ELECTROFLUIDMECHANICS

by

M. F. Italiano, Jr.

Submitted in Partial Fulfillment of the Requirements
for the Degree of
Master of Science in Engineering
in the
Mechanical Engineering
Program

Edwin R Pejack 3 June 77
Advisor Date

Leon Rand August 12, 1977
Dean of the Graduate School Date

YOUNGSTOWN STATE UNIVERSITY

August 27, 1977

ABSTRACT

ELECTROFLUIDMECHANICS

M. F. Italiano, Jr.
Master of Science in Engineering
Youngstown State University, 1977

The fundamental interactions of electrofluidmechanics are explored. A review of the necessary elements of electrical theory along with the specific effects of electric fields on fluids is presented. A review of the research is also undertaken for some specific processes affected by fields.

An unsuccessful attempt to measure the body force produced by a specific electrode geometry is presented. The use of high AC fields and very small pressures generated by the body force hampered the investigation.

ACKNOWLEDGEMENTS

I would like to thank Dr. E. R. Pejack for suggesting the topic of this thesis.

I would also like to thank my wife, Linda, for her encouragement and patience, and Michele Raidel for typing the rough and final drafts.

TABLE OF CONTENTS

PAGE

CHAPTER

PAGE

LIST OF FIGURES

FIGURE PAGE

CHAPTER I

INTRODUCTION

The purpose of this thesis is to explore the fundamental interactions between electric fields and fluids, known as, electrofluidmechanics. These interactions prove of consequence in such areas as fluid flow, heat transfer and combustion. One such phenomena is investigated experimentally. Although the experimental aspect proved unsuccessful, it provided insight as to the nature and complexity of electrofluidmechanics.

It is known that an electric field will influence both charged and uncharged particles. This influence leads to a controllable body force acting on the fluid as a whole. Along with this body force, properties of fluids such as viscosity can be appreciably affected. The changing of properties, of course, leads to changes in fluid flow behavior, boiling, heat transfer and thermodynamic properties.

The fact that electric fields exert some type of influence on fluids has been known for over one hundred years. It has only been during the last thirty years that researchers have had success explaining and putting to use the forces involved.

A review of the necessary elements of electrical theory is presented to provide a background for a study of electrofluidmechanics. A survey covering some of the many interactions involved is undertaken. Descriptions, explanations and theories are presented when possible.

CHAPTER II

ELECTRICITY REVIEW

Basic Concepts

For a better understanding of the material to follow, some fundamental concepts of electricity will be reviewed. This is not intended to cover all topics of electricity, but rather those necessary to provide background for discussing electrofluidmechanics.

It was Charles Augustin de Coulomb that first measured electrical attractions and repulsions quantitatively in 1785. The familiar law bearing his name for point charges is:

$$\vec{F} = (q_1 q_2 / 4\pi\epsilon_0 r^2)\vec{n} . \quad (1)$$

Where:

$\vec{F}$ = electric force (newtons)

ϵ_0 = permittivity (farad/meter)

q_1, q_2 = charge (coulomb)

r = distance between charges (meter)

$\vec{n}$ = unit vector

A coulomb is defined as the amount of charge that flows through a given cross section of wire in one second when there is a steady current of one ampere in the wire, Thus:

$$q = it .$$

Where:

i = current (ampere)

t = time (seconds)

Following from Coulomb's Law is the definition of the electric field. By placing a test charge, q, at a point in space and measuring the electric force that acts upon it, the electric field strength, $\vec{E}$, at the point is defined as:

$$\vec{E} = \vec{F}/q. \tag{2}$$

Thus the force on a charge in an electric field is $\vec{F} = q\vec{E}$. Since this is a vector equation the force is in the same direction as the field.

Thinking of the electric field in terms of lines of force rather than as a vector is a convenient way of visualizing electric field patterns. The relationship between these two concepts is that at any instant in time, $\vec{E}$ is tangent to and in the direction of the lines of force. The density of the lines of force is proportional to the magnitude of $\vec{E}$. Thus the lines of force give a physical significance to the electric field and helps one to visualize the effects of the field on its surroundings. As an example, consider a uniform field such as that found between a parallel plate capacitor as shown in figure 1.

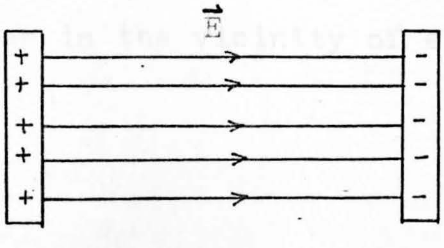


Fig. 1. Parallel Plate Capacitor

This simple system represents an electric field that is in a single direction; and, since the lines of force are equally spaced, of constant magnitude. Note that the lines of force are directed from positive to negative for convention.

Intimately related to the electric field strength is the electric potential difference, V. The electric potential is fundamentally defined as:

$$V_B - V_A = W_{AB}/q_o \, . \tag{3}$$

This equation states that a test charge q_o is moved from point A to point B being kept at equilibrium at all times. The work measured in moving the test charge from A to B is W_{AB}. Thus the potential difference between A and B is the work divided by the charge.

The electric field strength can be determined from the potential by the use of:

$$\vec{E} = -\vec{\nabla} V \, . \tag{4}$$

Now consider the electric dipole. The interaction of the dipole and the electric field is of fundamental importance in electrofluidmechanics. Two equal charges, of opposite sign, separated by a distance 2a define a dipole. Figure 2 illustrates the nonuniform field associated with the dipole. As can be seen the lines of force are not uniform and are much closer together in the vicinity of either charge.

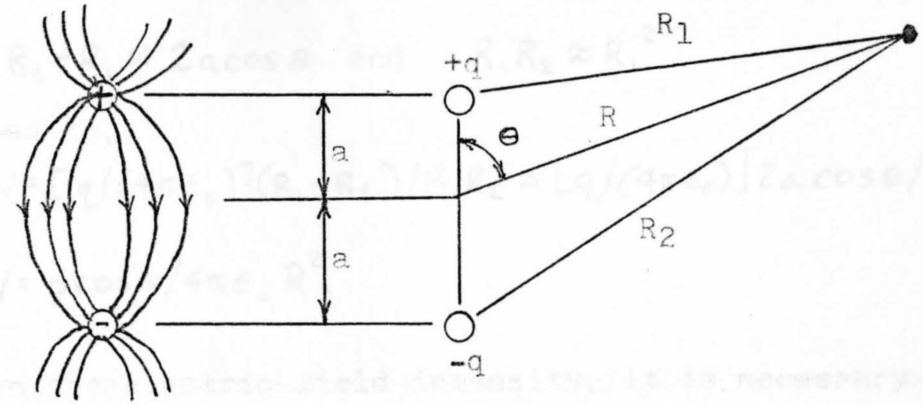


Fig. 2. Dipole

Thus the electric field strength increases near either charge.

The dipole has a moment associated with it equal to the product of the charge and the distance separating them. Thus the dipole moment is $p = 2aq$. There is also an electric potential, V, at any point in space due to the dipole. The potential at some point in space resulting from a group of point charges can be found using the following relation:

$$V = \sum_n V_n = (1/4\pi\epsilon_0)\sum_n q_n / R_n \quad , \tag{5}$$

where V_n is the potential due to each charge separately, q_n is the value of the n^{th} charge and R_n is the distance from the point in question to the charge.

Using this to determine the potential due to the dipole at point M in figure 2:

$$V = \sum_n V_n = V_1 + V_2 = (1/4\pi\epsilon_0)(q/R_1 - q/R_2).$$

Assuming point M to be far from the dipole, or $R \gg 2_a$, leads to the approximate relations:

$$R_2 - R_1 \approx 2a\cos\theta \quad \text{and} \quad R_1R_2 \approx R_1^2.$$

These lead to,

$$V = [q/(4\pi\epsilon_0)](R_1 - R_2)/R_1R_2 \approx [q/(4\pi\epsilon_0)]\, 2a\cos\theta/R^2$$

and

$$V = p\cos\theta/4\pi\epsilon_0 R^2.$$

To obtain the electric field intensity, it is necessary to evaluate the gradient of the potential:

$$\vec{E} = -\vec{\nabla} V.$$

Using spherical coordinates:

$$\vec{\nabla} V = \frac{\partial r}{\partial R}\vec{r} + \frac{1}{R}\frac{\partial v}{\partial \theta}\vec{\theta} + \frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}\vec{\phi},$$

$\vec{r}, \vec{\theta}$ & $\vec{\phi}$ being unit vectors.

Evaluating the partial derivatives gives,

$$-\vec{\nabla} V = \frac{2p\cos\theta}{4\pi\epsilon_0 R^3} + \frac{1}{R}\frac{p\sin\theta}{4\pi\epsilon_0 R^2}$$

Thus,

$$\vec{E} = \frac{p}{4\pi\epsilon_0 R^3}(2\cos\theta\,\vec{r} + \sin\theta\,\vec{\theta}).$$

The dipole then has an electric potential and field intensity associated with it. It is noted that the potential varies inversely as the square of the distance from the dipole while the field intensity varies inversely as the cube of this same distance.

When a dipole is under the influence of a uniform external electric field the result is that it will orient itself in the direction of the field.

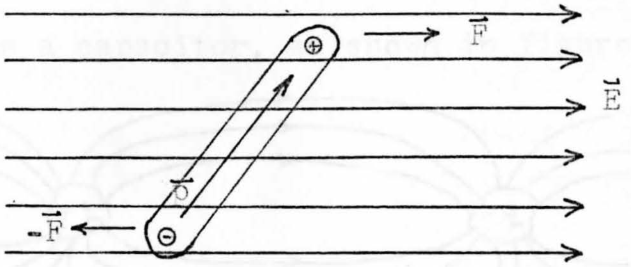


Fig. 3. Dipole in an Electric Field

As seen in figure 3, using vector notation the dipole experiences a torque tending to align it with the field. This torque is:

$$\vec{\tau} = \vec{p} \times \vec{E} \,, \tag{6}$$

$\vec{p}$ being the dipole moment.

As long as a collection of charges behaves like and exhibits the properties of a dipole, it can be treated as such. In this vein, many molecules have electric dipole moments. Molecules and atoms not having permanent dipoles can be induced to exhibit dipole behavior by placing them in an external electric field. This is known as polarization, the field tending to separate the negative and positive charge. This induced electric dipole moment is present only when the electric field is applied, and is proportional to the field strength.

Capacitance

Two insulated conductors carrying equal and opposite charges define a capacitor, as shown in figure 4.

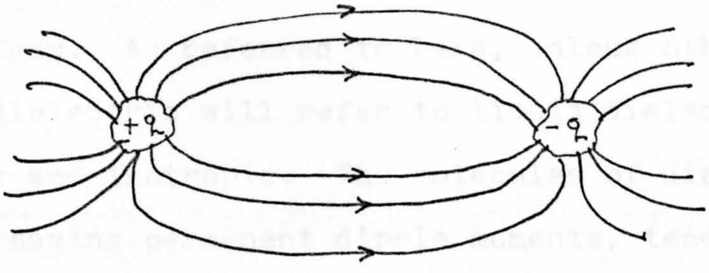


Fig. 4. Capacitor

The capacitance, C, of any capacitor in equation form is,

$$C = q/V. \tag{7}$$

Where q is the charge on either insulated conductor and V is the potential difference between them.

Consider the parallel plate capacitor shown below.

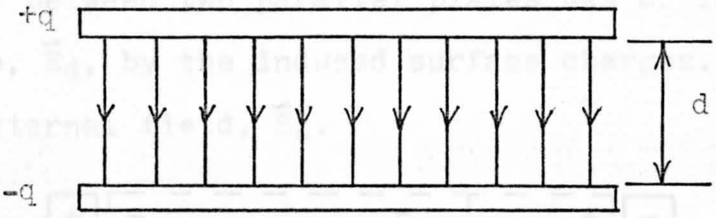


Fig. 5. Parallel Plate Capacitor

From Gauss' Law the charge on either plate is $q = \epsilon_0 EA$, A being the area of one of the plates. The voltage is $V = Ed$.

Thus the capacitance is:

$$C = \epsilon_0 EA/Ed = \epsilon_0 A/d. \tag{8}$$

Each capacitor of different geometry will have a different relation describing its capacitance.

Equation 8 is valid only when the capacitor is in a vacuum. When a dielectric is placed in the field of a capacitor, the capacitance increases. A dielectric is generally considered any insulator. Thus, it can be a solid, liquid or gas. As referred to here, unless otherwise stated, the term dielectric will refer to liquid dielectrics which are linear and isotropic. The molecules of dielectrics, such as water, having permanent dipole moments, tend to align themselves with an external field. The degree of alignment will increase as the field strength increases or as the temperature decreases. Dielectrics having molecules without permanent dipole moments acquire them by induction upon being exposed to such a field. When a dielectric is placed in an electric field, induced surface charges form which weaken the original field within the dielectric. Referring to figure 6, a dielectric between two parallel plates has an internal field set up, $\vec{E}_d$, by the induced surface charges. This opposes the external field, $\vec{E}_o$.

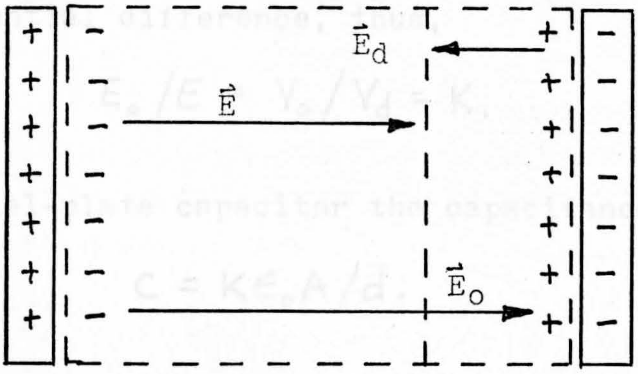


Fig. 6. Dielectric in a Uniform Electric Field

The resultant field in the dielectric is then $\vec{E}_o - \vec{E}_d = \vec{E}$.

The ratio of the external field to the resultant field is the dielectric constant, K:

$$K = E_o/E$$

The weakening of the electric field by the dielectric is evident when measuring the potential between two parallel-plate capacitors, one capacitor having no dielectric present and the other with a dielectric of constant K. This is illustrated in Figure 7,

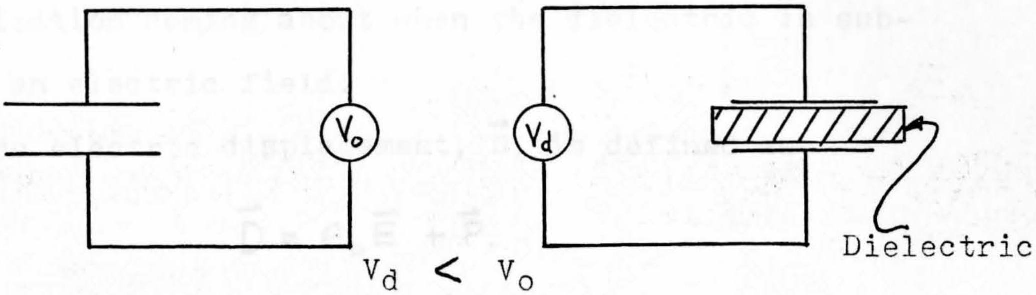


Fig. 7. Two Identical Parallel-plate Capacitors, except for the presence of the dielectric.

where it is shown that the introduction of a dielectric reduces the potential difference, thus,

$$E_o/E = V_o/V_d = K.$$

For the parallel-plate capacitor the capacitance becomes,

$$C = K\epsilon_o A/d.$$

Equation 8 is a special case of equation 11 where K = 1 for a vacuum.

In general, experiment shows that capacitance of all types of capacitors will increase when a dielectric is present, leading to a general equation governing capacitance of any capacitor,

$$C = K\epsilon_0 L, \tag{12}$$

where L is dependent on the geometric configuration of the capacitor and has dimensions of length.

The polarization of a dielectric, which is the dipole moment per unit volume, is defined as,

$$\vec{P} = (2aq/v)\vec{n}. \tag{13}$$

The polarization coming about when the dielectric is subjected to an electric field.

The electric displacement, $\vec{D}$, is defined as,

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}.$$

The electric displacement is associated with the free charge in a dielectric, the polarization with the polarization charge and $\epsilon_0 E$ is associated with all charges.

