INTERACTIVE COMPUTER PROGRAMS
FOR SHEET PILE DESIGN

by

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ABSTRACT

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This study was primarily concerned with the numerical solution to the free and fixed earth support methods of sheet pile design. Newmark's Numerical Method will be briefly reviewed as it pretains to the sheet pile problem, and some sample beam problems will be solved numerically. The fixed earth support and free earth support methods of sheet pile design will also be reviewed. The sheet pile problem will ultimately be reduced to that of a specially loaded beam. Interactive computer programs will then be introduced to numerically solve the sheet pile problem using the free earth support and fixed earth support methods of sheet pile design.

ACKNOWLEDGEMENTS

The author wishes to thank his wife for her patience, and Dr. Jack Bakos for his dedication to the teaching profession.

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LIST OF SYMBOLS

SYMBOL	DEFINITION	UNITS
E	Modulus of Elasticity	psi/lin. ft.
I	Moment of Inertia	in^4/lin. ft.
Ap	Active Soil Pressure	lbs/lin. ft.
Pp	Passive Soil Pressure	lbs/lin. ft.
Wn	Distributed loading Ordinate at Joint n	lbs.
Jn	Equivalent Load at Joint n	lbs.
Hn	Length of Increment n	ft.
R	Ratio of Adjacent Increments	none
M	Bending Moment Ordinate	ft-lb/lin. ft.
Sn	Average Shear over Hn	lb.
In	Moment Increment over Hn	ft.-lb.
D	Embeddment Depth	ft.
D'	Fixed Earth Trial Depth	ft.
Z	Distance from Anchor Point to Bottom of Pile	ft.
Vz	Shear at Bottom of Pile	lb./lin. ft.
H	Pile Height Above Dredge Line	ft.

LIST OF FIGURES

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CHAPTER I

NEWMARK'S NUMERICAL METHOD

1-1. Introduction

Numerical techniques have proven to be a powerful tool in structural analysis as a means to investigate the behavior of structures subjected to complex loading conditions. Depending upon the degree of complexity, numerical results may vary from exact to very close approximations. It is often the case that the only logical approach to a problem may be with the use of numerical procedures. Numerical analysis, however, has one major drawback; the repetitive calculations utilized to arrive at a solution, although not difficult, can become tedious and time consuming. As the complexity of a problem increases, so does the number of calculations and time required for solution. For this reason accuracy is usually sacrificed for time and other means are employed to arrive at the solution to a complex problem.

Modern computers perform calculations at a speed approaching that of light. The large number of repetitive calculations used in numerical analysis, therefore, makes the method ideally suited to computerization. This study was primarily concerned with the numerical solution to the free and fixed earth support methods of sheet pile design. Interactive computer programs were written to numerically analyze anchored sheet pile bulkheads to determine the required depth of embeddment.

1-2. Statically Equivalent Concentrated Loads.

Newmark's Numerical Method (1)* consists of dividing the span of a beam into increments called chords. The ends of the chords are called joints or nodes. The behavior of the loaded structure can be investigated only at the joints. A joint is located at the point of application of each concentrated load on a structure. The numerical analysis of a point loaded structure will yield exact results at the node points along the structure. Statically equivalent joint loads must be determined and applied to each joint on a structure subjected to a distributed loading. The ordinates of the distributed load are described by the equation of a curve. The accuracy of the numerical analysis of a structure having distributed loading depends upon the degree of the curve that describes the loading, and the length and number of increments.

To analyze a structure by Newmark's Numerical Method, it has already been pointed out that the loading must be comprised of point loads applied at the joints. The structure's behavior can then be exactly investigated only at the joints. This poses no problem for a point loaded structure. The behavior of a structure between the joints on a point loaded structure is also known since no load is applied between the joints. The exact behavior between the joints of a structure subjected to a distributed load cannot be determined with Newmark's Numerical Method. However, on a distributive loaded structure, the average change in behavior over the increment lengths can

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*Number in parenthesis indicates reference cited.

be found by converting the distributed load into statically equivalent concentrated loads applied at the joints. The load conversion makes it possible to predict the change in shear or moment across an increment subjected to a distributed load, thereby permitting very accurate analysis of the structure at the joints.

Concentration formulae have been derived that properly proportion the area under the loading curve over any two adjacent increments, such that the distributed load on the two increments is converted into statically equivalent concentrated loads acting at the appropriate joints. The method of converting a distributed load into equivalent joint loads using the concentration formulae is similar to the Trapezoidal Rule or Simpson's Rule of the calculus.

Refering to Figure 1-1, the problem is to determine the area under the curve

$$y=F(x) \tag{1-1}$$

from x=a to x=b. The Trapezoidal Rule states that this area may be divided into a number of trapezoids. The area of each trapezoid is then determined and the sum of these areas approximates the total area under the curve. The interval $[a,b]$ in Figure 1-1 was partitioned into subintervals and ordinates were erected to the curve from each of the partitioning points. The points in which successive ordinates met the curve were connected by straight line segments in the Trapezoidal Rule; in Simpson's Rule the points are connected by segments of parabolas.

The area under a linear or parabolic curve may be exactly determined by using the Trapezoidal Rule or Simpson's Rule respectively. The area under a third degree or higher order curve may be found by using either linear or parabolic approximations to the curve. The

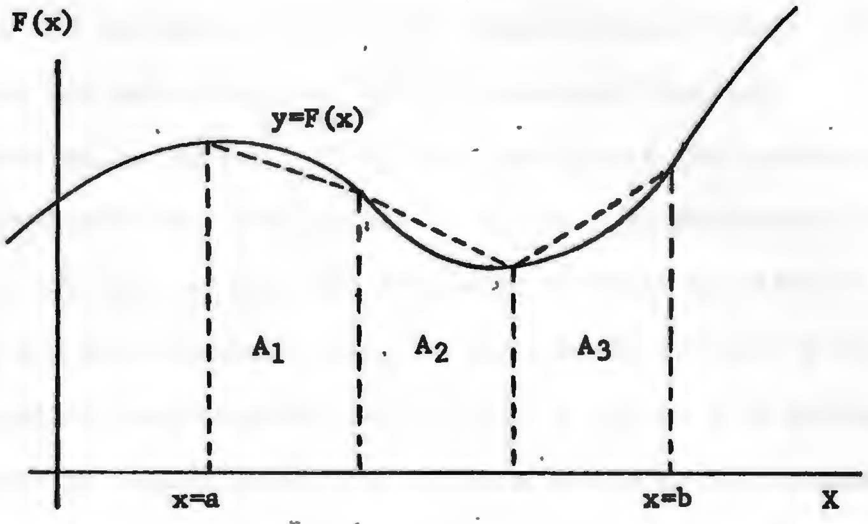


Figure 1-1. The Trapezoidal Rule.

accuracy of the results depends upon the selection of the number and size of the subintervals. This same reasoning applies when converting an Nth order loading curve into statically equivalent concentrated loads with the concentration formulae. The subintervals would be analogous to beam increments and the partitioning points may be considered as nodes or joints on the beam.

A distributed load must always be converted into equivalent joint loads before the numerical technique can procede. Concentration formulae are used for this purpose. Concentration formulae have been derived for both linear and parabolic load distributions. The Trapezoidal Rule and Simpson's Rule are respectively analogous to the linear and parabolic concentration formulae used in Newmark's Numerical Method. Concentration formulae may be derived for higher order curves, but for practical applications this is not necessary. The accuracy of the equivalent joint loads as computed by the concentration formulae is determined by the selection of the number and length of the beam increments. The derivation (1) of the concentration formulae will be avoided here and only their use will be presented herein.

Refering to Figure 1-2, W_a, W_b, and W_c are the loading ordinates of the distributed load W at joints a, b, and c respectively. The increment lengths are H_{ab} and H_{bc}. The following notation is used in Figure 1-2 to specify the concentrated values of the distributed load W at joint b:

J_{ba} = equivalent concentrated load at joint b due to W on increment ba

J_{bc} = equivalent concentrated load at joint b due to W on increment bc

$J_b = J_{ba}+J_{bc}$ = total statically equivalent joint load at b due to W

The linear concentration formulae used to compute the statically equivalent joint loads at joints b and c are as follows:

$$J_b = J_{ba}+J_{bc} \tag{1-2}$$

$$J_{ba} = \frac{H_{ba}}{6}(2W_b+W_a) \tag{1-3}$$

$$J_{bc} = \frac{H_{bc}}{6}(2W_b+W_c) \tag{1-4}$$

$$J_{cb} = J_c = \frac{H_{cb}}{6}(2W_c+W_b) \tag{1-5}$$

The parabolic concentration formulae used to compute statically equivalent joint loads at joints b and c are:

$$J_b = J_{ba}+J_{bc} \tag{1-6}$$

$$J_{ba} = \frac{H_{ba}}{12}\left[W_a\left(\frac{1}{1+R}+1\right)+W_b(R+4)-W_c\left(\frac{1}{1+R}+R-1\right)\right] \tag{1-7}$$

$$\text{where, } R = H_{ba}/H_{bc} \tag{1-8}$$

$$J_{bc} = \frac{H_{bc}}{12}\left[W_c\left(\frac{1}{1+R}+1\right)+W_b(R+4)-W_a\left(\frac{1}{1+R}+R-1\right)\right] \tag{1-9}$$

$$\text{where, } R = H_{bc}/H_{ba} \tag{1-10}$$

$$J_{cb} = J_c = \frac{H_{cb}}{12}\left[W_c\left(\frac{1}{1+R}+3\right)+W_b(R+2)-W_a\left(\frac{1}{1+R}+R-1\right)\right] \tag{1-11}$$

$$\text{where, } R = H_{bc}/H_{ba} \tag{1-12}$$

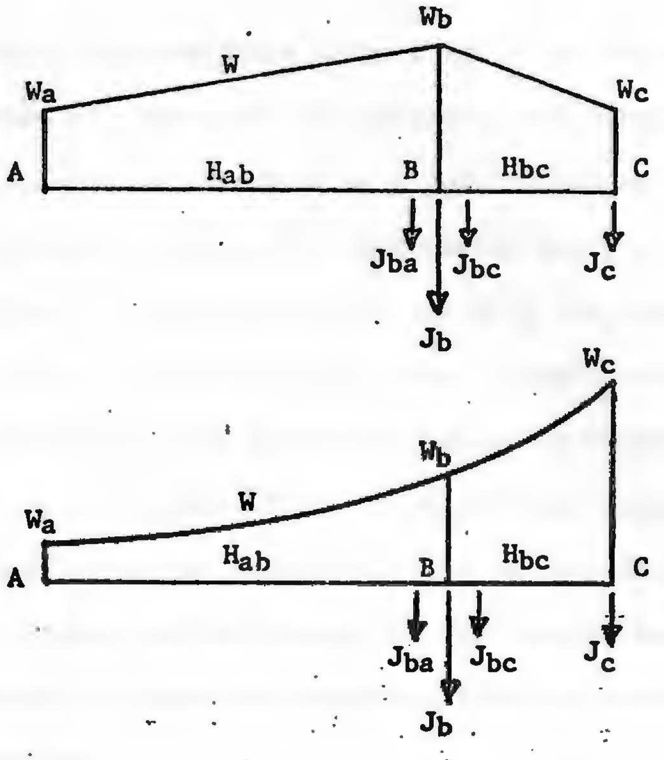


Figure 1-2. Linear and Parabolic Load Distribution

For equal chord lengths R equals unity, and the papabolic concentration formulae reduce to:

$$J_b = J_{ba} + J_{bc} \quad (1\text{-}13)$$

$$J_{ba} = \frac{H_{ba}}{24}(3W_a + 10W_b - W_c) \quad (1\text{-}14)$$

$$J_{bc} = \frac{H_{bc}}{24}(3W_c + 10W_b - W_a) \quad (1\text{-}15)$$

$$J_{cb} = J_c = \frac{H_{cb}}{24}(7W_c + 6W_b - W_a) \quad (1\text{-}16)$$

The computed shears and moments are exact at the joints of a point loaded structure analyzed by Newmark's Numerical Method. The accuracy of the computed shears and moments at the joints of a distributive loaded structure depends upon the accuracy of the statically equivalent concentrated joint loads as computed with the concentration formulae.

Deflections at the joints of a beam can be determined by loading a conjugate beam with an elastic load of intensity $M/E\underline{I}$. M is the moment distribution of the real beam while E and I are the modulus of elasticity and moment of inertia of the section. The moment diagram of a loaded beam will always be described by a curve of at least order one, i.e. the elastic load will always be a distributed load. Equivalent concentrated loads must be determined from the M/EI diagram and applied to the respective joints on the conjugate beam. Shears and moments in the joints of the conjugate beam are then computed by Newmark's Numerical Method. The shear in the joints of the conjugate beam equals the slope at the respective joints of the real beam. The moments in the joints of the conjugate beam equals the deflections at the respective joints of the real beam. Newmark's Numerical Method will now be explained and some example problems worked.

1-3. The Numerical Procedure

The following sign convention will be used throughout this study: positive moment will tend to bend an element of the beam concave upward, positive shear tends to rotate a beam element clockwise, positive loading is considered as acting upwards, and positive deflection is taken as upward.

The technique used in Newmark's Numerical Method is one of numerical integration. Taking into account the end conditions, integration is carried forward in a step-by-step manner from one joint to the next. The numerical procedure is shown in its general form in Figure 1-3. The equivalent joint loads J_a, J_b, J_c, and J_d are shown acting in the positive direction and are applied at the joints a, b, c, and d respectively. Increment lengths are H_{ab}, H_{bc}, and H_{cd}. To determine the shears and moments at the joints, two values must be

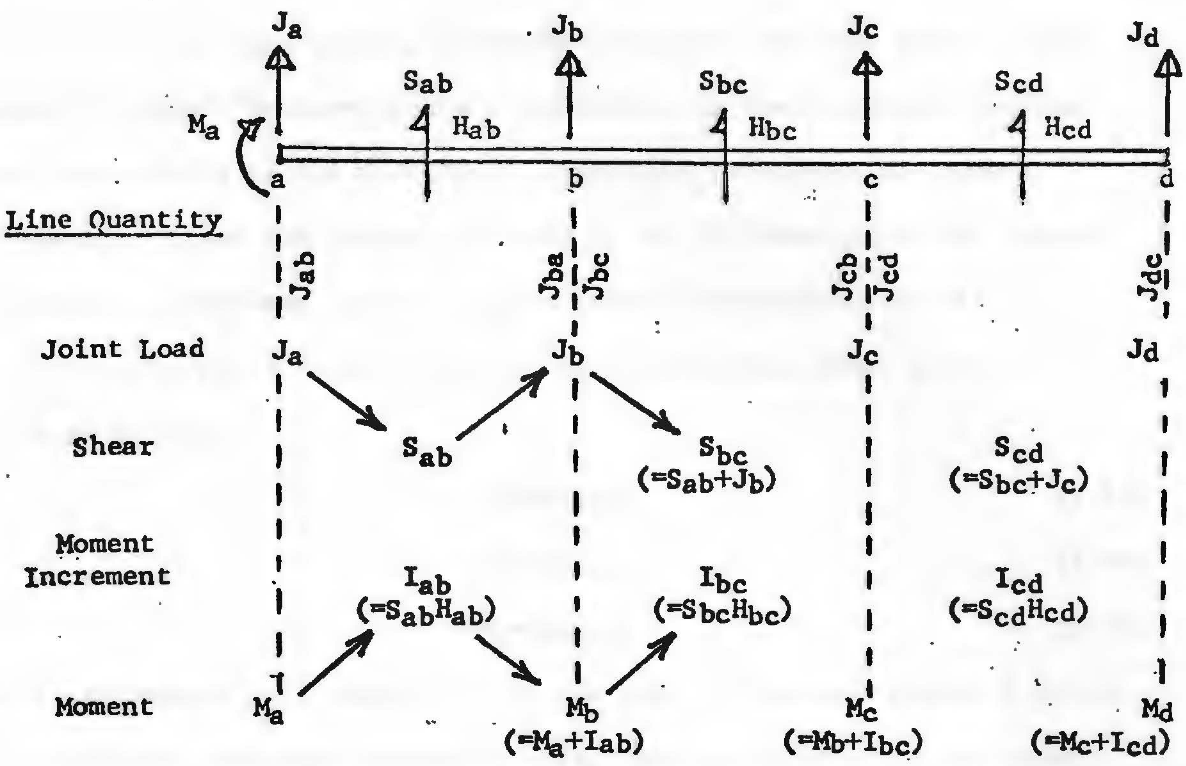


Figure 1-3. Forward Integration Procedure

known; at least one shear and one moment, or two moments. These values come directly from the end conditions of the beam and they are essential in order to integrate from load to shear and from shear to moment. These known values are, in fact, the constants of integration.

