# ELASTIC-PLASTIC STRESS ANALYSIS OF AN EDGE <br> C̈RACK SPECIMEN AND EVALUATION OF FRACTURE MECHANICS PARAMETERS 

## by

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# ABSTRACT <br> ELASTIC-PLASTIC STRESS ANALYSIS OF AN EDGE CRACK SPECIMEN AND THE EVALUATION OF FRACTURE MECHANICS PARAMETERS 

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Finite element stress analysis was used to calculate tne fracture mechanics parameters, tne so-called J-and Mintegrals, for a tensile edge notcn specimen (Figure 2).

A comparison of the finite element load-displacement was made witn tne experimental results obtained from tne Department of Macromolecular science at case western Reserve University. Also, for a class of problems wnere tne plastic zone remains localized near tne crack tip, estimation tecnniques were employed to determine tne J-integral values for the same tensile specimen.

The m-integral formulas suitable for numerical evaluation were derived. one set of equations employed tne concept of isoparametric element (a computer program could be written using tnese equations). Applying these equations, a manual calculation was performed to evaluate tne m-integral values. These values were compared witn tne values obtained

## from an estimation technique

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INTRODUCTION

## Hlstorical Backoround

The application of concepts of fracture mecnanics nas become the primary approacn in control ling the brittle fracture and the fatigue failures in structures. From 1913 until the early 1960s, very little research work was done in the area of fracture mechanics. In 1965 tne American society for Testing Material (ASTM) formed Committee E-24 on Fracture Testing of Metals. Since then, the literature on fracture mechanics has been growing at a very rapid pace [1,2].

The decade of the sixties can be cnracterized as a searcn for valid test methods and an exploration of fracture tneories. In the seventies, considerable retninking and regrouping occurea with tne introauction of new concepts sucn as the $R$-Curve and $J$-Integral [1,2].

## 1.2 concepts and Apolication of Eraciure Mechanics

The phenomenon of fracture has been observed by all on a daily oasis. operations that produce certain surface geometrics, such as cnopping of wooa and the cutting of glass to desired sizes illustrate intentionally induced fractures.

The accidental cracking of windsnields on automobiles and the catastrophic brēaking of large structures sucn as ships and oil tankers, typify undesirable fracture. Fracture process initiates not only with tne presence of a crack or a flaw somewhere in the structure, but also the stress level which can induce tne crack propagation leading to a catastrophic fai lure.

Brittle fracture is a type of catastrophic failure in structural materials tnat occurs at an extremely high speeds (as nign as 7,000ft/sec) [3]. It is characterized Dy.a flat fracture surface (cleavage) with little or no shear lips, and at average stress levels below tnose of general yieiding Stresses. Historical review illustrates the fact tnat brittle fractures can occur in engineering structures such as tanks, pressure vessels, snips, oridges, airplanes, etc Numerous factors such as the service temperatures, fracture toughness, welding, residual stresses, fatigue, constraint conditions, can contribute to brittle fractures in large welded structures. However, the primary factors tnat control the susceptibility of a structure to brittle fracture are FRACTURE TOUGHNESS ( $K_{C}$ ) of the material. CRACK SIZE (a) and STRESS LEVEL ( $\sigma$ ). The general relationship among material tougnness ( $K_{c}$ ), nominal stress ( ${ }^{\text {b }}$ ), ana crack size (a) is shown scnematically in Figure 1 [3]. It snows tnat tnere are many combinations of stresses and crack sizes which may cause fracture in a fabricated structure having a particular value Of $K_{C}$ at a particular service temperature, loading rate ana

```
plate thickness. conversely, there are many combinations of
stresses (Fo) and flaw sizes ( }\mp@subsup{O}{0}{}\mathrm{ ) that will not cause failure
Of a particular structural material. To prevent fracture,
the actual stresses and flaw sizes must be below the levels
Shown in Figure 1.
    If the tougnness of the material is sufficiently high,
brittle fracture will not occur, and the failure under
tensile loading will precede a large plastic
deformation. For large structures where the elastic-plastic
behavior is observed witn tne formation of a large plastic
zone prior to the failure, the linear elastic analysls used
to calculate the stress intensity factors is not app||cable.
Under this condition, analysis otner tnan linear elastic
fracture mechanics (LEFM) must be used. J-INTEGRAL, f-CUFVE
analysis and CRACK OPENING DISPLACEMENT (COD) are tne
extensions of LEFM into elastic-plastic fracture mechanics.
```

1.3 The oplectives and gcope of the stugy

Recently, however, it has Deen shown c 41 tnat a single field characterization using the J-integral is not sufficient to model tne crack growtn In the elastic-plastic region. some new parameters sucn as $L$, $M$, and $N$ integrals C53 have been introduced. Their application in predicting crack growtn in structures seems to be promising. The numerical evaluation of tnese fracture mechanics parameters is complicated aue to tne nature of the equations describing these parameters. One of tho objoctives of this stuay is to
evaluate some of these parameters sucn as the $J$ - and $M$ Integrals.

This calculation requires an accurate computation of the stress and strain fields near the crack tip. In the present study the finite element method (FEM) is cnosen to do sucn an analysis on a cracked retangular plate (Figure 2 ).

Since non-linear finite element analysis (FEA) depends upon the type of mesn (and loading step selection and tolerance), another objective of this work is to see the effect of different grid sizes on the FEA results. To check results, comparisons of tne results of $F E A$ are made witn experimental data.

```
chapter : :
```

Basic Theories and Their use in EPFM


[^0]necessary to account for the work of the internal forces as well as that of the external forces. Then equilibrium [7.3.9] is identified by
$$
w_{1}+w_{e}=0
$$

If a solid is subjected to a set of body forces $f$ tnen by the Virtual Work Principle we can write

$$
\begin{equation*}
\int_{v}[\delta t]^{\top} \sigma d v-\int_{v}[\delta u]^{\top}+d v-\int_{T_{t}}[\delta u]^{\top} t d \eta^{r}=0 \tag{2.1}
\end{equation*}
$$

wnere $\sigma$ is a vector of stresses, $t$ is a vector of boundary tractions, $\delta u$ is a vector of virtual displacements, $\delta \in$ is a vector of associated virtual strains and $v$ is the volume. $d \Gamma$ is an element of arc length along $T, T_{t}$ is tnat part of the boundary on wnicn boundary tractions are prescribed and $T_{u}$ is tnat part of the boundary on wnicn displacements are prescribed [7].

### 2.2.2 Rlsplacement Eunctions

The first step in tne analysis is to select a set of displacement functions tnat give tne displacements of every point witnin the element. In a two dimensional analysis tne cnosen form of tne displacement functions Is a polynomial as follows:

$$
[w]=\left[\begin{array}{ll}
u & (x, y)  \tag{2.2}\\
v & \\
v & (x, y)
\end{array}\right]
$$

wnere

$$
\begin{aligned}
& u(x, y)=a_{0}+a_{1} x+a_{2} y+a_{3} x^{2}+a_{4} x y+a_{5} y^{2}+\ldots \\
& v(x, y)=b_{0}+b_{1} x+b_{2} y+b_{3} x^{2}+b_{4} x y+b_{5} y^{2}+\ldots
\end{aligned}
$$

or

$$
\left\{\begin{array}{c}
u(x, y) \\
v(x, y)
\end{array}\right\}=\left[\begin{array}{ccc}
{[F(x, y)]} & 0 \\
0 & {[F} & (x, y)]
\end{array}\right]\left\{\begin{array}{c}
\{a\} \\
\{b\}
\end{array}\right\}
$$

In wnich $u(x, y)$ and $v(x, y)$ are tne two components of displacements in the $x$ and $y$ directions respectively and F (x,y) is a polynomial for displacement function. For a rectangular element, tne nodal displacement vector can be expressed as an eignt-component vector involving the $u$ and $v$ displacements at nodes 1,2,3, and 4.

$$
\begin{equation*}
[a]=\left[u_{1} v_{1} u_{2} v_{2} u_{3} v_{3} u_{4} v_{4}\right]^{\top} \tag{2.4}
\end{equation*}
$$

The aisplacement function [w] can be expressed [7.9] in terms of the nodal displacements

```
[w] =[A][a]
```

The strains are obtained in terms of the nodal
displacements and nave tne form

$$
\begin{equation*}
[\in]=[B][w]=[B][A][d] \tag{2.5}
\end{equation*}
$$

The elastic relationship between stress and strain components for plane stress is

$$
\left[\begin{array}{l}
\sigma_{x} \\
\sigma_{y} \\
\sigma_{x y}
\end{array}\right]=\left(E /\left(1-v^{2}\right)\right)\left[\begin{array}{ccc}
1 & v & 0 \\
v & 1 & 0 \\
0 & 0 & (1-v) / 2
\end{array}\right]\left[\begin{array}{l}
E_{x} \\
E_{y} \\
E_{x y}
\end{array}\right]
$$

which can be written in tne matrix form as

$$
\begin{equation*}
F=[D][\in]=[D][B][d] \tag{2.6}
\end{equation*}
$$

2.3 Lsoparametric Representation

In a finite element representation, tne displacements
and strains $[7,8,9$,$] for a solid mechanics applications may$
be expressed by

wnere for noae $i, d_{j}$ is the vector of nodal variables, $N_{i}$ is tne global snape function and $\theta_{i}$ is the global strainaisplacement matrix.

If (2.7) and (2.8) are substituted in the virtual work expression (2.1) tnen the following is obtainea


The displacement can be expressed in the usual way as

wnere, for local node $I$ of element e. $N_{i}^{(e)}$ is the global shape function and the vector of variables is $a_{i}^{(0)}$. There are $r$ local noaes in each element $e$.

According to Figure 3, the shape functions [6] for a typical 8 node isoparametric element is as follows:
(a) Corner nodes

$$
\begin{align*}
N_{i}^{(e)} & \left.=(1 / 4)\left(1+\xi \xi_{i}\right)\left(1+\eta \eta_{i}\right), \xi \xi_{i}+\eta \eta_{i}-1\right) \\
i & =1,3.5,7
\end{align*}
$$

(b) Midside nodes

$$
N_{i}^{(e)}=\left(\xi_{i}^{2}-2\right)\left(1+\xi \xi_{i}\right)\left(1-\eta^{2}\right)+\left(\eta_{i}^{2} / 2\right)
$$

$$
\begin{align*}
& \left(1+\eta \eta_{1}\right)\left(1-3^{2}\right) \\
i & =3,4,6,8 \tag{2.12}
\end{align*}
$$

In an isoparametric representation the following formula is used for the $x, y$ cooruinates within an element:

$$
\left\{\begin{array}{c}
x^{(e)} \\
y^{(e)}
\end{array}\right\}=\sum_{i=1}^{r}\left[\begin{array}{cc}
N_{i}^{(e)} & 0 \\
0 & N_{i}^{(e)}
\end{array}\right]\left\{\begin{array}{c}
x_{i}^{(e)} \\
y_{i}^{(e)}
\end{array}\right\}
$$

The Jacobian matrix'[6] is evaluated as

$$
J(e)=\left[\begin{array}{ll}
\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi}  \tag{2.14}\\
\frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta}
\end{array}\right]
$$

or

The strain displacement relationships are expressed as

$$
\begin{equation*}
E^{(\theta)}=\sum_{i=1}^{r} B_{i}^{(\theta)} a_{i}^{(\theta)} \tag{2.15}
\end{equation*}
$$

where $\mathrm{B}_{\mathrm{l}}^{\left.()^{( }\right)}$is the strain matrix
Since a linear stress-strain relationship is obtained
within each element

$$
\begin{equation*}
\sigma^{(e)}=D^{(e)} t^{(e)}=D^{(e)} \sum_{j=1}^{r} B_{j}^{(e)} a_{j}^{(e)} \tag{2.16}
\end{equation*}
$$

Tnen the contribution c61 from element e to the first term in (2.9) is given as

wnere $k_{i j}^{(e)}$ is tne submatrix of tne stiffness matrix $k^{(e)}$
The contribution from element e to the second term in
(2.9) is given as

$$
f_{B i}^{(e)}=\int_{Q}\left[N^{(e)}\right]^{T} D^{(e)} d Q
$$

For the thira term, the contribution from element e is

$$
f_{T_{i}}^{(e)}=\int_{-\infty|e|}^{\left.\mathrm{rN}^{(e)}\right]^{T} t^{(e)} d T \mid}
$$

wnere $T_{t}^{(e)}$ is that part whicn coincides witn a boundary of element e. of course, for many elements tnere wili be no Contribution to $f_{T_{i}}^{(e)}[6]$.

This leads to an assembly procedure and
$[k](d)=\{f\}$

Solution for (d) can be obtained by
$\{a\}=[k\}^{-1}(f\}$
$\{5\}=[D][B]\{0\}$
2.4 The Mathematical Ineory of Plasticity

For any three-dimensional stress state [10] there
exists a cubic equation wnose tnree roots are tne tnree
principal stresses. This equation is

$$
5^{3}-1,6^{2}-1_{2} 5-13=0
$$

Regardless of the coordinate system chosen, these three roots of the equation wnicn art principal stresses remain the same for a given state of stress. In other words, the coefficients $I_{1}, I_{2}$ and $I_{3}$ must not vary with a change in the coordinate system. These coefficients are referred to as the invariants of the stress tensor and are denoted as follows:

$$
\begin{aligned}
& 1_{1}=\left(\sigma_{x}+\sigma_{y}+\sigma_{x}\right) \\
& 1_{2}=-\left(\sigma_{x} \sigma_{y}+\sigma_{y} \sigma_{z}+\sigma_{z} \sigma_{x}\right) \\
& 1_{3}=\left(\sigma_{x} \nabla_{y} \sigma_{z}\right)
\end{aligned}
$$

The magnitude of the mean normal stress, $\mathrm{Fin}_{\mathrm{m}}$, is equal to one third the algebriac sum of the three normal stresses;
therefore, $1_{1}=3 \sqrt{m}$. Thus the first invariant is a function of the hydrostatic or mean component and should not influence yielding. The two most common yield criteria employed in the description of the behavior of metals are the Tresca criterion and the Non Bises criterion.

### 2.4.1 The Yield Eriterle

The Non wises criterion $[6,10]$ predicts that yielding occurs when:

$$
\left(\sigma_{1}-\sigma_{2}\right)^{2}+\left(\sigma_{2}-\sigma_{3}\right)^{2}+\left(\sqrt{3}-\sqrt{1}_{1}\right)^{2}=\text { constant }
$$

$\sigma_{1}, \sigma_{2}$ and $\sigma_{3}$ are principal stresses. In a more general form.

$$
\left(\sigma_{x}-\sigma_{y}\right)^{2}+\left(\sigma_{y}-\sigma_{z}\right)^{2}+\left(\sigma_{z}-\sigma_{x}\right)^{2}
$$

$$
+\sigma\left(\tau_{x y}^{2}+\tau_{y z}^{2}+\tau_{z x}^{2}=\right.\text { constant }
$$

Let - (effective stress) be a function of the applied
stresses. Whenever its magnituae reacnes the yield stress $\mathrm{F}_{y}$ for the material in uniaxial tension, then that applied stress state should cause yielding to occur. Thus: Von Mises: $\quad-(1 / \sqrt{2})\left[\left(-\sigma_{2}\right)^{2}+\left(\nabla_{2}-\sigma_{3}\right)^{2}+\left(\nabla_{3}-\sigma_{1}^{2}\right)^{1 / 2}\right.$ According to the Von Mises criterion when $\bar{\sigma}$ reaches a value of 3 K where K is the maximum allowable shear stress, yielding is predicted

2.4.2 Plastic work<br>After initial yielding, the stress level at which further plastic deformation occurs may be dependent on the current degree of plastic straining. For an isotropic nardening model, tne original yield surface expands uniformly witnout translation. The total plastic work or strain hardening is as follows:<br>$$
k=w_{p}=\int \sigma_{i j}\left(a t_{i j}\right)_{p} \quad i, j=1,2,3
$$<br>$K$ is the nardening parameter and $\left(d \epsilon_{i j}\right)_{P}$ are the plastic components of strain occuring during a strain increment.

### 2.4.3 Eloy Rules or Plastic gtress/straln Relation <br> After initial yielding the material benavior will be partly elastic and partly plastic. The cnanges of strain are assumed to be divided into elastic and plastic components during any increment of stress, so that

$$
\begin{equation*}
a \epsilon_{i j}=\langle a \epsilon i j)_{e}+(a \epsilon i j)_{P} \tag{2.18}
\end{equation*}
$$

The elastic strain increment is related to tne stress
increment by tne incremental form of $\bar{T}=D \in$. The
relationship between tne plastic strain component and the stress increment can be expressed as [e]

$$
\begin{equation*}
a\left(\epsilon_{i j}\right) p=a \lambda\left(\partial Q / \partial \sigma_{i j}\right) \tag{2.19}
\end{equation*}
$$

For associative flow rule, assume $f=0$ where $f$ is tne yiela function, then

$$
\begin{equation*}
d(\epsilon i j) p=d \lambda\left(\partial f / \partial \sigma_{i j}\right) \tag{2.20}
\end{equation*}
$$

d is a proportionality constant termed the plastic multiplier, and $Q$ is the plastic potential. $\partial f / \partial \sigma_{i j}^{i s}$ a vector and it provides the direction cosines of tne normal to the yield surface at the stress point under consideration. Equation (2.19) becomes tne flow rule since it governs the plastic flow after yielding.

### 2.5 The d-intearal cencept

### 2.5.1 Definition of l -Inteargl

Rice's J-integral definition [11] is applicable to a homogeneous body of I inear or nonlinear elastic material free of body forces. It is subjected to a two-dimensional deformation field so that all stresses $\nabla_{i j}$ depend only on two cartesion coordinates $x=\left(x_{i}{ }^{3},(i=1,2)\right.$.

For a tree notcn having fiat surfaces parallel to the $x$-axis and a rounded tip $T_{t}(f i g u r e 4)$, the J-integral is defined as a path integral for an open path surrounding the noten $t i p ;$
$J=\int_{F_{i}}\left(\right.$ wax $\left._{2}-\mathbf{T}_{i} \frac{\partial \varphi_{i}}{\partial x_{i}} d s\right)$
The J-integral has tne same value when computed by an integration along either $\prod_{1}$ or $T_{2}$ provided that the identical
positive normal airection is usea for botn paths. The patn independence of $-J$ integral for Iinear and non-linear material nas been proved theoretically in [12].

### 2.5.2 d-Intearal Evaluation Metneds

The evaluation of the $J$-integral can be acnieved by selecting an integral patn and evaluating equation (2.21) by numerical integration. There are two choices to select the integral paths: a path passing through the nodal points or a patn passing through the integration points. The J-integral evaluation, using a path passing through tne nodal points of the FE grid requires an extrapolation of known quantities such as stresses and derivatives of the displacements from the integration points to the nodal points. The advantage of this method is tnat there are no requirements for tne mesn to define a smootn patn. The disadvantage is, of course, the extra work required for extrapolation procedure. The accuracy might be less than tnat of the second method in which the contour is aiong a line passing througn the integration points of the elements. However, the mesh for the second method nas to be able to define a smooth integration path. AII tne common element types can be used in EPFM, but tne most common types used have been the higher order elements. These elements are well Known because of their flexibility to model complex structures and the capability to provide good accuracy with a coarse mesn.

```
If a 3x3 integration scneme is used to evaluate tne
```

```
element stiffness, ttlree patns can be defined tnrougn one
element. It nas been observed tnat the patns close to tne
boundary of tne element furnisn almost identical J-integrals
While the one tnrougn the middle gives approximately a 10%
higner value. it nas been demonstrated by Bakker [13] tnat
tne average value of all tnree paths of integration should be
considered. This average is tnen tneoretically equal to tne
J-integral found by the virtual crack extention method [11].
```


## CHAPTER III

## FINITE ELEMENT (FE) ANALYSIS

## 3.1

## introguction

In this investigation, an odge crack specimen is analyzed uslng tne Nonlinear Finite Element Program (NFAP) developed at the University of Akron [14]. The edge notcn geometry considered in thls study is snown in Figure 2. The specimen contains a $60^{\circ}$ " $V$ " notch of 1 mm length. The almensions of the specimen are:
$H=40 \mathrm{~mm}$
$2 W=20 \mathrm{~mm}$
$T=1.33 \mathrm{~mm}$
In which $H$ is tne length, $2 W$ is the width and $T$ is the tnickness of the specimen [Figure己l. For symmetry reasons only one-nalf of the test plate was analyzed using conventional 8-node isoparametric elements.

### 3.2 Model and Propertios ef the Spocimen

Figure $5(a)$ shows tne stress-strain rolation for tho material of the specimen and Figure 5(b) shows the applied stress with time along the upper and lower edges of the specimen. As yield criterion, the Von Mises condition was used. In the plasticstate, the associatea flow rule of the Prandtl-Reuss equations was assumed to analyze this specimen
having Young's modulus of elasticity $E=81357 \mathrm{~N} / \mathrm{mm}^{2}$, Poisson's ratio of $V=0.30$ and yiela stress of $\sigma_{y}=834.26 \mathrm{~N} / \mathrm{mm}^{2}$.

