ANALOG TRANSFER FUNCTION COMPUTER

by

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ABSTRACT

ANALOG TRANSFER FUNCTION COMPUTER

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Conventional methods used for the determination of the transfer function of an unknown four terminal network or control system involve time domain or frequency domain testing. System response in time domain can be obtained with little effort. The analysis of the transient waveform involves subjective interpretation. The frequency domain approach is more laborious and involves analysis of magnitude and phase v-s frequency data.

By employing the Transfer Function Computer, the transfer function of an unknown system can be determined directly by matching a reference and system responses. This is accomplished by systematically adjusting a series of potentiometers. Prior knownledge of system gain and general frequency response is needed. This information can be readily obtained from the Transfer Function Computer.

ACKNOWLEDGEMENTS

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CHAPTER I

Introduction

The object of this thesis is to study the operation of an analog transfer function computer.

The equipment under study was manufactured by Wayne Kerr Laboratories, Surrey, England and is designated Transfer Function Computer model SA 100. Two units, serial numbers 74 and 75 were purchased by the Electrical Engineering department of Youngstown State University in June, 1968. These were the last two units manufactured.

Theory of operation is not available from the manufacturer. Publications pertaining to this type equipment could not be found in a literature search.

The purpose of this thesis is twofold: to develop the theory of operation and to study the areas of application for the transfer function computer. It is also desirable to incorporate this equipment in undergraduate studies in networks and control theory. Laboratory experiments to introduce the student to this equipment will be developed.

The transfer function computer (T.F.C.) is used to directly determine the transfer function of active or passive four-terminal networks and control systems. The method of transfer function determination is by frequency response.

The input of the system to be tested is connected to system excitation. This voltage will depend upon the transfer function of the system under test and the excitation frequency. The maximum value of this voltage will not exceed 25 volts peak to peak. The minimum value will be in millivolts. This voltage will vary during the determination of the transfer function. For this reason, application of the T.F.C. should be restricted to linear systems.

The output of the system under test is connected to system response which is the Y axis input of a cathode ray tube. The reference voltage is connected to the X axis of the cathode ray tube.

The overall excitation frequency of the T.F.C. is from .0125 to 400 Hz. This is divided in 12 overlapping ranges, each covering 4 octaves. The frequency range used to determine the transfer function of a system will depend upon the pole-zero location of the system. Frequency within a range is varied by turning the wT dial. This dial is calibrated for wT values from .315 to 2.55. Dividing the wT setting by the T setting yields system frequency in radians per second.

The coefficients defining the transfer function are selected by varying coefficient potentiometers while frequency is varied by the wT dial in 10 steps over 4 octaves.

When the excitation matches the response in phase and magnitude, the display on the CRT will be a straight line with slope equal to one. The coefficients will define the system if the unity slope can be maintained while frequency is varied over the range of interest.

The transfer function as derived from the transfer function computer will be of the form

$$\frac{b_o + \sum_{n=1}^{4} b_n (pT)^n}{[a_o + \sum_{n=1}^{4} a_n (pT)^n](c_o + c_1 pT + c_2 p^2 T^2)} \qquad (1)$$

The p terms refer to the differential operator $\frac{d}{dt}$, a_n, b_n and c_n terms are the settings of the coefficient potentiometers. T defines the R-C time constant of the operational amplifiers, and it also determines the range of frequencies over which the system is tested.

CHAPTER II

PRINCIPLE OF OPERATION

The signals generated by the function generator consist of a sinusoidal output signal e_o and the first through fourth derivative terms of e_o. To generate the reference and excitation signals e_o and the derivative terms are selected proportional to the b_n and a_n coefficient potentiometer settings and summed. The $\sum b_n$ will be the reference signal and the $\sum a_n$ will be the excitation signal.

The reference and excitation signals will be sinusoidal that differ in magnitude and phase. Each can be represented, respectively, as

$$e_o(b_o + b_1p + b_2p^2 + b_3p^3 + b_4p^4) = D(p)R(p) \tag{2}$$

$$e_o(a_1 + a_1p + a_2p^2 + a_3p^3 + a_4p^4) = D(p)E(p) \tag{3}$$

The system under test can be represented by its transfer function

$$T(p) = \frac{\text{Response}}{\text{Excitation}} = \frac{\sum f_n(p)}{\sum f_d(p)} \tag{4}$$

When the system under test is driven by the excitation signal, the response will be

$$D(p)E(p)\frac{\sum f_n(p)}{\sum f_d(p)} \tag{5}$$

The reference signal is $D(p)R(p)$. The coefficient potentiometers are adjusted so that reference matches response. Then

$$D(p)R(p) = D(p)E(p)\frac{\sum f_n(p)}{\sum f_d(p)} \tag{6}$$

or

$$\frac{R(p)}{E(p)} = \frac{D(p)\sum f_n(p)}{D(p)\sum f_d(p)} = \frac{\sum f_n(p)}{\sum f_d(p)} \tag{7}$$

As $R(p)$ and $E(p)$ represent the coefficient potentiometer settings, the system under test will be fully defined by

$$\frac{b_o + \sum_{n=1}^{4} b_n p^n}{a_o + \sum_{n=1}^{4} a_n p^n} \tag{8}$$

A block diagram of the system is shown in Figure 1.

The $C_n p^n$ terms in equation (1) are generated by the C_n potentiometers shown in Fig. 1. They can be included in the excitation signal and are omitted in the above derivations.

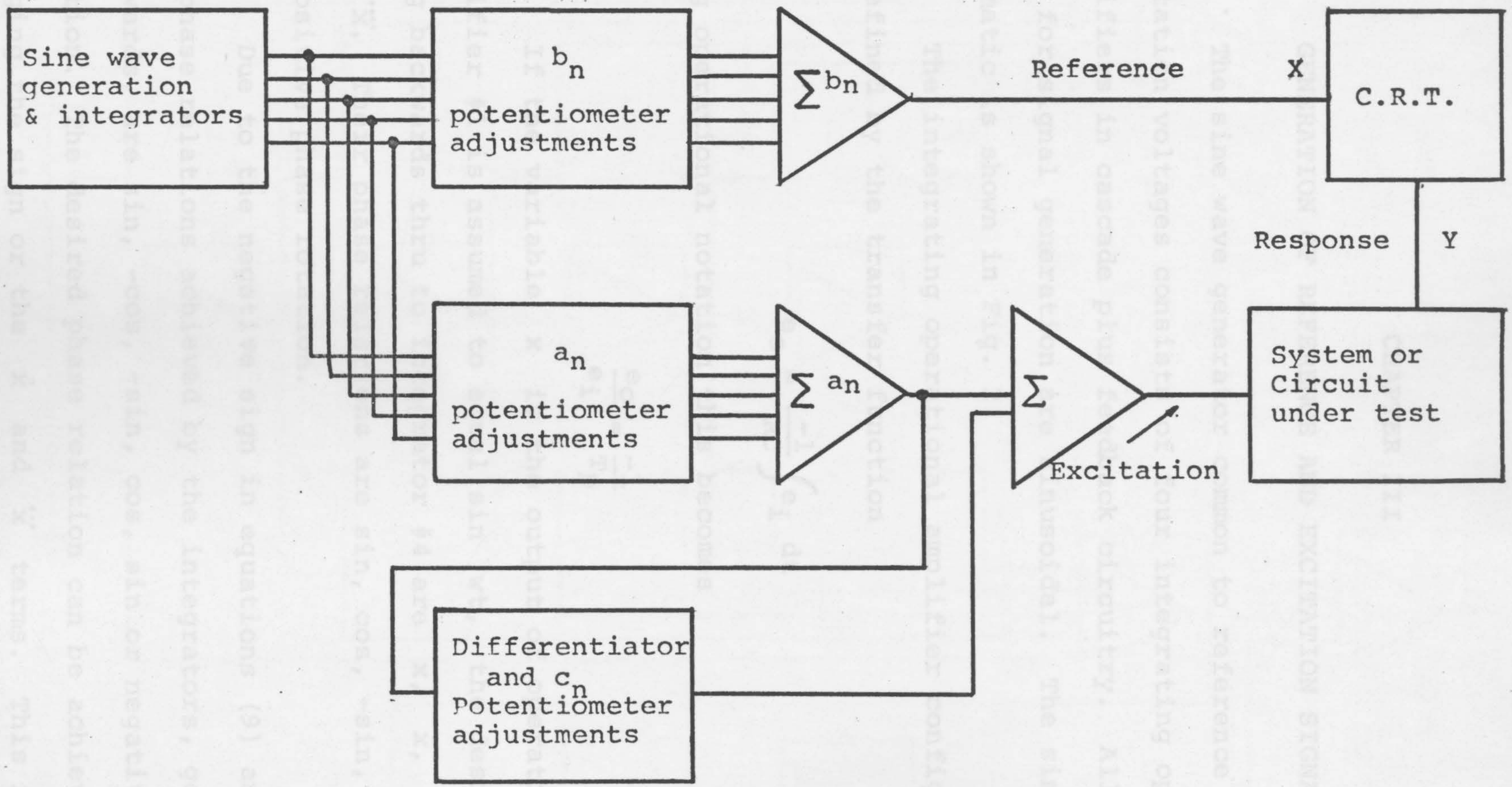


Fig. 1. System Block Diagram

CHAPTER III

GENERATION OF REFERENCE AND EXCITATION SIGNALS

The sine wave generator common to reference and excitation voltages consists of four integrating operational amplifiers in cascade plus feedback circuitry. All voltages used for signal generation are sinusoidal. The simplified schematic is shown in Fig. 2.

The integrating operational amplifier configuration is defined by the transfer function

$$e_o = \frac{-1}{RC}\int e_i \, dt \tag{9}$$

Using operational notation this becomes

$$\frac{e_o}{e_i} = \frac{-1}{Tp} \tag{10}$$

If the variable x is the output of operational amplifier #1 is assumed to equal sin wt, the desired signals, going backwards thru to integrator #4 are x, $\dot{x}$, $\ddot{x}$, $\dddot{x}$, and $\ddddot{x}$. Their phase relations are sin, cos, -sin, -cos, sin, or positive phase rotation.