Let it be assumed that the moment at joint a, M_a, and the change in shear over increment ab, S_{ab}, are known. The change in shear over all the other increments can be found by adding across as follows:

$$S_{bc} = S_{ab} + J_b \qquad (1\text{-}17)$$

$$S_{cd} = S_{bc} + J_c \qquad (1\text{-}18)$$

The known shear will usually be at one end of the beam due to a given end condition, although correct results can be obtained if the chord shear is known at any other location on the span.

The loading consists of point loads applied only at the joints, and there is no load acting on the beam segment between joints. The change in moment between joints, therefore, is the increment average shear multiplied by the increment length and is called the moment increment. Since the moment at joint a, M_a, is known, and the change in moment, I, between joints is also known, the moments at all the other joints can be found by adding across the beam from joint to joint as follows:

$$M_b=M_a+I_{ab} \quad (1\text{-}19)$$

$$M_c=M_b+I_{bc} \quad (1\text{-}20)$$

$$M_d=M_c+I_{cd} \quad (1\text{-}21)$$

The known moment will usually be at one end of the beam due to a given end condition, although correct results can be obtained if the moment is known at any other location on the span.

In order to determine the real shear V at a joint, Figure 1-3 is again utilized and the following procedure is used:

$$V_a=V_a \quad (1\text{-}22)$$

$$V_b=V_a+J_{ab}+J_{ba} \quad (1\text{-}23)$$

$$V_c=V_b+J_{bc}+J_{cb} \quad (1\text{-}24)$$

$$V_d=V_c+J_{cd}+J_{dc} \quad (1\text{-}25)$$

The real shear at joint a,or at any other joint, must be known.

The initial assumption in the foregoing discussion was that a known shear and a known moment exist, such as at the free end of a cantilever beam. Two end moments are readily known in the case of a simply supported beam. A shear value, i.e. an end shear, can be determined by summing moments but this is not necessary. When analyzing a simply supported beam, the average shear in any increment is assigned an arbitrary value. The shears and moments are then computed by the numerical procedure. The computed values at the joints will be in error unless the assumed shear value was correct. A linear correction can then be applied to the moments to make then conform to the two known moment conditions. The correct average shear values can then be obtained by working back from the corrected moment values.

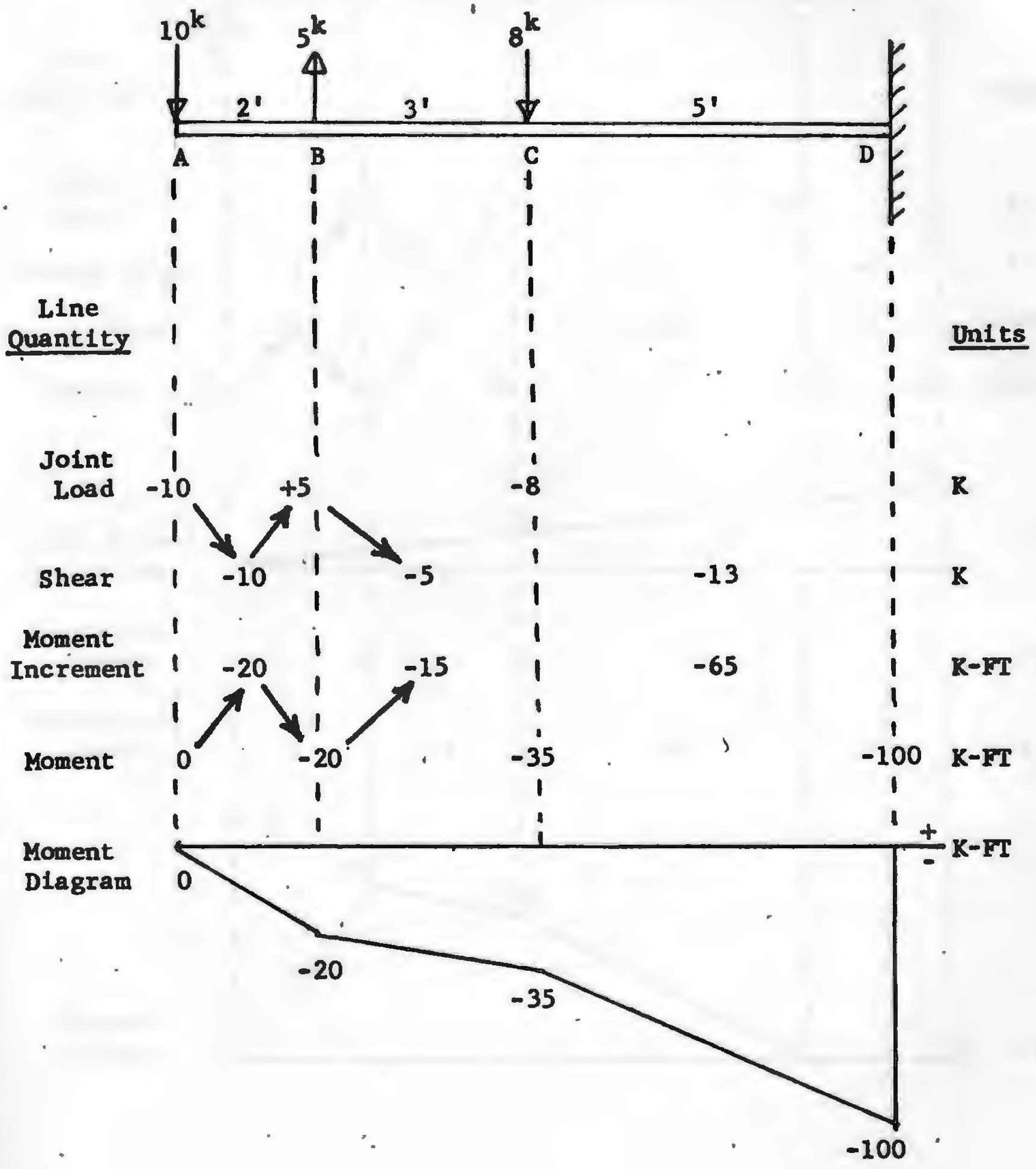
Some example beam problems will now be presented to illustrate the procedure and techniques involved in Newmark's Numerical Method:

Example Problem 1-1.

Given: The cantilevered beam shown below. $E = 29 \times 10^6$ psi, $\underline{I} = 100\ in^4$.

Find: Shear and moment at A, B, C, and D.

Solution:

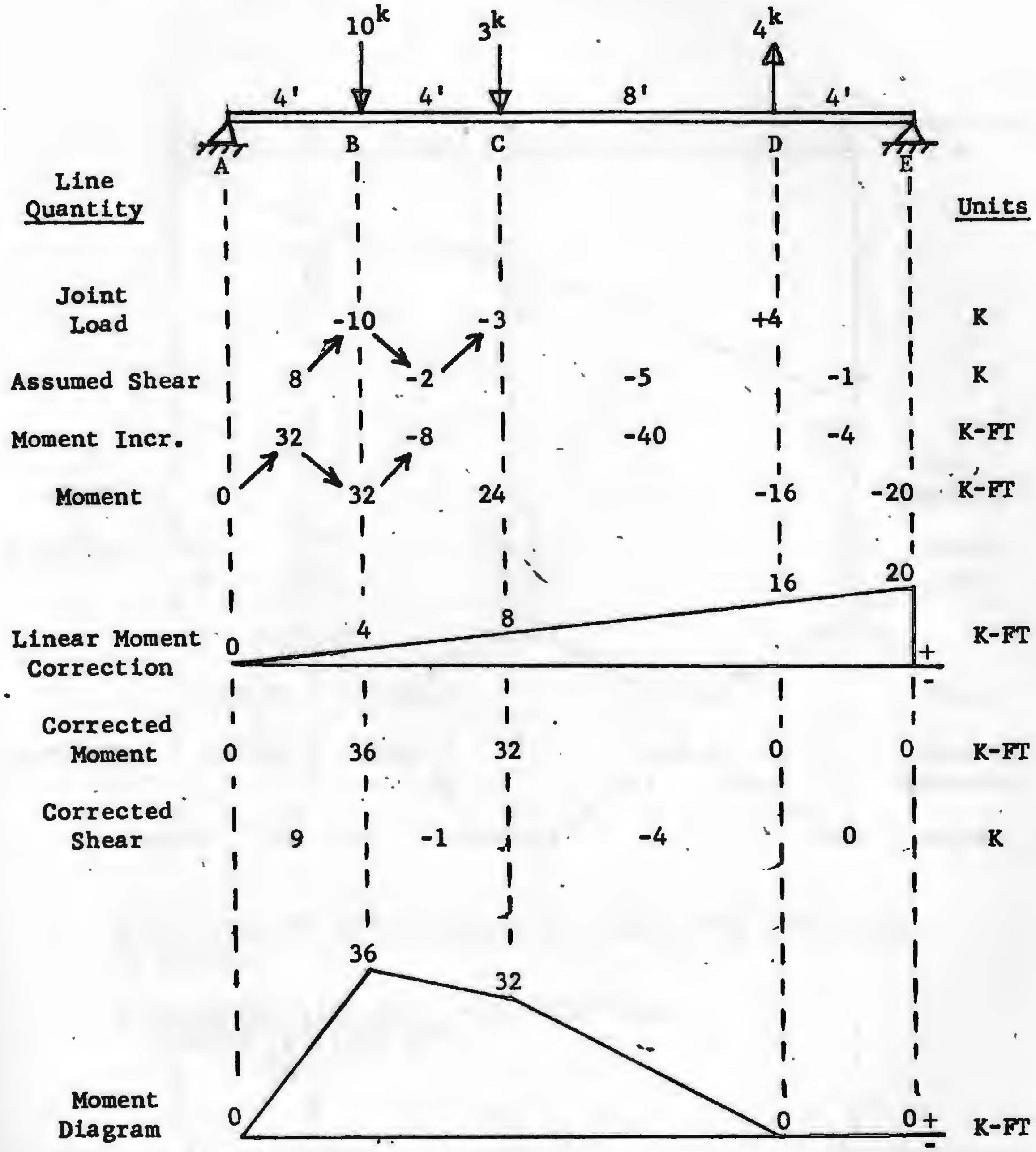


Example Problem 1-2.

Given: The simply supported beam loaded as shown below.

Find: Shear and moment at A, B, C, D, and E.

Solution:

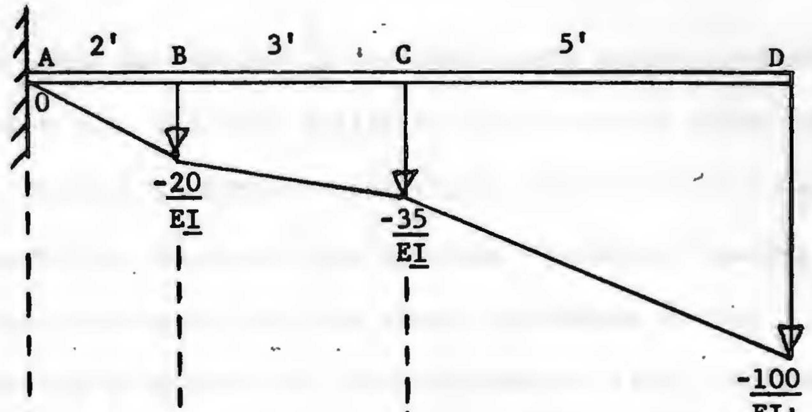


Example Problem 1-3.

Given: The cantilevered beam of Example Problem 1-1 and its associated M/EI diagram.

Find: Deflection at points A, B, C, and D.

Solution:



Conjugate Beam

Units								Line Quantity
K-FT^2/EI	-6.67		-13.33, -37.5		-45.0, -141.67		-195.83	Joint Load
"	-6.67		-50.83		-186.67		-195.83	
"		-431.3		-380.5		-193.83		Shear
K-FT^3/EI		-862.6		-1141.5		-969.85		Moment Increment
"	-2974		-2111.4		-969.85		0.0	Moment

Multiplying the above moments by C gives the deflection in inches.

$$C = \frac{(1000\ LB)(1728\ IN^3)}{(29X10^6\ \frac{LB}{IN^2})(100\ IN^4)} = 5.96X10^{-4}\ IN$$

Inches	-1.77	-1.26	-0.58	0.0	Deflection of the Real Beam

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CHAPTER II

THE FREE EARTH SUPPORT METHOD

2-1. General

The basic assumption in the free earth support method of anchored sheet pile bulkhead design is that the soil below the dredge line cannot develop sufficient restraint so as to produce negative bending moments in the sheet pile section. Negative bending can only occur in that section of the pile above the anchor point.

Refering to Figure 2-1, the bulkhead is first considered fixed at a depth Z, where Z is the distance from the anchor point to the bottom of the pile. The assumption of fixity at Z implies both moment (M_z) and shear (V_z) exist at that point. The resultant forces produced by the active and passive soil pressures are P_a and P_p respectively. The distance from the anchor point to P_a is Z_a and the distance from the anchor point to P_p is Z_p.

