and Krantz's (1999) approach to the chalkboard, I decided to use colored chalk for diagrams, yellow, white, and orange chalk for written notes, and mathematical tools for the chalkboard to do my presentations. Krantz's approach states that material must be organized and the chalkboard must be completely filled before the teacher considers erasing it. By using the chalkboard section by section, I could present and illustrate the different steps of a process more efficiently. The students had time to absorb the material and formulate questions about the material. I no longer found it cumbersome to put diagrams back on the overhead. With ease, I moved to the diagram to point to the section in question. Even though my students had all the notes written down, the chalkboard, itself, seemed to meld the material into a single entity for them.

As my students attempted to follow the process I had demonstrated on the board, I would take a break and sit on a chair in the corner for a few minutes in silence. I personally refer to this as my "commercial break" where I use my time to quickly recap the instruction and view the faces of my students for obvious questions. Krantz (1999), too, views this as standing before a movie camera and zooming in and out. Krantz considered this time important to the student so they could view this material as a whole process much as the Japanese view the board work as an entire lesson.

However, here is where I differ from the Japanese. The time frame for the Japanese class is approximately 40 minutes while the time frame for my class and Krantz's (1999) college classes is longer. I teach on a block schedule with 80 minutes each day for instruction. College classes can range from 50 minutes to 120 minutes depending on the institution. Though the total minutes of instruction for the year's work in high school is equivalent to a semester course in college, the manner in which the material and the activities are presented require the classroom lesson to be presented differently. My class adapts better to Krantz's approach to presenting material as more material will be presented during a class session than can be written on one covering of the blackboard. At the end of the lesson, the class will provide a written summary of the lesson on the blackboard for all to see. My students usually write down this summary in their notebooks. Most of the time, it is a written process for finding the solution to a problem.

I was pleased that I had restructured my geometry classes. I was so excited that I gladly informed the principal that the first test on proofs yielded 92.7% of the students receiving a passing grade. This was an unusual event. I was proud of my students' accomplishments and pleased that the restructure of the course had a positive affect on the students.

Up to this point in time, I have always followed the order of the textbook for geometry, while for other courses I have deviated the order of the material presented based on the academic needs of the students. Parents have stressed the use of the textbooks for homework assignments to the point that a creative project that does not contain numbers is often frowned upon. Conversations with these parents can be challenging, since they are steadfast in their belief that math is always computational problems containing numbers. They do not wish for their child(ren) to gain a comprehension of the formation of mathematics and how man discovered these computational processes. In an attempt to educate the parents, I have brought to the parent/teacher conferences samples of past work of students (names removed) on particular assignments. I explain that the understanding of mathematics is a key to unlock the workings of the physical world. I emphasize that understanding why some mathematics exists is based on humankind's need to improve the quality of life through inventions and knowledge of the universe. As for the structure of the course, the principal has given the parents a booklet/checklist of all the content standards for each subject that must be taught during the academic year. He encourages the parents to review their child's assignments and question him/her as to which content standards are being met. In addition, the principal requires that the content standards be written in each day's lesson plan.

As a high school student in mathematics, I always wondered where I would use the mathematics that I was being taught. I rarely had word problems to solve. Though I still use *Modern Introductory Analysis* (Dolciani, Beckenbach, Donnelly, Jurgensen, & Wooton, 1984) to obtain clear and concise computational problems that illustrate the particular procedure I wish to teach, this text has a minimal amount of "word problems." These are far from what Kline (1973) was speaking of when he stated that real life situations should be used to teach mathematics. Though his concept of the proper mathematics course was a radical change from traditional mathematics courses, I could not discard his approach. After reviewing many of the current high school mathematics textbooks over the years, I have seen a gradual trend toward including more word problems that model real life situations. Though many of the textbooks I viewed had few examples for Algebra I and Geometry, the Trigonometry and Pre-calculus textbooks contain many more problems. I personally chose Larson and Hostetler's (1997) *Precalculus (4th ed.)* for the majority of my classroom material for my trigonometry and pre-calculus classes. As a strong secondary source, I use Demana, Waits, Foley and Kennedy's (2001) text *Precalculus Graphical, Numerical, Algebraic (5th ed.)* (2001).

My peers have commented on my method of instruction. I have been told by our head mathematics teacher that he has never seen trigonometry taught the way I teach it. He has commented on the natural flow that exists from the initial presentation of the concepts to the final real life activity that the students do to demonstrate their knowledge of the section and how it applies to the physical world and society. He was especially interested in the real life navigation and construction problems that I used in this course. Our head mathematics teacher has communicated his observations to the principal and discussed his comments with me. Since his computer was located in his classroom and our school district was cramped for teaching space, he remained in the classroom during his planning period while I taught class. It is from this vantage point that our head mathematics teacher has been able to make his statements.

