COMPUTER SIMULATION OF A HYDRAULIC

RELIEF VALVE

by

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ABSTRACT

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Demands of the space age brought about new technologies to meet the variety of exacting problems that were encountered. One of these new concepts was the utilization of the computer to perform mathematical simulations of the systems or individual components. A typical industrial application of computer simulations will be demonstrated here by establishing a mathematical model of a hydraulic relief valve.

The method is based on the assumption that relief valves can be represented by a spring-mass system. An iterative type program was set up on a digital computer to calculate displacement, velocity, and acceleration of the poppet along with pressure and flow for each time increment following the occurrence of an initial blockage of flow in the hydraulic system. The procedure involves comparing an initially assumed displacement for each time increment, to the calculated displacement that was obtained from the information supplied by the assumed displacement for each time increment.

An existing relief valve design was utilized for this simulation to permit experimental verification of the assumptions and the computer results.

ACKNOWLEDGEMENTS

I wish to express my sincere thanks to Dr. Frank Tarantine for his guidance and assistance on this thesis, to Commercial Shearing Inc. for permitting me to use their laboratory facilities and to Mesta Machine Co. for the computer time.

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LIST OF SYMBOLS

SYMBOL	DEFINITION	UNITS
A	Area	in^2
A_F	Flow area of relief valve	in^2
A_P	Effective pressure area	in^2
C	Damping function	lb-sec/in
C_d	Discharge coefficient	
F	Force	lb.
F_D	Damping force	lb.
F_{MTG}	Spring force at mounting	lb.
F_S	Spring force	lb.
K	Spring rate	lb/in
M	Mass of poppet	$lb\text{-}sec^2/in$
N	Bulk modulus of elasticity	lb/in
P	Pressure	lb/in^2
Q	Flow	in^3/sec
Q_P	Pump flow	in^3/sec
T	Time	sec.
V	Velocity	in/sec
V'	Volume	in^3
X	Displacement or stroke of poppet	in.
$\dot{X}$	Velocity	in/sec
$\ddot{X}$	Acceleration	in/sec^2
ΔT	Change in time	sec
g	Gravitational constant	in/sec^2
γ	Weight density	lb/in^3
μ	Absolute viscosity	$lb\text{-}sec/in^2$

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CHAPTER I

INTRODUCTION

In the last thirty years, engineers have witnessed an enormous growth and change in technology. As a result, he must deal with increasing sophisticated machines and still meet all the requirements of safety and cost.

The engineer must use every skill and design tool available to accomplish the results expected of him.

One of the tools that he has at his disposal is the computer and its ability to perform simulations of physical systems. The computer was utilized extensively by the aerospace industries, whose goal was to land men on the moon. Computer simulation helps to optimize the design of the space hardware with the minimum of prototype construction and testing, thereby saving time and money as well as lives. The computers also ran simulators to train the astronauts. By having computerized simulations available, scientists and engineers were better equipped to handle unexpected problems by studying the results of their proposed solution before initiating them. This feature proved to be invaluable in bringing the men of Apollo 13 home after their near fatal accident in space.

The non-aerospace industries have accepted the computer but the technique of computer simulation has not been fully utilized. One industry that can benefit from computer simulation is the fluid power industry.

Fluid power refers to the utilization of pumped or compressed fluid as the working medium in producing force and motion to mechanisms. Liquid applications of fluid power are referred to as hydraulics while pneumatics involves gases. The hydraulic and pneumatic industries are involved to various degrees in the development of the generation, control and application of fluid power. Much of the design concepts that these industries rely on are based on experience, supported by extensive measurements for steady flow situations. In recent years, it has become necessary to analyze the dynamic behavior of the system or components.

This is the area where computer simulation can help. To demonstrate how a simulation program can be constructed, the hydraulic relief valve was selected to be the subject.

The relief valve function is to protect the hydraulic system from excessive pressure. Its design is usually based on static conditions and steady flow but it operates under dynamic conditions. Having the relief valve relieve at a designated pressure is not enough if the design is unstable and chattering results.

The simulation will be based on an existing relief valve to permit comparison between the computerized results and experimential data.

CHAPTER II

BASIC HYDRAULIC SYSTEM

The basic components of any hydraulic system are essentially the same regardless of their size or application. The five basic components are as follows:

1.--Reservoir.

2.--Pump.

3.--Tubing or piping (circulatory system).

4.--Control valve or selector valve.

5.--Actuating unit.

The primary purpose of a hydraulic reservoir is to provide storage space for the fluid being used in the hydraulic system. Additionally, the fluid temperature can be controlled by incorporating into the design of the reservoir the ability to dissipate energy in the form of heat.

The pump provides the means to circulate the fluid within the hydraulic system. The operating principle of the pump involves converting mechanical energy into fluid energy. The medium for transmitting the fluid is the circulatory system (tubing or piping). The control valve or selector valve is a device for directing and controlling the fluid flow or pressure within the system. The actuating unit of a hydraulic system converts the fluid energy supplied by the pump into mechanical energy so that useful work can be done.

These devices are all that are required for a basic system as shown in Fig. 1.

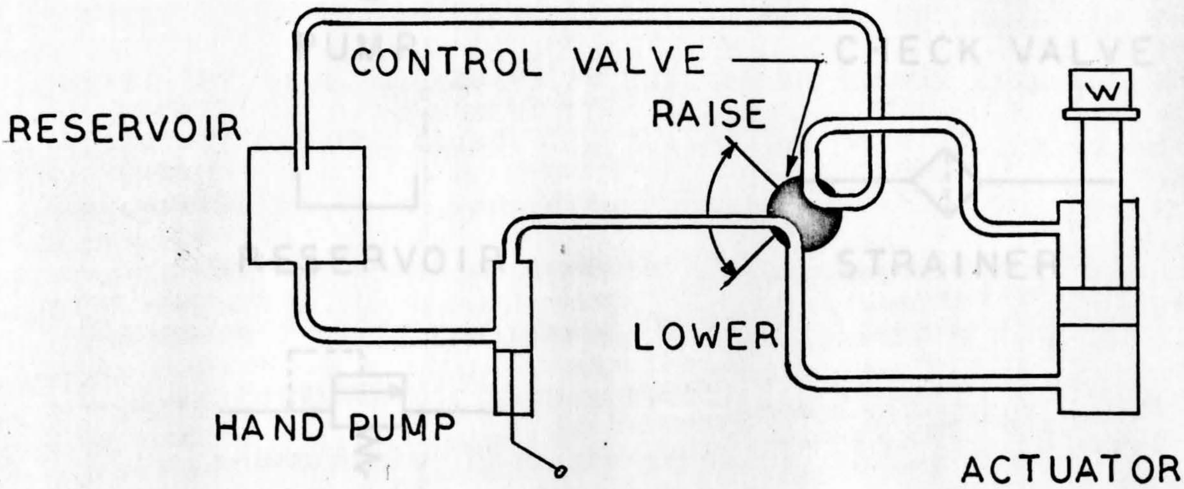


Fig. 1. Basic hydraulic system

The accessories to improve the system's function in advance applications are:

1. Relief valves to protect the system.
2. Filters for cleaning the fluid.
3. Pressure regulators to control pump pressure.
4. Accumulators to cushion shocks or maintain system pressure.
5. Check valves to permit flow in one direction only.

To aid in sketching a complex hydraulic circuit, designers use shorthand notations called J. I. C. (Joint Industrial Conference) symbols, some of which are shown in Fig. 2.

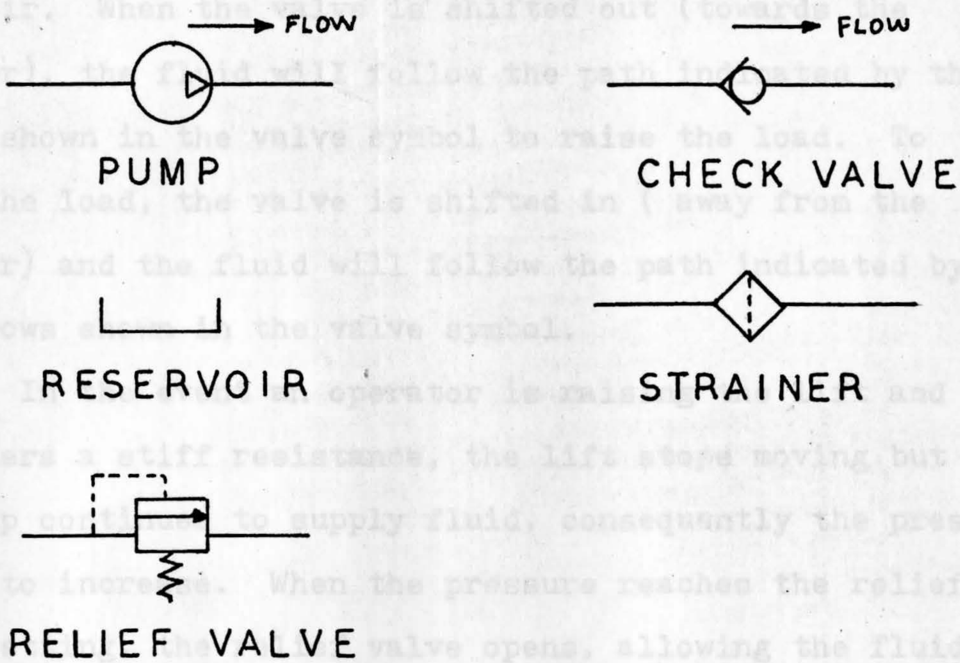


Fig. 2. J. I. C. symbols

The arrow in the relief valve symbol indicates direction of flow when relieving, see Fig. 3.

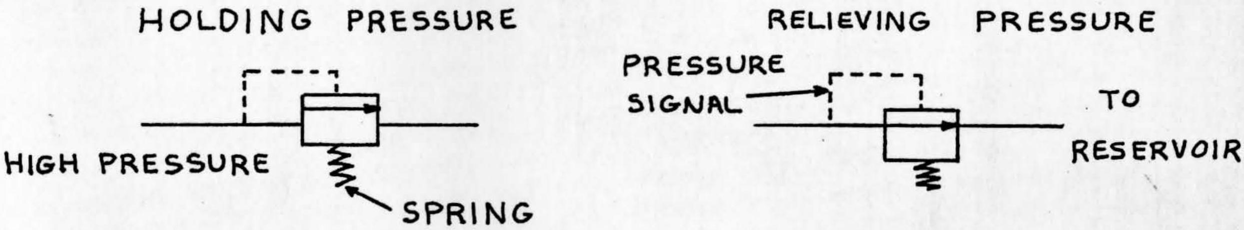


Fig. 3. Relief valve symbol

Using these accessories, a more functional system can be made such as for a lift truck shown on Fig. 4.

The pump will begin to function when the lift truck's engine is started and the power take-off engaged. The fluid will circulate from the pump, to the valve and back to the reservoir. When the valve is shifted out (towards the operator), the fluid will follow the path indicated by the arrows shown in the valve symbol to raise the load. To lower the load, the valve is shifted in (away from the operator) and the fluid will follow the path indicated by the arrows shown in the valve symbol.

In the event an operator is raising the lift and encounters a stiff resistance, the lift stops moving but the pump continues to supply fluid, consequently the pressure begins to increase. When the pressure reaches the relief valve setting, the relief valve opens, allowing the fluid to escape to tank. The check valve closes, preventing the load from dropping until the resistance is removed or the valve shifted. Once this occurs, the pressure drops and the relief valve closes allowing the lift truck to resume normal operation.

