DEFLECTION CURVES OF AN ELASTIC STRIP UNDER TENSION AND FRICTION FORCES

by

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ABSTRACT

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The purpose of this thesis is to develop the deflection curves resulting when a succession of forces and moments are applied to a semi-infinite beam which is under tension, and where displacement is resisted by distributed friction loads.

The problem applies to the study of web guidance, which is concerned with accurately passing long strips over a series of rollers. The area studied in this thesis can be interpreted as the region of contact between the strip and a roller.

The exact, general solution to the problem consists of an infinite number of waves, but a closed form solution is impossible. A method is described for calculating a solution accurate to any given number of waves.

This method is demonstrated, the two wave and one wave approximations being developed as examples.

A Fortran program is provided, which uses the one wave approximation to generate deflection curves in tabular form. This program may be incorporated as a subroutine into

a larger program to provide a precise computer simulation of an entire rolling strip system.

ACKNOWLEDGEMENTS

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Also, I must thank all those who patiently took time to listen to the myriads of problems that arose in this work, and helped me think them through. In particular, Mr. Robert Sorokach, and my good friends Bob Ramhoff and Carol Lee Novotny were of great help.

Finally, I thank my parents for their continuing support and encouragement.

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LIST OF SYMBOLS

SYMBOL		DEFINITION	UNITS
A	=	$\sqrt{T/EI}$	$ft.^{-1}$
c	=	T/EI	$ft.^{-2}$
e	=	End deflection of strip	ft.
E	=	Modulus of elasticity of strip	lb/in^2
F	=	Applied lateral force	lbs.
I	=	Moment of inertia of cross section	in^4
k	=	Distributed friction load	lb./ft.
L	=	Total length of n^{th} wave of deflection	ft.
M	=	Applied moment	ft.lb.
N_2	=	Nondimensional moment (M/EIA)	none
N_3	=	Nondimensional force (F/T)	none
N_4	=	Nondimensional friction load (k/EIA^3)	none
r	=	Roller radius	ft.
T	=	Longitudinal tension	lb.
μ	=	Coefficient of friction between roller and strip	none
θ_o	=	Slope at end of strip	radians

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The problem treated in this thesis is as follows: given a semi-infinite elastic beam (or strip) placed in contact with a friction surface, and held in constant longitudinal tension. If the end of the strip is subjected to lateral forces and moments, what deflections will result?

This problem arose from a detailed theoretical study of web guidance problems. Web guidance is concerned with systems wherein long strips of elastic material must be passed over many rollers. The metal strip industry is typical, where an essentially continuous web of metal must be uncoiled, passed through a roller system, perhaps through slitting and finishing processes, and recoiled once again.

In such processes, the lateral position of the web is often critical. If all rollers were perfectly aligned, and the strip material were perfectly uniform in cross section and perfectly straight, lateral position would never vary, as the strip would roll exactly straight ahead. But generally, this ideal case is not achieved, and imperfections cause lateral motions in the system.

The basic mechanism of lateral motion can be described as follows: consider a straight web running onto a roll which is aligned perpendicular to the web, as shown in figure 1A. As long as the approach angle remains at 90°, the web will keep the same position relative to the roll.

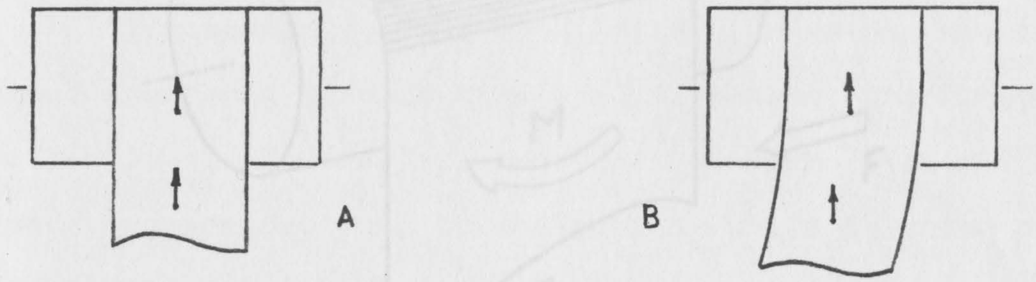

Fig. 1.--Rolls with approaching strips

However, if the web has an approach angle other than 90°, as shown in figure 1B, each succeeding point will contact the roll further to one side, and the web will be seen to displace laterally relarive to the roll, it's lateral velocity varying with the web speed and the approach angle. For this reason, the angle the strip makes relative to the roll is of great interest.

The physical situation treated in this thesis, then, is a minute but critical component of a roll-strip system, the portion shaded in figure 2. At any instant a force or moment applied to the edge of the contact area by the free span of entering strip, results in a lateral sliding of a certain portion of the contact area, and thus, non-perpendicularity between roll and strip. This thesis will relate the deflections and resulting angles to the forces and moments present.

In constructing the problem, it was assumed that

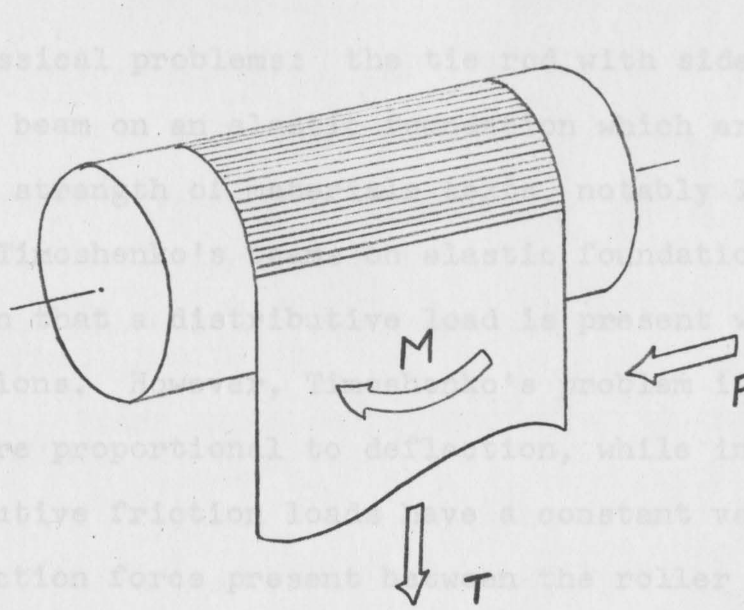

Fig.2.--Roll and strip, showing area of interest

there is a constant normal force between roll and strip, within the area of contact. Also, in keeping with the constant normal force, friction is assumed to have constant magnitude. The direction of the friction load is such that it always opposes displacement.

It is intuitively obvious that the greater the applied force, the longer the displaced portion of the strip will be. (This displaced portion is hereafter referred to as the "affected length".) Clearly, it must be assumed that the affected length is small enough to be contained on the roll. For simplicity, the strip treated in the problem is considered semi-infinite.

The physical basis for the problem has now been thoroughly defined. It may be seen that it has aspects of two classical problems: the tie rod with side loading, and the beam on an elastic foundation which are treated in many strength of materials texts, notably Timoshenko.[1]

Timoshenko's beams on elastic foundations are analagous in that a distributive load is present which opposes deflections. However, Timoshenko's problem is such that loads are proportional to deflection, while in this problem, distributive friction loads have a constant value, that of the friction force present between the roller and the strip. Only the sign of this load changes, depending on the direction of displacement. (An elementary statical analysis will show that the value of this friction load, k, can be related to roll system parameters by the equation, $k = T\mu/r$).

It will be seen that, as with the beam on an elastic foundation, the solution to this problem takes the form of an infinite series of diminishing waves.

Standard beam theory assumptions prevail in this work. They are as follows:

The strip is assumed initially straight and uniform. Although webs are often cambered or curved, the radius of curvature is generally very large, and our affected lengths will be seen to be very small. Thus, this imposes no serious limitation.

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[1] S. Timoshenko, Strength of Materials, Vol. II (Third edition; Princeton, N.J., D. VanNostrand Co., 1956), pp. 1-5, 41-46.

Also, loads must be applied slowly enough that inertia effects are negligible.

Other assumptions include:

1. Deflections are small.

2. The web is homogenous and uniform and the material obeys Hooke's law.

3. Plane sections perpendicular to the longitudinal axis remain plane after bending.

4. There must be no buckling or wrinkling of the web.

5. Shear deflections are negligible.

It is necessary to establish sign conventions for the problem. These are given in figure 3., and require no comment.

Thus, the problem is thoroughly defined, and the method of the general solution may be established.

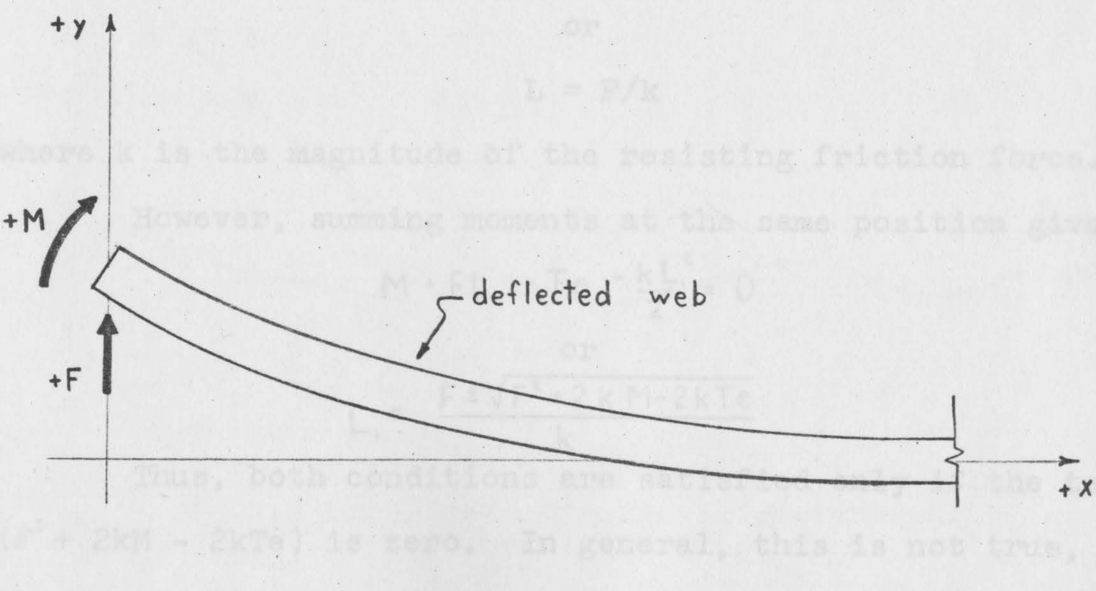


Fig.3.--Sign Conventions

CHAPTER II

METHOD FOR GENERAL SOLUTION

The exact solution to this problem will be shown to consist of a configuration of diminishing waves, as illustrated in figure 4. This can be demonstrated by examining the forces and moments present at $x_1 = L_1$ (which is the end of the first wave).

If the solution consisted of only one wave, followed by a straight strip, at the end of that wave there could be no residual forces or moments, since the presence of a moment implies a radius of curvature. Thus, summing vertical forces at this position shows

$$F = kL$$

or

$$L = F/k$$

where k is the magnitude of the resisting friction force.

However, summing moments at the same position gives

$$M + FL_1 - Te - \frac{kL_1^2}{2} = 0$$

or

$$L_1 = \frac{F \pm \sqrt{F^2 + 2kM - 2kTe}}{k}$$

Thus, both conditions are satisfied only if the term $(F^2 + 2kM - 2kTe)$ is zero. In general, this is not true, and the residual shear and moment at $x_1 = L_1$, result in deflections in the second wave. A similar analysis at the end of the second wave shows a similar imbalance, indicating a third wave present, and so on, ad infinitum.

It is, however, possible to achieve an approximate solution by imposing boundary conditions of zero slope and moment after a sufficient number of waves. The general equations for such a solution will be developed here.