Due to the negative sign in equations (9) and (10), the phase relations achieved by the integrators, going backwards, are sin, -cos, -sin, cos, sin or negative phase rotation. The desired phase relation can be achieved by changing the sign or the $\dot{x}$ and $\dddot{x}$ terms. This is

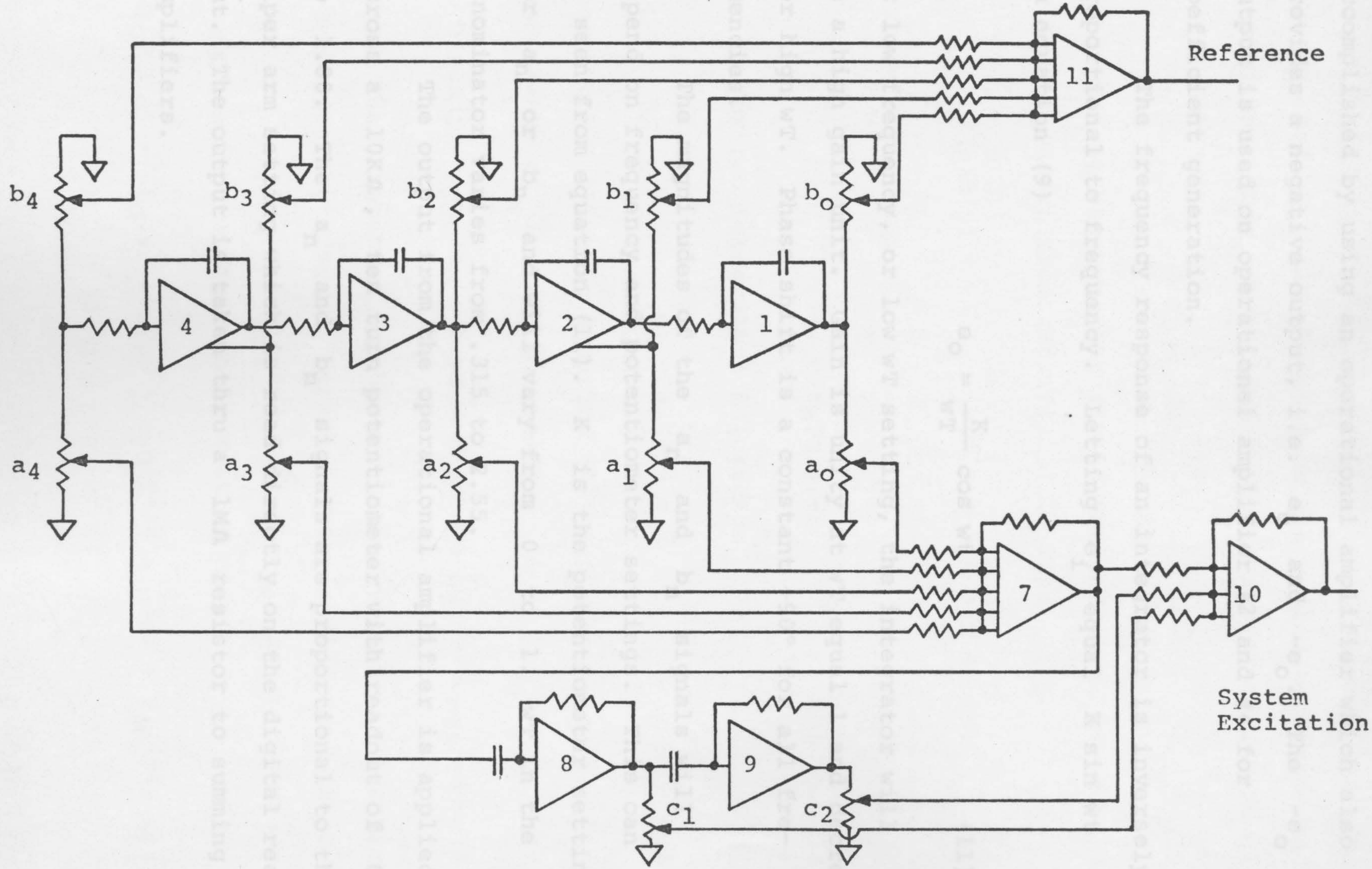


Fig. 2. Simplified System Schematic Diagram.

accomplished by using an operational amplifier which also provides a negative output, i.e. e_o and $-e_o$. The $-e_o$ output is used on operational amplifier #2 and #4 for coefficient generation.

The frequency response of an integrator is inversely proportional to frequency. Letting e_i equal K sin wt in equation (9)

$$e_o = \frac{K}{wT} \cos wt \qquad (11)$$

At low frequency, or low wT setting, the integrator will be a high gain unit. Gain is unity at wT equal 1 and decreases for high wT. Phase shift is a constant +90° for all frequencies.

The magnitudes of the a_n and b_n signals will depend on frequency and potentiometer settings. This can be seen from equation (11). K is the potentiometer setting for a_n or b_n and will vary from 0 to 1. wT in the denominator varies from .315 to 2.55.

The output from the operational amplifier is applied across a 10KΩ, ten turn potentiometer with readout of 0 to 1.00. The a_n and b_n signals are proportional to the wiper arm setting which is read directly on the digital readout. The output is taken thru a 1MΩ resistor to summing amplifiers.

Operational amplifiers #8 and #11 are used for summing the a_n and b_n signals to form the system reference and excitation. They are defined by the transfer function

$$e_o = -(Rf \sum \frac{e_i}{R_i}) \qquad (12)$$

To see how equation (1) is derived, let x equal sin wt be the output from amplifier #1. Then

$$D(p)R(p) = -(b_o x + b_1\dot{x} + b_2\ddot{x} + b_3\dddot{x} + b_4\ddddot{x}) \qquad (13)$$

For one integrator

$$e_i = -e_o Tp = -\dot{x} \qquad (14)$$

For two integrators in cascade

$$e_i = e_o T^2 p^2 = \ddot{x} \qquad (15)$$

Substituting these in equation (13) and accounting for sign change at $\dot{x}$ and $\dddot{x}$ yields

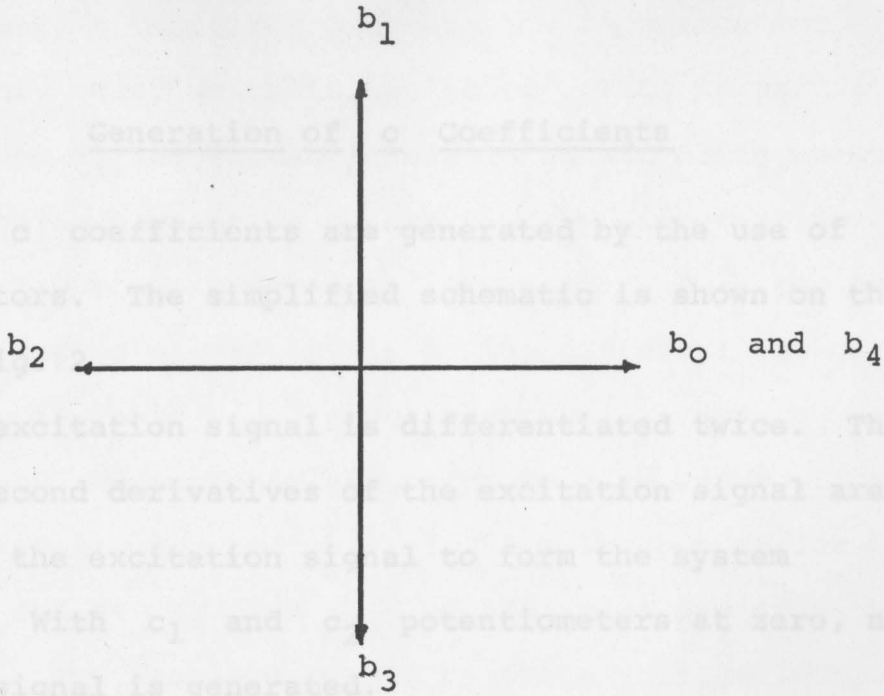
$$D(p)R(p) = -e_o(b_o + b_1 pT + b_2 p^2 T^2 + b_3 p^3 T^3 + b_4 p^4 T^4) \qquad (16)$$

The generation of the excitation voltage is identical to reference but for c_n terms. This is discussed later. With c_1 and c_2 equal 0, this circuit can be ignored. The excitation voltage can be written as

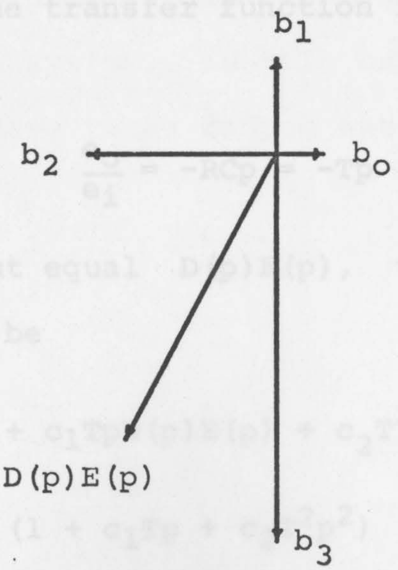
$$D(p)E(p) = -e_o(a_o + a_1 pT + a_2 p^2 T^2 + a_3 p^3 T^3 + a_4 p^4 T^4) \qquad (17)$$

The $-e_o$ term is common to both numerator and denominator in equation (7) and equation (1) will result.

The vector diagram showing relationship of the signals generated by the coefficient potentiometer is shown in Fig. 3a. The a_o and b_o voltages are in phase with a_4 or b_4 respectively. Their relative magnitudes will depend on the coefficient potentiometer settings and the value of wT. The phasor voltages are summed according to equation (12). The phaser diagram for a hypothetical D(p) R(p) is shown in Fig. 3b. The input is taken to be sin wt. T is equal to 1 and w is equal to 2. From equation (11) it can be seen that the signal magnitude at each integrator is halved. Coefficient potentiometer values b_o thru b_3 are taken to be 1, b_4 is assumed to be 0.



a. Phase relation of signals generated by b_n coefficient potentiometers.



b. Magnitude and phase for w equals 2, b_o through b_4 equal to 1, b_4 equals zero.

Fig. 3. Phasor Diagrams for D(p)R(p)

Generation of c Coefficients

The c coefficients are generated by the use of differentiators. The simplified schematic is shown on the bottom of Fig. 2.

The excitation signal is differentiated twice. The first and second derivatives of the excitation signal are summed with the excitation signal to form the system excitation. With c_1 and c_2 potentiometers at zero, no additional signal is generated.

For differentiation, the standard operational amplifier is used. The transfer function for the configuration is given by

$$\frac{e_o}{e_i} = -RCp = -Tp \tag{18}$$

Letting the input equal $D(p)E(p)$, the input to the summing amplifier would be

$$D(p)E(p) + c_1TpD(p)E(p) + c_2T^2p^2D(p)E(p) \tag{19}$$

$$= D(p)E(p)\ (1 + c_1Tp + c_2T^2p^2)$$

The $D(p)E(p)$ term is given in equation (17). The c_o term is always unity. The negative output is taken for the c_1 term generation to yield the correct polarity signal.

The series capacitor presents low impedance for high frequency noise which is amplified. This is particularly troublesome at low signal levels which are often encountered.

The obvious question to ask is, why differentiate? Why not add two more integrators in cascade? This can be answered by noting signal levels at the different integrator stages shown in Table I. The voltages at a_2 and b_2 are always 8Vp-p. Adding one integrator to each end would generate voltages at a_o and b_o of 254 volts p-p at low frequencies and at a_4 and b_4 of 133 volts p-p at high frequencies. The problems caused by these voltage levels are obvious.

Another problem would be sixth order characteristic equation for the system. In this case, two unwanted frequencies would have to be damped out.

TABLE I

MAXIMUM VOLTAGE VALUES AT COEFFICIENT POTENTIOMETERS

Selector Switch wT	a_o and b_o	a_1 and b_1	a_2 and b_2	a_3 and b_3	a_4 and b_4
A = .315	80.6	25.5	8.0	2.52	.8
B = .38	55.4	21.0	8.0	3.0	1.15
.50	32.0	16.0	8.0	4.0	2.0
C = .61	21.5	13.1	8.0	4.9	3.0
D = .80	12.5	10.0	8.0	6.4	5.1
1.0	8.0	8.0	8.0	8.0	8.0
E = 1.25	5.1	6.4	8.0	10.0	12.5
F = 1.60	3.1	5.0	8.0	12.8	20.5
2.0	2.0	4.0	8.0	16.0	32.0
G = 2.55	1.2	3.1	8.0	20.4	52.0

Magnitude Scaling

Coefficient magnitude scaling is an essential feature of the T.F.C. This allows for greated flexibility in coefficient selection and provides finer resolution for coefficient determination.

Table II gives maximum coefficient values as read on the a_n and b_n coefficient dials.