Stability requires that the sum of the moments about the anchor point equal zero. Neglecting M_z and V_z for the time being

$$M_{ap}=P_a Z_a-P_pZ_p=0 \tag{2-1}$$

Ignoring the anchor force temporarily, the moment at Z is found by summing moments about Z

$$M_z=P_a(Z-Z_a)-P_p(Z-Z_p)=(P_a-P_p)Z+P_pZ_p-P_aZ_a \tag{2-2}$$

Still ignoring the anchor force and summing forces in the horizontal direction determines the shear at Z

$$V_z=P_a-P_p \tag{2-3}$$

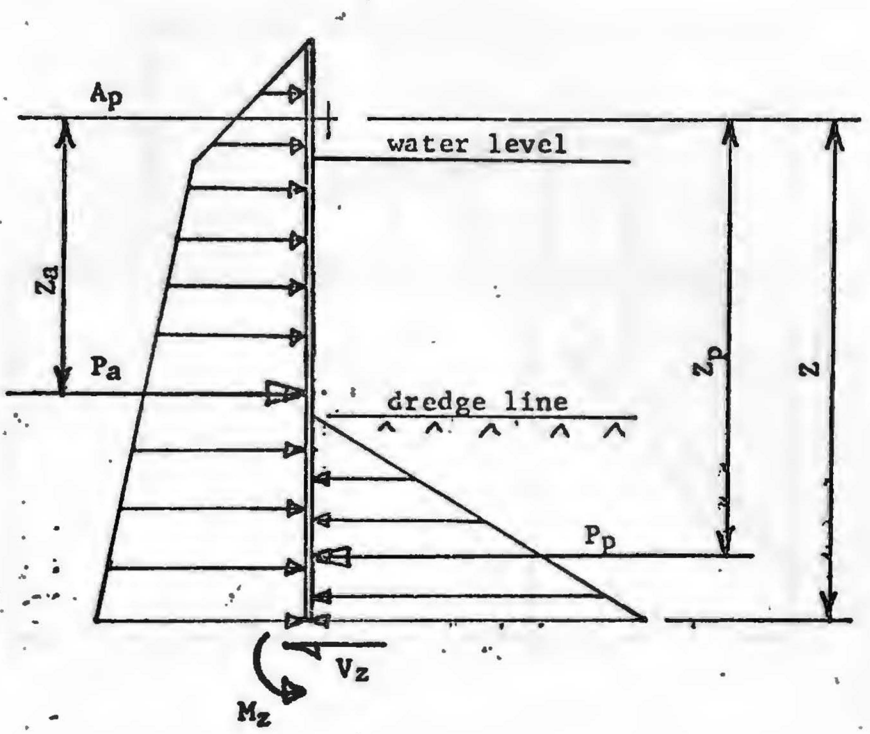


Figure 2-1. Free Earth Support Method

Substituting (2-3) into (2-2)

$$M_z = V_z(Z)+P_pZ_p-P_aZ_a \tag{2-4}$$

Substituting (2-1) into (2-4)

$$M_z = V_z(Z) \tag{2-5}$$

Thus the only Z for which equation (2-1) holds is the same Z which is required to satisfy (2-5).

The required depth of embeddment is determined by analyzing the cantilevered member shown in Figure 2-2. The rotated pile is subjected to distributed loading due to the active and passive soil pressures. Neglecting the anchor force, the shear V_z and moment M_z are computed at the support for various values of Z until equation (2-5) is satisfied, i.e. when M_z equals V_z multiplied by Z, the required depth of embeddment has been obtained. The bottom tip of the pile is at Z, and

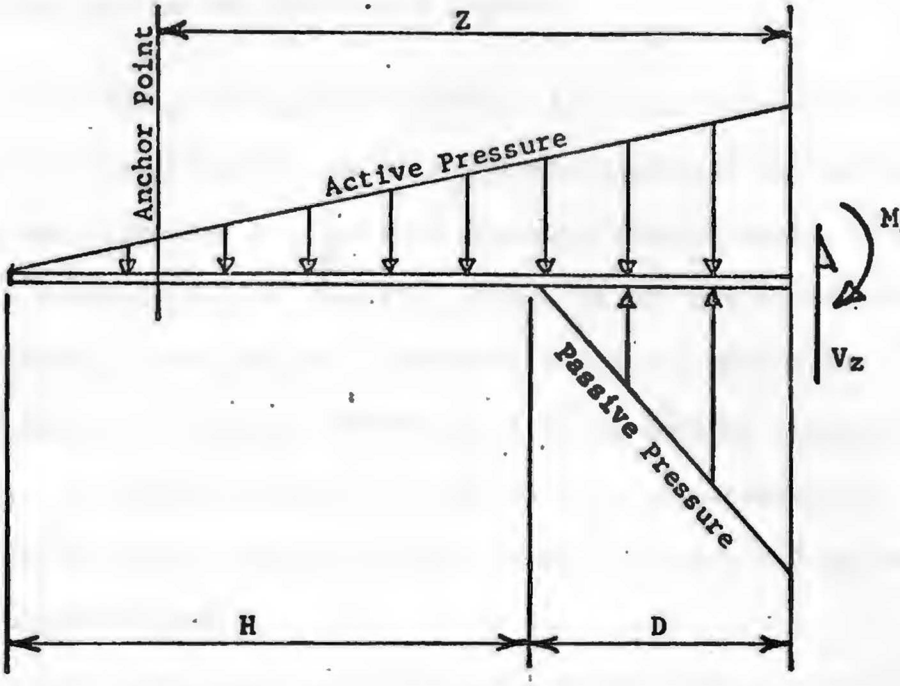


Figure 2-2. Cantilevered Beam with Distributed Load Due to Active and Passive Soil Pressures

since the free end of a member can carry no shear or moment, the unbalanced shear must be balanced by the anchor or

$$A_p = V_z \tag{2-6}$$

thus, the real shear at Z is zero. Consider now the anchor force applied to the member in Figure 2-2. Summing moments about Z, determines the real moment at Z. In equation form

$$\text{real } M_z = M_z - A_p(Z) = 0 \tag{2-7}$$

The real moment at Z is zero as it should be at a free end.

2-2. Numerical Method for Free Earth Support

The trial and error approach used to find the required depth of embeddment is facilitated by using Newmark's Numerical Method when computing M_Z and $V_Z(Z)$ for a given soil pressure distribution. Once this depth is determined, the numerical values of all the forces acting upon the bulkhead are then known. Newmark's Numerical Method can then be used to determine the actual shears and bending moments induced by these forces. The computational procedure will be illustrated by numerically analyzing the loaded bulkhead shown in Figure 2-3 by the free earth support method.

The active and passive soil pressures shown in Figure 2-3 are purely arbitrary. The distribution and intensity of the assumed soil pressures, although unrealistic, will expedite the hand solution to the problem by simplifying the calculations. The linear pressure distributions will also make it easier to check the results by summing moments and forces.

The first step in the free earth support method is to assume a depth of embeddment D. A cantilevered beam of length H+D is loaded with a distributed load due to the assumed active and passive soil pressures. The anchor force is neglected initially. The span is then divided into increments with joints at the ends of each increment. The increments need not be of equal length. A joint must, however, be located at the anchor point since the anchor force will be a point load. The joints are numbered for convenience. Concentration formulae can then be used to convert the distributed soil loads into equivalent concentrated loads applied at the joints. The shears and moments at

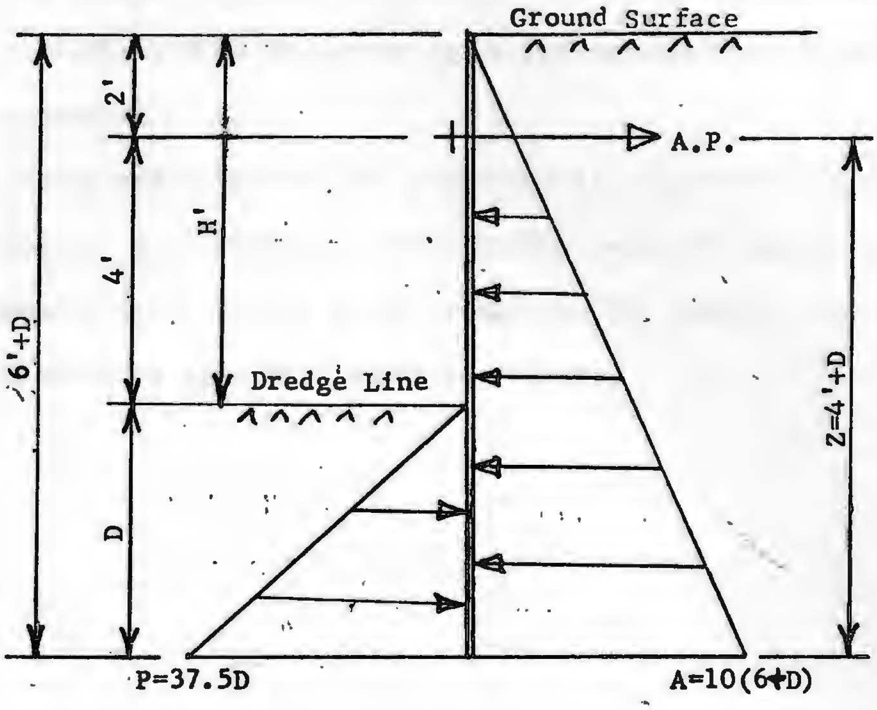


Figure 2-3. Example Problem for Sheet Pile Design.

the joints are computed by Newmark's Numerical Method beginning at a point of known moment or shear; in this case, at the free end of the cantilever pile section. The computed moment at the support, i.e. the embedded tip, is then compared to the shear at the same location multiplied by Z. The required depth of embeddment is obtained when these quantities are equal. A new embeddment depth is selected and the process is repeated if equality does not exist.

The loading used in the procedure thus far was that due only to soil loads. The anchor force was ignored and it must now be considered. Once the required depth of embeddment has been obtained, the computed shear force at the support, i.e. the embeddment tip, is equaled to the anchor force. The numerical procedure must then be repeated, but this time to include the anchor force. The resulting shears and moments computed at the joints will then be the actual shears and moments on the pile in accordance with the given loads and assumptions of the free earth support method.

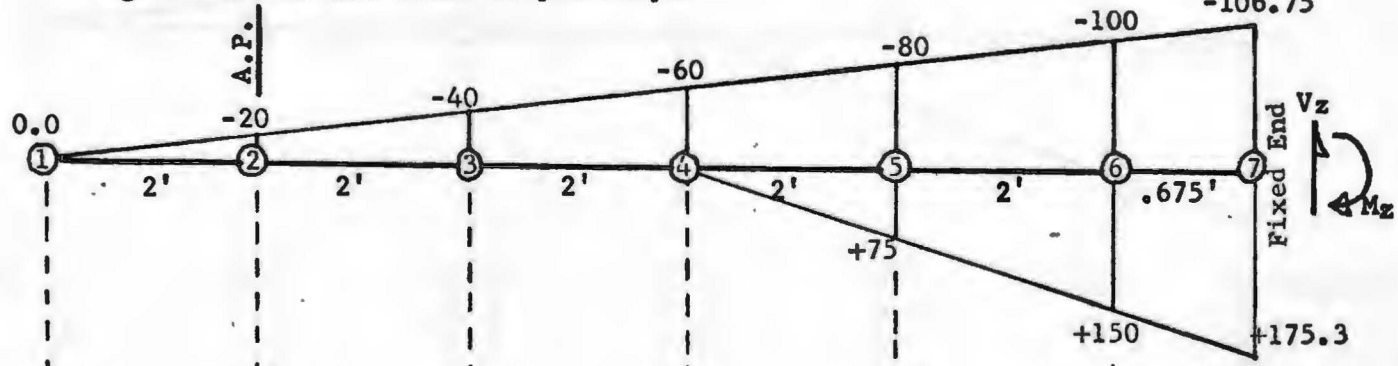
The set-up and computational procedure is illustrated in the following example. For convience, the required depth of embeddment has been predetermined with the aid of the computer. The check at the end of the procedure verifies that this depth is correct.

Example Problem 2-1.

Given: The loaded bulkhead shown in Figure 2-3.

Find: Required embeddment depth, shears, moments, and anchor force. Use the free earth support method.

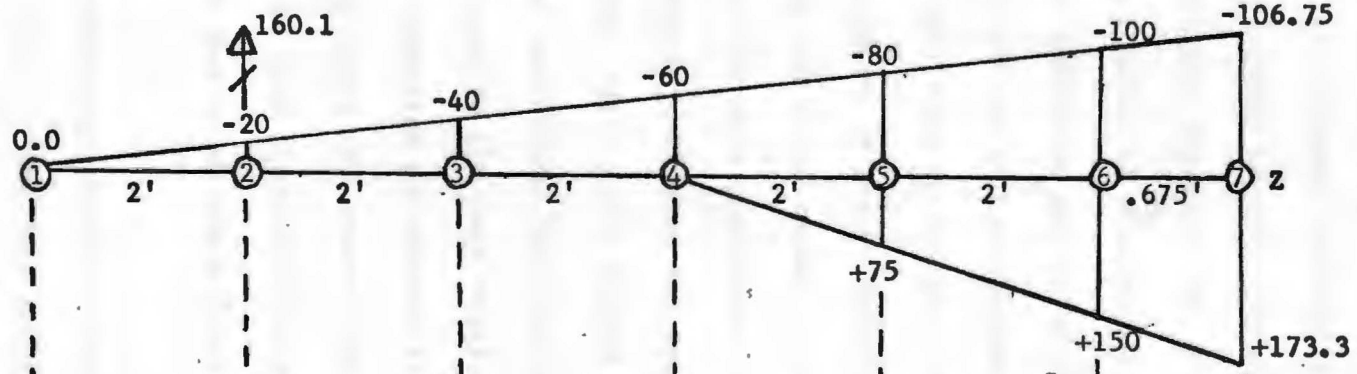
Solution: Assume a depth of embeddment of 4.675 ft. Use 2' increments.
Ignore the anchor force temporarily.



Line Quantity	-6.67		-13.33 / -26.67		-33.33 / -46.67		-53.33 / -41.67		-23.33 / +13.33		+31.67 / +18.96		+21.06	Units/Ft.
Joint Load	-6.67		-40		-80		-95		-10		+50.63		+21	LBS.
Ave. Shear		-6.67		-46.47		-126.67		-221.67		-231.67		-181		"
Moment Incr.		-13.34		-93.34		-253.34		-443.34		-463.34		-122.2		FT-LB
Moment	0.0		-13.34		-106.68		-360		-803.36		-1266.7		-1388.9	"
Shear	0.0		-20		-80		-180		-245		-200		-160.1	LBS.

Example Problem 2-1 (continued).

M_z = -1388.9, $V_z(Z)$ = (-160.1)(8.675) = -1388.9, therefore 4.675 ft. is the required embeddment depth. Now compute real shears and moments to include the anchor force.



	①		②		③		④		⑤		⑥		⑦	
Line Quantity	-6.67		-13.33 / +133.33		-33.33 / -46.67		-53.33 / -41.67		-23.33 / +13.33		+31.67 / +18.96		+21.05	Units/Ft.
Joint Load	-6.67		+120.1		-80		-95		-10		+50.63		+21	LBS.
Ave. Shear		-6.67		+113.43		+33.43		-61.57		-71.57		-20.9		"
Moment Incr.		-13.34		+226.86		+66.86		-123.14		-143.14		-14.1		FT/LBS
Moment	0.0		-13.34		+213.52		+280.38		+157.24		+14.1		0.0	"
Shear	0.0		-20		+80		-20		-85		-40		0.0	LBS.