To study the effect of coarse-mesh on tne elastoplastic analysis, four different finite element meshes were designed. Meshl, M1 (Figure 6) had as many distorted elements as Mesn2, ME (Figure 7) witn the exception that it had a layer of singular elements surrounding the crack tip Mesn3, M3 (Figure 8) had tne same rectangular layout as mesh4, M4 (Figure 9) with the only difference being that it had singular elements surrounding tne crack tip

The singular elements (Figure 10 ) were created by the-so-called "1/4 point technique" [15] in wnicn tne nodes of the crack tip are collapsed to form a triangular shape element. Tne midside nodes of the second order elements are placed at $1 / 4$ distance from the crack tip node, to obtain tne $1 / \sqrt{r}$ singularity.

### 3.3 Results and Cemparisen of the Meshes

It was observed tnat these results for crack opening displacements obtained from the meshes ald not differ significantly from eacn otner (Figure 11). The results of stresses obtained from MI and M3 were close to each other as were the results for M2 and M4 (Figure 12). It could be seen tnat tne results for tne grids witn singular elements predict higner stress near the crack tip. it shows the singular nature of stresses near the crack tip.

Figure 13 shows the plastic-zone growtn for different
mesnes employed in tnis investigation for $a / w=0.5$ and $\sigma / \sigma_{y}=$ 0.2. Plastic deformation occurs wnen the elastic stress in the vicinity of the crack tip is nigner tnan tne viela stress of the material. The effect of tne plastic zone is to increase the displacements and lower the stiffness of the plate. The length of the plastic-zone, $r_{y}$, under plane stress conditions can be estimated c33 by the following formula:

$$
r_{y}=(1 / 2 \pi)\left(K / \sigma_{y s}\right)
$$

on comparing tne plastic-zone sizes in tne neighbornood of tne crack in the test specimen, it was found tnat tne m3 produces result whicn is closer to tne estimated value accoraing to the Table 1.

Eight rectangular paths $[F i g u r e$ 14] passing tnrougn the Gauss integration points were used to calculate Rice's Jintegral (2.21). Tne a/w for all the finite element meshes was 0.5. Figure 15 snows tne comparison of J-values between M3 and M4 obtained from Table 2. For Ilnear-elastic benavior, tne J-integral whicn is tne energy release rate per unit crack extension, is identical to $\mathcal{G}$. Therefore, for linear-elastic plane-stress conditions the following formulas are employed to calculate tne stress intensity factor and the J-integral:
and

$$
\begin{aligned}
& K_{1}=\sigma \sqrt{\pi a} f(a / D) \\
& \sigma=J=(\alpha)^{2}\left(K_{1}^{2} / E\right)
\end{aligned}
$$

For a single eage notcn specimen c33f(a/b)=2.86 wnen a/w is equal to 0.5. It was found tnat the J-integrai value
obtained from M4 was closer to the theoretical value $[t a b l e$ 3]. As a result, the coarse-mesn procedure improves tne accuracy and also reduces the need for a large number of elements near tne crack tip. It was also found that the maximum relative deviation between tne $J$-values on tne patns obtained from M4 was $4.5 \%$ over the wnole range of loading. Thus, it can be stated that, in the case of monotonic loading, the J-integral (2.21) maintains its patn independent property within numerical accuracy (Figure 16) even if a flow tneory of plasticity is used.

Because tne J-integral value obtained from M4 is closer to the theoretical value (table 3), therefore, the finite element mesh4 was employed in tne study.
3.4 comparison of the EE Anslysis hood-Qisplacement with Experimental Result

A comparison is made between the load displacement obtained from finite element analysis and tne experimental data from tne Department of Macromolecular Science at Case Western Reserve University [18]. Tne oxperimental test was conducted with a compression molded sheet of Bisphenol Apolycarbonate with a MFI=2-3 suppiied by $D$ o enemical Company. Originally a specimen was milled at very low speed to the dimensions $120 \times 20 \times 0.33 \mathrm{~mm}$. A $60^{\circ}$ "v" notch of 1 mm depth was introduced at tne middle of one edge of the specimen. Fatigue tests are conducted in a laboratory atmosphere on an MTS-800 macnine using sinusoidal waveform at
a frequency of 0.5 Hz . The loading was tension-tension with a maximum stress of $33 \mathrm{~N} / \mathrm{mm}^{2}$ and a minimum to maxlmum load ratio $R=0.4$. Tne distance between the grips was 80 mm . The crack and the surrounding damage were followed visually using a traveling optical microscope attached to a video camera assembly, equipped with a visual display unit from which the entire history of the crack propagation was recorded.

A two-dimensional finite element program was employed to analyze the same experimental specimen using tne grid M4 (Figure 9) in which 383 nodes and 113 isoparametric elements were used. Figures $17(a)$ and $17(b)$ show the stress-strain relation and tne displacement controlled loading along the upper and lower edges of the specimen [Figure 2 ] with a length ( $H$ ) of 40 mm , wlath (2W) of 20 mm ana thickness ( $T$ ) of 0.33 mm . The specimen had an elastic modulus (E) of 2000 $\mathrm{N} / \mathrm{mm}^{2}$, a yield stress ( $\sigma_{y}$ ) of $70 \mathrm{~N} / \mathrm{mm}^{2}$ and Poisson's ratio (V) of 0.33.

Figure 18 snows a comparison between the experimental ly measured load-displacement and the loaddisplacement Obtained by finite element analysis loaddisplacement curve. The toad-displacement results for both tne experimental test and finite element analysis are close to each Other in the elastic range tapproximately $15 \%$ ). But tne deviation increases above the elastic range (approximately $20 \%$ ). However, the results of an elasticplastic FE analysis can differ considerably, depending on tne theoretical formulation used in the computer program and on
the structure. variables such as step size of tne loading,tolerance, and numerical schemes utilized in the equilibriumiterations also play an important role on tne accuracy of theresults obtained by $F E$ analysis.Figure 19 snows tne crack opening displacement atvarious stress levels for the specimen. Figures 20 ana 21snow the plastic zone growtns wnen $a / w=0.3$ and $a / w=0.7$.The $J$ values for different crack lengths at various stresslevels are snown in Figure 22 obtained from Tables 4,5,6,7,8and 9.

## ESTIMATION OF J-INTEGRAL BY

DIFFERENT TECHNIQUES
4. 1

## introduction

The application of tne FE metnod to practical problemsis usually time consuming and requires expensive computeranalysis wnicn can be pronibitive for tne engineer.Therefore, the use of simplified tecnniques for tnecalculation of the J-integral provides a useful alternative.These estimation techniques in combination with theexperimental load-displacement data are significantly easierand more economical to use tnan a conventional finite elementprocedure which models tne details of a geometry with cracklength a. These techniques which will be introduced in tniscnapter for determining tne J-integral are formulated for aciass of problems wnere tne plastic zone remains localizednear tne crack tip.4.2 Estimation Iecnnlaues to Eveluate j-Integral
The first approximate formula employed nere is based
upon the potential energy difference between two identical ly
loaded specimens having crack sizes that differ
infinitesimally. This formula is exact for linear loading
and rigid-plastic material response, and is shown to predict $J$ accurately in the elastic-plastic range of loading [17]. The basis of the second approximate J-integral formula is the extension of the elastic strain energy release rate to elastic-plastic loads. This is particularly suited for contained plasticity. When the global load-deflection curve exhibits non-linear benavior, this J-approximation formula deviates from the actual J-integral [17]

### 4.2.1 $\mathcal{L}$-Cglculated Erom Ine head=Rlsplacement curye Methed (Metnede).

In the elastic range, the J-integral is identical to tne elastic energy release rate $G$. Tnis is expresed by

$$
\begin{equation*}
J_{e}=G=K_{1}^{2} / E \tag{4.1}
\end{equation*}
$$

$K$ is the Mode I stress intensity and E is the Young's modulus. The J-integral formula for contained plasticity can De interpreted as the potential energy difference between two ldentically loaded specimens having different crack sizes [figure 23]. Tnat is,

$$
\begin{equation*}
J=-(1 / B) \frac{\partial U}{\partial a} \tag{4.2}
\end{equation*}
$$

U is the potential energy, $a$ is tne crack length, and 8 is tne specimen thickness.

Graphically, tne potential energy difference, du, for two specimens witn crack lengths a and $a+d a$ is equal to tne area between the load versus displacement curves as illustrated in Figure 23. This area equals BJda ssee equation (4.2). The following expression for $J$ is derived
[17] from equation (4.2).

P is any elastic load. In particular, $P$ can be cnosen to be tne limit load. The $J$ values are computed employing the equation (4.3) and the load-displacement curves for different crack sizes [Figure 23]. Interpolation by polynomials was performed to find tne polynomials of degree four for the load-displacement curves $f_{1}(\delta), f_{2}(\delta)$ and $f_{3}(\delta)$. The following polynomials are employed to calculate the areas Detween the load-displacement curves for different crack lengths,

$$
\begin{aligned}
& f_{1}(\delta)=-1.46 \delta^{4}+0.49 \delta^{3}-3.69 \delta^{2}+148.84 \delta \\
& f_{2}(\delta)=104.58 \delta^{4}-573 \delta^{3}+970.5 \delta^{2}-3735 \\
& f_{3}(\delta)=-7.73 \delta^{4}+35 \delta^{3}-56.72 \delta^{2}+185.79 \delta
\end{aligned}
$$

Table 10 snows tne $J$ values obtained by employing equation (4.3)for different crack lengths.

and $F$ is a function of displacement $\delta$ tnen force $f$ can be written as [17]:

$$
\begin{equation*}
f=H(a) F(5) \tag{4.7}
\end{equation*}
$$

It was found that tne maximum relative deviations between the $f_{1} / f_{2}$ and $f_{2} / f_{3}$ values [Figure 243 obtained from Tables 5.6. and 7 are $8 \%$ and $5 \%$ respectively over tne wnole range of loading. Thus the stiffness gradient metnod can be employed nere to estimate tne $J$ values.

Consider the force-displacement curves of a cracked specimen with crack lengths and $a+d a$, defined by $f_{1},{ }^{f}{ }_{2}$ and $f_{3}$. The function $f_{f}, f_{2}$ and $f_{3}$ can be subaivided into $n$ linear segments as snown in Figure 24. The J-integral is given by tne sum

where

$$
\alpha,=-\frac{1}{B m_{i}} \cdot \frac{d m ;}{d a}
$$

$W_{i}$ is the area under the load-displacement curve in the ith segment, $B$ is tne specimen thickness, $m_{i}$ defines the slope of tne itn linear segment of $f_{1}$ and $m_{i}^{\prime}$ defines tne slope of tne corresponding $f_{2}$ curve [17].

The areas under the load-displacement curves were
calculated using tne trapezoidal rule, that is

$$
w=\sum\left\{F_{i-1}+\frac{\Delta F_{i}}{2}\right\} \Delta u_{i}
$$

Table 10 snows the $J$ values obtained by employing equation (4.8) for different crack lengths


CHAPTER V

## NUMERICAL CALCULATION OF M-INTEGRAL

introguction
Tne path independent M-integral formulas are derived using tne isoparametric element. The M-values are numerically caculated by using the results from tne finite element analysis. A comparison is made between the numerically calculated M-values and the theoretical values obtained by using an estimation tecnnique.

### 5.2 Deflnition of M-Integral

The M-integral c41 represents tne potentlal energy release rates witn respect to isotropic expansion of the active zone [flgure 233. The M-integral [18] is defined as follows:

where $T_{1}$ is the traction vector defined accoraing to the outward normal $n$ along $\Gamma, T_{i}=\sigma_{i j} n_{j}$, and ds is an element of arc length along $\Gamma . x_{k}$ is a variable along tne patn on which tne integration takes place. Tne magnitude of $X_{k}$ depends on tne location of tne integration points along tne patn to tne origin where the crack tip is [Figure 25].

Knowles and E. Sternberg [18] applied Noether's tneorem to elastostatics and obtained the patn independency of tnc $M$ integral for a linear, nomogeneous and isotropic medium Shortly after tne $J . ~ K n o w l e s ~ a n d ~ E . ~ S t e r n b e r g ~ p u b l i c a t i o n . ~$ tne physical interpretation of $M$ integral was discussed by B. Budiansky and J. Rice [20].
5.3 Derlvation of 1 and M-lnteqrals Eormulas Bultabio for

## Numerical Evaluation.

The expression for $J$-integral can be written as:

$$
\begin{equation*}
J_{k}=\int_{T}+n_{k}-T_{i} u_{i, k}, \text { as } ; \quad k=1,2 \tag{5.2}
\end{equation*}
$$

$T_{i}$ is the traction vector defined according to the outward normal $n$ along $\Gamma, T_{i}=\sigma_{i j} n_{j}$, and ds is an element of arc length along $\Gamma$.

Figure 26 snows rectangular contours surrounding the crack tip of a crack in a two dimensional body. Derivation of formulas for tne $J$ - and $M$-integrals for tne patn $2-B$ is as follows:

$$
\begin{equation*}
J_{1}=\int_{\Gamma}\left(n_{1}-\sigma_{i j} n_{j} u_{i, 1}\right) d s \tag{5.3}
\end{equation*}
$$

Figure 26 snows that the integral along tne patn of the upper half-plane is equal to the sum of the individual line integrals. Therefore, due to symmetry

Herein, tne normals and tne arc lengtns, for tne patns of integration are

From A to 8

$$
\begin{array}{lll}
n_{1}=1 & n_{2}=0 & n_{3}=0 \\
a s=a x & a x_{1}=0 &
\end{array}
$$

From B to C

$$
\begin{array}{lll}
n_{1}=0 & n_{2}=1 & n_{3}=0 \\
d 3 & =-d x & d x_{2}=0
\end{array}
$$

from $C$ to $D$

$$
\begin{array}{lll}
n_{1}=-1 & n_{2}=0 & n_{3}=0 \\
a s=-a x & d x_{1}=0 &
\end{array}
$$

After expanding equation (5.4) and substituting the above values for the normal $n$, the following expression is obtained:

$$
\begin{align*}
J_{1}=2\left\{\begin{array}{l}
\int_{A}^{8}\left(1-\sigma_{11} t_{11}-\sigma_{21} u_{2,1}\right) d x_{2}+ \\
\\
\\
\left.\int_{B}^{B} C_{3} F_{12} t_{11}+\sigma_{22} u_{2,1}\right) d x_{1}
\end{array}+\right.
\end{align*}
$$

Using the same procedure, the following equation is
derived for $J$ when $k=2$ in equation (5.4). Therefore,

$$
\begin{align*}
& J_{2}=-2\left\{\int_{A}^{B} 5511 u_{1,2}+G_{21} t_{22}\right) d x_{2}+ \\
& a_{c_{3}}{ }^{r}-\sigma_{12} u_{1,2}-\sigma_{22} \epsilon_{22}, d x_{1} \\
& \left.\left.\int_{6}^{63} \sigma_{11} u_{1.2}+\sigma_{21} \not \epsilon_{22}\right) d x_{2}\right\} \tag{5.6}
\end{align*}
$$

Equation (5.1) can be expressed,

$$
\begin{align*}
& M=\int x_{1}\left(f n_{1}-E_{1 j} n_{j} u_{1,1}\right\rangle d s+ \\
&\left.\int x_{2}<f n_{2}-\sigma_{1 j} n_{j} u_{1,2}\right\rangle \text { as } \tag{5.7}
\end{align*}
$$

This equation for $M$ can be expressed and written as:

$$
M=M_{1}+M_{2}
$$

where

$$
\begin{aligned}
& M_{1}=2\left\{\int_{A}^{B} x_{1(A B)}^{B},-\sigma_{11} \epsilon_{11}-\sigma_{21} u_{2,1}, d x_{2}+\right. \\
& \left.\begin{array}{l}
\left.\left\{\begin{array}{l}
x_{1(B C)}\left(\sigma_{12} \epsilon_{11}+F_{22} u_{2,1}\right) d x_{1} \\
x_{1\left(C C_{3}\right)}
\end{array}\right)+\quad F_{11} \epsilon_{11}-F_{21} u_{2,1}\right) d x_{2}
\end{array}\right\}
\end{aligned}
$$

and

$$
m_{2}=2 \begin{cases}\int_{A}^{B} x_{2(A B)}, & \left(\sigma_{11} u_{1.2}+\sigma_{21} \epsilon_{22}\right) d x_{2}+ \\ \int_{B} x_{2(B C)}\left(f-\sigma_{12} u_{1,2}-\sigma_{22} t_{22}\right) d x_{1}+ \\ \left.\left.x_{2\left(C C_{3}\right)}\left(\sigma_{11} u_{1.2}+\sigma_{21} \epsilon_{22}\right) d x_{2}\right)\right\}(5.9)\end{cases}
$$

5.4 Derivation of Eormula for 1 and M-lntegrals using Lseparametric Elements

In this study, the following Rice formula is used to derive the $M$ formula by employing isoparametric elements

$$
\begin{equation*}
J=\int\left(u d y-t \frac{\partial d}{\partial x} d s\right) \tag{5.10}
\end{equation*}
$$

is tne strain energy density, tis the traction vector on a plane defined by tne outward drawn normal $n$, $d$ is tne displacement vector, and as is the element of arc along the path $\Gamma$. The path can be conveniently chosen to coincide witn the line $\xi=\}_{p}=$ constant.

The elemental arc length along the line $\xi=\xi_{p}$ is given oy

$$
\begin{equation*}
d s=\sqrt{d x^{2}+d y^{2}}=\sqrt{\left(\frac{\partial x}{\partial \eta}\right)^{2}+\left(\frac{\partial y}{\partial \eta}\right)^{2}} d \eta \tag{5.11}
\end{equation*}
$$

and similariy

$$
\begin{equation*}
d y=\frac{\partial y}{\partial \eta} d \eta \tag{5.12}
\end{equation*}
$$

Tne strain energy density for plane problem can we written

$$
\begin{equation*}
U=\frac{1}{2}\left(\sigma_{x x} \epsilon_{x x}+2 \sigma_{x y} \epsilon_{x y}+\sigma_{y y} \epsilon_{y y}\right) \tag{5.13}
\end{equation*}
$$

Also the traction vector is

$$
t=\left\{\begin{array}{l}
\sigma_{x x} \eta_{1}+\sigma_{x y} \eta_{z}  \tag{5.14}\\
\sigma_{x y} \eta_{1}+\sigma_{y y} \eta_{z}
\end{array}\right\}
$$

so tnat

$$
\begin{equation*}
t \frac{\partial d}{\partial x}=\left(\sigma_{x x} \eta_{1}+\sigma_{x y} \eta_{2}\right) \frac{\partial u}{\partial x}+\left(\sigma_{x y} \eta_{1}+\sigma_{y y} \eta_{2}\right) \frac{\partial v}{\partial x} \tag{5.15}
\end{equation*}
$$

The fol lowing expression c63 Is obtained by substituting equations (5.11), (5.121, (5.13), and (5.15) in equation

$$
\begin{gather*}
J^{(\text {(e) }}=\int_{-1}^{+1}\left\{\frac{1}{2}\left[\sigma_{x x} \frac{\partial u}{\partial x}+\sigma_{x y}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\sigma_{y y} \frac{\partial v}{\partial y}\right] \frac{\partial y}{\partial \eta}\right. \\
-\left[\left(\sigma_{x x} \eta_{1}+\sigma_{x y} \eta_{z}\right) \frac{\partial u}{\partial x}+\left(\sigma_{x y} \eta_{1}+\sigma_{y y} \eta_{z}\right) \frac{\partial u}{\partial x}\right] \\
 \tag{5.16}\\
\left.\sqrt{\left(\frac{\partial x}{\partial \eta}\right)^{2}+\left(\frac{\partial y}{\partial \eta}\right)^{2}}\right) d \eta=\int_{-1}^{+1} I d \eta
\end{gather*}
$$

Equation (5.16) gives the contribution to the J-integral from an individual element. The integrand $I$ in (5.16) is evaluated at tne Gaussian sampling points ( $\bar{\xi}_{F}, \eta_{F}$ ) for tne elements as follow:

$$
\begin{equation*}
J^{(e)}=\sum_{q=1}^{N G A U S} I\left(\xi_{p}, \eta_{q}\right) V_{q} \tag{5.17}
\end{equation*}
$$

$W_{q}$ is the weignting factor corresponding to $\eta_{q}$. The Cartesian derivatives of the displacement components required in (5.16) are given by

$$
\begin{align*}
& \frac{\partial(u, v)}{\partial x}=\sum_{i=1}^{n} \frac{\partial N_{i}^{(e)}}{\partial x}\left(u_{i}, v_{i}\right)  \tag{5.18}\\
& \frac{\partial(u, v)}{\partial y}=\sum_{i=1}^{n} \frac{\partial N_{i}^{(e)}}{\partial y}\left(u_{i}, v_{i}\right)
\end{align*}
$$

(5.19)
$u_{i}$ and $V_{i}$ are the displacement components of tne $n$ nodes of tne element. And the Cartesian derivatives of the element shape functions are given by

$$
\frac{\partial N_{i}^{(\rho)}}{\partial x}=\frac{\partial N_{i}^{(e)}}{\partial \xi} \cdot \frac{\partial \xi}{\partial x}+\frac{\partial N_{i}^{(e)}}{\partial \eta} \cdot \frac{\partial \eta}{\partial x}
$$

$$
\frac{\partial N_{i i}^{(e)}}{\partial y}=\frac{\partial N_{i i}^{(e)}}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}+\frac{\partial N_{i}^{(e)}}{\partial \xi} \cdot \frac{\partial \xi}{\partial y}
$$

The terms $\partial \xi / \partial x, \partial \eta / \partial x, \partial r / \partial y$ and $\partial \xi / \partial y$ may be obtainea from tne inverse of tne Jacobian matrix. The M-integral can be expressed as follow: t,

$$
\begin{equation*}
M^{(e)}=\int_{-1} x_{i} I d \eta, \quad i=1.2 \tag{5.22}
\end{equation*}
$$

In an isoparametric representation, we may use the equation (2.13) for $x i . \quad T n e r e f o r e$,

$r$ is tne number of nodes in eacn element e and $N_{i}$ is the matrix of shape functions calculated according to equations (2.11) and (2.12).
5.5 Results and pata Analyss

In tnis Investigation the $J$ - and M-integrals nave been calculated by employing the formulas derlved in section 5.3 To minimize the amount of calculation and therefore to maximize tne accuracy, patn 2-A and patn 2-8 (Figure 26) nave been cnosen. Figure 26 snows tne paths passing through tne integration points.

