THE EFFECT OF CREEP

ON THE STRESSES

IN THE WALLS OF A PRESSURE VESSEL

by

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Submitted in Partial Fulfillment of the Requirements

for the Degree of

Master of Science in Engineering

in the

Civil Engineering

Program

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YOUNGSTOWN STATE UNIVERSITY

March, 1972

ABSTRACT

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It is the purpose of this thesis to consider the effect of creep on the stresses in the walls of a pressure vessel caused by variations in temperature between the inside and outside walls of the vessel. A simple model whose behavior is the same as the wall of the pressure vessel when subjected to the imposed load is developed and equations relating to the solution of the problem are presented for the case when the material creep law is of the form $\dot{\varepsilon} = K_N \sigma^N$. The equations are then solved for N having values of 1 and 2. While the study does not attempt to solve equations for N = 3, 4, and 5, due to the mathematical complexity of the resulting equations, the author does discuss the determination of the initial stress on the model and also develops a formula to determine asymptotic values for the stress on the model when $t \rightarrow \infty$ for all values of N.

ACKNOWLEDGEMENTS

The author would like to extend his thanks to everyone who helped in the completion of this thesis. Special appreciation is extended to Dr. Russel Hibbeler, the author's advisor, and to Dr. John Cernica without whose helpful suggestions the work would not have been completed.

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LIST OF SYMBOLS

SYMBOL	DEFINITION
A	=Cross-sectional area of each bar of the two-bar mechanism
A_s	= Stress asymptote
E	= Young's modulus for the material
K	= Constant in the linear creep law
K_N	= Constant in the non-linear steady-state creep law
N	= Exponent in the non-linear steady-state creep law
P	= Net radial pressure difference between inner and outer wall of vessel
R	= Mean radius of vessel wall
t	= Wall thickness and time parameter
W	= Loading on the two-bar model
α	= Coefficient of thermal expansion
αT	= Thermal expansion in member
ΔT	= Temperature drop across wall thickness
ν	= Poisson's ratio for the vessel material
σ_o	= Constant in linear creep law

SUBSCRIPTS

A	= Reference to inside section A of the model
B	= Reference to outside section B of the model
r	= Radial direction
z	= Axial direction
θ	= Circumferential direction

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CHAPTER I

INTRODUCTION

It is the purpose of this thesis to consider the effect of creep on the stresses in the walls of a pressure vessel caused by variations in temperature between the inside and outside walls of the vessel.

This subject is not a new one. Any railroad workman knows that couplers between freight cars will break much more quickly on a cold day than on a hot one. Yet this may be a temperature differential of approximately only fifty degrees centigrade. If variation in performance is so noticeable with such a small temperature change, then it is reasonable to assume there will be more radical effects on materials that are subjected to temperature changes in the hundreds of degrees, as for example, a nuclear reactor whose fuel elements may have a temperature of seven hundred degrees centigrade inside the cladding wall.

A look at a modern jet engine can give more insight into the problems of creep. The engine is very hot on the inside where turbines are running at a rate of 25,000 rpm and at the same time is rather cool on the exterior surface. As the turbine blades are heated to high temperatures they begin to elongate or creep, eventually touching the engine casing. Subjecting the engine to repeated heating and cooling as the power settings are changed and the engine is shut down and started up again reduces engine life.

While the subject of creep is not new, it has become increasingly important as operating temperatures and pressures increase in mechanical and chemical equipment. In turbine blades, steam lines, high-pressure boilers, and other similar applications failure can result readily from creep unless the proper factors are considered in the original design and the proper materials are used.

Before 1925, little was known regarding the strength of metals at high temperatures. Ordinary tensile tests have been found to be of little value in determining the effects of creep because the problems of creep tend to be associated with moderate loads over long periods of time rather than those of higher loads, applied rapidly, as in the usual short-time tensile test.[1] At temperatures higher than 540 C, in particular, the discrepancy between short-time and long-time tests is very marked.[2]

Worthwhile tests involving creep are very time consuming, taking in the neighborhood of 10,000 hours to be considered reliable. At the present time no truly successful method has been found for speeding up the process of creep testing, nor is there a method yet known for predicting

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[1]Carl H. Samans, Metallic Materials in Engineering (New York, Macmillan Company, 1963), pp. 146-147

[2]Carl H. Samans, Engineering Metals and Their Alloys (New York, Macmillan Company, 1949), p. 228

results or data accurately, although mathematical analysis is attempting, with some success, to alleviate the problem.[3]

One such attempt was made by Dr. R. Hibbeler, assistant professor at Youngstown State University and T. Mura of Northwestern in 1968, in their paper entitled, "Viscous Creep Ratchetting of Nuclear Reactor Fuel Elements", which was concerned with the analysis of creep ratchetting in nuclear reactors under the influence of variable internal pressure forces and long-time cyclic thermal loading.[4] In their study, Hibbeler and Mura used a linear creep law of the form

$$\dot{\varepsilon}_c = K(\sigma - \sigma_o) \qquad (1)$$

where K and σ_o were assumed known constants. They argued that it is possible to correlate the constants K_N and N of a more general non-linear steady state creep law

$$\dot{\varepsilon} = K_N \sigma^N \qquad (2)$$

with those constants K and σ_o in the linear creep law they used by taking a linear approximation within the region between the stress bounds as shown in Fig. 1.

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[3] Samans, Metallic Materials, p. 147

[4] R. Hibbeler and T. Mura, "Viscous Creep Ratchetting of Nuclear Reactor Fuel Elements", Nuclear Engineering and Design, (Amsterdam, North-Holland Publishing Company, 1969) pp. 131-143

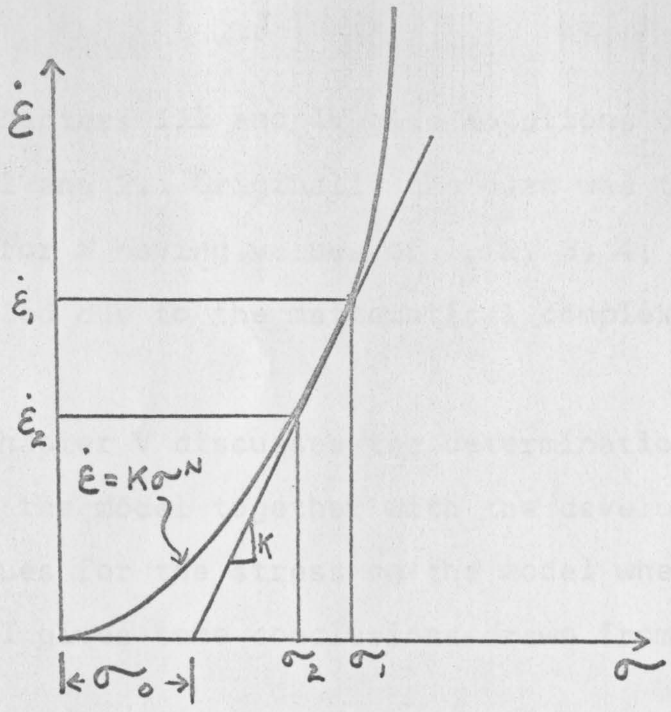

Fig 1.--Two-point Interpolation[5]

Comparing results obtained from examples using both the linear and non-linear forms of creep analysis for the bounded part of the curve, they showed that the linear analysis gives a close estimate of the radial strain in a pressure vessel.[6]

In their paper the authors suggested that a similar study might be conducted involving pressure vessels using, however, the non-linear creep law stated in equation (2). This thesis represents such an attempt.