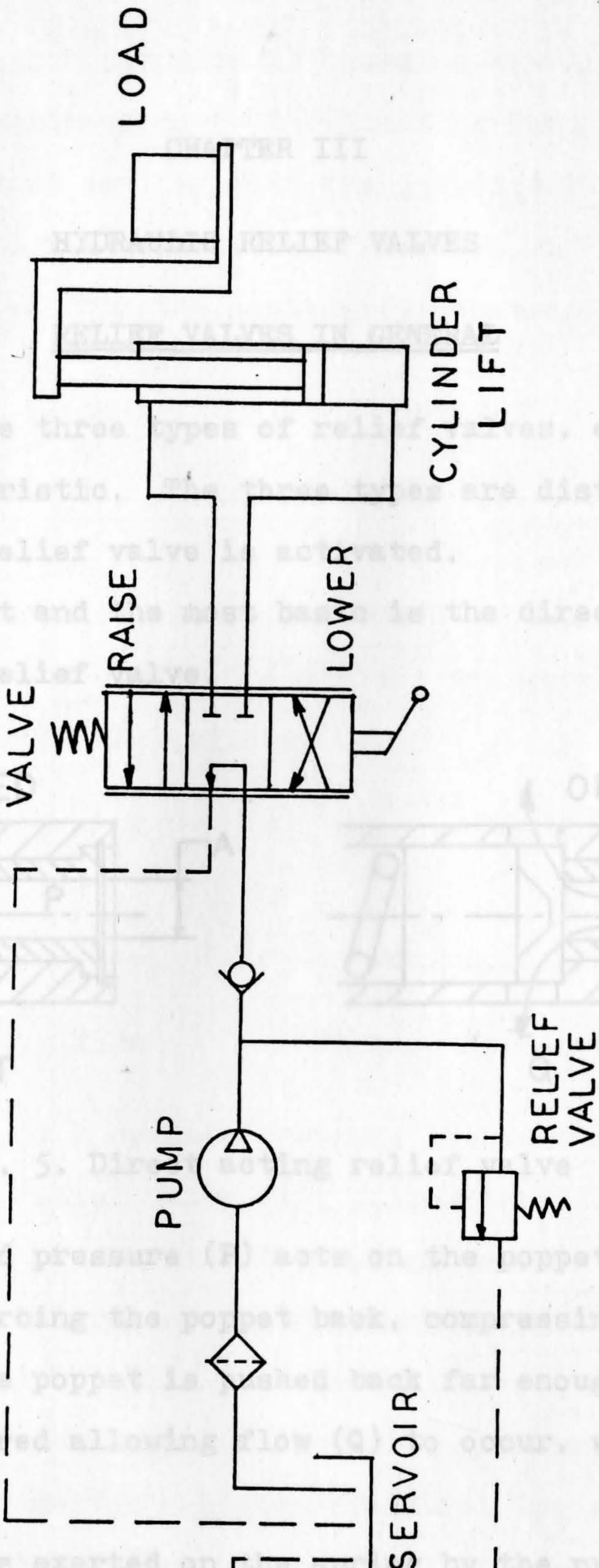


FIG. 4
LIFT TRUCK HYDRAULIC CIRCUIT

CHAPTER III

HYDRAULIC RELIEF VALVES

RELIEF VALVES IN GENERAL

There are three types of relief valves, each with its own characteristic. The three types are distinguished by the way the relief valve is activated.

The first and the most basic is the direct acting or brute force relief valve.

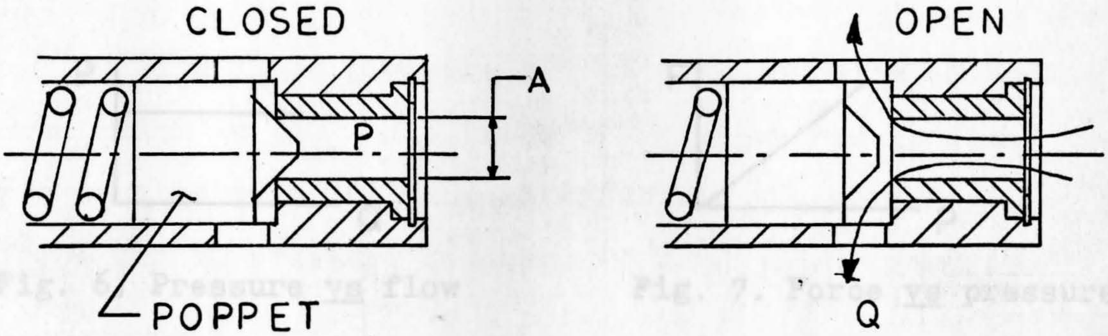


Fig. 5. Direct acting relief valve

The fluid pressure (P) acts on the poppet's effective area (A) forcing the poppet back, compressing the spring. When the poppet is pushed back far enough, the holes are uncovered allowing flow (Q) to occur, venting the system to tank.

The force exerted on the spring by the pressure is simply Pascal's Law, force is equal to pressure times area.

$$F = P A \tag{1}$$

The equation for flow is based on the entrance area (A) which is considered to be the limiting factor since it is usually designed smaller than the combined flow area of the exit holes on the side of the relief valve. The flow equation, obtained from the continuity equation, is given as :

$$Q = V A \tag{2}$$

where V is the velocity of the fluid through the entrance area.

The response of the relief valve to change in pressure is fast and direct. A typical pressure vs flow curve would resemble that of Fig. 6.

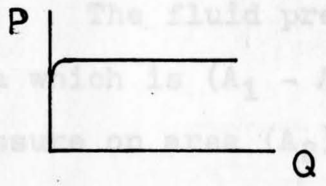


Fig. 6. Pressure <u>vs</u> flow

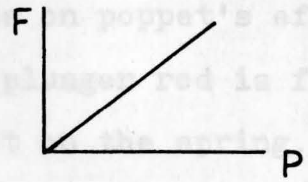


Fig. 7. Force <u>vs</u> pressure

If there is a dynamic pressure fluctuation within the system, this relief valve will follow that fluctuation due to the nature of the direct coupling between the pressure and the spring. Because of the pressure being directly proportional to force, Fig. 7., the relief valve must be limited to low pressure or low flow (low pressure drop) applications. To meet the higher pressure, the spring size must increase or the effective area must be decreased which in turn will increase the velocity of the fluid. This increase velocity can cause flow noise or instability.

The second type is a variation of the direct acting type and is called the differential relief valve. The design permits a higher pressure setting by reducing the effective area of the poppet and not changing the entrance area (A).

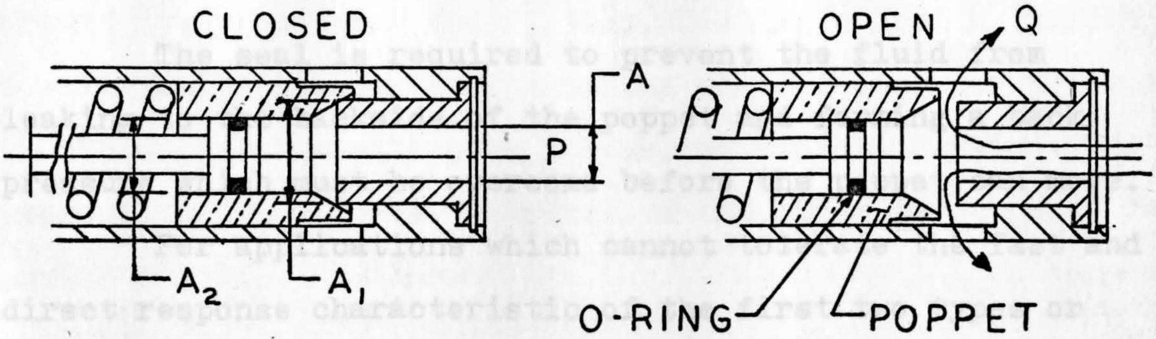


Fig. 8. Differential relief valve

The fluid pressure (P) acts on poppet's effective area which is (A_1 - A_2) since the plunger rod is fixed, the pressure on area (A_2) has no effect on the spring. The force due to pressure becomes,

$$F = P\ (A_1 - A_2) \tag{3}$$

while the flow will continue to be the same as the direct acting type,

$$Q = V\ A \tag{2}$$

where (V) is the velocity of the fluid through the orifice.

This relief valve has the same fast response time as the direct acting version. However, there is a hysteresis loop in the pressure vs flow curve due to the viscous drag of the o-ring seal on the plunger.

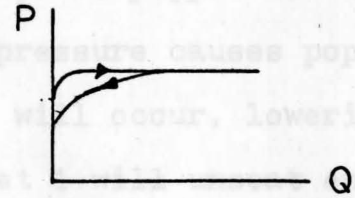


Fig. 9. Pressure vs flow

The seal is required to prevent the fluid from leaking to the backside of the poppet and forming a back pressure which must be overcome before the poppet can move.

For applications which cannot tolerate the fast and direct response characteristic of the first two types or require higher pressure setting in a smaller package, there is a third type called the pilot operated relief valve.

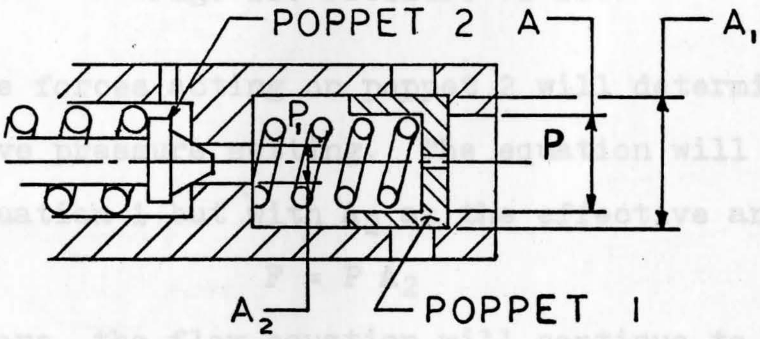


Fig. 10. Pilot operated relief valve

A hole has been drilled through poppet 1 which allows the fluid to flood the spring chamber of poppet 1. When there is no pressure drop across the orifice, pressure P_1 is equal to pressure P, poppet 1 is balanced and as a result, will not move. As pressure increases, it will be acting on the effective area (A_2) of poppet 2, which is a direct acting relief valve. When pressure causes poppet 2 to unseat, a small amount of flow will occur, lowering pressure P_1. With P_1 less than P, poppet 1 will unseat enough to establish flow out the side holes and attain equilibrium position. Should the pressure P and P_1 drop, seating poppet 2, poppet 1 will remain open until the spring can overcome the pressure P. This time lag will give the relief valve a flatter pressure vs flow curve, compared to the other two types.

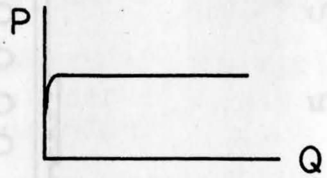


Fig. 11. Pressure vs flow

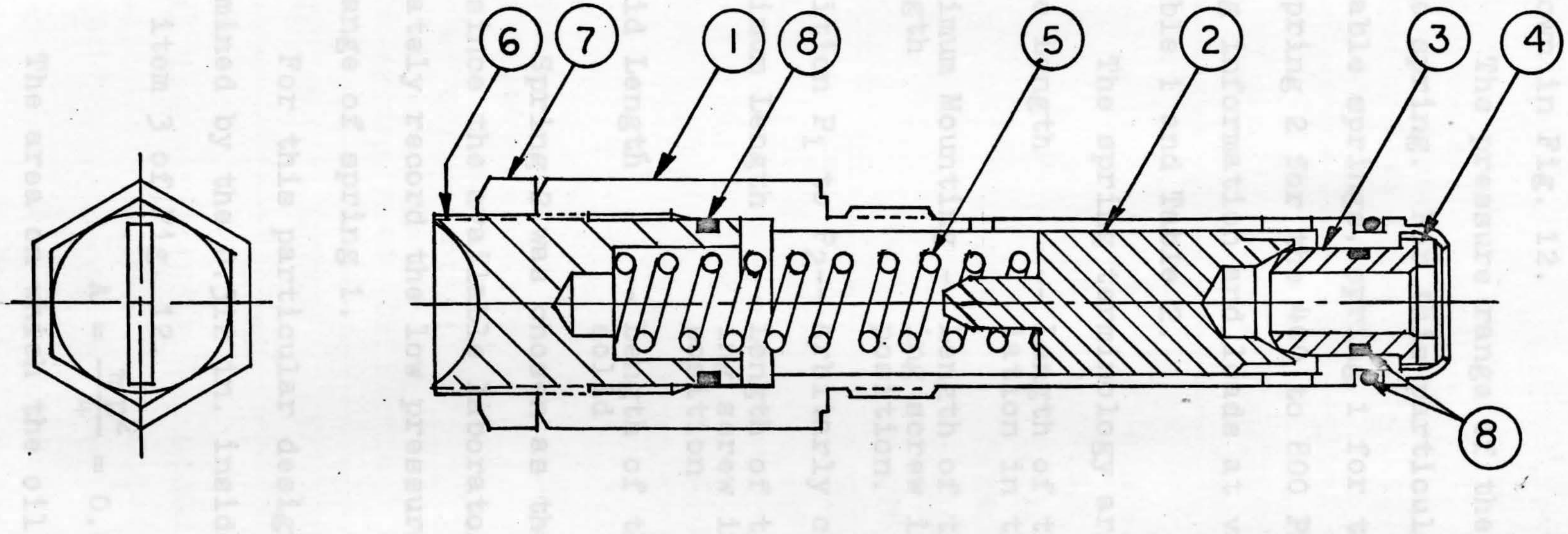
The forces acting on poppet 2 will determine the relief valve pressure setting. The equation will be the same as equation 1 but with A_2 as the effective area,

$$F = P A_2 \tag{4}$$

and as before, the flow equation will continue to be given by equation 2.