Check ΣM_z = 0.0:

$$M_z = 160.1(8.675)+\frac{(173.3)(4.675)}{(2)}\frac{(4.675)}{(3)}-\frac{(106.75)(10.675)^2}{(2)(3)} = -7.33 \text{ FT-LB}$$

The check of the results in the foregoing example problem reveals the presence of a small negative bending moment at the bottom of the pile. If the same problem is reworked using a slightly smaller depth D, the final computed moment at the bottom of the pile will be either a smaller negative one, or a small positive one. This would imply that a point of contraflexure must exist near the bottom of the pile. This situation may exist in reality, but for design purposes it is contrary to the initial assumption of the free earth support method, i.e., the soil into which the pile is driven cannot offer sufficient resistance so as to induce negative bending moments in the pile section. The sign of the final moment at the bottom tip of the pile is useful in determining the next embeddment depth during the trial procedure. A positive moment indicates the trial embeddment depth is too small and that the next trial depth should be larger. A negative moment indicates the present embeddment depth is too large and that a smaller depth should be used in the next trial. The exact depth of embeddment about which the summation of moments is uniquely zero may never be determined. However, by using the moment sign indicators, the required embeddment depth may be hand calculated to within a fraction of a foot in only a few trials, and to within a fraction of an inch using a high speed computer.

Restrictions have not been placed on displacements at the bottom tip of the pile in the free earth support method. This point may, in fact, displace. Compatibility conditions on deflection have not been imposed and, therefore, the conjugate beam method, or any other method, cannot be employed to calculate deflections at the remaining joints.

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CHAPTER III

THE FIXED EARTH SUPPORT METHOD

3-1. General

The basic assumption in the fixed earth support method is that the soil into which the pile is driven can offer sufficient resistance so as to induce negative bending moments in the pile below the dredge line. A point of contraflexure, therefore, exists and the bulkhead acts like a partially built-in beam. The fixed earth support method involves a number of simplifying assumptions. These assumptions will be explained in the following discussion of the procedure.

Refering to Figure 3-1 a depth of embeddment D' is selected and the active and passive soil pressure distributions are determined over the length H+D' to point t. To model the pile action below point t, the depth D' is extended by an additional amount equal to 0.2D'. A concentrated force R is placed on the bulkhead at point t in a direction such that it will tend to resist the passive earth pressures. The magnitude of R equals the resultant of the passive pressure distribution over the length of the additional 0.2D' below point t. The anchor force, Ap, is found by summing forces in the horizontal direction in Figure 3-1 to include the force R and the active and passive pressure distributions over the length H+D'. A deflection line of the bulkhead can then be determined for the known loading.

The elastic line of the bulkhead is assumed to be tangent to the vertical at point t and intersects the vertical at the anchor point,

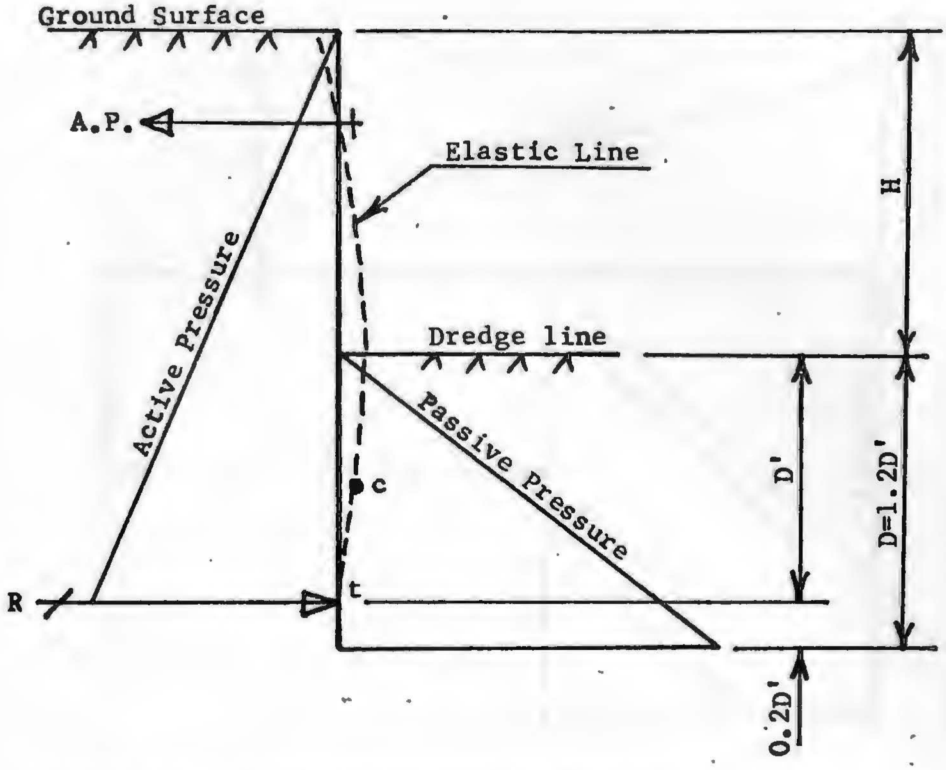


Figure 3-1. Fixed Earth Support Method.

i.e., the deflection at the anchor point is zero (Figure 3-1). If the elastic line thus determined does not intersect the vertical at the anchor point, then the depth D' has been estimated incorrectly and is not compatible with the conditions of equilibrium imposed. A new value must then be selected for D' and the entire procedure of determining the elastic line has to be repeated for the new depth. The required depth of embeddment has been obtained when the deflection of the elastic line is zero at the anchor point.

3-2. Numerical Method for Fixed Earth Support

A depth of embeddment D' is selected and a cantilever beam of length H+D' is loaded with the active and passive pressure distributions as shown in Figure 3-2. The span is then divided into increments with

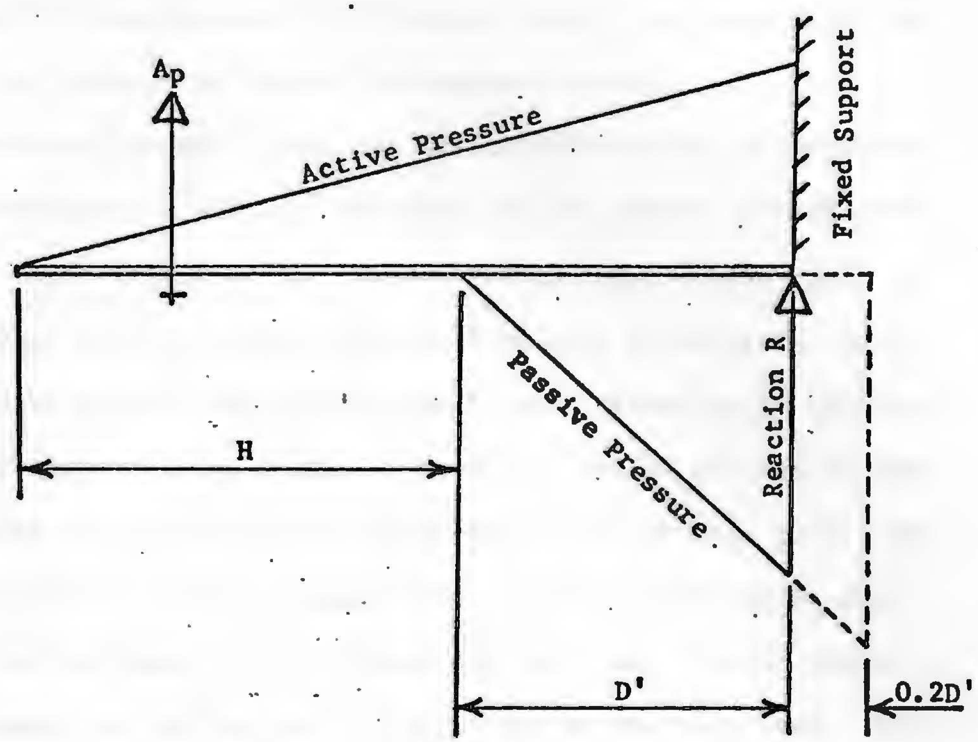


Figure 3-2. Cantilever Beam Used in Fixed Earth Support Method

joints at the ends of each increment. The joints are numbered for convenience. A joint must be located at the anchor point because the deflection of that point constitutes a design parameter and also because the anchor force at that point is a concentrated load. Concentration formulae can then be used to convert the active and passive pressure distributions into equivalent concentrated loads applied at the joints. Newmark's Numerical Method can then be used to compute the shear and moment at each joint due to the soil loads.

The reaction at the support of the cantilever pile equals the resultant of the passive pressure distribution over the additional length 0.2D' positioned at the bottom of the pile (the area enclosed by the dashed lines in Figure 3-2). The only remaining unknown is the anchor force and it is found by summing forces. All the forces acting upon the bulkhead are now known and the induced shears and moments at the joints can be computed by Newmark's Numerical Method.

Deflection at each joint can now be determined. A conjugate beam is loaded with an elastic load equal to the moment distribution of the real beam divided by EI. Concentration formulae can again be used to convert the distributed elastic load into a series of equivalent concentrated loads applied at the joints. Starting at the free end of the conjugate beam, Newmark's Numerical Method is used to compute the shear and moment at the joints due to the elastic load. The shear at the joints on the conjugate beam equals the slope at the joints on the real beam, and the moment at the joints on the conjugate beam equals the deflection at the joints on the real beam. The required depth of embeddment has been obtained if the computed deflection at the anchor point equals zero. It must be added that no deflection or slope is experienced at each assumed embedded end of the pile.

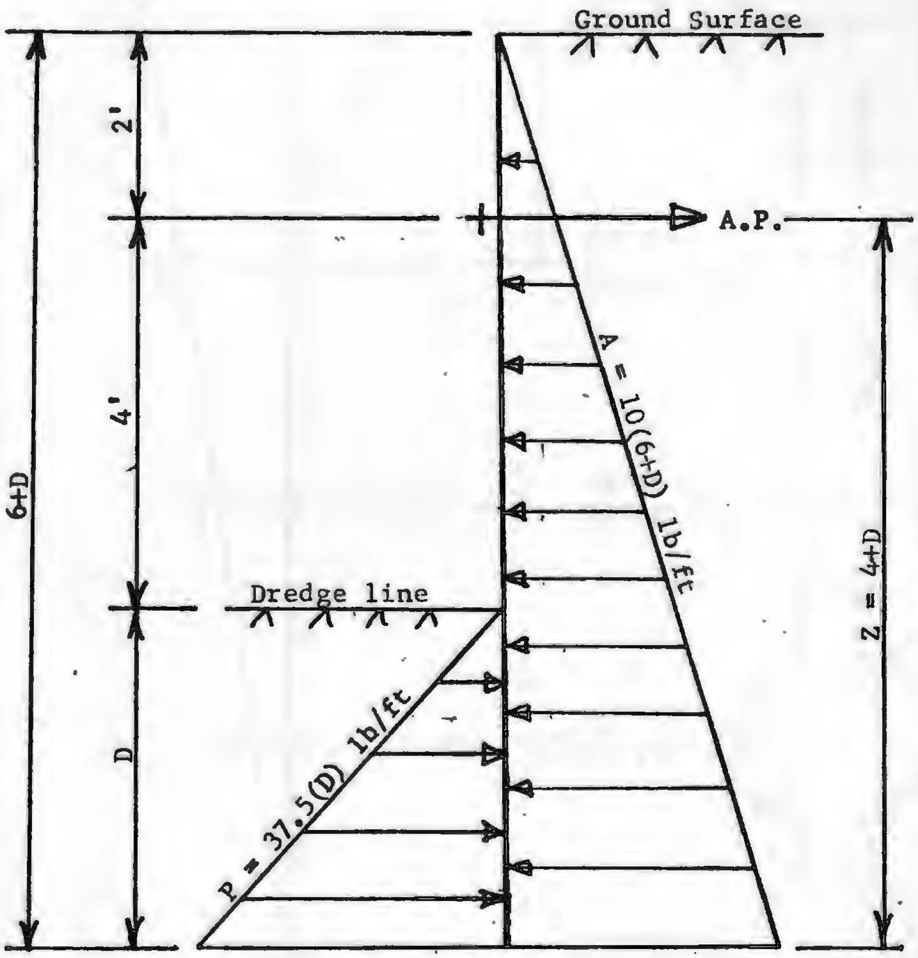
The foregoing procedure will now be illustrated with an example problem. The required embeddment depth for the bulkhead shown in Figure 2-3 will be computed by numerical procedures with the fixed earth support method. This same problem was worked by the free earth support method in example problem 2-1.

Example Problem 3-1.

Given: The anchored bulkhead shown in Figure 2-3 and reproduced below. E = 290,000 psi and I = 10 in.^{4}/ft.

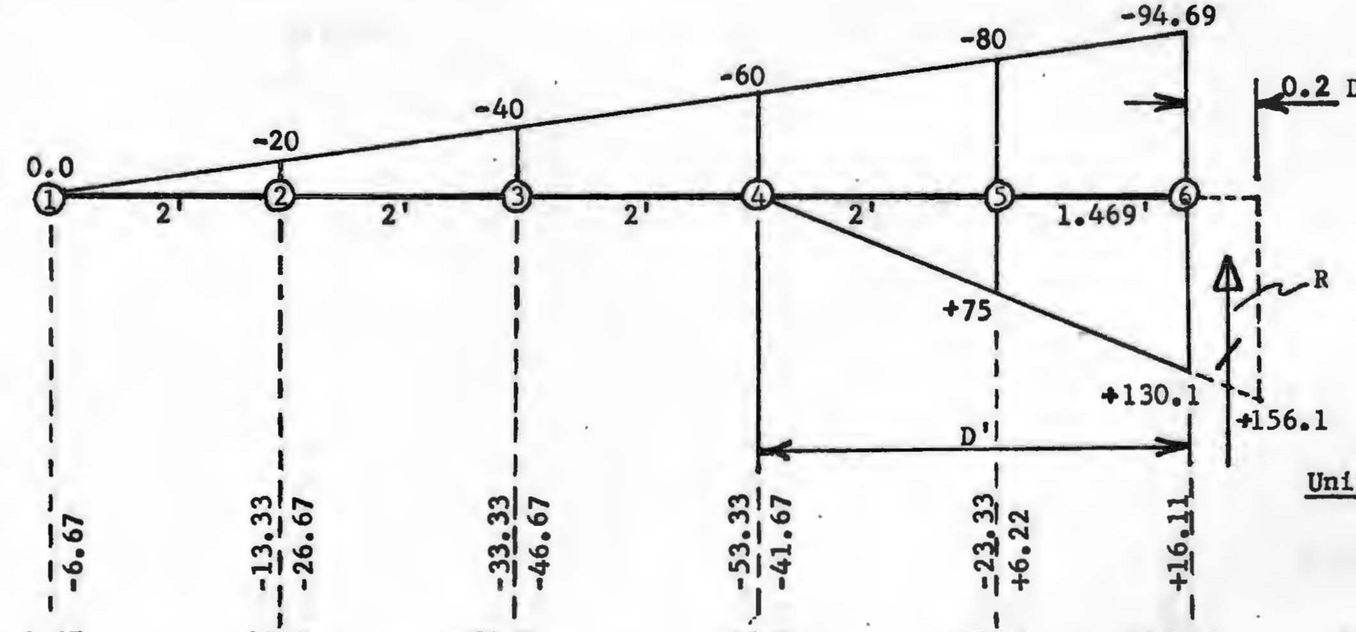
Find: Required embeddment depth, shear, moment, and deflection by the fixed earth support method.