The electric displacement is also equal to $K\epsilon_0 E$ thus,

$$\vec{D} = K\epsilon_0 \vec{E} = \epsilon_0 \vec{E} + \vec{P}$$

solving this for $\vec{P}$ yields,

$$\vec{P} = \epsilon_0 \vec{E}(K-1). \tag{14}$$

Of course all interactions involving electric fields will be governed by Maxwells' equations of electromagnetism. These relations are as follows:

$$
\begin{aligned}
\vec{\nabla}\cdot\vec{E} &= \rho/\epsilon_0 \\
\vec{\nabla}\cdot\vec{D} &= \rho \\
\vec{\nabla}\times\vec{E} &= -\partial\vec{B}/\partial t \\
\vec{\nabla}\times\vec{H} &= \vec{J} + \partial\vec{D}/\partial t \\
\vec{J} &= \sigma(\vec{E} + \vec{V}\times\vec{B}) + q\vec{V} \\
\vec{F} &= q\vec{E} + \vec{J}\times\vec{B} \\
\vec{D} &= \epsilon\vec{E} = \epsilon_0\vec{E} + \vec{P} \\
\vec{B} &= \mu\vec{H}
\end{aligned}
$$

Where:

$\vec{E}$ = electric field strength (volts/meter)

ρ = total charge density (coul/meter^3)

$\vec{D}$ = electric displacement (coul/inch^2)

$\vec{B}$ = magnetic flux density (weber/meter^2)

$\vec{H}$ = magnetic field (amp-turn/meter)

$\vec{J}$ = current density (amp/meter^2)

σ = electric conductivity (ohm-meter)$^{-1}$

$\vec{V}$ = velocity of charge (meter/second)

$\vec{F}$ = force (newtons)

ϵ = permittivity (farad/meter)

ϵ_0 = permittivity constant (farad/meter) = 8.84×10^{-12} farad/meter

$\vec{P}$ = electric dipole moment per unit volume (coul-cm/cm^3)

In the area of study defined as electrofluidmechanics there are no applied magnetic fields, and the magnetic fields produced by moving charges are very small. Therefore, as contrasted to magnetohydrodynamics the forces due to magnetic fields are negligibly small compared to electric forces. Thus the equations governing electrofluidmechanics are listed below in their "reduced" form:

$$
\begin{aligned}
\vec{\nabla}\cdot\vec{E} &= \rho\epsilon_0 \\
\vec{\nabla}\cdot\vec{D} &= \rho \\
\vec{\nabla}\times\vec{E} &= 0 \\
0 &= \vec{J} + \partial\vec{D}/\partial t \\
\vec{F} &= q\vec{E} \\
\vec{D} &= \epsilon\vec{E}_0 = \epsilon_0\vec{E} + \vec{P}
\end{aligned}
\qquad (15)
$$

CHAPTER III

EFFECTS OF ELECTRIC FIELDS

Electric fields can produce an effect on both charged and neutral particles. The most elementary action of the electric field is the force it imparts on a single charged particle. This is demonstrated by the classical case of an electron in a uniform field. Assume that the space considered is a vacuum, i.e. there are no collisions with other particles to consider.

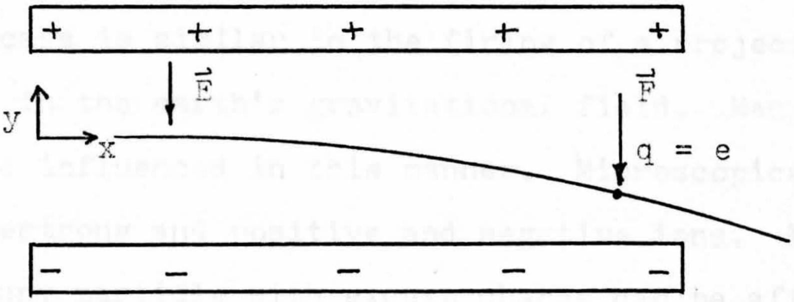


Fig. 8. Electron in a Uniform Electric Field.

The electron, with an initial velocity V_0, is acted upon by the electric field with a force, $\vec{F} = q\vec{E}$. The charge, q, is the charge of the electron, e. Considering only the force imparted by the electric field, and using Newton's Second Law,

$$\vec{F} = m\vec{a} = q\vec{E} = e\vec{E}, \tag{16}$$

with the boundary conditions at time zero of,

$$V_x = V_{x_0} \qquad x = x_0$$
$$V_y = V_{y_0} \qquad y = y_0$$

Thus the differential equation $m\vec{a} = e\vec{E}$ can be solved yielding two scalar equations of motion:

$$\sum F_x = ma_x = 0$$
$$\sum F_y = ma_y = eE \, .$$

The equations of motion are:

$$x = V_{x_0} t + x_0$$
$$y = (e/2m)Et^2 + V_{y_0} t + y_0 \, .$$

This simple case is similar to the firing of a projectile horizontally in the earth's gravitational field. Many particles can be influenced in this manner. Microscopically, there are electrons and positive and negative ions. Macroscopically, any particle with excess charge can be affected, such as dust, carbon in flames, liquid drops and impurities in solutions and colloids.

The effect that an electric field will have on a charged macroscopic particle varies considerably with the type of particle, size, charge and shape. Each would have to be considered separately for a detailed analysis. Of course, no matter what the particle, it will follow the basic law of being attracted or repelled by unlike and like charges respectively.

As an example of a macroscopic particle in an electric field, consider a spherical drop in an alternating field, as shown in figure 9.

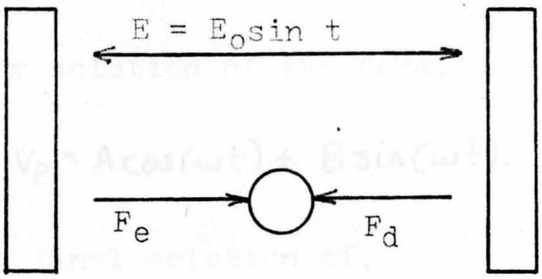


Fig. 9. Spherical Drop in a Uniform AC Field

Unlike the previous example, the electric field will be alternating with a frequency, ω. Thus the force acting on the drop due to the field is $F_e = qE_0\sin(\omega t)$. The force of gravity will be neglected, but a drag force, F_d, opposing F_e, will be taken into account.

Thus:

$$\Sigma F = F_e - F_d = ma$$

using Stoke's Law,

$$F_D = C_D V.$$

Where,

C_D = drag coefficient

V = velocity.

This results in the differential equation,

$$qE_0 \sin\omega t - C_D V = ma$$

where,

t = time

This equation can be solved using a complimentary solution of the form,

$$V_c = C_1 e^{-C_D t/m}$$

and a particular solution of the form,

$$V_P = A\cos(\omega t) + B\sin(\omega t).$$

These lead to a final solution of,

$$V = V_c + V_p = C_1 e^{-C_D t/m} + [E_0 q/(C_D^2 + m^2\omega^2)^{1/2}]\sin(\omega t - \phi).$$

This velocity equation can now be solved using the same technique where:

$$X_c = C_2$$

and

$$X_P = Ae^{-C_D t/m} + B\sin(\omega t - \phi) + C\cos(\omega t - \phi).$$

This yields the final equation of motion,

$$x = C_2 - (C_1 m/C_D)e^{-C_D t/m} - E_0 q\cos(\omega t - \phi)/\omega\,[C_D^2 + m^2\omega^2]^{1/2}$$

where $\phi = \mathrm{Tan}^{-1}(m\omega/C_D)$.

The constants in these equations will depend upon the boundary conditions. The equations illustrate the influence of the parameters involved on the drop while in the electric field. The effects of electric fields on drops and aerosols is a widely researched area as will be noted later.

In the previous example, the drag on a macroscopic particle was considered. Microscopic particles, such as ions, will also experience this drag in a dense fluid, due to collisions with neutral molecules. Thus collisions with other particles must be taken into account. As was stated in equation 16, an ion with the charge of one electron in an electric field will have its motion described by,

$$\vec{F} = m\vec{a} = e\vec{E}$$

in a vacuum. In a dense gas the ions will undergo numerous collisions causing a loss of energy and continuous changes in direction. Between collisions the acceleration of the ion is, $a = eE/m$, where e/m is the charge to mass ratio of the ion. Assuming the ion is subjected to a large number of collisions, it will have an average drift velocity proportional to the field strength and the density of the gas. This drift velocity is determined to be

$$V_d = eEL/m\bar{c} \tag{17}$$

where the terms not previously defined are:

L = mean free path of the ion

$\bar{c}$ = average thermal velocity of the gas particle.

Several simplifying assumptions are made in the derivation of equation 17, they are:

1. Perfectly elastic collisions occur.
2. Ions do not exert forces on neutral particles.
3. The mass of the ion is equal to the mass of the gas particles.
4. The concentration is uniform.
5. The energy gained by an ion between collisions is small compared to the average energy of agitation of the gas particles.

It has also been determined experimentally that the drift velocity remains directly proportional to the field strength only when the ion velocity due to the field is less than the average thermal velocity of the gas.

From the drift velocity is defined ion mobility, a reference velocity which is characteristic of the gas through which the ion moves.

$$\text{Ion Mobility} = k = v_d/E \qquad (18)$$

$$\text{or } k = eL/m\bar{c}. \qquad (18a)$$

This expression is approximate with more exact ones available.[1] It should be noted that impurities have a pronounced effect on reducing the value of k.

Even though not indicated by equation 18, it has been noted experimentally that mobilities of positive and negative ions of the same molecule are not the same. Negative ions usually move more quickly through a given field than positive ions, Thus the distribution in a uniform field will not be symmetric as may be expected.[2] Also, the purity of a gas has a large effect on the ion mobility, the greater the impurities the smaller the value of k obtained, as would be expected.

The concept of ion mobility is vital to the explanation of phenomena concerning electrically augmented flames.

Assuming the concept of ion mobility to be valid, that gives the ionic wind velocity as expressed in equation 18, as $v_d = kE$. Applying this to the positive and negative ions separately gives,

$$V_+ = k_+ E \quad \text{and} \quad V_- = k_- E \,. \tag{19}$$

These equations are valid for field strengths from zero to approximately 38,100 volts/inch for ions and up to approximately 5,000 volts/inch for electrons, both at 1 atm. pressure.[3]

The forces acting on a dipole in a uniform field have been discussed. The uniform field will exert a torque on the dipole but there is no net force acting on it. This is not true of a dipole in a nonuniform field, where a direct force will act upon the dipole. The nature of this force is now investigated.[4] A dipole in a nonuniform field will tend to orient itself in the direction of the field as shown in figure 10.

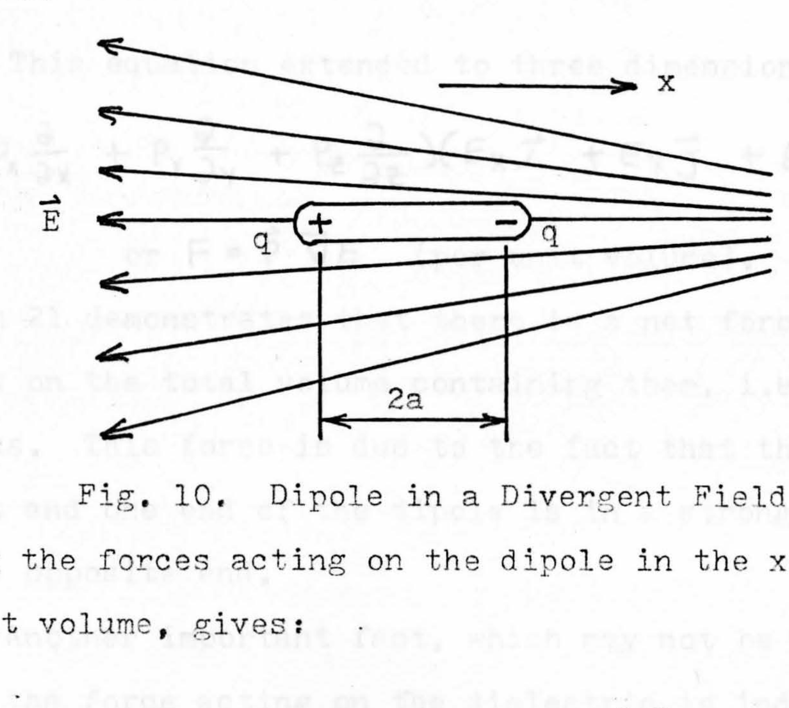


Fig. 10. Dipole in a Divergent Field

Summing the forces acting on the dipole in the x- direction, per unit volume, gives:

$$F_x = -qE_x + \left(qE_x + q\frac{\partial E_x}{\partial x}\,\delta x\right),$$

this reduces to

$$F_x = q \frac{\partial E_x}{\partial x} \delta x. \qquad (20)$$

It is obvious here that the force acting on the dipole is due to a change in E in the x- direction, i.e. the divergence produces the force on the dipole. The distance x between the charges of the dipole is 2a. Equation 20 becomes,

$$F_x = 2aq \frac{\partial E_x}{\partial x}$$

where F_x is the force per unit volume acting on the dipole. The quantity 2aq is the dipole moment. Since this is on a per unit volume basis this is the polarization as defined in equation 13:

$$F_x = P_x \frac{\partial E_x}{\partial x} .$$

This equation extended to three dimensions is:

$$F = (P_x \frac{\partial}{\partial x} + P_y \frac{\partial}{\partial y} + P_z \frac{\partial}{\partial z})(E_x \vec{i} + E_y \vec{j} + E_z \vec{k})$$

$$\text{or } F = \vec{P} \cdot \vec{\nabla} E \quad \text{(per unit volume)}. \qquad (21)$$

Equation 21 demonstrates that there is a net force on dipoles and thus on the total volume containing them, i.e. on dielectrics. This force is due to the fact that the field diverges and one end of the dipole is in a stronger field than the opposite end.

Another important fact, which may not be obvious, is that the force acting on the dielectric is independent of the direction of the field. This is more readily seen by substituting equation 14 into equation 21:

$$F = \epsilon_o (K-1) \vec{E} \cdot \vec{\nabla} E$$

which can be rewritten as[5]

$$F = \frac{1}{2} E_o (K-1) \nabla E^2.$$

Here it is seen that the force depends upon E^2, thus the direction of the field is not a factor. Thus, a time dependent and steady nonuniform field will have the same effect on a dielectric. The force will always be directed to the region of highest intensity.

In the same way that a single dipole is affected by an electric field, so is a polarized particle. Consider a spherical particle in a nonuniform field.[6]

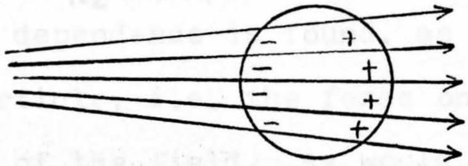


Fig. 11. Spherical Particle in Nonuniform Field

For a spherical particle the dipole moment is:

$$p = vP = 4/3 \pi a^3 P$$

where, v = volume

a = radius of the particle.

Assuming the particle is in a dielectric liquid,

K_1 = dielectric constant of the liquid

K_2 = dielectric constant of the sphere.

The excess polarization in the sphere is the,

$$\vec{P} = \epsilon_0 \vec{E}_s (K_2 - K_1). \tag{22}$$

$\vec{E}_s$ = internal field in the sphere in the direction of the external field

$$= \frac{3K_1}{K_2+2K_1}, \vec{E} = \text{external field.} \tag{23}$$

Thus the dipole moment becomes,

$$\vec{P} = \frac{4}{3}\pi a^3 [\epsilon_0 \frac{3K_1}{K_2+2K_1} \vec{E}(K_2 - K_1)$$

substituting this in equation 21 gives the total force on the particle due to the field as,

$$F = 4\pi a^3 \left(\frac{K_2 - K_1}{K_2 + 2K_1}\right) K_1 \epsilon_0 \vec{E} \cdot \vec{\nabla} E$$

or,

$$F = 2\pi a^3 \left(\frac{K_2 - K_1}{K_2 + 2K_1}\right) K_1 \epsilon_0 \nabla E^2.$$

The same dependence is found, as it should be, for the spherical particle, i.e. the force on it is independent of the direction of the field. As would be expected, the polarized sphere exhibits the properties of a dipole.

CHAPTER IV

BODY FORCES ON FLUIDS

As has already been discussed, a body force exists in a fluid when there is a gradient in the applied electric field. Neglecting gravity, this then is the only force acting on the liquid dielectric. A more rigorous derivation than that offered earlier is given by Stratton.[7] This is,

$$\vec{F} = -\frac{1}{2} E^2 \vec{\nabla}\epsilon + \frac{1}{2}\vec{\nabla}\left(E^2 \tau \frac{\partial \epsilon}{\partial \tau}\right) + \rho \vec{E} \tag{24}$$

where τ is the density of the liquid. The first two terms on the right hand side represent the nonuniform effects and the third term is due to the free charges present in the liquid. This equation has been derived with the assumption that ϵ is a function of position and density alone. This is valid for liquids and gases but not necessarily in the case of solids.