Appendix $A$ shows the stresses and strains at the integration points for patns 2-A and 2-8 (Figure 26) of tne plate. The stresses and strains are obtained from tne FE analysis in Chapter 111 wnen $a / w=0.4$ and $F=2.9 \mathrm{~N} / \mathrm{mm}^{2}$.

Location of tne integration points in tne global system are
computed (Figure 27) by employing tne equations (2.11), (2.12), and (2.73). Equation (2.2) is used to calculate $u_{i, 1}$ and $u_{i, 2}$ for eacn integration point along tne paths (Figure 26).

The calculated results in appendix $A$ are usea to obtain tne $J(K=1,2)$ and $M$ values for botn patns. The deviations between the $J$ values obtained from both patns and tne numerical calculation are witnin $10 \%$ error [Table 11$].$

However, patn $2-A$ gives a value closer to tne numerically obtained FE value ( $2 \%$ error). And the m values calculated from patns 2-A and 2-B are witnin $7 \%$ aifference. These values of $M$ are compared with tne values obtained by the formula given below [21]

$$
\begin{equation*}
M=\alpha J[(1 / 2)(l+w)] \tag{5.24}
\end{equation*}
$$

in wnicn $\ell$ and ware tne length and width of tne plastic zone respectively and $\alpha$ is a coefficient wnicn is approximatea as equal to $\mathrm{M} / \mathrm{J}$. This ratio is always less tnan one. Table 12 snows tne comparison of $M$ values between theoretical ana numerical calculation. As it can be seen, the magnitiude of $\alpha$ is less than one and tne values of $M$ differ siightly.

## conclusions

The J-integral values were determined by changing the crack length in the finite element grid for the edge notch specimen. A review of analysis of these various finite element (FE) modelings suggests that accurate results are obtained with collapsed isoparametric elements containing quarter nodes near the crack tip. Furthermore, the collapsed elements reduce the need for a large number of elements in the region near the crack tip.
comparison of the J-integral values obtained, through the integration of eight different patns, shows that the $J$ integral is path independent (Figure 16) even if a flow theory of plasticity is used (2.21).

Comparison was made between the experimentally measured load-displacement and the load-displacement obtained from the FE analysis(Figure 18). The results were close to each other in the elastic range but the deviation increased above tne elastic range. In addition, a comparison was made between the $J$ values from tne elastic-plastic FE method and tne approximate methods (Table 10). The numerically determined J value was within $5 \%$ with those Obtained from estimation techniques. The nigher deviation found may be due to a smaller crack length $a / w=0.2$, which is close to the edge notch. The estimation load-displacement curve provides better result than the stiffness gradient method.

The numerically determined $M$-values for two different

```
paths were computed manually and they were compared with tne
estimated value [TaDIE 123. It was found that the
coefficient }\alpha\mathrm{ is smaller than one for both paths and the M-
values have an error of 8. 2%.
```



Figure 1. Schematic relation between stress, flaw size and material tougnness


Figure 2. Eage notcn crack specimen

| Local node <br> number | $\xi_{i}$ | $\eta_{i}$ |
| :---: | :---: | :---: |
| 1 | -1 | -1 |
| 2 | 0 | -1 |
| 3 | 1 | -1 |
| 4 | 1 | 0 |
| 5 | 1 | 1 |
| 6 | 0 | 1 |
| 7 | -1 | 1 |
| 8 | -1 | 0 |

Global element
Figure 3. The node numbering and local coordinates of the nodes for an eight nodes isoparametric element


Figure 4. Contours surrounding the notch in a two dimensional body

a). Idealized stress-strain relation

Figure 5. Stress-strain relation and load curve for tne steel spec imen


Figure 6. Finite element gria Meshi, M1


Figure 7. Finite element gria Mesne


Figure 8. Finite element grid Mesh3. M3


Figure 9. Finite element gria Mesn4, M4


Figure 10. Crack modeling witn quadratic isoparametric element





Figure 14. Integration paths for J-integral evaluation $a / w=0.5$


Figure 15. A comparison Of J-values between $F E$ mesnes M3, M4


a). Idealized Stress-strain relation

b). Load curve for controlled displacement

Figure 17. Stress-strain relation and load curve for tne polycarbonate specimen


Figure 18. comparison between the experimental ly measured load-displacement (---) and FE analysis (——)


Figure 19. Crack opening oisplacements for various stress levels


Figure 20. Plastic zone growth for $a / w=0.3$


Figure 21. Plastic zone growth for $a / w=0.7$


Figure 22. J-values for different crack lengths at various stress levels


Figure 23. Load-displacement curves to determine $J$ value from the load-displacement curve method


Figure 24. Load-displacement curves to determine $J$ value from tne generalized stiffness gradient metnod


Figure 25. Active ana inert zone within a plate


Figure 26. Selected path to determine J-and M-integrals


$$
\eta=\xi= \pm 1 / \sqrt{3}
$$

$$
(a)
$$



Figure 27. Location of the integration points in local (a) ana global (b) systems

$$
u=a_{0}+a_{1} x+a_{2} y+a_{3} x y
$$

## Gauss point



Figure 28. An element representation to determine the displacement within the element

Table 1. Compar̄ison of tne plastic zone values from different mesnes witn tneoretical value, $a / w=0.5, \sigma / \sigma_{y}=0.2$


Table 2. The average J-values obtained from fe grias M3 and M4 at different load levels, $a / w=0.5$

| $\sigma / \sigma_{y}$ | J-values for $a / w=0.5$ |  |
| :---: | :---: | :---: |
|  | $\mathrm{N} . \mathrm{mm} / \mathrm{mm}^{2}$ | M4 <br> N.mm/mm ${ }^{2}$ |
| 0.01 | 0. $1850750+00$ | $0.2129200+00$ |
| 0.04 | $0.3121200+01$ | $0.3451480+01$ |
| 0.07 | $0.9558680+01$ | 0.10e8850 +02 |
| 0.10 | 0.1954400 + 02 | $0.2209490+02$ |
| 0.13 | $0.3043030+02$ | $0.4305050+02$ |
| 0.17 | $0.6232680+02$ | 0.740804D+02 |
| 0.20 | $0.1017680+03$ | 0.127919D+03 |
| 0.24 | 0.1225200+03 | 0.1593510+03 |
| 0.23 | $0.1484220+03$ | 0.2017190+03 |
| 0.24 | $0.1819930+03$ | $0.2682650+03$ |
| 0.25 | 0.227234D+03 | - |

Table 3. Comparison of the J-values obtained from Fe analysis and theoretical value

| J-values for $\mathrm{a} / \mathrm{w}=0.5 \cdot \sigma / \nabla_{y}=0.01$ |  |  |
| :---: | :---: | :---: |
| Theoretical$\mathrm{N} \cdot \mathrm{~mm} / \mathrm{mm}^{2}$ | From Fe analysis |  |
|  | M3 | M4 |
| 0.2197 | 0.1951 | 0.2129 |

Table 4. Elastic-plastic FE results for grid M4, a/w = 0. 2

| $\sigma / V_{y}$ | P.N | S.mm | J, N. mm/mm ${ }^{2}$ | $\sigma, N / m^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.048 | 22. 11 | 0.14 | 0.99400-01 | 3.35 |
| 0.259 | 119.72 | 0.76 | 0.3023D-01 | 18.14 |
| 0.467 | 215.75 | 1.38 | 0.10800-02 | 32.69 |
| 0.661 | 305.45 | 2.00 | 0.27170-02 | 46.28 |
| 0.757 | 349.80 | 2.40 | 0.4774D-02 | 53.00 |
| 0.798 | 368.54 | 2.80 | 0.75670-02 | 55.84 |

Table 5. Elastic-plastic FE results for grid M4, a/w = 0. 3

| $0 / 6 y$ | P, N | $\Sigma, \mathrm{mm}$ | $\mathrm{J}, \mathrm{N} . \mathrm{mm} / \mathrm{mm}^{2}$ | $\sigma \cdot N / \mathrm{mm}^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.045 | 20.79 | 0.14 | 0.16490-00 | 3.15 |
| 0.241 | 111.47 | 0.76 | $0.50870+01$ | 16.89 |
| 0.430 | 198.73 | 1.38 | $0.17930+02$ | 30.11 |
| 0.587 | 271.06 | 2.00 | $0.43810+02$ | 41.07 |
| 0.656 | 302.87 | 2.40 | $0.68900+02$ | 45.89 |
| 0.686 | 316.87 | 2.80 | $0.97160+02$ | 48.01 |

Table 6. Elastic-plastic Fe results for grid m4, $a / w=0.4$

| $\sigma / \sigma_{y}$ | P,N | $\delta, \mathrm{mm}$ | $\mathrm{J}, \mathrm{N} . \mathrm{mm} / \mathrm{mm}^{2}$ | T, $\mathrm{N} / \mathrm{mm}^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.041 | 18.94 | 0.14 | $0.2344 \mathrm{C}+01$ | 2.87 |
| 0.221 | 101.97 | 0.76 | 0.7035D+01 | 15.45 |
| 0.386 | 178.20 | 1.38 | 0. $24510+02$ | 27.00 |
| 0.514 | 237.27 | 2.00 | 0.5522D+02 | 35.95 |
| 0.569 | 283.01 | 2.40 | $0.80930+02$ | 39.85 |
| 0.590 | 276.73 | 2.80 | 0. $10890+03$ | 41.93 |
| 0.610 | 281.56 | 3.20 | 0. $13710+03$ | 42.66 |

Table 7. Elastic-plastic FE results for grid M4, $a / w=0.5$

| $\nabla / \sigma_{y}$ | P.N | $8 . \mathrm{mm}$ | S.N. mm/mm ${ }^{2}$ | $\sigma \cdot \mathrm{N} / \mathrm{mm}^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.027 | 12.24 | 0.14 | $0.26950+00$ | 1.86 |
| 0.187 | 86.20 | 0.70 | $0.8044 D+01$ | 13.06 |
| 0.322 | 148.76 | 1. 38 | $0.2692 D+02$ | 22.54 |
| 0.418 | 193.02 | 2.00 | $0.5647 \mathrm{D}+02$ | 29.25 |
| 0.463 | 213.79 | 2.40 | $0.79590+02$ | 32.40 |
| 0.496 | 228.91 | 2.80 | $0.10400+03$ | 34.68 |

Table 8. Elastic-plastic FE results for grid M4, $a / w=0.6$ -

| $\sigma / \sigma_{y}$ | P, N | $\delta . \mathrm{mm}$ | J,N. $\mathrm{mm} / \mathrm{mm}^{2}$ | $\mathrm{G}, \mathrm{N} / \mathrm{mm}^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.029 | 13.63 | 0.14 | $0.2833 \mathrm{D}+00$ | 2.07 |
| 0.156 | 72.27 | 0.78 | $0.83200+04$ | 10.95 |
| 0.265 | 122.33 | 1.38 | 0.27150+02 | 18.54 |
| 0.343 | 158.37 | 2.00 | $0.54790+02$ | 23.99 |
| 0.378 | 174.70 | 2.40 | $0.76000+02$ | 26.47 |
| 0.399 | 184.40 | 2.80 | $0.98970+03$ | 27.94 |
| 0.406 | 187.40 | 3.20 | 0.12210+03 | 28.40 |

Table 9. Elastic-plastic $F E$ results for grid M4, a/w $=0.7$

| $\sigma / V_{y}$ | P,N | $\delta . \mathrm{mm}$ | J.N. $\mathrm{mm} / \mathrm{mm}^{2}$ | T. $\mathrm{N} / \mathrm{mm}^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.024 | 10.86 | 0. 14 | $0.27130+00$ | 1.65 |
| 0.130 | 60.06 | 0.76 | $0.78840+01$ | 9.10 |
| 0.210 | 97.02 | 1.38 | $0.25390+02$ | 14.70 |
| 0.272 | 125.73 | 2.00 | $0.50800+02$ | 19.05 |
| 0.297 | 137.09 | 2.40 | $0.7091 \mathrm{D}+02$ | 20.77 |
| 0.306 | 141.50 | 2.80 | $0.93770+03$ | 21.44 |
| 0.309 | 142.96 | 3.20 | $0.11820+03$ | 21.66 |

Table 10. comparison between tne elastic-plastic FE analysis and the estimate tecnniques

| Metnods | Crack depth | Displacement ${ }_{\text {d }}$ | Failure load | J-value N.mm/mm | Error |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Method 1 | 4 | 2.8 | 368.54 | 75.67 | - |
| (2.21) | 6 | 2. 8 | 316.87 | 97. 16 | - |
| Met nodz | 4 | 2.8 | 368.54 | 70.90 | $\epsilon$ |
| (4.3) | 6 | 2.8 | 316.87 | 97.42 | 0.278 |
| Met nods | 4 | 2.8 | 368.54 | 94.00 | 24.40 |
| (4.8) | 6 | 2. 8 | 316.87 | 102.39 | 5.38 |

Table 11. Comparison of tne J-value between tne elasticplastic $F E$ analysis and numerical calculation

| FE | Numerical calculation |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | From path (2-A) |  |  | From patn (2-日) |  |  |
|  | J 1 | J2 | M | J 1 | J2 | M |
| 0.220112 | 0.223976 | 0.035303 | 0.078828 | 0.243221 | 0.066592 | 0.07308 |

Table 12. comparison of M-values between the theoretical value and numerical calculation

| $\begin{aligned} & \text { Theoretical } \\ & \text { value (5.24) } \end{aligned}$ | Numerical calculation |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\alpha$ | M from patn (2-A) | M from patn (2-B) | Error <br> $\%$ |
| 0.0770372 | 0.35 | 0.078828 | - | 2.3 |
| 0.0790422 | 0.30 | - | 0.0730483 | 8. 2 |

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## APPENDIX A

# Results for Calculation of $J$ - and M-integrals for patns (2-A) and (2-8) 

Stresses from_FE analysis (Path 2-A), a/w $=8 / 20,6=2.9$

| I.P | Vyy | $\sqrt{2 z}$ | $\sqrt{y z}$ | $\epsilon_{Y Y}$ | $\epsilon_{z z}$ | Eyz |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | -0.3 | 0.0 | -0. 1 | -0.1600-03 | 0.729D-04 | -0.1480-03 |
| $A_{1}$ | 1.0 | 0.6 | 0.9 | 0.3980-03 | 0.1320-04 | 0. 1260-03 |
| B | 0.8 | 0.8 | 0.8 | 0.2720-03 | 0.2510-03 | 0.1110-02 |
| ${ }^{8} 1$ | 1.2 | 2.7 | 2.4 | 0. 145D-03 | 0.1140-03 | 0.3140-02 |
| ${ }_{2}$ | 0.7 | 5.6 | 1.4 | -0. $5510-03$ | 0.2660-02 | 0.1920-02 |
| ${ }^{B} 3$ | 0.4 | 0.5 | 1.0 | -0.8800-03 | 0.3180-02 | 0. 1380-02 |
| $\mathrm{B}_{4}$ | 0.9 | 6.8 | -0.3 | -0.6560-03 | 0.3230-02 | -0.4050-03 |
| c | 0.6 | 5.9 | $-0.7$ | -0.6750-03 | 0.2860-02 | -0.8720-03 |
| $c_{1}$ | 1.7 | 5.9 | $-1.3$ | -0.1110-03 | 0.2690-02 | -0.1700-02 |
| $c_{2}$ | 3.7 | 6.0 | -0.5 | $0.8300-03$ | 0.2420-02 | -0.7150-03 |

stresses from-FE analysis (Path 2-81, $a / w=8 / 20, \sigma=2.9$

| I.P | $E_{y y}$ | $\mathrm{F}_{2}$ | $\sqrt{y z}$ | $\epsilon_{y y}$ | $t_{z z}$ | $t y z$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $-0.5$ | $-0.2$ | $-0.1$ | -0.2280-03 | -0.123D-04 | -0.1200-03 |
| $\mathrm{D}_{1}$ | 0.5 | 0.1 | 0.2 | 0.2480-03 | -0.5790-04 | 0.2590-03 |
| $\mathrm{D}_{2}$ | 0.5 | 1.5 | 0.5 | $0.1580-03$ | 0.1620-03 | $0.6690-03$ |
| E | 0.6 | 1.0 | 0.7 | 0.165D-03 | $0.3730-03$ | 0.9250-03 |
| $E_{1}$ | 0.8 | 3.7 | 1.3 | 0.100D-03 | $0.7250-03$ | 0.178D-02 |
| $E_{2}$ | 0.4 | 4. 1 | 1.7 | -0.294D-03 | 0.1480-02 | 0.2270-02 |
| $E_{3}$ | 0.0 | 5.7 | 1.4 | -0.7700-03 | 0.237D-02 | 0.1820-02 |
| $E_{4}$ | 0.1 | 5.4 | 0.8 | -0.9550-03 | $0.2730-02$ | 0.1010-02 |
| $E_{5}$ | 0.1 | 5.9 | -0. 1 | -0.1010-02 | $0.2960-02$ | -0.9620-04 |
| $E_{6}$ | 0.1 | 5.5 | -0.5 | -0.8420-03 | $0.2720-02$ | -0.717D-03 |
| $F$ | 0.3 | 5.0 | -0.8 | -0.6680-03 | $0.2440-02$ | -0.1100-02 |
| $F_{1}$ | 0.9 | 5. 1 | - 1.0 | -0.3790-03 | 0.2400-02 | -0.131D-02 |
| $\mathrm{F}_{2}$ | 1.7 | 4.9 | -1.0 | 0.4530-04 | $0.2180-02$ | -0.1280-02 |
| $\mathrm{F}_{3}$ | 2.5 | 4.7 | -0.4 | 0.4830-03 | $0.1930-02$ | -0.4770-03 |

J1 calculation (path 2-A), a/w=8/20, b $\quad=2.9$

| Path | $F$ | $u_{2,1}$ | $\nabla_{22} U_{2,1}$ | $\left[_{2}, u_{2}, 1\right.$ |
| :---: | :---: | :---: | :---: | :---: |
| $A-A_{1}$ | 0.0390-03 | 4.8800-03 | $0.0000-00$ | -0.4900-03 |
| $A_{1}-A_{2}$ | 1.3690-03 | 4.740D-03 | 2.844D-03 | 4.2700-03 |
| $A_{2}{ }^{-8}$ | 1.1000-03 | 4.507D-03 | 3.6900-03 | 3.6900-03 |
| $\mathrm{B}-\mathrm{B}_{1}$ | - | 6.035D-03 | 1.6300-02 | 1.4480-02 |
| $\mathrm{B}_{1}-\mathrm{B}_{2}$ | - | 6.034D-03 | 3.3790-02 | 8.4500-03 |
| $\mathrm{B}_{2} \mathrm{~B}_{3}$ | - | 3. $1410-03$ | 2.042D-02 | 3.1400-03 |
| $\mathrm{B}_{3}-\mathrm{B}_{4}$ | - | 3.1410-03 | 2.135D-02 | -0.940D-03 |
| $\mathrm{B}_{4}-\mathrm{C}$ | - | 0.993D-03 | 5.860D-03 | -0.695D-03 |
| $C-C_{1}$ | 1.0050-02 | 0.727D-03 | 4.2900-03 | -0.9500-03 |
| $c_{1}-c_{2}$ | 8.964D-03 | 0.3600-03 | 2. 16OD-03 | -0.180D-03 |