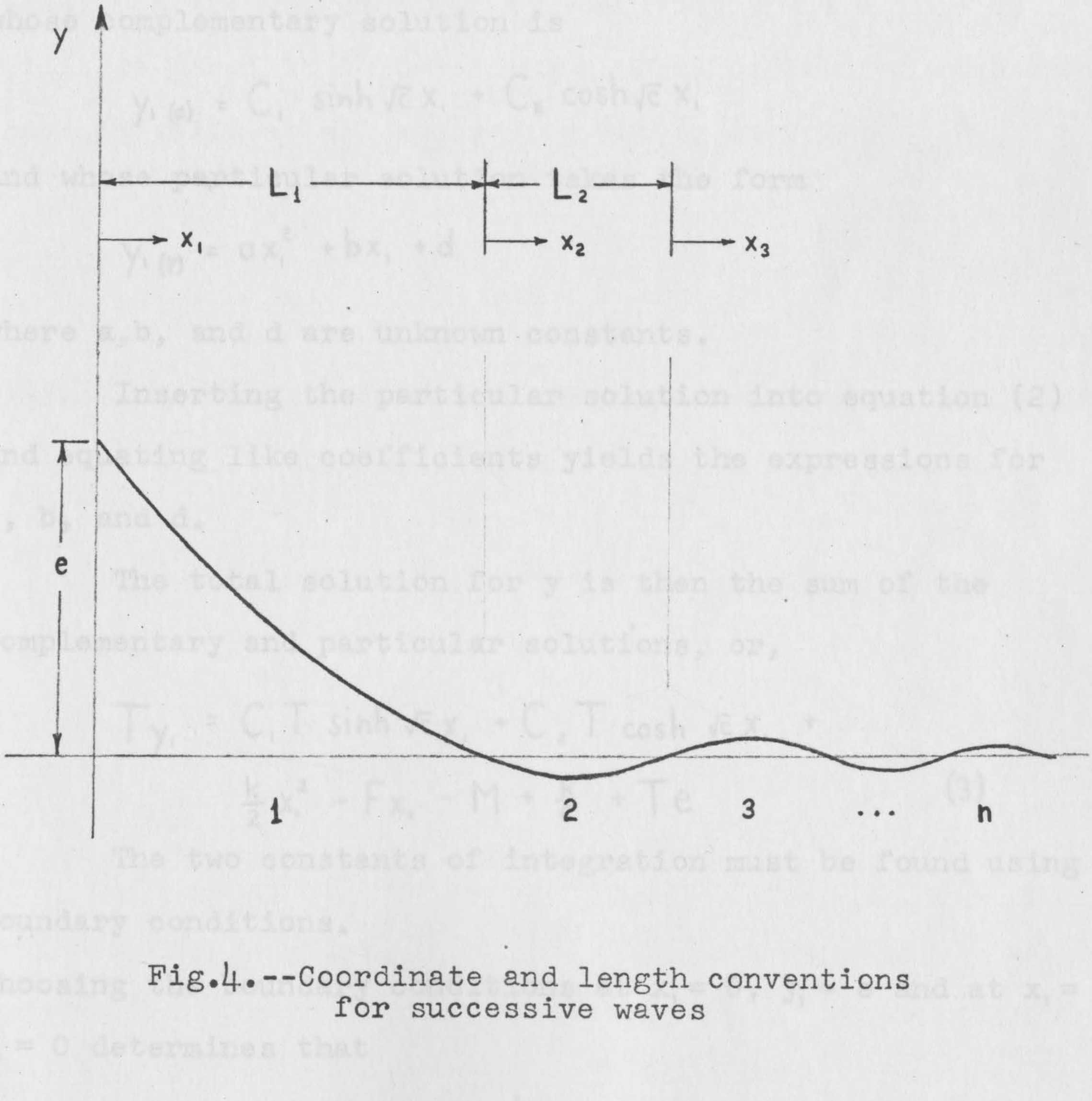


Fig.4.--Coordinate and length conventions for successive waves

2.1 Equations for the First Wave

Considering forces and moments on the first wave in figure 4 yields the differential equation

$$EI\,y_1'' = M + Fx_1 - \frac{kx_1^2}{2} - T(e-y) \qquad (1)$$

defining c = T/EI gives

$$y_1'' - cy = \frac{M - Te}{EI} + \frac{F}{EI}x_1 - \frac{k}{2EI}x_1^2 \qquad (2)$$

whose complementary solution is

$$y_{1\,(c)} = C_1 \sinh\sqrt{c}\,x_1 + C_2 \cosh\sqrt{c}\,x_1$$

and whose particular solution takes the form

$$y_{1\,(p)} = ax_1^2 + bx_1 + d$$

where a,b, and d are unknown constants.

Inserting the particular solution into equation (2) and equating like coefficients yields the expressions for a, b, and d.

The total solution for y is then the sum of the complementary and particular solutions, or,

$$Ty_1 = C_1 T \sinh\sqrt{c}\,x_1 + C_2 T \cosh\sqrt{c}\,x_1 + \frac{k}{2}x_1^2 - Fx_1 - M + \frac{k}{c} + Te \qquad (3)$$

The two constants of integration must be found using boundary conditions.

Choosing the boundary conditions at $x_1 = 0$, $y_1 = e$ and at $x_1 = L_1$, $y_1 = 0$ determines that

$$TC_2 = M - k/c$$

and

$$TC_1 = \frac{1}{\sinh\sqrt{c}L_1}\left[\left(\frac{k}{c} - M\right)\left(\cosh\sqrt{c}L_1 - 1\right) - \frac{kL_1^2}{2} + FL_1 - Te\right]$$

Thus, the equation valid for the first wave is

$$T y_1 = \frac{\sinh \sqrt{c}\, x_1}{\sinh \sqrt{c}\, L_1}\left[\left(\frac{k}{c} - M\right)\left(\cosh \sqrt{c}\, L_1 - 1\right) - \frac{k L_1^2}{2} + F L_1 - Te\right]$$

$$+ \left(M - \frac{k}{c}\right)\left(\cosh \sqrt{c}\, x_1 - 1\right) + \frac{k x_1^2}{2} - F x_1 + Te \tag{4}$$

The equation describing the first wave is unique, because of the unique boundary condition $y_1 = e$ at $x_1 = 0$, but it is possible to develop a general deflection equation which is valid at all succeeding waves, as follows.

2.2 Equations for a General Wave

By summing moments about some point (x_n, y_n) in the n^{th} wave, it can be said that for that wave,

$$EI\, y_n'' = F\left(x_n + \sum_{i=1}^{n-1} L_i\right) + M - T(e - y_n)$$
$$+ \sum_{m=1}^{n-1}\left((-1)^m k L_m\right)\left(\frac{L_m}{2} + x_n + \sum_{i=m+1}^{n-1} L_i\right) + (-1)^n \frac{k x_n^2}{2} \qquad (5)$$

Some clarification is no doubt necessary. The first three terms are obviously the moments due to the force, moment, and tension applied.

The last two terms are the moments due to the distributive friction loads. The last term accounts for the friction on the n^{th} wave, while the preceeding term is the sum of the friction effects in all previous waves. The subscript m refers to the particular wave whose friction effect is being accounted for. The term $\left((-1)^m k L_m\right)$ is the total force due to friction in the m^{th} wave, while the term $\left(\frac{L_m}{2} + x_n + \sum_{i=m+1}^{n-1} L_i\right)$ is the moment arm through which this force acts.

In this equation, and those following the convention is held that backwards summation limits go to zero (for example, $\sum_{n=3}^{2} L_n = 0$).

Following the procedure used for the first wave, the complementary and particular solutions are determined, giving the following equation, which is valid for any wave except the first.

$$Ty_n = C_{1(n)}T\sinh\sqrt{c}\,x_n + C_{2(n)}T\cosh\sqrt{c}\,x_n - \frac{k(-1)^n x_n^2}{2}$$
$$-Fx_n + x_n\sum_{m=1}^{n-1}((-1)^{m+1}kL_m) + Te - M - \frac{k}{c}(-1)^n$$
$$-\sum_{m=1}^{n-1}FL_m + \sum_{m=1}^{n-1}((-1)^{m+1}kL_m)\left(\frac{L_m}{2}+\sum_{i=m+1}^{n-1}L_i\right) \qquad (6)$$

As before, there are two constants of integration. The first boundary condition, $y_n = 0$ at $x_n = 0$, gives us $C_{2(n)}$ and the equations

$$Ty_n = C_{1(n)}T\sinh\sqrt{c}\,x_n + \left[M - Te + \frac{k(-1)^n}{c} + \sum_{m=1}^{n-1}FL_m + \sum_{m=1}^{n-1}((-1)^m kL_m)\left(\frac{L_m}{2}+\sum_{i=m+1}^{n-1}L_i\right)\right]*$$
$$(\cosh\sqrt{c}\,x_n) - \frac{k(-1)^n x_n^2}{2} - Fx_n - x_n\sum_{m=1}^{n-1}((-1)^m kL_m) + Te$$
$$-M - \frac{k}{c}(-1)^n - \sum_{m=1}^{n-1}((-1)^m kL_m)\left(\frac{L_m}{2}+\sum_{i=m+1}^{n-1}L_i\right) \qquad (7)$$

$$Ty_n' = C_{1(n)}T\sqrt{c}\cosh\sqrt{c}\,x_n$$
$$+\sqrt{c}\sinh\sqrt{c}\,x_n\left[M - Te + \frac{k(-1)^n}{c} + \sum_{m=1}^{n-1}FL_m + \sum_{m=1}^{n-1}((-1)^m kL_m)\left(\frac{L_m}{2}+\sum_{i=m+1}^{n-1}L_i\right)\right]$$
$$-k(-1)^n x_n - F - \sum_{m=1}^{n-1}((-1)^m kL_m) \qquad (8)$$

The constant $C_{1(n)}$ is still undetermined. The boundary condition to be used is the matching of slopes between waves. That is, y_n' at $x_n = 0$ must equal y_{n-1}' at $x_{n-1} = L_{n-1}$

Unfortunately, the determination of C_1 for the n^{th} wave necessitates knowing the deflection equation for the preceeding wave. Hence, $C_{1(n-1)}$ must be known. Similarly that constant can't be determined before calculating $C_{1(n-2)}$, and so on, back to the first wave.

The ability to write one general deflection equation, valid for all waves, has thus been lost. Still, the procedure for determining the deflection curve of any given wave should be clear. The equation for the first curve has been determined. Using this equation, and equating slopes at the junction of waves 1 and 2, the unknown constant for wave 2 can be found. (The deflection curve for wave 2 being given by the general formula, equation (7)). Once the second wave has been determined, the same procedure can be used to find the third, then fourth, and so on, to a sufficiently accurate approximate solution.

It will be noted that the general equation for wave n contains two other unknown quantities, namely e, the deflection at the end of the strip, and the quantities L_1, L_2, ... L_{n-1}, which are the lengths of the successive waves.

Thus, once the equations for y_1, y_2, ... y_n have been produced, n more equations will be needed to determine the n-1 lengths and the end deflection.

With the exception of the first wave, the fact that deflection at the end of a wave is zero has not been introduced. This can now be done, by inserting $y_2 = 0$ at $x_2 = L_2$, and so on, to $y_n = 0$ at $x_n = L_n$ into the n^{th} equation. It will be seen that this procedure yields n-1 of the necessary n equations. However, another unknown has been introduced, which is the previously unmentioned L_n term, the length of the last wave. Thus, two more equations are necessary.

These two equations can be found by considering conditions at the end of the last wave. The assumption that the solution consists of n waves implies the strip is a straight line beyond the n^{th} wave. Thus, it is desirable to impose conditions of zero slope and zero moment at $x_n = L_n$.

The first of these conditions is given in general terms by

$$0 = \sqrt{c}\, C_{1(n)} \cosh \sqrt{c} L_n + \sqrt{c} \sinh \sqrt{c} L_n \Big[M - Te + \frac{k(-1)^n}{c} + \sum_{m=1}^{n-1} F L_m + \sum_{m=1}^{n-1} \big((-1)^m k L_m\big)\Big(\frac{L_m}{2} + \sum_{i=m+1}^{n-1} L_i\Big) \Big] - k(-1)^n L_n - F - \sum_{m=1}^{n-1} \big((-1)^n k L_m\big) \qquad (9)$$

with the $C_{1(n)}$ term still present.

Next, summing moments about the end of the last wave gives:

$$F \sum_{m=1}^{n} L_m + M - Te + \sum_{m=1}^{n} k L_m \Big(\frac{L_m}{2} + \sum_{i=m+1}^{n} L_i\Big)(-1)^m = 0$$

or

$$Te = M + \sum_{m=1}^{n} L_m \left[F + (-1)^m k \left(\frac{L_m}{2} + \sum_{i=m+1}^{n} L_i\right)\right] \tag{10}$$

Which yields an expression for the unknown end deflection, e.

Methods have now been demonstrated for finding the necessary number of equations, making it possible to solve for all unknowns, and thus produce the deflection equation of any wave desired. Note that equations (9) and (10) are approximations. They are strictly true only for $n = \infty$, but can be considered accurate if n is sufficiently large.

2.3 Summary of the General Method

Summarizing the solution to the problem, the following equations have been found: the general equation for the curve of wave n, containing the unknown e, and the unknowns L_1, L_2, L_3, ... , L_{n-1}. That is, n unknowns in all.

There are the equations resulting from specifying y = 0 at x = L in each wave but the first. There are n-1 of these equations, but the last introduces another unknown, L_n. Thus, two unknowns remain, e and L_n.

There is the approximate equation specifying zero moment, which yields an expression for e.

There is equation (9), specifying zero slope, which makes it possible to remove the last of the unknowns.

This procedure is illustrated in the next chapter.