For an ideal fluid, the equation of equilibrium is,

$$\vec{F} = \vec{\nabla} P \tag{25}$$

where p is pressure.[8] Substituting equation 24 into equation 25 yields,

$$-\frac{1}{2} E^2 \vec{\nabla}\epsilon + \frac{1}{2}\vec{\nabla}\left(E^2 \tau \frac{\partial \epsilon}{\partial \tau}\right) + \rho \vec{E} - \vec{\nabla} P = 0$$

Assuming there are no free charge effects present, such as in an oscillating field or simply no free charges present, then, ρE = o. Thus the relation between pressure and fluid intensity at any point in the fluid is,

$$\vec{\nabla}\rho = -\frac{1}{2}E^2\vec{\nabla}\epsilon + \frac{1}{2}\vec{\nabla}\left(E^2\tau\frac{\partial\epsilon}{\partial\tau}\right). \qquad (26)$$

The Clausius-Mossotti law gives, to a good approximation, the relation between the dielectric constent, K, and density. This relation holds for gases, liquids and some solids,

$$\frac{K-1}{K+2} = C\tau \quad \text{or} \quad K-1 = \frac{3C\tau}{1-C\tau}. \qquad (27)$$

This leads to,

$$\tau\frac{\partial\epsilon}{\partial\tau} = \frac{\epsilon_0}{3}(K-1)(K+2) \qquad (28)$$

using K = ϵ/ϵ_0.

Thus,

$$\vec{\nabla}P = -\frac{1}{2}E^2\vec{\nabla}\epsilon + \frac{1}{2}\vec{\nabla}E^2\frac{\epsilon_0}{3}(K-1)(K+2)$$

or

$$\vec{\nabla}P = \frac{-\epsilon_0}{2}E^2\vec{\nabla}K + \frac{\epsilon_0}{6}\vec{\nabla}\left[E^2(K-1)(K+2)\right]. \qquad (29)$$

Assuming the dielectric constant, K, uniform through the liquid:

$$\vec{\nabla}P = \frac{\epsilon_0}{6}\vec{\nabla}E^2(K-1)(K+2)$$

or

$$P-P_0 = \frac{\epsilon_0}{6}\nabla E^2(K-1)(K+2).$$

Assuming p = o then,

$$P = \frac{\epsilon_0}{6}\nabla E^2(K-1)(K+2). \qquad (30)$$

This equation now provides a theoretical pressure produced by an electric field. As an example consider the parallel plate capacitor where V = Ed, V being the applied voltage, E the electric field and d is the distance between the plates.

Assuming p = o, the pressure generated between the electrodes of the capacitor will be,

$$P = \frac{\epsilon_0}{6} E^2 (K-1)(K+2)^{9}$$

or

$$P = \frac{\epsilon_0}{6} \frac{V^2}{d^2} (K-1)(K+2) \qquad (31)$$

Thus knowing d and K, the dielectric constant, and by measuring the applied voltage the pressure produced on the liquid can be determined.

If the liquid between the electrodes is transformer oil (K = 2.5) and a potential of 20,000 volts is applied across two centimeters the pressure induced is,

$$p = \frac{8.85 \times 10^{-12}}{6} \frac{(20{,}000)^2}{(.02)^2} (2.5-1)(2.5+2)$$

$$= 9.96 \ nt/m^2 = 14.5 \times 10^{-4} \ psi$$

As can be seen by the previous example, the pressure generated by the electric field is quite small. This is indicative of most interactions between dielectrics and electric fields, the effects being quite subtle. Also note that free charge effects are neglected, which in the case of a parallel plate capacitor could prove large. This would depend upon the free charge density of the dielectric present.

CHAPTER V

PHORESIS

Phoresis is a derivative of the Greek work for motion. In the present context, phoresis refers to the motion of particles in conjunction with an electric field. One such phoretic effect has already been described in detail, the effect of a nonuniform field on a fluid dielectric being dielectrophoresis. In contrast to this are the phoretic phenomenon explained by the double layer theory. This double layer occurs at the boundary between two phases. Consider the boundary between a solid and a liquid. Referring to figure 12, assume the solid has a positive charge built on its surface. The negatively charged particles in the liquid are attracted through Coulombic forces to the surface of the solid.

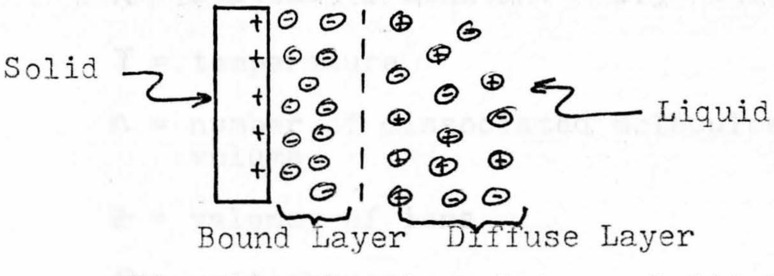


Fig. 12. Boundary Between Solid & Liquid

This forms a bound layer of negative charge at the phase boundary. A diffuse layer of predominantly positive particles is induced by the bound layer of negative charge. The diffuse layer diminishes and becomes neutral further into the liquid. The bound and diffuse layers result in the formation of an electrical double layer. This double layer is neutral as a whole and may be several hundred Angstroms or more thick. The diffuse layer can be separated from the bound layer causing the liquid to acquire the charge of the diffuse layer. The bound charge remains at the surface thus causing a charge difference which is dissipated quickly or slowly, depending upon the conductivity of the liquid.

The charge per unit area of the double layer is given by:

$$\alpha = \epsilon \frac{V_d}{\delta}$$

where δ is the thickness of the double layer,

$$\delta = \left[\frac{\epsilon K T}{2 n z e^2} \right]^{1/2}$$

the remaining quantities are,

α = charge per unit area
V_d = potential across double layer
K = Boltzman's constant = 1.38 x 10^{-23} joule/°K
T = temperature
n = number of dissociated molecules per unit volume
z = valency of ions
e = unit charge.

The potential, V_d, is usually given as the electrokinetic or Zeta potential defined as,

$$\zeta = \frac{4 \pi \delta \alpha}{\epsilon}$$

Electrophoretic mobility, similar to ion mobility, is given by,

$$K = \frac{\zeta\epsilon}{4\pi\mu}$$

where μ denotes viscosity.
Also the electrophoretic velocity is given by,

$$u = K\epsilon$$

Most phoretic phenomenon can be explained in terms of the double layer. Two such phenomenon are the Dorn effect and streaming potential. The Dorn effect is due to the settling of large quantities of suspended particles in a nonconducting fluid. The particles have their double layers sheared as they settle, causing a potential difference. This potential difference can be large and neutralizes with time depending upon the conductivity of the fluid.

The streaming potential occurs when a fluid of low conductivity flows through a tube. Analagous to the Dorn effect, the double layer is again sheared from the surface of particles causing a charge separation in the tube. This phenomenon has been blamed in causing fires where flammable liquids are transported in hoses.

Electrophoresis, sometimes known as cataphoresis, describes a motion which arises due to the attraction of charged electrodes for charged particles. The particles have a double layer associated with them at their surface. An electric field causes the outer, diffuse layer, to migrate toward the opposite electrode. The motion in the case of electrophoresis is dependent upon the direction of the applied field. Thus, in a uniform alternating field the particles would oscillate back and forth never migrating to either electrode. This is not always the case, however. Charged particles have been observed to migrate to sharp electrodes in rapidly alternating fields of high strength.[10] Electrophoresis can be observed in either divergent or uniform fields with particles of any molecular size and requires relatively small charges and low voltages.

In contrast to electrophoresis is dielectrophoresis which is not due to action on the double layer. Dielectrophoresis is due to the body force imposed upon the dielectric by the electric field. As discussed earler, this force is proportional to the gradient of the square of the field strength. Thus, the particle motion is independent of the direction of the field and should be most apparent in highly divergent fields. The effect will not only be felt in liquids with permanent dipoles but in others where polarization may result either in neutral molecules or by displacement of the double layer thus providing induced dipoles. Dielectrophoresis is a weak effect generally needing relatively large particles and high field strengths (e.g., 2,000 volts/cm). Also the viscosity of the liquid is a factor, lower viscosity liquids showing greater reactions to dielectrophoresis than more viscous ones. Dielectrophoresis can also be used to move materials in relation to each other, providing that there is a large difference in their dielectric constants, either induced or natural (e.g., $K_1 - K_2 = 2$ to 100).[11]

CHAPTER VI

EXAMPLES OF PHYSICAL PROCESSES AFFECTED BY FIELDS

The amount of research done pertaining to physical processes effected by fields is voluminous. The effects are quite varied and range from simple to complex. It is not intended to cover all such processes here, but rather to give an overview. Such an overview will emphasize the diversity of the effects of electric fields. Five general areas are represented, some of which cross boundries. These general areas are:

1. Dielectrophoresis and electrophoresis
2. Aerosols
3. Heat transfer
4. Fluid flow
5. Combustion

Dielectrophoresis and Electrophoresis

An excellent introduction to dielectrophoretic and electrophoretic effects are three papers by H. A. Pohl and one by Pohl and J. P. Schwar. Pohl shows theoretically how physical parameters effect a spherical particle in a nonuniform field.[12]

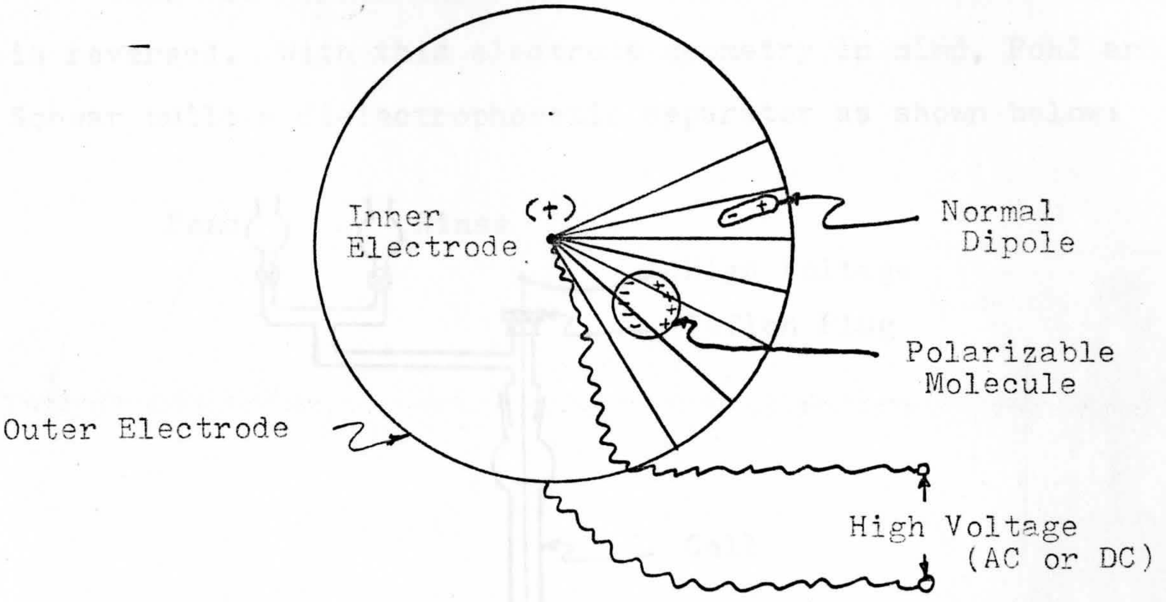


Fig. 13. Diagram of Forces Acting on Dipoles and Induced Dipoles Suspended in an Inhomogeneous Electric Field[a]

---

[a] H. A. Pohl, "Some Effects of Nonuniform Fields on Dielectrics", Journal of Applied Physics, XXIX (Aug., 1958), p. 1182

For the field shown above the dielectrophoretic force on a spherical particle in a dielectric medium is,

$$F = 2\pi a^3 K_1 \epsilon_0 \left[\frac{K_2 - K_1}{K_2 + 2K_1} \right] \nabla E^2.$$

Where: a = particle radius
K_1 = dielectric constant of the medium
K_2 = dielectric constant of the particle
E = electric field strength
ϵ_0 = permittivity of free space

This equation reveals that the force is proportional to the cube of the particle radius and to the difference between its dielectric constant and that of the medium. The

E^2 term also shows that the force is the same if the field is reversed. With this electrode geometry in mind, Pohl and Schwar built a dielectrophoretic separator as shown below:

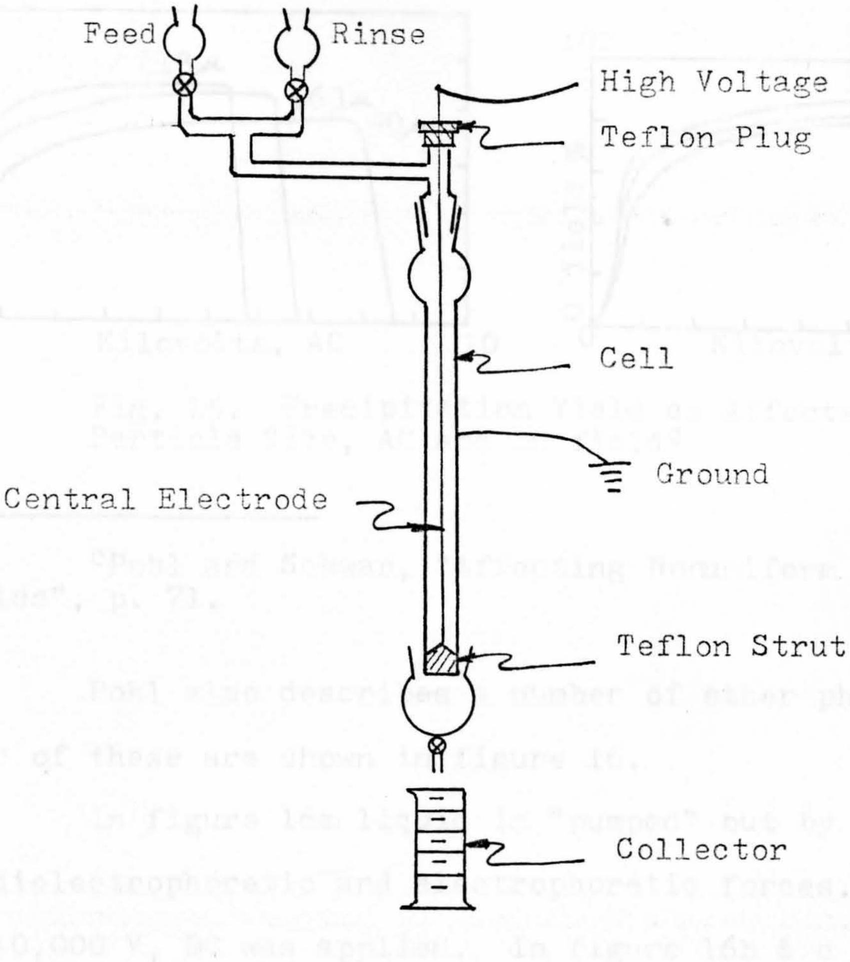


Fig. 14. Apparatus of Pohl and Schwar[b]

Using this apparatus they were able to seperate polar polyvinyl chloride from a nonpolar mixture of carbon tetrachloride and benzene. Using both direct and alternating voltages the P V C powder shimmed down the central electrode toward the

---

[b] H. A. Pohl and J. P. Schwar, "Factors Affecting Separations of Suspensions in Nonuniform Electric Fields", Journal of Applied Physics, XXX (Jan., 1959), P. 70.

collecting port.[13] Their tests showed the same yield of P V C for direct voltages and rms alternating voltages, as shown in Figure 15.