Furthermore, younger faculty members have asked how I structure my trigonometry/precalculus classes and geometry classes and why I plan and teach these courses using the methods I do. In fact, our newest teacher is a former student of mine. He worked with me to create the course of study for the College Preparatory Geometry and General Geometry courses during the past academic year. He has told me that he often listens during his planning period to my presentations, especially if his students are having a difficult time understanding the material as he presented to them. These younger teachers have questioned the processes I have used to teach a particular lesson. They have frequently found it easier to understand my lessons than suggestions from the text.

I believe that there is merit to Kline's (1973) approach to mathematics education. However, I am not willing to discard all of the traditional approach to trigonometry either. I find that need of comprehending what the values assigned to the trigonometric functions mean is important to the overall understanding of the course as it applies to the physical world. There are trigonometric identities that need to be memorized in order that the students can utilize them with ease and not have to constantly refer back to the text.

As a result of this conflict, I have tried to create a trigonometry course that incorporates the following techniques/methods of instruction. Within a unit, I may use several of these techniques to vary the process of instruction. They are cross-curricular activities, use of technology as an investigation tool, literary resources, tutorial sources, and demonstrations of computational skills of the processes that focus on real world applications. These methods of instruction must incorporate several of the mathematical skills that are being taught in the particular unit. I attempt to diversify my classroom instruction to accommodate the audio, the visual, and the kinesthetic learners within each lesson. I use a partner approach with allowances for group work when the project warrants it. I require that each member of the group submit his/her own paper. This alleviates potential conflicts that may arise in completing the assignment and controversy which may occur over methods of determining solutions. Each person gives his/her own solution.

For this trigonometry class, the first day is business day. Once the textbooks and graphing calculators have been assigned, the course syllabus and the assertive discipline sheets have been passed out for a parental signature, the instructions for the first written assignment are given to the students and discussed. Since I agree with Kline that the students need to have a focus or an understanding of real life use of trigonometry as they proceed in learning trigonometry, this written assignment appears imperative to the formation of a knowledge base. Plus, I believe there is credibility in Skemp's (1987) idea that a schema must be formed in the intellect that can provide a foundation to which concepts can be linked, expanded upon and even reconstructed as learning progresses. Unto those ends, the students are given the topic, *What is Trigonometry and What are its Application in the Real World?* The students are provided with four websites that they are to read and summarize. If they do not have access to the Internet at home, then a disc is provided to them and their work is to be done on the school computers. Since the students come in with little or no understanding of what trigonometry is, I feel that it is appropriate that they be provided with websites that have an introduction to the real world application. One of my favorite website for my students to use is http://www-spof.gsfc.nasa.gov/stargaze/Sintro.htm . Section *(M-7) Trigonometry—What is it good* *for?* is a section of a book-on-the-web, *Stargazers to Starships* by David P. Stern. (Stern, 2005) There have been times that several of my students have returned to read *Stargazers to Starships* in its entirety and enjoyed it.

Another excellent source of information is *Dave's Short Trig Course* by David E. Joyce of Clark University. This site provides the students with information about trigonometry and real life applications besides giving the students a brief course in the basic of trigonometry.

I thoroughly believe that vocabulary is essential to understanding any mathematics course content at any level. Therefore, my students receive a vocabulary list of the terms for each unit that requires a definition and are necessary to comprehending the material, as Krantz claims that it is necessary for a teacher to instruct the students in reading mathematics. Though he is referring to symbolism and notation, I believe that pertinent vocabulary is just as important for a complete understanding of the mathematics.

DeLong and Winter (2002) contend that students cannot read mathematics upon entry into college. Their suggested readings can assist the mathematics teacher in learning how to teach students to read mathematics. I have assigned some of their reading sections and subsequently began the following class with questions about the reading material. I have taken the time to read through textbooks with the students, teaching them how to focus on important concepts, and requiring students to solve the sample problem without looking at the solution. In the Calculus Enrichment Class each fall, I dedicate many lessons to reading the material from the textbook hoping that these students who will be advancing in mathematics and science in college will be better prepared through this attention to learning to read mathematics. I also know that the other calculus teacher is as concerned that the correct mathematical concepts can be transmitted from the book and his lectures to the student where it is assimilated and correct solutions are returned using proper terminology in acceptable mathematical format.

Furthermore, Skemp (1987) emphasizes that vocabulary is crucial to the understanding of mathematics. He states that a concept between two people must contain a common meaning in order to communicate an idea. An incorrect association due to miscommunication can interrupt the learning process and cause confusion within the student's mind. It is for these reasons that I require that my students complete a vocabulary worksheet, turn it in to be graded, and be tested on the understanding of the vocabulary words separately from the mathematical processes being learned.