RELIEF VALVE SIMULATION PARAMETERS

Of the three types of relief valves, the direct acting version is the most desirable for the initial simulation. Its operation is straightforward, it is sensitive to pressure fluctuations and the absence of the o-ring eliminates considering the causes of the hysteresis effect.



1	CARTRIDGE SHELL	5	SPRING
2	FLOATING SEAT OR POPPET	6	ADJUSTMENT SCREW
3	FIXED SEAT	7	LOCK NUT
4	RETAINER	8	SEALS

FIG. 12
DIRECT ACTING RELIEF VALVE

The internal makeup of the relief valve selected is shown in Fig. 12.

The pressure range of the relief valve is determined by the spring. For this particular model, there are two available springs, spring 1 for the 100 to 400 PSI range and spring 2 for the 400 to 800 PSI range. The complete spring information and loads at various positions are shown in Table 1 and Table 2.

The spring terminology are as follows:

Free Length	-- Length of the spring prior to installation in the relief valve.
Maximum Mounting Length	-- Length of the spring with the adjusting screw in its maximum allowable out position.
Position P_1 to P_2	-- Arbitarly chosen spring positions.
Minimum Length	-- Length of the spring with the adjusting screw in its maximum allowable in position
Solid Length	-- Length of the spring when compressed solid

Spring 2 was chosen as the standard for this simulation since the available laboratory instrumentation cannot accurately record the low pressure and flow for the operating range of spring 1.

For this particular design, the entrance area (A) is determined by the 0.312 in. inside diameter of the fixed seat, item 3 of Fig. 12.

$$A = \frac{\pi D^2}{4} = 0.077 \text{ in.}^2$$

The area on which the oil pressure acts on the poppet, is called the effective pressure area (A_p).

TABLE 1

RELIEF VALVE SPRING NO. 1

Recommended pressure range: 100 psi to 400 psi
Spring diameter: 0.562 in.
Spring wire diameter: 0.121 in.
Number of active coils: 10
Spring rate: 298 lb./in.
Force = spring rate (Deflection) = KX Hooke's Law (5)

POSITION	LENGTH (inch)	DEFLECTION (inch)	FORCE (lb.)	PRESSURE (lb./in.^{2})
free	2.281	-	-	--
max. lg.	2.207	0.074	22.2	100
p-1	2.150	0.131	39.0	176
p-2	2,100	0.181	53.9	243
p-3	2.050	0.231	68.8	310
p-4	2.000	0.281	83.7	377
p-5	1.983	0.298	88.2	400
min. lg.	1.893	0.388	115.6	520
solid	1.694	0.587	174.9	-

TABLE 2

RELIEF VALVE SPRING NO. 2

Recommended pressure range: 400 psi to 800 psi
Spring diameter: 0.578
Spring wire diameter: 0.156
Number of active coils: 10
Spring rate: 1030 lb./in.
Force = spring rate (Deflection) = KX Hooke's Law (5)

POSITION	LENGTH (inch)	DEFLECTION (inch)	FORCE (lb.)	PRESSURE (lb./in.^{2})
free	2.242	-	-	-
max. lg.	2.156	0.086	88.8	400
p-1	2.100	0.142	146.3	659
p-2	2.080	0.162	166.5	750
p-3	2.000	0.242	249.3	1122
min. lg.	1.893	0.349	359.5	1619
solid	1.872	0.370	381.1	-

This pressure area is based on the diameter of the contact or sealing ring between the poppet and the fixed seat,

$$A_P = \frac{\pi D^2}{4} = 0.222 \text{ in.}^2 \qquad (6)$$

where (D) is the 0.531 diameter shown in Fig. 13.

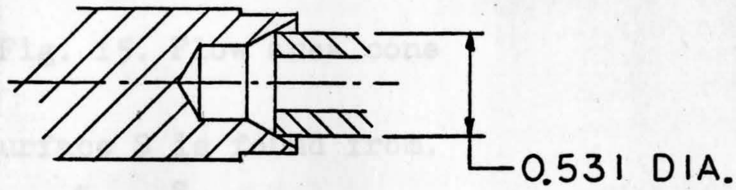


Fig. 13. Poppet sealing ring diameter

The flow area (A_F) will vary with the distance between the poppet and the seat. The area as a function of displacement can be developed in the following manner.

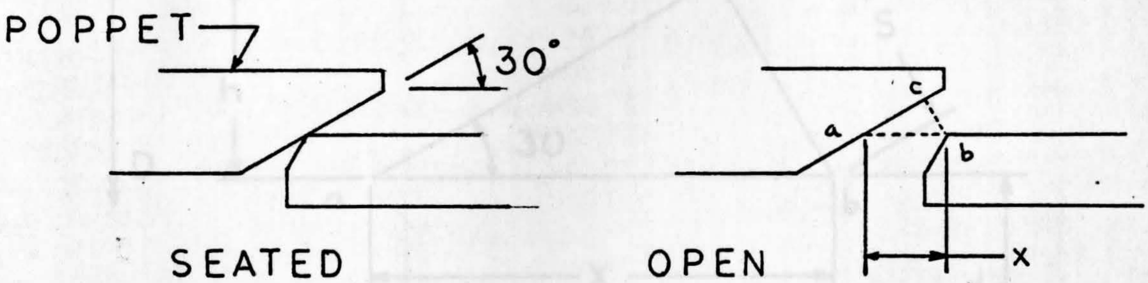


Fig. 14. Flow area location

The line bc, rotated 360° about the relief valve axis forms a truncated cone. The surface area of the side S of the cone is the flow area of the relief valve,

$$A_F = A_{(surface)} = \frac{1}{2}\pi S\ (D + d) \qquad (7)$$

where the diameter (d) is the contact ring diameter, 0.531 in.

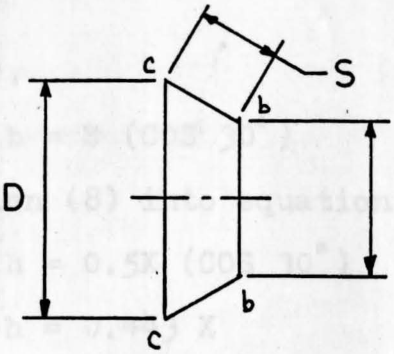


Fig. 15. Flow area cone

The slope surface S is found from,

$$\text{SIN } 30^\circ = \frac{S}{X}$$

rearranging, $S = X\ (\text{SIN } 30^\circ)$

$$S = .5\ X \qquad (8)$$

where (X) is the displacement of the poppet.

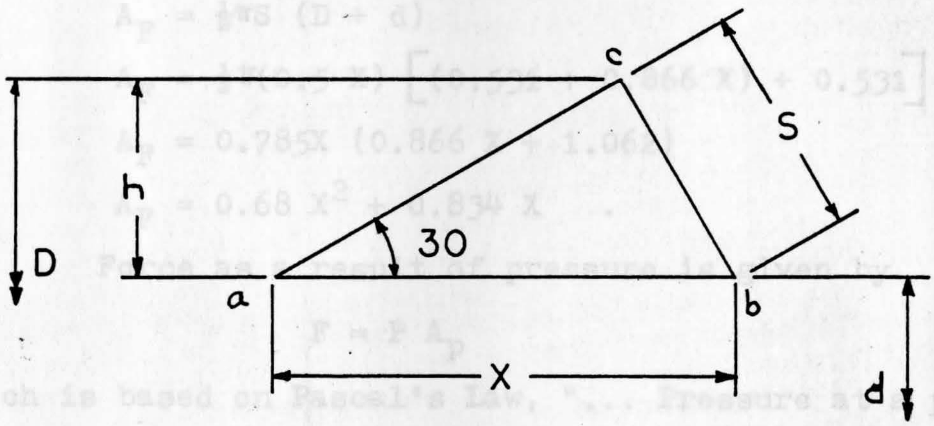


Fig. 16. Flow area cone detail

The expression for diameter (D) can be given as,

$$D = d + 2(h) \qquad (9)$$

where (h) is determined from the expression,

$$\text{COS } 30^\circ = \frac{h}{S}$$

rearranging for (h),

$$h = S\ (\cos 30^\circ) \tag{10}$$

Substituting equation (8) into equation (10),

$$h = 0.5X\ (\cos 30^\circ)$$

$$h = 0.443\ X \tag{11}$$

yields (h) as a function of (X). Then inserting equation (11) and d = 0.531 into equation (9),

$$D = d + 2\ (0.443\ X)$$

$$D = 0.531 + 0.866\ X \tag{12}$$

gives the expression for diameter (D).

The flow area (A_F) as a function of (X) can now be determined. Recalling equation (7) and substituting the appropriate terms,

$$A_F = \tfrac{1}{2}\pi S\ (D + d) \tag{7}$$

$$A_F = \tfrac{1}{2}\pi(0.5\ X)\left[(0.531 + 0.866\ X) + 0.531\right]$$

$$A_F = 0.785X\ (0.866\ X + 1.062)$$

$$A_F = 0.68\ X^2 + 0.834\ X\quad . \tag{13}$$

Force as a result of pressure is given by

$$F = P\ A_p \tag{14}$$

which is based on Pascal's Law, "... Pressure at a point in any static fluid is equal in magnitude in all directions..."[1]

This force acts on the poppet, compressing the spring. The spring deflection is described by Hooke's Law, "...the spring force is proportional to the spring deformation. The constant of proportionality, measured in force per unit

[1]Donald Gilbrech, Fluid Mechanics (Wadsworth Publishing Company, Inc., 1965), p. 20.

deflection, is called the spring constant (K)."[2] This was given earlier as equation (5), from Tables 1 and 2.

$$F_S = K\ X \tag{5}$$

[2]Tse, Morse, & Hinkle, <u>Mechanical Vibration</u> (Allyn and Bacon, Inc. 1968) , p. 5.

CHAPTER IV

FORMULATION OF THE RELIEF VALVE MODEL

Regardless of the type of relief valve chosen for the simulation, they all have the same two components, poppet and spring. This suggests that relief valves can be represented by a spring-mass system.

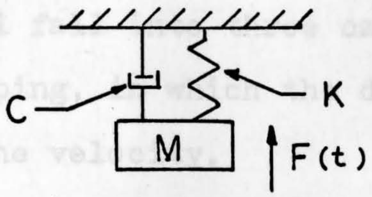


Fig. 17. Spring-mass system

M = Mass of the spring-poppet system.

K = Spring rate.

C = Damping function.

F(t) = Forcing function.

A free body diagram will appear as shown in the Figure below.

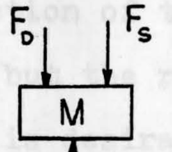


Fig. 18. Free-body diagram

Applying Newton's Law for the summation of forces on the poppet, and letting $\ddot{X} = \frac{d^2X}{dT^2}$ be the acceleration of the poppet, then

$$\Sigma F = M\ddot{X} \qquad \text{(Newton's Law)}$$

or $$F(t) - F_D - F_S = M\ddot{X} \; . \qquad (15)$$

The spring force (F_S)is determined by equation (5).

The damping force (F_D) will depend on the relief valve design and the assumptions that will be made governing these forces. The factors that will influence damping are the rubbing of two surfaces, the moving of a body through a fluid and the forcing of a fluid through an orfice. The damping force will fall into three categories:

1. Viscous damping, in which the damping force is proportional to the velocity.
2. Coulomb damping, in which the damping force is proportional to the normal force between the two sliding bodies.
3. Velocity square damping, in which the damping force is proportional to the square of the velocity.