J2 calculation (path 2-A), $a / w=8 / 20, \quad \zeta=2.9$

| Path | $F$ | $u_{2,1}$ | $V_{22} u_{2,1}$ | $1 u_{2,1}$ |
| :---: | :---: | :---: | :---: | :---: |
| $A-A_{1}$ | - | 0.5740-03 | -0.172D-03 | 0.5740-03 |
| $A_{1}-\mathrm{A} 2$ | - | 0.5400-03 | 0.5400-03 | 0.5400-03 |
| $A_{2}{ }^{-B}$ | - | 0.1000-03 | 0.0800-03 | 0.1000-03 |
| $\mathrm{B}_{-\mathrm{B}_{1}}$ | - 167D-03 | 0.1000-03 | 0.1200-03 | 0.100D-03 |
| $\mathrm{B}_{1}-\mathrm{B}_{2}$ | 9.9420-03 | 0.1000-03 | $0.0700-03$ | 0.1000-03 |
| $\mathrm{B}_{2}-\mathrm{B}_{3}$ | 1. 154D-02 | 1.2860-03 | 0.514D-03 | 1.2860-03 |
| $B_{3}-8_{4}$ | $1.0810-02$ | 1.2850-03 | 1. 1570-03 | 1.2850-03 |
| $\mathrm{B}_{4}-\mathrm{C}$ | 8.9430-03 | 0.0750-03 | $0.0450-03$ | 0.0750-03 |
| $\mathrm{c}-\mathrm{C}_{1}$ | - | 0.1840-03 | $0.3130-03$ | 0.184D-03 |
| $c_{1}-c_{2}$ | - | 0.5080-03 | $1.8800-03$ | 0.508D-03 |

J1 calculation (path 2-B), $a / w=8 / 20, \quad b=2.9$

| Path | $F$ | $u_{2,1}$ | $\Gamma_{22} U_{2,1}$ | 21142,1 |
| :---: | :---: | :---: | :---: | :---: |
| $D-D_{1}$ | 0.0820-03 | 0.4900-02 | -0.098D-02 | -0.0490-02 |
| $D_{1}-D_{2}$ | 0.1090-03 | $0.4700-02$ | $0.0470-02$ | 0.0940-02 |
| $D_{2}-E$ | 0.1600-03 | $0.4810-02$ | . 0.2300-02 | 0.2310-02 |
| $E-E_{1}$ | 0.3600-03 | 0.4390-02 | $0.4390-02$ | 0.3070-02 |
| $1^{-E_{2}}$ | - | 0.439D-02 | $0.7460-02$ | $0.5710-02$ |
| $E_{2}-E_{3}$ | - | $0.483 D-02$ | 1.497D-02 | $0.8210-02$ |
| $E_{3}-E_{4}$ | - | $0.4830-02$ | 2.2700-02 | $0.6760-02$ |
| $E_{4}-E_{5}$ | - | 0.312D-02 | 1.6800-02 | 0.2500-02 |
| $E_{5}-E_{6}$ | - | $0.3000-02$ | 1.7700-02 | -0.0300-02 |
| $E_{6}-E$ | - | $0.1400-02$ | $0.7700-02$ | -0.0700-02 |
| $E_{7}-F$ | - | $0.1400-02$ | $0.7000-02$ | -0.112D-02 |
| F-F | 6.3300-03 | 0.100D-02 | $0.5100-02$ | -0.100D-02 |
| $F_{1}-F_{2}$ | 5.3000-03 | 0.0680-02 | $0.3300-02$ | -0.0080-02 |
| $F_{2}{ }^{-F_{3}}$ | 4.8800-03 | $0.019 \mathrm{D}-02$ | 0.0900-02 | -0.0080-02 |

$$
(A-5)
$$

J2 calculation (path 2-B), $a / w=8 / 20, \quad \overline{8}=2.9$

| Poth | $F$ | $u_{2,1}$ | $V_{22} U_{1}, 1$ | $1_{2}, U_{2,1}$ |
| :---: | :---: | :---: | :---: | :---: |
| $D-D_{1}$ | - | $0.5730-03$ | -0.2870-03 | -0.0570-03 |
| $D_{1}-D_{2}$ | - | $0.0540-03$ | 0.027D-03 | $0.0110-03$ |
| $D_{2}-D_{3}$ | - | 0.104D-03 | 0.0520-03 | 0.0520-03 |
| $D_{3}-E$ | - | 0.2100-03 | 0.126D-03 | 0.147D-03 |
| ${\mathrm{E}-\mathrm{E}_{1}}^{1}$ | 2.9700-03 | 0.0220-03 | $0.018 \mathrm{D}-03$ | $0.0290-03$ |
| $E_{1}-E_{2}$ | 6.0900-03 | 0.4100-03 | 1.162D-03 | 0.6970-03 |
| $E_{2}-E_{3}$ | 8.1200-03 | 0.4300-03 | $0.0000-00$ | 0.6020-03 |
| $E_{3}-E_{4}$ | 8.2300-03 | 2.2330-03 | -0.223D-03 | 1.7900-03 |
| $E_{4}-E_{5}$ | 8.7900-03 | 3.1320-03 | -0.3130-03 | -0.3130-03 |
| $E_{5}-E_{6}$ | 7.4700-03 | 1.4790-03 | 0.1480-03 | -0.7400-03 |
| $E_{6}-F$ | 6.8800-03 | 1.4800-03 | 0.444D-03 | -1.1800-03 |
| $\mathrm{F}_{-} \mathrm{F}_{1}$ | - | 0.0790-03 | 0.0710-03 | -0.079D-03 |
| $F_{1}-F_{2}$ | - | 0.1820-03 | 0.3100-03 | -0.1830-03 |
| $F_{2}{ }^{-F_{3}}$ | - | 0.0910-03 | 0.2300-03 | -0.0360-03 |

## APPENDIX B

```
Input and output data for computer program (NFAP)
```

8.1 Sample input data for tensile specimen of constantly applied stress along tne upper and lower eages.
B. 2 Sample output data for tensile specimen of constantly appl led stress along tne upper and lower edges.
8.3 Sample input data for tensile specimen of constantly applied displacement along the upper and lower eages

INPUTCABDIMAGES










$\begin{array}{r}\text { CARD } \\ \mathrm{NO} \\ \hline\end{array}$

.50........ . 60


.70.
70.
......... . .90
Non

(B. 1)

(B.1)

```
edge notch specimen mesh ts
CONTMOL I NFORMATION
    HUMaER OF yOOAL POINTS . . . . . . . . . (NUMNP) = 383
    gaseer x-translazton Code . . . . . . . . (Idof(1)) = 1
    master y-trayslation code . . . . . . . (foor(2)) = 0
    gastek z-tanislation COd& . . . . . . . (Idof(3)) = 0
    majtér X-ROTATION CODE . . . . . . . (IDOF(4)) = 1
    master y-hotation CODE . . . . . . . (IDOF(5)) = 1
    mater z-zotatIon CODE . . . . . . . (IDOF(o)) = 1
    number of limeaz element groups . . . . . . (negl) = 0
    nutaez of nonliyear element groups . . . . (negal) = 1
    SULUTION MOJE . . . . . . . . . . . . (MODEX) = 1
        E?:?: DATAGCHECK
        Ej:2; RESTART
    TOTAL tImE STEP InCaRMEyT . . . . . . (NSTE) = 2
    painting intenval.............. (IPRI) - - . - 1
    restakt Save inteñal............ (IGint) = 9999
    SPECIFIED BLOこK LENGTH . . . . . . . . . .(ISTOTE) = 0
```




```
        Ev:2: REAJ NODAL TEMPERATURES FROM CARDS
    TEMPEASTURE histofy INDICATOR . . . . . . (nTEMP) = 0
        E:OO, UNIFORM TEMPERATUAE
        EG:1, STEEADY STATE TEMPERATURE (EMPERATURE BEOYES STEAJY STATE AFTER STEP K
```



```
    NUMBER OF ALLOUABLE STIFFNESS REFORNATIONS
    IM EACH TIME STEP . . . . . . . . . . . . (NUMREF) = 0
    NUMBER OF TIEE STEPS RETMEE:
    EQUILIBFIURITEFATIONS . . . . . . . . . (IEQUIT) ■ 1
    HAXIMUM NUMEEZOF ERUTLIBRIUM
    ACCELEFATION CODE • . . . . . . . . . . . . (IACC) - - 1
    EYO, NO ACCELERATION PERFORMED
    CONVEFGENCE TOLERANCE. . . . . . . . . . . (ETOL) = 0.10D-02
A H A L Y S I S T Y P E
    TIMEDEPENCESCY CODE * . . . . . . . . . . (IJTAT) = 0
        EGOO, STAEICANALYSIS
    NONLINEARITY CODEE . . . . . . . . . . . . (ÖLIN) - - 1
        EQ.0. LINEAR ANALYSIS
        EQ.i: MOMLINEARAMMALYSIS
DIS P/VEL/| C C P R I N T O U T C O D D E
    HUMEER OF BLOCKS OF NODAL PRINTOUT . . . . . . (NPZ) m 1
    DIS?LACEMENT PRINTOUT CODZ * * * (IDC) = 1
        EQ.0. NO PRIHTIHG OF DISPLACEMENTS
        E\.1: PRINT DISPLACEMENTS
```



```
    EQ.O, NO PEINTING OF YELOCİIIES
```



```
        EQ.0, NO PRINTINGOF ACCELERATIONS
        Ev.1; PRIGT ACCELERATIONS
    SLOCK 1
        FIRST NODE OF THIS BLOCK . . . . (IPNODE(1,1) ■ 1
        LAST NGDE OF THIS BLOCK . . . . . (IPNODE(2,1) = 383
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AUTOMATILC LOADINAG
( FOR ELASTOPLASTIC ANALYSIS )
    AUTOMETIC SET THE INCREMENTAL LOAD STEP ..(IAUTU) = 0
    EQQ1 NOTTIYCTED
    AUTOYETIC LOAD ADJUSTAENT . . . . . . . . (IADJ) = 0
    EQ.O NOTACTIVATED
    LOAD ADJUSTMENT SEGMENTATION . . . . . . . (YSES) = 0
```



```
LO A D C O N T R O L D A T A
    NUNSEA OF NODAL LOADS =
    NU^3ER OF LOAD CURVES =
    MAY NO. OF POINTS IN LOAD CURIES = 10
    GaAVITY LOADIMG CODE = 0
            E.0 NO GRANITY LOAOING
            &`.2 CONSISTENT GEAVITY LOADING
    MAX PEESSUFE LOADING SUBGROUP = 0
    NO Of FAESSURE LOAD CURYE = 0
    &O. Of PreSSURE TABLE = 0
    PRESSUFE-GEOMETAT-DEPENDENT FLAG = 0
        E\.0 HO
    :O. OF UNLOAJING STEP = 0
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    \JMडER OF DISP. JOUDARY CONDITIONS - 0
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ELEaEmTGGOUPDDATA
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```
    EQ:1, TRUSS ELEMEMTS
numgea of eleremts. . . . . . . . . ( Npar(2) ). . = 112
```



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    EQ:1: LAGEAIALINONLNEARITY ONL
    E`:2: LAGEANGIAY FORMULATTON
Elemext SUbTypE. . . . . . . . . . ( SpAp(5) ). . = 2
    Ev.O: AXISYMMETRIC ELEMENTS
    EV:2: %LARE STRESS ELEAENTS
hay mumzegof nodes deschietng
    cyr one eleamet .e.0.....( spar(7)). . = s
wUSEER OF INTEGRATIOY POIMTS FOR
    ELEMENT STIFFNESS GENERATION. . . .( NPAR(10)). . = 2
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    &2.0, PAIKTIIT INTEGRATION PÖINTSS (NPaR(13)). " = 0
j-Ihtejall akAlysis . . . . . . . . ( MPAR(14)). . = 1
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    Ej:2', YES WITH CIACK EXTEMSION
AateaIAL DEFINITION
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rateaikl hodel. . - . - . . . . . ( NPAR(15)). . =
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rateaikl hodel. . - . - . . . . . ( NPAR(15)). . =
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    Q.3. ELASTOPLASTIC (VON MISES (VOTEOPIC PERFACTLY AND BILINEAR AaRDEMING
    5%.4, ELASTOPLASTIC ( von mises) (rana
    0.5 RINGMATIC BILEAR HARDENING
    EQ.S. ELASTOPLASTIC ( YON RISES)
    &Q.%, CONCREPIC MONLINEAR HANDENMNGG
    Ev:%, THEAHO-ELASIIC-CREE
guaber of differeat sets of material
    comstants...........(NPar(16))..= = 1
nunger of gaterial conStants per Set. .( Npar(17)). . = u
dinemsioy of storage drray (wa)
    PER INTEGRATION POİ\T.!.....( NPAR(I 8)). . . }1






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tofal number of integãal paths = 8



|  | $\begin{aligned} & 1 \\ & 0 \end{aligned}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 0 | 5 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $102$ | 109 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ELEAENTS IN PATH HO <br> d 3 - $4^{2}$ 35 a 6 | 3 d | 98 | 33 | 93 | 103 | 110 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | $7{ }^{4}$ | 77 | 78 | 79 | a 2 | 89 | 32 | 99 | 102 | 111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ELEMEATS In PATH NO <br> ol 026364 | 5 65 | 66 | 57 | 68 | 69 | 70 | 71 | 80 | 81 | 90 | 31 | 100 | 101 | 112 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | $55$ | 56 | 57 | 58 | 59 | 60 | 61 | 70 | 71 | 80 | 31 | 90 | 31 | 100 | 101 | 112 | 0 | 0 | 0 | 0 |
| $\text { ELEEENTS } \text { IN PAT3 }_{4} \mathrm{H}_{4} \mathrm{NO}$ | $\begin{array}{r} 7 \\ \text { US } \end{array}$ | 46 | 47 | 48 | 49 | 50 | 51 | 60 | 61 | 70 | 71 | 80 | 31 | 90 | 31 | 100 | 101 | 112 | 0 | 0 |
| $\text { ELEHENTS IN PATH } 33_{3} \mathrm{NO}$ | $35$ | 36 | 37 | 33 | 39 | 40 | 41 | 50 | 51 | 60 | 61 | 70 | 71 | 80 | 31 | 90 | 91 | 100 | 101 | 112 |

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[^4]| 1CDE | X-displacement | y-displajement | z-displacement | X-Rotation | I- Rotaiton | z-rotation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 99 | 0.0 | $0.2801820+00$ | 0.8437940-02 | 0.0 | 0.0 | 0.0 |
| 1 | 0.0 | $0.2795230+00$ $0.2768640+00$ | $0.2410060-01$ $0.3973500-01$ | 0.0 | 0.8 | 0:0 |
| 102 | 0 | $0.2751760+00$ | $0 \cdot 553433 \mathrm{D}-01$ | 0.0 | 0.0 | 0.0 |
| 103 | 0.0 0.0 | $0.2774700+00$ $0.2767420+00$ | O.709341D-01 | 0.0 | 0.0 | 0.0 |
| 105 | 0.0 | $0.2759930+00$ | -. $0.102099 \mathrm{D}+00$ | 0.0 | 3.0 | 0.0 |
| 106 | 3.0 | $0.2752420+00$ | $0 \cdot 117689 \mathrm{D}+00$ | 0.0 | 0.0 | 0.0 |
| 107 | 9.0 | $0.274490 \mathrm{D}+00$ | 0.133285D+00 | 0.0 | 0.0 | 0.0 |
| 143 | 3.0 | $0.2729650+00$ | $0.164513 \mathrm{D}+00$ | 0.0 | 3.8 | 0.0 |
| 110 | 0.0 | $0.2722270+00$ | $0.1801380+00$ | 0.0 | 0.0 | 0.0 |
| 1112 | ${ }^{C} .0$ | $0.2715025+00$ $0.2704000+00$ | 0:1957010 +00 | -0:0 | -0:0 | 0.0 |
| 113 | 3.0 | $\bigcirc .270116 \mathrm{D}+00$ | $0.2269770+00$ | 0.0 | 0.0 | 0 |
| $11 \frac{4}{5}$ | \% 0 | $0.263451 \mathrm{D}+00$ | $0.242553 \mathrm{D}+00$ | 0.9 0.0 | 0.0 | $\bigcirc$ |
| 110 | 0.0 | $0.263150 \mathrm{D}+00$ | $0.273613 \mathrm{D}+00$ | 0.0 | 0.0 | 0.0 |
| 117 | 0.0 | $0.2675130+00$ | $0.2890980+00$ | 0.0 | 0.0 | 0.0 |
| 118 | 3.0 0.0 | ${ }_{0} .3130330+00$ | $-0.269942 D-01$ $0.460790-02$ | 0:0 | 0.0 0.0 | -0.0 |
| 120 | 0.0 | $3.310423 \mathrm{D}+00$ | $0.3536010-01$ | 0.0 | 0.0 | 0.0 |
| 121 | 0.0 | $0.308 \rightarrow 590+00$ | $0.5695990-01$ | 0.0 | 0.0 | 0.0 |
| 122 | 0.3 | 0.307355D+00 | $0.7803760-01$ | 0.0 | 0.0 | 0 |
| 124 | 0.0 | $0.3040110+00$ | $0: 1604580+00$ | 0.0 | $0: 0$ | 0.0 |
| 125 | 0.0 | $0.3024310+00$ | $0.191792 \mathrm{D}+00$ | 0.0 | 0.0 | 0.0 |
| 127 | 3.0 | -. $300380 \mathrm{D}+00$ | O.223101D*00 | 0:0 | 0.8 | 0.0 |
| 129 | 0.0 | $0.239387 \mathrm{D}+00$ | 0.2852110 .00 | 0.0 | 0.0 | 0.0 |
| 129 | 0.0 | 0.3447520+00 | -0.306669D-01 | 0.0 | 0.0 | 0.0 |
| 131 | 0.3 | 3.3443529D+00 | -0.1483330-01 | 0.0 | 0.0 | O:0 |
| 132 | 0.0 0.0 | $0.342872 \mathrm{D}+00$ | $0.164923 \mathrm{D}-01$ | 0.0 | 0.0 | 0.0 |
| 133 | 0.0 | $0.342169 \mathrm{D}+00$ $0.341400 \mathrm{D}+00$ | 0.3199890-01 | -0.0 | 0.0 | 0.0 |
| 135 | 3.0 | $0.340575 \mathrm{D}+00$ | $0.628958 \mathrm{D}-01$ | 0.0 | $0: 0$ | 0.0 |
| 136 | 3.0 | O.339692D+00 | $0.793419 \mathrm{D}-01$ | 0.0 | 0.0 | 0.0 |
| 138 | 0.0 | $0.337816 \mathrm{D}+00$ | $0.938142 D-01$ 0.1093410 | 0.0 | 0.0 | 0.0 |
| 139 | 0.0 | $0.3303540+00$ | $3.1249210+00$ | 0.0 | 0.0 | 0.0 |
| 140 | 0.0 | $0.335397 \mathrm{D}+00$ | $0.140567 \mathrm{D}+00$ | 0.0 | 0.0 | 0.0 |
| 142 | 0.0 | -. 3349570 | 0.156261 $0+00$ | 0.0 | 3.0 | 0 |
| 143 | 0.0 | $0.3332260+00$ | $0 \cdot 1877580+00$ | 0.0 | 0.0 | 0.0 |
| 144 | 0.0 | 0.3324330+00 | $0.203516 \mathrm{D}+00$ $3.219248 \mathrm{D}+00$ | 0.0 | 0.0 3.0 | 0.0 |
| 145 | 0.0 | $0.331004 D+00$ | 0.2349280+00 | 0.0 | 0.0 | 0.0 |
| 147 | 0.0 | $0.3303579+00$ | 0. $250536 \mathrm{D}+00$ | 0.0 | 3.0 | 0.0 |
| 143 | 0.0 | $0.3291320+00$ | $0.281444 \mathrm{D}+00$ | 0.0 | 0.0 | 0.0 |
| 150 | 0.0 | $0.3696620+00$ | -0.3336640-01 | 0.0 | 0.0 | 0.0 |