Solution: Assume D' equals 3.469 ft. Use 2 ft. increments.



Example Problem 3-1(continued).

Temporarily ignoring the anchor force, compute joint shears and moments for the assumed D' by Newmark's Numerical method:



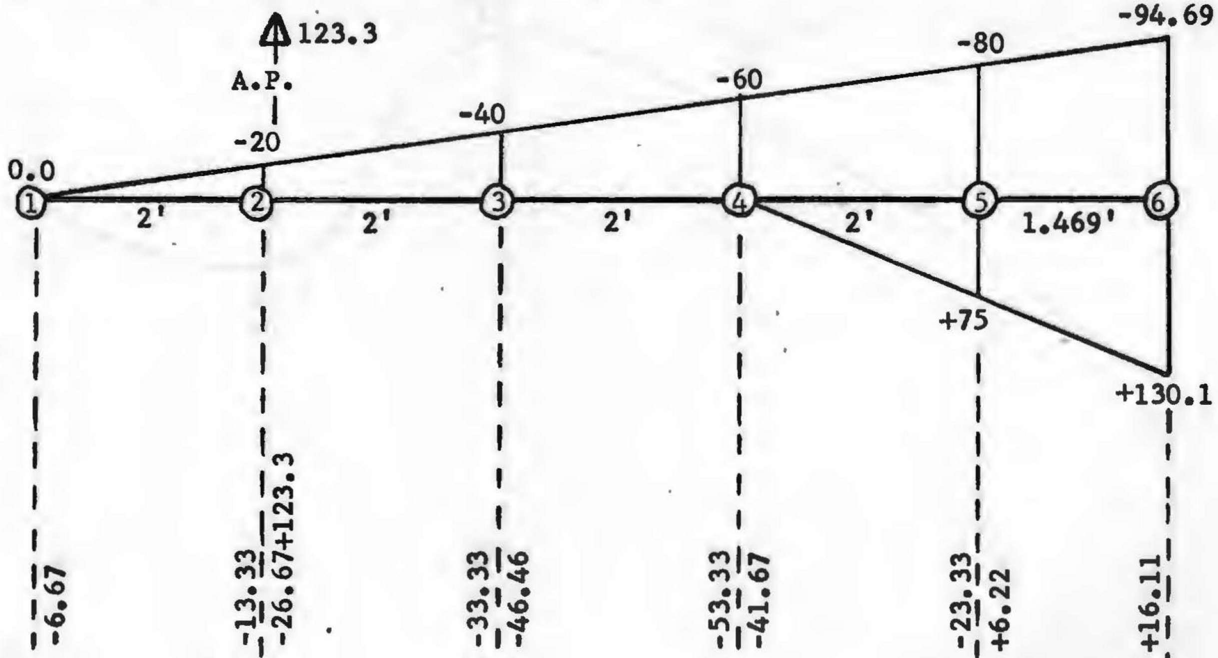
Use Linear Concentration Formulae

Line Quantity	①		②		③		④		⑤		⑥	Units/Ft
Joint Load	-6.67		-40.0		-80.0		-95.0		-17.11		+16.11	LBS.
Ave. Shear		-6.67		-46.67		-126.67		-221.67		-238.78		"
Moment Incr.		-13.34		-93.34		-253.34		-443.34		-350.77		FT-LBS
Moment	0.0		-13.34		-106.68		-360.0		-803.36		-1154.13	"
Shear	0.0		-20		-80		-180		-245		-222.67	LBS.

Example Problem 3-1 (continued).

The reaction $R = \frac{(130.1+156.1)}{2}(0.6938) = 99.28$ LB/FT

The anchor force $= -V_z - R = (-)(-222.67) - 99.28 = +123.3$ LBS/FT

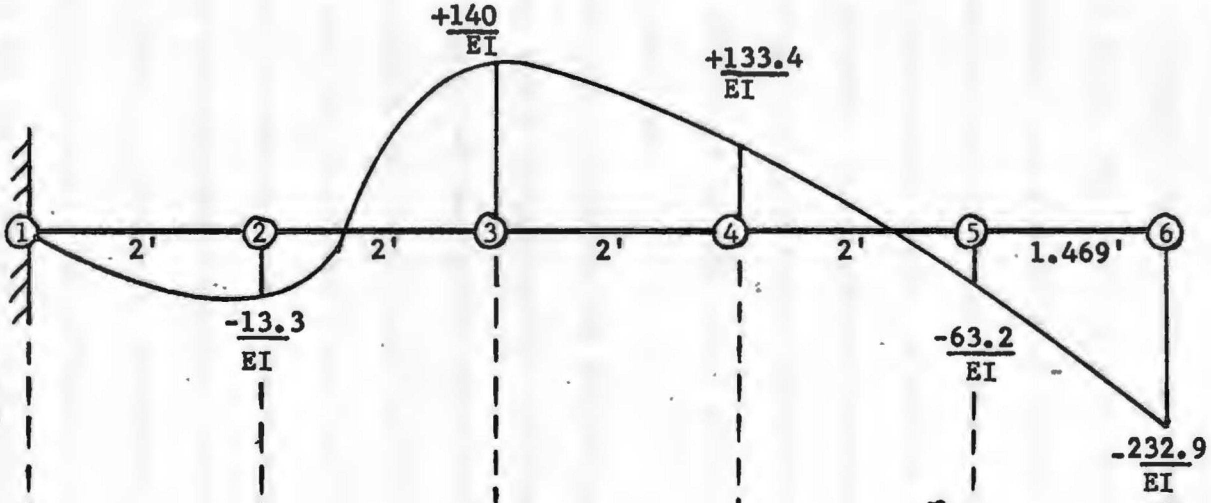


Use Linear Concentrarion Formulae

Line Quantity												Units/Ft
Joint Load	-6.67		+83.3		-80		-95		-17.1		+16.1	LBS.
Ave. Shear		-6.67		+76.6		-3.3		-98.3		-115.4		LBS.
Moment Incr.		-13.3		+153.3		-6.6		-196.6		-169.6		FT-LBS
Moment	0.0		-13.3		+140.0		+133.4		-63.2		-232.9	"
Shear	0.0		-20		+43.3		-56.7		-121.7		-99.3	LBS.

Example Problem 3-1 (continued).

Deflection equals moment on a conjugate beam loaded as defined by the M/EI diagram.



Use Parabolic Concentration Formulae

	1		2		3		4		5		6	Units/Ft
Line Quantity	-0.91		-1.13 / +1.18		+5.08 / +7.5		+7.5 / +4.15		$+4.5X10^{-3}$ / -4.45		-6.5	$X10^3$
Joint Load	-0.91		0.055		12.58		11.65		-4.44		-6.5	$X10^3$
Ave. Shear		13.34		13.29		0.71		-10.9		-6.5		$X10^3$
Moment Incr.		320.16		318.96		17.04		-261.6		-114.58		$X10^3$ INCH
Moment (Real Deflection)	+0.279		-0.04		-0.359		-0.376		-0.114		0.0	INCH

The computed deflection at the anchor point in the foregoing example problem is 4.0×10^{-2} inch for the given depth of embeddment. A positive deflection indicates displacement in the direction of the anchor force, that is, opposite to the direction of the active pressure forces. If example problem 3-1 is reworked using a slightly larger embeddment depth, the computed deflection at the anchor point will be either smaller, i.e. positive or reversed, i.e. negative. This change in sign can be used, therefore, as an indicator when selecting the next trial embeddment depth. A positive sign for the computed deflection at the anchor point implies the assumed embeddment depth is too small, and an increased length should be used for the next trial. A negative deflection at the anchor point implies the trial embeddment depth is too large.

The depth at which the deflection at the anchor point is uniquely zero may never be determined. In order to illustrate this point, example problem 3-1 was worked using the computer and the deflection at the anchor point was found to be 5.052×10^{-4} inch for a D equal to 4.16352 feet, and -3.499×10^{-4} inch for D equal to 4.16364 feet. The deflection at the anchor point changed sign from positive to negative by increasing the embeddment depth an additional .00012 feet. It would not be feasible to attempt to determine the embeddment depth D to closer a tolerance than this. The difference in magnitude of the shears and moments in the pile section over a change in embeddment depth this small is insignificant.

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CHAPTER IV

CONCLUSIONS

4-1. Selection of Computational Method

A conservative embeddment depth will always be obtained when designing a bulkhead by the free earth support method. The design parameters of example problems 2-1 and 3-1 are identical but example problem 2-1 was worked by the free earth support method, and example problem 3-1 was worked by the fixed earth support method. A comparison of the required embeddment depths for the two example problems reveals that a larger embeddment depth was determined for the bulkhead designed by the free earth support method. Bulkheads embedded in soft clay or soils having questionable loading characteristics should, therefore, be designed by the free earth support method.

The fixed earth support method may be used to design bulkheads embedded in sand or predominately granular soils. Field measurements indicate that stiff, overconsolidated clays also provide sheet pile fixation below the dredge line just as effectively as do sands. No data is available for clays of medium stiffness, nor for complex types of soils such as silt or mixtures of silt with sand and clay(2). Engineering judgement must be used to estimate the extent of sheet pile fixation in such soils.

4-2. Accuracy of Results.

The design of sheet pile bulkheads using numerical procedures is nearly exact in accordance with the assumed soil pressure distributions and the assumptions of the particular design method being used. The assumptions in the free and fixed earth support methods are based on theoretical and experimental results, but they cannot be applied specifically to every situation. Factors such as soil moisture content, soil type, density, angle of internal friction, wall friction, etc. make each sheet pile design problem unique. For example, in the fixed earth support method, it may be that the true length over which the resultant of the passive soil pressure is determined, and applied to the bottom of the pile as a concentrated reaction, is equal to a value other than .2 D'. This length may even vary with different soil types or soil properties. The criteria of no negative bending of the bulkhead in the free earth support method is also a design assumption. Investigation of an existing bulkhead designed by the free earth support method may, in fact, reveal the existance of a point of contraflexure in the bulkhead below the dredge line. Error may also be introduced in assuming the type of curve which describes the soil pressures acting upon the bulkhead. Errors of the above nature, rather than inherent errors in the numerical technique itself, will govern the accuracy of a bulkhead designed using Newmark's Numerical Method with the free or fixed earth support methods.

It must be pointed out that no soil constraints, such as permissable soil displacement,are imposed with the free or fixed earth support methods. This is advantageous in that, once the required depth of embeddment is obtained by the fixed earth support method, the designer can vary the pile section modulus and compute deflections by Newmark's Numerical Method until any desired deflection is obtained. Also, by using Newmark's Numerical Method, a very accurate analysis can be obtained for any given soil pressure distribution or loading condition.

4-3. Summary

Except for the work of a few individuals such as Rowe, Blum, and Tschebotarioff (2), relatively little experimental work has been done to correlate theoretical results, nor to supplement or alter existing assumptions of the free or fixed earth support methods of sheet pile design. This lack of experimentation is probably due to the complexity of the problem with respect to the large number of variables involved. Full scale tests to include all combinations of these variables would be practically and economically infeasible.

This study is not intended to criticize or make recommendations to the existing assumptions of the free or fixed earth support methods. The main objective here is to introduce more proficient computational techniques and methods for the existing design criteria. This is accomplished with the aid of Newmark's Numerical Method which has already been presented, and interactive computer programs which will be discussed later. Regardless of the fact that some basic assumptions may be questionable, the free and fixed earth support methods have proven to furnish reliable design criteria for anchored flexible sheet pile bulkheads. Nevertheless, safety factors and good engineering judgement should be included in every sheet pile design.

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CHAPTER V

INTERACTIVE COMPUTER PROGRAMS

5-1. Description of Programs

The programs presented herein have been written in the BASIC language to facilitate on-line user interaction with the computer. The computer will "ask" for input of data and variables as the program procedes. The user will supply this information at the terminal, as "called for" by the computer. A knowledge of the BASIC language is, therefore, useful, but not necessary to design anchored bulkheads with these programs.

The trial and error approach for finding the required depth of embeddment by the free or fixed earth support methods of sheet pile design, and computation of the pile section's behavior by Newmark's Numerical Method, is greatly facilitated by computer programming. The computational techniques used in the programs are identical with those illustrated in example problems 2-1 and 3-1, with the exception that results are obtained with greater speed and accuracy.

To set up a problem for computer solution, the designer must first assume an embeddment depth D for the free earth support method, or D' for the fixed earth support method. The active and passive soil pressure distributions are then assumed, taking into consideration soil densities, surcharge loads, etc. The pile is then divided into increments, and the joints at the ends of each increment are numbered consecutively from top to bottom of the bulkhead. A joint must be located at the anchor point on the bulkhead. The number of joints thus determined will be refered to henceforth as "the original number of joints". It is not necessary for the increment lengths to be equal or for the pressure distributions to be linear. The active and passive loading ordinates corresponding to each joint are determined by the designer in pounds per linear foot of bulkhead and read into the computer for calculation of the equivalent joint loads.

It is recommended that the initial assumption for the embeddment depth be larger than what is felt by the designer to be actually required. This reasoning can be justified through the use of Figure 5-1, which illustrates the portion of a bulkhead below the dredge line. D0 represents the initial assumed embeddment depth. The embeddment depths used in the next two succeeding trials are D1 and D2. A4 and P4 are the active and passive loading ordinates corresponding to joint 4. The increment length at the bottom of the pile between the last two joints (3&4) is designated by L3. It can be seen in Figure 5-1 that the last increment length will change as the embeddment depth changes. This will cause the last joint at the bottom of the pile to be relocated. For the smaller embeddment depth D' in Figure 5-1, the last increment length L3 will be smaller, and joint 4 will be repositioned between its original location and joint 3.