In order to consider the stresses on the walls of a pressure vessel, a simple model whose behavior is the same is developed in Chapter II.

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[5]Hibbeler, p. 140

[6]Hibbeler, p. 141

Chapters III and IV give solutions of equation (2) when N = 1 and 2. Originally the plan was to solve equation (2) for N having values of 1, 2, 3, 4, and 5, but this was modified due to the mathematical complexity for N = 3, 4, and 5.

Chapter V discusses the determination of the initial stress on the model together with the development of asymptotic values for the stress on the model when $t \to \infty$, while Chapter VI gives some conclusions drawn from the analysis.

CHAPTER II

DEVELOPMENT OF THE MODEL

Creep strain is only one of three types of strain which compose the total strain on a body, the other two being thermal and elastic strain. Creep strain is defined as the plastic elongation of a material with time and is normally temperature-dependent; that is, the higher the temperature, the faster the rate of creep strain.[7] Therefore, when the interior surface of a pressure vessel is very hot and the exterior cool by comparison, the creep rate for the material varies from the inside to the outside and has a much faster creep rate on the inside.

In order to develop the necessary equations needed to consider the effect of creep strain on the stresses in the walls of a pressure vessel, a simple model must be produced, whose properties are similar to a section of the pressure vessel. Two initial assumptions must be made:

1. The wall of the pressure vessel is thin compared to the diameter of the vessel.
2. Plane strain holds.

The stress distribution across the thickness of the vessel wall becomes uniform when subjected to a net internal

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[7] J. D. Lubahn and R. P. Felger, Plasticity and Creep of Metals (New York, John Wiley and Sons, Inc., 1961), p. 129

pressure and has a magnitude of

$$\sigma_\theta = \frac{PR}{t} \qquad (3)$$

$$\sigma_z = \frac{PR}{2t} \qquad (4)$$

in the hoop and axial directions, respectively, where "P" is the net difference between the radial contact pressure at the inside and outside surfaces of the wall, "t" is the thickness of the wall, and "R" represents its mean radius. Noting that the value of σ_θ is always twice that of σ_z and, therefore a more dominant stress, the influence of σ_z will be ignored in this study. In addition, because the wall of the vessel is thin, the effect of σ_r can be neglected.

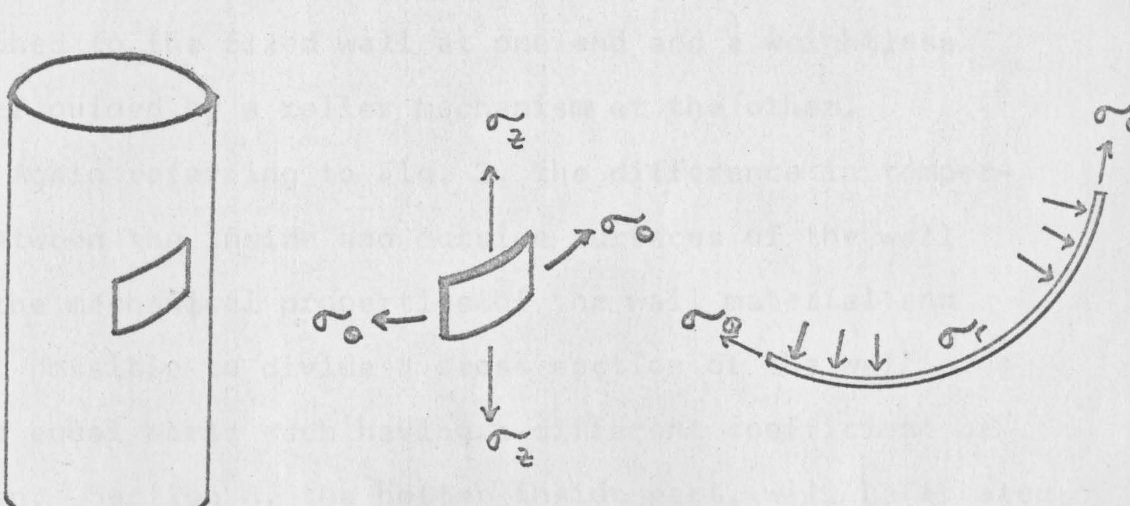

Fig. 2--Section of the Pressure Vessel

For a thin cylinder having a temperature drop of ΔT across its walls, the thermal stress distribution is linear, and can be represented by the equation

$$\sigma_\theta = \pm \frac{E\alpha\Delta T}{2(1-\nu)} \tag{5}$$

where E, α, and ν are the modulus of elasticity, the coefficient of thermal expansion, and Poisson's ratio, respectively, for the material composing the wall of the pressure vessel.

Consider a section of the wall material as shown in Fig. 3. The vessel wall will expand uniformly outwards due to the symmetry of internal pressure. Therefore, elements lying along radial lines through the thickness will experience a constant hoop strain. This uniform strain behavior can be preserved in the model by requiring that the model be attached to the fixed wall at one end and a weightless rigid bar guided by a roller mechanism at the other.