| node | x-displaceaent | :-displacememt | z-displacement | x-rotation | y-rotaizon | z-rotation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 151 | 0.0 | 0.3674910 .00 | -0.1810490-02 | 0.0 | 0.0 | 0.0 |
| 152 153 15 | 0.0 0.0 | $0.3061070+00$ $0.3643840+03$ | 0.291127001 | 0.0 | 0.0 | 0.0 |
| 154 | 0.0 | $0.3643840+00$ | 0.5775930-01 | -0.0 | 0 | 0.0 |
| 155 | 0.0 | $3.3602120+05$ | $0.1215530+00$ | 0.0 | $0 \cdot 0$ | ${ }_{0} 0$ |
| 156 | 0.0 | 3.3581110+00 | 0.1529860+00 | 0.0 | 0.0 | 0.0 |
| 157 153 | 0 | $0.3562190+00$ | $0.1846743+03$ $0.2163820+00$ | 0.0 0.0 | 3.0 | 0.0 0.0 |
| 153 | 0 | 3:3532770+00 | $0.247927 \mathrm{j}+00$ | 0.9 | 0.0 | $0 \cdot 0$ |
| 160 | 0.0 | 0.352104D+00 | -0.2737430009 | 0.0 | 0.0 | 0.0 |
| 102 | 0.0 | 3.392518D 39200 | -0.359501D-01 | 0.0 | 0.0 | 0.0 |
| 103 | 0.0 | $3.391594 D+C 0$ | -0.4357790-02 | 0.0 | 0.0 | 0.0 |
| 15 | 0.3 | -. $39099400+00$ | 0.1101470-01 | 0.3 | 0.0 | ${ }_{0}^{0} 0$ |
| 106 | c.0 | 3.389290D+00 | $0.4137010-01$ | 0.0 | 0.0 | 0.0 |
| 167 | 0.0 | - 3082800000 | $0.564942 \mathrm{J-01}$ | 0.0 | 0.0 | 0.0 |
| 108 169 | 0.0 | $0.387169 \mathrm{D}+00$ | $0.7168260-01$ $0.869650-01$ | $0 \cdot 0$ | 0.0 | 0.0 |
| 170 | 0.0 | $0.3847510+00$ | 3.102398D+00 | 0.0 | 0.0 | 0.0 |
| 171 | 0.0 | 3.3935270+00 | 0.117972J+00 | 0.0 | 0.0 | 0.0 |
| 173 | 0.0 | 0.3911700+00 | $0.1439590+00$ | 0.0 | O.0 | 0.0 |
| 174 | 30 | $0.3800960+00$ | $0.1655130+00$ | 0.0 | 0.0 | 0.0 |
| 175 | 0.0 | -.379110D+00 | $0.181528 \mathrm{D}+00$ | 0.0 | 0.0 | 0.0 |
| 177 | 0.0 | $0.379223 \mathrm{D}+00$ | 0.1975560+00 | 0.0 | 0.0 | 0.0 |
| 173 | 0.0 | $0.176733 \mathrm{D}+00$ | $0.2294620+00$ | 0.0 | 0.0 | 0.0 |
| 179 | 0.0 | $0.3761110+00$ | $0.2452420+00$ | 0.0 | 0.0 | 0.0 |
| $1{ }^{130} 1$ | 0.0 | $0 \cdot 375547 \mathrm{D}+00$ | 0.260842D*00 | 0.0 | 0:0 | 0:0 |
| $1{ }^{1} 2$ | 0.0 | $0.416792 \mathrm{D}+\mathrm{CO}$ | -0.3303720-01 | 0.0 | 0.0 | 0.0 |
| 103 | 0.0 | $0.415830 \mathrm{D}+\mathrm{CO}$ | -0.6715580-02 | 0.0 | 0.0 | $0: 0$ |
| 105 | 0.0 | 0.414375D+00 | $0.2335490-01$ $0.535920-01$ | 0.0 | 0.0 | 0.0 |
| 135 | 0.0 | $0.4096050+00$ | 0.8317370-01 | 0.0 | 0.0 | 0.0 |
| 157 13 | 0.0 | $0.406780 \mathrm{D}+05$ | $0.11141270+00$ | 0.0 | 0.0 | 0.0 |
| 139 | 0.0 | 0.4018980+00 | $0.1783110+00$ | 0.0 | 0.0 | 0.0 |
| 130 | 0.0 | $0.400157 \mathrm{P}+00$ | $0 \cdot 210773 D+00$ | 0.0 | 0.0 | 0.0 |
| $1 \begin{aligned} & 191 \\ & 192\end{aligned}$ | 0.0 | $0.398833 \mathrm{D}+00$ | $0.2428210+00$ | 0.0 | 0.0 | 0 |
| 193 | 0.0 | $0.440951 \mathrm{D}+30$ | -0.3973090-01 | 0.0 | 0.0 | 0.0 |
| 178 | 0.0 0.0 | 0.440114J+00 | -0.2397760-01 | 0.0 | 0.0 | 0 |
| 135 | 0.0 | $0.4395320+00$ | -0.5964310-02 | 0.3 | 0.0 | 0.0 |
| 197 | 0.0 | -. $4385730+09$ | $0.2046090-01$ | 0.0 | 0.0 | 0.0 |
| 193 | 0.3 | 0.4375710+30 | $0.3447940-01$ | 0.0 | 0.0 | 0.0 |
| 200 | 0.0 | $0.4347740+00$ | $\begin{aligned} & 0.4935360001 \\ & 0.6403760-01 \end{aligned}$ | 0.0 | 0.0 | 0 |
| 201 202 | 0:0 | $0.4331780+0 . J$ $0.4315390+00$ | $0.789899 \mathrm{D}-01$ $0.9429280-01$ | 0.0 0.0 | 0.0 0.0 | 0.0 0.0 |

(B.2)

| YODE | I- DISPLACEFEN1 | I-DISPLACEMENT | 2-DISPLACEMENT | I-ROTATIOH | I- ROTAITON | 2-rotation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 203 | 0.0 | $0.4299200 \cdot 00$ | $0.1099230+00$ | 0.0 | 0.0 | 0.0 |
| 204 | 0.0 | $0.428386 \mathrm{D}+00$ | 0.1258840 +00 | 0.0 | 0.0 | 0.0 |
| 205 | 0.0 | $0.426967 \mathrm{D}+00$ | $0.1420990+00$ | 0.0 | 0.0 | 0.0 |
| 206 | 0.0 | $0.4257050+00$ | $0.1585130+00$ | 0.0 | 0.0 | 0.0 |
| 207 | 0.5 | $0.424598 \mathrm{D}+00$ | $0.1750350+00$ | 0.0 | 0.0 | 0.0 |
| 209 | 0.0 0.0 | 0.4236560*00 | 0.1915960+00 | 0.0 | 3.0 0.0 | 0.0 0.0 |
| 209 | 0.0 0.0 | $0.4223630+00$ $0.4222040+00$ | 0.2030870 0.00 | 0.0 0.0 | 0.0 0.0 | 0.0 0.0 |
| 211 | 0.0 | $0.4215590+00$ | $0.240619 \mathrm{D}+00$ | 0.0 | 3.0 | 0.0 |
| 212 | 0.0 | $0.4212010+00$ | 0. 2565020.00 | 0.0 | 0.0 | 0.0 |
| $\bigcirc 13$ | 0.0 | $0.420801 \mathrm{D}+00$ | $0.272018 \mathrm{D}+00$ | 0.0 | 0.0 | 0.0 |
| 214 | 0.0 | $0.4655550+00$ | -0.4075920-01 | 0.0 | 0.0 | 0.0 |
| 215 | 0.0 | $0.4645400+00$ | -0.104947D-01 | 0.0 | 3.0 | 0.0 |
| 216 | 0.0 | 3. $462997 \mathrm{D}+00$ | 0.1753820-01 | 0.0 | 0.0 | 0.0 |
| 217 | 0.0 | C. $4602350+00$ | 3.4530040-01 | 0.0 | 0.0 | 0.0 |
| 218 | 0.0 | $0.4560430+00$ | 0.7431880-01 | 0.0 | 0.0 | 0.0 |
| $\bigcirc 17$ | 9.0 | $0.452903 D+00$ | $0.1052922+00$ | 0.0 | 0.0 | 0.0 |
| 220 | 0.9 | $0.4496710+00$ | $0.1380170+00$ | 0.0 | 0.0 | 0.0 |
| 221 | 0.0 | $0.4472440+00$ | $0.1717250+00$ | 0.0 | 0.0 | 0.0 |
| 222 | 0.0 | $0.445595 D+00$ | $0.2055350+00$ | 0.0 | 0.0 | 0.0 |
| 223 | 0.0 | $0.444533 \mathrm{D}+00$ | $0.2396970+00$ | 0.0 | 3.0 | 0.0 |
| 224 | J. 0 | $0.443859 D+00$ | $0.270524 \mathrm{D}+00$ | 0.0 | 0.0 | 0.0 |
| 225 | 0.0 | C. $4888930+00$ | -0.4076500-01 | 0.0 | 0.0 | 0.0 |
| 220 | 0.0 | $0.4887430+00$ | -0.2569390-01 | 0.0 | 0.0 | 0.0 |
|  | 0.0 | $0.4887400+00$ | -0.1169260-01 | 0.0 | 0.0 | 0.0 |
| 22 d | 0.0 | $0.488169 \mathrm{D}+00$ | $0.1600500-02$ | 0.0 | 0.0 | 0.0 |
| 229 | 0.0 | $0.4871900+00$ | $0.1454030-01$ | 0.0 | 0.0 | $0 \cdot 0$ |
| 2331 | 0.0 | $0.4357940+00$ $0.450530+00$ | $0.274983 \mathrm{D}-01$ | 0.0 | 0.0 | 0.0 |
| 231 231 | 0.0 0.0 | $0.4340530+00$ $0.4320370+00$ | $0.407195 \mathrm{D}-01$ $0.5451983-01$ | 0.0 0.0 | 0.0 0.0 | 0.0 0.0 |
| 233 | 0.0 | $0.479892 \mathrm{D}+00$ | $0.689513 \mathrm{D}-01$ | 0.0 | 0.0 | 0.0 |
| 234 | 0.0 | $0.4777190+00$ | $0.841911 \mathrm{D}-01$ | 0.0 | 0.0 | 0.0 |
| 235 | 0.0 | $0.4756470+00$ | $0.100112 \mathrm{D}+00$ | 0.0 | 3.0 | 0.0 |
| 235 | 0.3 | -. $+738150+00$ | $0.1166740+00$ | 0.0 | 3.0 | 0.0 |
| 237 | 0.0 | $0.4722390+90$ | $0.1337030+00$ | 0.0 | 0.0 | 0.0 |
| 23 239 | 0.0 | $0.4799452+00$ $0.4699260+00$ | $0.151097 D+00$ $0.1684730+00$ | 0.0 0.0 | 0.0 | 0.0 0.0 |
| 240 | 0.0 | $0.469124 \mathrm{D}+00$ | $0.1858910+00$ | 0.0 | 0.0 | 0.0 |
| 241 | 0.0 | $0.4685180+00$ | $0.2032070+00$ | 0.0 | 0.0 | 0.0 |
|  | 0.0 | $0.463039 D+00$ | $0.2203010+00$ | 0.0 | 0.0 | 0.0 |
| 243 | U. 0 | $0.4676810+00$ | $0.2371070+00$ | 0.0 | 0.0 | 0.0 |
| 244 | 0.0 | $0.467406 \mathrm{D}+00$ | 0. $2535620+00$ | 0.0 | 0.0 | 0.0 |
| 245 | 0.0 | $0.4672140+00$ | $0.2695340+00$ | 0.0 | 0.0 | 0.0 |
| 246 | 0.0 | $0.5121040+00$ | -0.3939050-01 | 0.0 | 0.0 | 0.0 |
| 247 | J. 0 | $0.5124710+00$ | -0.1224220-01 | 0.0 | 0.0 | 0.0 |
| 248 | 0.0 | 0. $510990 \mathrm{D}+30$ | $0.1146100-01$ | $0 \cdot 0$ | 0.0 | 0.0 |
| 243 | 0.0 | $0.507527 \mathrm{D}+\mathrm{CO}$ | 0.355065 - 01 | $0 \cdot 0$ | 0.0 | 0.0 |
| 250 | 2.0 | $0.502768 \mathrm{D}+30$ | $0.627650 \mathrm{D}-01$ | $0 \cdot 0$ | 0.0 | 0.0 |
| 251 252 | - ${ }_{0}$ | $0.4981070+00$ | $0.9432 \mathrm{C} 30-01$ | $0 \cdot 0$ | $\bigcirc .0$ | 0.0 |
| 252 253 | 0.0 0.0 | $0.494636 D+00$ $0.492756 D+00$ | $0.1291910+00$ $0.165360+00$ | 0.3 | 0.0 0.0 | 0.0 |
| 254 | 0.0 | 0.491805D+00 | $0.2011950+00$ | 0.0 | 0.0 | 0.0 |

NODE ..... $X$-DISPLACEMENT Y-DISPLACEA

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(B.2)



|  | STRESS STATE | Staess-yy | ATton | STRESS-Y2 | L M | $G$ | $\begin{aligned} & \text { (2/D continuua) } \\ & \text { (PLANE STRESS) } \end{aligned}$ |  | EFFECTIVESTAMIN |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | Strain-zz | STRAIN- YZ | EFFECTIVE STRESS |  |
| 1 ( 1 |  |  |  |  |  |  |  |  |  |
| 23 <br> 4 | ELASTIC | 0.0 | 153.5 | 0.1 | -0.643D-03 | $0.1950-02$ | 0.1930-05 | 158.5 | 0.1760-02 |
|  | ELASTIC | 0.0 0.0 | 153.8 | 0.2 | -0.6440-03 | 0.195002 | $0.6380-05$ | 158.7 | 0.1760-02 |
|  | ELASTIC | 0.0 | 159.5 | 0.0 | - $0.643 \mathrm{D}-03$ $-0.644 \mathrm{D}-03$ | $0.1950-02$ $C .195 D-02 ~$ | $0.423 D-06$ $0.2310-05$ | 158 | 0.176002 $0.1760-02$ |
|  |  |  |  |  |  |  |  |  |  |
| 3 | Elastic | 0.2 | 153.5 | 0.1 | -0.6410-03 | $0.195 \mathrm{D}-02$ | 0.3740-05 | 158.4 | 0.1750-02 |
|  | ELASTIC | 0.1 | 153.6 | 0.3 | -0.6420-03 | 0.195002 | $0.103 \mathrm{D}-\mathrm{CH}$ | 158.5 | 0.1760-02 |
|  | ELASTIC | 0.1 | 158.5 | 0.1 | -0.642J-03 | $0.1950-02$ $0.195 D-02$ | $0.296 D-05$ $0.8330-05$ | 158 | $0.1750-02$ $0.1760-02$ |
|  |  |  |  |  |  |  |  |  |  |
| 1234 | ELajic | 0.3 | 153.5 | 0.1 | -0.6390-03 | $0.1950-02$ | $0.3830-05$ | 158.3 | $0.1750-02$ |
|  | ELASTIC | $0 \cdot 2$ | 158 | 0.3 | $-0.6410-03$ $-0.6400-03$ | 0.1950-02 | $0.9490-05$ $0.399 D-05$ | 158.3 | 0.1750-02 |
|  | Enらすic | 0.1 | 153.5 | 0.3 | -0.6410-03 | $0.1950-02$ | $0.102 \mathrm{D}-04$ | 158.4 | $0.1750-02$ |
| 1 | elajitc | 0.5 | 158.5 | 0.1 | -0.6370-03 | 0.1950-02 | 0.2530-05 | 158.2 | 0.1750-02 |
|  | ELASTIC | 0.2 | 158.3 | 0.2 | -0.6400-03 | $0.195 D-02$ | 0.25050-05 | 158.2 | $0.1750-02$ |
|  | ELAETIC | 0.4 0.2 | 153.5 158.4 | 0.1 0.3 | $-0.6338 D-03$ $-0.640 D-03$ | $0.1950-02$ $0.1950-02$ | $0.3430-05$ $0.8190-05$ | 158.3 | $0.1750-02$ $0.1750-02$ |
|  |  |  |  |  |  |  |  |  |  |
| 6 | EListic | 0.5 | 158.5 | 0.0 | -0.636D-03 | $0.195 \mathrm{D}-02$ | 0.4580-06 | 158.2 | $0.175 \mathrm{D}-02$ |
|  | ELASTIC | 3.2 | 153.3 | 0.0 | -0.6400-03 | 0.1940002 | 0.112D-05 | 158.2 | $0.1750-02$ |
|  | ELASTIC | 0.5 | 158.5 | 0.1 | $-0.6370-03$ $-0.6405-03$ | 0.195002 $0.1940-02$ | $0.173 D-05$ 0.39505 | 158.2 158.2 | - $0.1750-02$ |
|  |  |  | - |  |  |  |  |  |  |
| 7 | Elasic | 0.5 | 150.5 | -0.1 |  | $0.195 \mathrm{D}-02$ |  |  | $0.1750-02$ |
|  | ELiSSTIC | 0.5 0.5 | 159.3 | $=0.1$ -0.0 | $-0.64400-03$ $-0.5300-03$ | $0.194 D D-02$ $0.195 D-02$ | $=0.4040-05$ $-0.475 D-06$ | 158.2 159.2 | $0.1750-02$ $0.1750-02$ |
|  | ELASTIC | $\stackrel{0}{0.5}$ | 159.5 153.3 | -0.0 | $-0.5300-03$ $-0.6400-03$ | 0.1950-02 | $-0.475 D-06$ $-0.122 D-05$ | 159.2 158.2 | $0.1750-02$ $0.175 D-02$ |
|  |  |  |  |  |  |  |  |  |  |
| 4 | Elastic | 0.4 | 158.5 | -0.1 | -0.6380-03 | $0.1950-02$ | -0.343D-05 | 158.3 | $0.1750-02$ |
|  | ELASTIC | 0.2 0.5 | 153.4 158.5 | -0.3 | $=0.6400-03$ $-0.6370-03$ | $0.1950-02$ $0.1950-02$ | $-0.322 D-05$ $-0.254 D-05$ | 158.3 158.2 | $0.175 D-02$ $0.175 D-02$ |
|  | ELastic | 0.5 | 158.5 158.3 | -0.1 | $-0.637 \mathrm{D}-03$ $-0.6400-03$ | $0.195 D-02$ $0.195 D-02$ | $-0.254 D-05$ $-0.612 \mathrm{D}-05$ | 158.2 153.2 | $0.1750-02$ |
|  |  |  |  |  |  |  |  |  |  |
| 4 | ELASTIC | 0.2 | 153.5 | -0.1 | -0.6400-03 | $0.1950-02$ | -0.4030-05 | 158.4 | $0.1750-02$ |
|  | ELiSTIC | 0.1 | 153.5 | -0.3 | $-0.6415-03$ $-0.6390-03$ | $0.1950-02$ | $=0.1010-04$ -0.384 | 158.4 | 0.1750-02 |
|  | SLASTIC | 0.3 0.2 | 153.5 153.4 | -0.1 -0.3 | $-0.639 D-03$ $-0.641 D-03$ | $0.1950-02$ $0.1950-02$ | $-0.3340-05$ $-0.9460-05$ | 158.3 158.3 | $0.1750-02$ $0.1750-02$ |
|  |  |  | 1 - |  |  |  |  |  |  |
| $10^{4}$ | ELASTIC | 0.1 | 159.5 | -0.1 | -0.6420-03 | $0.1950-02$ | -0.3140-05 | 158.5 | $0.1760-02$ |
|  | ELASTIC | 0.1 | 158.7 | -0.2 | $=0.6430-03$ $=0.6410-03$ | $0.1950-02$ | -0.3050-05 | 158.6 | 0.1760-02 |
|  | ELASTIC | 0.2 | 153.5 159.6 | -0.1 -0.3 | $=0.6410-03$ $-0.642 \mathrm{D}-03$ | 0.1950-02 | $-0.3820-05$ $-0.9970-05$ | 158.4 158.5 | $0.1750-02$ $0.1760-02$ |
|  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 3 \end{aligned}$ | Elastic | 0.0 | 158.5 | -0.0 | -0.6430-03 | 0.1950-02 | -0.3720-06 |  |  |
|  | ELASTIC | 0.0 -0.0 0.0 | 158.8 158.6 | -0.1 -0.0 | $=0.6440-03$ $-0.643 D-03$ | 0.1950002 | $0.3730-05$ $-0.1730-05$ | 158.8 | 0.1760002 |
|  | ELASTIC | 0.0 0.1 | $1 \begin{aligned} & 158.7 \\ & 158.7\end{aligned}$ | -0.0 -0.2 | $=0.643 \mathrm{D}-03$ $-0.643 \mathrm{D}-03$ | $0.195 D-02$ 0.195020 | $-0.1500-05$ $-0.609 D-05$ | 158.5 | 0.1760-02 |