This is desirable since a second degree approximation to the third order M/EI diagram is used to compute deflections in the fixed earth support method, and small increment lengths will increase the accuracy of the computed deflections. The last increment length L3 in Figure 5-1 will be larger for the larger embeddment depth D2, i.e. joint 4 will be repositioned at a greater distance from joint 3. The

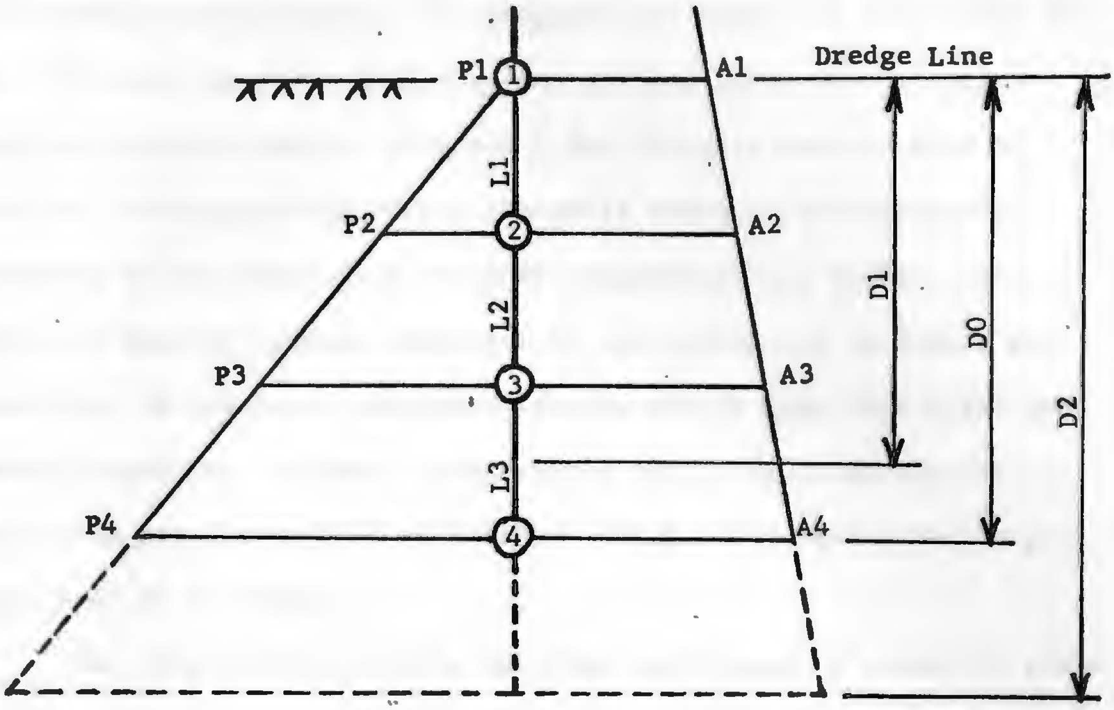


Figure 5-1. Relocation of Last Joint for Various Embeddment Depths

initial selection of the embeddment depth should be large enough to provide an ample number of increments, such that for succeeding trial depths, the location of the bottom joint on the pile will always fall between two originally existing joints.

Parameters such as the original number of joints, increment lengths, and the active and passive loading ordinates that defined the original problem are automatically reset by the computer before any computations are performed for a new trial depth. The computer will also calculate the active and passive loading ordinates associated with the newly relocated bottom joint for each new trial embeddment depth before any other computation procedes.

Two seperate computer programs have been written; one facilitates sheet pile design by the free earth support method and the other by the fixed earth support method. The programs are currently titled FRE and FIX. The input data for both programs is identical, the exception being that relative values of E and I for the pile section must be input for the program FIX which eventually computes displacements. Dimensions of the input data for both programs are as follows: the active and passive loading ordinates at each joint are in pounds per linear foot of bulkhead, increment lengths are in feet, and trial embeddment depths are in feet. For program FIX, E is in pounds per square inch per linear foot of bulkhead and I is in inches fourth per linear foot of bulkhead.

The program FRE utilizes the free earth support method of sheet pile design which was explained in Chapter II. With this program, the designer interacts with the computer by inputting various values for D (embeddment depth), and comparing relative values of M and $V_z(Z)$. When M equals $V_z(Z)$ the required depth of embeddment has been reached. The computer will then print, if directed by the designer, joint load, shear, and moment at each joint. The anchor force will also be printed, and the program terminates.

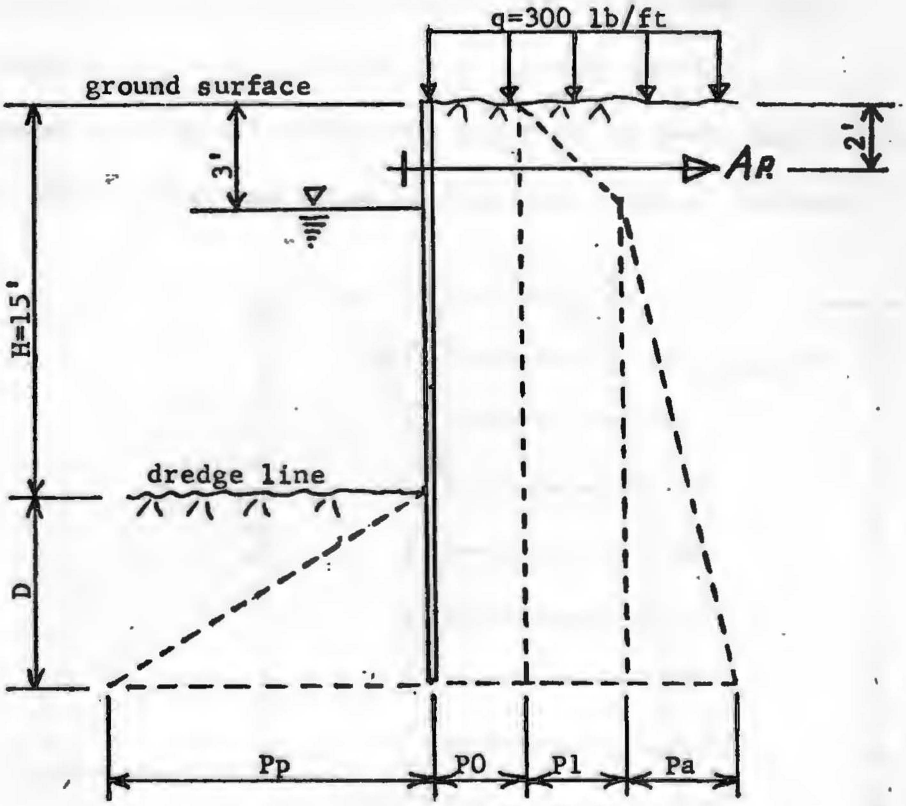
The program FIX utilizes the fixed earth support method of sheet pile design which was explained in Chapter III. Designer interaction consists of inputting various values for D', and observing the computed relative deflection of the bulkhead at the anchor point. Relative deflections are due to relative E and I values originally input by the designer. When the relative deflection at the anchor is very small or zero, the computer will print, if directed by the designer, equivalent joint load, shear, and moment at each joint. The anchor force will also be printed. The computer will then "ask" for real values of E and I. When these are input by the designer, real deflection in inches will be printed for each joint. The computer will then "ask" for another I value. The designer can then terminate the program, or the designer can continue to input various values for I until satisfied that the computed joint deflections obtained are tolerable.

The normal procedure for terminating the FIX program is to input the letter N (which stands for no) when the computer "asks" for another I value. Both the FIX and FRE programs will be terminated whenever a value of zero is input for D' or D.

APPENDIX A

Example Anchored Bulkhead Design Problems and Their Computer Solutions

Example Problem 1-A.



Given: The bulkhead shown above. γ=120 pcf, G=2.65, ϕ=30°. Factor of safety = 2.

Find: Computer bulkhead design by both free and fixed earth support methods.

Example Problem 1-A (continued).

Solution:

$\gamma sub. = \gamma-(\gamma/G) = 120-(120/2.65) = 75.0$ pcf

$Ka = TAN^2(45-\phi/2) = TAN^2(30) = 0.333$

$Kp = 1/Ka = 1/0.333 = 3.0$

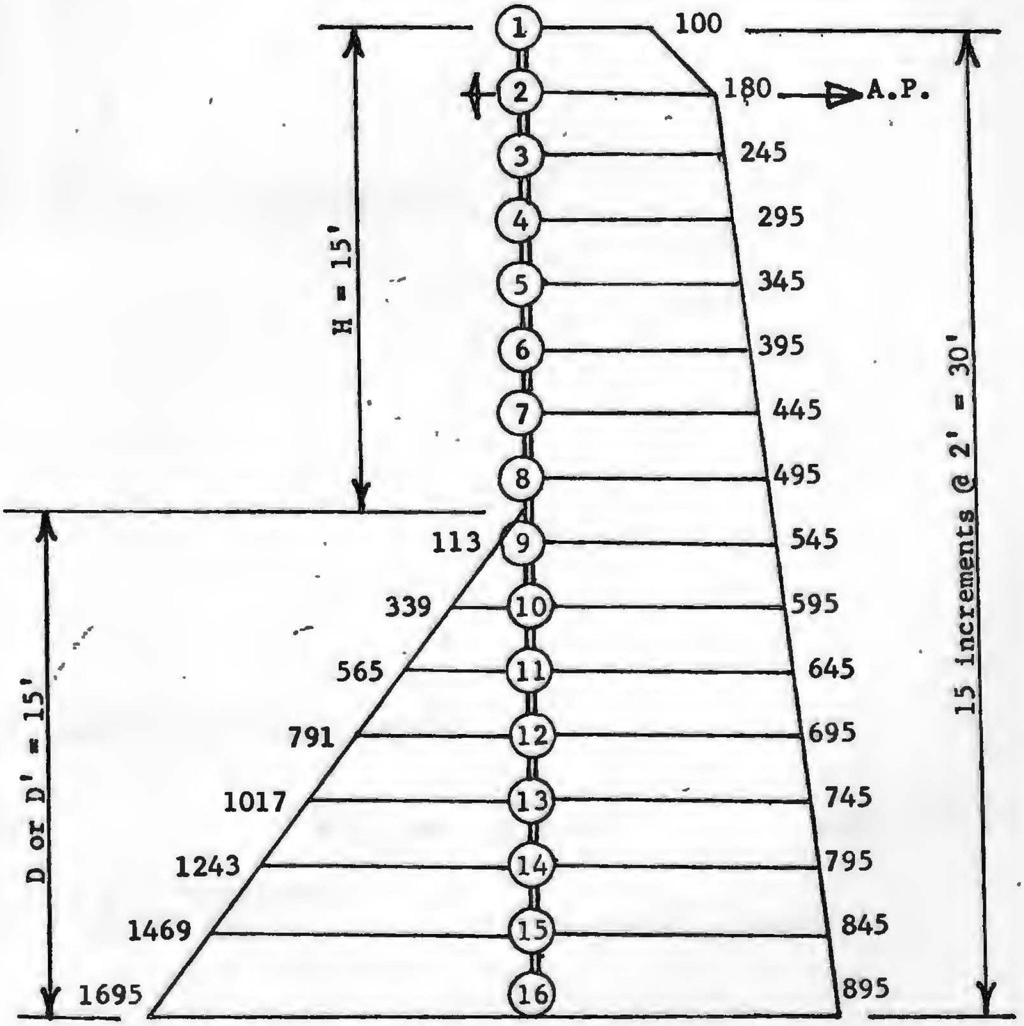
$PO = qb(Ka) = (300)(1)(0.333) = 100.0$ lb/ft

$P1 = b\gamma(3.0)(Ka) = (1)(120)(3)(0.333) = 120.0$ lb/ft

$Pa = b\gamma sub.(H+D-3)(Ka) = (1)(75)(12+D)(0.333) = 25(12+D)$ lb/ft

$Pp = \gamma sub.bD(Kp/F.S.) = (75)(1)D(3)/2 = 113.0(D)$ lb/ft

Assume an original embeddment depth of 15 feet. Use 2 foot increments. The loading ordinates at the joints are as follows:



Example Problem 1-A (continued).

Free earth support computer solution.

```
basic fre
INPUT ANCHOR POINT JOINT NUMBER.
? 2
INPUT ORIGINAL NUMBER OF JOINTS.
? 16
INPUT ACTIVE AND PASSIVE LOADING ORDINATES (LBS/FT)
? 100,0
? 180,0
? 245,0
? 295,0
? 345,0
? 395,0
? 445,0
? 495,0
? 545,113
? 595,339
? 645,565
? 695,791
? 745,1017
? 795,1243
? 845,1469
? 895,1695
INPUT INCREMENT LENGTHS (FT)
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
INPUT IMBEDDMENT DEPTH D (FT)
? 13

D= 13                    REL. M= 10.4185               REL. V(Z)= 10.8678

D SHOULD BE INCREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n
```

```
INPUT IMBEDDMENT DEPTH D (FT)
? 14

D= 14                REL. M= 10.8038          REL. V(Z)= 9.48223

D SHOULD BE DECREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n

INPUT IMBEDDMENT DEPTH D (FT)
? 13.25

D= 13.25             REL. M= 10.521           REL. V(Z)= 10.5556

D SHOULD BE INCREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n

INPUT IMBEDDMENT DEPTH D (FT)
? 13.27

D= 13.27             REL. M= 10.5291          REL. V(Z)= 10.5297

D SHOULD BE INCREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n

INPUT IMBEDDMENT DEPTH D (FT)
? 13.275

D= 13.275            REL. M= 10.5311          REL. V(Z)= 10.5232

D SHOULD BE DECREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n

INPUT IMBEDDMENT DEPTH D (FT)
? 13.272

D= 13.272            REL. M= 10.5299          REL. V(Z)= 10.5271

D SHOULD BE DECREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n
```

```
INPUT IMBEDDMENT DEPTH D (FT)
? 13.2705

D= 13.2705            REL. M= 10.5293            REL. V(Z)= 10.529

D SHOULD BE DECREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n

INPUT IMBEDDMENT DEPTH D (FT)
? 13.2704

D= 13.2704            REL. M= 10.5292            REL. V(Z)= 10.5292

D SHOULD BE DECREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? y

FOR IMBEDDMENT DEPTH (FT)= 13.2704
```

NODE NO.	JOINT LOAD (LB)	SHEAR (LB)	MOMENT
1	-126.667	0	0
2	3653.	-280.	-253.333
3	-485.	3303.	6799.33
4	-590.	2763	12882.
5	-690.	2123.	17784.7
6	-790.	1383.	21307.3
7	-890.	543.001	23250.
8	-952.333	-396.999	23412.7
9	-826.333	-1324.	21670.7
10	-512	-2012.	18276.
11	-160	-2348.	13857.3
12	192	-2332.	9118.67
13	544	-1964.	4764.01
14	896	-1244.	1497.34
15	650.766	-171.998	22.6807
16	86.5048	-6.09131E-02	-.725586

ANCHOR FORCE (LBS)= 4008.

Example Problem 1-A (continued).