Again referring to Fig. 3, the difference in temperature between the inside and outside surfaces of the wall alters the mechanical properties of the wall material and makes it possible to divide a cross section of the wall area into two equal parts each having a different coefficient of expansion. Section A, the hotter inside part, will be treated as a different material, having a different creep rate, than the cooler outside section B. The correspondence of the material behavior in the model is preserved by requiring

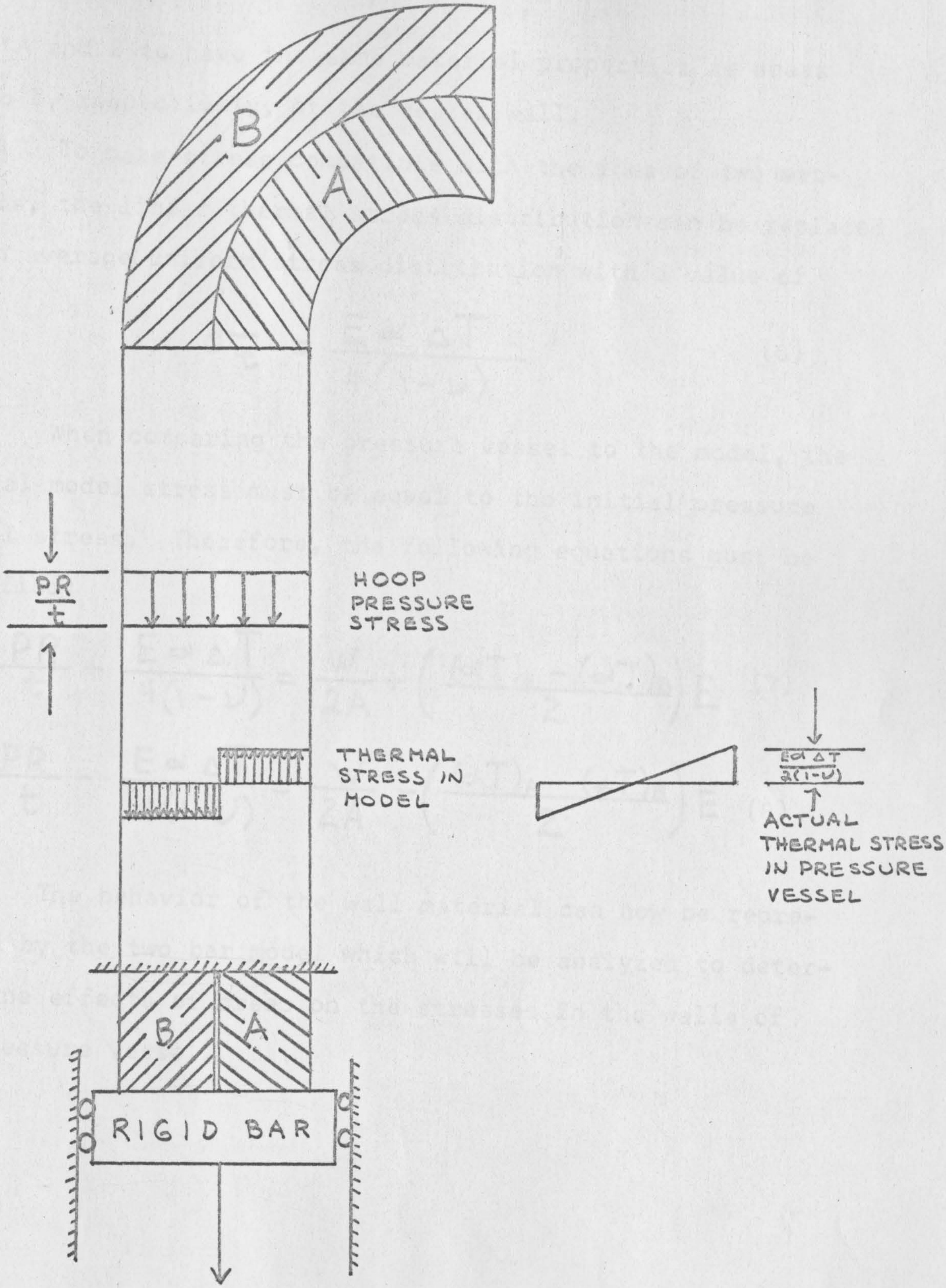

Fig. 3 - Proposed two-bar model showing the internal stresses as developed in the wall of the pressure vessel

bars A and B to have the same material properties as areas A and B, respectively, of the vessel wall.

To make stress compatible with the idea of two materials, the linear thermal stress distribution can be replaced by an average uniform stress distribution with a value of

$$\sigma_t = \frac{E\alpha\Delta T}{4(1-\nu)} \tag{6}$$

When comparing the pressure vessel to the model, the initial model stress must be equal to the initial pressure vessel stress. Therefore, the following equations must be satisfied:

$$\frac{PR}{t} + \frac{E\alpha\Delta T}{4(1-\nu)} = \frac{W}{2A} + \left(\frac{(\alpha T)_A - (\alpha T)_B}{2}\right)E \tag{7}$$

$$\frac{PR}{t} - \frac{E\alpha\Delta T}{4(1-\nu)} = \frac{W}{2A} - \left(\frac{(\alpha T)_A - (\alpha T)_B}{2}\right)E \tag{8}$$

The behavior of the wall material can now be represented by the two bar model which will be analyzed to determine the effects of creep on the stresses in the walls of the pressure vessel.

CHAPTER III

N = 1

The creep law which is to be considered in this analysis is of the form

$$\frac{d\varepsilon_c}{dt} = \dot{\varepsilon}_c = K\sigma^N \tag{9}$$

where K and N are known constants for a given material and σ represents the stress. A quick look at the formula tells us that the creep rate increases exponentially. Therefore, our analysis will be limited to values of N = 1 and N = 2, because the resulting equations using larger values of N are unreasonably complex to solve by the method used here.

Consideration of our model yields several basic equations that are required for a further look into the subject:

equilibrium equation: $\sigma_A + \sigma_B = \frac{W}{A}$ (10)

compatibility equation: $\varepsilon_A = \varepsilon_B$ (11)

total strain (ε) for each member:

$$\varepsilon = \frac{\sigma}{E} + \alpha\Delta T + \varepsilon_c \tag{12}$$

in which αT represents thermal strain, ε_c is the creep strain as developed in equation (9), and $\frac{\sigma}{E}$ is the strain resulting from the pressure in the vessel.

The preceding equations (10 through 12) make it possible to analyze the trend of stresses on the two sections of the model and ultimately to determine the initial and final stress on the wall of the pressure vessel as a result of the effects of creep.