CHAPTER III

TWO WAVE AND ONE WAVE SOLUTIONS

As an example, the procedure will be demonstrated for a case where two waves are considered sufficiently accurate... i.e., after the second wave the strip is straight.

From equation (4), the first wave equations are

$$T y_1 = \frac{\sinh \sqrt{c}\, x_1}{\sinh \sqrt{c}\, L_1}\left[\left(\frac{k}{c}-M\right)\left(\cosh \sqrt{c}\, L_1 - 1\right) - \frac{kL_1^2}{2} + FL_1 - Te\right]$$
$$+ \left(M - \frac{k}{c}\right)\left(\cosh \sqrt{c}\, x_1 - 1\right) + \frac{kx_1^2}{2} - Fx_1 + Te$$

$$T y_1' = \frac{\sqrt{c}\cosh \sqrt{c}\, x_1}{\sinh \sqrt{c}\, L_1}\left[\left(\frac{k}{c}-M\right)\left(\cosh \sqrt{c}\, L_1 - 1\right) - \frac{kL_1^2}{2} + FL_1 - Te\right]$$
$$+ \left(M - \frac{k}{c}\right)\left(\sqrt{c}\sinh \sqrt{c}\, x_1\right) + kx_1 - F \qquad (11)$$

and for the second wave, from equations (7) and (8) with n = 2,

$$T y_2 = C_{1\,(2)}\, T \sinh \sqrt{c}\, x_2 + \cosh \sqrt{c}\, x_2 \left(M + \frac{k}{c} + FL_1 - \frac{kL_1^2}{2} - Te\right)$$
$$- \frac{kx_2^2}{2} - Fx_2 + kL_1 x_2 - M - \frac{k}{c} - FL_1 + \frac{kL_1^2}{2} + Te \qquad (12)$$

$$T y_2' = \sqrt{c}\, C_{1\,(2)}\, T \cosh \sqrt{c}\, x_2 + \sqrt{c}\sinh \sqrt{c}\, x_2 \left[M + \frac{k}{c} + FL_1 - \frac{kL_1^2}{2} - Te\right]$$
$$- kx_2 - F + kL_1 \qquad (13)$$

It is now possible to determine the constant C_1 for the second wave by equating y_2' at $x_2 = 0$ to y_1' at $x_1 = L_1$.

Thus, combining equations (11) and (13) gives

$$C_1 T = \frac{\cosh\sqrt{c}L_1}{\sinh\sqrt{c}L_1}\left[\left(\frac{k}{c}-M\right)\left(\cosh\sqrt{c}L_1 - 1\right) - \frac{kL_1^2}{2} + FL_1 - Te\right] + \left(M - \frac{k}{c}\right)\sinh\sqrt{c}L_1$$

Using this expression for the constant in the equations for the second wave yields

$$\begin{aligned} Ty_2 = {} & \sinh\sqrt{c}x_2\left[\frac{\cosh\sqrt{c}L_1}{\sinh\sqrt{c}L_1}\left(\left(\frac{k}{c}-M\right)\left(\cosh\sqrt{c}L_1 - 1\right) - \frac{kL_1^2}{2} + FL_1 - Te\right) + \left(M - \frac{k}{c}\right)\sinh\sqrt{c}L_1\right] \\ & + \cosh\sqrt{c}x_2\left[M + \frac{k}{c} + FL_1 - \frac{kL_1^2}{2} - Te\right] - \frac{kx_2^2}{2} - Fx_2 \\ & + kL_1x_2 - M - \frac{k}{c} - FL_1 + \frac{kL_1^2}{2} + Te \end{aligned} \tag{14}$$

Turning to the moment criterion, equation (10), to find an expression for e, and setting n = 2 yields

$$Te = M + FL_1 - \frac{kL_1^2}{2} - kL_1L_2 + FL_2 + \frac{kL_2^2}{2}$$

Substituting into (4) and (14) produces

$$\begin{aligned} Ty_1 = {} & \frac{\sinh\sqrt{c}x_1}{\sinh\sqrt{c}L_1}\left[\left(\frac{k}{c}-M\right)\left(\cosh\sqrt{c}L_1\right) - \frac{k}{c} - FL_2 - \frac{kL_2^2}{2} + kL_1L_2\right] \\ & + \left(M - \frac{k}{c}\right)\cosh\sqrt{c}x_1 + \frac{k}{c} + \frac{kx_1^2}{2} - Fx_1 - \frac{kL_1^2}{2} \\ & + FL_1 + FL_2 - kL_1L_2 + \frac{kL_2^2}{2} \end{aligned} \tag{15}$$

$$\begin{aligned} Ty_2 = {} & \sinh\sqrt{c}x_2\left\{\frac{\cosh\sqrt{c}L_1}{\sinh\sqrt{c}L_1}\left[\left(\frac{k}{c}-M\right)\left(\cosh\sqrt{c}L_1\right) - \frac{k}{c} - FL_2 - \frac{kL_2^2}{2} + kL_1L_2\right]\right. \\ & \left. + \left(M - \frac{k}{c}\right)\sinh\sqrt{c}L_1\right\} \\ & + \cosh\sqrt{c}x_2\left\{\frac{k}{c} - FL_2 - \frac{kL_2^2}{2} + kL_1L_2\right\} - \frac{kx_2^2}{2} + \left(kL_1 - F\right)x_2 \\ & - \frac{k}{c} + FL_2 + \frac{kL_2^2}{2} - kL_1L_2 \end{aligned}$$

The unknown lengths L_1 and L_2 must now be found. This is done by making use of the boundary conditions $y_2 = 0$ and $y_2' = 0$ at the end of the last wave.

Using the first boundary condition yields the equation

$$0 = \sinh\sqrt{c}L_2 \left\{ \frac{\cosh\sqrt{c}L_1}{\sinh\sqrt{c}L_1}\left[\left(\frac{k}{c}-M\right)\left(\cosh\sqrt{c}L_1\right)-\frac{k}{c}-FL_2-\frac{kL_2^2}{2}+kL_1L_2\right] + \left(M-\frac{k}{c}\right)\sinh\sqrt{c}L_2\right\} + \cosh\sqrt{c}L_2\left\{\frac{k}{c}-FL_2-\frac{kL_2^2}{2}+kL_1L_2\right\}-\frac{k}{c} \qquad (16)$$

Using the $y_2' = 0$ boundary condition yields

$$0 = \sqrt{c}\cosh\sqrt{c}L_2 \left\{ \frac{\cosh\sqrt{c}L_1}{\sinh\sqrt{c}L_1}\left[\left(\frac{k}{c}-M\right)\left(\cosh\sqrt{c}L_1\right)-\frac{k}{c}-FL_2-\frac{kL_2^2}{2}+kL_1L_2\right] + \left(M-\frac{k}{c}\right)\sinh\sqrt{c}L_2\right\} + \sqrt{c}\sinh\sqrt{c}L_2\left\{\frac{k}{c}-FL_2-\frac{kL_2^2}{2}+kL_1L_2\right\}-kL_2+kL_1-F$$

There are now two equations for the two unknowns, L_1 and L_2. Unfortunately, although it is theoretically possible to solve these equations, it is very difficult in practice. A closed form solution is in fact impossible, and the author has found that even a computer solution for a given physical case can involve a great degree of difficulty. For solutions accurate to three waves or greater, the difficulty increases enormously.

For this reason, the bulk of the work on this problem consisted of the development and use of a one wave approximation to the problem. The development of a one wave model follows.

With this model, it is assumed that one wave is sufficient to predict the response of the system. As before, moments are assumed to balance at the end of the first wave, and the slope of the curve is assumed zero there.

The development of the equation for the first wave is identical, and equation (4) remains valid. It is

$$Ty_1 = \frac{\sinh\sqrt{c}\,x_1}{\sinh\sqrt{c}\,L_1}\left[\left(\frac{k}{c}-M\right)\left(\cosh\sqrt{c}\,L_1\right)-\frac{k}{c}+M-\frac{kL_1^2}{2}+FL_1-Te\right]$$
$$+\left(M-\frac{k}{c}\right)\cosh\sqrt{c}\,x_1 - M + \frac{k}{c} + \frac{kx_1^2}{2} - Fx_1 + Te \qquad (4)$$

The moment equation, equation (10), for n = 1, becomes

$$Te = M + FL_1 - \frac{kL_1^2}{2}$$

which, when combined with equation (4), gives

$$Ty_1 = \frac{\sinh\sqrt{c}\,x_1}{\sinh\sqrt{c}\,L_1}\left[\left(\frac{k}{c}-M\right)\cosh\sqrt{c}\,L_1-\frac{k}{c}\right]+\left(M-\frac{k}{c}\right)\cosh\sqrt{c}\,x_1+\frac{k}{c}+F(L_1-x_1)-\frac{k}{2}\left(L_1^2-x_1^2\right)$$
$$(17)$$
$$Ty_1' = \frac{\sqrt{c}\cosh\sqrt{c}\,x_1}{\sinh\sqrt{c}\,L_1}\left[\left(\frac{k}{c}-M\right)\cosh\sqrt{c}\,L_1-\frac{k}{c}\right]+\left(M-\frac{k}{c}\right)\left(\sqrt{c}\sinh\sqrt{c}\,x_1\right)-F+kx_1$$

It will be noted that the only quantity yet to be determined is L_1. This is found by making use of the imposed boundary condition, that y_1' is zero at $x_1 = L_1$.

Applying this condition to equation (17) gives

$$0 = \frac{\sqrt{c}\cosh\sqrt{c}\,L_1}{\sinh\sqrt{c}\,L_1}\left[\left(\frac{k}{c}-M\right)\cosh\sqrt{c}\,L_1-\frac{k}{c}\right]+\left(M+\frac{k}{c}\right)\sqrt{c}\sinh\sqrt{c}\,L_1-F+kL_1$$

which can be simplified to

$$0 = (kL_1-F)\sinh\sqrt{c}\,L_1-\frac{k}{\sqrt{c}}\cosh\sqrt{c}\,L_1+\sqrt{c}\left(\frac{k}{c}-M\right) \qquad (18)$$

which is the implicit equation for L, in terms of system parameters.

Again, even for the simple one wave approximation, it is impossible to find a closed form algebraic expression for the affected length, L. However, it is not difficult to program the computer to find roots to the above equation to fit any given physical case. This has been done, and the program, along with sample results, will be presented.

CHAPTER IV

COMPUTER SOLUTION TO THE PROBLEM

One of the more interesting aspects of this problem is that the shape assumed by the deflected strip at any time depends not only on the forces acting on the strip at that time, but on those that have gone before. That is, the shape is also affected by the strip's history.

To illustrate, if an initially straight strip is deflected with a force of 100 pounds, the deflected shape will be somewhat as shown in figure 5, if a one wave approximation is used.

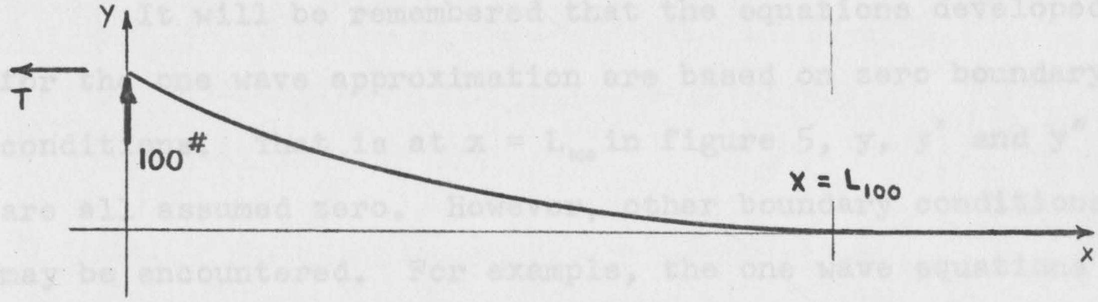


Fig.5.--Deflection Due to 100 lb. Force

If the 100 pound force is then removed, and replaced with a 50 pound force in the opposite direction, the result would be as shown in figure 6.

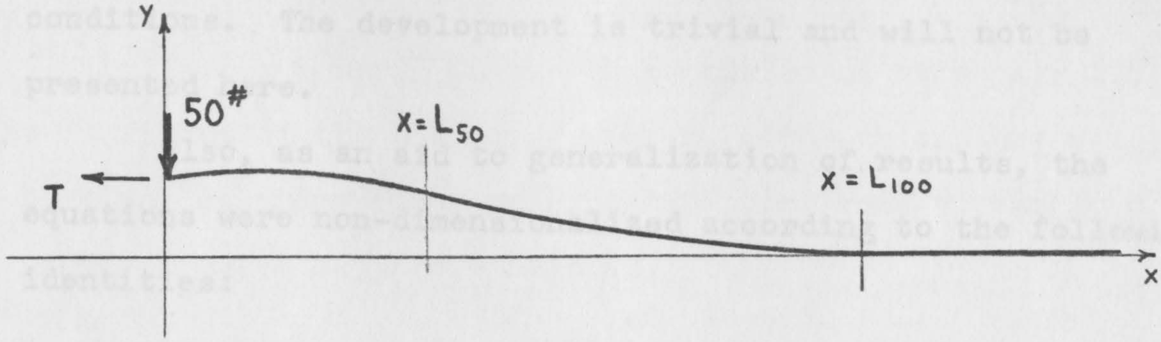


Fig.6.--Strip Deflected by 100 lb., then -50 lb. Forces

Note that a certain portion of the length affected by the original force is undisturbed by the second force.