Let me turn from my trigonometry classes to my Honors Geometry class for an example of the confusion that can occur. Much to my surprise, I did discover that one of my geometry students became confused with the symbol used for the radius of an inscribed circle in a regular polygon and with the symbol used for the radius of the circumscribed circle about the same regular polygon along with the general formula for the area if a circle. To him, these circles should have used the same symbol all the time. To bring the reader abreast of the situation, this student had been absent during a critical part of the presentation. He missed the explanation that the radius of the inscribed circle was the same as the apothem of the regular polygon. Plus, he missed the class discussion on why it was necessary to change the symbol of the radius of the inscribed circle to match the symbol for the apothem of the regular polygon. It is for this reason that Stigler and Hiebert (1999) pointed out that the Japanese educators who were evaluating the film found it inexcusable that classroom instruction is interrupted for any reason on a regular basis in American schools. As I found out, this bright young student had an extremely unsatisfactory and unproductive time that evening at home while attempting to complete the assignment. His being called out of class by the guidance office caused him to miss the complete instruction. Thus, I agree with the Japanese approach to class work. Instruction needs to take priority over anything other that an emergency.

Once the vocabulary work has been done, a presentation of historical information concerning the current topic is given. For example, a discussion of the sexagesimal numbering system of the Babylonians and their positional system, which is still in use today, is used to introduce the unit circle and degree measurement. This is in line with Morris Kline's (1973) belief that students should be made aware of the cultural aspect and the influence it has had on society.

Though graphing calculators are used to obtain the values of the trigonometric functions for use in computational and application problems, I believe that a complete understanding of the degree measure, radian measure and the trigonometric table of the quadrantal values is still a valuable tool to know. Therefore, once the quadrantal values for the six trigonometric functions have been obtained using the 30-60-90 triangle and the 45-45-90 triangle in each quadrant, the student will use the word processor to create his/her personal chart. The exact values as displayed in the trigonometric table help the student calculate the exact values for double and half-angle trigonometric identities and other circular and trigonometric functions. The students' use of the word processor allows students to employ the Insert tool and the Symbol tool containing many math symbols and Greek letters often used in mathematics. This use of the computer is more of a liberal arts approach to the class (an allowance of the use of tools not normally used in mathematics classes) and does expand the students' scope of tools that are associated with mathematics, although this is not the use of the computer that Krantz (1999) refers to in his text.

At this point in the course, I find it necessary to incorporate a review of the basic concepts of the Pythagorean Theorem and the applications of the trigonometric functions as they pertain to obtaining angle and sides of any right triangle. The Ohio Academic Content Standards introduce the basic trigonometric ratios of sine, cosine, and tangent in the ninth grade geometry standard. By the tenth grade, the use of these trigonometric ratios is incorporated in the plane geometry courses. Subsequently, by the eleventh grade, the geometry and spatial sense standard states that the use of trigonometric relationships must be used to determine lengths and angles measures. As Skemp (1987) so aptly states, a review of the previously learned concepts is appropriate since a solid foundation must be present allowing the retrieval of old ideas to be associated with new ideas.

I have been successful at expanding the trigonometric ratios from the basic three to include their reciprocals. This comprises the six trigonometric functions which naturally lead to the defining relationships between the trigonometric functions. The most basic of the relationships are the reciprocal identities, quotient identities, and the Pythagorean Identities. Without a review of the Pythagorean Theorem and the utilization of the Unit Circle as a means to define the trigonometric functions, the students would not readily grasp this material.

Normally at this point, I feel comfortable with distributing a worksheet of five real world problems. It is here that I begin to share with my students their summaries of the articles they have read, discuss their thoughts on trigonometry and its applications, and try to grasp where they are in understanding the material. This process begins when I pass back the summaries and allow the students to review them for a few minutes reading my comments. Then, I place the question on the board, *What is Trigonometry?* I give them a minute or two to collect their thoughts. I ask for a few volunteers to quickly write a short answer under the question. Once everyone has a chance to look at the responses, the students are asked to explain their answer and tell which website(s) was the source of their response. At this point, I ask for questions and/or comments from the students to their peers responses. This procedure is then repeated for the question, *What are the Real World Applications of Trigonometry?* Instead of the question/comment section, I have the students get into their groups of four. Here, they discuss these applications. Each person provides a statement to their group writer about trigonometry and its function in the real world. This process engages more students to give their understanding of trigonometry. Although no grades are given, these papers are turned in so I can read their comments and glean an overview of the students understanding of trigonometry.

Once our discussion seems to be winding down, I refer to the five real world problems worksheet passed out at the beginning of the lesson and inform students that these are due at the end of the section. They can work on these gradually, since they will need additional information which has not yet been taught. Upon completion of the section, they should have all the knowledge necessary to complete the required problems. As a culminating activity for my trigonometry classes, I assign a group project that is a navigational course design for a boat race or an aviation trip that utilizes wind velocity and requires use of straightedge, protractor, and calculator for a scale drawing of this design. Since most of these students have not had physics, the project cannot be too complicated. Frequently, I hear comments that this course is so interesting and usable in everyday projects.

Krantz (1999) and I also think along the same lines in reference to the teacher's attire and personal appearance. As a student, I had excellent examples of educators who considered their occupation of teaching as a true profession. Thus, they dressed and acted accordingly. I believe that the teacher's personal presentation lends an air of importance to the class environment. It informs the student that business will be conducted and preparedness is expected. Of course, the attire needs to fit the activity, the teacher's budget, and the type of course taught (woodworking, chemistry, and foods lab needing a more task-appropriate attire).