An equation for stress versus time can be derived by taking the derivative of equation (12)

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{d\varepsilon_c}{dt} \tag{13}$$

and substituting equation (9) into (13)

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + K\sigma^N \tag{14}$$

Relating equation (14) to material A yields

$$\frac{d\varepsilon_A}{dt} = \frac{1}{E}\frac{d\sigma_A}{dt} + K_A\sigma^N \tag{15}$$

Similarly, for material B we get

$$\frac{d\varepsilon_B}{dt} = \frac{1}{E}\frac{d\sigma_B}{dt} + K_B\sigma^N \tag{16}$$

The derivative of equation (11) gives

$$\frac{d\varepsilon_A}{dt} = \frac{d\varepsilon_B}{dt} \qquad (17)$$

Therefore, substituting (15) and (16) into (17) yields

$$\frac{1}{E}\,\frac{d\sigma_A}{dt} + K_A\,\sigma_A^{\,N} = \frac{1}{E}\,\frac{d\sigma_B}{dt} + K_B\,\sigma_B^{\,N} \qquad (18)$$

Rewriting equation (10) and taking its derivative we get

$$\sigma_A = \frac{W}{A} - \sigma_B \qquad (19)$$

$$\frac{d\sigma_A}{dt} = -\,\frac{d\sigma_B}{dt} \qquad (20)$$

Substitution of these values into (18) produces

$$\frac{d\sigma_B}{dt} + EK_B\,\sigma_B^{\,N} = -\,\frac{d\sigma_B}{dt} + EK_A\left(\frac{W}{A} - \sigma_B\right)^N \qquad (21)$$

which can be rewritten as

$$\frac{d\sigma_B}{K_A\left(\frac{W}{A} - \sigma_B\right)^N - K_B\,\sigma_B^{\,N}} = \frac{E}{2}\,dt \qquad (22)$$

Taking the integral of equation (22) yields

$$\int \frac{d\sigma_B}{K_A\left(\frac{W}{A} - \sigma_B\right)^N - K_B\,\sigma_B^{\,N}} = \frac{E}{2}\,t + C_1 \qquad (23)$$

The following represents the solution of equation (23) when N = 1.

$$\int \frac{d\sigma_B}{\frac{W}{A}K_A - \sigma_B(K_A+K_B)} = \frac{E}{2}t + C_1 \quad (24)$$

$$-\int \frac{d\sigma_B}{(K_A+K_B)\sigma_B + \left(-\frac{W}{A}K_A\right)} = \frac{Et}{2} + C_1 \quad (25)$$

From a table of integrals

$$\int \frac{dx}{ax+b} = \frac{1}{a} Ln\,(ax+b) \quad (26)$$

$$\therefore -\frac{1}{K_A+K_B} Ln\left[(K_A+K_B)\sigma_B + \left(-\frac{W}{A}K_A\right)\right] \quad (27)$$

$$= \frac{Et}{2} + C_1$$

At t = 0, $\sigma_B = \sigma_{B1}$

$$\therefore C_1 = \quad (28)$$

$$-\frac{1}{K_A+K_B} Ln\left[(K_A+K_B)\sigma_{B1} - \frac{W}{A}K_A\right]$$

$$\therefore Ln\left[(K_A+K_B)\sigma_B - \frac{W}{A}K_A\right] = \tag{29}$$

$$-\frac{Et}{2}(K_A+K_B) + Ln\left[(K_A+K_B)\sigma_{B1} - \frac{W}{A}K_A\right]$$

$$Ln\left[\frac{(K_A+K_B)\sigma_B - \frac{W}{A}K_A}{(K_A+K_B)\sigma_{B1} - \frac{W}{A}K_A}\right] = -\frac{Et}{2}(K_A+K_B) \tag{30}$$

$$\frac{(K_A+K_B)\sigma_B - \frac{W}{A}K_A}{(K_A+K_B)\sigma_{B1} - \frac{W}{A}K_A} = e^{-\frac{Et}{2}(K_A+K_B)} \tag{31}$$

$$(K_A+K_B)\sigma_B - \frac{W}{A}K_A = \tag{32}$$

$$(K_A+K_B)\sigma_{B1}e^{-\frac{Et}{2}(K_A+K_B)} -$$

$$\frac{W}{A}K_A e^{-\frac{Et}{2}(K_A+K_B)}$$

$$\sigma_B = \frac{W}{A}\frac{K_A}{(K_A+K_B)} - \frac{W}{A}\frac{K_A}{(K_A+K_B)} e^{-\frac{Et}{2}(K_A+K_B)} + \sigma_{B1} e^{-\frac{Et}{2}(K_A+K_B)} \tag{33}$$

Equation (33) gives a value for the stress on section B of the model as time varies. To compute simulated values for σ_B, substitutions in the equation can be made for the constants as indicated in Table 1.

Elastic Modulus (E) (ksi)	20×10^3
Poisson's ratio (ν)	0.3
Constant K in creep law (in./in.-hr-ksi)	
K_A	5.2×10^{-8}
K_B	6.4×10^{-9}
Load (W) (kips)	10
Area (A) (in^2)	1.0

Table 1

In actual computation, the initial stress on section B (σ_{B1}), was varied from 7 psi. to 12 psi. As "t" became large for each value of σ_{B1}, σ_B was found to reach a final value of 8.904 psi. (See Figure 4.) This indicates that section B of the model takes a stress of approximately 9 psi. or .9 of the load placed on the wall of the cylinder.

It must be emphasized that these results do not correspond to any actual case, but merely indicate that the stress on section B will eventually reach some constant value.

Time	$\sigma_{B1}=7$	$\sigma_{B1}=8$	$\sigma_{B1}=9$	$\sigma_{B1}=10$	$\sigma_{B1}=12$
0.0	7.00	8.00	9.000	10.00	12.00
5.0	7.00	8.00	9.000	10.00	12.00
10.0	7.01	8.00	9.000	10.00	12.98
50.0	7.05	8.03	8.997	9.97	12.91
100.0	7.11	8.05	8.995	9.94	11.82
200.0	7.21	8.10	8.989	9.88	11.66
250.0	7.26	8.12	8.987	9.85	11.58
300.0	7.31	8.15	8.985	9.82	11.50
400.0	7.40	8.19	8.980	9.77	11.36
500.0	7.48	8.23	8.976	9.72	11.22
750.0	7.68	8.32	8.966	9.61	10.90
1000.	7.84	8.40	8.958	9.52	10.63
1250.	7.99	8.47	8.950	9.43	10.39
1500.	8.11	8.53	8.944	9.36	10.10
2000.	8.30	8.62	8.934	9.34	9.87
2500.	8.46	8.69	8.926	9.16	9.62
3000.	8.57	8.48	8.921	9.09	9.44
5000.	8.80	8.86	8.909	8.96	9.07
7500.	8.88	8.89	8.905	8.92	8.94
10000.	8.90	8.90	8.904	8.91	8.91