Using values of 1, 10, and 100 will not alter the differential equation as both numerator and denominator are multiplied by a constant. The respective a_n and b_n terms also have a T^n multiplier to determine terms in the differential equation. Using values larger than 1.0 will facilitate equation manipulations as numbers less than 1 can be more cumbersome. Also, working with frequency response, the a_o coefficient is usually 1.0.

Scaling is accomplished by using voltage dividers. The largest number represents full output voltage at the respective operational amplifier. The lower order values are achieved by dropping 90% and 99% of the voltage across a series resistor. With scaling switch in position 1, full output is applied across a_1, a_2, and a_3 potentiometers, 10% to a_o and a_4 potentiometers. In scaling switch position #3, full voltage is applied only across potentiometer a_2, 10% of the voltage across a_1 and a_3 and 1% across a_o

and a_4 potentiometers. In scaling switch positions 2 and 3, signal levels at all positions but a_2 and b_2 are attenuated by a factor of 10. The resulting reference and excitation signal levels will be low and close to the threshhold of noise.

TABLE II

MAXIMUM COEFFICIENT VALUES

Switch Position	a_o	a_1	a_2	a_3	a_4	b_o	b_1	b_2	b_3	b_4	c_1	c_2	c_3
1	1	10	10	10	1	1	10	10	10	1	1	1	.1
2	1	10	10	10	1	1	10	100	10	1	1	10	1.0
3	1	10	100	10	1	1	10	100	10	1	1	1	.1

CHAPTER IV

OSCILLATOR

The T.F.C. does not have an external oscillator. The four integrator chain can sustain oscillations through proper feedback.

The simplified schematic of the oscillator circuit is shown in Fig. 4. Variable R_o and C_o provide the RC time constants of the integrators. C_o consists of four capacitors from .001 μF through 1 μF. R_o consists of four resistors of 1MΩ through 4MΩ in series. These can be switched independently by a four position switch and provide time constants from .001 to 4 sec. These are switched to select the frequency range over which the unknown system is to be excited. R_1, R_2, R_3, and R_4 are ganged resistors in the feedback path. These are mounted on multi position wT dial and are switched to achieve fine frequency control during coefficient determination.

For sinusoidal input voltage, sin wt at X_5, each integrator is defined by equations (9) and (11). These are given by

$$e_o = \frac{-K}{RC} \int \sin wt = \frac{K}{wT} \cos wt$$

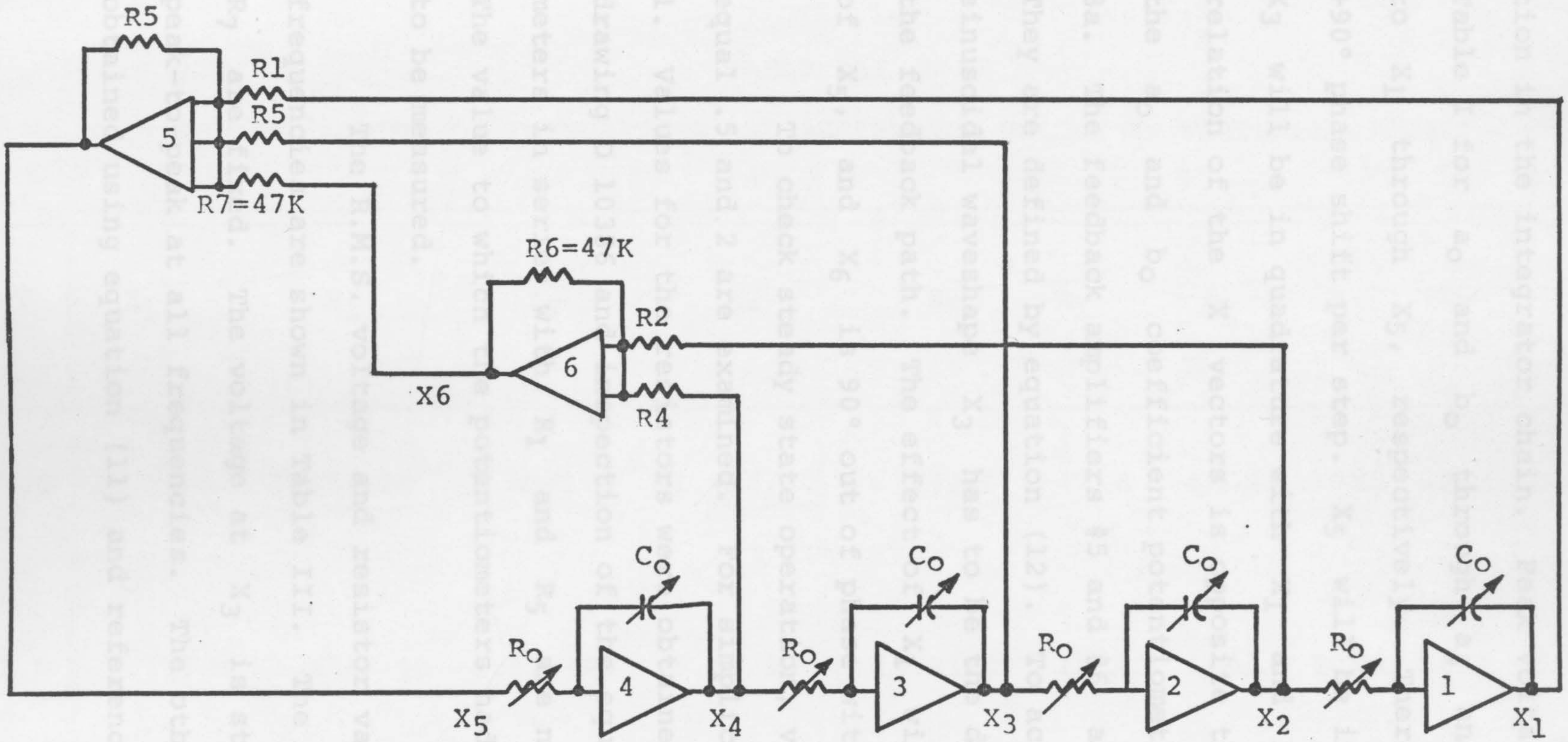


Fig. 4. Simplified Schematic Diagram for Oscillator Circuit

The value of K will depend on wT setting and signal location in the integrator chain. Peak voltage values given in Table I for a_o and b_o through a_4 and b_4 correspond to X_1 through X_5, respectively. There will be a constant +90° phase shift per step. X_5 will be in phase with X_1, X_3 will be in quadrature with X_1 and X_5. The phase relation of the X vectors is opposite that generated by the a_o and b_o coefficient potentiometers shown in Fig. 3a. The feedback amplifiers #5 and #6 are used for summing. They are defined by equation (12). To achieve undistorted sinusoidal waveshape X_3 has to be the dominant voltage in the feedback path. The effect of X_1 will be cancellation of X_5, and X_6 is 90° out of phase with X_5.

To check steady state operation, voltages at wT equal .5 and 2 are examined. For simplicity, let T equal 1. Values for the resistors were obtained from Wayne Kerr drawing D 10386 and inspection of the equipment. Potentiometers in series with R_1 and R_5 are not shown in Fig. 4. The value to which the potentiometers had been adjusted had to be measured.

The R.M.S. voltage and resistor values for the two frequencies are shown in Table III. The values of R_6 and R_7 are fixed. The voltage at X_3 is stabilized at 8V peak-to-peak at all frequencies. The other voltages were obtained using equation (11) and reference voltage at X_3.

TABLE III

VOLTAGES AT OUTPUTS OF INTEGRATORS AND RESISTANCE VALUES IN FEEDBACK CIRCUIT

wT	X_1	X_2	X_3	X_4	X_5	R_1	R_2	R_3	R_4	R_5
.5	11.31/0	5.68/-90	2.83/180	1.414/90	.71/0	49KΩ	47KΩ	10KΩ	12KΩ	12KΩ
2.0	.707/0	1.414/-90	2.83/180	5.67/90	11.31/0	11KΩ	14KΩ	10KΩ	56KΩ	51KΩ

Substituting proper resistor and voltage values in equation (12), it is seen that the steady state voltage at X_6 is zero. For wT equal .5

$$X_6 = -\left(\frac{R6}{R4} X_4 \underline{/90} + \frac{R6}{R2} X_2 \underline{/-90} \right)$$

$$X_6 = -[(3.92)(1.414 \underline{/90}) + (.1)(5.656 \underline{/-90})]$$

$$X_6 = .08 \underline{/90} = 0$$

$$X_5 = -\left(\frac{R5}{R1} X_1 \underline{/0} + \frac{R5}{R3} X_3 \underline{/180} \right)$$

$$X_5 = -[(.245)(11.32) \underline{/0} + (1.2)(2.828 \underline{/-180})]$$

$$X_5 = .66 \underline{/0}$$

For wT equal 2

$$X_6 = -[(.84)(5.656 \underline{/90}) + (3.36)(1.414 \underline{/-90})]$$

$$X_6 = 0$$

$$X_5 = -[(4.63)(.707 \underline{/0}) + (5.1)(2.828 \underline{/180})]$$

$$X_5 = 11.2 \underline{/0}$$

The voltage magnitudes at X_5 compare well with values of 2.0V and 32V p-p obtained in Table I. The steady state voltages can be expected to be undistorted sinusoidals if no other frequencies are present in the system.

To determine the system frequencies, the characteristic equation for the system must be found. This can be done using state variable methods. The state variables are identified by:

$$\dot{X}_1 = \frac{-X_2}{T}$$

$$\dot{X}_2 = \frac{-X_3}{T}$$

$$\dot{X}_3 = \frac{-X_4}{T}$$

$$\dot{X}_4 = \frac{-X_5}{T}$$

These are shown in Fig. 4. The values of X_5 and X_6 can be found in terms of the state variables by using equation (12).

$$X_6 = -\frac{R6}{R2} X_2 - \frac{R6}{R4} X_4$$

$$X_5 = -\frac{R5}{R3} X_3 - \frac{R5}{R1} X_1 + \frac{R5R6}{R7R2} X_2 + \frac{R5R6}{R7R4} X_4$$

In matrix form, the state variable equation becomes

$$\begin{vmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{vmatrix} = \begin{vmatrix} 0 & -\frac{1}{T} & 0 & 0 \\ 0 & 0 & -\frac{1}{T} & 0 \\ 0 & 0 & 0 & -\frac{1}{T} \\ \frac{R5}{R1}\frac{1}{T} & -\frac{R5R6}{R7R2}\frac{1}{T} & -\frac{R5R6}{R7R4} & -\frac{R5R6}{R7R4}\frac{1}{T} \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{vmatrix}$$

This is of the form $\overline{X} = \overline{AX}$. Let a equal $\frac{R5}{R1}$, b equal $\frac{R5R6}{R7R2}$, c equal $\frac{R5}{R3}$, and d equal $\frac{R5R6}{R7R4}$. The $\overline{A}$ matrix becomes

$$\overline{A} = \begin{vmatrix} 0 & -\frac{1}{T} & 0 & 0 \\ 0 & 0 & -\frac{1}{T} & 0 \\ 0 & 0 & 0 & -\frac{1}{T} \\ \frac{a}{T} & -\frac{b}{T} & \frac{c}{T} & -\frac{d}{T} \end{vmatrix}$$

To find the characteristic equation of the system, the determinant of the matrix $[s\overline{I}-\overline{A}]$ is found

$$[s\overline{I}-\overline{A}] = \begin{vmatrix} s & \frac{1}{T} & 0 & 0 \\ 0 & 0 & \frac{1}{T} & 0 \\ 0 & 0 & s & \frac{1}{T} \\ -\frac{a}{T} & \frac{b}{T} & -\frac{c}{T} & s + \frac{d}{T} \end{vmatrix} \qquad (20)$$

Determinant of $[s\overline{I}-\overline{A}]$ is found to be

$$s^4 + \frac{d}{T} s^3 + \frac{c}{T^2} s^2 + \frac{b}{T^3} s + \frac{a}{T^4} \qquad (21)$$

It is assumed that the characteristic equation can be factored into

$$(s^2 + w_o^{\ 2})(s^2 + 2\zeta w + w^2) \tag{22}$$

To check validity of this assumption, resistor values for wT equal .5 and 2 are substituted in equation (21). For simplicity, let T equal 1. At wT equals .5, the characteristic equation is

$$s^4 + \frac{s^3}{1} + \frac{1.2s^2}{1^2} + \frac{.255s}{1^3} + \frac{.245}{1^4} \tag{23}$$

$$= \quad (s^2 + .25)(s^2 + s + .95)$$

At wT equals 2

$$(s^4 + .91s^3 + 5.1s^2 + 3.54s + 4.63) \tag{24}$$

$$= \quad (s^2 + 4)(s^2 + .9s + 1.1)$$

In both instances the characteristic equation yields the system frequency and a damped frequency.