The instruction of the trigonometry course along with the geometry courses is modeled, in part, after Krantz's (1999) approach. While he believes that a teacher should arouse a student's interest in mathematics, he holds that a student must be responsible for his/her learning. As I already stated, I do attempt to arouse my students' interest in trigonometry and geometry by presenting material and assignments concerning the history of the particular discipline of mathematics being taught and its current use in the real world. However, I firmly believe that the each student is responsible for his/her own education. By this I mean that it is the student's responsibility to come prepared for class with the reading of the new material and any written assignment completed. I believe it is mandatory that both students and teachers come prepared to class. It is for this reason that homework counts as a test grade. For the college bound student and the honors student, failure to complete the homework assignments will lower a grade. For most of these students, grades are an extrinsic motivation.

Along with DeLong and Winter (2002), I believe that student motivation can assist learning mathematics. To intrinsically motivate my students, I have taken extensive time in compiling scenarios that are familiar to them based on my knowledge of them and their interests. Based on my observation, I believe I am able to spark their interest with this method of instruction. A fellow mathematics teacher within the district, Brian D. Harper, has such a clever way of creating humorous scenarios that his students enjoy reading. Students are amused when they can recognize a character. This enjoyment sets the class at ease and allows them to become focused on learning how to find the solution [See Appendix B: Harper, 2004]. I personally enjoy reading his worksheets. Occasionally, a story line can be observed traveling through the homework problems. I believe that beginning high school students can be receptive of this type of instructions as well as senior students. I occasionally employ this technique when I have freshman. It does grasp their attention and gives them a personal interest, thus intrinsically motivating my students to work on the task at hand. Since my freshmen are normally accelerated, their assignments are not quite as humorous as they are challenging.

Since the school district provides time before, during, and after school for students to get help on assignments, I find it disturbing that a student would not be motivated enough to attempt to get help in order to understand the assignment. He/she may indeed have difficulty with the homework problems, but not reading the text material or the class notes and summary along with reading the problems and jotting down questions concerning a particular problem under consideration, leads me to believe that they are unprepared and do not have enough intrinsic motivation. To provide these students with encouragement to work, I give full and/or partial credit on homework assignments. Some may disagree with my philosophy, but I give credit if the problem has been attempted though not correct or if the student has written down questions concerning the problem that he/she will need answered to go on with solving the assigned problem. These questions must reflect the student's thinking and not be generic in nature.

As Stigler and Hiebert (1999) report in the TIMSS study, the Japanese do not assign homework. At the same time the students appear to be prepared to do the next assignment. In Germany, the teachers act as though all students have their homework complete by reading off the answers. This is an ideal situation in both foreign school systems, but it is not generally the case in my college preparatory classes, though it does hold true for my honors classes. As an educator, I have received excuses from my students that range from the fact that the school bus for the sporting or the academic extracurricular activity did not return until late and the student had to go to bed or the student had to do another assignment for another class which was also due. I believe there is a time management situation that arises for U.S. students, mine included, who are involved in extracurricular school activities. Many may not even consider doing anything else during the travel time or waiting time between their event and other events. The time management skill needs to be stressed. On the other hand, there are students who have mastered the time management skill quite well. For example, other teachers and I have had students come up into the bleachers at a sporting event and ask questions about the assignment while younger students are playing their match. There is time to do homework and play sports. Sadly, these students are not the majority and hence a well prepared lesson is ruined by an unprepared class. Our Japanese and German counterparts do not have these school sponsored activities. Therefore, I believe that the structure of the school itself has posed a problem for education in the United States. Thus, I provide the extrinsic motivation of grades to assist the student in being prepared for class.

Krantz (1999) holds that computer use should be restricted to reinforcement labs. Since I do not have my own computer lab and must share the computers with the entire school, I find it difficult, but not impossible, to incorporate computer labs into my class instruction. When I am able to use the computer lab for my trigonometry students, it is usually a review lesson from a tutorial website such as Joyce's (2002) *Dave's short trig course*. Several years back the high school purchased *Geometer's Sketchpad* for the geometry classes. Several of the teachers, including myself, try to use this software to enhance our student's understanding of mathematics. There is a unit that is easily adapted to trigonometry that I have used. There are also tutorial lessons on the internet that I have used as review exercises or supplemental work for students.