Fixed earth support computer solution:

```
basic fix

INPUT ANCHOR POINT JOINT NUMBER
? 2
INPUT RELATIVE VALUES FOR E,I
? 30000000,50
INPUT ORIGINAL NUMBER OF JOINTS
? 16
INPUT ACTIVE AND PASSIVE LOADING ORDINATES (LBS/FT. OF WALL)
? 100,0
? 180,0
? 245,0
? 295,0
? 345,0
? 395,0
? 445,0
? 495,0
? 545,113
? 595,339
? 645,565
? 695,791
? 745,1017
? 795,1243
? 845,1469
? 895,1695
INPUT INCREMENT LENGTHS.
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
? 2
```

```
INPUT D PRIME (FT).
? 10

D PRIME = 10                    REL. DEFL. AT A.P. = .65822

D PRIME SHOULD BE INCREASED
SHALL I PRINT ALL JL, V, M, DEFL? (TYPE N FOR NO, Y FOR YES.)
? n


INPUT D PRIME (FT).
? 10.25

D PRIME = 10.25                 REL. DEFL. AT A.P. =-.507456

D PRIME SHOULD BE DECREASED
SHALL I PRINT ALL JL, V, M, DEFL? (TYPE N FOR NO, Y FOR YES.)
? n


INPUT D PRIME (FT).
? 10.15

D PRIME = 10.15                 REL. DEFL. AT A.P. =-2.74763E-02

D PRIME SHOULD BE DECREASED
SHALL I PRINT ALL JL, V, M, DEFL? (TYPE N FOR NO, Y FOR YES.)
? n


INPUT D PRIME (FT).
? 10.14

D PRIME = 10.14                 REL. DEFL. AT A.P. = 1.94519E-02

D PRIME SHOULD BE INCREASED
SHALL I PRINT ALL JL, V, M, DEFL? (TYPE N FOR NO, Y FOR YES.)
? n


INPUT D PRIME (FT).
? 10.145

D PRIME = 10.145                REL. DEFL. AT A.P. =-3.95447E-03

D PRIME SHOULD BE DECREASED
SHALL I PRINT ALL JL, V, M, DEFL? (TYPE N FOR NO, Y FOR YES.)
? n
```

```
INPUT D PRIME (FT).
? 10.144

D PRIME = 10.144               REL. DEFL. AT A.P. = 7.21037E-04

D PRIME SHOULD BE INCREASED
SHALL I PRINT ALL JL, V, M, DEFL? (TYPE N FOR NO, Y FOR YES.)
? y


FOR D PRIME (FT)= 10.144
```

JOINT NO.	JOINT LOAD (LBS)	SHEAR (LBS)	MOMENT (ft-lb)
1	-126.667	0	0
2	2690.09	-280.	-253.333
3	-485.	2340.09	4873.51
4	-590.	1800.09	9030.35
5	-690.	1160.09	12007.2
6	-790.	420.091	13604.
7	-890.	-419.909	13620.9
8	-952.333	-1359.91	11857.7
9	-826.333	-2286.91	8189.92
10	-512	-2974.91	2869.44
11	-160	-3310.91	-3475.04
12	192	-3294.91	-10139.5
13	388.112	-2926.91	-16420
14	193.973	-2558.16	-19568.4

```
ANCHOR FORCE (LBS)= 3045.09
```

```
INPUT REAL VALUES FOR E, I  (PSI & INCHES FOURTH)
? 29000000,50

FOR I = 50                 D PRIME = 10.144
```

JOINT NO.	REAL DEFLECTION (IN)
1	.211303
2	7.46727E-04
3	-.20888
4	-.39566
5	-.539862
6	-.627376
7	-.650668
8	-.609738
9	-.51304
10	-.377958
11	-.229604
12	-9.79425E-02
13	-1.44625E-02
14	0

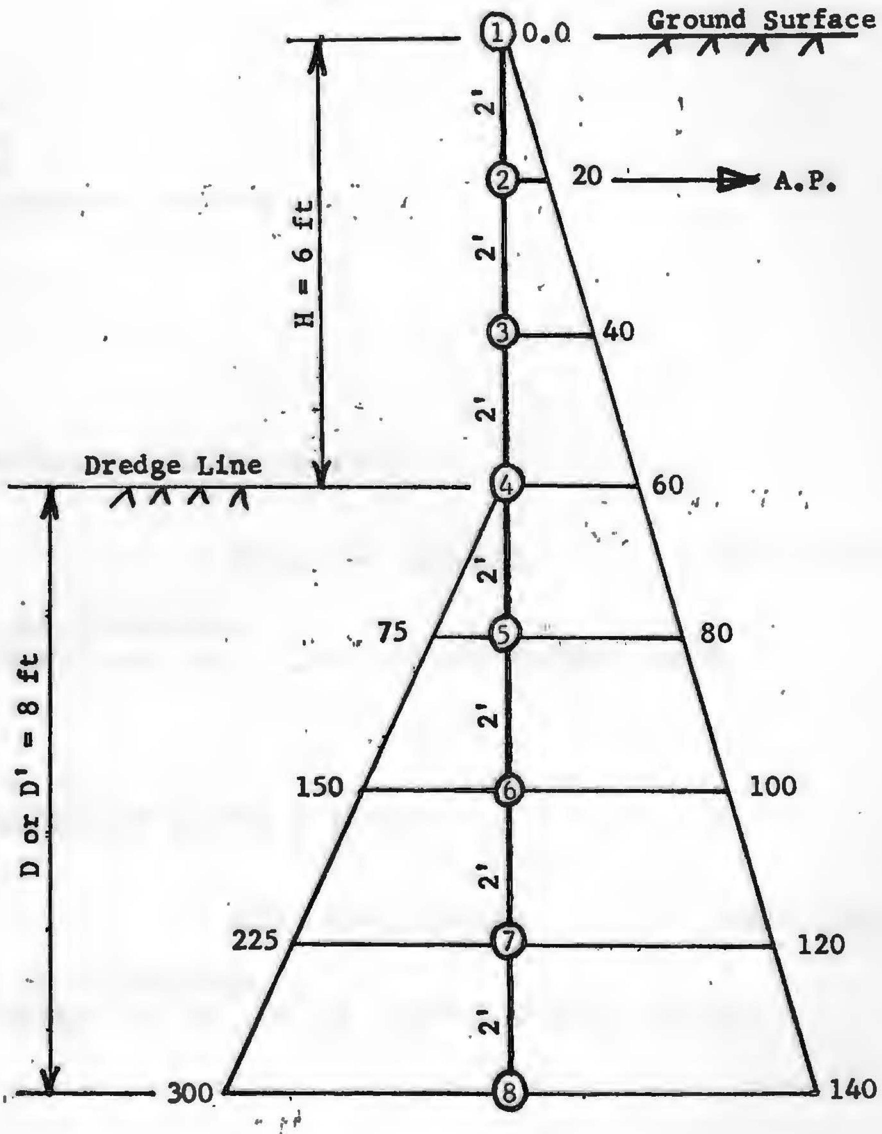
```
WOULD YOU LIKE TO TRY ANOTHER I???(TYPE YES OR NO)
? yes
INPUT I
? 25
FOR I = 25                D PRIME = 10.144

JOINT NO.           REAL DEFLECTION (IN)
 1                   .422608
 2                   1.49584E-03
 3                  -.417757
 4                  -.791318
 5                  -1.07972
 6                  -1.25475
 7                  -1.30133
 8                  -1.21947
 9                  -1.02608
 10                 -.755915
 11                 -.459208
 12                 -.195885
 13                 -2.89250E-02
 14                  0


WOULD YOU LIKE TO TRY ANOTHER I???(TYPE YES OR NO)
? no


DONT FORGET, YOUVE BEEN INPUTING D PRIME. THE REAL D=1.2(D')

FINAL IMBEDDMENT DEPTH (FT) = 1.2(D PRIME) = 12.1728
```

Example Problem 2-A.

Given: The anchored bulkhead of Example Problems 2-1 and 3-1. E=290,000 psi and $\underline{I}$ = 10 in^4/ft.

Find: Computer bulkhead design by both free and fixed earth support methods.

Solution: Use 2' increments.



Example Problem 2-A (continued).

Free earth support computer solution.

```
basic fre
INPUT ANCHOR POINT JOINT NUMBER.
? 2
INPUT ORIGINAL NUMBER OF JOINTS.
? 8
INPUT ACTIVE AND PASSIVE LOADING ORDINATES (LBS/FT)
? 0,0
? 20,0
? 40,0
? 60,0
? 80,75
? 100,150
? 120,225
? 140,300
INPUT INCREMENT LENGTHS (FT)
? 2
? 2
? 2
? 2
? 2
? 2
? 2
INPUT IMBEDDMENT DEPTH D (FT)
? 3

D= 3                    REL. M= .104625           REL. V(Z)= .165375

D SHOULD BE INCREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n


INPUT IMBEDDMENT DEPTH D (FT)
? 4

D= 4                    REL. M= .126667           REL. V(Z)= .16

D SHOULD BE INCREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n
```

```
INPUT IMBEDDMENT DEPTH D (FT)
? 5

D= 5                    REL. M= .143708            REL. V(Z)= .122625

D SHOULD BE DECREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n


INPUT IMBEDDMENT DEPTH D (FT)
? 4.5

D= 4.5                  REL. M= .135984            REL. V(Z)= .145828

D SHOULD BE INCREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n


INPUT IMBEDDMENT DEPTH D (FT)
? 4.6

D= 4.6                  REL. M= .137668            REL. V(Z)= .141943

D SHOULD BE INCREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n


INPUT IMBEDDMENT DEPTH D (FT)
? 4.75

D= 4.75                 REL. M= .140067            REL. V(Z)= .135419

D SHOULD BE DECREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n


INPUT IMBEDDMENT DEPTH D (FT)
? 4.6733

D= 4.6733               REL. M= .138859            REL. V(Z)= .138861

D SHOULD BE INCREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n
```

```
INPUT IMBEDDMENT DEPTH D (FT)
? 4.67335

D= 4.67335           REL. M= .13886           REL. V(Z)= .138858

D SHOULD BE DECREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? n


INPUT IMBEDDMENT DEPTH D (FT)
? 4.67334

D= 4.67334           REL. M= .13886           REL. V(Z)= .138859

D SHOULD BE DECREASED.
SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).
? y


FOR IMBEDDMENT DEPTH (FT)= 4.67334
NODE NO.          JOINT LOAD (LB)   SHEAR (LB)        MOMENT
 1                -6.66667           0                 0
 2                 120.098          -20.              -13.3333
 3                -80.               80.0985           213.53
 4                -95               -19.9015           280.394
 5                -10.              -84.9015           157.257
 6                 50.5782          -39.9015           14.1211
 7                 20.9895          -4.42505E-04      -1.22824E-02

ANCHOR FORCE (LBS)= 160.098
```

Example Problem 2-A (continued).

Fixed earth support computer solution.

```
basic fix


INPUT ANCHOR POINT JOINT NUMBER
? 2
INPUT RELATIVE VALUES FOR E,I
? 30000000,5
INPUT ORIGINAL NUMBER OF JOINTS
? 8
INPUT ACTIVE AND PASSIVE LOADING ORDINATES (LBS/FT. OF WALL)
? 0,0
? 20,0
? 40,0
? 60,0
? 80,75
? 100,150
? 120,225
? 140,300
INPUT INCREMENT LENGTHS.
? 2
? 2
? 2
? 2
? 2
? 2
? 2


INPUT D PRIME (FT).
? 4

D PRIME = 4                   REL. DEFL. AT A.P. =-.120105

D PRIME SHOULD BE DECREASED
SHALL I PRINT ALL JL, V, M, DEFL? (TYPE N FOR NO, Y FOR YES.)
? n


INPUT D PRIME (FT).
? 3.5

D PRIME = 3.5                 REL. DEFL. AT A.P. =-5.17007E-03

D PRIME SHOULD BE DECREASED
SHALL I PRINT ALL JL, V, M, DEFL? (TYPE N FOR NO, Y FOR YES.)
? n
```

```
INPUT D PRIME (FT).
? 3.45

D PRIME = 3.45                    REL. DEFL. AT A.P. = 3.25252E-03

D PRIME SHOULD BE INCREASED
SHALL I PRINT ALL JL, V, M, DEFL? (TYPE N FOR NO, Y FOR YES.)
? n


INPUT D PRIME (FT).
? 3.46

D PRIME = 3.46                    REL. DEFL. AT A.P. = 1.60715E-03

D PRIME SHOULD BE INCREASED
SHALL I PRINT ALL JL, V, M, DEFL? (TYPE N FOR NO, Y FOR YES.)
? n


INPUT D PRIME (FT).
? 3.467

D PRIME = 3.467                   REL. DEFL. AT A.P. = 4.43920E-04

D PRIME SHOULD BE INCREASED
SHALL I PRINT ALL JL, V, M, DEFL? (TYPE N FOR NO, Y FOR YES.)
? y


FOR D PRIME (FT)= 3.467

JOINT NO.         JOINT LOAD (LBS)  SHEAR (LBS)       MOMENT
 1                -6.66667           0                 0
 2                 83.5775          -20.              -13.3333
 3                -80.               43.5775           140.488
 4                -95               -56.4225           134.31
 5                -17.1371          -121.422          -61.8683
 6                 16.06            -99.1662          -230.905

ANCHOR FORCE (LBS)= 123.577
```

```
INPUT REAL VALUES FOR E, I  (PSI & INCHES FOURTH)
? 290000,10

FOR I = 10                    D PRIME = 3.467

JOINT NO.             REAL DEFLECTION (IN)
 1                     .365138
 2                     2.29619E-02
 3                    -.317793
 4                    -.355481
 5                    -.110786
 6                     0



WOULD YOU LIKE TO TRY ANOTHER I???(TYPE YES OR NO)
? yes
INPUT I
? 500
FOR I = 5                     D PRIME = 3.467

JOINT NO.             REAL DEFLECTION (IN)
 1                     .730275
 2                     4.59227E-02
 3                    -.635587
 4                    -.710962
 5                    -.221572
 6                     0


WOULD YOU LIKE TO TRY ANOTHER I???(TYPE YES OR NO)
? no

DONT FORGET, YOUVE BEEN INPUTING D PRIME. THE REAL D=1.2(D PRIME)

FINAL IMBEDDMENT DEPTH (FT) = 1.2(D PRIME) = 4.1604
```