Checking the characteristic equation for other values of T, let T equal 0.1 in equation (23). This yields

$$s^4 + 10s^3 + 120s^2 + 255s + 2450 \tag{25}$$

$$(s^2 + 25)(s^2 + 10s + 100)$$

This corresponds to a system frequency of 5 radians per second which is 10 times the frequency found in equation (23).

A unique design feature can be seen from the calculations above. The wT dial, which selects resistors in the feedback circuit varies wT from .315 to 2.55. This value divided by the integrator time constant T yields the system frequency. The wT ratio is always constant for all values of T.

CHAPTER V

DETERMINATION OF UNKNOWN COEFFICIENTS

The coefficients for an unknown system are determined in an ascending order of p.

The first to be determined are the $\frac{b_o}{a_o}$ coefficients. These represent the D.C. ratio as $p \to 0$, i.e., attenuation in a passive system or Kp, Kv, or Ka for a feedback system. In either case, these are determined by applying a step input and observing response as $t \to \infty$.

The procedure can be summarized as:

1) Set Function selector to Step.
2) Set Step selector to Zero.
3) Set Mode selector to Null and center dot on CRT.
4) Set a_o and b_o to 1, all others to 0. Set X Gain and Y Gain to 10.
5) Switch Step to +V or -V and observe motion of dot. Adjust b_o potentiometer leaving a_o at 1 or, a_o leaving b_o at 1, to bring dot to center of screen. For a passive network b_o is less than 1, a_o is equal to 1. For active network with gain, the reverse will hold.

6) Switch X Gain and Y Gain to a greater sensitivity as the values are finalized.

7) Switch Step selector to +V and -V and check symmetry of response.

The magnitude of the applied step input is 0 to 4V. This is changed by the excitation level control. In addition, dc bias is available to bias the signal approximately 100%, i.e., at maximum signal level the zero output can be biased to +4V and switched from .5V to 8V.

The selection of the other terms can be best understood by examining the voltage levels throughout the integrating loop. At all frequencies, the integrating operational amplifier has gain of $\frac{e_o}{e_i}$ which is $\frac{1}{wT}$. The wT ratio is contant for all values of T. For wT equal .315, the gain for one amplifier is 3.17. At higher frequencies, the integrator attenuates the signal level. For wT equal 2.55, the $\frac{e_o}{e_i}$ ratio is .39.

The voltage at the output of integrator #3 is 8V p-p regardless of frequency. The voltage levels at the different stages for all values of wT is shown in Table I. These will have the same values for all T settings.

It can be seen that at low frequencies the larger voltage levels exist at the low order coefficient stages. With wT set at A, the dominant voltage will be at a_o and a_1 stages. The voltages at stages 3 and 4 will have negligible effect on $D(p)E(p)$. The converse is true for high frequencies where the dominant voltages exist at the higher order coefficient stages.

The general procedure can be summarized as follows:

1) Determine $\frac{b_o}{a_o}$ by step response.
2) Determine setting for T. This is explained in next section.
3) Set Magnitude scaling to 1.
4) Set CRO to 45° trace.
5) Set wT to A and adjust a_1 and b_1 coefficient potentiometers to obtain a 45° trace.
6) Move wT to .5 or C and adjust a_2 and b_2 potentiometers to approximate 45° trace.
7) Return to A and readjust a_1 and b_1.
8) Return to C and readjust a_2 and b_2.
9) Go to wT equal 1, and set a_3 and b_3.
10) Repeat steps 5 through 9 until a straight 45° trace is obtained for all frequencies from wT equal A through G.

The procedure is definitely trial and error, expecially when zeros are encountered in the transfer function.

Determination of T

Examination of the system frequency response will yield information on the characteristics of the system and minimize the guesswork in coefficient determination.

The $\frac{b_o}{a_o}$ coefficient ratio must first be determined to insure that low frequency response will yield a slope of one.

Starting at wT equal .5, T equal to 4, the T values are sequentially decreased to increase frequency. The reference and excitation voltages do not change with varying T, and changes in the lissajous pattern will be easy to determine. The result will be the standard frequency response. Clockwise tilt of the pattern will indicate all zeros at infinity and poles on negative real axes. Complex conjugate poles will exhibit a magnitude peak at w_m. Zeros on negative real axes will cause counterclockwise tilt. This inspection is visual.

Selection of T that yields excitation frequency range corresponding to break frequencies for system response will yield best discrimination in coefficient determination. A higher or lower value of T will cause low or high order coefficient terms in transfer function to dominate. The value of T that indicates system response close to the first break frequency at low wT will yield coefficient values of equal magnitude if the poles and zeros are within 4 octaves.

To illustrate this, choose a transfer function with a double pole at s equal -10. This can be written as

$$\frac{1}{1 + .2p + .01p^2} \tag{26}$$

A double break or -40 db per decade slope will occur at w equal to 10. With wT set at .5, this will occur at T equal to .05. As this is not available on the T.F.C., pick T equal to .04. The frequency range obtained by varying wT will be from approximately 8 to 63 radians per second and the break occurs within this range. This will appear on the T.F.C. as

$$\frac{1}{1 + 5pT + 6.25p^2T^2} \tag{27}$$

Both a_1 and a_2 are of the same order of magnitude and neither will dominate the high or low frequency response. Using T equal .02 would result in frequency range of 16 to 168 radians. This yields the following coefficient settings

$$\frac{1}{1 + 10pT + 25p^2T^2} \tag{28}$$

Since this would require a scale position 3 and a resulting decrease of excitation and reference voltages, rescale it to

$$\frac{.4}{.4 + 4pT + 10p^2T^2}$$

The a_2 coefficient setting is much larger than a_1. The voltage generated by the a_2 coefficient potentiometer will be the dominant voltage in D(p)E(p).

To determine the sensitivity of the display to change in coefficient value, the corresponding change in response voltage must be determined. This calculated for the two values of T for a 10% change in the a_1 potentiometer setting in equations (27) and (29). Since low order coefficient terms are determined at low frequencies, wT of .315 is used in the calculations. Values for the different parameters are shown in Table IV.

The voltages generated by the coefficient potentiometers can be determined by:

$$Va_n = Vx \times S.F. \times P.S. \qquad (29)$$

Where Va_n is the voltage at the coefficient potentiometer, Vx is the voltage at the output of the integrator (see Table I), S.F. is the scale factor and P.S. is the potentiometer setting.

The value of D(p)E(p) is determined by vectorially adding the voltages generated by the coefficient potentiometers. D(p)R(p) is the voltage generated by the b_o coefficient potentiometer and is equal to Va_o.

The transfer function for the system under test is found by letting p equal jw in equation (26). System response is given in equation (4) as the product of excitation and the transfer function.

TABLE IV

PARAMETER VALUES TO DETERMINE CHANGES IN LISSAJOUS PATTERN

T	.04	.02
w radians	8	16
Va_o	$2.83/\underline{0}$	$1.13/\underline{0}$
Va_1	$4.5/\underline{90}$	$3.6/\underline{90}$
Va_2	$1.77/\underline{180}$	$3.98/\underline{180}$
D(p)R(p) Reference	$2.83/\underline{0}$	$1.13/\underline{0}$
D(p)E(p) Excitation	$4.62/\underline{76.7}$	$3.98/\underline{114.25}$
T.F.	$\frac{1}{1.64/\underline{76.7}}$	$\frac{1}{3.58/\underline{114.25}}$
Response	$2.82/\underline{0}$	$1.12/\underline{0}$
$V(a_1 + \Delta a_1)$	$4.94/\underline{90}$	$3.96/\underline{90}$
D(p)R(p)	$5.07/\underline{78}$	$4.32/\underline{113.35}$
Response	$3.1/\underline{1.3}$	$1.24/\underline{-1.9}$
Change in Display	2.5°	1.6°

The top part of the table gives voltage values for coefficient settings in equations (27) and (28). The dominant voltage in D(p)E(p) for T equal .04 is generated by the a_1 coefficient potentiometer. For T equal to .02 both a_1 and a_2 generate voltages of equal order of magnitude. For both T settings, the excitation voltage is equal to the response voltage and C.R.T. display will be a straight line with slope of one. The last four rows in Table IV give parameter values for a_1 increased by 10%. The response voltage magnitude has increased by almost 10% for T equal .04. This is to be expected since a_1 generates the dominant voltage and change in D(p)E(p) will be proportional to change in a_1 coefficient. The slope of the display will change by 2.5°. The change in response for T equal .02 is 6%. This decrease is caused by the large voltage generated by the a_2 coefficient. The resulting change in the slope will be 1.6°.

Similar calculations show that at higher frequencies the change in display versus the change in a_2 will be nearly identical for the two values of T. It is therefore desireable to keep all coefficient values at the same order of magnitude. This does not include a_o or b_o which are determined by a different method. The coefficient magnitudes will be closest when T is selected to include all the break frequencies in frequency band covered by the wT dial.

CHAPTER VI

EXPERIMENTS

Active Network

To examine the operation of the equipment with respect to pole-zero locations, a third order differential equation with poles on the negative real axis was used. A Donner Analog Computer Model 5400 was used to generate the transfer function. The circuit to generate each pole is shown in Fig. 5. The transfer function for each active network is:

$$\frac{e_o}{e_i} = \frac{-R_2}{R_1\ (1 + R_2Cs)} \tag{30}$$

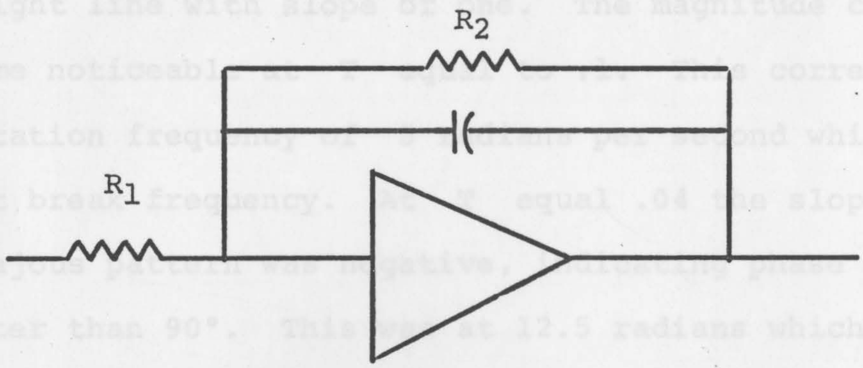


Fig. 5. Generation of Transfer Function.