Obviously, if the 50 pound downward force were applied to a straight strip, all deflection would be negative. Thus, the history of the strip profoundly affects the final shape. The program developed to generate deflection curves incorporates this phenomenon.

It will be remembered that the equations developed for the one wave approximation are based on zero boundary conditions. That is at $x = L_{100}$ in figure 5, y, y' and y'' are all assumed zero. However, other boundary conditions may be encountered. For example, the one wave equations describing the portion of the curve deflected by the 50 pound force in figure 6 must take their boundary conditions from the values of the first curve at $x = L_{50}$ (since at this length and beyond, values of the first curve are unchanged).

For this reason, equations (17) and (18) were modified for use by the computer, to include non-zero boundary conditions. The development is trivial and will not be presented here.

Also, as an aid to generalization of results, the equations were non-dimensionalized according to the following identities:

$$N_2 \equiv M/EIA \qquad N_3 \equiv F/EIA^2 = F/T \qquad N_4 \equiv k/EIA^3$$

$$\text{where} \quad A \equiv \sqrt{C} = \sqrt{T/EI}$$

Making use of these definitions, and the boundary conditions,

$$B = y(L)$$

$$D = y'(L)$$

$$G = y''(L)$$

Equations (17) and (18), modified for computer use, are:

```
AY =(SINH(AX)/SINH(AL))*(N4-(N2+N4)*COSH(AL)+G/A)
    +(N2+N4)*COSH(AX)-(N4/2)*((AX)**2-(AL)**2)
    -N3*(AX-AL)-N4+A*B-G/A)

Y'= (COSH(AX)/SINH(AL))*(N4-(N2+N4)*COSH(AL)+G/A)          (19)
    +(N2+N4)*SINH(AX)-N4*AX-N3

Y"/A=(SINH(AX)/SINH(AL))*(N4-(N2+N4)*COSH(AL)+G/A
    +(N2+N4)*COSH(AX)-N4

FCN = COSH(AX)*(N4+G/A)-SINH(AX)*(N4*AX+N3+D)-N2-N4        (20)
```

In the program, when FCN equals zero, Ax equals AL. A flowchart of the one wave program, with explanatory notes, will follow. But first an explanation of some of the logic of the program may prove helpful.

First, it should be noted that the values of N2 and N3 read into the program are the net values acting on the strip, not the change in the value. For example, if data is entered with N2 equal to .3, then equal to -.05, the second curve calculated is for N2 = -.05, <u>not</u> N2 = +.25.

When checking values of the implicit equation, FCN, for the affected length, roots are located by noting changes in sign of the function between successive values of x. Each time a sign change is noted, the increment between successive x values is altered to determine the root more precisely. AL values used for computation are accurate to six decimal places.

It is conceivable that smaller and smaller alternating forces, for one example, may divide the strip into numerous regions, as shown in figure 7 (with friction loads indicated).

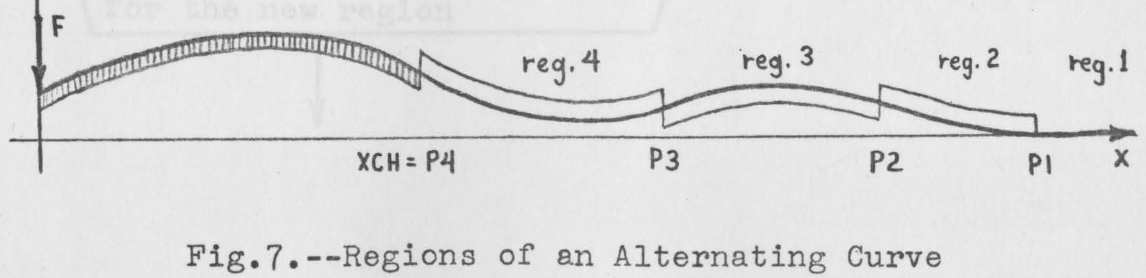
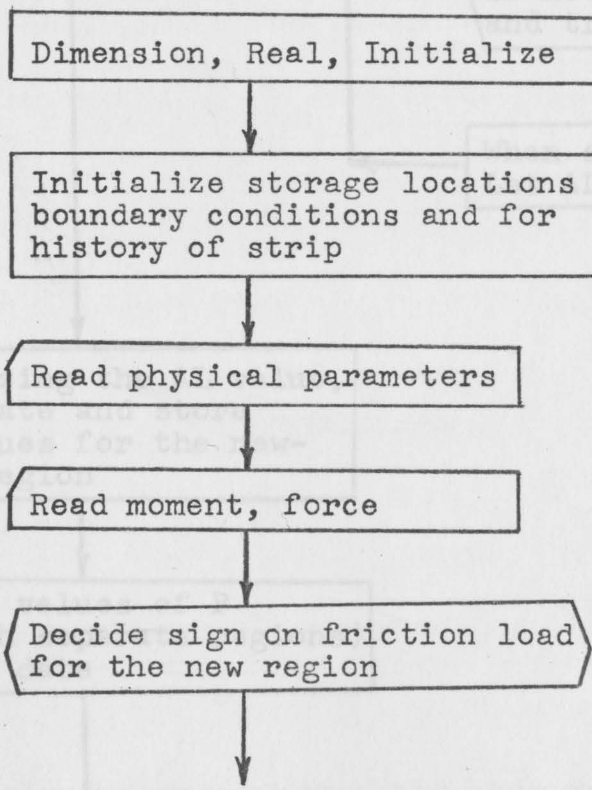


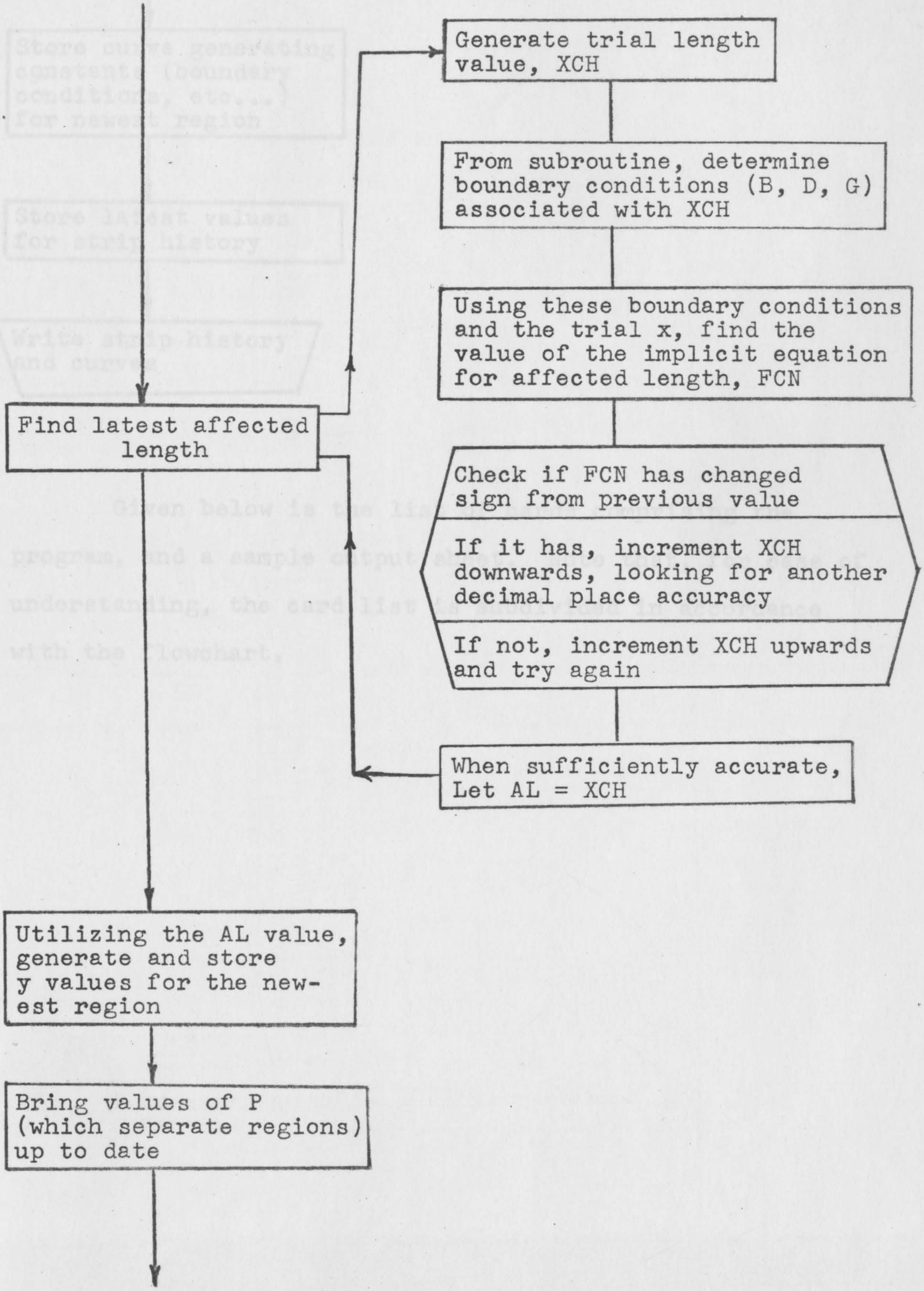
Fig.7.--Regions of an Alternating Curve

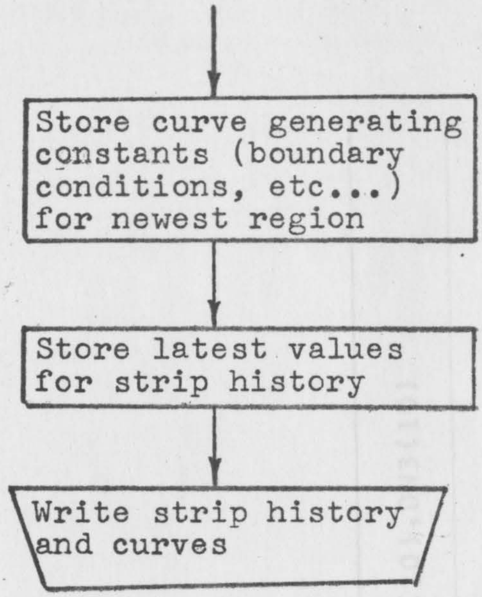
When the length associated with the current force F is being searched for, the boundary conditions must be taken from the proper region. The x values separating the various regions are remembered, and stored as the P values shown.

Once the proper value for affected length is determined, it is stored as a P value separating the newest region from the others. If there had been other P values smaller than the new one, they must be removed.

The general flowchart is given below.