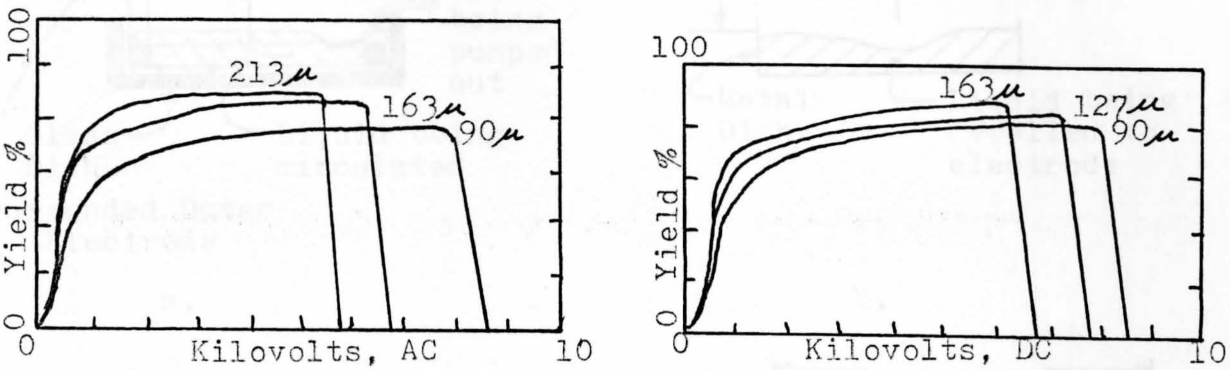


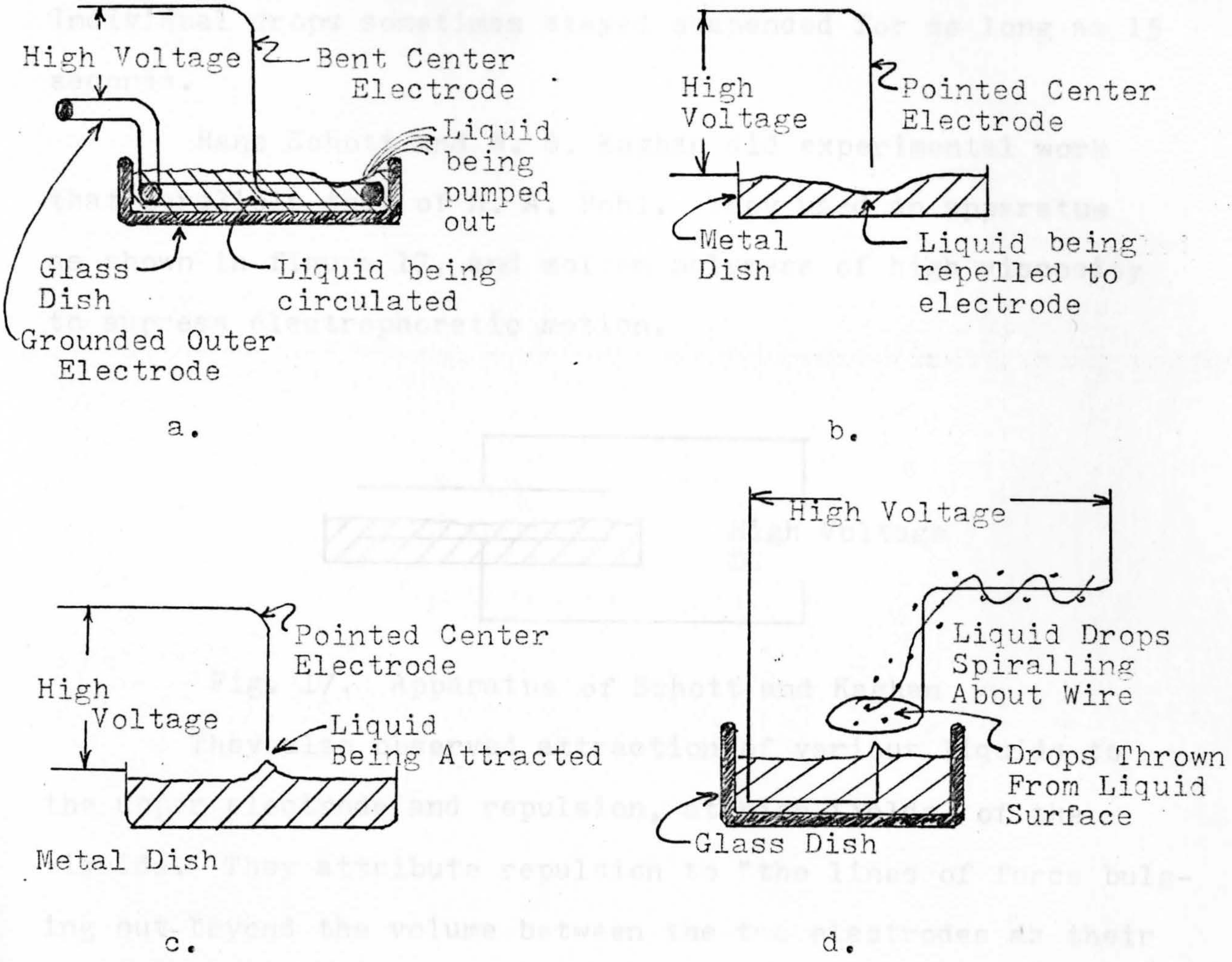
Fig. 15. Precipitation Yield as Affected by Particle Size, AC and DC field[c]

---

[c] Pohl and Schwar, "Affecting Nonuniform Electric Fields", p. 71.

Pohl also describes a number of other phenomenom.[14] Four of these are shown in figure 16.

In figure 16a liquid is "pumped" out by a combination of dielectrophoretic and electrophoretic forces. A voltage of 10,000 V, DC was applied. In figure 16b & c the same electrode geometry provides two different effects. Pohl attributes the repulsion to the charging of air molecules. He assumes dielectrophoretic forces attract air molecules to the point, then charged at the electrode, they are subsequently repelled strongly causing the liquid to be pushed aside. E. T. Pierce[15] disputes this theory, saying that the repulsion is primarily electrophoretic in nature due to conventional corona discharge. That is, ions are accelerated toward the



a. b. c. d.

Fig. 16. Actions of Strong Divergent Fields[d]

---

[d] Pohl, "Effects of Nonuniform Fields on Dielectrics", pp. 1186 - 1187.

sharp point, thus producing more ions by collision and ultimately being repelled causing the "repulsion" of the liquid upon impact.

By moving the pointed electrode closer the liquid is now attracted as shown in figure 16c. Figure 16d shows drops leaving the dish and "hanging" in the air around the electrode. Individual drops sometimes stayed suspended for as long as 15 seconds.

Hans Schott and W. S. Kaghan did experimental work that parallels that of H. A. Pohl. They used an apparatus as shown in figure 17, and molten polymers of high viscosity to supress electrophoretic motion.

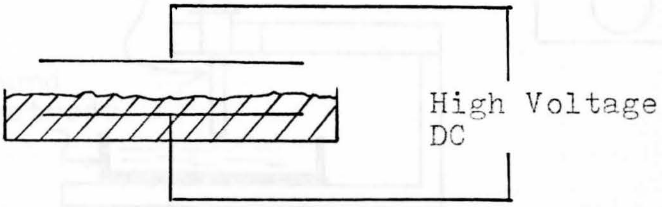


Fig. 17. Apparatus of Schott and Kaghan

They also observed attraction of various liquids to the upper electrode and repulsion, at high fields, of the liquids. They attribute repulsion to "the lines of force bulging out beyond the volume between the two electrodes as their number increased and they became more crowded. Since the dipoles are always oriented parallel to the lines of force, they would finally be tilted at such angles with respect to the surface that the melt below the top electrode would be pushed aside."[16]

E. P. Damm, Jr. investigated the attraction of liquids by electric fields[17] which was demonstrated by Pohl. Damm used response time to measure the effects of various system parameters. Response time as defined by Damm is the time between application of the electric field and liquid contact with the electrode. The test electrode geometry is shown in figure 18.

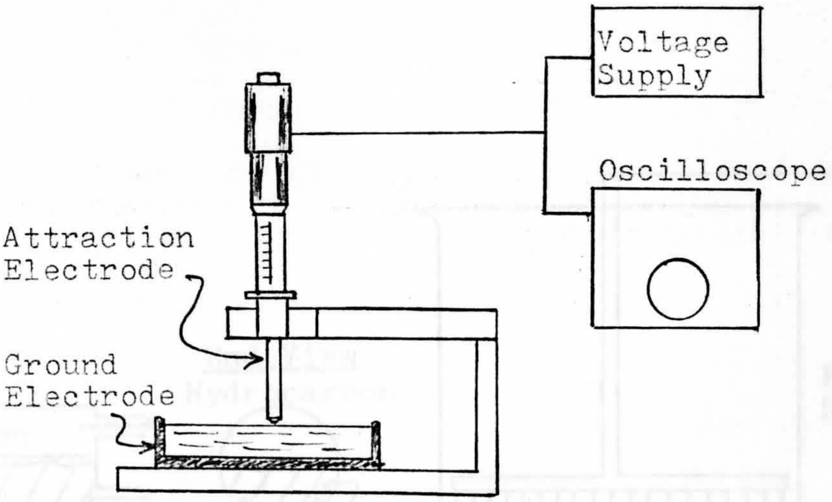


Fig. 18. Responsometer[e]

Damm found that surface tension and viscosity were determining factors up to a point. However, above certain field strengths this did not prove to be true. Liquids with high dielectric constants had lower response times than those with low dielectric constants. Also, the evaporation rate appeared to increase response time. Damm found that electrical polarity within the liquid had little effect on response time, based upon this and the other results Damm concluded the phenomenon involved was largely dielectrophoretic.

Whereas Pohl and Schwar experimented with seperating mixtures using dielectrophoresis, W. P. Cropper and H. S. Seelig[18]

---

[e] E. P. Damm, Jr., "Attractions of Liquids by Electric Fields", Dielectrophoretic Deposition (New York: Electrodeposition Division Electrochemical Society, 1969) pp. 96 - 101.

turned their attention to the mixing process affected by this phenomenon. They used two types of cells to study the effects. Flow experiments were carried out in the cell shown in figure 19a and batch experiments in cell 19b.

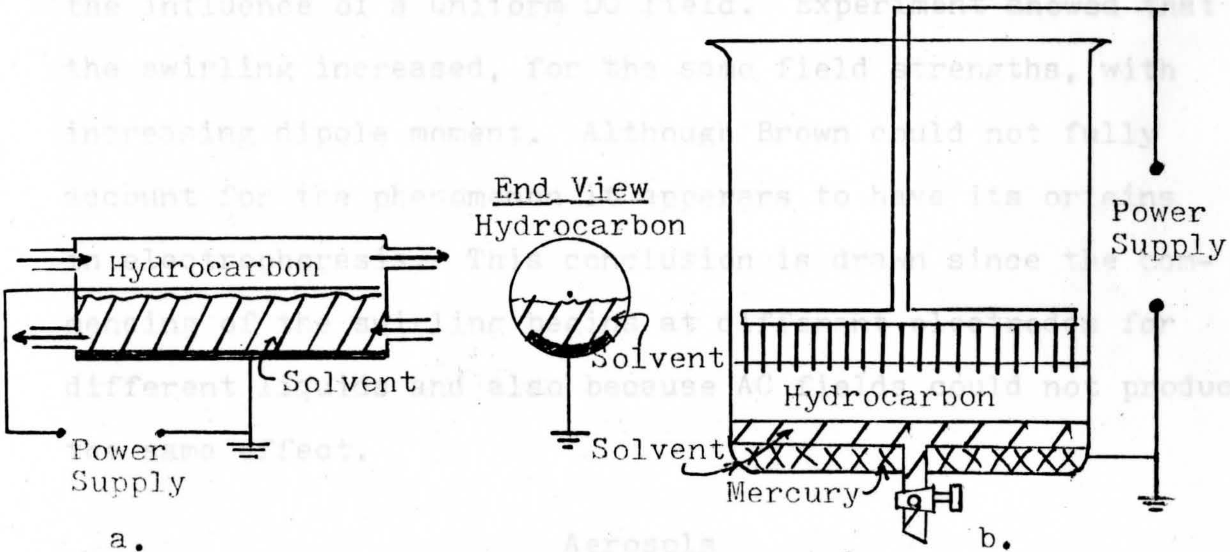


Fig. 19. Mixing Cells^f

W. P. Cropper and H. S. Seelig, "Mixing With an Electrostatic Field", I & EC Fundamentals, I (Feb. 1962), p. 49.

Cropper and Seelig found the nonhomogeneous fields were particularly effective in mixing hydrocarbons and nonaqueous solvents. In small scale apparatus, power consumption is similar to that of conventional mixers but with the advantage of no moving parts. For both types of mixing, AC voltage was applied leaving the possibility that both electrophoretic and dielectrophoretic forces are involved. Field strengths ranged from 4.5 to 11.5 Kv depending upon the liquids involved. Also of interest is the fact that Pohl and Schwar's investigation centered on macroscopic particles (P V C) interacting with nonhomogeneous fields while Cropper and Seelig produced their mixing with an interaction between molecules and nonhomogeneous fields.

D. R. Brown[19] noted a swirling of liquids while under the influence of a uniform DC field. Experiment showed that the swirling increased, for the same field strengths, with increasing dipole moment. Although Brown could not fully account for the phenomenon it apperars to have its origins in electrophoresis. This conclusion is drawn since the commencing of the swirling begins at different electrodes for different liquids and also because AC fields could not produce the same effect.

Aerosols

The increased rate of disintegration of a liquid jet while under the influence of electrostatic fields can find an application whenever atomization of a liquid is important. According to Drozin[20] this phenomenom was discovered by Bose in 1745, and rediscovered in 1914 by Zeleny. Drozin also reported the stages of dispersion shown in figure 20 by increasing the applied potential from zero to 12 Kv.

This same type of dispersion was reported by Vonnegut and Neubauer[21] using both AC and DC potentials. They also noted an interesting result. The stream of drops formed by the DC field were charged while those formed by the AC field were not.

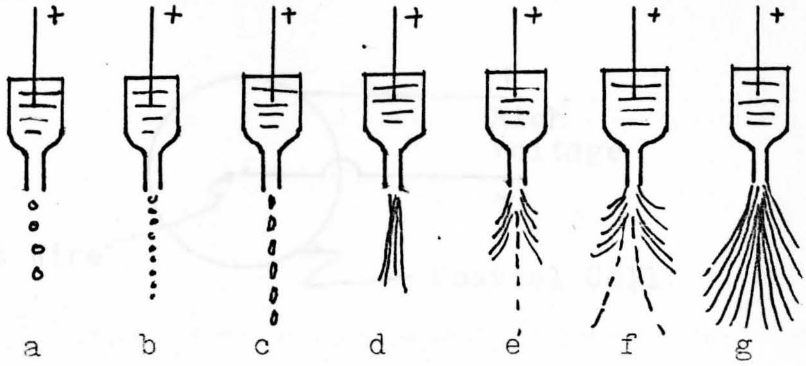


Fig. 20. Stages of Dispersion Observed by Increasing the Applied Potential[g]

---

[g] Vadim G. Drozin, "The Electrical Dispersion of Liquids as Aerosols", Journal of Colloid Science, X (1955), p. 159.

F. J. Weinberg[22] reports on the control over the formation and behavior of aerosols. He uses this phenomenom to control fuel dispersion and the solid and intermediate products of combustion. Along these same lines Fujimoto, et al[23] reported on the atomization of distilled water in an electrostatic field. Hogan and Hendricks[24] present the complex nature of the electrical atomization of liquids and analyze the factors contributing to radius and specific charge of emitted particles.

Heat Transfer

Electric fields can effect heat transfer in many different ways. Watson[25] reports of increasing heat transfer by using nonuniform fields and inhomogeneous dielectric liquids. He increased the heat transfer from a hot wire to an insulating liquid, hexane, using the electrode geometry shown below.

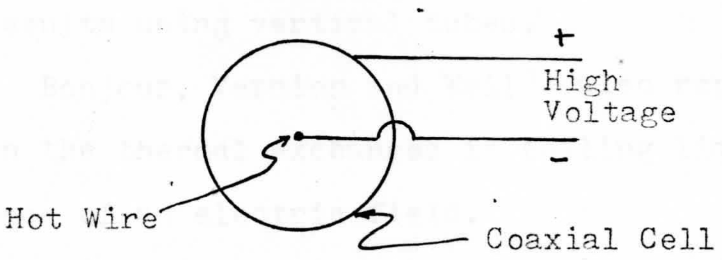


Fig. 21. Apparatus of Watson

Gross and Porter[26] also increased heat transfer with the following electrode geometry.

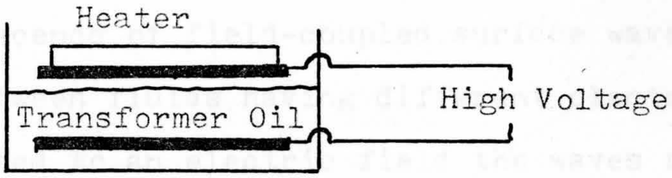


Fig. 22. Apparatus of Gross and Porter

Without an electric field, heating the upper plate produced no convection. With a positive potential applied at the lower plate convection patterns appeared in the transformer oil. The patterns were visable with the aid of a Schlieren system.

H. R. Velkoff[27] and M. E. Franke[28] both showed that convective heat transfer could be appreciably increased from a flat plate. The primary cause is the corona wind from the various electrode geometries used.

As would be expected electric fields also effect such phenomenom as condensation and boiling. Velkoff and

Miller[29] found a large increase in heat transfer with a 150% increase in cooling on a copper plate. Choi[30] found the same results using vertical tubes.

Bonjour, Verdier and Weil[31] also reported an improvement in the thermal exchanges in boiling liquids, under the influence of an electric field.

Fluid Flow

Some of the most interesting phenomenom produced by the interaction of electric fields and fluids have been brought to light by J. R. Melcher. Devitt and Melcher[32] demonstrated the prescence of field-coupled surface waves. When an interface between fluids having different electrical properties is subjected to an electric field the waves result. Using AC fields with a frequency of 50 Kc/sec waves were induced at liquid dielectric and air interfaces. Also using high frequencies they showed that in this limit the dynamics of the surface waves are predicted by using rms fields with theory based on constant fields. Melcher, Guttman and Hurwitz[33] extended this phenomenom to the control of liquids in low-gravity environments. The electric field essentially replacing gravity. They show that it would be possible to suppress slosh of liquids, orient liquids and pump liquids. Melcher, Hurwitz and Fax[34] demonstrated the practicality of a light weight pump that could be used in space missions.