For this application, the coulomb damping force is insignificant due to the light weight of the poppet (2.4 oz.) and the absence of the o-ring to provide for any force perpendicular to the direction of travel. The velocity square damping can be applied but the results would lead to a nonlinear function and it is desirable to keep the initial simulation linear.

With the assumption that this simulation will involve linear functions, the viscous damping force is considered to be proportional to the velocity, that is,

$$F_D = C\dot{X} \tag{16}$$

where $\dot{X}$ is the velocity of the poppet and C is the damping coefficient.

The forcing function F(t) is made up of the forces exerted by the hydraulic system on the relief valve. The operating characteristics of the pumps, valves and the other components will determine the extent of how these forces will react with respect to time. In general, they will fall into the following three categories:

a rectangular step function which is a suddenly applied load with instaneous reaction time;

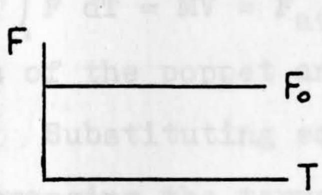


Fig. 19. Step function

a combination of the ramp function and the step function;

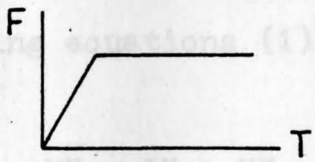


Fig. 20. Ramp function

and a combination of the exponential decaying function and the step function.

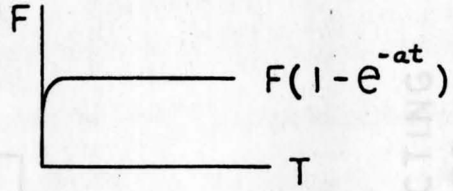


Fig. 21. Step & exponential decay function

The forces that affect the relief valve are due to pressure and flow forces or momentum.

Pressure is given by Pascal's Law, equation (1).

$$F = PA$$

The momentum term is derived from Newton's Law and is given by,

$$\int_1^2 F\,dT = MV = F_{avg}\,\Delta T \qquad (17)$$

where M is the mass of the poppet and V is the change in velocity $(V_2 - V_1)$. Substituting equations (5) and (16) into equation (15), rearranging the terms,

$$M\ddot{X} + C\dot{X} + KX = F(t)\ . \qquad (18)$$

The above (18), is nonhomogeneous, second order, linear differential equation and is a linear spring-mass system. Substituting equations (1) and (17) for the forcing function,

$$M\ddot{X} + C\dot{X} + KX = PA + \frac{MV}{\Delta T}\ . \qquad (19)$$

The hydraulic circuit for the relief valve simulation will be assumed to consist of the basic hydraulic components shown in Fig. 22 which follows.

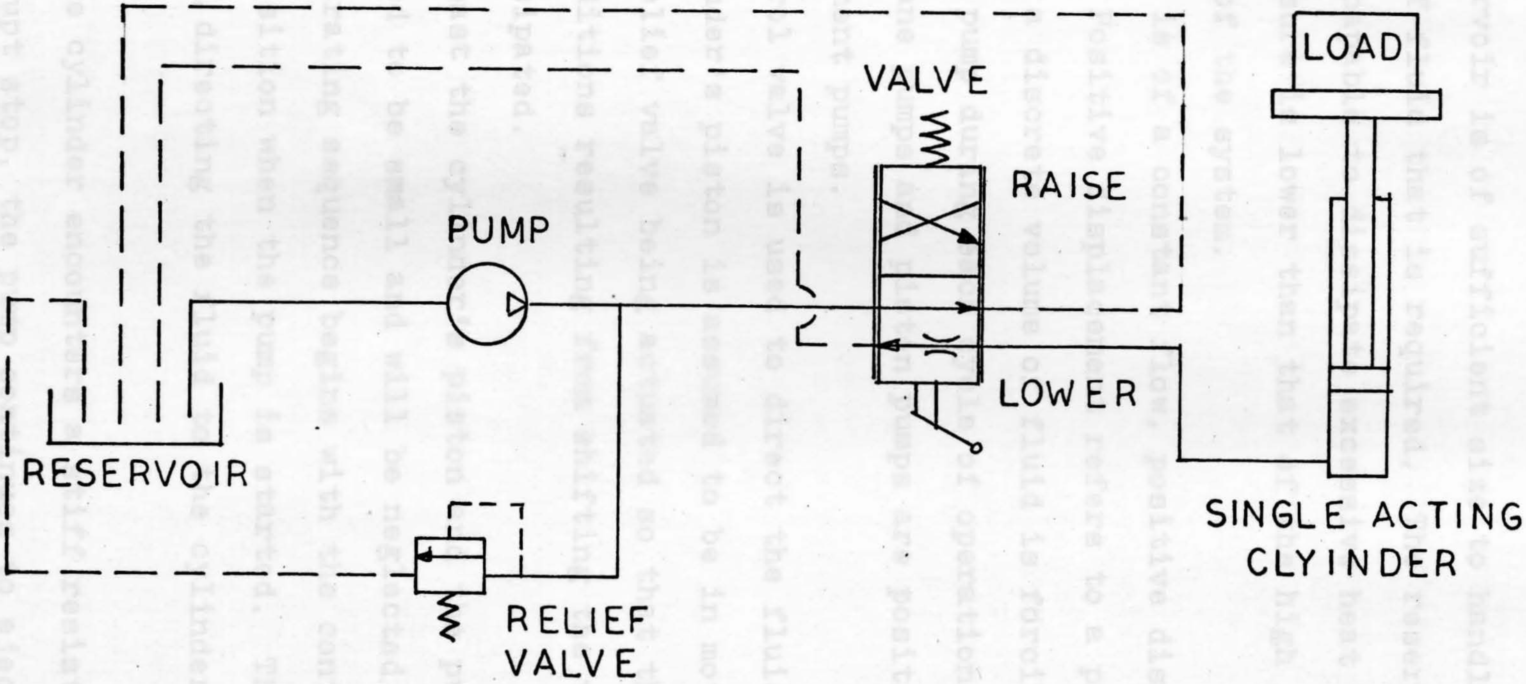


Fig. 22. Basic relief valve circuit.

With regards to this system, the following assumptions will be made:

1. The reservoir is of sufficient size to handle the amount of fluid that is required. The reservoir will also be capable to dissipate excessive heat and that its pressure is lower than that of the high pressure portion of the system.
2. The pump is of a constant flow, positive displacement design. Positive displacement refers to a pump design in which a discrete volume of fluid is forcibly ejected from the pump during each cycle of operation. Gear pumps, vane pumps and piston pumps are positive displacement pumps.
3. The control valve is used to direct the fluid flow. The cylinder's piston is assumed to be in motion prior to the relief valve being actuated so that the transient conditions resulting from shifting the valve have been dissipated.
4. Leakage past the cylinder's piston and the pump is considered to be small and will be neglected.

The operating sequence begins with the control valve in the drain position when the pump is started. The valve is then shifted, directing the fluid to the cylinder, causing it to move.

When the cylinder encounters a stiff resistance and comes to an abrupt stop, the pump continues to eject the same discrete volume of fluid per unit time. This results in an increase in pressure and the pressure will continue to build up until it reaches the relief valve setting. At this point, the relief valve is unseated, diverting the flow to the reservoir and preventing any further increases in pressure. The flow will continue to be dumped into the reservoir until the cylinder is allowed to resume motion or the valve shifted.

CHAPTER V

COMPUTER SIMULATION

PRELIMINARY CALCULATIONS

Analogies are often made between hydraulic systems and electrical networks. A hydraulic component will resist fluid flow much in the same manner that a resistor resists the flow of current.

By programming an equivalent electrical network on an analog computer, an effective simulation of the hydraulic system or component can be performed.

The analog computer differs from the more common place digital computer in the sense that instead of working with discrete numbers, the analog solves an equation through the programmed electrical network. The results are then put in the form of a continuous time-wise graph or chart instead of a numerical printout.

In spite of the advantages of the analog computer, the final simulation was conducted on the digital computer. Programming the relief valve equation (equation 19) into the Donner model 3430 analog computer yielded a curve describing the spring-mass system's natural frequency with the proper amplitude. However, the relief valve, as designed, permitted only a limited poppet travel as shown in Fig. 23.

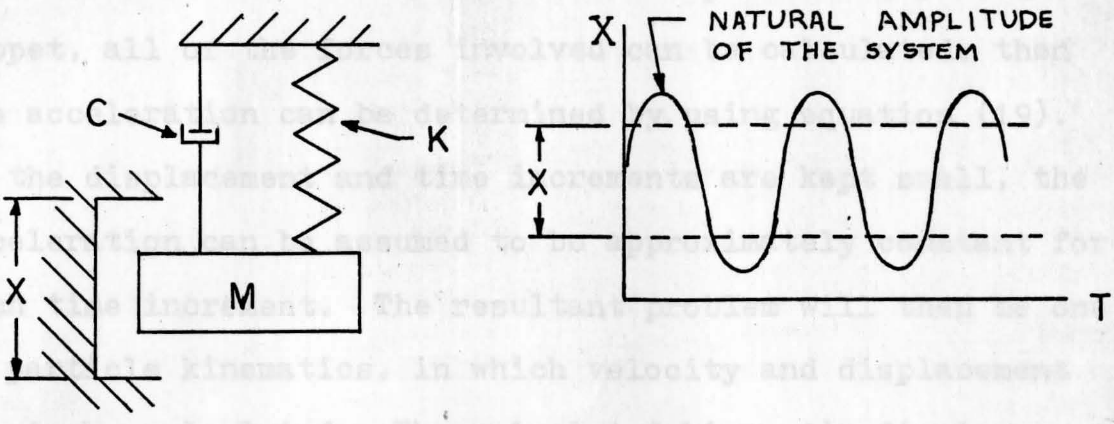


Fig. 23. Limit displacement spring-mass system

By treating the mechanical stop as an elastic spring with a large spring rate (based on the elasticity of steel), the impact and rebound of the poppet can be taken into account. The resulting relief valve model would then have two equations, one equation for displacements to the point of impact and a second for impact.

The Donner model 3430 analog computer did not have the capacity for switching from one programmed circuit to the other and back again. Additionally, all the attempted methods of using an external switching circuit proved to be unsatisfactory.

The relief valve model will not change when using the digital computer but the method of simulation does change. The differential equation (equation 19) describing the relief valve which could have been solved directly on the analog computer, will now have to be solved by a series of approximations in using the digital computer.

If it is assumed that for each position (X) of the poppet, all of the forces involved can be calculated, then the acceleration can be determined by using equation (19). If the displacement and time increments are kept small, the acceleration can be assumed to be approximately constant for each time increment. The resultant problem will then be one of particle kinematics, in which velocity and displacement are to be calculated. The calculated kinematic displacement once computed, is compared to the originally assumed displacement. If the two displacements do not match, then a new trial displacement is assumed and the calculations are repeated. This process is continued until a reasonable match in displacements is obtained. When this occurs, all the forces that have been calculated for the given displacement are characteristic to the relief valve at that point.

There are several additional assumptions that can be made concerning the subject relief valve so that equation (19) can be simplified further.

First, it can be assumed that the starting point for displacement (X = 0) will be at the spring mounting length rather than at the free length as suggested by equation (19). This would change the spring force equation (5) to,

$$F_S = K\ (X + X_{MTG}) \tag{20}$$

where X_{MTG} is the deflection of the spring to mounting position. This term is called the preload or mounting load, where $F_{MTG} = K\ X_{MTG}$, and equation (20) becomes,

$$F_S = K\ X + F_{MTG}\ . \tag{21}$$

This change makes it easier to keep track of the opening between the poppet and the seat for the flow area calculation. Equation (19) can then be written as follows:

$$M\ddot{X} + C\dot{X} + KX = PA + MV - F_{MTG} \; . \qquad (22)$$

Second, the damping term ($C\dot{X}$) can be neglected. The only drag forces on the poppet are those of fluid friction. It has been found that when the two moving surfaces are separated from each other by a very thin film of lubricant, the friction is that of boundary lubrication and the load on these surfaces is carried entirely by the hydrodynamic pressure in the film. In this case, the frictional loss is due solely to the internal fluid friction in the film.