Three of these networks were used in cascade to generate the overall transfer function.

For the first simulation poles at s equal -5, -10, and -20 were chosen. This yields a transfer function of

$$\frac{-1}{(1 + .2s)(1 + .1s)(1 + .05s)} \tag{31}$$

$$= \frac{-1}{1 + .35s + .035s^2 + .001s^3}$$

The log frequency asymptote plot for this is shown in Fig. 6.

A step input was applied with b_o and a_o equal to one, all other coefficient terms set at zero. It was seen that the spot on the cathode ray tube returned to center after the transient response. Therefore the $\frac{b_o}{a_o}$ ratio was unity.

Frequency response was checked with wT fixed at .5, T was varied from 4 to .001. The initial response was a straight line with slope of one. The magnitude change became noticeable at T equal to .1. This corresponds to excitation frequency of 5 radians per second which is the first break frequency. At T equal .04 the slope of the lissajous pattern was negative, indicating phase shift greater than 90°. This was at 12.5 radians which is close to the second break frequency. For T less than .04 the magnitude of the response was too small to notice any further change.

Excitation frequency band corresponding to T equal .04 was used for coefficient determination. This was the T setting that indicated the second break frequency. The

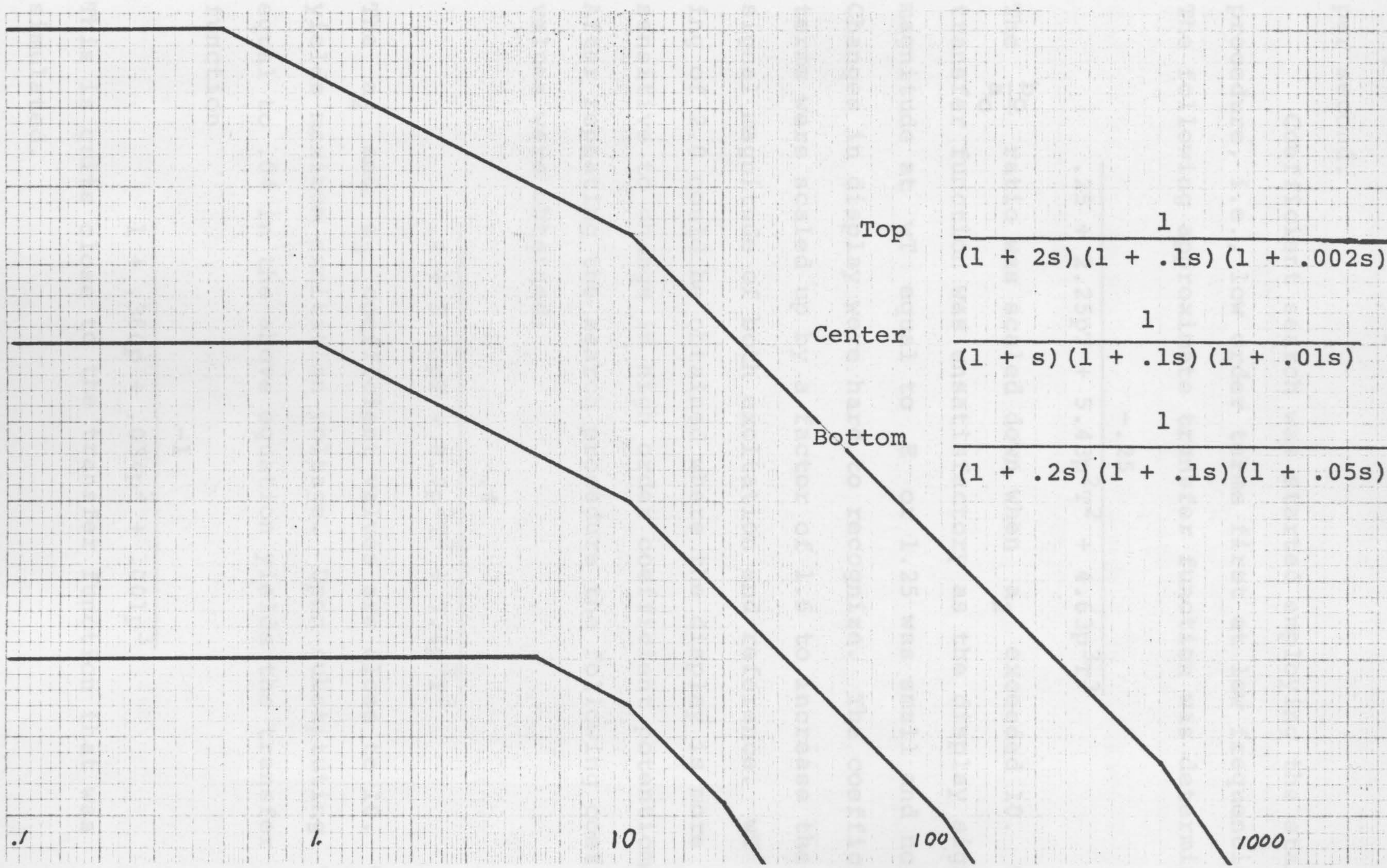


Fig. 6. Frequency Asymptote Plots for Simulated Transfer Functions.

frequencies covered by this band are from 8 to 65 radians per second.

Coefficient search was started employing the standard procedure, i.e., low order terms first at low frequencies. The following approximate transfer function was determined:

$$\frac{-.25}{.25 + 2.25pT + 5.43p^2T^2 + 4.63p^3T^3}$$

The $\frac{b_o}{a_o}$ ratio was scaled down when a_2 exceeded 10. This transfer function was unsatisfactory as the display signal magnitude at wT equal to E or 1.25 was small and noisy. Changes in display were hard to recognize. The coefficient terms were scaled up by a factor of 1.6 to increase the signal magnitude of both excitation and reference. wT setting of 2.0 could be obtained where the display is more sensitive to change in high order coefficient potentiometers. After repeating the search procedure the following coefficient values were obtained:

$$\frac{-.4}{.4 + 3.60pT + 8.7p^2T^2 + 7.4p^3T^3}$$

The a_2 and a_3 coefficient values are close to 10. This yields maximum excitation voltage. Upon substituting T equal to .04 in the above equation yields the transfer function

$$\frac{-1}{1 + .360p + .034p^2 + .001p^3}$$

This is quite close to the transfer function that was simulated.

The cathode ray tube on the T.F.C. has a long persistance screen which is helpful. However, it has only a 4" diameter which makes changes in slope hard to determine. A Hewlett-Packard Oscilloscope Model #120B with a 10" screen was used for coefficient determination. The horizontal input had to be amplified 10 times to achieve 10 millivolts per centimeter sensitivity on both inputs.

Picture of noise on the lissajous pattern at wT equal 2 is shown in Fig. 7. Deflection was at 10 mV/cm. At wT equal to 2.55 the trace was 2/3 the magnitude and small changes in slope were impossible to determine.

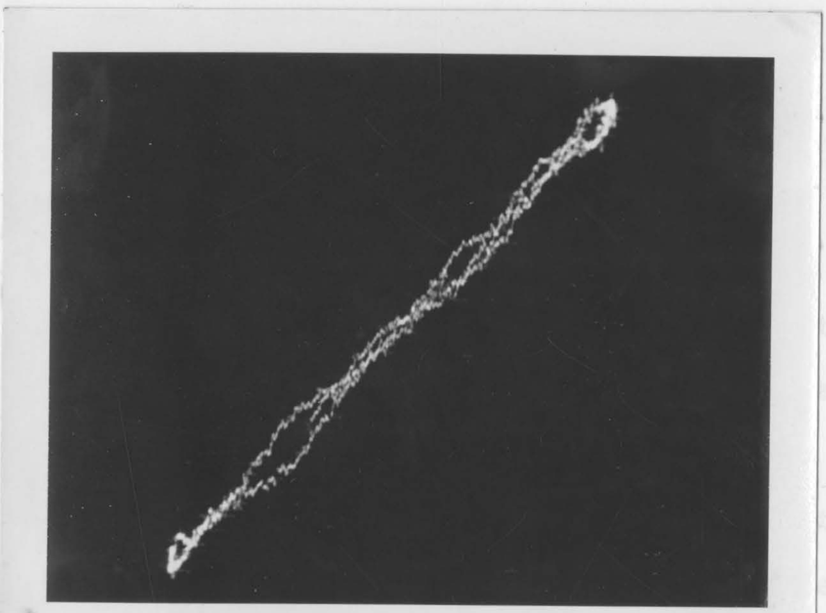


Fig. 7. Noise on Lissajous Pattern

To study the effect of greater pole seperation on transfer function determination, poles at s equal -1,-10, and-100 were chosen. The resulting transfer function is

$$\frac{-1}{(1 + s)(1 + .1s)(1 + .01s)} \tag{32}$$

$$= \frac{-1}{1 + 1.11p + .111p^2 + .001p^3}$$

The results obtained from application of step function were the same as before. Frequency response indicated the first break frequency at 1 radian and phase shift exceeded 90° at 12.5 radians. Coefficient values at T equal .04 were found to be

$$\frac{-.15}{.15 + 4pT + 10p^2T^2 + 3p^3T^3}$$

$$= \frac{-1}{1 + 1.11 + .107p^2 + .00128p^3}$$

The a_3 coefficient is in error by 28%. Again, the maximum value of wT that gave discernable results was E or 1.25.

Simple scaling cannot be used for this particular transfer function since the a_2 coefficient value is already 10. One way to achieve higher value for the a_3 term is to change T to .02. This will yield coefficient values of:

$$\frac{-1}{1 + 55pT + 277.5p^2T^2 + 125p^3T^3}$$

$$= \frac{-.08}{.08 + 4.4pT + 22.2p^2T^2 + 10p^3T^3}$$

To get these potentiometer values on the T.F.C., magnitude position 3 would have to be used. But signal levels are attenuated in this position and the attempt would be self-defeating. The magnitude of the response signal would be smalled than before.

The only way to increase the signal levels in this instance is to modify the transfer function. In many open loop systems such as filters or transfer functions, the pole locations are independent of gain. Increasing system gain will increase the reference signal. The magnitude of the response signal will increase by the same amount (see equations 6 and 7). The value of b_o as determined before was .15. This can be increased by a factor of 5. Using this approach, the following transfer function was obtained.

$$\frac{-.6}{.12 + 3.30pT + 8.45p^2T^2 + 1.90p^3T^3}$$

$$= \frac{-5}{1 + 1.1p + .113p^2 + .001p^3}$$

The results are quite acceptable. Excitation frequency corresponding to wT equal 2 could be used. The display was noisy, but covered 10cm. on the oscilloscope.

For the last simulation of this type, transfer function poles at s equal -.5, -10, and -500 were chosen. The frequency asymptote plot is shown in Fig. 6. The resulting transfer function is

$$\frac{-1}{(1+2s)(1+.1s)(1+.002s)} \tag{33}$$

$$= \frac{-1}{1+2.102p+.2042p^2+.0004p^3}$$

Choosing T equal .04 and following the standard procedure yielded the following coefficient values and transfer function

$$\frac{-.075}{.075+5.4pT+10p^2T^2+2.6p^3T^3}$$

$$= \frac{-1}{1+2.9p+.21p^2+.002p^3}$$

Frequencies above wT equal to 1 yielded small and noisy display which accounts for the large error in the a_3 coefficient value. The a_2 coefficient value was much larger than a_3 and at low frequencies generated the dominant voltage in the excitation signal.