Table 2 - Values of σ_B using the constants from Table 1

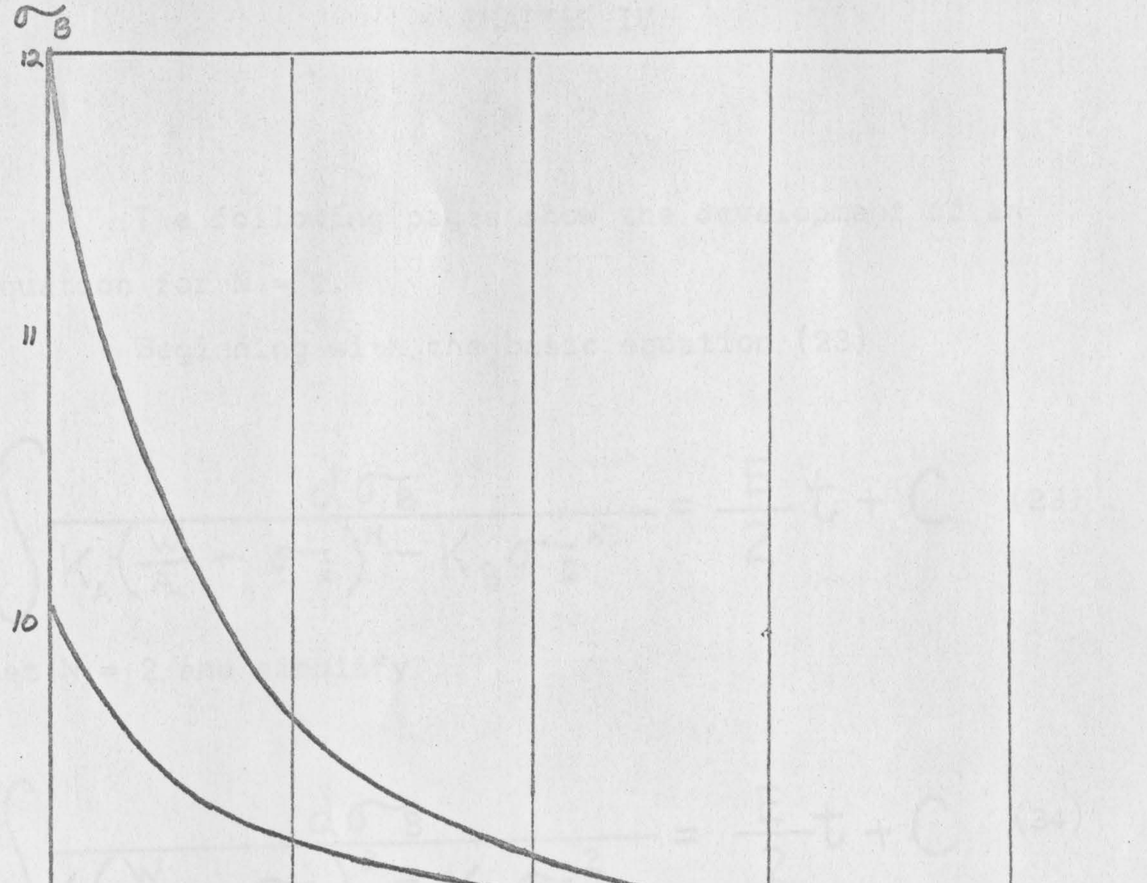

Fig. 4 - Graph of σ_B versus time for N = 1

CHAPTER IV

N = 2

The following pages show the development of an equation for N = 2.

Beginning with the basic equation (23)

$$\int \frac{d\sigma_B}{K_A\left(\frac{W}{A}-\sigma_B\right)^N - K_B\sigma_B^{\,N}} = \frac{E}{2}t + C \qquad (23)$$

Let N = 2 and simplify

$$\int \frac{d\sigma_B}{K_A\left(\frac{W}{A}-\sigma_B\right)^2 - K_B\sigma_B^{\,2}} = \frac{E}{2}t + C \qquad (34)$$

$$\int \frac{d\sigma_B}{K_A\left(\frac{W^2}{A^2}-2\sigma_B\frac{W}{A}+\sigma_B^{\,2}\right) - K_B\sigma_B^{\,2}} = \frac{E}{2}t + C \qquad (35)$$

$$\int \frac{d\sigma_B}{(K_A-K_B)\sigma_B^{\,2} - 2K_A\frac{W}{A}\sigma_B + K_A\frac{W^2}{A^2}} = \frac{E}{2}t + C \qquad (36)$$

Because section A of the model is hotter than section B, we may conclude that $K_A > K_B$.

$$\therefore 4K_A^2 \frac{W^2}{A^2} > 4(K_A - K_B) K_A \frac{W^2}{A^2} \tag{37}$$

and $K_A > K_A - K_B$.

Relating the general formula $ax^2 + bx + c$ to equation (36), we can see that $b^2 > 4ac$ by equation (37). From mathematical tables the general equation for integrating an equation like (36), in which $b^2 > 4ac$, is

$$\int \frac{dx}{ax^2 + bx + c} = \frac{1}{\sqrt{b^2 - 4ac}} \times \mathrm{Ln}\left[\frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}}\right] \tag{38}$$

The left side of equation (36) then becomes

$$\frac{1}{4\left(K_A^2 \frac{W^2}{A^2} - 4(K_A - K_B) K_A \frac{W^2}{A^2}\right)} \times \mathrm{Ln}\left[\frac{2(K_A - K_B)\sigma_B - 2K_A \frac{W}{A} - 2\frac{W}{A}\sqrt{K_A K_B}}{2(K_A - K_B)\sigma_B - 2K_A \frac{W}{A} + 2\frac{W}{A}\sqrt{K_A K_B}}\right]$$

which simplifies as

$$\frac{1}{2\frac{W}{A}\sqrt{K_A K_B}} \times$$

$$Ln\left[\frac{2(K_A - K_B)\sigma_B - 2K_A\frac{W}{A} - 2\frac{W}{A}\sqrt{K_A K_B}}{2(K_A - K_B)\sigma_B - 2K_A\frac{W}{A} + 2\frac{W}{A}\sqrt{K_A K_B}}\right]$$

and the total equation is

$$C + \frac{Et}{2} = \frac{1}{2\frac{W}{A}\sqrt{K_A K_B}} \times$$

$$Ln\left[\frac{K_A\sigma_B - K_B\sigma_B - K_A\frac{W}{A} - \frac{W}{A}\sqrt{K_A K_B}}{K_A\sigma_B - K_B\sigma_B - K_A\frac{W}{A} + \frac{W}{A}\sqrt{K_A K_B}}\right] \quad (39)$$