Combustion

Heinsohn's and Becker's[35] paper is an excellent introduction to the electrical aspects of combustion. In it they describe three ways in which fields may influence flames:

1. Since positive and negative ions are formed during combustion, the migration of positive particles toward the cathode produces an "electric wind".
2. On a microscopic scale the wind may alter the concentrations of neutral species and thus change combustion chemistry.
3. The combustion chemistry may also be influenced by the initiation of new reactions.

The paper also reviews historical and up-to-date research in this area.

CHAPTER VII

EXPERIMENTAL INVESTIGATION

With the body force produced by a divergent field apparent, its demonstration and prediction was attempted experimentally. First the theoretical model is reviewed, then an experimental apparatus built to deliver a high voltage electric field. A device to measure the body force is also incorporated to try to correlate theoretical and experimental results.

The measurement of the body force produced by an electric field on a liquid dielectric imposes two major problems. First, the voltage must be very high and second, even with high voltage, only a very small effect, detected as a pressure change, could be expected. With this and the dielectric itself in mind, the following characteristics were considered necessary for successful testing:

1. The liquid dielectric should have a high dielectric constant and be relatively pure.
2. Both AC and DC fields would be advantageous, and possibly a combination of the two.
3. A simple electrode geometry to produce a divergent field would be convenient.
4. An accurate measurement of the force produced by the field is necessary.
5. An accurate measurement of the applied voltage is necessary.

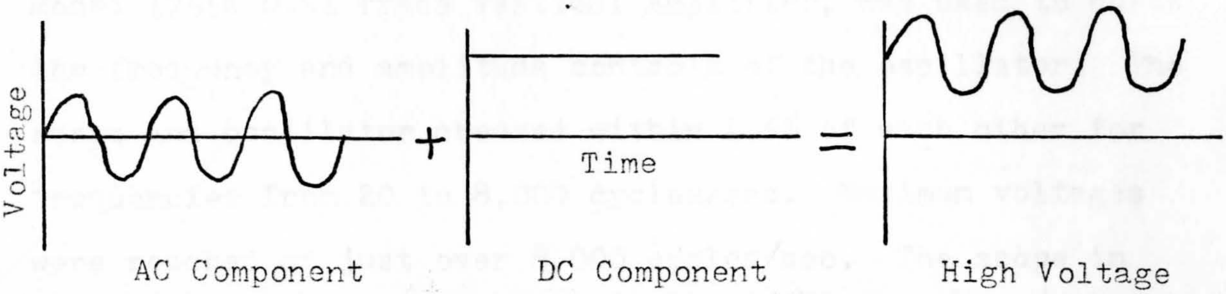
The design of the apparatus followed these desirable characteristics, beginning with the dielectric liquid. The dielectric constant is of prime importance since any pressure generated will be proportional to it. Also of importance is availability.

The dielectrics were thus narrowed to the following:

Dielectric	Dielectric Constant
Acetone	21.3
Silicone Oil	2.8
Toluene	2.4
Carbon Tetrachloride	2.3
Transformer Oil	2.5

At first glance, acetone appears to be a good candidate but in actual practice it proved unsafe. Since it is highly flammable any arcing across the electrodes, will and did, ignite it. Carbon tetrachloride was ruled out for the same reason and because of the noxious fumes produced. Silicone oil proved to be expensive and in short supply. Also, its exact dielectric constant could not be verified. Toluene fails since it attacked the plastic tubing to be used elsewhere in the apparatus. Thus, transformer oil emerges as the test liquid. It is safe to work with, abundant and relatively inexpensive. It also has no visible impurities.

The high voltage power supply was initially conceived as having both an AC and DC component which would combine to give a higher voltage than either acting alone, as shown below.



Since a DC power supply was available attention was turned to an AC supply. Two factors were considered important here, high voltage and good frequency control. The frequency control, being used to eliminate the effect of charged particles, thus dielectrophoresis would predominate. Figure 23 shows schematically the AC source.

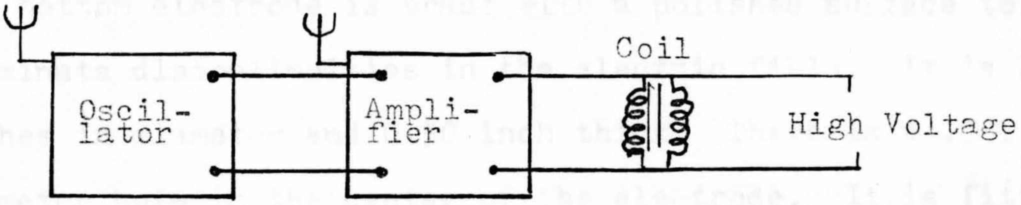


Fig. 23. High Voltage AC Source

The oscillator is a Hewlett-Packard, Model 200C, audio oscillator. It has a frequency range of 20 to 200,000 cycles/sec. The signal from the oscillator is amplified by a Marantz, Model 5, power amplifier and then fed into a Bendix Ignition Coil, Model UC-45X. The coil was a standard automobile ignition coil. An audio transformer was tried initially in place of the coil. The transformer had a frequency response of 20, to 20,000 cycles/sec. and a maximum audio power of 50 watts. In its "step up" mode the transformer could produce no more than 5,000 volts. The coil, though, produced voltages in the range of 20,000 to 25,000 volts.— An oscilloscope, Hewlett-Packard, Model 175A, with Model 1755A Dual Trace Vertical Amplifier, was used to check the frequency and amplitude controls of the oscillator. The scope and oscillator checked within 1.5% of each other for frequencies from 20 to 8,000 cycles/sec. Maximum voltages were reached at just over 8,000 cycles/sec. The scope in conjunction with a voltage divider was used for voltage readings.

The electrode configuration and manometer to measure pressure are shown in figure 24. The two cells, connected by .066 inches I.D., clear tubing, act as a manometer. The left cell is a dummy and the right contains the electrodes. The bottom electrode is brass with a polished surface to eliminate discontinuities in the electric field. It is 1.75 inches in diameter and 0.50 inch thick. There is a .031 inch diameter hole in the center of the electrode. It is fitted into a clear Plexiglas tube and sealed against leaks from the outside with Dow Corning Silicone Rubber Sealer. Glass tubing was originally used in place of the Plexiglas but proved too fragile. The dummy cell is constructed the same except the bottom is made of Plexiglas instead of brass. The upper electrode in the test cell is a ballbearing, 0.125 inch in diameter, soldered to a steel rod which is then connected to the power supply. The distance between the two electrodes is adjusted by means of a set screw in the cap of the test cell. The cap in both cells is made of Plexiglas. By inducing a bubble in the tubing joining the two cells, any

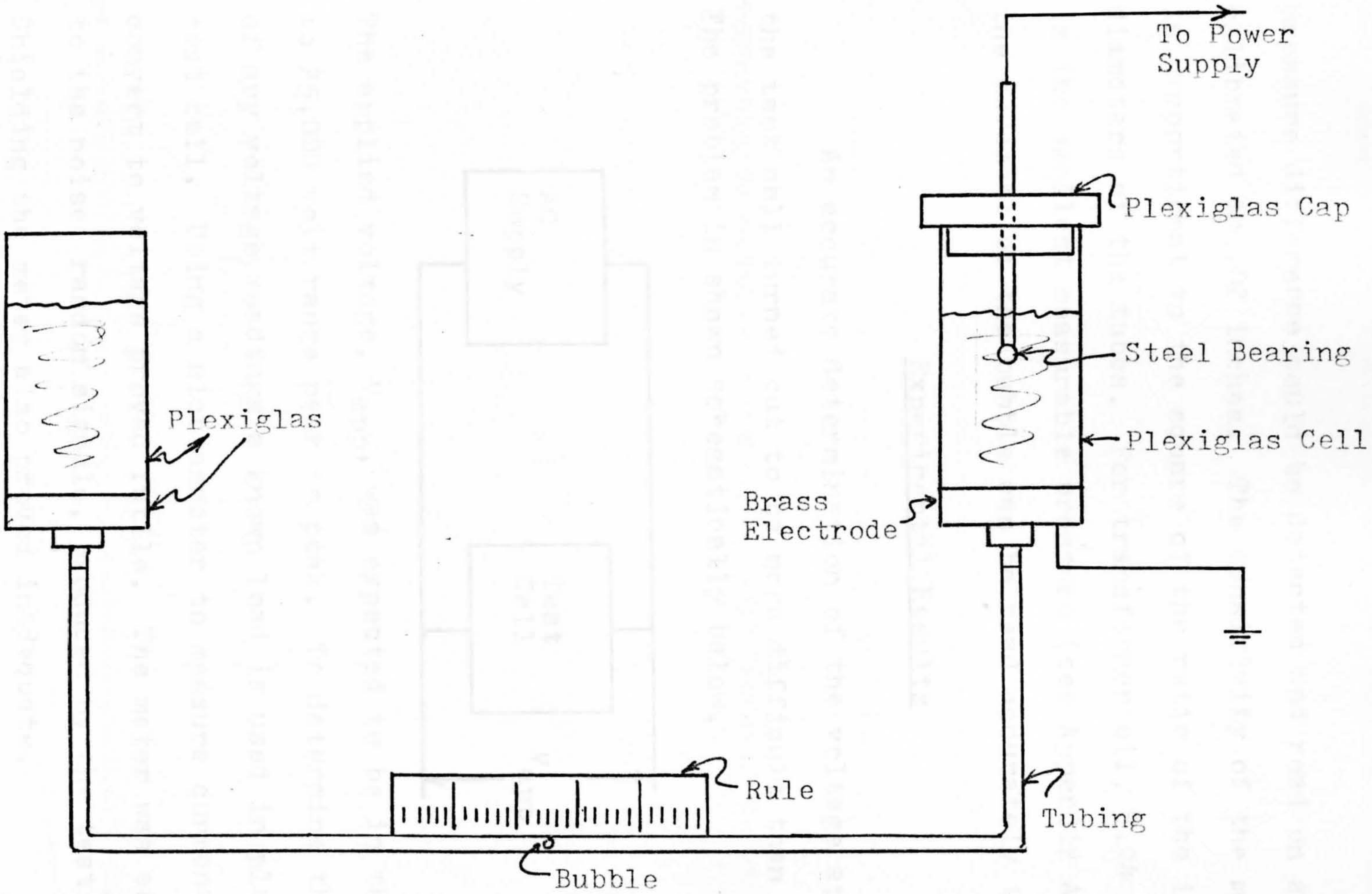
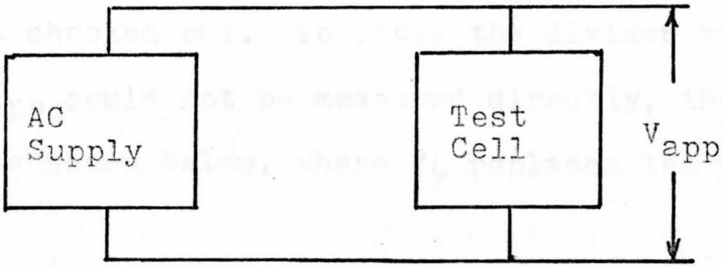


Fig. 24. Electrode and Manometer Apparatus

pressure difference could be detected and read on a rule calibrated in .02 inches. The sensitivity of the manometer is proportional to the square of the ratio of the inside diameters of the tubes. For transformer oil, 8.94×10^{-7}psi is the smallest measurable pressure (see Appendix A) assuming the movement of the bubble can be read accurately to .02 inch.

Experimental Results

An accurate determination of the voltage applied to the test cell turned out to be more difficult than expected. The problem is shown schematically below.



The applied voltage, V_{app}, was expected to be in the 20,000 to 25,000 volt range peak to peak. To determine the accuracy of any voltage readings a known load is used in place of the test cell. Using a microammeter to measure current and then convert to voltage proved futile. The meter was erratic due to the noise, random signals, produced by the test set up. Shielding the meter also proved inadequate.

A voltage divider, shown in figure 25, was incorporated using high precision resistors.

For this circuit,

$$V_{app} = \frac{R_2 + R_1}{R_1} V_1$$

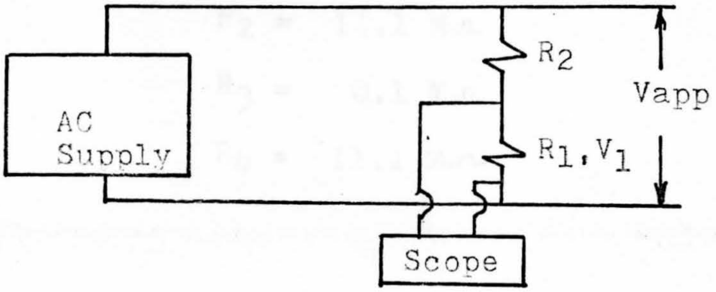
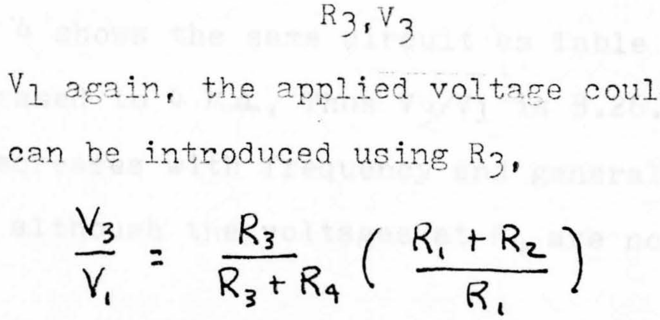


Fig. 25. Voltage Divider

Using low voltage, so that V_{app} could be measured directly, two tests were run. One with an 11 to 1 division and the other 112 to 1. For low voltage, less than 330 volts, both dividers checked out. To check the divider at higher voltages, where V_{app} could not be measured directly, the circuit was set up as shown below, where R_4 replaces the test cell.



By measuring V_1 again, the applied voltage could be calculated. Now, a ratio can be introduced using R_3,

$$\frac{V_3}{V_1} = \frac{R_3}{R_3 + R_4}\left(\frac{R_1 + R_2}{R_1}\right)$$

V_3 and V_1 can be measured directly and their ratio can be used as a check on voltage measurements.

A low voltage setup was tried first using the following resistances,

$R_1 = 0.1\ M\Omega$

$R_2 = 11.1\ M\Omega$

$R_3 = 0.1\ M\Omega$

$R_4 = 11.1\ M\Omega$

The results are shown in Table 1 in Appendix B. The ratio of V_3 to V_1 is very good but the applied voltage begins to deviate from that calculated, as the frequency increases, by as much as 32%.

Table 2 shows the same circuit with R_4 increased to 22.2 MΩ and $V_3/V_1 = .502$. Here the V_3/V_1 ratio is within experimental limits and the actual and calculated voltages compare favorably at low voltage and frequency. As the frequency and voltage increase the V_3/V_1 ratio becomes erratic and the per cent of error is too high to be reliable.

Once more increasing R_4 to 50.22 MΩ and changing R_3 to 1 gives a V_3/V_1 of 2.19. Table 3 shows V_3/V_1 again increasing with frequency and voltage.

Table 4 shows the same circuit as Table 3 except R_3 has been increased to 4 MΩ, Thus V_3/V_1 is 8.26. This time the V_3/V_1 ratio decreases with frequency and generally increases with voltage, although the voltages at R_4 are not off by that much.

Using low voltages and removing R_4 and R_3 from the circuit, the applied and calculated voltages deviated considerably as shown in Table 5.

Tests were also run for the circuit in figure 26.

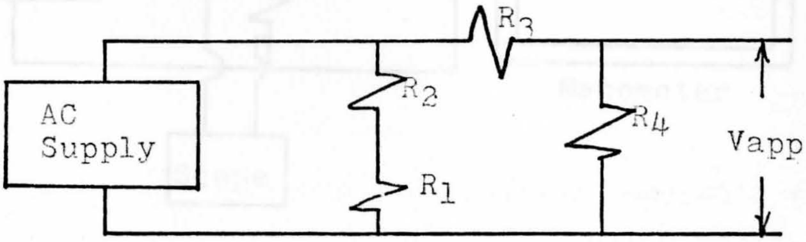


Fig. 26. Test Circuit

The position of R_3 was changed just to see how the results would compare with previous tests. Again, the same problem occurred.

Voltages were also read on a Honeywell Digitest voltmeter and they agreed with the oscilloscope readings.