To study this function in more detail, the coefficient values of the transfer function for several values of T are calculated

$$\frac{-1}{1+21.02pT+20.42p^2T^2+.4p^3T^3} \quad \text{for T equal .1}$$

$$\frac{-1}{1+52.54pT+127.5p^2T^2+6.25p^3T^3} \quad \text{for T equal .04}$$

$$\frac{-1}{1+105.1pT+511p^2T^2+50p^3T^3} \quad \text{for T equal .02}$$

It is desired to maximize the a_3 coefficient value. This value is seen to increase with decreasing T. The scaled magnitude of the a_3 term can be 10. The magnitude of the a_o term will be $\frac{10}{a_3}$. For T equal .004 this would yield $\frac{1}{625}$ or .0016. The last place would be lost in the potentiometer readout. Scaling the coefficient values for T equal to .02 and .01 so they fit the coefficient values yields:

$$\frac{-.095}{.095 + 10pT + 46.5p^2T^2 + 4.75p^3T^3} \quad \text{for T equal .02}$$

$$\frac{-.025}{.025 + 5.27pT + 50.85p^2T^2 + 10p^3T^3} \quad \text{for T equal .01}$$

Magnitude scale position 3 will have to be used for either case because the a_2 coefficient value is much larger than a_3. The coefficient values for T equal .01 look reasonable. The low order coefficient values are always relatively easy to determine. The high order terms are difficult. The largest value of D(p) at high wT settings appears at the a_4 coefficient potentiometer which cannot be used due to scaling problems. Without the use of a_4 potentiometer, the excitation voltage decreases at high frequencies.

The phasor diagram for excitation voltage at wT equal to 2, T equal to .01 is shown in Fig. 8. The reference voltage will be .177 millivolts. The simulated network can be represented by

$$\frac{1}{1 + 2.102jw + .2042(jw)^2 + (.0004jw)^3}$$

$$= \frac{1}{8620 \angle -161}$$

The response will be

$$\frac{1.53 \angle -161}{8620 \angle -161} = .177 \text{ mV.}$$

The resultant display will be a 45° trace. The magnitude of this signal would be too small and noisy to yield any information. Increasing the gain by 40 gives b_o coefficient value of 1 and a reference voltage of 7.1 mV.

The coefficient values for T equal .01 were set up on the T.F.C. A 20% change in a_2 and a_3 coefficient potentiometer values caused a visual change in the display. Pictures of excitation voltage and C.R.T. display are shown in Fig. 9.

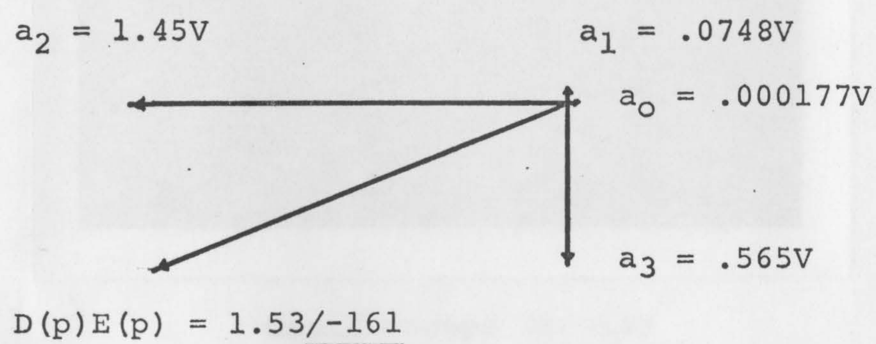
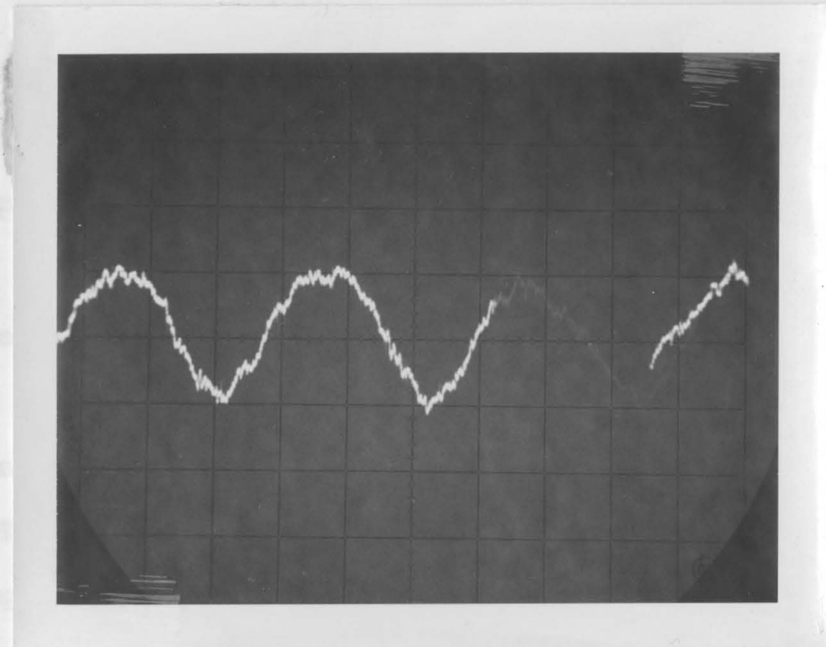
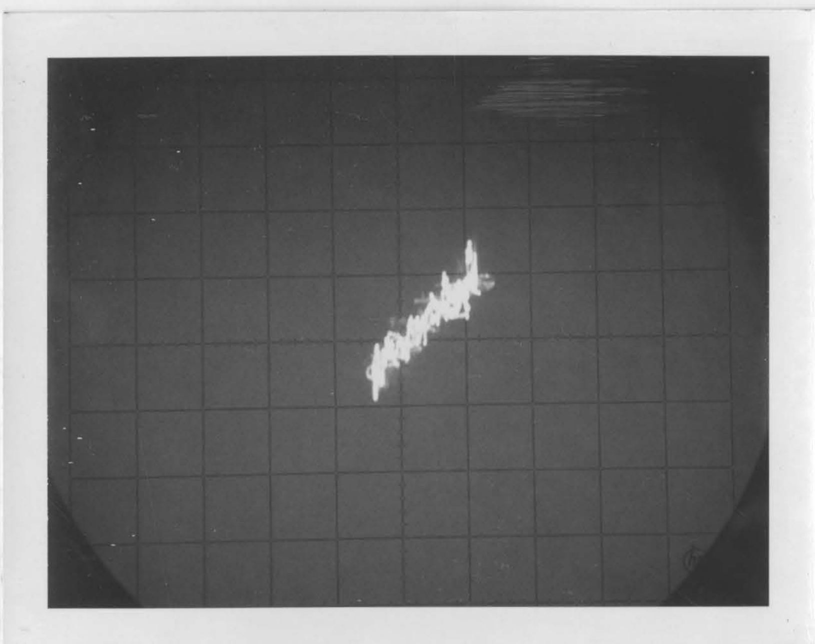


Fig. 8. Phasor Diagram for Excitation Voltage



Excitation Voltage
Y = 10mV/cm   w = 200 rad/sec



Oscilloscope Display
X = 10mV/cm   Y = 10mV/cm

Fig. 9.
Excitation Voltage and Oscilloscope Display for

$$\frac{-40}{(1 + 2s)\ (1 + .1s)\ (1 + .002s)}$$

Feedback Control System

A position control system was set up to check on the limitations imposed by lack of high frequency response. The components were manufactured by Feedback Inc. A simplified block diagram of the system as shown in Fig. 10. A single turn potentiometer was used for both reference and output. A linearizing circuit was employed to yield a linear signal time constant response for the $\frac{1}{20}$ H.P. motor.

The system was first tested open loop. The excitation signal was applied to the operational amplifier. The output signal was taken from a shaft mounted D.C. tachometer. The expected transfer function was

$$\frac{b_o}{a_o + a_1 Tp} \tag{34}$$

Where b_o is the overall gain in $\frac{\text{volts out}}{\text{volts in}}$ and $a_1 T$ is the motor time constant. Better resolution for the a_1 term could be expected by adjusting the gain to get b_o equal to 1. This was done by setting the $\frac{b_o}{a_o}$ ratio to unity and adjusting the system gain to yield $\frac{e_o}{e_i}$ equal to one for steady state step response. For a system with a single time constant the transfer function can be determined at a constant frequency. The frequency must be chosen to maximize the a_1 coefficient. Frequency response indicated a break at approximately 2.5 radians. Coefficient search was made with T equal to .2 and wT equal to A, and indicated a

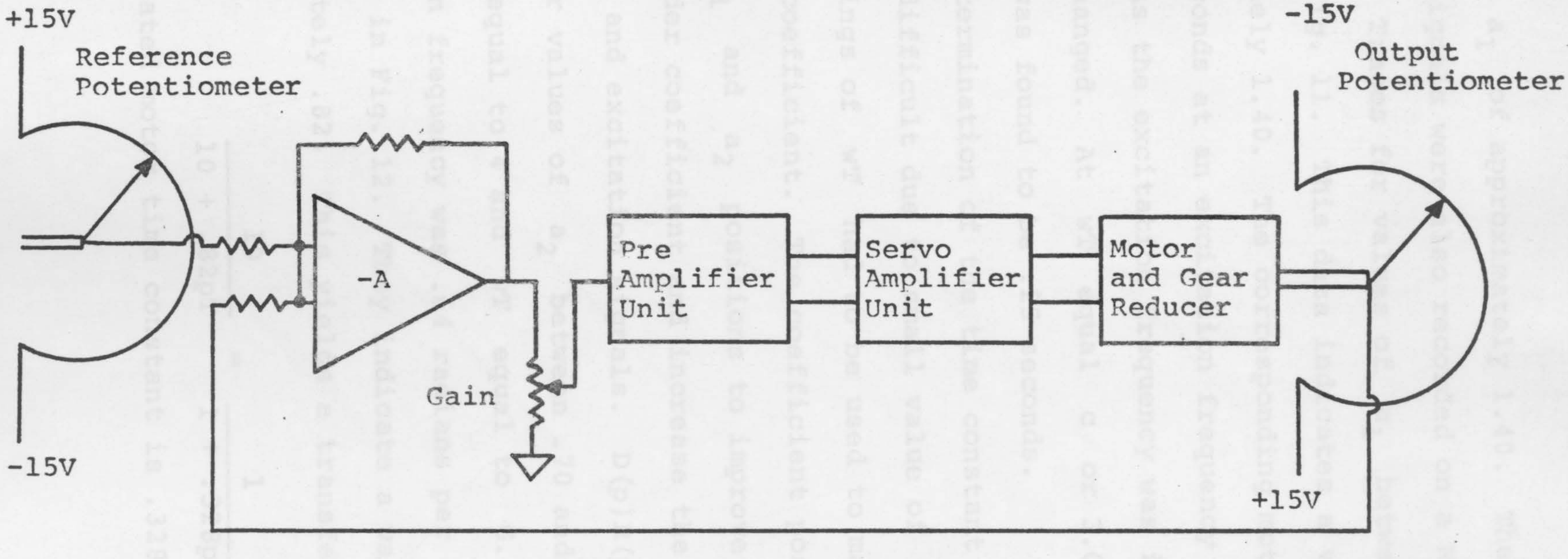


Fig. 10. Position Control System.

value for a_1 of approximately 1.40. The D(p)R(p) v.s. Response signals were also recorded on a Moseley 2D-2A X-Y recorder. Traces for values of a_1 between 1.2 and 1.7 are shown in Fig. 11. This data indicates a value for a_1 of approximately 1.40. The corresponding motor time constant is .28 seconds at an excitation frequency of 1.56 radians per second. As the excitation frequency was increased, the value of a_1 changed. At wT equal c or 3.05 radians, the time constant was found to be .25 seconds.