APPENDIX B

FRE and FIX Computer Program Listings

<u>FRE Listing.</u>

```
type fre basic

10 PRINT 'INPUT ANCHOR POINT JOINT NUMBER.'
20 INPUT Q9
30 A$='N'
40 DIMA(30),C(30),P(30),U(30),H(30),L(30),R(30),J(30),X(30),Y(30),S(30),I(30)
50 DIM M(30),V(30),E(30)
60 PRINT'INPUT ORIGINAL NUMBER OF JOINTS.'
70 INPUT W
80 PRINT 'INPUT ACTIVE AND PASSIVE LOADING ORDINATES (LBS/FT)'
90 FOR N=1 TO W
100 INPUT A(N),P(N)
110 NEXT N
120 PRINT 'INPUT INCREMENT LENGTHS (FT) '
130 FOR N=1 TO W-1
140 INPUT H(N)
150 NEXT N
160 D1=(P(W)*H(W-1))/(P(W)-P(W-1))
170 T1=0
180 FOR N=1 TO W-1
190 T1=T1+H(N)
200 NEXT N
210 G=T1-D1
220 IF Q9=1THEN250
230 IF Q9=2THEN270
240 IF Q9>2THEN290
250 Q1=0
```

```
260 GOTO330
270 Q1=H(1)
280 GOTO330
290 Q1=0
300 FOR N=1 TO Q9-1
310 Q1=Q1+H(N)
320 NEXT N
330 PRINT 'INPUT IMBEDDMENT DEPTH D (FT)'
340 INPUT D2
350 IF D2=0 THEN 1650
360 IF D2=D1 THEN 390
370 IF D2>D1 THEN 1160
380 IF D2<D1 THEN 1280
390 Z=W
400 FOR N=1 TO Z
410 C(N)=A(N)
420 U(N)=P(N)
430 NEXT N
440 FOR N=1 TO Z-1
450 E(N)=H(N)
460 NEXT N
470 L(1)=0
480 X(1)=0
490 R(Z)=0
500 Y(Z)=0
510 FOR N=1 TO Z-1
520 R(N)=(E(N)/6)*(2*C(N)+C(N+1))*(-1)
530 Y(N)=(E(N)/6)*(2*U(N)+U(N+1))
540 NEXT N
550 FOR N= 2 TO Z
560 L(N)=(E(N-1)/6)*(2*C(N)+C(N-1))*(-1)
570 X(N)=(E(N-1)/6)*(2*U(N)+U(N-1))
580 NEXT N
590 FOR N=1 TO Z
600 J(N)=L(N)+R(N)+X(N)+Y(N)
610 NEXT N
```

```
620 S(1)=J(1)
630 FOR N=2 TO Z-1
640 S(N)=S(N-1)+J(N)
650 NEXT N
660 FOR N=1 TO Z-1
670 I(N)=S(N)*E(N)
680 NEXT N
690 M(1)=0
700 FOR N=2 TO Z
710 M(N)=M(N-1)+I(N-1)
720 NEXT N
730 V(1)=L(1)+X(1)
740 FOR N=2 TO Z
750 V(N)=V(N-1)+R(N-1)+Y(N-1)+L(N)+X(N)
760 NEXT N
770 T=0
780 FOR N=1 TO Z-1
790 T=T+E(N)
800 NEXT N
810 Q2=T-01
820 IF A$ <> 'N' THEN 960
830 PRINT
840 PRINT'D=';D2,'REL. M=';(-1)*M(Z)/10000,'REL. V(Z)=';(-1)*V(Z)*Q2/10000
850 PRINT
860 IF ABS(M(Z))<ABS(V(Z)*Q2) THEN 890
870 PRINT 'D SHOULD BE DECREASED.'
880 GO TO 900
890 PRINT 'D SHOULD BE INCREASED.'
900 PRINT 'SHALL I PRINT ALL JL, V, M  (TYPE Y=YES, N=NO).'
910 INPUT A$
920 IF A$ = 'N' THEN 1040
930 R(Q9)=R(Q9)+(V(Z)*(-1))
940 V=V(Z)
950 GO TO 590
960 PRINT
970 PRINT
```

```
980 PRINT 'FOR IMBEDDMENT DEPTH (FT)=';D2
990 PRINT 'NODE NO.','JOINT LOAD (LB)','SHEAR (LB)','MOMENT (LB FT)'
1000 FOR N =1 TO Z
1010 PRINT N,J(N),V(N),M(N)
1020 NEXT N
1030 GO TO 1620
1040 PRINT
1050 PRINT
1060 PRINT
1070 FOR N=1 TO W
1080 C(N)=A(N)
1090 U(N)=P(N)
1100 NEXT N
1110 FOR N=1 TO W-1
1120 E(N)=H(N)
1130 NEXT N
1140 Z=W
1150 GOTO330
1160 Z=W+1
1170 E(Z-1)=D2-D1
1180 C(Z)=((A(W)-A(W-1))*(E(Z-1)+H(W-1))/H(W-1))+A(W-1)
1190 U(Z)=((P(W)-P(W-1))*(E(Z-1)+H(W-1))/H(W-1))+P(W-1)
1200 FOR N=1 TO Z-1
1210 C(N)=A(N)
1220 U(N)=P(N)
1230 NEXT N
1240 FOR N=1 TO Z-2
1250 E(N)=H(N)
1260 NEXT N
1270 GOTO470
1280 T=0
1290 FOR N=1 TO W-1
1300 T=T+H(N)
1310 IF T=G+D2THEN1340
1320 IF T>G+D2THEN1460
1330 NEXT N
```

```
1340 Z=N+1
1350 E(Z-1)=H(Z-1)
1360 C(Z)=A(Z)
1370 U(Z)=P(Z)
1380 FOR N=1 TO Z-1
1390 C(N)=A(N)
1400 U(N)=P(N)
1410 NEXT N
1420 FOR N=1 TO Z-2
1430 E(N)=H(N)
1440 NEXT N
1450 GOTO470
1460 Z=N+1
1470 W1=0
1480 FOR N=1 TO Z-2
1490 W1=W1+H(N)
1500 NEXT N
1510 E(Z-1)=G+D2-W1
1520 FOR N=1 TO Z-1
1530 C(N)=A(N)
1540 U(N)=P(N)
1550 NEXT N
1560 FOR N=1 TO Z-2
1570 E(N)=H(N)
1580 NEXT N
1590 C(Z)=((A(Z-1)-A(Z-2))*(H(Z-2)+E(Z-1))/H(Z-2))+A(Z-2)
1600 U(Z)=((P(Z-1)-P(Z-2))*(H(Z-2)+E(Z-1))/H(Z-2))+P(Z-2)
1610 GOTO470
1620 PRINT
1630 PRINT 'ANCHOR FORCE (LBS)=';V*(-1)
1640 PRINT
1650 END

R; T=0.86/4.08 14·22·00
```

FIX Listing.

```
type fix basic

  10 PRINT
  20 PRINT
  30 PRINT 'INPUT ANCHOR POINT JOINT NUMBER'
  40 INPUT Q9
  50 PRINT 'INPUT RELATIVE VALUES FOR E,I'
  60 INPUT B2,B3
  70 DIM A(30),P(30),H(30),C(30),U(30),E(30),L(30),X(30),R(30),Y(30)
  80 DIM B(30),F(30),K(30),O(30),D(30),G(30),Q(30),T(30)
  90 DIM V(30),J(30),S(30),I(30),M(30)
 100 P$='N'
 110 A$='NO'
 120 PRINT 'INPUT ORIGINAL NUMBER OF JOINTS'
 130 INPUT W
 140 PRINT 'INPUT ACTIVE AND PASSIVE LOADING ORDINATES (LBS/FT. OF WALL)'
 150 FOR N=1 TO W
 160 INPUT A(N),P(N)
 170 NEXT N
 180 PRINT 'INPUT INCREMENT LENGTHS.'
 190 FOR N=1 TO W-1
 200 INPUT H(N)
 210 NEXT N
 220 D1=(P(W)*H(W-1))/(P(W)-P(W-1))
 230 T1=0
 240 FOR N=1 TO W-1
 250 T1=T1+H(N)
 260 NEXT N
 270 G=T1-D1
 280 PRINT
 290 PRINT
```

```
300 PRINT 'INPUT D PRIME (FT).'
310 INPUT D2
320 IF D2= 0 THEN 2280
330 IF D2=D1 THEN 360
340 IF D2>D1 THEN 1740
350 IF D2<D1 THEN 1860
360 Z=W
370 FOR N=1 TO Z
380 C(N)=A(N)
390 U(N)=P(N)
400 NEXT N
410 FOR N=1 TO Z-1
420 E(N)=H(N)
430 NEXT N
440 Q8=((U(Z)-U(Z-1))*(E(Z-1)+(.2*D2))/E(Z-1))+U(Z-1)
450 Q7=((U(Z)+Q8)/2)*(.2*D2)
460 L(1)=0
470 X(1)=0
480 R(Z)=0
490 Y(Z)=0
500 FOR N=1 TO Z-1
510 R(N)=(E(N)/6)*(2*C(N)+C(N+1))*(-1)
520 Y(N)=(E(N)/6)*(2*U(N)+U(N+1))
530 NEXT N
540 FOR N=2 TO Z
550 L(N)=(E(N-1)/6)*(2*C(N)+C(N-1))*(-1)
560 X(N)=(E(N-1)/6)*(2*U(N)+U(N-1))
570 NEXT N
580 V(1)=L(1)+X(1)
590 FOR N=2 TO Z
600 V(N)=V(N-1)+R(N-1)+Y(N-1)+L(N)+X(N)
610 NEXT N
620 Q6=(Q7+V(Z))*(-1)
630 R(Q9)=R(Q9)+Q6
640 FOR N=1 TO Z
650 J(N)=L(N)+R(N)+X(N)+Y(N)
```

```
660 NEXT N
670 S(1)=J(1)
680 FOR N=2 TO Z-1
690 S(N)=S(N-1)+J(N)
700 NEXT N
710 FOR N=1 TO Z-1
720 I(N)=S(N)*E(N)
730 NEXT N
740 M(1)=0
750 FOR N=2 TO Z
760 M(N)=M(N-1)+I(N-1)
770 NEXT N
780 B(1)=L(1)+X(1)
790 FOR N=2 TO Z
800 B(N)=B(N-1)+R(N-1)+Y(N-1)+L(N)+X(N)
810 NEXT N
820 REM B IS REAL SHEAR TO INCLUDE ANCHOR FORCE
830 REM B1=S.M., B2=E, B3=I
840 F(1)=0
850 K(Z)=0
860 FOR N=1 TO Z
870 O(N)=(M(N)*12)/(B2*B3)
880 NEXT N
890 REM O IS M/EI
900 R=E(1)/E(2)
910 K(1)=E(1)*((((1/(1+R))+3)*O(1))+((R+2)*O(2))-(((1/(1+R))+R-1)*O(3)))
920 R=E(Z-1)/E(Z-2)
930 F(Z)=E(Z-1)*((((1/(1+R))+3)*O(Z))+((R+2)*O(Z-1))-(((1/(1+R))+R-1)*O(Z-2)))
940 FOR N=2 TO Z-1
950 R=E(N-1)/E(N)
960 F(N)=E(N-1)*((((1/(1+R))+1)*O(N-1))+((R+4)*O(N))-(((1/(1+R))+R-1)*O(N+1)))
970 R=E(N)/E(N-1)
980 K(N)=E(N)*((((1/(1+R))+1)*O(N+1))+((R+4)*O(N))-(((1/(1+R))+R-1)*O(N-1)))
990 NEXT N
1000 FOR N=Z TO 1 STEP -1
1010 D(N)=F(N)+K(N)
```

```
1020 NEXT N
1030 G(Z-1)=D(Z)
1040 FOR N=Z-2 TO 1 STEP -1
1050 G(N)=G(N+1)+D(N+1)
1060 NEXT N
1070 REM D AND G ARE J.L. AND AV.V. FOR CONJ. BEAM
1080 FOR N= Z-1 TO 1 STEP -1
1090 Q(N)=G(N)*E(N)*12
1100 NEXT N
1110 REM Q IS M.I. FOR CONJ. BEAM
1120 T(Z)=0
1130 FOR N= Z-1 TO 1 STEP -1
1140 T(N)=T(N+1)+Q(N)
1150 NEXT N
1160 REM T IS DEFLECTION OF REAL BEAM
1170 IF A$<>'NO' THEN 1470
1180 IF P$ <> 'N' THEN 1470
1190 PRINT
1200 PRINT 'D PRIME =';D2,'REL. DEFL. AT A.P. =';T(Q9)
1210 PRINT
1220 IF T(Q9)<0 THEN 1250
1230 PRINT 'D PRIME SHOULD BE INCREASED'
1240 GO TO 1260
1250 PRINT 'D PRIME SHOULD BE DECREASED'
1260 PRINT 'SHALL I PRINT ALL JL, V, M, DEFL? (TYPE N FOR NO, Y FOR YES.)'
1270 INPUT P$
1280 IF P$='N' THEN 1650
1290 PRINT
1300 PRINT
1310 PRINT
1320 PRINT 'FOR D PRIME (FT)=';D2
1330 PRINT
1340 PRINT 'JOINT NO.','JOINT LOAD (LBS)','SHEAR (LBS)','MOMENT (FT-LB)'
1350 FOR N=1 TO Z
1360 PRINT N, J(N),B(N),M(N)
1370 NEXT N
```

```
1380 PRINT
1390 PRINT 'ANCHOR FORCE (LBS)=';Q6
1400 PRINT
1410 PRINT'____________________________________________________________'
1420 PRINT
1430 PRINT 'INPUT REAL VALUES FOR E, I  (PSI & INCHES FOURTH)'
1440 INPUT B2,B3
1450 PRINT
1460 GO TO 860
1470 PRINT 'FOR I =';B3,'D PRIME =';D2
1480 PRINT
1490 PRINT 'JOINT NO.','REAL DEFLECTION (IN)'
1500 FOR N=1 TO Z
1510 PRINT N,T(N)
1520 NEXT N
1530 PRINT
1540 PRINT
1550 PRINT
1560 IF A$<>'NO' THEN 1580
1570 PRINT
1580 PRINT 'WOULD YOU LIKE TO TRY ANOTHER I???(TYPE YES OR NO)'
1590 INPUT A$
1600 IF A$='NO'THEN 2200
1610 PRINT 'INPUT I'
1620 INPUT B3
1630 GO TO 860
1640 REM T IS DEFLECTION
1650 FOR N=1 TO W
1660 C(N)=A(N)
1670 U(N)=P(N)
1680 NEXT N
1690 FOR N=1 TO W-1
1700 E(N)=H(N)
1710 NEXT N
1720 Z=W
1730 GO TO 280
```

```
1740 Z=W+1
1750 E(Z-1)=D2-D1
1760 C(Z)=((A(W)-A(W-1))*(E(Z-1)+H(W-1))/H(W-1))+A(W-1)
1770 U(Z)=((P(W)-P(W-1))*(E(Z-1)+H(W-1))/H(W-1))+P(W-1)
1780 FOR N=1 TO Z-1
1790 C(N)=A(N)
1800 U(N)=P(N)
1810 NEXT N
1820 FOR N=1 TO Z-2
1830 E(N)=H(N)
1840 NEXT N
1850 GO TO 440
1860 T=0
1870 FOR N=1 TO W-1
1880 T=T+H(N)
1890 IF T=G+D2 THEN 1920
1900 IF T>G+D2 THEN 2040
1910 NEXT N
1920 Z=N+1
1930 E(Z-1)=H(Z-1)
1940 C(Z)=A(Z)
1950 U(Z)=P(Z)
1960 FOR N=1 TO Z-1
1970 C(N)=A(N)
1980 U(N)=P(N)
1990 NEXT N
2000 FOR N=1 TO Z-2
2010 E(N)=H(N)
2020 NEXT N
2030 GO TO 440
2040 Z=N+1
2050 W1=0
2060 FOR N=1 TO Z-2
2070 W1=W1+H(N)
2080 NEXT N
```

```
2090 E(Z-1)=G+D2-W1
2100 FOR N=1 TO Z-1
2110 C(N)=A(N)
2120 U(N)=P(N)
2130 NEXT N
2140 FOR N=1 TO Z-2
2150 E(N)=H(N)
2160 NEXT N
2170 C(Z)=((A(Z-1)-A(Z-2))*(H(Z-2)+E(Z-1))/H(Z-2))+A(Z-2)
2180 U(Z)=((P(Z-1)-P(Z-2))*(H(Z-2)+E(Z-1))/H(Z-2))+P(Z-2)
2190 GO TO 440
2200 PRINT
2210 PRINT
2220 PRINT'DONT FORGET, YOUVE BEEN INPUTING D PRIME. THE REAL D=1.2(D PRIME)'
2230 D3=1.2*D2
2240 PRINT
2250 PRINT 'FINAL IMBEDDMENT DEPTH (FT) = 1.2(D PRIME) =';D3
2260 PRINT
2270 PRINT
2280 END

R; T=1.12/4.47 12:07:21
```

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