Find latest affected length
Generate trial length value, XCH
From subroutine, determine boundary conditions (B, D, G) associated with XCH
Using these boundary conditions and the trial x, find the value of the implicit equation for affected length, FCN
Check if FCN has changed sign from previous value
If it has, increment XCH downwards, looking for another decimal place accuracy
If not, increment XCH upwards and try again
When sufficiently accurate, Let AL = XCH
Utilizing the AL value, generate and store y values for the newest region
Bring values of P (which separate regions) up to date



Given below is the list of cards comprising the program, and a sample output sheet. Note that, for ease of understanding, the card list is subdivided in accordance with the flowchart.

```
C  DIMENSION, REAL, INITIALIZE.
      DIMENSION AX(200), Y(200),YPR(200),YDPR(200)
      DIMENSION DEN4(10),DAL(10),DB(10),DD(10),DG(10),DN2(10),DN3(10)
      DIMENSION CN2(15),CN3(15),MI(15),CAL(15)
      REAL N2,N3,N4,N2S,N3S
  111 N2S=0.
      N3S=0.
      P1=0.
      P2=0.
      P3=0.
      P4=0.
      P5=0.
      P6=0.
      P7=0.
      P8=0.
      P9=0.
C
C
C
C
```

```
C
C  INITIALIZE STORAGE FOR BOUNDARY COND'S AND STRIP HISTORY.
C
      DO 400 M=1,10
      DEN4(M)=0.
      DAL(M)=1.
      DB(M)=0.
      DD(M)=0.
      DG(M)=0.
      DN2(M)=0.
      DN3(M)=0.
  400 CONTINUE
      DO 453 J=1,15
      CN2(J)=0.
      CN3(J)=0.
      MI(J)=0
      CAL(J)=0.
  453 CONTINUE
C
C
C
C
```

```
C
C  READ PHYSICAL PARAMETERS.
C
      READ(5,11)A,N4
   11 FORMAT(2F20.6)
      J=1
C
C
C  THIS IS THE START OF THE MAIN LOOP......................................
C
C
C
C
C
C  READ MOMENT AND FORCE.
C
    1 READ(5,12,END=1000)N2,N3
   12 FORMAT(2F20.6)
      WRITE(3,24)N2,N3
   24 FORMAT('1',' HERE ARE N2 AND N3 ',2F20.8)
C
C
C
C
```

```
C
C  DECIDE SIGN ON FRICTION LOAD FOR THE NEW REGION.
C
      IF(N2.EQ.N2S) GO TO 505
      IF(N3.EQ.N3S) GO TO 505
      IF(((N2-N2S)*(N3-N3S)).LT.0.) GO TO 776
      GO TO 505
  776 WRITE(3,777)
  777 FORMAT('  SIMULTANEOUS, OPPOSING N2,N3. KILL THE RUN.')
      GO TO 1000
  505 CONTINUE
C IF THE NEW FORCE OR MOMENT IS A REDUCTION, THE FRICTION LOAD IS POSITIVE
      IF(N2.LT.N2S)GO TO 500
      IF(N3.LT.N3S)GO TO 500
      SN=-1.
      GO TO 501
  500 SN=1.
  501 EN4=SN*N4
      N2S=N2
      N3S=N3
      WRITE(3,77)EN4,A
   77 FORMAT(/,' EN4 IS ',F12.8,'  AND A IS',F12.8,//)
C
C
C
C
```

```
C
C THIS STARTS A SEARCH FOR THE AFFECTED LENGTH.
C
C
C
C
C
C  GENERATE TRIAL X VALUE, XCH.
C
      XCH=0.
      FRAC=(-.1)
C ### ACCURACY LOOP
      DO 98 IAC=1,5
      FRAC=(-.1)*FRAC
C *** SEARCH LOOP
    9 DO 99 I=1,150
      XCH=XCH+FRAC
C
C
C
C
```

```
C
C  DETERMINE PROPER BOUNDARY CONDITIONS ASSOCIATED WITH XCH.
C  ( WE MUST FIRST DETERMINE WHICH REGION OF THE CURVE WE ARE IN. )
C
C   THIS STARTS THE REGION FINDER. . . . . . . . . . .
      IF(XCH.GT.P9)N=9
      IF(XCH.GT.P8)N=8
      IF(XCH.GT.P7)N=7
      IF(XCH.GT.P6)N=6
      IF(XCH.GT.P5)N=5
      IF(XCH.GT.P4)N=4
      IF(XCH.GT.P3)N=3
      IF(XCH.GT.P2)N=2
      IF(XCH.GT.P1)N=1
      EEN4=DEN4(N)
      EAL=DAL(N)
      EB=DB(N)
      ED=DD(N)
      EG=DG(N)
      EN2=DN2(N)
      EN3=DN3(N)
C   THIS ENDS THE REGION FINDER . . . . . . . . .
      CALL EQS(XCH,EAL,EN2,EN3,EEN4,A,EB,ED,EG,B,D,G)
C
C
C
```

```
C
C
C  FIND THE VALUE OF FCN.
C........HERE IS THE IMPLICIT EQUATION FOR THE AFFECTED LENGTH...
      FCN=COSH(XCH)*(EN4+G/A)-SINH(XCH)*(EN4*XCH+N3+D)-N2-EN4
C
C
C
C
C
C  CHECK IF FCN HAS CHANGED SIGN SINCE THE LAST VALUE.
C     CHANGE XCH APPROPRIATELY.
C
      IF(HOLD.EQ.0.)WRITE(3,79)XCH,FCN
   79 FORMAT(' HOLD IS ZERO. XCH IS',F12.6,' AND FCN IS',F12.6,'.....')
      IF((IAC.EQ.1.AND.I.EQ.1).OR.HOLD.EQ.0.)HOLD=FCN
  445 IF(FCN*HOLD)97,96,95
   95 CONTINUE
      HOLD=FCN
   99 CONTINUE
C   IF IT COMES OUT THE BOTTOM WE GOT NO ROOT LESS THAN 150.
      WRITE(3,75)
   75 FORMAT(///,'  SEARCH ROUTINE FOUND NO ROOT. KILL THE RUN. ')
      GO TO 1000
   97 HOLD=FCN
   98 CONTINUE
C
C
C
C
```

```
C
C  WHEN SUFFICIENTLY ACCURATE, LET AL=XCH
C
   96 CONTINUE
      AL=XCH
      WRITE(3,22)AL,FCN
   22 FORMAT(   ' HERE IS AN AL VALUE, ',F20.8,'  AND ITS FCN VALUE,',
     19F20.8,///)
C THIS IS THE END OF THE SEARCH FOR AL.
C
C
C
C
C
C  GENERATE AND STORE Y VALUES FOR THE NEWEST REGION.
C
      L=AL*100.+2.
C EACH L IS ONE TENTH OF A UNITLESS LENGTH.
C
      DO 900 K=1,L
      XK=K-1
      EX=.01*XK                                                  FINE
C EX IS ACTUALLY THE DIMENSIONLESS LENGTH A*X.
      CALL EQS(EX,AL,N2,N3,EN4,A,B,D,G,YY,YYPR,YYDPR)
      AX(K)=EX
      Y(K)=YY*A
      YPR(K)=YYPR
      YDPR(K)=YYDPR/A
  900 CONTINUE
C
C
C
```

```
C
C
C   BRING P VALUES UP TO DATE.
C
C IF AL.LT.P9 THERE'S TOO MANY REGIONS FOR FURTHER WORK.
      IF(AL.LT.P9)WRITE(3,71)
   71 FORMAT(///'.....GOT TOO MANY REGIONS. WRITE, THEN STOP.....')
      IF(AL.LT.P9)GO TO 199
C
  206 IF(AL.GE.P8)GO TO 207
      P9=AL
      M=10
      GO TO 188
  207 IF(AL.GE.P7)GO TO 208
      P9=0.
      P8=AL
      M=9
      GO TO 188
  208 IF(AL.GE.P6)GO TO 209
      P9=0.
      P8=0.
      P7=AL
      M=8
      GO TO 188
  209 IF(AL.GE.P5)GO TO 210
      P9=0.
      P8=0.
      P7=0.
      P6=AL
      M=7
      GO TO 188
```

```
  210 IF(AL.GE.P4)GO TO 211
      P9=0.
      P8=0.
      P7=0.
      P6=0.
      P5=AL
      M=6
      GO TO 188
  211 IF(AL.GE.P3)GO TO 212
      P9=0.
      P8=0.
      P7=0.
      P6=0.
      P5=0.
      P4=AL
      M=5
      GO TO 188
  212 IF(AL.GE.P2)GO TO 213
      P9=0.
      P8=0.
      P7=0.
      P6=0.
      P5=0.
      P4=0.
      P3=AL
      M=4
      GO TO 188
  213 IF(AL.GE.P1)GO TO 214
      P9=0.
      P8=0.
      P7=0.
      P6=0.
      P5=0.
      P4=0.
      P3=0.
      P2=AL
```

```
      M=3
      GO TO 188
  214 CONTINUE
      P9=0.
      P8=0.
      P7=0.
      P6=0.
      P5=0.
      P4=0.
      P3=0.
      P2=0.
      P1=AL
      M=2
      GO TO 188
C
C
C
C
```

```
C  STORE NEWEST CURVE-GENERATING CONSTANTS.
C
   188 CONTINUE
       DEN4(M)=EN4
       DAL(M)=AL
       DB(M)=B
       DD(M)=D
       DG(M)=G
       DN2(M)=N2
       DN3(M)=N3
   199 CONTINUE
       LMAX=(P1*100.)+1.
C
C
C
C
C
C  STORE NEWEST STRIP HISTORY VALUES.
C
       CN2(J)=N2S
       CN3(J)=N3S
       MI(J)=M
       CAL(J)=AL
C
C
C
C
C
```

```
C  WRITE HISTORY AND CURVES.
C
         WRITE(3,30)
      30 FORMAT(' HERE IS THE STRIP HISTORY..A LIST OF PAST N2 AND N3 VALUE
        9S. THE LAST VALUES ARE CURRENT. ')
         WRITE(3,31)
      31 FORMAT(/ ,' N2=MOMENT  N3=FORCE      M      AL VALUE')
         DO 460 K=1,J
         WRITE(3,33)CN2(K),CN3(K),MI(K),CAL(K)
      33 FORMAT(2F10.5,I6,F14.8)
     460 CONTINUE
C OUTPUT UP TO WHERE ITS ZERO.
         WRITE(3,23)
      23 FORMAT(//,' HERE COME THE VALUES OF  X,Y,Y-PRIME, AND Y-DOUBLE-PRI
        9ME.',/)
         WRITE(3,34)
      34 FORMAT(   12X,'..AX..',14X,'..AY..',14X,'..YPR..',11X,'.YDPR/A.',/
        9)
         DO 600 K=1,LMAX
         WRITE(3,21)AX(K),Y(K),YPR(K),YDPR(K)
      21 FORMAT(F20.2,3F20.8)
     600 CONTINUE
         IF(M.EQ.10) WRITE(3,71)
         IF(AL.LT.P9)GO TO 1000
         IF(M.EQ.10) GO TO 1000
C
C
C
C
C
C  REPEAT WITH NEXT FORCE AND MOMENT.
C
         J=J+1
         GO TO 1
    1000 STOP
         END
```

```
C THIS IS THE SUBROUTINE CONTAINING THE BASIC EQUATIONS FOR THE CURVES.
      SUBROUTINE EQS(EX,AL,N2,N3,EN4,A,B,D,G,YY,YYPR,YYDPR)
      REAL N2,N3
C
C
      YY=((SINH(EX)/SINH(AL))*(EN4-(N2+EN4)*COSH(AL)+G/A)
     9+(N2+EN4)*COSH(EX)-(EN4/2)*((EX)**2-(AL)**2)
     9-N3*(EX-AL)-EN4+A*B-G/A)/A
C
      YYPR=(COSH(EX)/SINH(AL))*(EN4-(N2+EN4)*COSH(AL)+G/A)
     9+(N2+EN4)*SINH(EX)-EN4*EX-N3
C
      YYDPR=A*((SINH(EX)/SINH(AL))*(EN4-(N2+EN4)*COSH(AL)+G/A)
     9+(N2+EN4)*COSH(EX)-EN4)
      RETURN
      END
```

HERE ARE N2 AND N3 0.0 0.0

EN4 IS 0.44278300 AND A IS 0.11313695

HERE IS AN AL VALUE, 0.23475194 AND ITS FCN VALUE, 0.0

HERE IS THE STRIP HISTORY..A LIST OF PAST N2 AND N3 VALUES. THE LAST VALUES ARE CURRENT.

N2=MOMENT	N3=FORCE	M	AL VALUE
0.0	0.02000	2	0.09026307
0.02000	0.02000	2	0.34426683
0.0	0.0	3	0.23475194

HERE COME THE VALUES OF X,Y,Y-PRIME, AND Y-DOUBLE-PRIME.

..AX..	..AY..	..YPR..	.YDPR/A.
0.0	0.00035220	-0.00161320	0.0
0.01	0.00033581	-0.00161321	0.00000578
0.02	0.00031954	-0.00161293	0.00005591
0.03	0.00030375	-0.00161193	0.00015080
0.04	0.00028765	-0.00160977	0.00028968
0.05	0.00027156	-0.00160599	0.00047284
0.06	0.00025552	-0.00160016	0.00070035
0.07	0.00023955	-0.00159184	0.00097215
0.08	0.00022399	-0.00158057	0.00128871
0.09	0.00020772	-0.00156592	0.00164878
0.10	0.00019228	-0.00154745	0.00205404
0.11	0.00017726	-0.00152468	0.00250393
0.12	0.00016195	-0.00149722	0.00299782
0.13	0.00014716	-0.00146459	0.00353646

CHAPTER V

SAMPLE COMPUTER RESULTS

The above program is extremely versatile, and allows easy access to deflection curves for any sequence of configurations of forces and moments. Some sample curves have been plotted below.

The values of N4 and A used in these results are taken from a hypothetical roll-strip system, consisiting of an aluminum strip six inches wide and 1/16 inch thick, a roll radius of 2 feet, a roll-to-strip friction coefficient of .1, and tension of 1000 pounds. These parameters yield a friction load value of k = 50 lb./ft.

The computer variables associated with these dimensions are:

$$A = .11313/\text{ft.}$$

$$N4 = .442783$$

$$N3 = F/1000 \text{ lbs.}$$

$$N2 = M/8838.8 \text{ ft. lbs.}$$

The curves follow.