Determination of the time constant at lower frequencies was more difficult due to small value of a_1 for large T. High settings of wT had to be used to maximize sensitivity for a_1 coefficient. The coefficient positions were moved to the a_1 and a_2 positions to improve resolution for the higher order coefficient and increase the magnitudes of the reference and excitation signals. D(p)R(p) v.s. Response traces for values of a_2 between .70 and .95 were obtained with T equal to 4 and wT equal to G. The corresponding excitation frequency was .64 radians per second. The traces are shown in Fig. 12. They indicate a value for a_2 of approximately .82. This yields a transfer function of

$$\frac{10}{10 + .82pT} = \frac{1}{1 + .328p}$$

The indicated motor time constant is .328 seconds.

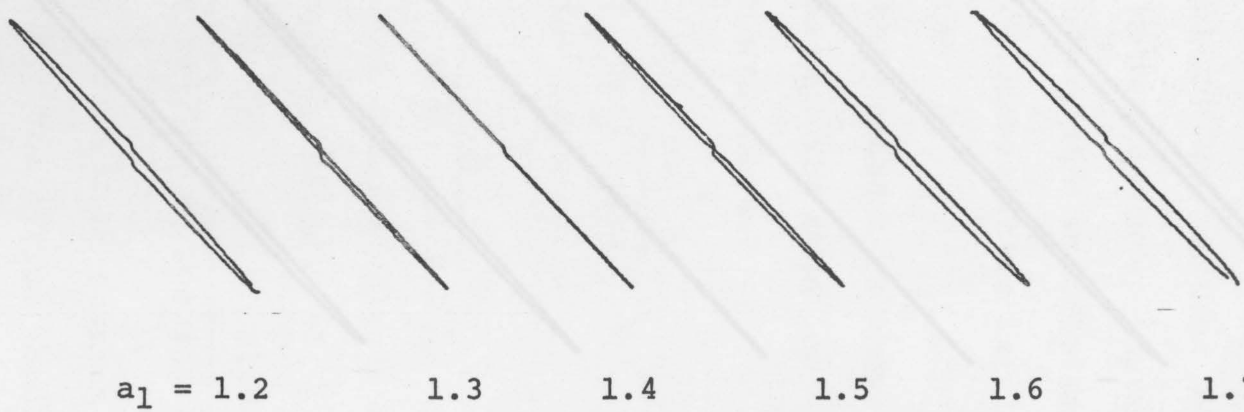


Fig. 11. Open loop Frequency Response.

Excitation frequency equals 1.56 radians per second. Indicated time constant equals .28 seconds.

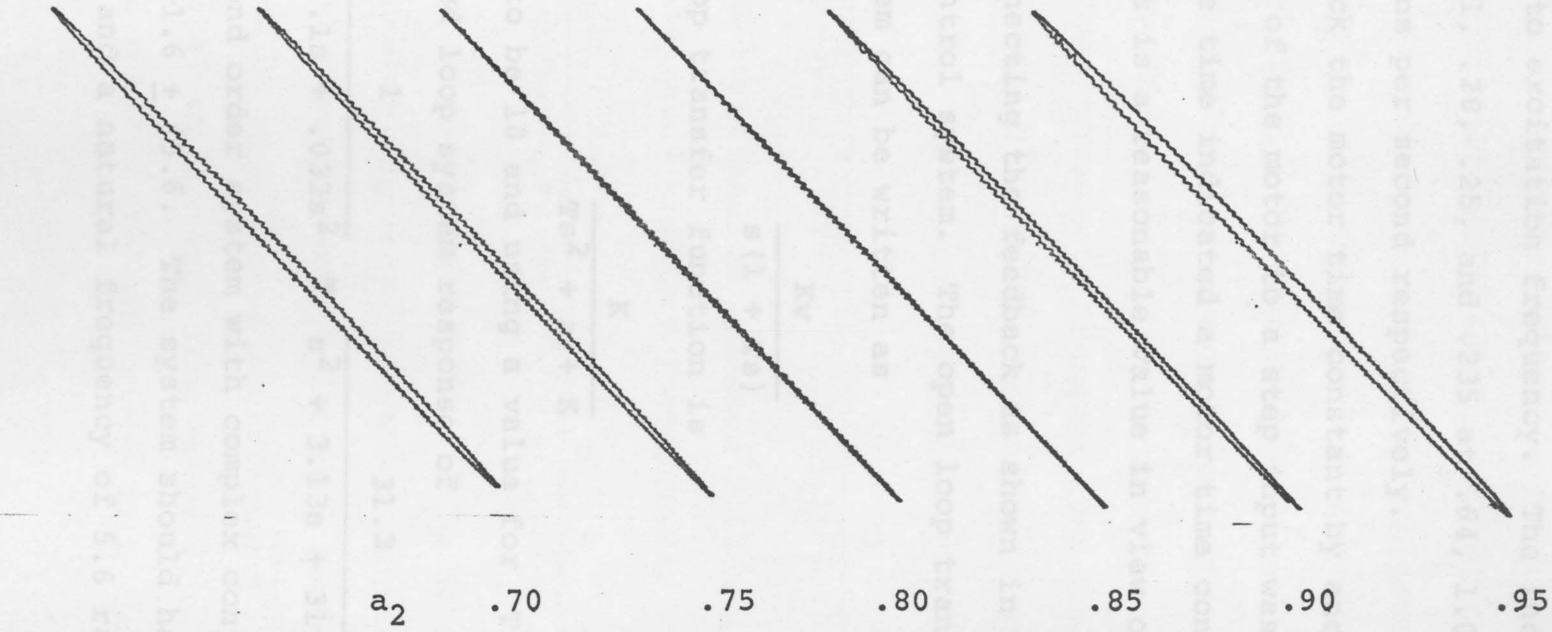


Fig. 12. Open Loop Frequency Response.

Excitation frequency equals .64 radians per second. Indicated time constant equals .328 seconds.

The motor time constant was found to be inversely proportional to excitation frequency. The indicated values were .328, .31, .28, .25, and .235 at .64, 1.0, 1.56, 3.05, and 5.0 radians per second respectively.

To check the motor time constant by another method, time response of the motor to a step input was recorded. The 62.8% rise time indicated a motor time constant of .32 seconds. This is a reasonable value in view of the above findings.

By connecting the feedback as shown in Fig. 10, yields a position control system. The open loop transfer function for this system can be written as

$$\frac{Kv}{s(1 + Ts)} \tag{35}$$

The closed loop transfer function is

$$\frac{K}{Ts^2 + s + K} \tag{36}$$

Choosing K to be 10 and using a value for T of .32 yields a closed loop system response of

$$\frac{1}{1 + .1s + .032s^2} = \frac{31.3}{s^2 + 3.13s + 31.3}$$

This is a second order system with complex conjugate poles at s equal $-1.6 \pm j5.6$. The system should have a damping factor of .28 and a natural frequency of 5.6 radians per second.

The response of a system with complex conjugate poles will have small changes in magnitude or phase for excitation frequencies below $\frac{w_n}{2}$. A value of 1 was tried for T. This setting yields a maximum excitation frequency of 2.55 radians. There was no discernable change in the display for any value of wT for changes in a_2 coefficient potentiometer.

The approximate transfer function was found at two lower values of T. The magnitude of the excitation voltage was kept low to minimize change in position. wT was varied over small range of frequencies to minimize changes in time constant. The approximate transfer function at T equal .2 and w between 1.56 and 2.5 radians per second was found to be

$$\frac{1}{1 + .5pT + .75p^2T^2} = \frac{1}{1 + .1p + .03p^2}$$

For T equal to .1 and w between 3.12 and 5 radians per second the indicated transfer function was

$$\frac{1}{1 + 1.2pT + 2.8p^2T^2} = \frac{1}{1 + .12p + .028p^2}$$

The coefficient values were difficult to determine due to noisy and somewhat nonlinear lissajous pattern. The approximate transfer function does indicate a decreasing p^2 coefficient value with increasing frequency. This term is directly related to the open loop motor time constant (see eq. 36). The results are consistant with those found for the open loop system.

Analog Simulation

To study this particular function further, analog simulation of $\frac{10}{10 + s + 0.32s^2}$ was used. The resulting circuit configuration is shown in Fig. 13.

Frequency response indicated a peaking in response past T equal .2. This value of T gives a frequency band of approximately 1.6 to 13 radians per second and includes the natural and peak frequencies. The a_1, a_2, and a_3 coefficient potentiometers were used to maximize coefficient values and improve resolution. With little effort, the coefficient values and resulting transfer function were

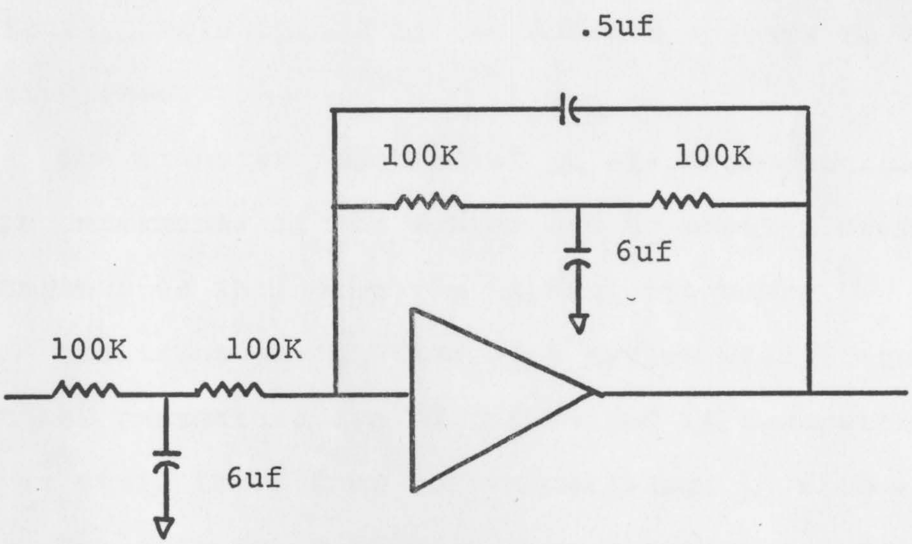


Fig. 13. Generation of Comples Conjugate Poles.

found to be

$$\frac{10}{10 + 5pT + 8p^2T^2} = \frac{10}{10 + p + .32p^2}$$

The sensitivity of the display to change in coefficient value was good. By using the a_1, a_2, and a_3 coefficient positions, the magnitude of the display did not decrease for high wT settings.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions can be drawn from the experiments studied:

The transfer function of an active network or system with poles in the negative left half plane can be determined without much difficulty if the pole spread is within one decade. This will also hold for a passive network.

Determination of transfer function for network or system with pole spread greater than one decade becomes more difficult. Pole spread of two decades appears to be the limiting case.

The transfer function of an electromechanical system can be determined if the system can be excited over a range of frequencies including its natural frequency.

The transfer function of a system with frequency dependent parameters can be determined if parameter variation is small for a four octave variation in frequency.

The greatest difficult encountered in coefficient determination was the decrease in $D(p)E(p)$ at high wT settings. This can be avoided if all coefficients are moved to higher order coefficient potentiometers. This cannot be done for all third order differential equations. The maximum scaled value for the a_1 and a_4 coefficient potentiometers is 1 vs. 10 for a_1 through a_3 coefficient potentiometers. If the coefficient values are scaled down to let the highest order term equal one, the lowest order term might be much less than one. It will mow appear in the a_1 position, which has two decimal places, v.s. three in the a_o position. Significant information will be lost and the transfer function will be altered.