The force to overcome this friction is very small as shown by solving equation (23)[3] as follows:

$$F = \mu A \left(\frac{V}{h}\right) \qquad (23)$$

μ = Absolute viscosity = $2.55 \times 10^{-6} \; \frac{\text{lb.-sec}^2}{\text{in}^2}$

(For mineral oil of 100 ssu at 120 F.)

A = Bearing area = π (diameter) (length of bearing)

= π (0.750 in.) (1.000 in,) = 2.356 in.^{2} .

h = Fluid film thickness = 0.005 in.

(Based on average machined tolerance)

V = Velocity

where it is assumed that the maximum velocity the poppet can attain is that of the oil entering the relief valve because of the spring resistance.

[3]Dudly Fuller, <u>Theory & Practice of Lubrication For Engineers</u>, (John Wiley & Sons, Inc.), p. 6

The pump flow is 20 GPM or 77 $\frac{in^3}{sec}$ and the entrance area is 0.222 in^2. Therefore,

$$V = \frac{FLOW}{AREA} = \frac{77}{.222} = 347 \frac{in}{sec} .$$

Substituting the above values into equation (23) gives

$$F = (2.55 \times 10^{-6})\ (2.356)\ (\frac{347}{.005}) = 0.4 \text{ lb.}$$

The damping force as computed is very small compared to the other forces, and can therefore be neglected.

The third assumption is that the change in momentum (MV) can also be neglected because of the small mass of the poppet. The combined weight of the poppet and spring is only 3.7 ounces or 0.232 lb. The resulting mass would be $6 \times 10^{-4}\ \frac{lb\text{-}sec^2}{in}$, and would require a very large change in velocity to develop a significant force.

Using the above assumptions, the relief valve's equation of motion now becomes,

$$M\ddot{X} + KX = PA - F_{MTG} \tag{24}$$

from which the acceleration can be calculated:

$$\ddot{X} = \frac{1}{M}\ (PA - F_{MTG} - KX)\ . \tag{25}$$

To establish P as a function of X, it was necessary to relate P to flow area (A_F). This was done by applying the continuity equation to the relief valve circuit shown in Fig. 22. When the cylinder's piston stops, the continuity equation states that the flow from the pump (Q_p) is equal to the flow out the relief valve (Q_{rv}). But during the time interval between the cylinder stopping and full relief valve flow, the pump continues to discharge the same quantity of fluid based on the assumption of a positive displacement pump (Q_p = constant). To account for this fluid, a term called compressibility factor (Q_c) must be considered.

$$Q_p = Q_{rv} + Q_c \tag{26}$$

This compressibility factor is a general term to account for expansion of the hydraulic system such as hoses stretching, loose couplings, compression of the fluid, and the compression of any trapped air in the fluid.

From the definition of the bulk modulus of elasticity,[4]

$$N = V' \frac{\Delta P}{\Delta V'}, \tag{27}$$

where V' is the volume of the hydraulic system and $\Delta P / \Delta V'$ is the change in pressure with respect to a change in volume. Rearranging equation (27) gives:

$$\Delta V' = \frac{V'}{N} \, (\Delta P) \;, \tag{28}$$

and noting that the change in volume can be equal to flow times time; i.e.,

$$\Delta V' = Q_c \, (\Delta T) \quad .$$

Now, substituting into equation (28), and rearranging gives;

$$Q_c = \frac{V'}{N} \left(\frac{\Delta P}{\Delta T}\right) \tag{29}$$

which is the desired compressibility term.

The expression for (Q_{rv}) in equation (26) can be obtained by applying Bernoulli's equation to the relief valve; i.e.,

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} \quad . \tag{30}$$

Rearranging equation (30) leads to:

$$\frac{1}{\gamma} \, (P_1 - P_2) = \frac{1}{2\,g} \, ({V_2}^2 - {V_1}^2) \quad . \tag{31}$$

[4]Gilbrech, <u>Fluid Mechanics</u>, p. 380

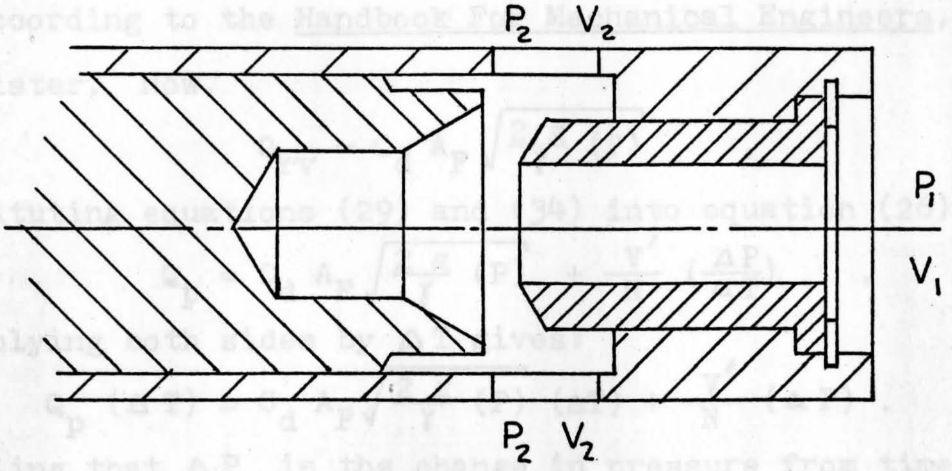


Fig. 24. Detail relief valve poppet & seat

By selecting P_1 and V_1 away from the entrance area, V_1 can be assumed to be very small compared to the exit velocity V_2 and can therefore be neglected.

Letting $P = P_1 - P_2$, equation (31) becomes:

$$\frac{1}{\gamma} P = \frac{1}{2\,g} (V_2^{\,2}) \quad . \qquad (32)$$

Recalling the continuity equation for volume flow rate, and applying it to the relief valve gives:

$$Q_{rv} = V\, A_F \quad .$$

Substituting for $V = V_2 = Q_{rv}/A_F$ in equation (32), and rearranging in order to solve for Q_{rv} gives:

$$\frac{1}{\gamma} P = \frac{1}{2\,g} \left(\frac{Q_{rv}^{2}}{A_F^{\,2}} \right) \quad ,$$

$$Q_{rv}^{\;2} = \frac{2\,g}{\gamma} (P)\,(A_F^{\,2}) \; ,$$

$$Q_{rv} = A_F \sqrt{\frac{2\,g}{\gamma} (P)} \; . \qquad (33)$$

The expression given by equation (33) is similar to that of an orifice for ideal flow. To obtain a realistic expression for Q_{rv}, a discharge coefficient (C_d) must be included. By definition C_d is the ratio of actual flow to ideal flow and has a value, as determined empirically, of 0.6 according to the Handbook For Mechanical Engineers, Baumeister. Now,

$$Q_{rv} = C_d A_F \sqrt{\frac{2g}{\gamma}(P)} \quad , \tag{34}$$

substituting equations (29) and (34) into equation (26), gives

$$Q_p = C_d A_F \sqrt{\frac{2g}{\gamma}(P)} + \frac{V'}{N}\left(\frac{\Delta P}{\Delta T}\right) ,$$

multiplying both sides by ΔT gives:

$$Q_p (\Delta T) = C_d A_F \sqrt{\frac{2g}{\gamma}(P)}\,(\Delta T) + \frac{V'}{N}(\Delta P) \ . \tag{35}$$

Recalling that ΔP_n is the change in pressure from time T_{n-1} to T_n, gives

$$\Delta P_n = P_n - P_{n-1} = P - P_o \quad , \tag{36}$$

where P is the pressure at the existing time interval and P_o is the pressure from the previous time interval. Thus,

$$Q_p (\Delta T) = C_d A_F \sqrt{\frac{2g}{\gamma}(P)}\,(\Delta T) + \frac{V'}{N}(P - P_o). \tag{37}$$

By letting $R = \sqrt{P}$, the above expression reduces to a quadratic equation from which P can be determined as a function of the flow area A_F. Hence,

$$Q_p (\Delta T) = C_d A_F \sqrt{\frac{2g}{\gamma}}\,(R)\,(\Delta T) + \left(\frac{V'}{N}\right) R^2 - \frac{V'}{N}(P_o)$$

$$\left(\frac{V'}{N}\right) R^2 + C_d A_F \sqrt{\frac{2g}{\gamma}}\,(\Delta T) R - \left(Q_p(\Delta T) + \frac{V'}{N}(P_o)\right), \tag{38}$$

where

$$P = R^2 \ . \tag{39}$$

PROGRAMING FOR SIMULATION

In review, the equations required for simulation are:

$$\ddot{X} = \frac{1}{M}\,(PA - F_{MTG} - KX) \tag{25}$$

with

$$P = R^2 \quad , \tag{39}$$

where R is determined by solving the quadratic equation:

$$\left(\frac{V'}{N}\right) R^2 + C_d A_F \sqrt{\frac{2g}{\gamma}} (\Delta T) R - \left(Q_p (\Delta T) + \frac{V'}{N}(P_o)\right) = 0 \quad (38)$$

with $$A_F = 0.68 X^2 + 0.834 X \quad . \quad (13)$$

The following parameters will apply:

1. Relief valve set at 750 PSI.
2. $Q_p = 20$ GPM $= 77 \frac{in^3}{sec}$ pump flow rate.
3. Spring no. 2 will be used, with a spring rate of $K = 1030\ ^{lb}/_{in}$.
4. For 750 PSI, the deflection of the spring to mount was measured to be, $X_{MTG} = 0.131$ in. Therefore $F_{MTG} = K X_{MTG} = (1030\ ^{lb}/_{in})(0.131 \text{ in.}) = 135$ lb.
5. The discharge coefficient (C_d) was given as 0.6 .
6. The expression $\frac{\gamma}{g}$ is the mass density of the oil, where g = gravitation constant = $386.4\ ^{in}/_{sec^2}$ and, γ = weight density = $0.031\ ^{lb}/_{in^3}$ for the oil. Substituting into the given expression and solving gives $\sqrt{\frac{2g}{\gamma}} = 157.89$.
7. The bulk modulus (N) is equal to $225{,}000\ ^{lb}/_{in^2}$ for oil (Reference, Handbook For Mechanical Engineers, Baumeister). The volume (V') is the volume of the hydraulic test circuit which is subjected to system's pressure. The volume was estimated to be 2 gallons or 462 in^3 which is typical for lift trucks and small excavators. Thus, $\frac{V'}{N} = \frac{462 \text{ in}^3}{225{,}000 \text{ lb/in}^2} = 0.002 \text{ in}^5/\text{lb}$.
8. The mass of the poppet (M) is $0.0006 \frac{lb/sec^2}{in}$ as determined previously.
9. The effective pressure area was computed to be 0.222 in^2 .

Substituting these parameters into the above relief valve equation (25) and (38) and simplifying, gives:

$$\ddot{X} = \frac{1}{.0006} (0.222\ P - 135 - 1030\ X) \qquad (39)$$

and $$.002\ R^2 + 94.9\ A_F\ (\Delta T)\ R - (77\ (\Delta T) + .002 P_o) = 0 \qquad (40)$$

From the two equations given above, the simulation program can be formulated.