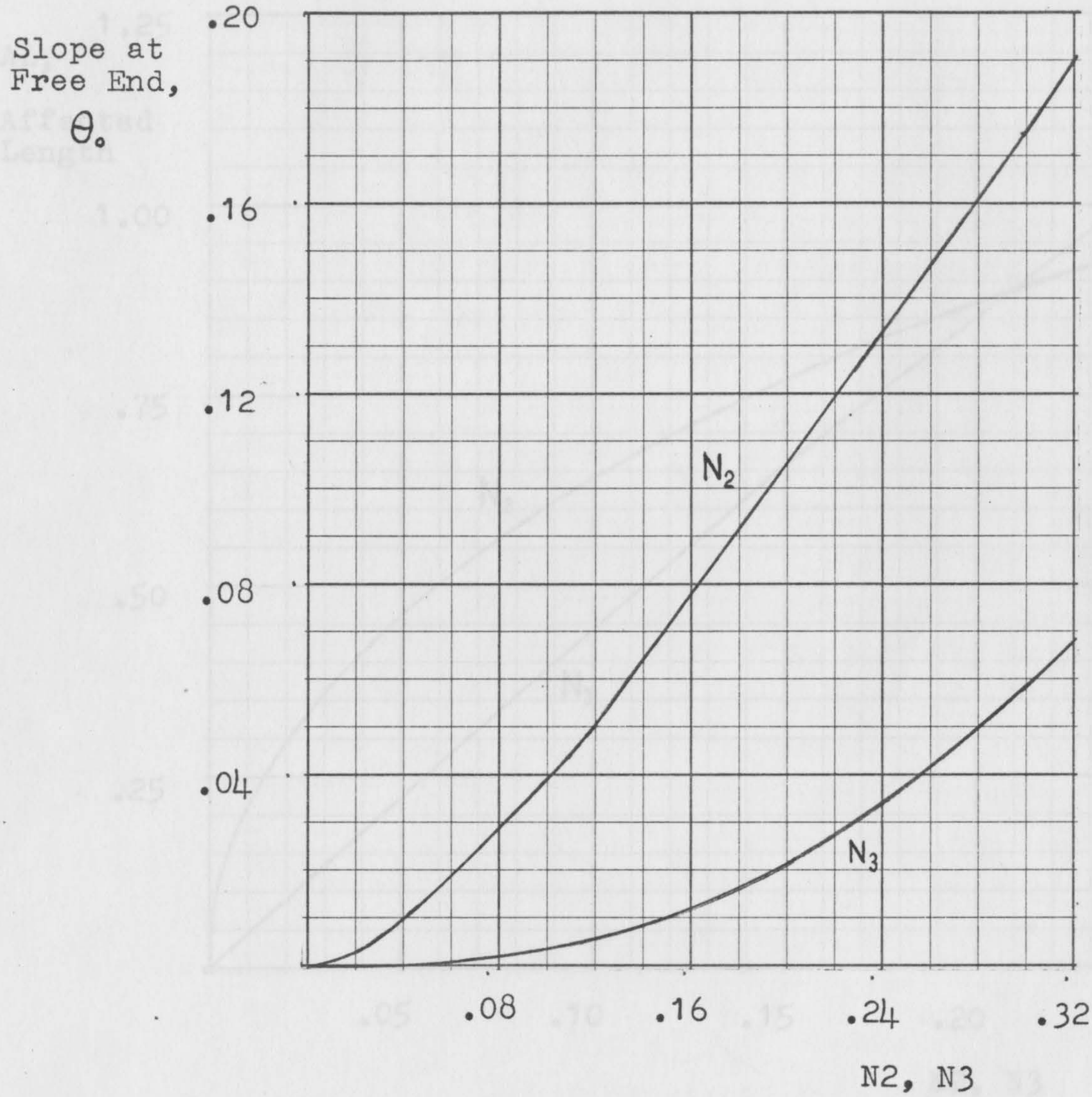


Fig.8.--θ_o vs. N2,N3 for N4=.442783, A=.11313/ft.
(Strip initially straight.)

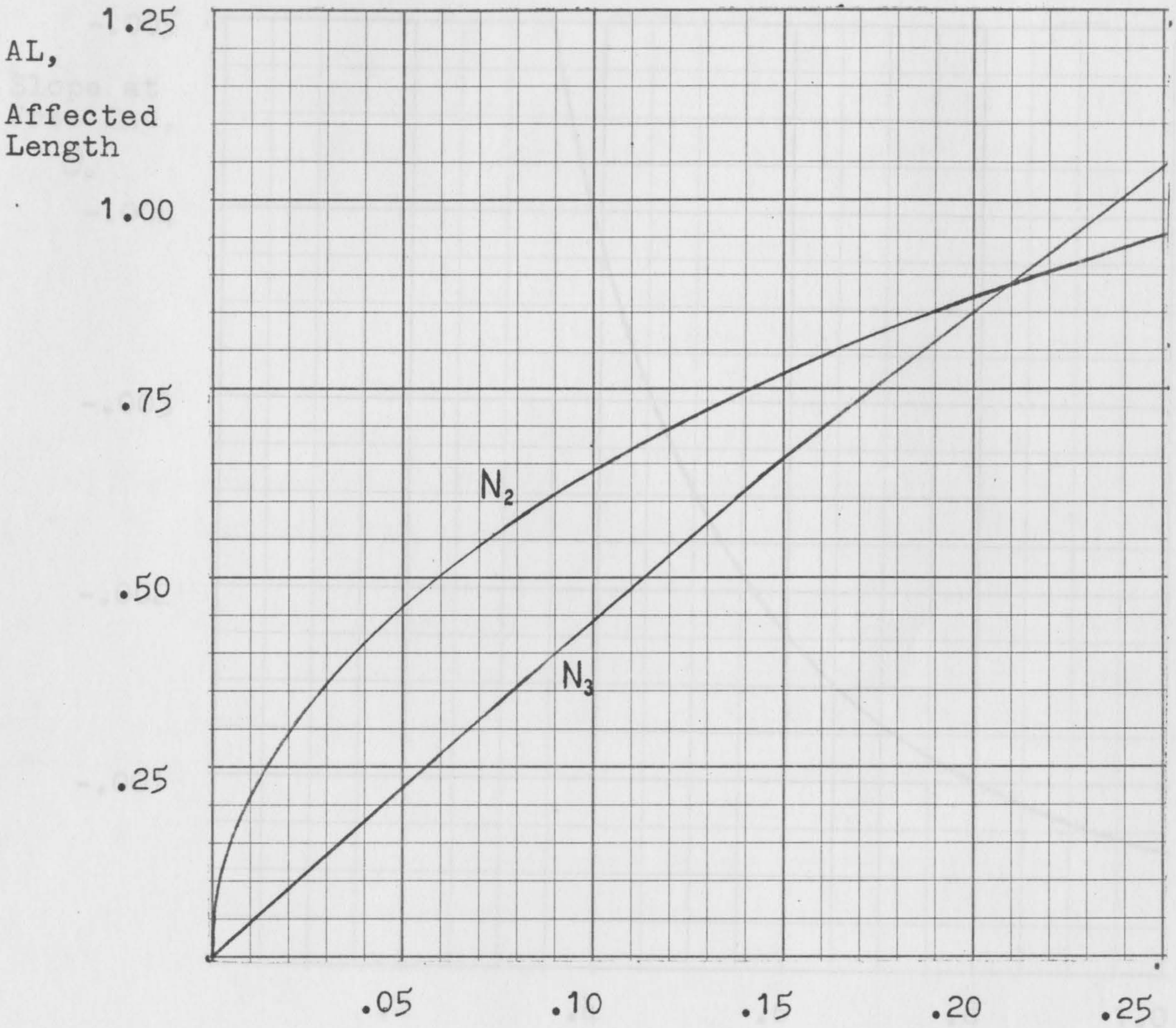


Fig.9.--Affected Length vs. N2, N3 for N4=.442783
A=.11313/ft. (Strip initially straight.)

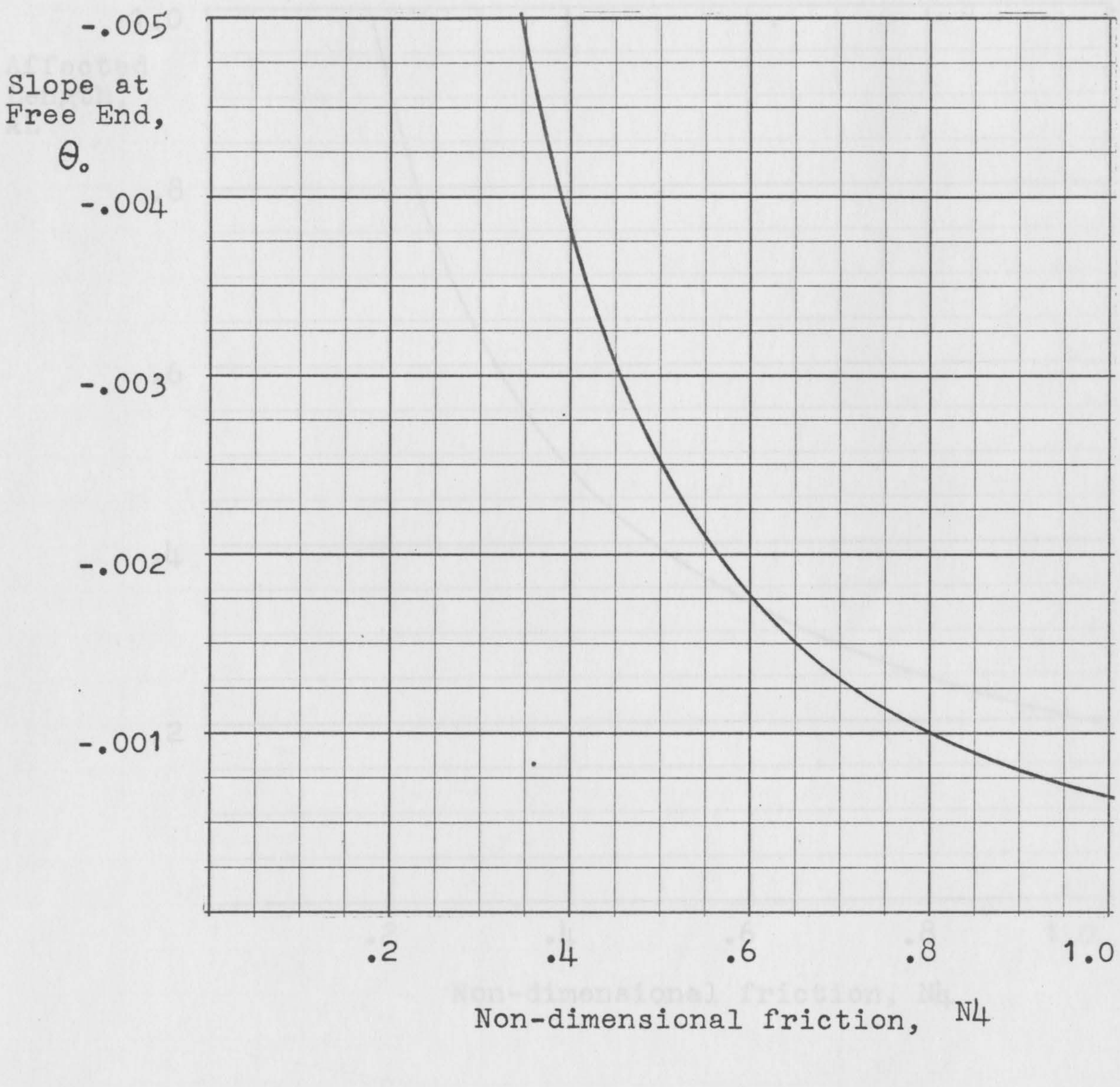


Fig.10.--Slope at Free End vs.N4 for N2=0, N3=.1,
A=.11313/ft. (Strip initially straight.)