|  | 1 2 3 4 | $\begin{aligned} & \text { ELASTIC } \\ & \text { ELASTIC } \\ & \text { ELASTIC } \end{aligned}$ | $\begin{array}{r} 0.0 \\ -0.0 \\ -0.0 \\ -0.0 \end{array}$ |  | $\begin{aligned} & 0.3 \\ & 0 . \\ & 0: 1 \\ & 0.1 \end{aligned}$ | $-0.6450-03$ <br> $-0.646 \mathrm{D}-03$ <br> $-0.646 D-03$ $-0.649 D-03$ | $\begin{aligned} & 0.195 \mathrm{D}-02 \\ & 0.190 \mathrm{D}-02 \\ & 0.196 \mathrm{D}-02 \\ & 0.197 \mathrm{D}-02 \end{aligned}$ | $\begin{aligned} & 0.8630-05 \\ & 0.8060-05 \\ & 0.28305-05 \\ & 0.233 D-05 \end{aligned}$ |  | $\begin{aligned} & 0.1760-02 \\ & 0.176 \mathrm{D}-02 \\ & 0.1760-02 \\ & 0.177 \mathrm{D}-02 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12 |  |  |  |  |  |  |  |  |  |  |
|  |  | ELASTIC | 0.0 | 158.6 | 0.4 | -0.6430-03 | $0.1950-02$ | 0.122D-04 | 158.6 | $0.1760-02$ |
|  | 2 | ZLASTIC | -0.1 | 158.0 | 0.2 | -0.6450-03 | $0.1950-02$ | $0.693 \mathrm{D}-05$ | 158.5 | $0.1760-02$ |
|  | 3 | ELAS=IC | C. 0 | 158.9 | 0.3 | -0.044D-03 | $0.1950-02$ | $0.114 \mathrm{D}-04$ | 158.9 | 0.1760-02 |
|  | 4 | ELASIC | -0.1 | 158.7 | 0.2 | -0.6450-03 | $0.1950-02$ | $0.708 \mathrm{D}-05$ | 159.0 | 0.176D-02 |
| 13 |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 0.0 | 153.3 | 0.3 | -0.6420-03 |  | 0.1070-04 | 158.3 |  |
|  | 2 | ELASTIC | -0.4 | 15 a.1 | 0.1 | -0.647D-03 | $0.1950-02$ | $0 \cdot 2290-05$ | $158 \cdot 3$ | $0.1750-02$ |
|  | 3 | ELSSTIC | -0.1 | 15.5 | 0.4 | -0.6420-03 | 0.195D-02 | -0.1220-04 | 158.5 | $0.1750-02$ $0.1750-02$ |
|  | 4 | ELASTIC | -0.3 | 153.3 | 0.1 | -0.6400-03 | 0.1950-02 | 0.4530-05 | 158.5 | 0.1750-02 |
| 1 u | 1 | ELASTIC | 0.0 | 158.1 | 0.2 | -0.641 D-03 | 0.1940-02 | 0.638D-05 | 158.1 | $0.1750-02$ |
|  | 2 | ELASTIC | -0.7 | 153.0 | -0.0 | -0.0490-03 | $0.1940-02$ | -0.6860-06 | 158.3 |  |
|  | 3 | ELASTIC | 0.1 | 153.2 | 0.3 | -0.641 - 03 | $0.1940-02$ | $0.9040-05$ | 158.2 | $0.1750-02$ |
|  | 4 | ELASTIC | -0.6 | 158.0 | 0.0 | -0.648D-03 | $0.1940-02$ | 0.101 D-05 | 158.3 | $0.1750-02$ |
| 15 |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 0.0 |  | 0.0 |  |  |  |  |  |
|  | 2 | ELAJTYC | -0.8 | 157.9 | -0.0 | -0.65 $510-03$ | $0.1940-02$ | -0.937D-06 | 158.3 | $0.1750-02$ |
|  | 3 | ELASTIC | -0.0 | 153.1 | -0.1 | -0.6410-03 | $0.1940-02$ | $0.407 \mathrm{D}-05$ $-0.204 \mathrm{D}-06$ | 158.0 | $0.1750-02$ |
|  | 4 | ELASTIC | -0.3 | 157.9 | -0.0 | -0.650D-03 | 0.1940-02 | -0.204D-06 | 158.3 | $0.1750-02$ |
| 16 |  | ELASTIC | 0.0 | 159.1 | -0.1 | -0.641D-03 | $0.194 \mathrm{D-02}$ | -0.432D-05 | 158.1 | $0.1750-02$ |
|  | 2 |  | -0.8 | 157.9 | -0.0 |  |  | -0.3100-06 |  |  |
|  | 3 | ilastic | 0.0 | 159.0 | -0.0 | -0.641D-03 | $0.194 \mathrm{D}-02$ | -0.139D-05 | 158.0 | $0.1750-02$ |
|  | 4 | ELASTIC | -0.8 | 157.7 | 0.0 | -0.651D-03 | $0.194 \mathrm{D-02}$ | 0.327D-06 | 158.3 | $0.1750-02$ |
| 17 |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 0.0 |  | -0.3 |  |  |  |  |  |
|  | 2 | ELASTIC | -0.0 | 153.0 | -0.0 | -0.649D-03 | $0.1940-02$ | -0.1230-05 | 158.3 | $0.1750-02$ |
|  | 3 4 4 | ELAJTIC ELASTIC | 0.0 -0.7 | 153.1 153.0 | -0.2 | $-0.6410-03$ $-0.6490-03$ | $0.1940-02$ $0.1940-02$ | $-0.656 D-05$ $0.238 D-06$ | 158.1 158.3 | $0.1750-02$ $0.1750-02$ |
| 18 |  |  |  |  |  |  |  |  |  |  |
|  |  | SLASTIC | 0.1 | 159.5 | -0.4 | -0.6420-03 | $0.1950-02$ |  |  |  |
|  | 2 | ELiSTIC | $-0.3$ | 159.3 | -0.1 | -0.646D-03 | $0.1950-02$ | -0.436D-05 | 158.5 | $0.1760-02$ |
|  | 3 | ELASTIC | 0.3 | 158.3 | $-0.3$ | -0.642D-03 | $0.1950-02$ | -0.106D-04 | $158 \cdot 3$ | $\begin{aligned} & 0.1750-02 \\ & 0.1750-07 \end{aligned}$ |
|  |  | ELastIC | -0.4 | 158.2 | -0.1 | -0.647D-03 | $0.1950-02$ | -0.231D-05 |  |  |
| 19 |  |  |  |  |  |  |  |  |  |  |
|  | 1 | ELdSIIC | 0.0 | 153.9 | -0.3 | -0.6440-03 | $0.1950-02$ | -0.112D-04 | 158.9 | 0.1760-02 |
|  | 2 | ELASTIC | -0.1 | 158.9 | -0.2 | -0.6400-03 | $0.1950-02$ | -0.6630-05 | 159.0 | $0.176 \mathrm{D}-02$ |
|  | 3 | SLASTIC | 0.0 | 150.6 | -0.4 | -0.643D-03 | $0.1950-02$ | -0.119D-04 | 158.6 | $0.176 \mathrm{D}-02$ |
|  | 4 | ELASTIC | -0.2 | 153.6 | -0.2 | -0.6450-03 | $0.1950-02$ | -0.649D-05 | 158.7 | 0.1760-02 |
| 20 | 1 | L A S T I C | 0.0 | 157.3 | -0.1 | -0.646D-03 | 0.1960-02 | -0.3390-05 | 159.3 | $0.1760-02$ |
|  | 2 | ELASTIC | -0.0 | 159.9 | -0.1 | -0.649D-03 | $0.1970-02$ | -0.216D-05 | 159.9 | $0.1770-02$ |
|  | 3 | ELASTIC | -0.0 | 159.0 | -0.2 | -0.645D-03 | $0.1959-02$ | -0.804D-05 | 159.1 | $0.1760-02$ |
|  | 4 | ELASTIC | -0.0 | 153.3 | -0.2 | $-0.647 \mathrm{D}-03$ | $0.1960-02$ | -0.747D-05 | 159.3 | $0.1760-02$ |
| 21 |  |  |  |  |  | -0.6480-03 | $0.196 \mathrm{D}-02$ | $0.159 \mathrm{D}-05$ |  | $0.1770-02$ |
|  | 2 | ELASTIC | -0.0.4 | 158.1 | -1.0 | -0.0.046D-03 | $0.1940-02$ | -0.328D-04 | 158.3 | $0.1750-02$ |
|  | 3 | ELASTIC | 0.0 | 100.5 | 0.1 | -0.651D-03 | $0.197 \mathrm{D}-02$ | $0.238 \mathrm{D}-05$ | 160.5 | $0.178 \mathrm{D}-02$ |
|  | 4 | ELASTIC | -0.0 | 159.6 | -0.2 | -0.648D-03 | $0.196 \mathrm{D}-02$ | -0.794D-05 | 159.6 | 0.1770-02 |


|  | 1 2 3 4 |  | -0.6 -1.9 -0.2 -0.9 | 158.3 157.3 15398 157.4 | -0.2 -2.3 -0.0 -1.7 | $=0.649 \mathrm{D}-03$ $=0.6610-03$ $=0.647 \mathrm{D}-03$ $-0.6500-03$ | $0.1950-02$ 0.1940002 0.195002 $0.1940-02$ | $\begin{aligned} & -0.6650-05 \\ & =0.762 D-04 \\ & -0.1940-06 \\ & -0.5400-04 \end{aligned}$ | 158.6 158.2 159.0 157.9 | $\begin{aligned} & 0.1760-02 \\ & 0.1750-02 \\ & 0.176002 \\ & 0.17500-02 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 23 |  |  |  |  |  |  |  |  |  |  |
|  | 1 | ELASTIC | -1.2 | 157.9 | -9.4 | -0.6550-03 | $0.1950-02$ | -0.138D-04 | 158.5 | 0.1760-02 |
|  | $\frac{2}{3}$ | ELASTIC | -3.9 | 154.0 | -2.7 | -0.68890-03 | 0.190002 | -0.8990-04 | 160.1 | $0.1770-02$ |
|  | 4 | ELAStic | -2.8 | 157:4 | - 2.6 | $-0.6510-03$ $-0.6720-03$ | 0.1950-02 | -0.193D-04 | 158.5 158.9 | $0.1760-02$ $0.1760-02$ |
| $\therefore$ |  | Elastre | -1.3 | 157.9 | -0.4 |  |  |  |  |  |
|  | 2 | ELASTIC | -5.6 | 159:2 | -2.0 | -0.7150-03 | $0.1950-02$ | -0.1230-04 | 158.8 | $0.1760-02$ $0.179 D-02$ |
|  | 3 | ELASTIC | -1.4 | 157.9 | -0.5 | -0.6500 0.03 | $0.1950-02$ | -0.1400-04 | 158.6 | $0.1700-02$ |
|  | 4 | elastic | -4.7 | 153.5 | -2.5 | -0.7310-03 | $0.197 \mathrm{D}-02$ | -0.8300-04 | 101.0 | 0.1780-02 |
| 25 | 1 | Elastic | -2.1 | 158.0 | -0.1 | -0.6670-03 | $0.1950-02$ | -0.2910-05 | 159.1 | $0.1760-02$ |
|  | $\frac{2}{3}$ | ELASTIC | -6.4 | 159.9 | -0.4 | -0.7220-03 | $0.1990-02$ | -0.11190-04 | 153.2 | $0.1800-02$ |
|  | 4 | ELASTIC | -6.1 | 159.6 | -1.3 | -0.7220-03 | $0.199 \mathrm{~d}-02$ | -0.939D-05 | 159.0 | ${ }_{0} .176 \mathrm{D}-02$ |
| 26 | 1 | Elastic | -2.0 | 158.0 | 0.3 | -0.665D-03 | 0.1950-02 | 0.8340-05 | 159.0 | 0.1760-02 |
|  | $\frac{2}{3}$ | ELASTICK | -6.0 | 154.6 | 1.4 | $=0.7210-03$ $-0.6570-03$ | 0.1990002 | 0.4500-04 | 162.7 | $0.1800-02$ |
|  |  | ELASTIC | -2.1 | 153.0 159.9 | 0.0 0.4 | $-0.6670-03$ $-0.7270-03$ | $0.1950-02$ $0.1990-02$ | $0.1190-95$ $0.1390-04$ | 159.1 163.1 | $0.1760-02$ $0.1800-02$ |
| 2 |  |  |  |  |  |  |  |  |  |  |
|  |  | ELASTIC | -1.5 | 159.0 | 0.4 | -0.659D-03 | $0.195 \mathrm{D}-02$ | $0.1420-04$ | 158.7 | $0.1760-02$ |
|  | ${ }_{3}^{2}$ | ELASIIC | -4.6 | 150.8 | 2.5 | $-0.6990-03$ $-0.6630-03$ | $0.1970-02$ $0.1950-02$ | $0.8230-04$ $0.1120-04$ | 160.9 158.9 | $0.1780-02$ $0.1760-02$ |
| 28 | ( | Elastic | -5.5 | 159.2 | 2.0 | -0.7130-03 | $0.1980-02$ | $0.648 \mathrm{D}-04$ | 162.1 | $0.179 \mathrm{D}-02$ |
|  |  | ELASTIC | -0.8 | 158.2 | 0.3 | -0.6510-03 | 0.1950-02 | 0.1100-04 | 158.6 | $0.1760-02$ |
|  | $\frac{2}{3}$ | ELASBIC | -2:6 | 157.5 153.0 | 2.0 | $-0.6710-03$ -0.650003 | 0.1950-02 | $0.8500-04$ $0.1360-04$ | 158.9 | $0.1760-02$ |
| 29 | 4 | FLASTIC | -3.7 | 153.1 | 2.7 | -0.637c-03 | $0.1960-02$ | 0.8825-04 | 160.1 | $0.177 \mathrm{D}-02$ |
|  |  | Elastic | -0.2 |  | 0.1 | -0.647D-03 | 0.195D-02 | $0.1710-05$ |  |  |
|  | $\frac{2}{3}$ | ELASTC | -0.9 | 157.5 | 1.6 | -0.6500-03 | 0.194002 | 0.539004 | 158.0 | $0.1750-02$ |
|  |  | ELASSic | -1.8 | 157:3 | $2: 3$ | -0.630D-03 | 0:195D-02 | $0.7750-05$ $0.7510-04$ | 158.7 158.3 | 0.1760-02 |
|  |  | ELASTIC | 0.0 | 160.4 | -0.1 | -0.6500-03 | 0.1970-02 | -0.1730-05 | 160.4 | 0.1780-02 |
|  | $\frac{2}{3}$ | ELASTIC | -3.0 | 159.4 | -0.3 | -0.647D-03 | $0.1960-02$ | 0.8380-05 | 159.4 | $0.1770-02$ |
|  | 4 | ELASTic | -0.0 | 159.0 | -1.0 | -0.6460-03 | 0.1940-02 | -0.6130-0. | 158.2 | $0.1770-02$ $0.1750-02$ |
| 31 |  |  |  |  |  |  |  |  |  |  |
|  |  | ELASTIC | -0.7 | 155.9 | -2.4 | -0.641D-03 | 0.1920-02 | -0.771D-04 | 156.3 | 0.173D-02 |
|  | $\frac{2}{3}$ | ELASTIC | -1.4 -0.0 | 150.5 | -5.3 | $-0.627 D-93$ $-0.0400-07$ | 0.1060-02 | $=0.1750-03$ $-0.2040-04$ | 151.5 | 0.168002 |
|  | 4 | ELASTIC | -0.1 | 152:0 | -1.5 | -0.0400-03 | - 0.1940002 | $-0.2040-04$ $-0.4870-04$ |  | - 0.1750002 |
| 32 |  |  |  |  |  |  |  |  |  |  |
|  |  | ELASTIC | -3.4 | 155.9 | -10.9 | -0.674D-03 | $0.1930-02$ | -0.1590-03 | 157.8 | $0.1750-02$ |
|  | 3 | ELASTIC | -1.7 | 155.6 | --3:5 | -0.6510-03 | - $0.1910 \mathrm{D}-02$ | $=0.3300-03$ $-0.1130-03$ | 157.5 | 0.1740002 |
| 33 | 4 | elastic | -3.1 | 151.0 | -7.8 | -0.651 D-03 | $0.1870-02$ | -0.256D-03 | 153.2 | 0.1690-02 |
|  | , | ELASTIC | -12.8 | 158.3 159.7 | -5.9 -10.5 | $-0.7260-03$ $-0.7970-03$ | $0.197 \mathrm{D}-02$ $0.201 \mathrm{D}-02$ | $=0.1760-03$ $-0.346 D-03$ | 162:1 | 0.1790-02 |
|  |  |  |  |  |  | (B) |  |  |  |  |


| 34 | 4 | $\begin{aligned} & \text { ELASTIC } \\ & \text { ELASTIC } \end{aligned}$ | -4.8 -8.7 | 15008 | $\begin{array}{r} -5.4 \\ -10.9 \end{array}$ | $\begin{aligned} & -0.695 D-03 \\ & -0.740 D-03 \end{aligned}$ | $\begin{aligned} & 0.1950-02 \\ & 0.195 \mathrm{D}-02 \end{aligned}$ | $\begin{aligned} & -0.1760-03 \\ & -0.3550-03 \end{aligned}$ | 159.6 | $\begin{aligned} & 0.1760-02 \\ & 0.178 \mathrm{D}-02 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 35 | 1 | Elastic | -9.6 | 151.0 | -3.7 | -0.771 D-03 | 0.2020-02 | -0.1210-03 |  |  |
|  | 2 | ELASTPC | -10.7 | 165.6 | -6:7 | -0. $-0770-03$ | 0.2145002 | -0.2250-03 | 174.9 | 0.193002 |
|  | 4 | ELASEIC | -14.3 | 162.5 | -4.9 | $-0.744 D-03$ $-0.835 D-03$ | $0.1990-02$ $0.2000-02$ | $-0.1610-03$ $-0.3080-03$ | 164.0 | $0.1810-02$ $0.1890-02$ |
|  | 1 | elastic | -10.8 | 162.4 | -0.6 | -0.7920-03 | 0.2040-02 | -0.194D-04 | 168.1 | 0.1860-02 |
| 36 | 2 | ELASTIC | -19.4 | 109.2 | -0.7 | -0.9080-03 | $0.2140 \mathrm{D-02}$ | -0.239D-04 | 178.1 | $0.1960-02$ |
|  | 3 | ELASTIC | -17.4 | 161:8 107 | -2.5 | -0.7940-03 $-0.8970-03$ | $0.2030 \mathrm{D}-02$ $0.2130-02$ | -0.23150-04 $-0.1440-03$ | 167.3 | 0.1850002 |
|  |  |  |  |  |  | -0.8970-33 | -. $213 \mathrm{D}-02$ | -0.144D-03 | 176.9 | 0.1950-02 |
| 37 | 1 | ELASTIC | -10.1 | 161.8 | 2.7 | -0.790D-03 | $0.203 \mathrm{D}-02$ | $0.8810-04$ | 167.1 | $0.1850-02$ |
|  | 2 | ELASTIC | -15.8 | 160.6 | 5.4 | -0.8820-03 | $0.212 \mathrm{D}-02$ | $0.178 \mathrm{D}-03$ | 175.8 | 0.1940007 |
|  | 4 | ELASTIC | -10.1 | 1050 | 2.0 | -0.9030-03 | $0.214 \mathrm{D}-02$ | $0.2850-04$ $0.6660-04$ | 1687 | - $0.1960-02$ |
| 38 | 1 | elajtic | -7.7 | 159.5 | 4.9 | -0.741 D-03 | 0.1990-02 | 0.1600-03 | 163.7 | $0.1810-02$ |
|  | $\frac{2}{3}$ | ELASTIC | -12.7 | 101.7 163.9 | 9.5 | $-0.9120-03$ $-0.7060-03$ | $0.204 D-02$ $0.2020-02$ | $0.3100-03$ $0.1250-03$ | 169.2 | 0.1870-02 |
|  | 4 | elastic | -15.2 | 104.8 | 7.5 | -0.6550-03 | 0.2090-02 | $0.2400-03$ | 173.4 |  |
| 39 | 1 | ELASTIC | -4.5 | 156.9 | 5.2 | -0.6910-03 | 0.1950-02 | $0.1700-03$ | 159.5 | 0.1760-02 |
|  | $\frac{2}{3}$ | ELASTIC |  | 155.9 153.3 | 10.1 | $-0.7220-03$ $-0.7200-03$ | 0.1940-02 | 0.1320-03 | 160.5 | 0.1770-02 |
|  | 3 | ELASTIC | $-6: 3$ -10.4 | 153.3 | 10.2 | $-0.720 \mathrm{D}-03$ $-0.7740-03$ | $0.1970-02$ $0.2000-02$ | $0.1730-03$ $0.3350-03$ | 161.9 165.6 | $0.1790-02$ $0.1830-02$ |
|  |  | Elastic | -1.6 | 155.7 | 3.5 | -0.6510-03 | 0.1920-02 | 0.1150-03 | 156.6 | 0.1730-02 |
| 43 | 2 | ELASEIC | -2.6 -3.1 | 151.5 150.1 | 7.1 | -0.6460003 $-0.6710-03$ | 0.1870-02 | $0.233 \mathrm{D}-03$ | 153.3 | 0.170002 |
|  | 4 | ELASTIC | -5:2 | 153.6 | $9: 3$ | -0.6800-03 | 0.1930-02 | $0 \cdot 15304003$ | 157.9 | $0.1750-02$ $0.1740-02$ |
|  |  |  |  |  |  |  |  |  |  |  |
| 41 |  | ELASTIC | -0.0 | 157.5 | 0.6 | $-0.639 \mathrm{D}-03$ <br> -0.6180 O <br> 0 l | $0.194 D-02$ $0.1970-02$ | $0.2030-04$ $0.4550-04$ | 157.5 | 0.174D-02 |
|  | 3 | ELASTIC | -0.7 | $155: 9$ | $2: 3$ | -0.6410 ${ }^{-0.63}$ | 0.1920002 | 0.761 D-OU | 156:3 | -17880-02 |
|  | 4 | Elastic | -1.2 | 151.0 | 4.3 | -0.626D-03 | 0.186D-02 | 0.161 D-03 | 151.8 | 0.1680-02 |
| 42 |  | Eldssic | $-2.0$ | 144.8 | -13.3 | -0.6120-03 | $0.1790-02$ | -0.2700-03 | 146.5 | 0.1620-02 |
|  | $\frac{2}{3}$ | ELASTIC | -3.0 | 134.4 | -13.5 | $-0.581 \mathrm{D}-03$ $-0.5930-03$ | $0.1660-02$ | -0.4410-03 | 137.9 | $0.1520-02$ |
|  | U | ELASTIC | -0.3 | 133.1 | -4.0 | -0.5440-03 | $0.1640-02$ | -0.1308-03 | 133.4 | 0.1480 -02 |
| 43 |  | Elastic |  | 150.9 |  | -0.7200-03 | 0.1890-02 | -0.493D-03 | 157.7 | 0.1740-02 |
|  | 2 | ELASTIC | -12.9 | 147.4 | -23.6 | -0.75 ${ }^{\text {a }}$ - 0.03 | $0.1860-02$ | -0.772D-03 | 159.5 | $0.1760-02$ |
|  | 3 | ELASTIC | -4.5 | 145.7 | -11.9 | -0.6500-03 | $0.182 \mathrm{D}-02$ | -0.389D-03 | 150.4 | $0.1660-02$ |
|  | 4 | Elastic | -6. 8 | 138.6 | -19.1 | -0.646D-03 | 0.173D-02 | -0.623D-03 | 145.9 | $0.161 \mathrm{D}-02$ |
| 44 |  | ELASTIC | -16.9 | 161.4 | -15.3 | -0.8620-03 | $0.2050-02$ | -0.502D-03 | 172.5 | $0.190 \mathrm{D}-02$ |
|  | 2 | ELASTIC | -24.3 | 105.2 155.4 | -23.1 | - $0.9699 \mathrm{D}=03$ $-0.7910 \mathrm{l}-03$ | 0.2130-02 | $-0.7560-03$ $-0.5240-03$ | 183.0 | 0.2010-02 |
|  | 4 | Elastic | -18.0 | 154.9 | -24.0 | - $0.791 \mathrm{D}-03$ $-0.8500-03$ | - 0.196002 | -0.524D-03 | 164:2 | $0.1810-02$ $0.1870-02$ |
|  |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \end{aligned}$ |  |  |  |  |  |  |  | $-0.3150-03$ |  |  |
|  |  | ELASTIC | - 12.8 | 179.1 | -13.7 -13.5 | $-0.1120-02$ -0.916003 | $0.233 \mathrm{D}-02$ $0.212 \mathrm{D}-02$ | $=0.447 \mathrm{D}-03$ -0.441003 | 198.6 178.0 | $0.2180-02$ $0.1960-02$ |
|  |  | ELASTIC | -28.4 | 172.0 | -19.9 | -0.105D-02 | $0.223 \mathrm{D}-02$ | -0.6490-03 | 190.9 | 0.210002 |
|  |  |  |  |  |  | (3.2) |  |  |  |  |