At t = 0, $\sigma_B = \sigma_{B1}$

$$C = \frac{1}{2\frac{W}{A}\sqrt{K_A K_B}} \times$$

$$Ln\,\frac{(K_A - K_B)\sigma_{B1} - K_A\frac{W}{A} - \frac{W}{A}\sqrt{K_A K_B}}{(K_A - K_B)\sigma_{B1} - K_A\frac{W}{A} + \frac{W}{A}\sqrt{K_A K_B}} \quad (40)$$

$$\therefore \frac{Et}{2} = \frac{1}{2\frac{W}{A}\sqrt{K_A K_B}} \times$$

$$\operatorname{Ln}\left[\frac{(K_A-K_B)\sigma_B - K_A\frac{W}{A} - \frac{W}{A}\sqrt{K_A K_B}}{(K_A-K_B)\sigma_B - K_A\frac{W}{A} + \frac{W}{A}\sqrt{K_A K_B}}\right] - \tag{41}$$

$$\frac{1}{2\frac{W}{A}\sqrt{K_A K_B}}\operatorname{Ln}\left[\frac{(K_A-K_B)\sigma_{B1} - K_A\frac{W}{A} - \frac{W}{A}\sqrt{K_A K_B}}{(K_A-K_B)\sigma_{B1} - K_A\frac{W}{A} + \frac{W}{A}\sqrt{K_A K_B}}\right]$$

and

$$\frac{WEt}{A}\sqrt{K_A K_B} = \operatorname{Ln}\left[\frac{\left[(K_A-K_B)\sigma_B - K_A\frac{W}{A} - \frac{W}{A}\sqrt{K_A K_B}\right]}{\left[(K_A-K_B)\sigma_B - K_A\frac{W}{A} + \frac{W}{A}\sqrt{K_A K_B}\right]} \times \right. \tag{42}$$

$$\left.\frac{\left[(K_A-K_B)\sigma_{B1} - K_A\frac{W}{A} + \frac{W}{A}\sqrt{K_A K_B}\right]}{\left[(K_A-K_B)\sigma_{B1} - K_A\frac{W}{A} - \frac{W}{A}\sqrt{K_A K_B}\right]}\right]$$

$$e^{\frac{W}{A}Et\sqrt{K_A K_B}} = \frac{\left[(K_A-K_B)\sigma_B - K_A\frac{W}{A} - \frac{W}{A}\sqrt{K_A K_B}\right]}{\left[(K_A-K_B)\sigma_B - K_A\frac{W}{A} + \frac{W}{A}\sqrt{K_A K_B}\right]} \times \tag{43}$$

$$\frac{\left[(K_A-K_B)\sigma_{B1} - K_A\frac{W}{A} + \frac{W}{A}\sqrt{K_A K_B}\right]}{\left[(K_A-K_B)\sigma_{B1} - K_A\frac{W}{A} - \frac{W}{A}\sqrt{K_A K_B}\right]}$$

Therefore

$$\left[(K_A-K_B)\sigma_B - K_A\frac{W}{A} + \frac{W}{A}\sqrt{K_AK_B}\right]\times$$
$$\left[(K_A-K_B)\sigma_{B1} - K_A\frac{W}{A} - \frac{W}{A}\sqrt{K_AK_B}\right]e^{\frac{W}{A}Et\sqrt{K_AK_B}} = \quad (44)$$
$$\left[(K_A-K_B)\sigma_B - K_A\frac{W}{A} - \frac{W}{A}\sqrt{K_AK_B}\right]\times$$
$$\left[(K_A-K_B)\sigma_{B1} - K_A\frac{W}{A} + \frac{W}{A}\sqrt{K_AK_B}\right]$$

and

$$\Big\{(K_A-K_B)^2\sigma_B\sigma_{B1} - K_A(K_A-K_B)\frac{W}{A}\sigma_{B1} +$$
$$\frac{W}{A}(K_A-K_B)\sqrt{K_AK_B}\,\sigma_{B1} - K_A(K_A-K_B)\frac{W}{A}\sigma_B +$$
$$K_A^2\frac{W^2}{A^2} - \frac{W^2}{A^2}K_A\sqrt{K_AK_B} - \frac{W}{A}\sqrt{K_AK_B}(K_A-K_B)\sigma_B +$$
$$\frac{W^2}{A^2}K_A\sqrt{K_AK_B} - \frac{W^2}{A^2}K_AK_B\Big\}e^{\frac{W}{A}Et\sqrt{K_AK_B}} = \quad (45)$$
$$\Big\{(K_A-K_B)^2\sigma_B\sigma_{B1} - K_A(K_A-K_B)\frac{W}{A}\sigma_{B1} -$$
$$\frac{W}{A}(K_A-K_B)\sqrt{K_AK_B}\,\sigma_{B1} - K_A(K_A-K_B)\frac{W}{A}\sigma_B +$$
$$K_A^2\frac{W^2}{A^2} + \frac{W^2}{A^2}K_A\sqrt{K_AK_B} + \frac{W}{A}\sqrt{K_AK_B}(K_A-K_B)\sigma_B -$$
$$\frac{W^2}{A^2}K_A\sqrt{K_AK_B} - \frac{W^2}{A^2}K_AK_B\Big\}$$

$$
\begin{aligned}
&\left\{(K_A-K_B)^2\sigma_{B1} - K_A(K_A-K_B)\frac{W}{A} - \right.\\
&\left.\frac{W}{A}\sqrt{K_A K_B}\,(K_A-K_B)\right\}\sigma_B\, e^{\frac{W}{A}Et\sqrt{K_A K_B}} - \\
&\left\{(K_A-K_B)^2\sigma_{B1} - K_A(K_A-K_B)\frac{W}{A} + \right.\\
&\left.\frac{W}{A}\sqrt{K_A K_B}\,(K_A-K_B)\right\}\sigma_B = \\
&-\left\{-K_A(K_A-K_B)\frac{W}{A}\sigma_{B1} + \frac{W}{A}(K_A-K_B)\times \right.\\
&\sqrt{K_A K_B}\,\sigma_{B1} + \frac{K_A^2 W^2}{A^2} - \frac{W^2}{A^2}K_A\sqrt{K_A K_B} + \qquad (46)\\
&\left.\frac{W^2}{A^2}K_A\sqrt{K_A K_B} - \frac{W^2}{A^2}K_A K_B\right\} e^{\frac{W}{A}Et\sqrt{K_A K_B}} + \\
&\left\{-K_A(K_A-K_B)\frac{W}{A}\sigma_{B1} - \frac{W}{A}(K_A-K_B)\times \right.\\
&\sqrt{K_A K_B}\,\sigma_{B1} + K_A^2\frac{W^2}{A^2} + \frac{W^2}{A^2}K_A^2\sqrt{K_A K_B} - \\
&\left.\frac{W^2}{A^2}K_A\sqrt{K_A K_B} - \frac{W^2}{A^2}(K_A K_B)\right\}
\end{aligned}
$$