I do not limit my use of the computer to packaged programs. In trigonometry, I require that the students do a cross-curricular project using the internet. This project is on pi whose symbol is π. In accordance with Kline's (1973) philosophy and with Skemp's (1987) position of the development of a schema to assist students with numerical concepts, I believe that it is necessary to assist the students in their understanding of the place of mathematics in the development of the world cultures. I choose this transcendental number, π, because of the plethora of information that can be accessed on the internet. My students are required to locate a minimum of ten different useful websites. An annotative bibliography is required for each of the web sites three weeks into the project. The report is at least 1,000 words in length covering the history of pi using six different mathematicians involved in the development of its value. These mathematicians are to represent at least four different countries. The report must also include information about pi's occurrences in nature and its influence on today's society whether in the work place or in recreation. The students are given a six page packet titled, *Now Tell Me Exactly What You Are...Numerically! (A Study of the History of Pi – its values and its uses)* (Obhof, 2004). This packet, modeled after Bernie Dodge's Webquest (Dodge, 1998) format, gives an introduction to the project with helpful hints on where to look for information [See Appendix A: Obhof, 2004]. The task is defined and the process is presented with guideline questions to assist the high school student in the research process thus making a thorough investigation of the topic possible. The rubric is included in the packet since I believe that the students should be aware of the grading process and point spread.

I am not alone in believing that the students need to be aware of the rubrics for quizzes, tests and assignments. Our principal has recently required that teachers inform the students of the point spread for each question on a graded assignment. Furthermore, our principal believes that there needs to be a grade given each day for each student's work. Hence, Krantz' (1999) philosophy of using small quizzes once or twice a week is easily accepted and employed in my classroom. This also helps me gather information on the students' progress in understanding the unit being presented. Though I agree with Krantz that the handwritten exam is very good, I am obliged to develop tests and quizzes that model the Ohio Graduation Test in style, directions, typographical format, and assessment of the each student's work. This strategy is designed to increase the student's level of comfort, generating a positive mental attitude and familiarity with the instructions. The panic attitude is believed to be reduced and the students can then perform to their full potential. This helps students to be successful when taking the Ohio Graduation Test and set a format for testing within the high school.

In the past I developed my review sessions much like Krantz'(1999) process. I never have my students reprove theorems or regurgitate homework problems. Neither do I give them the problems ahead of time for an in-class exam. Nor do I use this tool to teach more material that I have not covered prior to the exam. I believe that an exam or test is meant to reflect the student's knowledge acquired in the mathematics course.

Krantz (1999) believes that a consistent and fair grading system should be in place at the commencement of the course. I hold this same philosophy and distribute the grading scale and course requirements on the first day of class. Parents and students need to be aware of the expectations and to know that the grading scale and course requirements will be adhered to as fairly as possible. I have always done this and will continue to require a signature from the parents and students on these forms. For the high school approach, I also inform parents of any unusual transgressions of these agreements often to the particular student's great displeasure.

Though I have only briefly touched on the subject of "math anxiety" thus far, I believe that it exists. This inability to comprehend mathematics by intelligent people can be disabling to students from the first day in the classroom. Since I am extremely concerned with this phenomenon, over the past years I have questioned my family, friends, and some non-math peers concerning their feelings of mathematics. The conversations were held in a relaxed atmosphere and questions were general in nature. Most of these people had some terrifying experience or a long tail of woes that reiterated Skemp's (1987) opinion that the anxiety has been caused by classroom experience or a particular classroom teacher. This idea of math anxiety has been supported by my own students who entered my classroom disliking mathematics. For several years, I have asked my students to write for me their feelings on mathematics. Like the adults I have spoken to, these students can relate a situation that precipitated these feelings. Polya (1973) cited an article in the preface of his book that came to the same conclusion. However, Polya states that this anxiety comes from the teacher's dislike of the subject or a particular topic in mathematics. This cyclic effect multiplies when more and more students are exposed to educators who dislike mathematics and then become teachers themselves. Knowing that this phenomenon cripples the education process in high school, I have spent five years deliberately discussing my purpose and defining my techniques to my students. Furthermore, I gather information on my students' opinions of mathematics through brief essays written on the first couple of days of school. This way I can focus on the student's particular difficulties as I am teaching a concept. Often times, a review of the skills prior to the introduction of a new concept can ease some of this math anxiety feeling.

Math anxiety can affect student motivation. DeLong and Winter (2002) emphasize that allowing students to use calculators on tests and permitting the use of a cheat sheet can eliminate some of this anxiety. I took this information to heart and passed out a note card or a small post-it note sheet to the students to let them write down anything they thought would help them with the test. They were required to staple this to the test. I found that the students actually spent time studying for the test in order to collect information they thought would be useful. Eventually, the students did not need to use the card. By the end of the semester, many students said they studied so much making the cheat sheet that they ended up learning the material.

Our district's mathematics committee was given the task to improve K-12 mathematics instruction within our district. As part of the process necessary to complete this task, the committee needed us to examine student achievement data using the Ohio Proficiency Test results. These results were examined first within the district for 1999 through 2003. They were then evaluated as to the distance from the state standard. Lastly, it was decided to compare the scores of similar school districts. Under the guidance of the curriculum director, guidelines for a list of similar school districts were created. Some of the criteria employed in determining similar school systems were based on an environmental make-up of rural/farm homes and residential dwellings of a small urban town mix, size of the enrollment, and the number of free and reduced lunches. Once the list of similar districts was created, the comparison was done. As a committee, we felt that we needed to observe and speak to the successful teachers within our specific grade level within the similar successful districts.