Transfer Function Computer Serial No. 74 has excessive harmonics present at the higher order coefficient values. These result from improperly adjusted potentiometers in the feedback circuit.

Recommendations for future work:

Modify equipment to provide independent scaled values of 1 and 10 for a_o through a_4 potentiometers.

Adjust resistors in feedback circuit of unit No. 74 to eliminate harmonics.

APPENDIX

LABORATORY EXPERIMENT

Object: To study the operation of the Transfer-Function Computer (T.F.C.).

Principle of Operation

The T.F.C. is, essentially, a four integrator loop. The output x and the four derivative terms x, $\dot{x}$, etc. are shown in Fig. 1.

Connected as an integrator, the transfer function of each operational amplifier is given by

$$e_o = \frac{-1}{RC} \int e_i dt \quad \text{or} \quad \frac{e_o}{e_i} = \frac{-1}{pT} \tag{1}$$

For sinusoidal excitation K sin wT, the output is given by

$$e_o = \frac{K}{wT} \cos wt \tag{2}$$

Phase shift is +90° per step. The desired phase shift to achieve x, $\dot{x}$ through $\ddddot{x}$ is -90° per step. To obtain that, magnitude of outputs from operational amplifier #2 and #4 are inverted.

The final a_n and b_n coefficient values are equal to - within a multiplier T^n - the coefficient values of the transfer function to be determined. The voltages

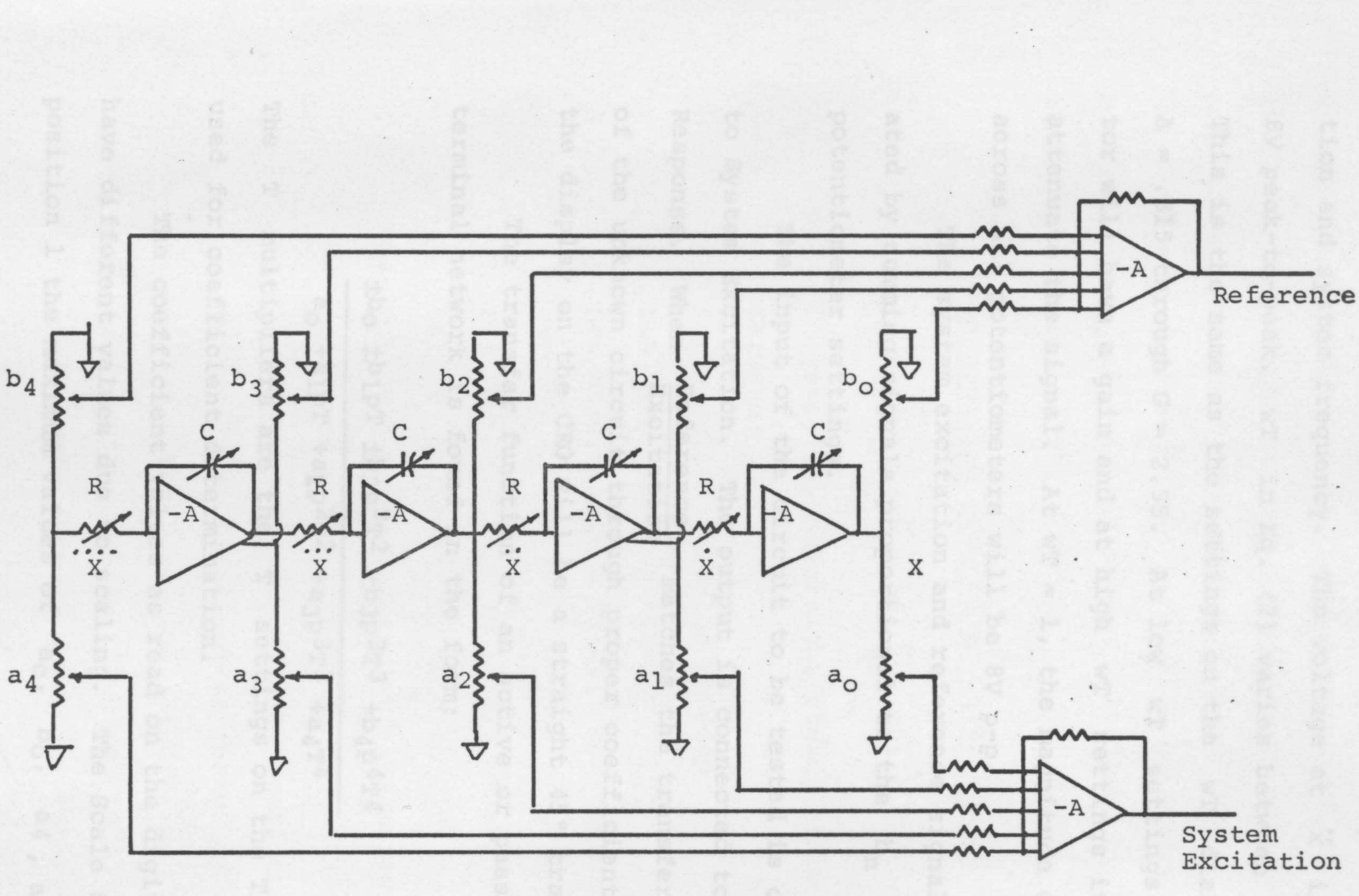


Fig. 1. Simplified System Schematic Diagram.

across these potentiometers will depend on the op-amp location and system frequency. The voltage at $\ddot{x}$ is always 8V peak-to-peak. wT in Eq. (2) varies between .315 and 2.55. This is the same as the settings on the wT dial, i.e., A = .315 through G = 2.55. At low wT settings the integrator will have a gain and at high wT settings it will attenuate the signal. At wT = 1, the magnitude of signals across all potentiometers will be 8V p-p.

The system excitation and reference signals are generated by summing signals proportional to the b_n and a_n potentiometer settings.

The input of the circuit to be tested is connected to System Excitation. The output is connected to System Response. When $\frac{\text{Reference}}{\text{Excitation}}$ matches the transfer function of the unknown circuit through proper coefficient settings, the display on the CRO will be a straight 45° trace.

The transfer function of an active or passive four terminal network is found in the form:

$$\frac{\pm b_o \pm b_1 pT \pm b_2 p^2T^2 \pm b_3 p^3T^3 + b_4 p^4T^4}{a_o + a_1 pT + a_2 p^2T^2 + a_3 p^3T^3 + a_4 T^4} \tag{3}$$

The T multipliers are the T settings on the T.F.C. used for coefficient determination.

The coefficient values as read on the digital readouts have different values due to scaling. The Scale Selector position 1 the maximum values of a_o, b_o, a_4, and b_4

are 1. The maximum value of all other potentiometers is 10. Only 10% of the x and $\ddddot{x}$ signal is applied to the potentiometers. Full signal magnitude of $\dot{x}$, $\ddot{x}$, and $\dddot{x}$ signals is applied to the respective potentiometers. Both numerator and denominator are multiplied by 10 and the transfer function is not altered.

Determination of Unknown Coefficients

Determination of $\frac{b_o}{a_o}$ Ratio

The $\frac{b_o}{a_o}$ ratio represents the steady state gain as $t \to \infty$. To determine $\frac{b_o}{a_o}$

1) Set Function Selector to Step.
2) Set Step Selector to Zero.
3) Set Mode Selector to Null and center dot on CRT.
4) Set a_o and b_o to 1, all others to 0. Set X Gain and Y Gain to 10.
5) Switch Step to +V or -V and observe motion of dot. Adjust b_o potentiometer leaving a_o at 1, or a_o, leaving b_o at 1, to bring dot to center of screen. For a passive network b_o is less than 1, a_o is equal to 1. For active network with gain, the reverse will hold.
6) Switch X Gain and Y Gain to greater

sensitivity as the values are finalized.

7) Switch Step Selector to +V and -V and check symmetry of response.

Determination of T

The value of T selects the frequency range over which the system is excited during coefficient selection. This range should include all the break frequencies of the system.

1) Set Excitation to Sinusoidal, Display to 45°.
2) Set wT to .5, T to 4.
3) Set X Gain and Y Gain to 10 and adjust X and Y Shift to obtain 45° trace or lissajous pattern close to 45°.
4) Switch T to larger values to increase frequency, i.e., 4-2-1-.4, etc.
5) Visually observe resulting lissajous pattern. CW rotation indicates poles, C.C.W. indicate zeros. 45° tilt in pattern from original indicates 90° phase shift or first singularity.
6) Set T to value where system indicates maximum change, i.e., between break frequencies.

Determination of p^n Coefficients

The coefficients of an unknown system are determined in an ascending order of p.

1) Determine $\frac{b_o}{a_o}$ by step response.
2) Determine setting for T.
3) Set Magnitude scaling to 1.
4) Set CRO to 45° trace.
5) Set wT to A and adjust a_1 and b_1 coefficient potentiometers to obtain a 45° trace.
6) Move wT to .5 or C and adjust a_2 and b_2 potentiometers to approximate 45° trace.
7) Return to A and readjust a_1 and b_1.
8) Return to C and readjust a_2 and b_2.
9) Go to wT = 1, and set a_3 and b_3.
10) Repeat steps 5 through 9 until a straight 45° trace is obtained for all frequencies from wT = A through G.
11) The coefficient potentiometers should read toward full scale, i.e., 10 for a_1, a_2, and a_3, for maximum sensitivity.

If any high order coefficient has low value repeat procedure with lower T.

12) If any coefficient value exceeds maximum potentiometer setting, rescale potentiometer values by dividing numerator and denominator by a constant.

13) Multiply a_n and b_n coefficients by the value of T used to obtain transfer function as in Equation (3).

Procedure

1) Connect Excitation to Response, Reference to Reference. Set $a_o = b_o = 1$. All other coefficient potentiometers to zero.

2) a) Vary frequency by varying wT for T = .001. Check that wT = A, .5, 1, 2, and G yields radian frequency of $w = \frac{A}{.001}, \frac{.5}{.001}$, etc. Radian frequency = $\frac{\text{wT setting}}{\text{T setting}}$.

   b) Repeat a) for different T.

3) a) Set wT = A, T = .001, measure p-p voltage of b_o for wT = A, .5, 1, 2, and G.

   b) Set $b_o = 0$, $b_1 = 10$, and repeat a).

3) c) Repeat a) for b_2, b_3, b_4.
Remember you are measuring 10% of full voltage at b_o and b_4.
d) Repeat a) through c) for different value of T.

4) Set $a_o = b_o = 1$, all others to zero. Use dual trace oscilloscope. Check phase relations of all the coefficient voltages by keeping a_o as reference. Reduce b_o to zero and increase b_1 sufficiently to read phase relation. Return b_1 to 0 and increase b_2, etc.

Report

1) What are the radian frequency ranges for all T Settings, i.e., keeping T constant and varying wT?

2) Make a chart for b_o through b_4 coefficient voltages for wT = .5, 1 and 2. Where are the dominant voltages at low and high frequencies!

3) What is the phase relation of the b_o and a_o voltages to the other coefficient voltages! Draw phasor diagrams.

4) Draw phasor diagram of reference and excitation voltages for given transfer

function at $wT = 1, T = 1$

$$\frac{1}{1 + 5p + 10p^2}$$

Numerator is reference or b_o which is equal 1. Only 10% of full voltage is applied to this position. Denominator is excitation. $a_o = 1$, $a_1 = 5$ or 50% full voltage, $a_2 = 10$ or full voltage of the coefficient.

5) Check that excitation times transfer function equals reference by letting $p = jw$ in transfer function above.

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