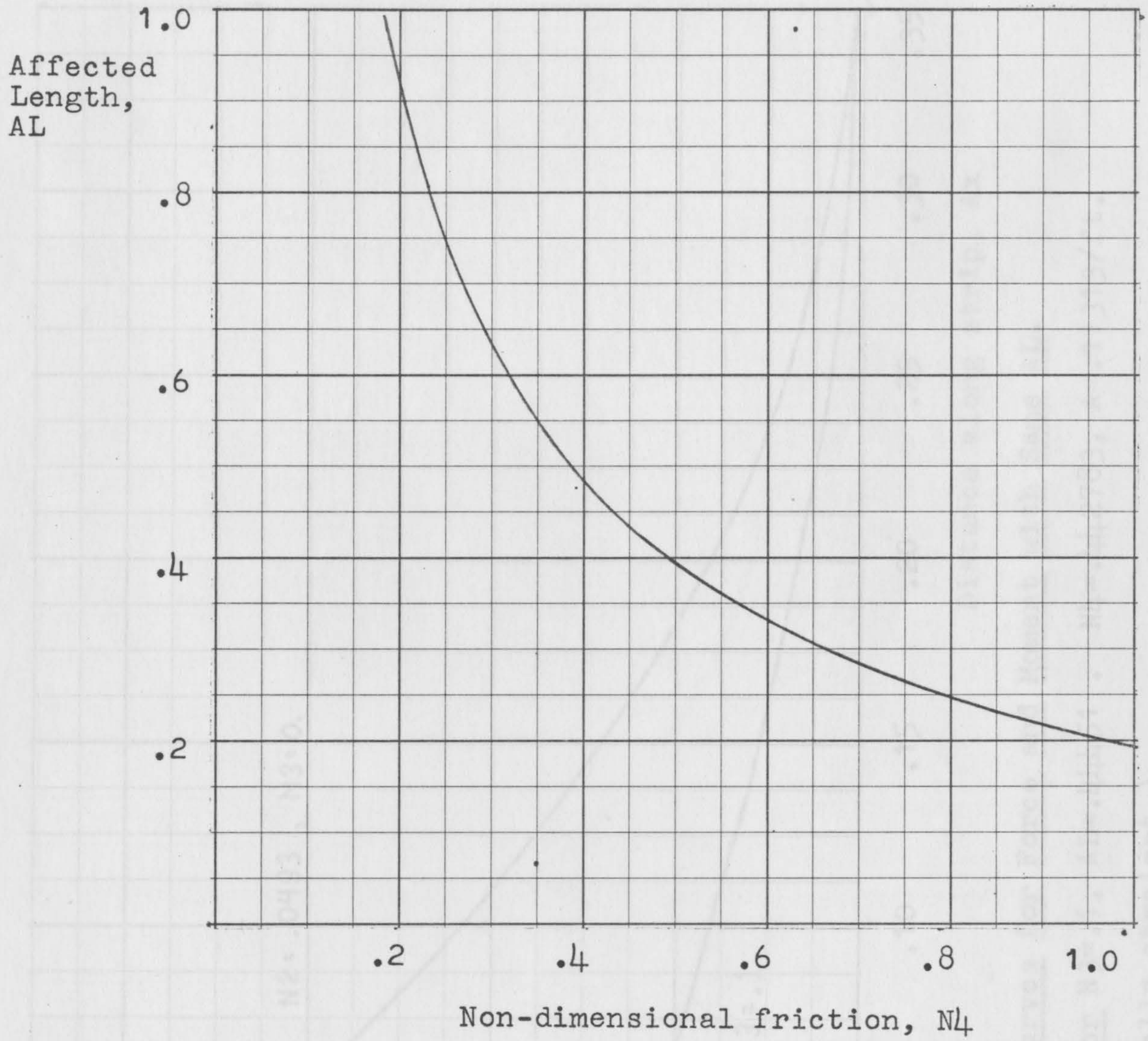


Fig.11.--Affected Length vs. N4 for N2=0, N3=.1, A=.11313/ft. (Strip initially straight.)

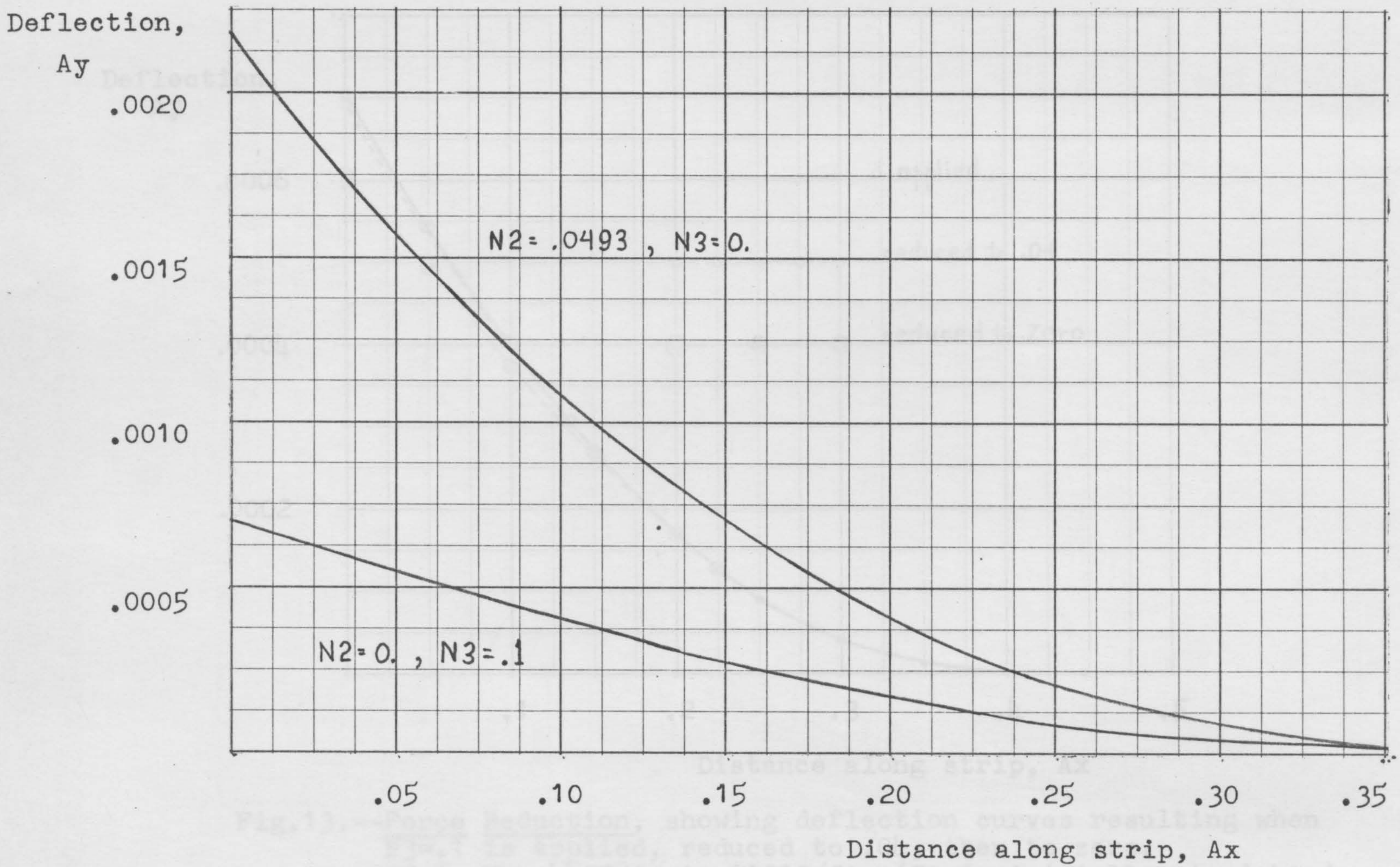


Fig.12.--Deflection Curves for Force and Moment with Same AL.

For N2=.0493 or N3=.1, AL=.44451 . N4=.442783, A=.11313/ft.

(Strip initially straight.)

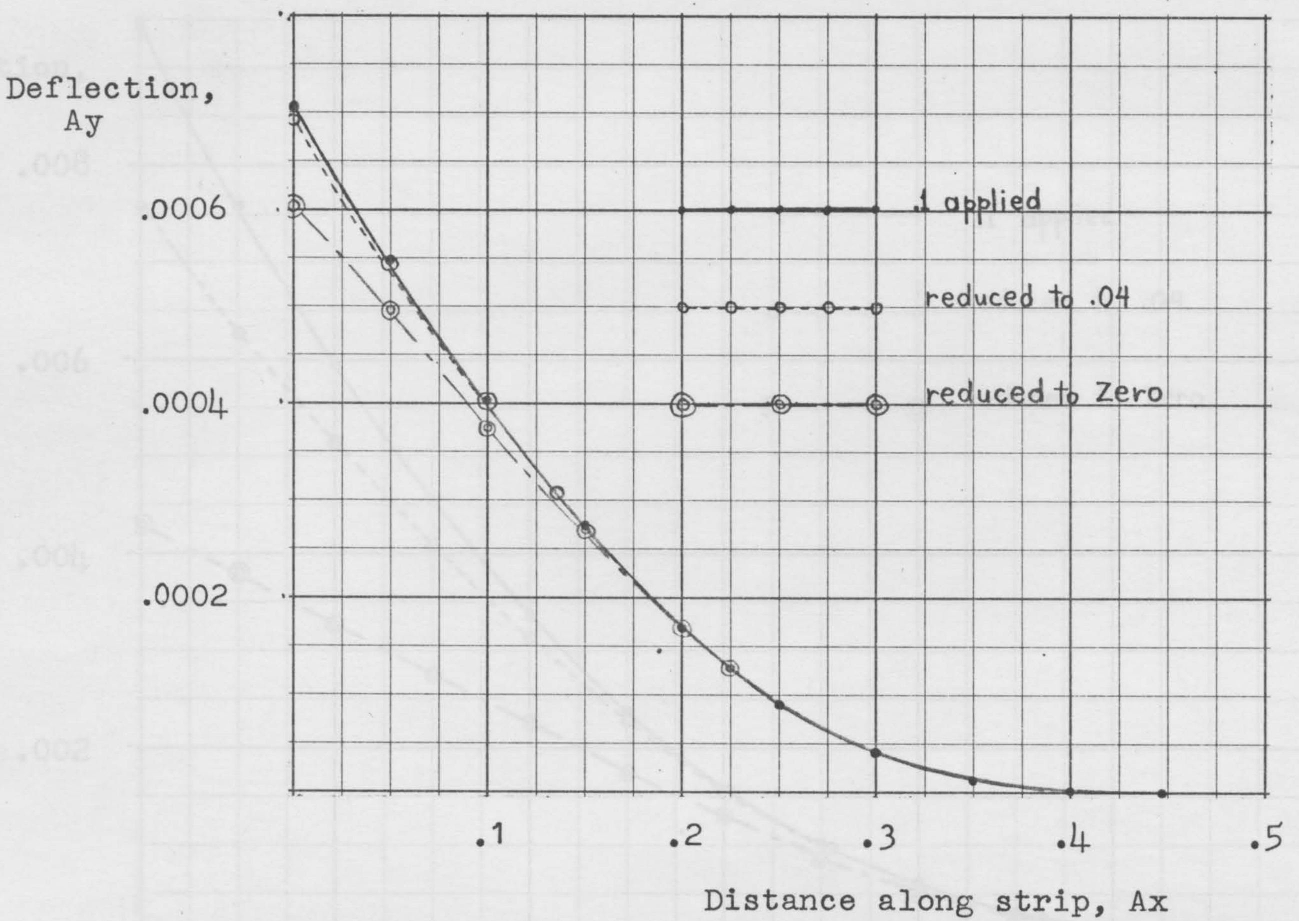


Fig.13.--Force Reduction, showing deflection curves resulting when N3=.1 is applied, reduced to .04, then to zero. N2=0, N4=.442783, A=.11313/ft. (Strip initially straight.)