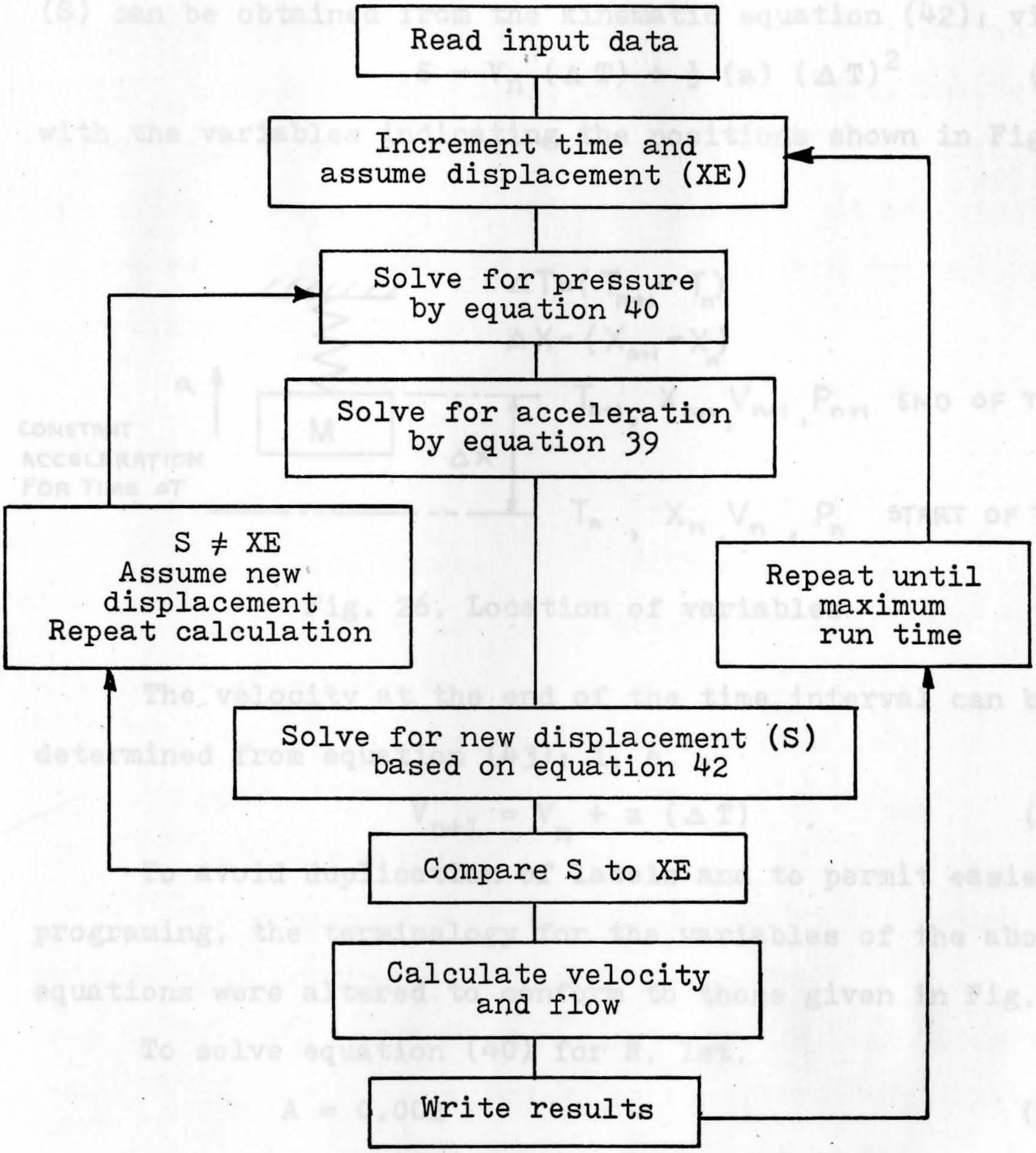


Fig. 25. Computer flow chart

Based on the iterative procedure mentioned earlier in this chapter, where the calculated displacement is compared to the initially assumed displacement, the program shown on the flow chart in Fig. 25 was constructed.

By calculating the acceleration from equation (39) for each time increment (ΔT), the calculated displacement (S) can be obtained from the kinematic equation (42); viz

$$S = V_n\,(\Delta T) + \tfrac{1}{2}\,(a)\,(\Delta T)^2 \qquad (42)$$

with the variables indicating the positions shown in Fig. 26.

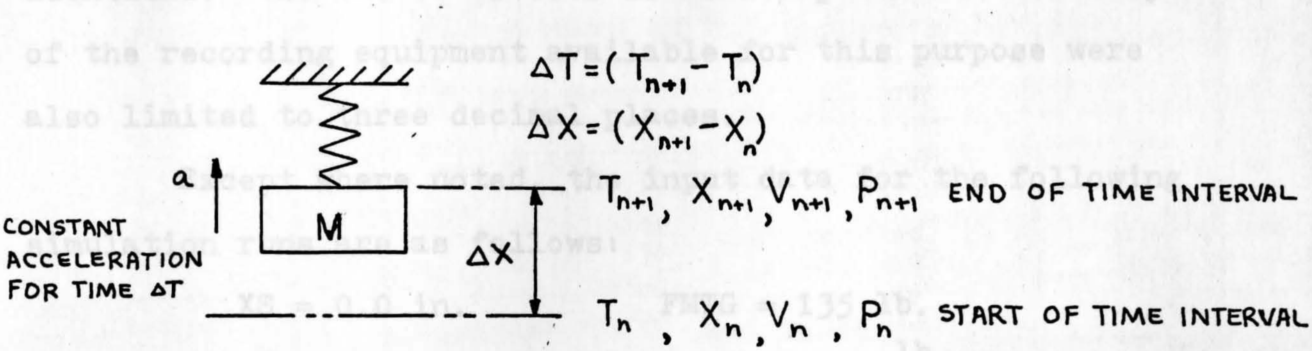


Fig. 26. Location of variables

The velocity at the end of the time interval can be determined from equation (43); i. e. ,

$$V_{n+1} = V_n + a\,(\Delta T) \quad . \qquad (43)$$

To avoid duplication of labels and to permit easier programing, the terminology for the variables of the above equations were altered to conform to those given in Fig. 27.

To solve equation (40) for R, let,

$$A = 0.002 \qquad (44)$$

$$B = 94.9\,(ARFLO)\,(DT) \qquad (45)$$

$$C = (PMTG) + (PUMP)\,(DT) \qquad (46)$$

and then equation (40) takes the form of

$$A\,R^2 + B\,R - C = 0$$

whose solution is;

$$R = \frac{-B + \sqrt{B^2 + 4AC}}{2A} \quad . \qquad (47)$$

The complete program is shown in Fig. 28.

To speed the convergence between the assumed and calculated displacements the step. "If (Y. LT. .001) go to 11", was added. This means that if the difference between S and XE is less than .001, they were to be considered identical. The choice of .001 is arbitary but the accuracy of the recording equipment available for this purpose were also limited to three decimal places.

Except where noted, the input data for the following simulation runs are as follows:

XS = 0.0 in.	FMTG = 135 lb.
TMAX = 60.0 sec.	RATE = 1030 $^{lb}/_{in.}$
XMAX = 0.239 in.	SPRMA = .0006 $^{lb\text{-}sec^2}/_{in}$
DT = .001 sec.	COMP = .002 $^{in^5}/_{lb}$
DX = .005 sec.	AREA = .222 in^2 .
PUMP = 77 $^{in^3}/_{sec}$	VEL = 0.0 $^{in}/_{sec}$
PMTG = 608 PSI	

The preciseness of the simulation to the actual relief valve is discussed in the next chapter.

Run no. 1 - This run represents the basic relief simulation, table 3. The input data are for 750 PSI setting at 20 GPM. The computerized result showed a setting of 766.81 PSI at 19.57 GPM which is in error of only 2.2% based on experimental data. However, the program does not give a true indication of the velocity as the acceleration approaches zero. This problem results from using equation (43) to calculate the final velocity. The program makes no provisions to permit the velocity to decrease. Other equivalent types of velocity equations that were attempted but proved to be unsatisfactory are:

$$V_{n+1} = \sqrt{V_n^{\,2} + 2\,a\,(X_{n+1} - X_n)} \qquad (48)$$

and,

$$V_{n+1} = (X_{n+1} - X_n)/(\Delta T) + \tfrac{1}{2}\,a(\Delta T)\ . \qquad (49)$$

Equation (48) (see table 4) could not handle the return stroke of the poppet because of a negative sign under the radical resulting from X_n being larger than X_{n+1} . Equation (49) (see table 5) has a periodic fluctuation during the poppet travel due to the reaction between the spring and pressure forces. This fluctuation is known not to be a characteristic of the subject relief valve.

This issue will be discussed in greater detail in the conclusion.

Run no. 2 - This run investigates the reaction of the relief valve when the poppet reaches the end of its stroke and rebounds. The XMAX term was reduced to 0.029 in.

The velocity of the rebound is given by the impulse equation, "VEL = .7 * VEL". The coefficient of restitution of .7 was selected because it permitted a reasonable recoil velocity against the pressure force and still remain in the elastic collision range. The data given in table 6, indicates that the relief valve is stable and the pressure fluctuations are not severe enough to cause flow noise, although there would be the sound of the poppet rapping against its seat or of spring fatigue resulting from it being fully compressed (solid).

Run no. 3 - This run shows how the relief valve would react for a different spring. In this case, the spring rate was reduced from 1030 $\frac{lb}{in}$ to 500 $\frac{lb}{in}$ with XMAX =0.029 in. The results given in table 7 show that reducing the spring rate is not recommended due to the resulting erratic behavior of the relief valve. The fluctuations in displacement are large and negative in sign at T = .013 sec.

indicating that the relief valve has closed.

An evaluation of the program is discussed in chapter 7.

LIST OF COMPUTER SYMBOLS

SYMBOL	DEFINITION
ACC	Acceleration
AREA	Effective pressure area
ARFLO	Flow area
COMP	Compressibility factor
DT	Time increment
DX	Displacement increment
FMTG	Spring mounting force
P	Pressure
PMTG	Pressure at the start of each new time interval
PUMP	Pump flow
Q	Flow
RATE	Spring rate
S	Calculated displacement
SPRMA	Mass of spring and poppet
T	Time
TMAX	Maximum run time
VEL	Final velocity at the end of a time interval
XE	Position of poppet at end of a time interval
XMAX	Maximum displacement
XS	Position of poppet at start of a time interval

Fig. 27. List Of Computer Symbols

COMPUTER PROGRAM

```
      READ(1,95)XS,TMAX,XMAX,DT,DX,PUMP,PMTG,FMTG,RATE,SPRMA,COMP
      READ(1,96)AREA,VEL
   95 FORMAT(5F7.4,3F7.3,F7.2,2F7.5)
   96 FORMAT(2F7.4,F7.3)
      WRITE(3,74)
   74 FORMAT(7X,4HTIME,5X,6HTRAVEL,3X,8HVELOCITY,5X,6HACCEL.,3X,
     1 8HPRESSURE,5X,4HFLOW,5X,9HFLOW AREA)
      WRITE(3,80)
   80 FORMAT(7X,4HSEC.,6X,4HINCH,5X,6HIN/SEC,3X,9HIN/SEC SQ,
     15X,3HPSI,8X,3HGPM,7X,5HSQ IN)
      T=0
      V= 0
    1 T=T + DT
      IF(T-TMAX)2,32,32
    2 XE = XS + DX
    3 X = XS + (XE-XS)/2
      ARFLO=.66*X*X + .834*X
      A = COMP
      B = 94.9*ARFLO*DT
      C= +(A*PMTG + PUMP*DT)
      R =((-B + SQRT(B**2+4*A*C))/(2*A)) **2
      ACC= (AREA*R-FMTG-RATE*X) / SPRMA
      S = VEL*DT + (ACC*DT**2)/2
      S=ABS(S)
      Y=ABS(S-XE)
      IF(Y.LT. .001000) GO TO 11
      IF(S-XE)10,11,12
   10 XE=XE-Y/2
      GO TO 3
   12 XE=XE+Y/2
      GO TO 3
   11 IF(XE-XMAX)30,31,31
   30 V = VEL
      VEL=V+ACC*DT
   33 P=R
      Q = 24.12*ARFLO*SQRT(P)
      WRITE(3,75)T,XE,VEL,ACC,P,Q,ARFLO
   75 FORMAT(2X,F9.4,F11.4,F11.4,F11.2,F11.2,F9.4,F14.4)
      XS=XE
      PMTG=P
      GO TO 1
```

Fig. 28. Computer Program

COMPUTER PROGRAM,
Cont.

```
 31 XE= XMAX
    ARFLO=.66*X*X + .834*X
    A = COMP
    B = 94.9*ARFLO*DT
    C = +(A*PMTG + PUMP*DT)
    R =((-B + SQRT(B**2+4*A*C))/(2*A)) **2
    ACC= (AREA*R-FMTG-RATE*X) / SPRMA
    V = VEL
    VEL=V+ACC*DT
    P=R
    Q = 24.12*ARFLO*SQRT(P)
    WRITE(3,76)T,XE,VEL,ACC,P,Q,ARFLC
 76 FORMAT(2X,F9.4,F11.4,F11.4,F11.2,F11.2,F9.4,F14.4)
    XS=XE
    PMTG=P
    XE=XS-DX
    VEL=-.7* VEL
    GO TO 1
 32 WRITE(3,77)
 77 FORMAT(6X,'MAXIMUM RUN TIME')
999 STOP
    END
```

Fig. 28. Computer Program (cont.)