I observed geometry classes in similar school systems with high academic scores, conversed with these educators about techniques utilized, examined their technology and discussed the time allotted during the class time for instructing the students and for using the technology as a necessary part of the course.

After viewing the classes from these similar schools, I determined that the approach to learning for middle school was more successful using an organized problem approach that resembled the Japanese lessons from the TIMSS study or unit project lesson. The approach taken by the high school teachers resembled my own instruction. However, they had an active group of students who would attend additional instructional time provided by the teacher during his/her preparation time. These students came from study halls and/or the alternate days of science lab classes.

After the observations were complete, my peers and I were taken to the computer labs. We were given a small demonstration of the equipment and discussed the use of the TI-83Plus calculators and the Ohio Graduation Test calculators as implemented in the courses of study. Furthermore, the mathematics teachers and the administrators of our host schools along with our committee members met and discussed the successes and failures of the techniques that had been tried to help improve student learning. In my opinion, our host schools were a little farther along in the realignment process having spent a considerable amount of time developing programs that worked for each teacher and his/her students.

Prior to the visits of similar schools, the curriculum director facilitated discussions on current research and practices as presented in various articles from professional journals at the bimonthly meetings. Of these articles, I found that *Educational Leadership February 2004* issue to be the most interesting. I was impressed by William H. Schmidt's (2004) belief that "curriculum does not refer to a particular textbook, learning activity, or pedagogical style, but to content expectations or standards" (p. 7). Across the United States, tracking of students still exists. This practice deprives students of equal exposure to the content expectations for any particular grade level. Schmidt analyzed the results of the TIMSS studies and believes that lack of exposure to the established international standards for eighth grade students deprives one-third of U.S. eighth grade students an equal opportunity to advance. He also believes that the teacher's knowledge of mathematics and his/her ability to implement a challenging curriculum are at a lower level than teachers of high-achieving countries. For real hope to compete on an international basis, Schmidt feels the curriculum alignment must be the first step. It needs to be in a common alignment across the all districts, understandable, and challenging.

Amazingly, Stigler and Hiebert (2004) also reviewed the TIMSS studies. Though they know that the emphasis is being placed on the recruitment of qualified teachers in order to correct the lack of knowledge base of current teachers that some people believe exist, Stigler and Hiebert "believe that the focus on the improvement of teaching-the methods that teachers use in the classroom-will yield greater returns" (p. 16). In focusing on teaching, the manner in which the students and teacher interact can be more powerful than any curriculum material. They believe that any attempt at implementing reform will be fatal and fruitless without first analyzing current teaching practices. Stigler and Hiebert have expressed their belief that many educators have never seen the use of problems in instruction done effectively.

With the research in mind, I discussed the dilemma of lower scores in mathematics with my peers through an examination of the students' achievement data comparing our current practices with those of high achieving institutions of similar districts. In order to reach this point, our committee needed input on current practices from within our district. Thus, the committee developed a teacher survey on instructional, curricular, and assessment practices which focused on attitude and classroom implementation of mathematics K-12. An analysis of the responses of this survey and formulation of recommendations for district consideration were completed by the committee and the curriculum director. The supporting data for these recommendations was assembled and presented to the district administrators and mathematics teachers.

The year's work brought the committee to a point of sharing the information with the mathematics teachers throughout the district. This second year brought about discussions among teachers by grade level, by departments within each school, and by departments among schools of adjoining grade levels. This placed a larger burden on the middle school mathematics teachers for their input was needed by the sixth grade teachers and the ninth grade teachers. Eventually, the process for aligning the K-12 mathematics courses and writing a course of study for each mathematics course based on the Ohio *Academic Content Standards for Mathematics* (2002) was required.

Subsequently, I created a course of study for plane geometry based on the Ohio *Academic Content Standards for Mathematics* (2002). I implemented this course of study in my geometry classes to field test its effectiveness. This information was utilized in generating a district course of study for plane geometry. I have willingly shared my methods, techniques, and actual lesson plans with another current plane geometry teacher with the hope that my success in instruction of my geometry students will prove to be beneficial to the department. Near the end of the 2004-2005 academic year, I was assigned to realign the Trigonometry/ Pre-Calculus course of study for both honors and college-bound programs. This was an easier task as I had begun to realign this course with the Ohio *Academic Content Standards for Mathematics* on my own a few years ago when the district permitted and gave me the responsibility for the development of an enrichment class for the future calculus students as an introduction to Calculus through a study of Limits.