Therefore

$$\sigma_B = \Big[\Big\{-K_A(K_A-K_B)\frac{W}{A}\sigma_{B1} - \frac{W}{A}(K_A-K_B)\sqrt{K_AK_B}\,\sigma_{B1} + K_A^{2}\frac{W^2}{A^2} - \frac{W^2}{A^2}K_AK_B\Big\} - \Big\{-K_A(K_A-K_B)\frac{W}{A}\sigma_{B1} + \frac{W}{A}(K_A-K_B)\sqrt{K_AK_B}\,\sigma_{B1} + K_A^{2}\frac{W^2}{A^2} - \frac{W^2}{A^2}K_AK_B\Big\}e^{\frac{W}{A}Et\sqrt{K_AK_B}}\Big] \div \Big[\Big\{(K_A-K_B)^2\sigma_{B1} - K_A(K_A-K_B)\frac{W}{A} - \frac{W}{A}\sqrt{K_AK_B}(K_A-K_B)\Big\}e^{\frac{W}{A}Et\sqrt{K_AK_B}} - \Big\{(K_A-K_B)^2\sigma_{B1} - K_A(K_A-K_B)\frac{W}{A} + \frac{W}{A}\sqrt{K_AK_B}(K_A-K_B)\Big\}\Big] \tag{47}$$

To simplify, let $C_2 = K_A - K_B$; $C_3 = K_A$; $C_4 = \sqrt{K_AK_B}$

Substituting, we get:

$$\sigma_B = \left[\left\{ -C_2 C_3 \sigma_{B1} - C_2 \frac{W}{A} C_4 \sigma_{B1} + C_3^2 - \frac{W}{A} C_3 K_B \right\} - \left\{ -C_2 C_3 \sigma_{B1} - C_2 \frac{W}{A} C_4 \sigma_{B1} + C_3^2 - C_3 \frac{W}{A} K_B \right\} e^{\frac{W}{A} E t C_4} \right] \div \left[\left\{ C_2^2 \sigma_{B1} - C_2 C_3 - \frac{W}{A} C_4 C_2 \right\} e^{\frac{W}{A} E t C_4} - \left\{ C_2^2 \sigma_{B1} - C_2 C_3 + \frac{W}{A} C_4 C_2 \right\} \right] \tag{48}$$

Just as in Chapter III, the constants from Table 1 were substituted in equation (48). For all values of σ_{B1}, as "t" became large, σ_B gradually reached a constant value. (See Table 3.) Figure 4 illustrates a graph of σ_B versus time. As was anticipated, the curve reaches an asymptote more rapidly for N = 2.

Time	$\sigma_{BI}=7$	$\sigma_{BI}=8$	$\sigma_{BI}=9$	$\sigma_{BI}=10$	$\sigma_{BI}=12$
0.0	7.00	8.00	9.00	10.00	12.00
5.0	7.01	8.00	8.98	9.97	11.96
10.0	7.02	7.98	8.95	9.94	11.93
100.0	7.13	7.82	8.58	9.40	11.27
200.0	7.21	7.70	8.26	8.91	10.56
250.0	7.25	7.65	8.13	8.70	10.22
300.0	7.27	7.61	8.02	8.51	9.89
400.0	7.31	7.55	7.84	8.21	9.31
500.0	7.34	7.51	7.71	7.98	8.83
750.0	7.38	7.44	7.53	7.64	8.05
1000.	7.39	7.42	7.45	7.50	7.67
1250.	7.40	7.41	7.42	7.44	7.51
5000.	7.40	7.40	7.40	7.40	7.40
10000.	7.40	7.40	7.40	7.40	7.40