TIME SEC.	TRAVEL INCH	VELOCITY IN/SEC	ACCEL. IN/SEC SQ	PRESSURE PSI	FLOW GPM	FLOW AREA SQ IN
0.0010	0.0050	8.9821	8982.11	643.98	1.2787	0.0021
0.0020	0.0130	17.5934	8611.25	673.17	4.7342	0.0076
0.0030	0.0192	21.9563	4362.95	694.65	8.6507	0.0136
0.0040	0.0232	23.4283	1472.02	710.42	11.5540	0.0180
0.0050	0.0245	24.9986	1570.31	723.05	13.1492	0.0203
0.0060	0.0266	27.5787	2580.16	733.61	14.2008	0.0217
0.0070	0.0288	29.4887	1909.99	741.64	15.4890	0.0236
0.0080	0.0302	30.3817	893.02	747.46	16.6113	0.0252
0.0090	0.0310	30.9693	587.57	751.89	17.3157	0.0262
0.0100	0.0317	31.6308	661.49	755.43	17.7733	0.0268
0.0110	0.0323	32.2224	591.61	758.18	18.1736	0.0274
0.0120	0.0328	32.6067	384.37	760.23	18.5265	0.0279
0.0130	0.0331	32.8506	243.84	761.77	18.7870	0.0282
0.0140	0.0334	33.0535	202.89	762.96	18.9655	0.0285
0.0150	0.0336	33.2320	178.58	763.88	19.1015	0.0287
0.0160	0.0337	33.3666	134.56	764.58	19.2138	0.0288
0.0170	0.0338	33.4591	92.57	765.11	19.3016	0.0289
0.0180	0.0339	33.5280	68.84	765.51	19.3655	0.0290
0.0190	0.0340	33.5842	56.25	765.82	19.4128	0.0291
0.0200	0.0340	33.6289	44.71	766.06	19.4498	0.0291
0.0210	0.0341	33.6615	32.58	766.24	19.4788	0.0292
0.0220	0.0341	33.6852	23.73	766.37	19.5008	0.0292
0.0230	0.0341	33.7038	18.59	766.48	19.5170	0.0292
0.0240	0.0342	33.7185	14.78	766.56	19.5294	0.0292
0.0250	0.0342	33.7297	11.19	766.62	19.5391	0.0293
0.0260	0.0342	33.7381	8.34	766.66	19.5465	0.0293
0.0270	0.0342	33.7442	6.21	766.70	19.5521	0.0293
0.0280	0.0342	33.7491	4.83	766.73	19.5563	0.0293
0.0290	0.0342	33.7528	3.71	766.75	19.5595	0.0293
0.0300	0.0342	33.7557	2.98	766.76	19.5620	0.0293

TABLE 3

Computer Run No. 1

0.0310	0.0342	33.7576	1.91	766.77	19.5639	0.0293
0.0320	0.0342	33.7593	1.65	766.78	19.5653	0.0293
0.0330	0.0342	33.7606	1.30	766.79	19.5663	0.0293
0.0340	0.0342	33.7614	0.89	766.79	19.5672	0.0293
0.0350	0.0342	33.7625	1.07	766.80	19.5679	0.0293
0.0360	0.0342	33.7633	0.76	766.80	19.5684	0.0293
0.0370	0.0342	33.7637	0.48	766.81	19.5689	0.0293
0.0380	0.0342	33.7640	0.28	766.81	19.5692	0.0293
0.0390	0.0342	33.7641	0.05	766.81	19.5694	0.0293
0.0400	0.0342	33.7638	-0.23	766.81	19.5695	0.0293
0.0410	0.0342	33.7636	-0.23	766.81	19.5695	0.0293
0.0420	0.0342	33.7640	0.38	766.81	19.5694	0.0293
0.0430	0.0342	33.7641	0.18	766.81	19.5694	0.0293
0.0440	0.0342	33.7640	-0.15	766.81	19.5696	0.0293
0.0450	0.0342	33.7639	-0.08	766.81	19.5695	0.0293
0.0460	0.0342	33.7639	0.03	766.81	19.5695	0.0293
0.0470	0.0342	33.7640	0.05	766.81	19.5695	0.0293
0.0480	0.0342	33.7640	0.0	766.81	19.5695	0.0293
0.0490	0.0342	33.7640	0.0	766.81	19.5695	0.0293
0.0500	0.0342	33.7640	0.0	766.81	19.5695	0.0293
0.0510	0.0342	33.7640	0.0	766.81	19.5695	0.0293
0.0520	0.0342	33.7640	0.0	766.81	19.5695	0.0293
0.0530	0.0342	33.7640	0.0	766.81	19.5695	0.0293
0.0540	0.0342	33.7640	0.0	766.81	19.5695	0.0293
0.0550	0.0342	33.7640	0.0	766.81	19.5695	0.0293
0.0560	0.0342	33.7640	0.0	766.81	19.5695	0.0293
0.0570	0.0342	33.7640	0.0	766.81	19.5695	0.0293
0.0580	0.0342	33.7640	0.0	766.81	19.5695	0.0293
0.0590	0.0342	33.7640	0.0	766.81	19.5695	0.0293
0.0600	0.0342	33.7640	0.0	766.81	19.5695	0.0293

TABLE 3

Computer Run No. 1, Cont.

TIME	TRAVEL	VELOCITY	ACCEL.	PRESSURE	FLOW	FLOW AREA
0.0010	0.0045	8.9821	8982.11	643.98	1.2787	0.0021
0.0020	0.0136	15.7192	9168.52	673.44	4.5946	0.0073
0.0030	0.0180	16.9554	4557.42	694.97	8.6284	0.0136
0.0040	0.0198	17.5528	5701.37	712.82	10.4950	0.0163
0.0050	0.0217	18.4550	8381.93	728.45	11.6225	0.0179
0.0060	0.0234	19.3474	9963.71	741.72	12.8264	0.0195
0.0070	0.0249	20.1994	11189.88	752.89	13.8924	0.0210
0.0080	0.0265	21.1392	12575.30	762.46	14.7057	0.0221
0.0090	0.0275	21.7520	12756.37	770.11	15.6781	0.0234
0.0100	0.0287	22.4849	13868.38	776.74	16.2022	0.0241
0.0110	0.0294	22.9009	13773.65	781.91	16.9435	0.0251
0.0120	0.0300	23.2813	14180.32	786.19	17.3947	0.0257
0.0130	0.0308	23.7642	14940.29	789.96	17.6552	0.0260
0.0140	0.0311	23.9802	14677.18	792.77	18.1383	0.0267
0.0150	0.0315	24.1970	14944.18	795.12	18.3744	0.0270
0.0160	0.0317	24.3631	15046.72	797.02	18.6048	0.0273
0.0170	0.0319	24.5011	15159.33	798.57	18.7830	0.0276
0.0180	0.0321	24.6126	15242.49	799.83	18.9308	0.0278
0.0190	0.0323	24.7034	15312.20	800.85	19.0504	0.0279
0.0200	0.0324	24.7772	15368.17	801.68	19.1479	0.0280
0.0210	0.0325	24.8371	15413.31	802.35	19.2271	0.0281
0.0220	0.0326	24.8858	15450.31	802.90	19.2915	0.0282
0.0230	0.0326	24.9253	15480.30	803.34	19.3438	0.0283
0.0240	0.0327	24.9575	15504.76	803.71	19.3864	0.0284
0.0250	0.0327	24.9837	15524.67	804.00	19.4211	0.0284
0.0260	0.0328	25.0049	15540.41	804.24	19.4492	0.0284
0.0270	0.0328	25.0221	15553.44	804.43	19.4720	0.0285
0.0280	0.0328	25.0359	15563.53	804.59	19.4905	0.0285
0.0290	0.0328	25.0471	15572.15	804.71	19.5054	0.0285
0.0300	0.0328	25.0563	15579.17	804.82	19.5175	0.0285
0.0310	0.0328	25.0638	15584.89	804.90	19.5274	0.0285

TABLE 4

Computer Run No. 1-A

TIME SEC.	TRAVEL INCH	VELOCITY IN/SEC	ACCEL. IN/SEC SQ	PRESSURE PSI	FLOW GPM	FLOW AREA SQ IN
0.0010	0.0050	9.4911	8982.11	643.98	1.2787	0.0021
0.0020	0.0133	12.4683	8280.86	673.01	4.8169	0.0077
0.0030	0.0166	6.6437	6696.79	695.69	8.0399	0.0126
0.0040	0.0143	4.6569	13976.89	717.60	8.4314	0.0130
0.0050	0.0162	13.1155	22471.07	739.51	8.4327	0.0129
0.0060	0.0229	17.4150	21298.52	756.40	10.9859	0.0166
0.0070	0.0264	11.7474	16554.72	767.32	14.0165	0.0210
0.0080	0.0228	6.7412	20643.90	778.12	14.0825	0.0209
0.0090	0.0219	13.7808	29347.61	791.26	12.8881	0.0190
0.0100	0.0278	20.2695	28821.59	801.36	14.4369	0.0211
0.0110	0.0317	14.9437	21980.70	805.63	17.3976	0.0254
0.0120	0.0276	7.7250	23728.46	809.93	17.3882	0.0253
0.0130	0.0248	13.5523	32750.86	818.16	15.3847	0.0223
0.0140	0.0298	21.7026	33405.26	824.98	16.1042	0.0232
0.0150	0.0348	17.5678	25135.55	825.83	19.1391	0.0276
0.0160	0.0306	8.1127	24532.78	826.16	19.4002	0.0280
0.0170	0.0260	12.4157	34095.64	831.62	16.7929	0.0241
0.0180	0.0310	22.8409	35681.84	836.76	16.9566	0.0243
0.0190	0.0360	18.3932	26786.40	835.92	19.9987	0.0287
0.0200	0.0319	8.6681	25485.79	834.55	20.2655	0.0291
0.0210	0.0269	12.3739	34760.64	838.55	17.5363	0.0251
0.0220	0.0312	22.7778	36989.19	842.90	17.3608	0.0248
0.0230	0.0362	19.2842	28568.50	841.67	20.1915	0.0289
0.0240	0.0331	9.9194	26129.84	839.39	20.7337	0.0297
0.0250	0.0279	11.9693	34225.36	842.04	18.2225	0.0260
0.0260	0.0313	21.9149	37101.16	845.68	17.7168	0.0253
0.0270	0.0363	19.7062	29412.44	844.31	20.2695	0.0289
0.0280	0.0336	10.5234	26426.82	841.59	20.9476	0.0299
0.0290	0.0284	11.7696	33947.68	843.61	18.5462	0.0265
0.0300	0.0313	21.4878	37138.17	846.93	17.8815	0.0255