At the present time this committee is still a critical part of the school district's quest for successful mathematics courses. Prior to the creation of this committee, a fellow mathematics teacher from my school district and I developed Short Cycle Assessments grades 9, 10, 11 based on the Ohio *Academic Content Standards for Mathematics* (2002) under the supervision of the Ashtabula County Educational Service Center. The assessments were field tested county wide. Many other mathematics teachers assisted us in examining the results and restructuring particular questions. After two years, each district was given the assessments with permission to revise them or utilize them as each district saw fit. This was the Ashtabula County Educational Service Center's first attempt at providing an examination which tested each standard in the Ohio *Academic Content Standards for Mathematics.* This activity did stir emotions within the county.

During the second summer session, more mathematics teachers took an active role in trying to develop questions for the examination. It is here that I spoke to other mathematics teachers about their approach to classroom instruction and their response to the state's requirement that the Short Cycle Assessments be based on multiple choice problems, short answer responses, and extended answer responses. Creation of the Short Cycle Assessments forced me to become familiar with all of the standards sooner than most teachers within my district and within my county. The first collaborative effort to create a pool of questions for the mathematics teachers was remarkable. Sadly, this collaborative effort between school districts has ceased and once again each district is on its own. The program did not continue as planned because the driving force for this program left the Educational Service Center for a new challenge. Each school district was given all of the Short Cycle Assessments to use and modify as they saw fit. The elementary and junior high grades within the district still use the assessments to monitor the progress of student learning. The high school teachers utilize the test problems by unit. The problems have been restructured to follow the patterns of test problems given on the Ohio Graduation Test. This serves a two fold purpose for the high school teachers. Besides assessing the progress the students are making at learning the standards, these problems prepare the students for the type of questions, the format for these questions, and how the responses should be formulated for the Ohio Graduation Test.

Plans are being put into place for the high school teachers at Buckeye Local Schools to evaluate the results of the Ohio Graduation Test to see which academic content standards are still weak areas and try to discern why the students are having difficulty in these areas. Collaborative meetings have been planned for the mathematics department for discussion on these weak areas. A lead teacher has been chosen to obtain information for the department and summarize the collaborative meetings. Besides all these efforts to improve instruction, the mathematics committee is still in place and will examine the overall district efforts.

Conclusion

Mathematics instruction is a key element in improving student mathematics education. Under normal conditions, mathematics instruction comes from the mathematics teachers. It has been established that teaching is a life experience taught to today's teachers by their teachers from the time the current teachers were children and entered elementary school. This cyclic action results in the repetition of the style of instruction. Not only do the students fail to advance as knowledge increases, they demonstrate a "fear of mathematics." This fear of mathematics is part of the hidden curriculum passed on to them by some teachers as Polya remarked or by parents and/or even older siblings who have had a negative experience with mathematics sometime during their lives.

Over the past 60 years, there has been a concentrated effort to improve mathematics education. Both reform movements and research of individual mathematicians have proved fruitless at increasing student performance. Polya's method for proving theorem instruction, Kline's philosophy that mathematics must be pertinent to the student's environment, Krantz's effort of telling teachers of his teaching successes, and DeLong's and Winter's effort at professional development for current educators does not produce results unless the techniques are implemented in the classroom. Though some of these mathematicians could have been my professors, my contemporaries or even my students, what is important is the incorporation of their techniques in my instruction. Improvement of classroom instruction can only occur if the information available is examined and utilized in the classroom. This takes a lifetime of reading and implementing techniques into the teaching style of each teacher. All of the information that is obtained by an individual teacher is lost to the U. S. students at the retirement of a mathematics teacher. Although there is abundant research available, there is insufficient prepared information through lessons and instruction of particular mathematical concepts within each discipline for new U.S. teachers to reap the benefit of successful instructional lessons. Japan has spent 50 years creating, collecting, testing and re-circulating lessons prepared by the ordinary classroom teachers. It is time that U.S. teachers pass on their successes through publications or even within departments in any school district. Successful teaching should not be for only a few. It should be a shared responsibility of all educators to assist their fellow mathematics teachers at successful mathematics instruction.

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Appendix A

Now Tell Me Exactly What You Are...Numerically!

(A Study of the History of Pi – its values and its uses)

A WebQuest for Trigonometry

Designed by

A.M. Obhof

$$\pi$$

Introduction / Task / Process / Evaluation Page / Conclusion / Web Site Page/ Rubric

Introduction

You are the investigator…You are the inspector…You are the detective assigned to uncover the true identity of a member of an elite group of Greeks who are moving in on you. They are invading your thoughts and the thoughts of your peers as they did Archimedes, Ptolemy, Tsu Ch'ung Chi, al-Khwarizmi, Lambert, and Lindemann, to name only a few. This particular member has been called by various names over the years. This elusive member has surfaced every so often for more than 3,000 years. It has not been confined to just one area of the globe. Its reputation is world renown. Though William Jones gave it the name of Pi, it was Euler who popularized it. Being the sleuth that you are, you must investigate the reports that Pi has been hiding in rainbows and in people's eyes. Alas, a biologist has discovered Pi in his lab. It is your mission to discover the truth about Pi, to report about how it has influenced man, appears in nature, and determine its true numerical value, if possible.