Table 3 - Values of σ_B using the constants from Table 1

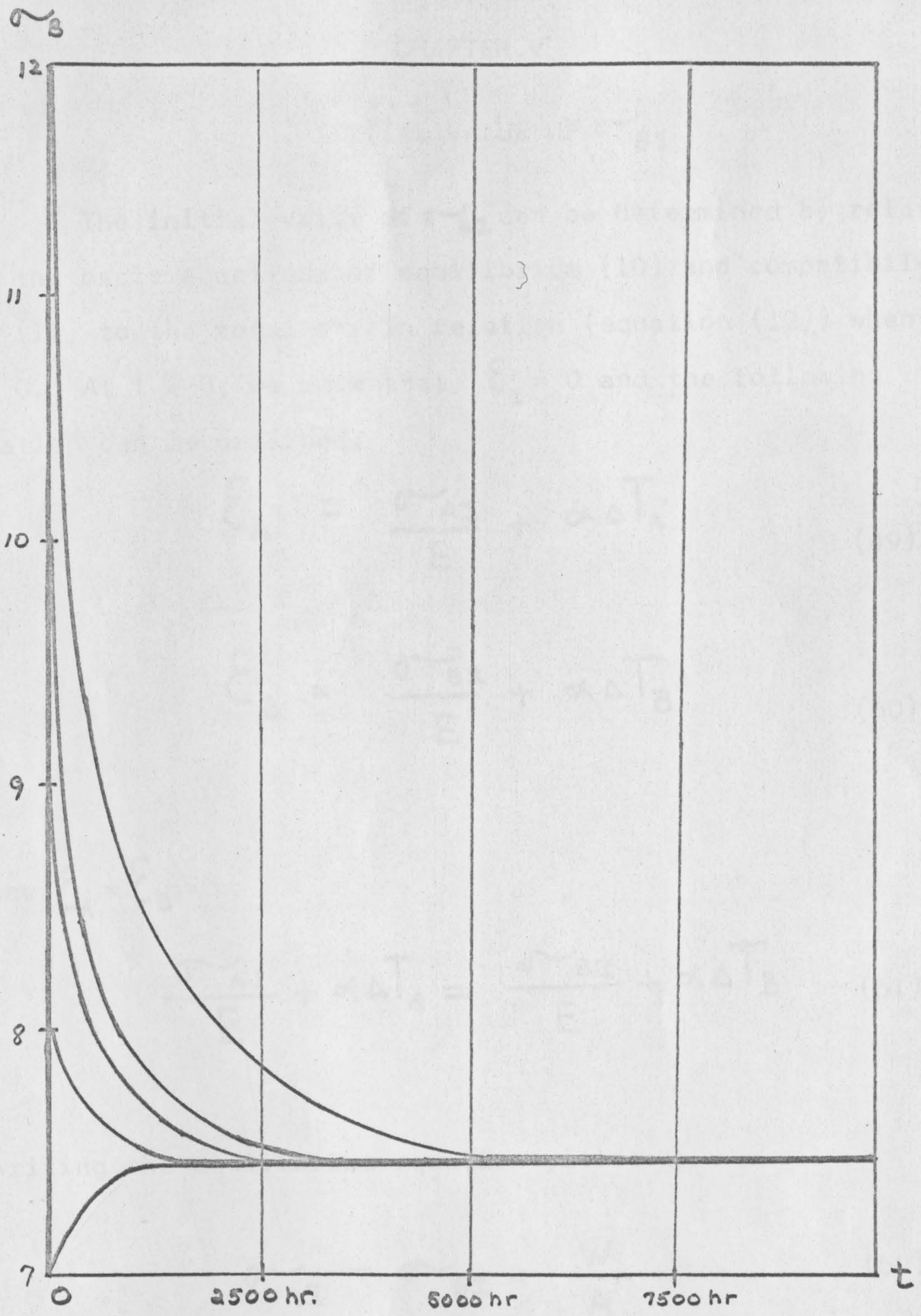

Fig. 5 - Graph of σ_B versus time for N = 2

CHAPTER V

INITIAL VALUE OF σ_{B1}

The initial value of σ_{B1} can be determined by relating the basic equations of equilibrium (10) and compatibility (11) to the total strain relation (equation (12)) when $t = 0$. At $t = 0$, we know that $\varepsilon_c = 0$ and the following equation can be obtained.

$$\varepsilon_A = \frac{\sigma_{A1}}{E} + \alpha \Delta T_A \tag{49}$$

$$\varepsilon_B = \frac{\sigma_{B1}}{E} + \alpha \Delta T_B \tag{50}$$

Since $\varepsilon_A = \varepsilon_B$,

$$\frac{\sigma_{A1}}{E} + \alpha \Delta T_A = \frac{\sigma_{B1}}{E} + \alpha \Delta T_B \tag{51}$$

Rewriting the equilibrium equation yields

$$\sigma_{A1} + \sigma_{B1} = \frac{W}{A} \tag{52}$$

Substituting equation (52) into equation (51), we get

$$-\frac{\sigma_{B1}}{E}+\frac{W}{AE}=\frac{\sigma_{B1}}{E}+\alpha\Delta T_B-\alpha\Delta T_A \qquad (53)$$

which when solved for σ_{B1} yields

$$\sigma_{B1}=\frac{1}{2}\left(\alpha\Delta T_A-\alpha\Delta T_B\right)+\frac{W}{A} \qquad (54)$$

Equation (54) can now be used to obtain an initial value for σ_{B1}. If $\alpha_A = \alpha_B$ and $T_A = T_B$, it follows from equation (53) that $\sigma_{B1} = \frac{W}{2A}$. Since α_A increases with ΔT_A, we may conclude that $\alpha_A\Delta T_A \geq \alpha_B\Delta T_B$, or using the constants in Table 1, $\alpha_A\Delta T_A \geq 5$. Values of σ_{B1} from seven to twelve were then substituted.

DETERMINING THE ASYMPTOTIC VALUES

OF σ_B FOR VARIOUS VALUES OF N

Even though this paper does not solve equations for $N > 2$, it is possible to conclude on the basis of the behavior of the material for $N = 1$ and $N = 2$ that σ_B will eventually reach an asymptote for each value of N. The following equation can be used to compute these asymptotic values of σ_B (See Table 4) using the constants from Table 1.

$$As_B = \frac{\frac{W}{A}}{1 + \left(\frac{K_B}{K_A}\right)^{\frac{1}{N}}} \tag{55}$$

It appears from the values given for σ_B in Table 4 that the asymptotic values of σ_B decrease but at a decreasing rate as N increases. We might also conclude from the indicated values, that a material following the creep law considered in this paper will tend to let σ_B reach an asymptote of value just above 5 ksi. for large values of N; that is, as $N \rightarrow \infty$, σ_B will have a value only slightly greater than σ_A.

Observing equation (55), if W = 0, then $A_{sb} = 0$, indicating that at t = infinity, the vessel would not have any stress in its walls. Sections A and B of the vessel would creep in such a manner that $\sigma_A = \sigma_B = 0$, or both would return to the conditions that existed before the temperature difference was applied.

N	As_B
1.00	8.90
2.00	7.40
3.00	6.68
4.00	6.28
5.00	6.03
6.00	5.86
7.00	5.74
8.00	5.65
9.00	5.58
10.00	5.52
11.00	5.47
12.00	5.44
13.00	5.40
14.00	5.38
15.00	5.35

Table 4 - Asymptotes for Various Values of N

CHAPTER IV

CONCLUSION

It was the purpose of this thesis to solve the general creep equation

$$\dot{\mathcal{E}}_c = K_N \sigma^N \tag{2}$$

and find the stress in the walls of a pressure vessel by means of mathematical analysis in an attempt to analyze in a general way the effects of creep strain on the walls of a pressure vessel. Useful insight into the problems of creep can be gained from the consideration of a practical situation. For example, consider a pressure vessel made of stainless steel which uses a value of N = 5 in the general creep equation considered here. Consulting Table 4 on page 33, we find that σ_B, the stress on the outer section of the model, reaches a value of 6.03 ksi. as $t \rightarrow \infty$, thus determining a stress on the inner section of the wall of 3.97 ksi. This information can be interpreted to mean that sixty per cent of the stress falls in the outer one-half of the vessel wall.

Because stress is actually linear across the wall and not a rectangular approximation as was assumed in order to use a model, it can be seen that the outermost part of the outer skin of the vessel carries the highest stress. It is therefore evident that flaws, nicks, or scratches in the materials used in construction the pressure vessel will have a profound effect on its performance.

Because this analysis solved the general creep equation using only values of N = 1 or N = 2, stress values for arbitrary times can be determined for those materials having a value of N = 1 or N = 2 in the creep equation. However, by applying equations (54) and (55) respectively, the initial and final stresses in the model can be established for any value of N. Values of σ_{θ} as $t \rightarrow \infty$ which are shown in Table 4, page 33, give indications of the stress distribution in the walls of the vessel after long periods of use. The initial values are the result of the difference in temperature from the inside to the outside wall of the vessel as the hotter inside expands and stretches the outside.

The pressure in the vessel also contributes to the hoop stresses (equation (3), page 7). Since stresses from the internal pressure and the stress caused by temperature differences must be added together we see a higher stress created in the outside wall of the vessel.

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