|  |  | Elasilc | -24.8 | 173.9 | -0.7 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\frac{2}{3}$ | ELASTIC | - 34.2 | 174.0 | -0:0 | -0.117D-02 | $0.224 D-02$ $0.240 \mathrm{D}-02$ | $-0.2230-04$ $0.1075-05$ | 187.5 | 0.2060-02 |
|  | 4 | ELASTIC | -24.3 | 172:6 | -6.0 | -0.998D-03 $-0.116 D-02$ | $0.22200-02$ $0.2380-02$ | -0.1960-03 | 186.2 | $0.2230-02$ 0.205002 |
| 46 |  |  |  |  |  |  |  |  | 202.0 | 0.2220-02 |
|  | 1 | ELASTIC | - 22.4 | 171.3 | 8.1 | -0.970D-03 | 0.2200-02 |  |  |  |
|  | $\frac{1}{3}$ | ELASTIC | -30.0 -24.3 | 1773.2 | 12.8 | -0.1100-02 | $0.2320-02$ | $0.418 D=03$ | 197:2 | 0.217D-02 |
| 47 | 4 | ELASTE | -32:9 | 183.1 | $5 \cdot 3$ | $-0.100 D-02$ $-0.11>0-02$ | $0.2230-02$ $0.2380-02$ | $0.1070-03$ $0.1930-03$ | 186.9 201.9 | 0.206002 $0.2220-02$ |
|  | 1 | ELASTIC | -10.7 | 163.9 | 13.7 |  |  |  |  |  |
|  | 2 | ELASTIC | -21.4 | 167.4 | 20.5 | -0.943D-03 | 0.208D-02 | $0.4490-03$ $0.6750-03$ | 174.5 | $0.1920-02$ $0.2010-02$ |
|  | 4 | Elastic | -20.9 | 174.8 | 11.0 16.9 | -0.933D-03 | $0.215 D-02$ $0.226 D-02$ | $0.3600-03$ $0.5520-03$ | 190.5 191.9 | $0.1990-02$ $0.2110-02$ |
| 48 |  | Elastic |  |  |  |  |  |  |  | - |
|  | 2 | ELAS $=1 \mathrm{C}$ | -12:5 | 152.9 | 14.5 | $-0.7475-03$ $-0.7740-03$ | $0.1940-02$ $0.1930-02$ | $0.4790-03$ | 161.8 | $0.179 \mathrm{D}-02$ |
|  |  | Elastic | -13.7 | 160.0 | 14:9 | -0.8150 03 | -0.2020-02 | $0.7070-03$ $0.4820-03$ | 163.8 | 0.1810-02 |
|  |  |  | -17.9 | 101.2 | 21.9 | -0.8740-03 | $0.2050-02$ | $0.716 \mathrm{D}-03$ | 175.1 | $\bigcirc 0.1870002$ |
| 49 | 1 | CLhSTIC | -3.5 | 147.5 | 10.5 | -0.6410-03 | 0.1830-02 |  |  |  |
|  | $\frac{2}{3}$ | ELhSTIC | -4.4 | 140.6 | 15.5 | -0.6255-03 | $0.1750-02$ | $0.3420-03$ $0.508 D-03$ | 150.4 | 0.1660-02 |
|  |  | ELAS运 | -6.8 | 151.1 | 13.5 | -0.6960-03 $-0.7040-03$ | 0.189D-02 | $0.4400-03$ | 156.4 | $0.1730-02$ |
| 50 |  |  |  |  |  |  | 0.1840-02 | 0.6490-03 | 155.5 | 0.1723-02 |
|  | 1 | ELASTIC | -0.1 | 146.2 | 2.1 | -0.595D-03 | 0.1800-02 | 0.6920-04 | 146.3 | 0.1620-02 |
|  | 3 | ELRSTIC | -0.2 | 135.6 | 7:3 | $-0.5520-03$ $-0.6110-03$ | 0.1670002 | $0.1060-03$ | 135.3 | $0.1500-02$ |
|  |  | elastic | -2:0 | 137.2 | 11:0 | -0.58ido-03 | $0.180 D-02$ $0.1690-02$ | $0.2380-03$ $0.3590-03$ | 1478.2 | $0.1630-02$ $0.1540-02$ |
| 51 | 1 | ELASTIC | -4.1 | 123.3 | -13.7 | -0.5500-03 | 0.1530-02 | -0.6130-03 |  |  |
|  | 3 | ELASTEC | -5.7 -0.3 | 102.1 | -28.7 | $-0.4840-03$ $-0.4900-03$ | 0.128002 | -0.94400-03 | 116.3 | $0.1280-02$ |
| 52 |  | ELastic | -0.7 | 91.5 | - -8.5 | -0.4900-03 $-0.3790-03$ | 0.147002 | -0.152003 $-0.2800-03$ | 120.3 93.1 | $0.1330-02$ $0.1030-02$ |
|  | $\frac{1}{2}$ | ELASTIE | -17.0 | 143.8 | -31.9 | -0.7920-03 | 0.184D-02 | -0.104D-02 |  |  |
|  | $\frac{2}{3}$ | EiASTIC | -23.6 | 139.7 131.0 | -47.1 | $=0.8530-03$ $-0.6400-03$ | 0.180002 | -0.1540-02 | 172.4 | $0 \cdot 1790-02$ |
|  | 4 | Elastic | -12.8 | 115.7 | -26.1 | $-0.640 \mathrm{D}-03$ $-0.627 \mathrm{D}-03$ | $0.165 D-02$ $0.1470-02$ | -0.8530-03 $-0.1290-02$ | 142.9 | $0 \cdot 1580 \mathrm{D}-02$ |
| 53 |  |  |  |  |  |  | 0.1470-02 | -0.1290-02 | 140.3 | 0.1540-02 |
|  | $\frac{1}{2}$ | ELASTIC | - 31.3 | 169.3 | $-30.3$ | -0.107D-02 | 0.2210-02 | -0.9910-03 |  |  |
|  | 3 | ELAS IC |  | 155.1 | -42.9 | $-0.125 D-02$ $-0.9150-03$ | $0.2370-02$ | -0.140.0-02 | 217:0 | $0.2380-02$ |
|  |  | Elasiic | -32.4 | 156.5 | -47:8 | -0:1030-02 | 0.2060 -02 | $-0.1030-02$ $-0.1500-02$ | 1777.4 | 0.1950-02 |
| 54 | 1 | ELASFIC | - 40.7 | 183.2 | -17.1 | -0.1200-02 | $0.2480-02$ | -0.558D-03 |  |  |
|  | 3 | ELisiIC | -36.1 | 178:8 | -22.0 | -0.150D-02 $-0.1170-02$ | $0.2760-02$ $0.2340-02$ | $-0.7210-03$ $-0.8420-03$ | 241.4 | $0.2650-02$ |
| 55 | 4 | elastic | -48.8 | 193.1 | - 35.3 | -0.1380-02 | 0.257D-02 | $-0.8420-03$ $-0.1150-02$ | 20493 | 0.2240002 $0.2520-02$ |
|  |  | Elasfic | -42.1 | 194.2 | 1.2 |  |  |  |  |  |
|  |  | ELASIIC | -53.8 | 213.2 | 4:8 | -0.1 $-0.1500-02$ $-0.1500-02$ | 0.2540-02 | $0.3890-04$ $0.1560-03$ | 218.3 244.8 | $0.2400-02$ 0.26902 |
|  |  | Elastic | -53.4 | 212.2 | -10.5 | -0.1300-02 | 0.254D-02 | $-0.3100-03$ $-0.3450-03$ | 217.3 | 0.2390-02 |

(B.2)

| 57 | 1 2 3 4 | ELASTIC ELASTIC ELAJIC ELASIC | -36.1 -44.0 -40.4 -50.4 | 187.0 200.2 192.4 209.8 | 17.5 26.9 8.7 15.3 | $-0.1200-02$ $-0.1350-02$ $-0.1290-02$ $-0.1470-02$ | $\begin{aligned} & 0.244 \mathrm{D}-02 \\ & 0.264 \mathrm{D}-02 \\ & 0.253 \mathrm{D}-02 \\ & 0.276 \mathrm{D}-02 \end{aligned}$ | $\begin{aligned} & 0.5730-03 \\ & 0.880003 \\ & 0.2850-03 \\ & 0.5010-03 \end{aligned}$ |  | $\begin{aligned} & 0.2300-02 \\ & 0.2530-02 \\ & 0.2370-02 \\ & 0.2640-02 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 | 1 | ELASTIC | -25.8 | 170.6 | 27.1 | -0.1010-02 | 0.2200-02 |  |  |  |
|  | 2 | ELAETIC | -29.8 | 175.2 | 39.4 | -0.1080-02 | $0.2270-02$ | 0.8800-03 | 190.7 | $0.2100-02$ 0.22302 |
|  | 3 | ELAJIIC | - 32.1 | 180.6 | 22.5 | -0.1130-02 | $0.235 \mathrm{D}-02$ | $0.1739 D-03$ 0.7390 .02 | 202.5 | 0.223D-02 |
|  | 4 | ELASTIC | -37.9 | 190.6 | 33.3 | -0.124D-02 | $0.2500-02$ | $0.1090-02$ | 219.3 | 0.2410-02 |
| 59 | 1 | ELASIIC | -14.5 | 151.0 | 29.0 | -0.791 $5-03$ | 0.1910-02 | $0.9140-03$ | 166.0 | $0.183 \mathrm{D-02}$ |
|  | 2 | ELASIC | -15.8 | 146.9 | 38.0 | -0.7900-03 | $0.1970-02$ | $0.1240-02$ | 166.0 | $0.1830-02$ $0.1860-02$ |
|  | 3 | ELASTIC | -20.9 | 102.3 | 23.0 | -0.9155-03 | $0.203 \mathrm{D}-\mathrm{C} 2$ | $0.93 \div 0-03$ | 180.6 | 0.1990-02 |
|  | 4 | ELAEIIC | -23.4 | 103.2 | 39.7 | -0.7505-03 | $0.210 \mathrm{D}-02$ | 0.1300-02 | 189.0 | 0.2080-02 |
|  | 1 | ELASTIC | -5.2 | 133.9 | 20.1 | -0.6Ј5J-03 | 0.167D-02 |  |  |  |
| 60 | 2 | ELASEIC | -5.4 | 121.7 | 20.9 | -0.0.5635-03 | $0.157 \mathrm{D}-02$ | $0.6570-03$ $0.8780-03$ | 140.9 132.9 | $0.1560-02$ $0.1470-02$ |
|  | 3 | ELASTIC | -10.1 | 143.1 | 25.7 | -0.7040-03 | $0.1800-02$ | 0.8390-03 | 132.9 154.9 | $0.1470-02$ $0.1710-02$ |
|  | 4 | ELASTIC | -10.9 | 135.4 | 34.5 | -0.683D-03 | $0.171 \mathrm{D}-\mathrm{C} 2$ | $0.11130-02$ | 153.3 | $0.1690-02$ |
| 51 | , | ELASTIC | -0. 2 | 124.9 | 4.2 | -0.5090-03 | 0.154D-02 | 0.1390-03 | 125.2 | $0.1390-02$ |
|  | 2 | ELASEIC | -0.2 | 105.2 | 5.9 | -0.4230-03 | $0.1290-02$ | $0.1880-03$ | 105.7 | $0.1170-02$ |
|  | 3 | ELdSTIC | -2.3 | 120.0 | 14.3 | -0.5490-03 | $0.1590-02$ | $0.4650-03$ | 132.1 | $0.1460-02$ |
|  | 4 | ELASIIC | -2.5 | 113.1 | 19.2 | -0.4590-03 | $0.1400-02$ | $0.629 \mathrm{D}-03$ | 119.1 | $0.1320-02$ |
| 02 |  | ELASTIC | -7.4 | 90.4 | -39.6 | -0.4170-03 |  |  |  |  |
|  | $\frac{2}{3}$ | ELASTIC | -9.7 | $40 \cdot 9$ | - 55.8 | -0.2860-03 | $0.5420-03$ | -0.1250-02 | 107.6 | $\begin{aligned} & 0.1180-02 \\ & 0.1170-02 \end{aligned}$ |
|  | 4 | ELASEIC | -0.6 | 62.6 | -11.9 | -0.2610-03 | $0.772 \mathrm{D}-03$ | -0.3880-03 | 166.2 | $0.7320-03$ |
|  | 4 | ELASIIC | -1.1 | 5.2 | -17.6 | -0.342D-04 | 0.683D-04 | -0.5740-03 | 31.0 | $0.3380-03$ |
| 63 | 1 | ELASTIC | -29.9 | 134.0 | -51.4 | -0.9110-03 | $0.1770-02$ | -0.2010-02 | 184.8 | 0.2020-02 |
|  | $\frac{2}{3}$ | ELASTIC | - 30.1 | 124.3 | - 35.7 | -0.983D-03 | $0.173 \mathrm{D}-02$ | -0.2800-02 | 211.8 | $0.231 \mathrm{D}-02$ |
|  | 4 | ELASTIC | -16.1 | 102.0 | -52.2 | -0.6110-03 | $0.132 \mathrm{D}-02$ | -0.1710-02 | 143.1 | $0.1570-02$ |
|  |  | -ras!-C | -20.6 | 76.3 | -74.0 | -0.563D-03 | $0.102 \mathrm{D}-02$ | -0.2420-02 | 155.3 | $0.1700-02$ |
| 04 | 1 | ELASTIC | -52.7 | 189.5 | -54.0 | -0.142D-02 | $0.2540-02$ | -0.1760-02 |  |  |
|  | 2 | ELASTIC | -05.9 | 212.3 | -72.2 | -0.167D-02 | $0.288 \mathrm{D}-02$ | -0.2360-02 | 239.6 | $0.2630-02$ $0.308 D-02$ |
|  | 3 | ELASTIC | -40.3 | 159.3 | -61.5 | -0.114D-02 | $0.212 \mathrm{D}-02$ | -0.2010-02 | 211.6 | $0.2320-02$ |
|  | 4 | ELASTIC | -51.2 | 166.5 | -84.4 | -0.130D-02 | $0.225 \mathrm{D}-02$ | -0.2760-02 | 245.4 | $0.268 \mathrm{D}-02$ |
| 05 | 1 | ELASTIC | -65.0 |  |  |  |  |  |  |  |
|  | 2 | ELASTIC | -73.3 | 262.0 | -29.4 | -0.2040-02 | 0.3540-02 | $-0.8320-03$ $-0.9500-03$ |  |  |
|  | 3 | ELASTIC | -59.6 | 200.2 | - 43.0 | -0.20.1580-02 | -0.2800-02 | -0.1420 | 313.5 254.9 | $0.3440-02$ $0.2790-02$ |
|  | 4 | ELASTIC | -74.4 | 233.6 | $-55.6$ | -0.1880-02 | 0.3230-02 | -0.1820-02 | 299.2 | $0.3280-02$ |
| 60 | 1 | ELASTIC | $-62.5$ | 231.8 | 9.2 | -0.1710-02 | 0.3100-02 | 0.301 D-03 |  |  |
|  | 2 | ELASEIC | -72.5 ${ }^{\text {1 }}$ | 264.8 | 20. 5 | -0.1970-02 | $0.3550-02$ | $0.6710-03$ | 269.0 | 0.2950-02 |
|  | 3 | ELASTIC | -65.8 | 231.6 | -10.5 | -0.1750-02 | $0.311 \mathrm{D}-02$ | -0.343D-03 | 271.1 | 0. $297 \mathrm{D}-02$ |
|  | - | ELASTIC | -78.2 | 269.2 | -6.3 | -0.205D-02 | $0.363 \mathrm{D}-02$ | -0.222D-03 | 315.8 | $0.346 \mathrm{D}-02$ |
| 67 |  | ELASEIC | -48.6 |  |  | -0.146D-02 |  |  |  |  |
|  | 2 | EASTIC | $\begin{aligned} & -49.6 \end{aligned}$ | 230.5 | 54.9 | $-0.1540-02$ | $0.3030-02$ | $\begin{aligned} & 0.1190-02 \\ & 0.1790-02 \end{aligned}$ | 248.6 275.8 | $\begin{aligned} & 0.2730-02 \\ & 0.3030-02 \end{aligned}$ |
|  | 3 | ELASTIC | -57.8 | 225.8 | 22.3 | $-0.163 \mathrm{D}-02$ | 0.3010-02 | $0.7290-03$ | 262.4 | $0.2880-02$ |
|  |  | ELASTIC | -63.0 | 254.2 | 37.9 | -0.1800-02 | $0.338 \mathrm{D}-02$ | $0.1240-02$ | 298.1 | 0.327D-02 |
| 1 |  | ELASTIC | -30.8 | 179.7 | 48.7 | -0.1100-02 | 0.2320-02* | $0.159 \mathrm{D-02}$ |  |  |
|  |  | ELASTIC | -25.5 | 180.6 | 65.4 | -0.1050-02 | 0.2320-02 | $0.2140-02$ | 225.2 | $\begin{aligned} & 0.234 D-02 \\ & 0.247 D-02 \end{aligned}$ |