Three steps were taken to alleviate the problem of inconsistant readings. First, all grounds were double checked and found to be adequate. Second, in an attempt to eliminate noise, the coil was shielded. This was not successful since an appreciable amount of noise was still picked up by the scope. Third, all wires were made as short as possible to eliminate noise and showed no effect.

As a final check, the apparatus was set up and a test was run to see how the actual and theoretical pressure values would compare. To do this, two parallel plate electrodes were used. The 112 to 1 voltage divider was used to determine the applied voltage and the manometer, to measure the pressure, as shown in figure 27. A photograph of the parallel plates and test cells appears in Appendix C.

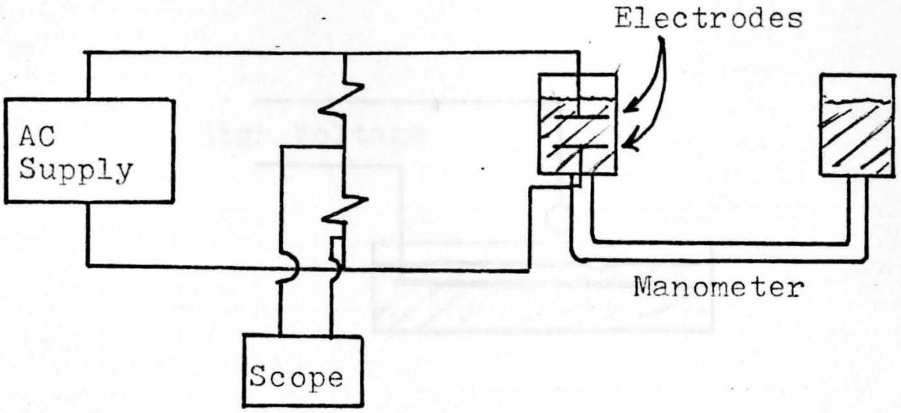


Fig. 27. Pressure Measurement

The distance between the plates was set at 0.75 inch. Using transformer oil, with a specific gravity of 0.87 and a dielectric constant of 2.5, the results are shown below,

Indicated Voltage (rms)	Bubble Displacement (inch)	Actual Pressure (psi)	Calculated Pressure (psi)
7,071	.10	4.47×10^{-6}	2.0×10^{-4}
10,607	.10	4.47×10^{-6}	4.5×10^{-4}
14,142	.16	7.15×10^{-6}	8.0×10^{-4}

The pressures are errant by as much as 10^{-2} psi. The measured pressures also do not indicate the fact that the pressure is proportional to V^2. This reinforces the fact that the voltage measurements are in error.

Although the attempt to measure the body force due to dielectrophoresis proved to be futile, some other phenomena were observed. The first of which is shown in figure 28. The upper electrode is spherical and the lower is a plate. Upon

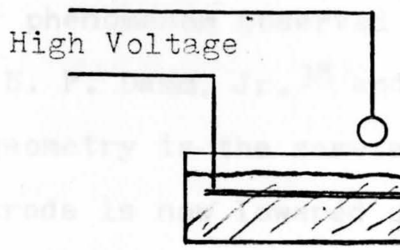


Fig. 28. Sphere and Plate Electrodes.

applying a high AC voltage (approximately 20,000 volts at 8,000 cycles/sec), repulsion of the transformer oil appeared to occur. This is shown in figure 29 and by a photograph in Appendix C. This appears to be an attraction caused by the divergent field, rather than a repulsion of the liquid as one might expect from a DC source as reported by H. A. Pohl[36] and E. T. Pierce[37]. The crater being created, as the liquid is dielectrophoretically attracted towards the spherical electrode. Thus the rim of the crater is actually rising creating the crater rather than there being a force on the liquid creating the rim. The fact that the liquid level appears to be the same on either side of the rim verifies

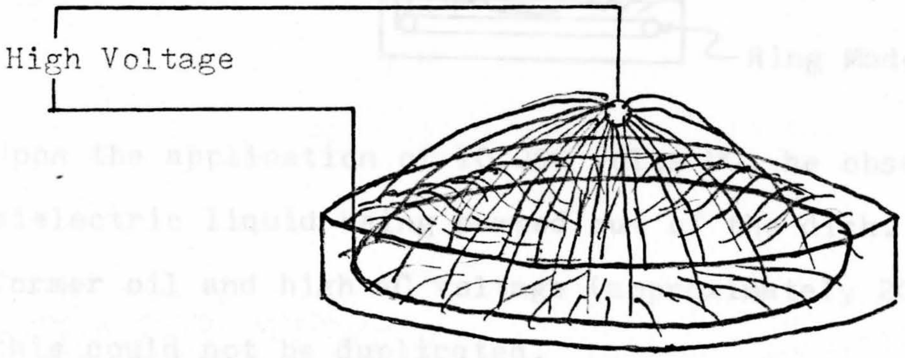
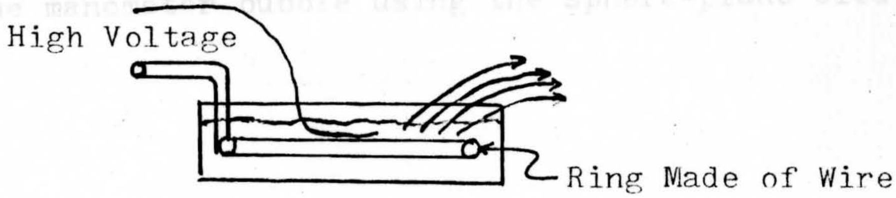


Fig. 29. Sphere and Plate Electrodes with Applied Field

this. It should also be noted that the rim rose as the voltage increased.

Another phenomenom observed is that investigated extensively by E. P. Damm, Jr.[38] and also noted by H. A. Pohl[39]. The electrode geometry is the same as the previous example. The upper electrode is now lowered until the liquid jumps the gap and clings to the electrode. This is shown by a photograph in Appendix C. Gaps of up to .50 inch were jumped in this manner using transformer oil. H. A. Pohl also reported a pumping action using the electrode geometry shown below.



Upon the application of 10,000 volts DC, he observed the dielectric liquid being pumped out of the dish. Using transformer oil and high AC voltage (approximately 20,000 volts), this could not be duplicated.

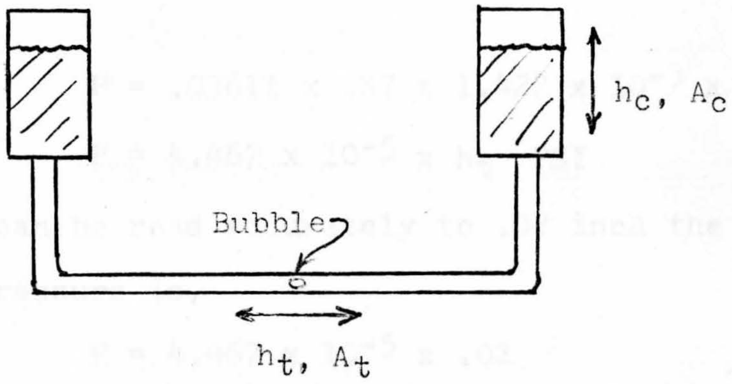
Summary

Although the experimental aspect proved unsuccessful, considerable insight was gained as to the nature of electrofluidmechanics. The field is very diverse and consists of very simple to very complex and subtle reactions. The specific problem encountered here, that of inaccurate measurements, with more time, better equipment and expertise can be overcome. Measuring the body force by means of a pressure difference has not been attempted in as much as I have been able to determine. That such a force exists was apparent by movement of the manometer bubble using the sphere-plane electrode geometry.

APPENDIX A

Pressure Calculation

The manometer sensitivity is determined as follows:



h_c = Change in liquid level in cell

h_t = Distance bubble moves in tube

A_c = Area of test cell

A_t = Area of tube

D_t = I.D. of tube = .066 inch

D_c = I.D. of cell = 1.75 inches

Knowing $A_1 h_1 = A_2 h_2$

$$\frac{\pi D_c^2 h_c}{4} = \frac{\pi D_t^2 h_t}{4}$$

$$\frac{h_c}{h_t} = \left(\frac{D_t}{D_c}\right)^2$$

Thus

$$\frac{h_c}{h_t} = \left(\frac{.066}{1.75}\right)^2 = 1.422 \times 10^{-3}.$$

The increase in pressure from that at the free surface is,

$$P = .036\, S h_c \text{ PSI}$$

$$S = \text{Specific Gravity.}$$

For transformer oil,

$$S = .87$$

$$h_c = 1.422 \times 10^{-3} \times h_t$$

thus,

$$P = .03611 \times .87 \times 1.422 \times 10^{-3} \times h_t$$

$$P = 4.467 \times 10^{-5} \times h_t \text{ PSI}$$

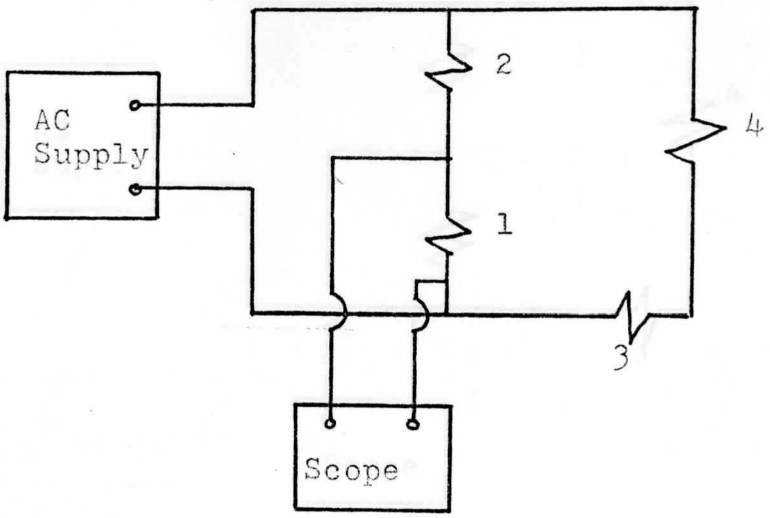
Assuming h_t can be read accurately to .02 inch the smallest detectable pressure is,

$$P = 4.467 \times 10^{-5} \times .02$$

$$P = 8.934 \times 10^{-7} \text{ PSI.}$$

APPENDIX B

Tables



$$V_4 = \frac{R_4}{R_3+R_4}\left[\frac{R_1+R_2}{R_1}\right]V_1$$

$$\frac{V_3}{V_1} = \frac{R_3}{R_3+R_4}\left[\frac{R_1+R_2}{R_1}\right]$$

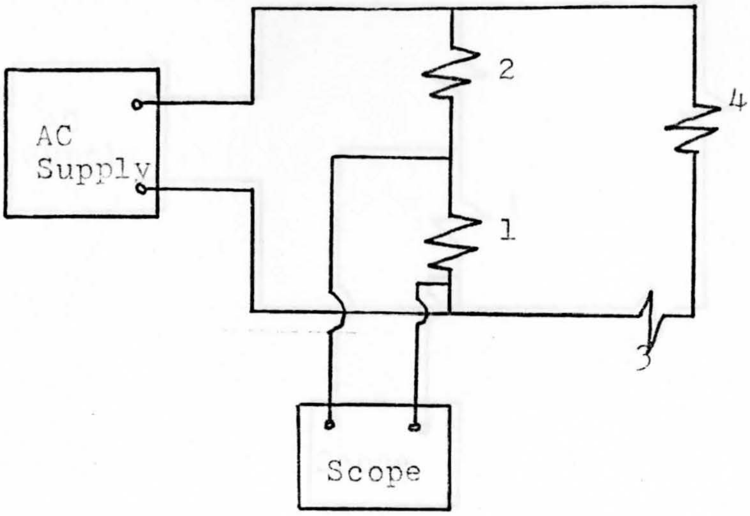
R_1 = .1 MΩ R_4 = 11.1 MΩ

R_2 = 11.1 MΩ $V_4 = 111\ V_1$

R_3 = .1 MΩ $V_3/V_1 = 1$

TABLE 1

Freq. (Hertz)	V_1 (Volts)	V_3 (volts)	V_3/V_1	V_4 (Calc.)	V_4 (Act.)	% Error In V_4
2000	1.12	1.12	1	124	130	- 4.8
2000	2.35	2.35	1	261	270	- 3.4
4000	1.35	1.35	1	150	160	- 6.7
6000	2.3	2.3	1	255	300	-17.6
6500	2.25	2.25	1	250	305	-22
6750	2.3	2.3	1	255	310	-21.6
8000	3.5	3.5	1	389	300	+22.9
8000	3.6	3.6	1	400	300	+25.0
8250	4.2	4.2	1	466	315	+32.4



$$V_4 = \frac{R_4}{R_3+R_4}\left[\frac{R_1+R_2}{R_1}\right]V_1$$

$$\frac{V_3}{V_1} = \frac{R_3}{R_3+R_4}\left[\frac{R_1+R_2}{R_1}\right]$$

$R_1 = .1\ \text{M}\Omega$ $R_4 = 22.2\ \text{M}\Omega$

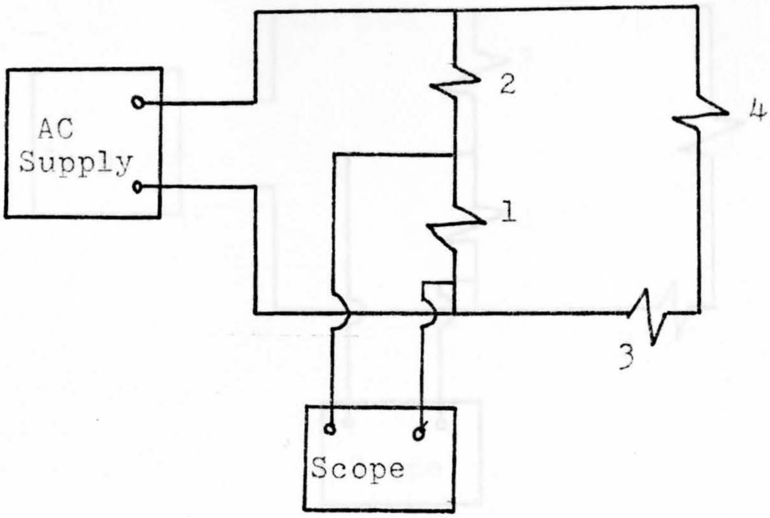
$R_2 = 11.1\ \text{M}\Omega$ $V_4 = 111\ V_1$

$R_3 = .1\ \text{M}\Omega$ $V_3/V_1 = .502$

TABLE 2

Freq. (Hertz)	V_1 (volts)	V_3 (volts)	V_3/V_1	V_4 (calc.)	V_4 (act.)	% Error In V_4
1000	1.04	.54	.52	115	118	-2.6
1000	1.65	.84	.51	183	190	-3.8
1000	2.2	1.1	.5	244	255	-4.5
1000	2.55	1.3	.51	283	295	-4.2
2000	.84	.54	.64	93	122	-31.2
2000	1.6	.84	.53	178	190	-6.7
2000	2.55	1.3	.51	283	295	-4.2
4000	1.3	.72	.55	144	155	-7.6
4000	1.95	1.04	.53	216	230	-6.5
4000	2.5	1.35	.54	278	295	-6.1
8000	5.8	4.1	.71	644	*	-
8000	11.0	7.8	.71	1221	*	-
8000	3.1	22.5	.73	3441	*	-

*Readings out of range of scope



$$V_4 = \frac{R_4}{R_3+R_4}\left[\frac{R_1+R_2}{R_1}\right]V_1$$

$$\frac{V_3}{V_1} = \frac{R_3}{R_3+R_4}\left[\frac{R_1+R_2}{R_1}\right]$$

$R_1 = .1\ M\Omega$ $R_4 = 50.22\ M\Omega$

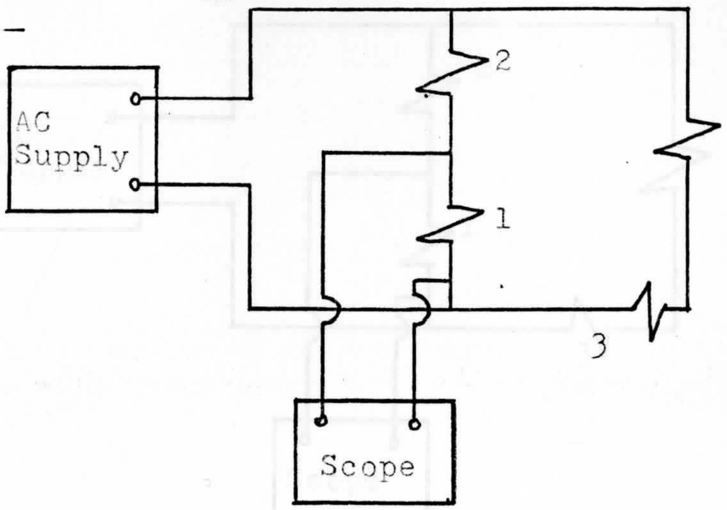
$R_2 = 11.1\ M\Omega$ $V_4 = 110\ V_1$

$R_3 = 1\ M\Omega$ $V_3/V_1 = 2.19$

TABLE 3

Freq. (Hertz)	V_1 (Volts)	V_3 (Volts)	V_3/V_1	V_4 (Calc.)	V_4* (Act.)	% Error In V_4
1000	25	53	2.12	2750		
1000	49	110	2.24	5390		
1000	82	200	2.44	9020		
1000	96	240	2.50	10560		
2000	2.25	4.8	2.13	248		
2000	9	20	2.22	990		
2000	13	29	2.23	1430		
2000	20	45	2.25	2200		
4000	10.8	29	2.67	1188		
4000	21	54	2.57	2310		
4000	34	88	2.59	3740		
4000	47	130	4.9	5170		
6000	20	62	3.10	2200		
6000	27	84	3.11	2970		
6000	35	110	3.14	3850		
6000	48	155	3.23	5280		