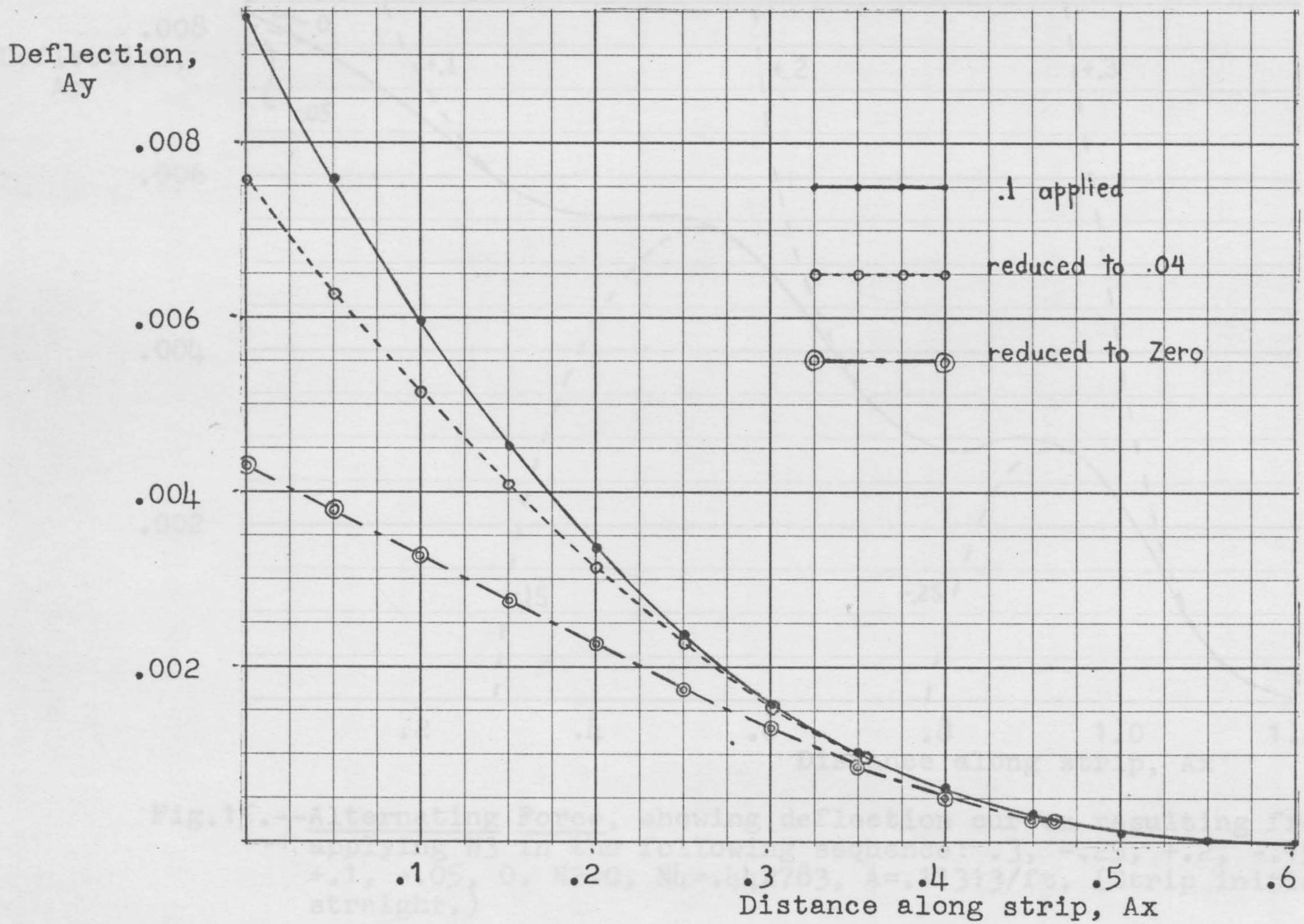


Fig.14.--<u>Moment Reduction</u>, showing deflection curves resulting when $N2=.1$ is applied, released to .04, then to zero. $N3=0$, $N4=.442783$, $A=.11313$/ft. (Strip initially straight.)

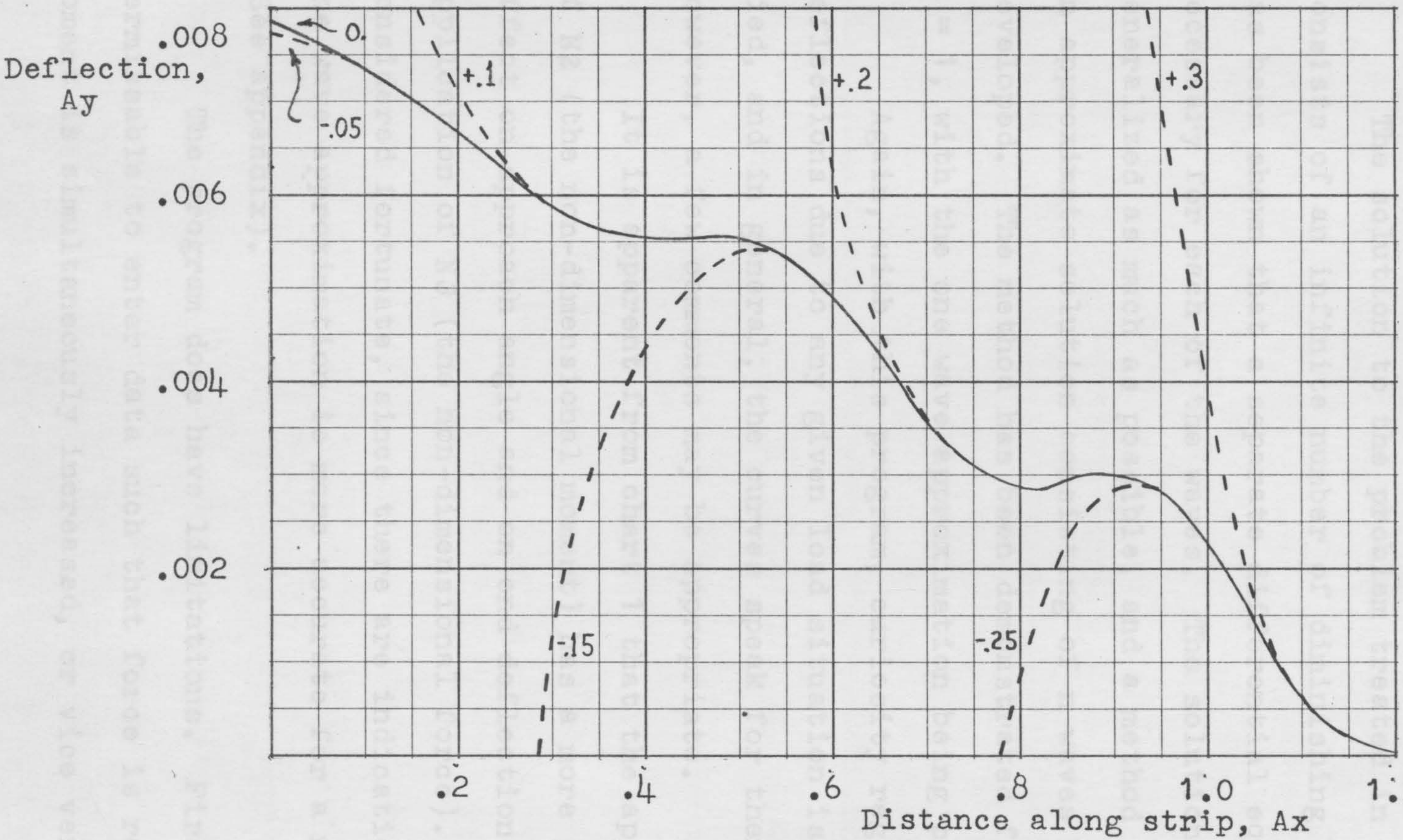


Fig.15.--Alternating Force, showing deflection curves resulting from applying N3 in the following sequence: .3, -.25, +.2, -.15, +.1, -.05, 0. N2=0, N4=.442783, A=.11313/ft. (Strip initially straight.)

CHAPTER VI

SUMMARY AND CONCLUSIONS

The solution to the problem treated in this thesis consists of an infinite number of diminishing waves. It has been shown that a separate differential equation is necessary for each of the waves. The solution has been generalized as much as possible, and a method for finding an approximate solution consisting of n waves has been developed. The method has been demonstrated for $n = 2$ and $n = 1$, with the one wave approximation being programmed.

Again, with this program, curiosity regarding deflections due to any given load situation is easily satisfied, and in general, the curves speak for themselves. However, a few comments may be appropriate.

It is apparent from chart 1 that the application of N2 (the non-dimensional moment) has a more pronounced effect on approach angle and on end deflection than does the application of N3 (the non-dimensional force). This can be considered fortunate, since there are indications that the one wave approximation is more accurate for a moment. (See appendix).

The program does have limitations. First, it is not permissable to enter data such that force is reduced while moment is simultaneously increased, or vice versa.

The reason for this restriction is that for such a case, it is conceivable that the one wave solution would be very inaccurate. As shown in figure 16, the second wave of deflection can be very pronounced, giving a large slope unreasonable.

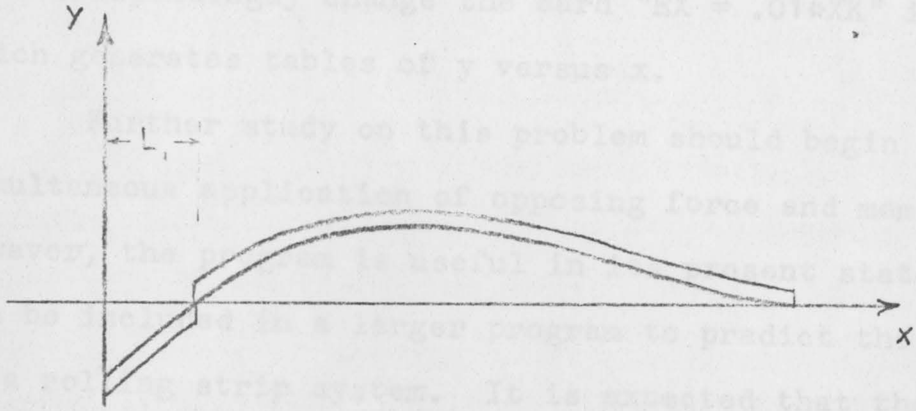


Fig.16.--Simultaneous, opposed force and moment

Thus, a one wave solution is inapplicable to this problem, and the one wave program model cannot be used to solve it.

It should be remembered that all of this presupposes a suitable definition of simultaneous, gradual application (since beam theory assumptions forbid sudden application).

Also, when utilizing the results of this program, it should be remembered that the assumption of a semi-infinite strip must be reconciled with the finite length of contact area available on a roll. Should the affected length exceed the contact area, the entire strip would displace along the roll, until reduced forces and moments reduced the affected length to suitable size.

The only other difficulty with the program is a minor one. Should the AL value of a particular situation be less than .01 or greater than 1.5, it will be necessary to alter the search increment card ("FRAC = -.1", the second card in the search routine) to suit. It may also be desirable to correspondingly change the card "EX = .01*XK" in the loop which generates tables of y versus x.

Further study on this problem should begin with the simultaneous application of opposing force and moment. However, the program is useful in its present state, and can be included in a larger program to predict the performance of a rolling strip system. It is expected that the action of the system will be divided into small time increments during which conditions may be assumed constant. Forces and moment, would produce an approach angle, which would change the strip position, resulting in new forces and moments.

Observation of such a computer model, incorporating the program developed in this thesis, can be expected to simulate very accurately the performance of a rolling strip system.

APPENDIX

REMARKS ON THE ACCURACY OF A ONE WAVE APPROXIMATION

As has been stated, the exact solution to this problem takes the form of an infinite number of waves, but the approximate solution arrived at consists of only one wave.

The primary reason for this simplification is the immense degree of difficulty involved in gaining any greater accuracy. Since there does exist a strong possibility that this computer program will become part of a larger program to predict performance of an entire rolling strip system, it was important to achieve a computer solution which would consistently give results, and if possible use a minimum of computer time. The program given here does achieve these ends. The question now to be examined is, to what degree of accuracy?

Ideally, the one wave model would be compared to the exact solution. This however, seems impossible. Consequently it was decided to solve a two-wave approximation, and compare the results of the one wave and two wave.

The two wave equations have been developed elsewhere in this thesis. It will be remembered that the main problem in achieving a two wave solution is the solution of two nonlinear, implicit simultaneous equations (16) for the affected lengths L_1 and L_2.

Since a closed form solution is obviously impossible, it was decided to program the computer to check the accuracy of some typical cases. Using an iterative procedure (specifically, a Newton-Raphson approximation[2]) the roots to equations (16) were arrived at for 8 different force-moment situations, accurate to four decimal places. The following table lists the affected lengths found, along with those determined by the one wave approximation.

For application to web guidance, perhaps the most important criterion is the slope at the end of the strip. This is also listed in the table.

As can be seen, the angles compare very favorably, especially for application of moments (which cause a very small second wave). It is reasonable to assume, then, that the effects of the third wave and those beyond are in fact not important.

It may be necessary to emphasize that there was a high degree of difficulty involved in obtaining the solutions given in the table. The Newton-Raphson technique involves guessing trial values of the roots to the equations, and iterating to precise roots. The difficulty arose from the fact that the equations being solved had many roots. Various solutions could be found, depending on the accuracy of the first trial values, and a certain amount of judgment was

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[2]Daniel D. McCracken and William Dorn, Numerical Methods and Fortran Programming (New York, John Wiley and Sons, 1964) pp.144,145.

necessary to decide which pair of roots corresponded to the physical situation. Thus, adapting the technique to a completely general computer program could prove to be very difficult.

Since the results obtained by a one wave approximation are certainly accurate enough to be useful, it seems that a one wave approximation is indeed the most reasonable approach.

N2	N3	ONE WAVE		TWO WAVE			DIFFERENCE in y′
		AL	y′	AL_1	AL_2	y′	(per cent)
.05	0.	.462852	-.014952	.4623	.00053	-.014953	≈ zero
.1	0.	.639472	-.040197	.6392	.00031	-.040917	"
.2	0.	.868817	-.104302	.8686	.00025	-.104304	"
.3	0.	1.028695	-.178325	1.0285	.00016	-.178327	"
0.	.05	.224898	-.000419	.1944	.10862	-.000422	.700
0.	.1	.444506	-.003180	.3849	.21203	-.003196	.500
0.	.2	.854859	-.021483	.7448	.39053	-.021574	.422
0.	.3	1.222561	-.058671	1.0737	.52739	-.058838	.284

Comparison of Lengths and Slopes
by One Wave and Two Wave Solutions
for N4=.442783