|  | $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 3 \end{aligned}$ | elastic |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ELASTIC | 0.7 | -13.2 | 1.95 | $0.8740-04$ 0.808004 | $-0.1210-03$ $-0.2260-03$ | 0.1440003 0.495004 | 12.6 | $0.1380-03$ 0.2077003 0.1050 |
|  |  | ELASTIC | 5:7 7.5 | 19 <br> 0.3 <br> .9 | 17.5 8.6 | $-0.1030-04$ $0.5730-04$ | $0.22110-03$ $0.7230-04$ | $0.5710-03$ $0.2810-03$ | $35: 0$ 16.3 | 0.3850-03 |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  | 1 | ELASTIC | 7.4 | -202.8 | -107.7 | $0.913 \mathrm{D-03}$ | -0.2520-02 | -0.3520-02 | 278.3 | 0.3060-02 |
|  | 2 | ELASTIC | 20.1 | -254.2 | -89.1 | $0.128 \mathrm{D}-02$ | -0.3210-02 | -0.2910-02 | 306.5 | $0.3330-02$ |
|  | 4 | ELASIIC | 0.0 | -387.8 | -35.4 | $0.1580-02$ $0.1940-02$ | $-0.4770-02$ $-0.5780-02$ | $-0.116 D-02$ -0.95503 | 393 473 | $0.435 D-02$ $0.5240-02$ |
|  |  |  |  |  |  |  |  |  |  |  |
|  | 2 | ELASTIC | 24.5 | 91.4 | -169.7 | -0.6910-04 | $0.1020-02$ | -0.555D-02 | 305.2 | $0.333 \mathrm{D}-02$ |
|  | $\frac{2}{3}$ | ELASTIC | 81.1 | 67.5 -73.7 | -159.2 -140.6 | $0.7230-03$ $0.5050-03$ | $0.501 D-03$ $-0.1030-02$ | $-0.4910-02$ $-0.4600-02$ | 270.8 258.6 | 0.2970-02 |
|  | 4 | ELAStic | 41.3 | -115:0 | -117:7 | $0.3740-03$ | -0.158D-02 | -0.385D-02 | 247:7 | 0.2700-02 |
| 9 |  |  |  |  |  |  |  |  |  | - |
|  |  | elastic | 16.2 | 377.2 | -170.9 | -0.133D-02 | 0.457D-02 | -0.5590-02 | 473.4 | 0.522D-02 |
|  | 2 | ELASTIC | 115.5 | 389.0 | -168.6 | -0.140.0-03 | $0.4310-02$ | -0.5510-02 | 452.6 | $0.5030-02$ |
|  | 4 | \%LASITC | 106.7 | 213.9 204 | -173.6 -148.2 | -0.551 0.03 | 0.2520-02 | $-0.5370-02$ $-0.5500-02$ | 371.1 341.0 | 0.4070-02 |
| 94 |  |  |  |  |  |  |  |  |  |  |
|  | 1 | ELASTIC | -40.3 | 540.7 | -25.3 | -0.2950-02 | $0.730 \mathrm{D}-02$ | -0.8290-03 | 603.4 | $0.666 \mathrm{D}-02$ |
|  | 2 | ELASTIC | 83.3 -14.7 | 733.7 | -59.1 -131.4 | $-0.189 D-02$ $-0.2220-02$ | 0.865D-02 | $-0.1930-02$ $-0.4300-02$ | 700.3 | $0.78 C D-02$ $0.615 D-02$ |
|  | 4 | ELASTIC | 113.3 | 561.7 | -171.0 | -0.83SD-03 | $0.62440-02$ | -0.5590-02 | 593:6 | 0.650002 |
| 95 |  |  |  |  |  |  |  |  |  |  |
|  |  | ELASTIC | 11.3 | 524.8 | 151.8 | $=0.1990-02$ | $0.6400-02$ | $\begin{aligned} & 0.495 \mathrm{D}-02 \\ & 0.4090-02 \end{aligned}$ | 582.0 | $\begin{aligned} & 0.6430-02 \\ & 0 . \\ & 7550-07 \end{aligned}$ |
|  | $\frac{2}{3}$ | ELASTIC | 37.7 -23.5 | 538.9 594.0 | 216.7 61.0 | $\begin{aligned} & -0.1930-02 \\ & -0.2660-02 \end{aligned}$ | $\begin{aligned} & 0.7090002 \\ & 0.7270-02 \end{aligned}$ | $\begin{aligned} & 0.7090-02 \\ & 0.1990-02 \end{aligned}$ | 633.4 605.4 | $\begin{aligned} & 0.7550-02 \\ & 0.6690-02 \end{aligned}$ |
|  |  | ELAS:IC | -10.5 | 714:9 | 47.4 | -0.2770-02 | $0.3750-02$ | $0.1550-02$ | 714:5 | 0:7920-02 |
| 30 |  | ELASTIC |  |  |  |  |  |  |  |  |
|  | 2 | ELASTIC | $173: 2$ | 169.1 | 154.7 | $0.1450-02$ | $0.1360-02$ | $0.5390-02$ | 332.5 | 0.3690-02 |
|  | 3 | Elastic | 40.3 | 357.9 | 189.7 | -0.9890-03 | $0.436 \mathrm{D}-02$ | $0.6200-02$ | 479.5 | $0.5290-02$ |
| 97 | 4 | ELASTIC | 133.0 | 414.2 | 295.7 | $0.5750-03$ | $0.435 \mathrm{D}-02$ | 0.9670-02 | 625.8 | 0.6920-02 |
|  |  | Elastic |  |  |  | 0.7100-03 | 0.6690-03 | 0.3180-02 |  | 0.2080-02 |
|  | 2 | ELASTIC | -99.0 | 177.2 | 59.7 | $0.9430-03$ | $0.9620-04$ | $0.1950-02$ | 129.1 | 0.143D-02 |
|  | 4 | ELASTIC | 101.7 | 174.7 86.4 | 148.4 | $0.541 \mathrm{D}-03$ 0.13802 | $0.173 \mathrm{D}-02$ $0.4900-03$ | 0.4850-02 | 298.6 | 0.330D-02 |
| 98 |  |  |  |  |  |  |  |  |  |  |
|  |  | Elastic | 45.5 | 35.6 | 36.7 | $0.4150-03$ | $0.253 \mathrm{D}-03$ | 0.1200-02 | 75.8 | 0.3410-03 |
|  | 3 | ELASTIC | $\frac{33}{53} 9$ | 14.7 50.8 | 15.6 64.3 | $0.3520-03$ $0.580 D-03$ | $0.451 \mathrm{D}-04$ | $0.5110-03$ | 139:7 | 0.4430-03 |
|  | 4 | Elajilc | 60.8 | 30.8 | 37.7 | $0.521 \mathrm{D}-03$ | $0.137 \mathrm{D}-03$ | $0.1230-02$ | 129:9 | 0.9320-03 |
| 9 |  |  |  |  |  |  |  |  |  |  |
|  |  | ELASTIC | 15.6, | 12.6 | 7.7 | 0.1400-03 | $0.923 \mathrm{D-04}$ | $0.2510-03$ | 19.5 | 0.2200-03 |
|  | $\frac{2}{3}$ | ELASTIC |  | 23.5 | -31.4 | $0.9620-04$ | 0.6290-05 | $0.110 D-03$ $-0.113 D-03$ | 47.8 | $0.1090-03$ $0.524 D-03$ |
|  | 4 | ELASTIC | 22:3 | 10.9 | 2.1 | $0.237 \mathrm{D}-03$ | $0.413 \mathrm{D}-04$ | $0.199 \mathrm{D}-03$ | 22.4 | $0.253 \mathrm{D}-03$ |
| 10 |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | -1.1 | $0.901 . \mathrm{D}-\mathrm{OU}$ | -0.2400-03 | -0.354D-04 | 19.8 | $0.219 \mathrm{D}-03$ |
|  | 2 | ELASTIC | 0.9 | -14.8 | -3.9 | 0.7940-04 | $=0.1850-03$ $-0.6170-05$ | -0.123D-03 | 16.6 | 0.183 DD 03 |
|  | 3 | ELASTIC | 4.3 | -1.6 | -6.1 | 0.5890-04 | -0.367D-04 | -0.1990-03 | 11.8 | - $0.129 \mathrm{D}-03$ |
| 101 |  |  |  |  |  |  |  |  |  |  |
|  | 1 | ELASTIC | 23.9 | -282.0 -305 | -64.4 | $0.1500-02$ $0.1700-02$ | $-0.3580-02$. $-0.3910-02$. | -0.2110002 $-0.647 D-03$ | 317.7 | $0.3500-02$ $0.3610-02$ |
|  | 2 | LLASTIC | 37.6 | $-305.6$ | -19.8 | $0.1700-02$ | -0.3910-02 |  | 327.8 | $0.3610-02$ |


| 102 | J | ELASTIC | 2:6 | -513.6 | $-21.1$ | $0.2140-02$ $0.2290-02$ | $-0.6380-02$ $-0.0810-02$ | $\begin{aligned} & -0.6390-03 \\ & -0.2060-03 \end{aligned}$ | 521.2 5551 | $0.5770-02$ $0.6150-02$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | ELASTIC | 122.0 | 41.4 | -111.6 | $0.133 \mathrm{D}-02$ | 0.1350-04 | -0.3550-02 |  |  |
|  | 2 | Etasilc | 150.7 | 34.6 | -31:7 | $0.1610-02$ | -0.2190-03 | -0.1040-02 | 154.6 | 0.1730-02 |
|  | 4 | einsisc | 60.2 80.9 | -140.1 -157.4 | -85.1 | 0.133D-02 | -0.199D-02 | -0.273D-02 | 234.5 | 0.2560002 |
| 103 |  |  |  |  |  |  |  |  |  | C.2340-02 |
|  | 2 | ELASTIC | 237.1 | 425.5 | -153.5 | 0.1190-02 | 0.4280-02 | -0.5020-02 | 455.7 | $0.514 \mathrm{D}-02$ |
|  | 3 | ELAJTIC | 1359.7 | 337.7 1310 | 1559.8 -134.7 | $0.2970-02$ 0.140020 | 0.2940-02 | -0.2150-02 | 376.4 | $0.434 \mathrm{D}-02$ |
|  | 4 | Elastic | 231.0 | 157:8 | -44:3 | 0.2200-02 | $0.100 \mathrm{D}-02$ | -0.1450-02 | 218.4 | 0.2500-02 |
| 104 |  | elaseic | 307.9 | 791.8 | -100.5 | 0.5720-03 | $0.84 \mathrm{CD}-02$ | -0.3293-02 | 713.0 | 0.8070-02 |
|  | 2 | PLASTIC | 577.7 | 934.4 | -133.5 | 0.3650.02 | 0.128001 | -0.1653-02 | 834.6 | $0.1370-61$ |
|  |  | ELASTIC | 240.6 403.0 | 543.5 534.3 | -149.8 -47 | 0.1230-02 | 0.5530-02 | -0.4900-02 | 54.202 | 0.6130002 |
| 105 |  |  |  |  |  |  |  | -0.1540-02 |  | 0.639D-02 |
|  | 1 | PLASTIC | 514.7 | 962.9 | 4.3 | $0.953 \mathrm{D}-02$ | $0.1770+00$ | 0.3390-03 | 834.5 | $0.2070 \times 00$ |
|  | $\frac{1}{3}$ | PLASTiC | 6437.4 | 939.1 | $-34.5$ | -0.6270-01 | $0.2050 \times 00$ | -0.3530-01 | 834.6 | $0.2770+00$ |
|  |  | PListic | 578:6 | 900.0 | 17.9 | -0.02510-04 | O. 30 CD -01 | - $0.4250-02$ | 834.5 834.6 | $0.2290-01$ $0.3440-01$ |
| 106 |  | Finstic |  | 737.7 |  | 0.5790-03 | 0.9660-02 | -1020-01 |  |  |
|  | 2 | PLASTIC | 337.2 | 925.5 | 129.3 | -0.1350-01 | $0.1500+00$ | $0.8010-01$ | 834:0 | $0.1690+00$ |
|  |  | PLASIIC | 370.2 | 933.0 | 107.1 | -0.2970-03 | $0 \cdot 1600-01$ | $0.5940-02$ | 834.6 | $0.1590-01$ |
| 107 | 4 |  |  | 956.3 | 58.7 | -0.324D-02 | $0.1730+00$ | $0.3300-01$ | 834.6 | $0.1960+00$ |
|  |  | ELASTIC | 209.7 | 33.0 | 165.4 | $0.2440-02$ | -0.440D-03 | 0.5410-02 | 346.7 | $0.3810-02$ |
|  | ${ }_{3}$ | PLASTIC | 293.2 256.7 | 913.7 | 112.0 | -0.2050-01 | 0.1120000 | 0.5520-01 | 834.6 | $0.1210+00$ |
|  | 4 | flasilc | 390.3 | 342.8 | -36.9 | -0.1310-01 | $0.1270+00$ | $0.5050-01$ | 834.5 | $0.6719 \mathrm{D}+00$ |
| 108 |  | blajtic | 153.6 | 45.0 | 104.2 | 0.1710-02 | -0.698D-04 | 0.3410-02 |  | 0.2500-02 |
|  | ${ }_{3}^{2}$ | ELASEIC | 33.9 526.6 | 311:4 |  | $0.4670-03$ | $-0.1910-.33$ $-0.7810-04$ | 0.3155-03 | 42.1 | $0.4640-03$ |
|  | 4 | eiasioc | 230:8 | 313:0 |  | $0.2700 \mathrm{D}-02$ | $-0.7300-03$ | - 0.1190 -02 | 834.6 225.2 | $0.6630-01$ $0.2510-02$ |
| 109 |  | Elastic | 55.4 |  |  |  |  |  |  |  |
|  | 2 | ELASTIC | -25.1 | 21.0 | 26.9 | -0.3410-03 | $0.334 D-04$ $0.1670-03$ | $0.8790-03$ $0.1270-03$ | 67.2 29.7 | $0.7480-03$ $0.3260-03$ |
|  | 4 | ELASTIC | 109.1 | 3 O .5 | 61.9 18.1 | $0.1170-02$ $0.3482-05$ | $0.343 D-04$ $0.782 \mathrm{D}-07$ | 0. 20202002 | 143.1 | 0. 0.1590002 |
| 110 |  |  |  |  |  |  |  |  |  | 0.3420-03 |
|  |  | ELASTIC | 15.4 | 5.4 | 0.9 | 0.1070-03 | 0.364D-05 | $0.2640-04$ | 13.6 | $0.1540-03$ |
|  | 3 | ELASTIC | -31.1 34.3 | -0.9 | -2.3 | $-0.3720-03$ $0.3900-03$ | $0.1150-03$ $-0.4430-04$ | $-0.7423-04$ $0.5640-03$ | 30.9 | $0.342 \mathrm{D}-03$ |
|  |  | Elastic | -31.1 | 0.1 | $-2.1$ | -0.3330-03 | -0.1280-03 | -0.7020-04 | 31.4 | -0.348D-03 |
| 111 |  |  |  |  |  | -0.7230-06 | 0.8140-05 | -0.2640-03 | 14.0 | 0.1530 |
|  | 2 | ELASTIC | -10.2 | -1.1 | -5.0 | -0.1210-03 | 0.2810-04 | -0.163D-03 | 13.0 | $0.1440-03$ |
|  |  | ELASTIC | - 23.7 | 1.9 | -4:8 | $0.3750-04$ $-0.2940-03$ | $0.4120-04$ $0.1080-03$ | $-0.1580-03$ $-0.1300-03$ | 25.7 | 0.107D-03 |
| 112 |  |  |  | 1.0 | -4.2 | -0.2940-03 | $0.108 \mathrm{D}-03$ | -0.1380-03 | 25.2 | 0.278D-03 |
|  | 1 | ELASTIC | -1.5 | -8.4 | -3.6 | 0.1550-04 | -0.975D-04 | -0.1170-03 | 10.0 | 0.1100-03 |
|  | $\frac{2}{3}$ | ELASTIC | -0.3 | -2.5 | -1.7 | 0.6420-05 | $=0.294 D-04$ $-0.418 D-04$ | $=0.5490-04$ $-0.2930-03$ | 13.7 | 0.4110004 |
|  |  | Elastic | -3.4 | -1.2 | -3.7 | -0.3630-04 | -0.1190-05 | -0.1200-03 | 7.0 | 0.7700004 |

```
V A L U E S O F J - I N T E G R A L
    INTEGAATION PATH NO J
    1 0.6533840+02
    2 0.1103230+03
    3 0.114524D+03
    4 0.113701D+03
    S 0.1163160+03
    6 0.1175560+03
    7 0.1132440+03
    8 2.113374D+03
AVREAGE J-INTEGRAL (IGNORING THE FIRST 1 PATYS) = 0.1155770+03
```



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S O L U T I O H T I M E L O G
            FOR probley
Edge NOTCH SPECIMEN MESH :5
IMPUT ह̈nse . . . . . . . . . . . . . . . . . . 0.0
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talangularization OF EFFECtIVE StIFFNESS mataIx 0.0
SIEP-aY-STER SOLUTION ( 2 TIAE STEPS)
    CALCULATION Of EFFECTIVE LORD VECTOZS • 0.0
    UPDATING EFFECTIVE STIFENESS MATRICES
    SONDIOAD VECTORSNOR NONIINEARITIESS: 0. 0.0
    EJUILIEFIUM ITERATIONS : 0.0. 0.0
    CALCULATION AHD PAINTING"OF DISPLACEE "
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                        STEP-BY-STEZ TOTAL 0.0
T O T A L S O L U T I O N T I & & . . . . . 0.0
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40.1 EOL.
451

Chad

CASD




[^0]:    2.2.1. Virtual Work Expression

    When tne principles of virtual work and virtual complementary work are applied to deformable solids, it is

[^1]:    
    

[^2]:    
    

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[^4]:    DI
    NCDE

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    X-DISPLACEMEST
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    | DSPLACEMEX | Z-DISPLACEAEH |
    | :---: | :---: |
    | 0.8467560-01 | $0.2191030+00$ |
    | 0.840344001 | $0.2347540+00$ |
    | $0.8337253-01$ | $0.2503430+00$ |
    | $0.8274940-01$ | $0.265933 \mathrm{D}+00$ |
    | $0.321055 \mathrm{D}-01$ | $0.281523 \mathrm{D}+00$ |
    | $0.5140130-01$ | $0.2971160+00$ |
    | C.83a173D-01 | $0.3127110+00$ |
    | $0.1404950+00$ | -0.5246690-02 |
    | $0.1302010+00$ | 0.2599780-01 |
    | $0.1379120+00$ | 0.572501D-01 |
    | $3.136525 c+00$ | $0.884963 \mathrm{D}-01$ |
    | $0.13533 \mathrm{c}+00$ | 0.119729 +00 |
    | $0.1340490+00$ | $0.1509450+00$ |
    | $0.1327610+00$ | $0.1821440+00$ |
    | $0.1314730+00$ | $0.2133270+00$ |
    | 0. 130185000 | $0.2444970+00$ |
    | $0.1203970 \rightarrow 00$ | $0.275661 \mathrm{D}+00$ |
    | $0.1275030+00$ | $0.306833 \mathrm{D}+00$ |
    | $0.1873720+00$ | -0.1115030-01 |
    | 0.186723 D 00 | 0.448757D-02 |
    | $0.1863750+00$ | $0.2013050-01$ |
    | $0.185429 \mathrm{D}+00$ | 0.3576890-01 |
    | $0.1847530+00$ | $0.5140580-01$ |
    | $0.184134 \mathrm{D}+00$ | $0.670362 \mathrm{D}-01$ |
    | 0.183484000 | $0.826617 \mathrm{D}-01$ |
    | $0.1828310+00$ | $0.982819 \mathrm{D}-01$ |
    | $0.1821760+00$ | $0.113896 \mathrm{D}+00$ |
    | $0.1815190+00$ | $0.129507 \mathrm{D}+00$ |
    | $0.180851 \mathrm{D}+00$ | $0.145112 \mathrm{D}+00$ |
    | $0.1802040+00$ | $0.1607140+00$ |
    | $0.1795470+00$ | $0.176311 \mathrm{D}+00$ |
    | $0.178392 \mathrm{D}+00$ | $0.1919040+00$ |
    | $0.1782380+50$ | $0.207492 \mathrm{D}+00$ |
    | $0.1775890+00$ | $0.2230740+00$ |
    | $0.175939 \mathrm{D}+00$ | $0.2386520+00$ |
    | $0.1752730+00$ | $0.2542240+00$ |
    | $0.1750450+00$ | $0.263794 \mathrm{D}+00$ |
    | $0.1749990 \cdot 00$ | $0.285360 \mathrm{D}+00$ |
    | $0.1743500+00$ | $0.300931 \mathrm{D}+00$ |
    | $0.2343360+00$ | -0.170838D-01 |
    | $0.2330390+00$ | $0.1426110-01$ |
    | $0.231737 \mathrm{D}+00$ | $0.455640 \mathrm{D}-01$ |
    | $0.2304095+00$ | $0.758196 \mathrm{D}-01$ |
    | $0.229051 \mathrm{D}+00$ | $0.108043 \mathrm{~J}+00$ |
    | $0.2276740+00$ | $0.1392530+00$ |
    | $0.2262950+00$ | 0.170456D*00 |
    | $0.2249380+00$ | $0.2016470+00$ |
    | $0.2230100+00$ | $0.232808 \mathrm{D}+00$ |
    | $0.2223080+00$ | 0. $2639260+00$ |
    | $0.2210090+00$ | $0.2950010+00$ |
    | $0.2314690+00$ | -0.2299530-01 |
    | $0.2808250+00$ | -0.7265940-02 |

    
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    2-ROTATION