*Readings out of range of scope



$$V_4 = \frac{R_4}{R_3+R_4}\left[\frac{R_1+R_2}{R_1}\right]V_1$$

$$\frac{V_3}{V_1} = \frac{R_3}{R_3+R_4}\left[\frac{R_1+R_2}{R_1}\right]$$

R_1 = .1 MΩ R_4 = 50.22 MΩ

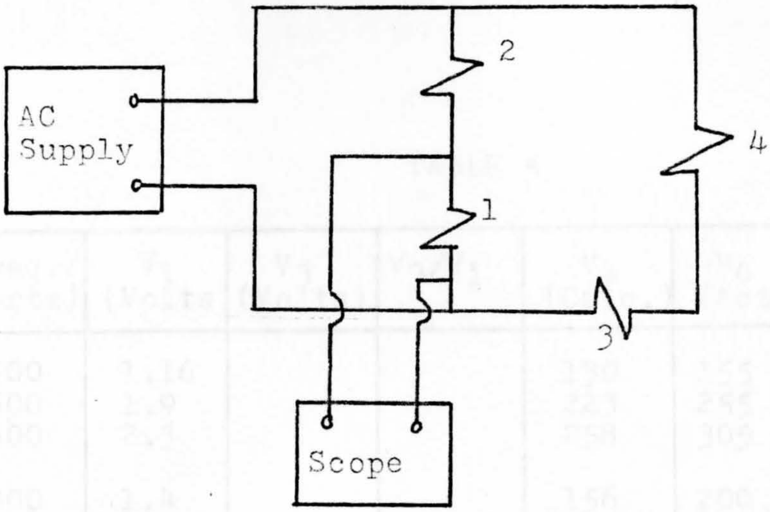
R_2 = 11.1 MΩ V_4 = 104 V_1

R_3 = 4 MΩ V_3/V_1 = 8.26

TABLE 4

Freq. (Hertz)	V_1 (Volts)	V_3 (Volts)	V_3/V_1	V_4 (Calc.)	V_4 (Act.)	% Error In 4
1000	1.08	6.2	5.7	112	125	-11.6
1000	1.6	9.2	5.8	166	185	-11.4
1000	2.1	10.4	5.0	218	240	-10.1
1000	2.45	14	5.7	255	285	-11.8
2000	1.04	6.6	6.3	108	120	-11.1
2000	1.85	10.2	5.5	192	215	-12.0
2000	2.45	13.0	5.3	255	290	-13.7
4000	1.3	5.7	4.4	135	150	-11.1
4000	2.65	11.8	4.5	276	295	- 6.9
6000	2	7.4	3.7	208	250	-20.2
6000	2.35	8.8	3.7	244	300	-23.0
8000	6	22.5	3.8	624	*	-
8000	14.5	52	3.6	1508	*	-
8000	45	165	3.7	4680	*	-

*Readings out of range of scope



$$V_4 = \frac{R_4}{R_3+R_4}\left[\frac{R_1+R_2}{R_1}\right]V_1$$

$$\frac{V_3}{V_1} = \frac{R_3}{R_3+R_4}\left[\frac{R_1+R_2}{R_1}\right]$$

$R_1 = .1\ \mathrm{M}\Omega$ $R_4 = \infty$

$R_2 = 11.1\ \mathrm{M}\Omega$ $V_4 = 112\ V_1$

$R_3 = \infty$

TABLE 5

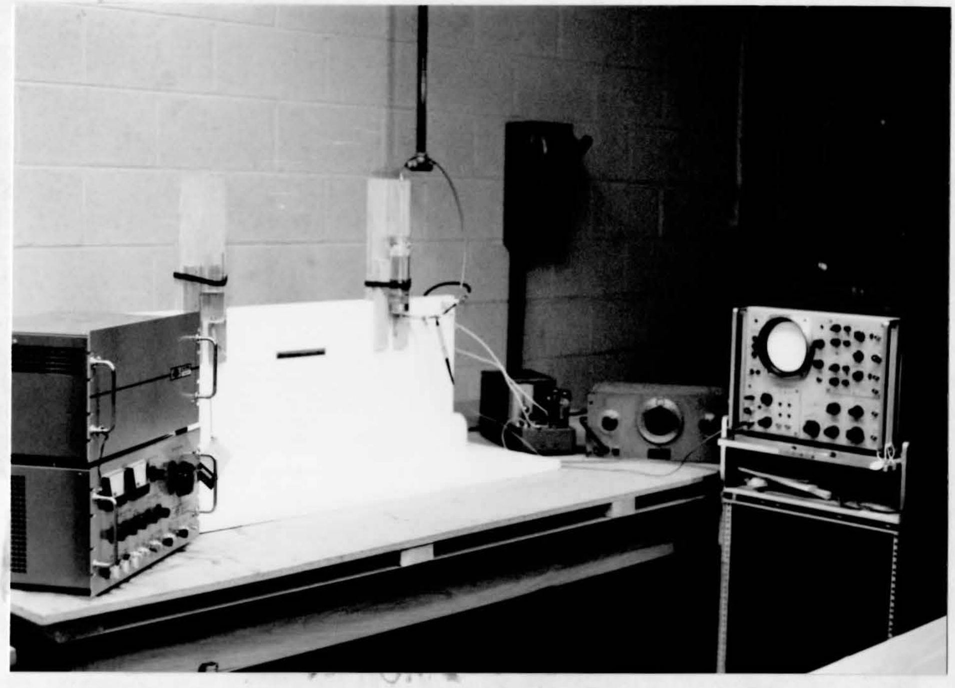
Freq. (Hertz)	V_1 (Volts)	V_3 (Volts)	V_3/V_1	V_4 (Calc.)	V_4 (Act.)	% Error In V_4
2000	1.04			117	120	-2.5
2000	2.05			230	240	-4.3
2000	2.6			291	305	-4.5
3000	1.08			121	130	-6.9
3000	1.9			223	225	-0.8
3000	2.6			291	305	-4.5
4000	1.2			134	145	-8.2
4000	2			224	240	-7.1
4000	2.5			280	300	-7.1
5000	1.4			156	175	-12.2
5000	1.9			223	235	-5.4
5000	2.45			274	300	-9.5
5500	1.16			130	150	-15.4
5500	1.6			179	200	-11.7
5500	2.05			230	260	-13.0
5500	2.35			263	300	-17.9
6000	1			112	130	-16.1
6000	1.6			179	205	-14.5
6000	2.35			263	305	-16.0

TABLE 5

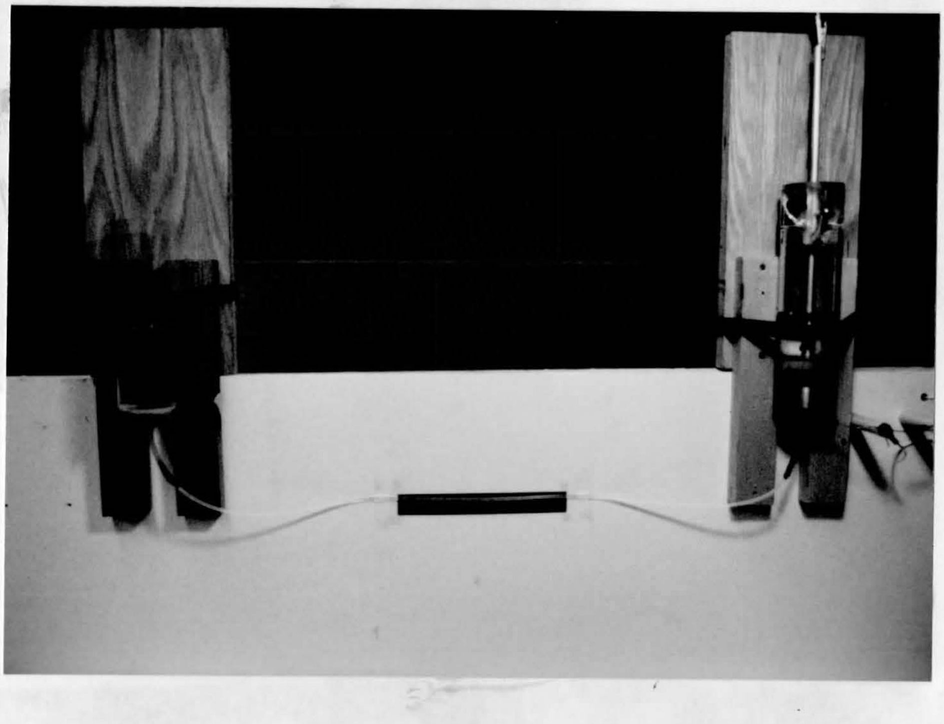
Freq. (Hertz)	V_1 (Volts)	V_3 (Volts)	V_3/V_1	V_4 (Calc.)	V_4 (Act.)	% Error In V_4
6500	1.16			130	155	-19.2
6500	1.9			223	255	-14.3
6500	2.3			258	305	-18.2
7000	1.4			156	200	-28.2
7000	2.15			241	300	-24.5
7500	1.85			207	275	-32.9
7500	2.2			246	320	-30.1
8000	2.4			269	320	-19.0
8500	3.6			403	305	+24.3