The Task

Your task will be to write a report about the history of Pi, its values through the ages, its appearance in nature, and its application in other areas such as society or recreation.

The Process

To accomplish your task, please follow the directions carefully. You must do your research on the internet.

Research

1. You must define what Pi is. Any search engine will help you find information. Take notes!
   An example: About Pi
   http://mathforum.org/dr.math/faq/faq.pi.html

2. You need to find information about the history of Pi.
   An example: Pi
   http://mathworld.wolfram.com/Pi.html

3. You need to find information on the mathematicians who computed the values of Pi.
   An example: Archimedes
   http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Archimedes.html

4. You need to find web sites that will give some applications of Pi in nature.
   An example: Earth Mysteries: Notes on Pi (π)
   http://witcombe.sbc.edu/earthmysteries/EMPi.html

5. You need to find web sites that will give some application in society such as recreation.
   An example: Pi Jokes
   http://www.pen.k12.va.us/Div/Winchester/jhhs/math/humor/pijokes.html

Web Site Research Questions

- Did you write down the web site addresses that you visited and the date you visited the websites?
- Did you write down the title of the particular article(s) you read?
- Did you write down the author's name and the date of the last revision of the article(s)?
- Did you remember to write down comments about the article(s)? This the information you will used to write your paper.
- Did you remember to write down information about the web site? Was the web site useful? What type of information did it contain? Did it contain useful links?
- Did you find at least ten useful web sites? (You may use three of the initial five websites from the preceding page.)

Parameters of the Report

- The length of the report will be at least 1000 words.
- The report will include information on what Pi is and who helped identify it.
- The report must thoroughly discuss the history of Pi incorporating in six mathematicians from four different countries that contributed in some way to the history of Pi.
- The report must include information about Pi's occurrence in nature. Be as specific as possible.
- The report must give ways that people are using or dealing with Pi in today's society whether in the work place or in recreation.
- Directions for typing the report.
    1. Double space using font 12
    2. Correct grammar and spelling
    3. Reference page done in MLA style (school requirement)
    4. Web site addresses - Due:
    5. Annotated Bibliography - Due:
    6. Final paper - Due:

Evaluation

Your report will be graded as follows: (150 points)

- Variety and usefulness of the web sites
- Research information will be evaluated on the annotated bibliography.
- Thoroughness of report. The report must include, at least, all mandatory information requested in the parameters of the report.
- Following directions and creativity in the presentation of the material and report

Conclusion

Pi has a key role in trigonometry. A solid foundation in comprehending the concept of Pi is a must for any student of trigonometry. Applications in nature have always led to an improvement in mankind's mathematical skills. Hopefully, as your understanding of trigonometry increases, so will your appreciation of the beauty of mathematics in the world.

This project is meant to increase your ability to use the Internet in your research and make you aware of some of the technical web sites that can be accessed through the educational network. These web sites range from the actual instruction of the math course to the history of the particular mathematical topic and even further to the applications in nature and daily life. Good luck on your research.

Point Sheet for Pi Project

Name ______________

150 pts. Distribution

Report – Mechanics

_______ double spaced/ font 12/ legible font type (10 pts)
_______ length (10 pts)
_______ grammar/spelling (10 pts)
_______ creativity (10 pts)

Report – Thoroughness

_______ Information on Pi (17 pts)
_______ Pi's occurrences in nature (7 pts)
_______ Pi in today's society (7 pts)
_______ Mathematician Names (6 pts) Countries (4 pts)

Mathematician Names	Countries
________________	________________
________________	________________
________________	
________________	
________________	

Annotated Bibliography

_______ Web Site Addresses (10 pts)
_______ Research information summaries (40 pts)
_______ Rating Scale /evaluation (10 pts)

Reference Page

_______ Proper format, etc. (10 pts)

_______ TOTAL

_______ GRADE

Appendix B

Algebra 1

Thursday, October 14, 2004

☺ Word problem Warm Up ☺

Gee Whiz, Thanks Mr. " H "

Erkle the outlaw, a.k.a. *The High Pants Drifter,* is wanted by the FBI,... Fashion Bureau of Investigation: the charge: "Wearing his chaps to high!"

Erkle has taken refuge in Math City in hopes of escaping the LAW, but soon learns that Marshall "Morgana Erp" is on his trail in hot pursuit. (Yes! The Morgana Erp, the fastest talker in the West!)

Erkle leaves Math City at 6:30 PM on horseback riding North at 6 mph along Word Problem Trail toward the Fraction Mountains, 18 miles North of Math City. Marshall Erp enters Math City at 6:30 PM from the South end of town not knowing the notorious Erkle just rode out of town.

Our good Marshall takes time to eat dinner, fill her canteen, and change horses, then at 7:30 PM, rides out of town on horseback at 10 mph heading North along Word Problem Trail toward Fraction Mountains.

Can our good Marshall catch up with the notorious Erkle before he can hide in the Fraction Mountains?