APPENDIX C

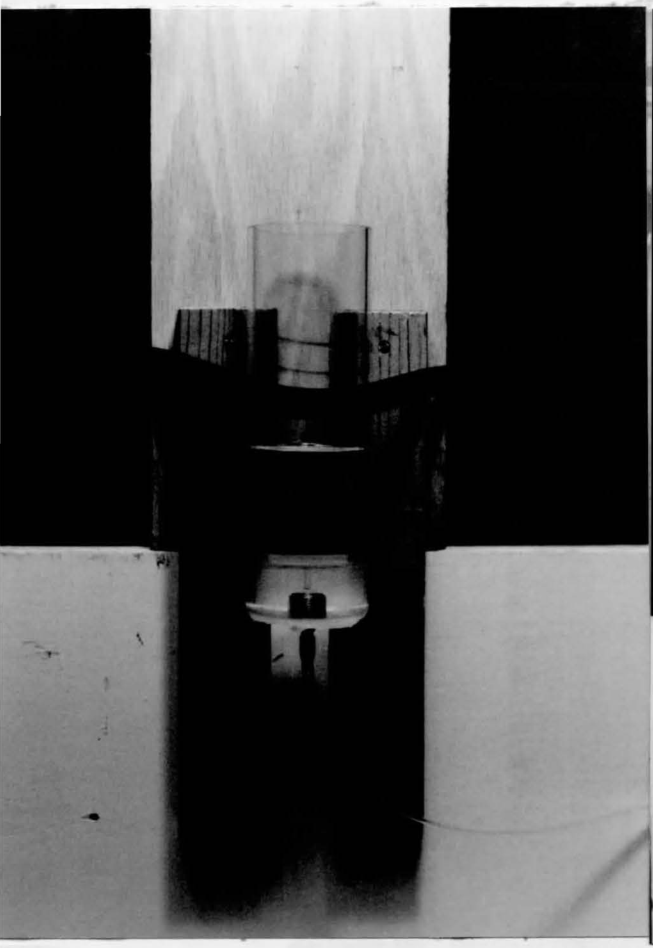
Photographs



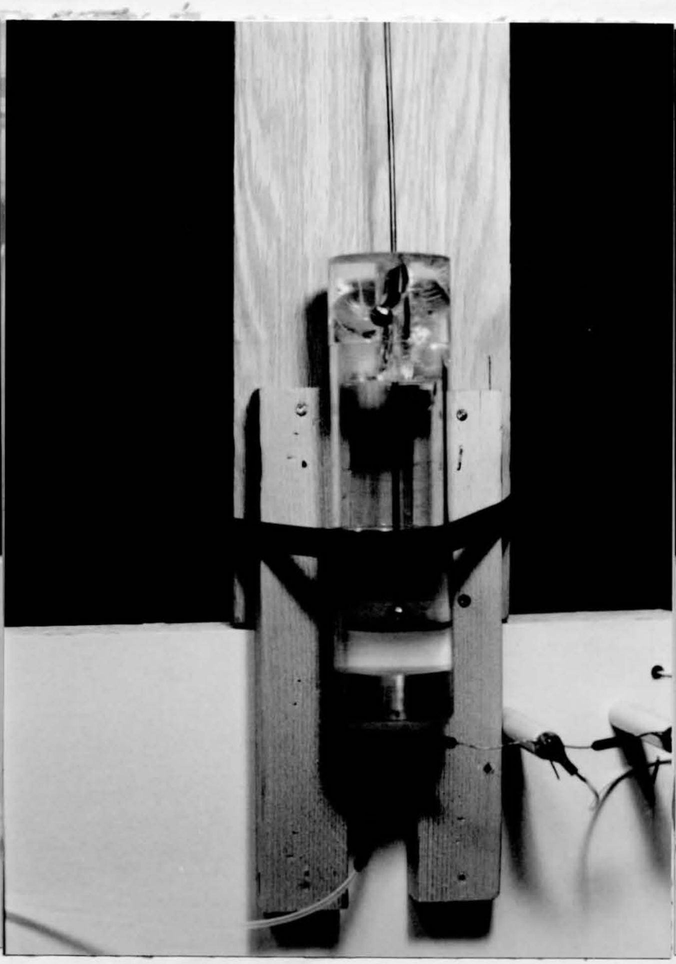
Experimental Apparatus



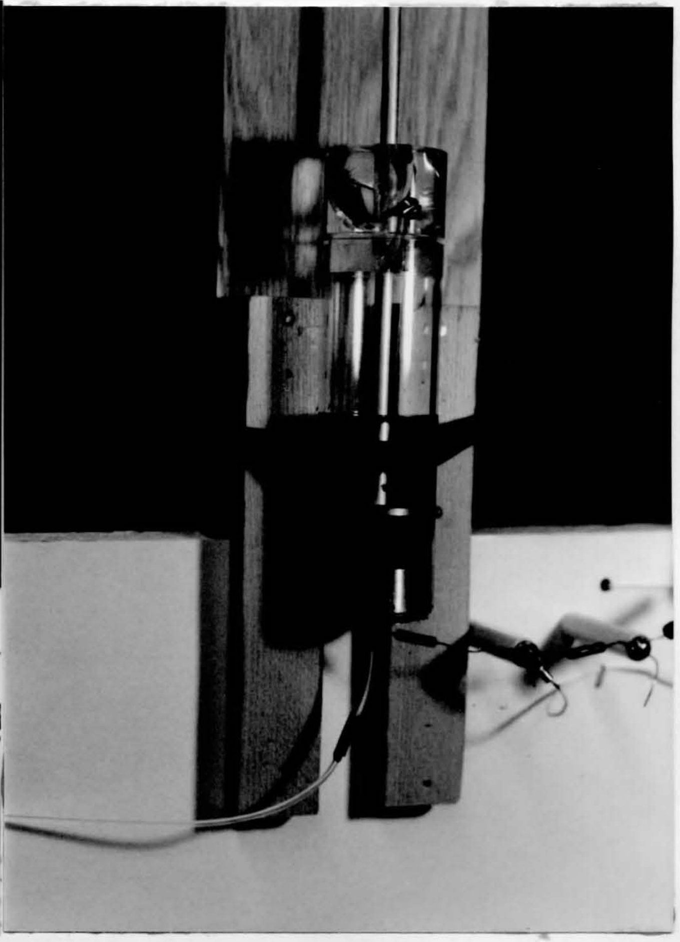
Manometer and Electrode Close-up



Dummy Cell

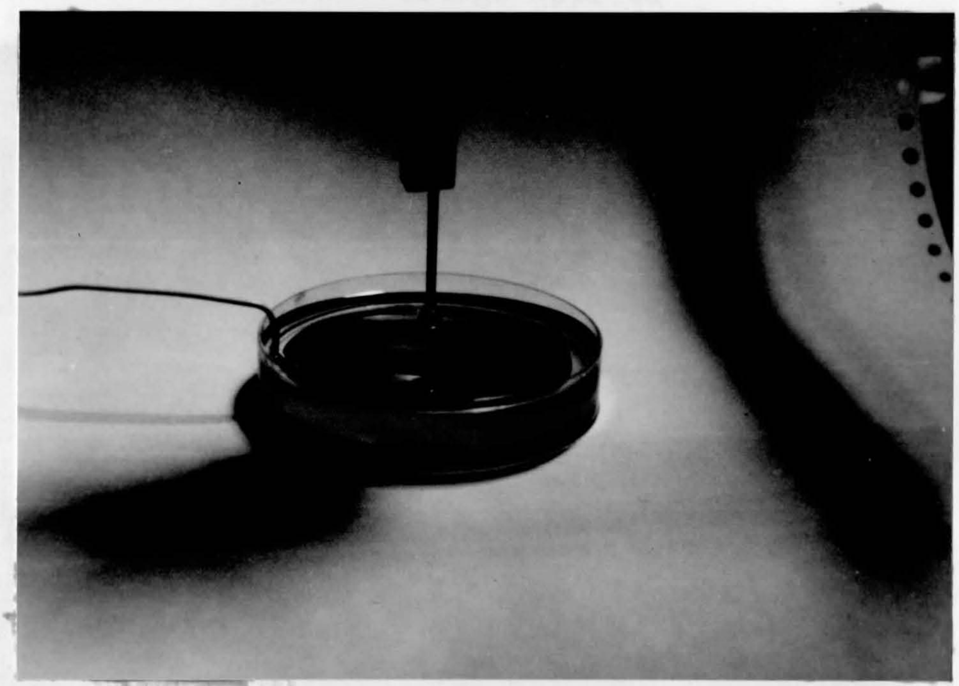


Test Cell With Sphere Plane Electrodes

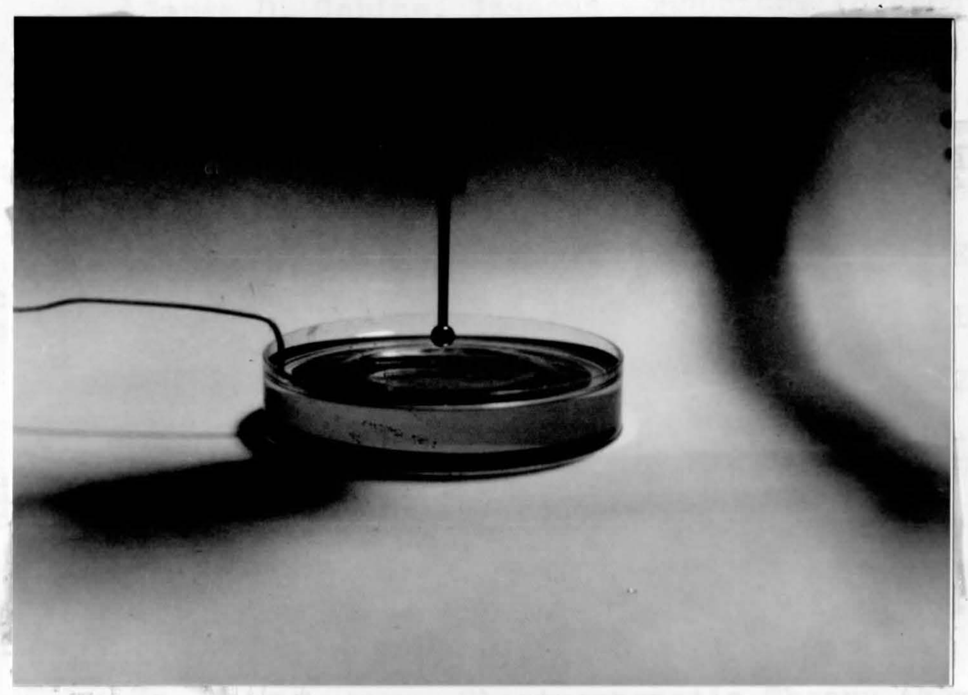


Test Cell With Parallel Plate Electrodes

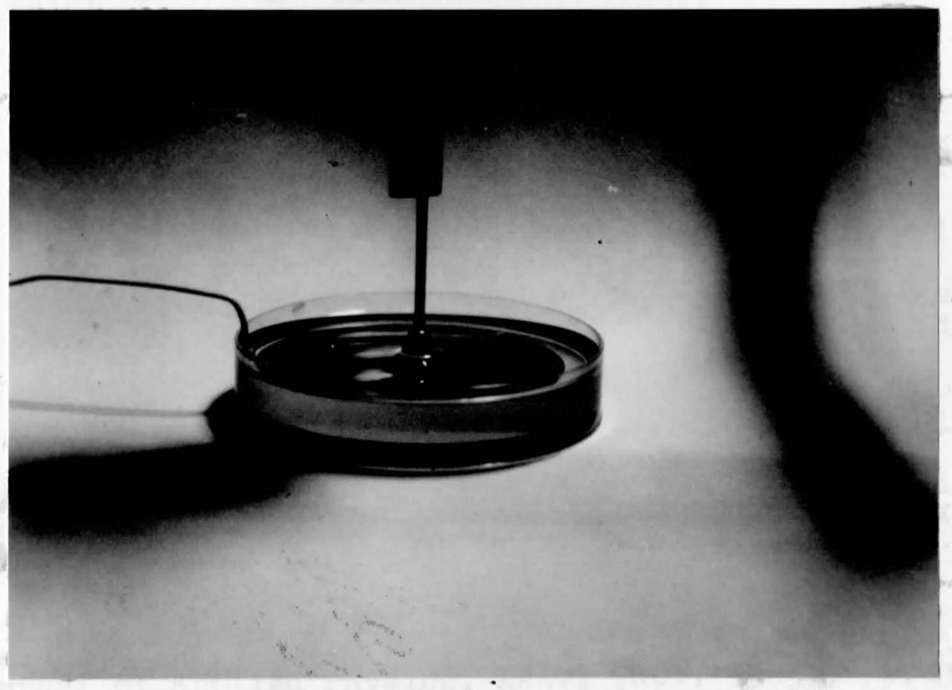
Sphere-Plate Electrodes



Sphere-Plate Electrodes with
Low AC Voltage Applied



Sphere-Plate Electrodes with
High AC Voltage Applied



Sphere-Plate Electrodes Showing
Attraction of Dielectric Liquid

FOOTNOTES

[1] James D. Cobine, Gaseous Conductors (Second Edition: New York, N. Y.: Dover Publications. Inc., 1958), pp. 33-41.

[2] Henry R. Velkoff, "Electrofluidmechanics: A Study of Electrokinetic Actions in Fluids", ADK-TR-61-642. Propulsion Laboratory, ASK, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio (Feb., 1962), p. 13.

[3] Robert J. Heinsohn and Philip M. Becker, "Electrically Augmented Flame", Center for Air Environment Studies, The Pennsylvania State University, (July 1969), pp. 10-12.

[4] Velkoff, "Electrofluidmechanics", pp. 21-22.

[5] Paul Lorrain and Dale Corson, Electromagnetic Fields and Waves (Second Edition: San Francisco, Calif.: W. H. Freeman and Company, 1970), pp. 125-126.

[6] H. A. Pohl, "Some Effects of Nonuniform Fields on Dielectrics", Journal of Applied Physics, XXIX (Aug., 1958), pp. 1182-1183.

[7] J. A. Stratton, Electromagnetic Theory (New York: McGraw Hill, 1941), p. 139.

[8] Stratton, Electromagnetic Theory, p. 95.

[9] Stratton, p. 151.

[10] Pohl, "Effects on Dielectrics", pp. 1182-1188.

[11] Pohl, "Effects on Dielectrics", pp. 1182-1188.

[12] Pohl, "Effects on Dielectrics", pp. 1182-1188.

[13] H. A. Pohl and J. P. Schwar, "Factors Affecting Separations of Suspensions in Nonuniform Electric Fields", Journal of Applied Physics, XXX (Jan., 1959), pp. 69-73.

[14] Pohl, "Effects on Dielectrics", pp. 1184-1187.

[15] E. T. Pierce, "Effects of High Electric Fields on Dielectric Liquids", Journal of Applied Physics, XXX (1959), pp. 445-446.

[16] Hans Schott and W. X. Kaghan, "Effects High Electric Fields on Dielectric Liquids of High and Low Viscosity", Journal of Applied Physics, XXXVI (Nov. 1965), pp. 3399-3401.

[17] E. P. Damm, Jr., "Attractions of Liquids by Electric Fields", *Dielectrophoretic Deposition*, (New York: Electrodeposition Division Electrochemical Society, 1969), pp. 96-101.

[18] W. P. Cropper and H. S. Seelig, "Mixing With an Electrostatic Field", *I & EC Fundamentals*, I (Feb. 1962), pp. 48-52.

[19] D. R. Brown, "Swirling of a Liquid Under the Influence of a Uniform Electric Field", *Nature*, CCII (May, 1964, pp. 868-870.

[20] Vadim G. Drozin, "The Electrical Dispersion of Liquids as Aerosols", *Journal of Colloid Science*, X (1955), pp. 158-164.

[21] Bernard Vonnegut and Raymond L. Neubauer, "Production of Monodisperse Liquid Particles by Electrical Atomization", *Journal of Colloid Science*, VII (Oct. 1952), pp. 616-622.

[22] F. J. Weinberg, "Electrical Aspects of Aerosol Formation and Control", *Pro. Roy. Soc.*, A CCCVII (Oct. 1968), pp. 195-208.

[23] Hajime Fujimoto, et al, "Liquid Atomization in Electrostatic Field", *Japan Society of Mechanical Engineers Bulletin*, X (Feb. 1967), pp. 155-163.

[24] J. J. Hogan and C. D. Hendricks, "Investigation of the Charge-to-Mass Ratio of Electrically Sprayed Liquid Particles", *AIAA Journal*, III (Feb. 1965), pp. 296-301.

[25] Keith Watson, "Influence of an Electric Field upon the Heat Transfer from a Hot Wire to an Insurlating Liquid", *Nature*, CXIC (Feb. 1961), pp. 563-564.

[26] M. J. Gross and J. E. Porter, "Electrically Induced Convection in Dielectric Liquids", *Nature*, CCXII, (Dec. 1966), pp. 1343-1345.

[27] H. R. Velkoff, "Electrofluidmechanics: Investigation of the Effects of Electrostatic Fields on Heat Transfer and Boundary Layers", ASD-TDR-62-650., Aeronautical Systems Division, Air Force Systems Command, WPAFB, Ohio, Sept. 1962

[28] M. E. Franke, "Effect of Vortices Induced by Corona Discharge on Free-Convection Heat Transfer From a Vertical Plate", *Journal of Heat Transfer*, (Aug. 1969), pp. 427-433.

[29] H. R. Velkoff and J. Miller, "Condensation of Vapor on a Vertical Plate with a Transverse Electrostatic Field", *Journal of Heat Transfer*, XIIC (May 1965).

[30] H. Y. Choi, "Electrohydrodynamic Condensation Heat Transfer", *Journal of Heat Transfer*, XC (Feb. 1968), pp. 98-102.

[31] E. Bonjour, J. Verdier, and L. Weil, "Electroconvection Effects on Heat Transfer", *Chemical Engineering Progress*, LVII (July 1962), pp. 63-66.

[32] E. B. Devitt and Jr. R. Melcher, "Surface Electrohydrodynamics with High-Frequency Fields", *Research Notes*, (1965), pp. 1193-1195.

[33] James R. Melcher, David S. Guttman, and Mathew Hurwitz, "Dielectrophoretic Orientation", *Journal of Spacecraft and Rockets*, VI, (Jan. 1969), pp. 25-32.

[34] James R. Melcher, Mathew Hurwitz, and Ruth Fax, "Dielectrophoretic Liquid Expulsion", *Journal of Spacecraft and Rockets*, VI (Sept. 1969), pp. 961-967

[35] R. J. Heinsohn and P. M. Becker, "Electrically Augmented Flame", Center for Environment Studies, Pennsylvania State University, Pub. No. 117-69, July 1969.

[36] Pohl, "Effects on Dielectrics", pp. 1182-1188.

[37] Pierce, Effects of Fields on Dielectric Liquids", pp. 445-446.

[38] Damm, pp. 96-101.

[39] Pohl, "Effects on Dielectrics", pp. 1182-1188.

BIBLIOGRAPHY

Books

Bier, M. (ed). Electrophoresis. New York: Academic Press, 1959.

Cobine, James D. Gaseous Conductors. Second Edition. New York: Dover Pub. Inc., 1958.

Halliday, David and Resnick, Robert. Physics. Second Edition. Vol. II. New York: John Wiley and Sons, Inc., 1959.

Lorrain, P. and Corson, D. Electromagnetic Fields and Waves. San Francisco: W. H. Freehan and Co., 1970.

Melcher, J. R. Field Coupled Surface Waves. M. I. T. Press, 1963.

Stratton, J. A. Electromagnetic Theory. New York: McGraw Hill, 1941.

Woodson, H. H. and Melcher, Jr. R. Electromechanical Dynamics. New York: John Wiley and Sons, 1968

Technical Papers

Bonjour, E., Verdier, J., and Weil, L. "Electroconvection Effects on Heat Transfer." Chemical Engineering Progress, LVIII (1962).

Brown, D. R. "Swirling of a Liquid Under the Influence of a Uniform Electric Field." Nature, CCII (1964), 868 - 870.

Choi, H. Y. "Electrohydrodynamic Condensation Heat Transfer." Journal of Heat Transfer, XC (1968, 98 - 102.

Cropper, W. P. and Seelig, H. S. "Mixing With An Electrostatic Field." I. and E. C. Fundamentals, I (1962), 48 - 52.

Damm, E. P. Jr. "Attractions of Liquids by Electric Fields." Dielectrophoretic Deposition, (1969), 96 - 101.

Devitt E. B. and Melcher J. R. "Surface Electrohydrodynamics With High Frequency Fields." Research Notes, (1965), 1193 - 1195.

Drozin, Vadim G. "The Electrical Dispersion of Liquids as Aerosols." Journal of Colloid Science, X (1955), 158 - 164.

Franke, M. E. "Effect of Vortices Induced by Corona Discharge on Free-Convection Heat Transfer From a Verticle Plate." Journal of Heat Transfer, XC (1968, 98 - 102.

Fujimoto, Hajime, Et. Al. "Liquid Atomization in Electrostatic Field." Japan Society of Mech. Engineers Bulletin, S (1967), 155 - 163.

Gross M. J. and Porter, J. E. "Electrically Induced Convection in Dielectric Liquids." Nature, CCXII (1966), 1343 - 1345.

Heinsohn, Robert J. and Becker, Philip M. "Electrically Augmented Flame." Pennsylvania State University. July, 1969.

Hogan, J. J. and Hendricks, C. D. "Investigation of the Charge-to-Mass Ratio of Electrically Sprayed Liquid Particles." AIAA Journal, III (1965), 296 - 301.

Melcher, James R., Guttman, David S., and Hurivity, Mathew. "Dielectrophoretic Orientation." Journal of Spacecraft and Rockets, VI (1969), 25 - 32.

Melcher James, R., Hurwitz, Mathew, and Fax, Ruth. "Dielectrophoretic Liquid Expulsion." Journal of Spacecraft and Rockets, VI (1969), 961 - 967.

Pierce, E. T. "Effects of High Electric Fields on Dielectric Liquids." Journal of Applied Physics, XXX (1959), 96 - 101.

Pohl, H. A. "Some Effects of Nonuniform Fields on Dielectrics." Journal of Applied Physics, XXIX (1958), 1182 - 1188.

Pohl H. A. and Schwar J. P. "Factors Affecting Separations of Suspensions in Nonuniform Electric Fields." Journal of Applied Physics, XXX (1959), 69 - 73.

Schott, Hans and Kaghan, W. X. "Effects of High Electric Fields on Dielectric Liquids of High and Low Viscosity." Journal of Applied Physics, XXXVI (1965), 3399 - 3401.

Velkoff, Henry R. "Electrofluidmechanics: A Strudy of Electrokinetic Actions in Fluids." ASK-TR-61-642. Wright-Patterson Air Force Base, (1962).

Velkoff, H. R. "Electrofluidmechanics: Investigation of the Effects of Electrostatic Fields on Heat Transfer and Boundry Layers." ASD-TDR-62-650. Wright-Patterson Air Force Base, (1962).

Vonnegut, Bernard and Neubauer, Raymond L. "Production of Monodisperse Liquid Particles by Electrical Atomization." Journal of Colloid Science, VII (1952), 616 - 622.

Watson, P. Kieth. "Influence of an Electric Field Upon the Heat Transfer from a Hot Wire to an Insulating Liquid." Nature, CXIC (1961), 563 - 564.

Weinberg, F. J. "Electrical Aspects of Aersol Formation and Control." Pro. Roy. Soc., ACCCVIII (1968), 195 - 208.