TABLE 5

Computer Run No. 1-B

TIME SEC.	TRAVEL INCH	VELOCITY IN/SEC	ACCEL. IN/SEC SQ	PRESSURE PSI	FLOW GPM	FLOW AREA SQ IN
0.0010	0.0050	8.9821	8932.11	643.98	1.2787	0.0021
0.0020	0.0130	17.5934	8611.25	673.17	4.7342	0.0076
0.0030	0.0192	21.9563	4362.95	694.65	8.6507	0.0136
0.0040	0.0232	23.4283	1472.02	710.42	11.5540	0.0180
0.0050	0.0245	24.9985	1570.31	723.05	13.1492	0.0203
0.0060	0.0266	27.5787	2530.16	733.61	14.2008	0.0217
0.0070	0.0268	29.4887	1909.99	741.64	15.4890	0.0236
0.0080	0.0290	30.3817	893.02	747.46	16.6113	0.0252
0.0090	0.0099	3.1535	24420.73	764.35	10.9836	0.0165
0.0100	0.0217	41.4973	38343.82	785.10	9.0235	0.0134
0.0110	0.0290	49.6694	8172.07	785.49	19.3707	0.0287
0.0120	0.0240	-11.5863	23182.27	793.80	15.3454	0.0226
0.0130	0.0113	33.5354	45121.75	812.03	10.2765	0.0150
0.0140	0.0290	61.0341	27498.63	817.02	17.0623	0.0247
0.0150	0.0289	-13.2806	29443.23	821.94	17.0698	0.0247
0.0160	0.0119	36.2677	49546.29	836.70	12.0654	0.0173
0.0170	0.0290	67.2717	31004.03	838.18	18.8190	0.0269
0.0180	0.0290	-11.3893	35700.38	841.90	17.6777	0.0253
0.0190	0.0281	47.2489	39276.07	846.77	17.0957	0.0244
0.0200	0.0290	61.4226	14173.71	836.77	24.6498	0.0353
0.0210	0.0216	1.4806	44476.40	845.60	15.0831	0.0215
0.0220	0.0266	51.7965	50315.94	855.72	14.4266	0.0204
0.0230	0.0290	66.0482	14251.73	843.74	25.6591	0.0366
0.0240	0.0261	-4.1650	42068.71	849.74	16.5215	0.0235
0.0250	0.0223	47.2794	51444.45	859.58	14.5663	0.0206
0.0260	0.0290	68.6116	21332.17	850.64	24.1154	0.0343
0.0270	0.0271	-4.4614	43566.72	855.95	16.8686	0.0239
0.0280	0.0226	47.9660	52427.38	864.99	14.9766	0.0211
0.0290	0.0293	69.9131	21947.13	855.16	24.5654	0.0348
0.0300	0.0272	-3.8984	45040.80	860.30	16.9594	0.0240

TABLE 6

Computer Run No. 2

Shorter Stroke

TIME	TRAVEL	VELOCITY	ACCEL.	PRESSURE	FLOW	FLOW AREA
0.0010	0.0050	10.5952	11190.44	643.98	-1.2787	0.0021
0.0020	0.0170	19.0715	14143.11	671.11	5.7822	0.0093
0.0030	0.0245	13.5820	12163.96	687.72	11.1260	0.0176
0.0040	0.0210	4.3788	15757.70	701.94	12.3430	0.0193
0.0050	0.0155	3.4292	25353.44	720.23	10.2724	0.0159
0.0060	0.0235	22.4930	30986.00	736.90	11.0942	0.0169
0.0070	0.0290	-16.5993	25837.33	743.39	16.2705	0.0247
0.0080	0.0055	-2.0738	42852.46	762.78	9.7145	0.0146
0.0090	0.0240	45.0299	53059.93	784.73	8.4088	0.0124
0.0100	0.0290	-17.0298	27343.91	776.27	23.3733	0.0355
0.0110	0.0095	6.6818	52363.57	792.99	11.0706	0.0163
0.0120	0.0290	-42.4461	55324.77	806.62	12.6400	0.0185
0.0130	-0.0055	1.9474	72895.00	831.59	6.8794	0.0099
0.0140	0.0290	-65.1265	75725.56	850.50	9.9566	0.0142
0.0150	-0.0165	2.4947	95989.31	881.62	3.7515	0.0052
0.0160	0.0290	-83.5510	94668.94	900.57	9.9358	0.0137
0.0170	-0.0235	5.9647	118929.38	935.73	1.6959	0.0023
0.0180	0.0290	-97.1664	110925.50	950.70	11.9618	0.0161
0.0190	-0.0265	14.1929	139385.69	987.64	0.7910	0.0010
0.0200	0.0290	-105.3213	123947.50	995.29	15.6846	0.0206
0.0210	-0.0265	22.4350	155869.94	1032.19	0.8087	0.0010
0.0220	0.0290	-108.9249	131055.44	1031.00	20.1751	0.0261
0.0230	-0.0245	38.3816	167763.19	1066.59	1.4808	0.0019
0.0240	0.0290	-109.1667	135592.31	1056.78	24.5571	0.0313
0.0250	-0.0210	37.4898	174979.63	1090.04	2.6650	0.0033
0.0260	0.0290	-106.5385	136752.44	1072.30	28.5846	0.0362
0.0270	-0.0170	43.0324	178064.75	1102.88	4.0273	0.0050
0.0280	0.0290	-102.3198	135377.44	1078.72	31.8512	0.0402
0.0290	-0.0130	46.3911	177782.19	1106.62	5.3873	0.0067
0.0300	0.0290	-97.3710	132380.00	1077.94	34.1495	0.0431

TABLE 7

Computer Run No. 3

Changed Spring Rate

CHAPTER VI

EVALUATION OF COMPUTER SIMULATION

To judge the validity of the assumptions that were made and the preciseness of the simulation to represent actual relief valve operation, the following tests were conducted:

1. Record pressure vs flow rate to determine the characteristic relationship between these two variables.
2. Record pressure vs time under dynamic loading to determine the forcing function characteristic.
3. Record pressure vs displacement to check the preciseness of the computer simulation.

The test circuit is shown in Fig. 29.

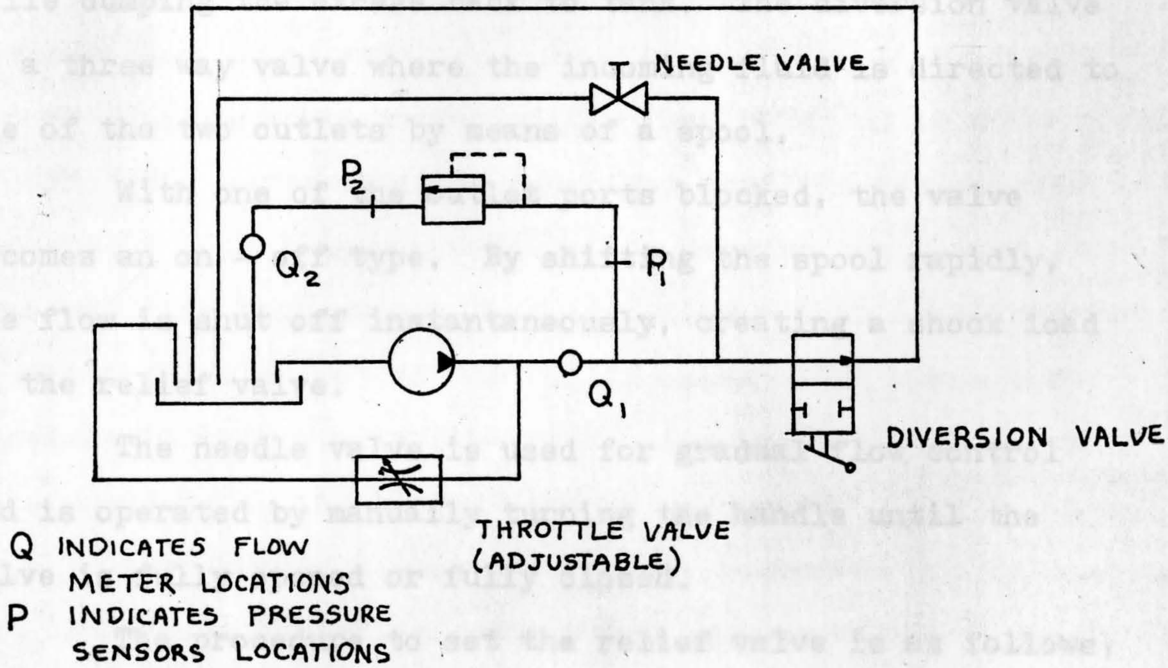


Fig. 29 Relief valve test circuit

The following components were used for the test:

1. 30 GPM gear pump, 1 in. gear width, rotating at 1800 rpm, Commercial Shearing model P-30.
2. Ambrex 97K, (Mobil Oil Co.) was the hydraulic fluid used. It is a straight mineral oil with a mild rust inhibitive. The specific gravity is 0.8708 and the density is 0.031 $\frac{lb}{in^3}$.
3. BLH Electronics pressure transducers, model GP-CG for 10,000 PSI service.
4. Cox turbine flow meter, model AN 20, 1.0 in. diameter.
5. Consolidated Electro Dynamic Corp. oscillograph, model 5 - 124A.
6. Hewlett - Packard X-Y recorder.

The throttle valve was built into the test stand and its function is to control the flow to the test circuit while dumping the excess back to tank. The diversion valve is a three way valve where the incoming fluid is directed to one of the two outlets by means of a spool.

With one of the outlet ports blocked, the valve becomes an on - off type. By shifting the spool rapidly, the flow is shut off instantaneously, creating a shock load on the relief valve.

The needle valve is used for gradual flow control and is operated by manually turning the handle until the valve is fully opened or fully closed.

The procedure to set the relief valve is as follows; (drawing of the relief valve is shown in Fig. 12).

1. Turn the adjusting screw out to relieve the spring load.
2. Start pump and then shut off the diversion valve and needle valve, allowing full flow to pass through the relief valve.
3. Screw in the relief valve adjusting screw and adjust the throttle valve until the desired pressure and flow are obtained. The pressure is indicated by the gauge at P_1 and the flow by the meter Q_2. The setting for this test was 750 PSI at 20 GPM flow.

Test number 1 was conducted to record the operating characteristic of the relief valve by establishing the relationship between pressure and flow. The diversion valve was closed and the needle valve was used to control the flow by screwing the valve shut at a slow, steady rate. The pressure drop ($P_1 - P_2$) and the flow rate, Q_2, were plotted on the X-Y recorder, Fig. 30.

The test was conducted for flow rates of 10 GPM and 20 GPM. The curve obtained indicates that the relief valve has stable operating characteristics despite the slight pressure rise from cracking to full flow. This is attributed to the orifice effect of the relief valve which means that pressure increases with flow rate. For an ideal relief valve this would be a flat line at the set pressure.

Test number 2 was conducted to determine the forcing function characteristics. The shock loading approach was used since the needle valve response is too slow to stop the flow quickly. With the needle valve closed and the diversion valve open, a pressure transducer was attached to the hydraulic circuit at point P_1. The diversion valve was then suddenly closed and the pressure rise was recorded by the oscillograph, see Fig. 31.

After a series of runs, the average time required to reach 750 PSI was determined to be 0.08 seconds. From this short time span and the shape of the curve, the step function assumption of equation (24), i. e. $(PA - F_{MTG})$ is justifiable. Although this curve shows no ramp function characteristics, the combination of a step and exponential decaying assumption can nonetheless be made.

Test number 3 was conducted to verify the computer simulation results. The given relief valve design permitted a direct measurement of the poppet travel in order to record pressure vs displacement, if this were not possible, then the displacement would have to be calculated based on pressure and flow rate. To record the very small flows that result when the relief valve first starts to open would have required special instrumentation.

In order to measure the poppet's position, a stiff, 0.062 in. diameter rod was welded to the back of the poppet and extended through a hole drilled in the adjusting screw as shown in Fig. 32. Since the pressure in this back chamber is low, the leakage through the small diameter hole was negligible. The locknut was pinned to the adjusting screw to permit using a wrench to adjust the pressure without disturbing the rod.

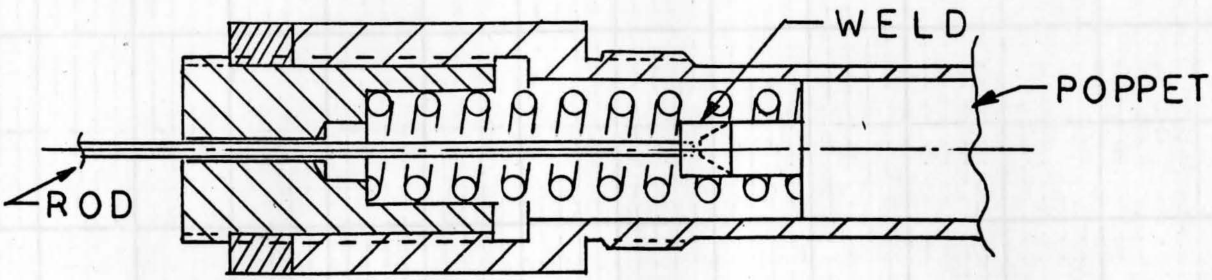


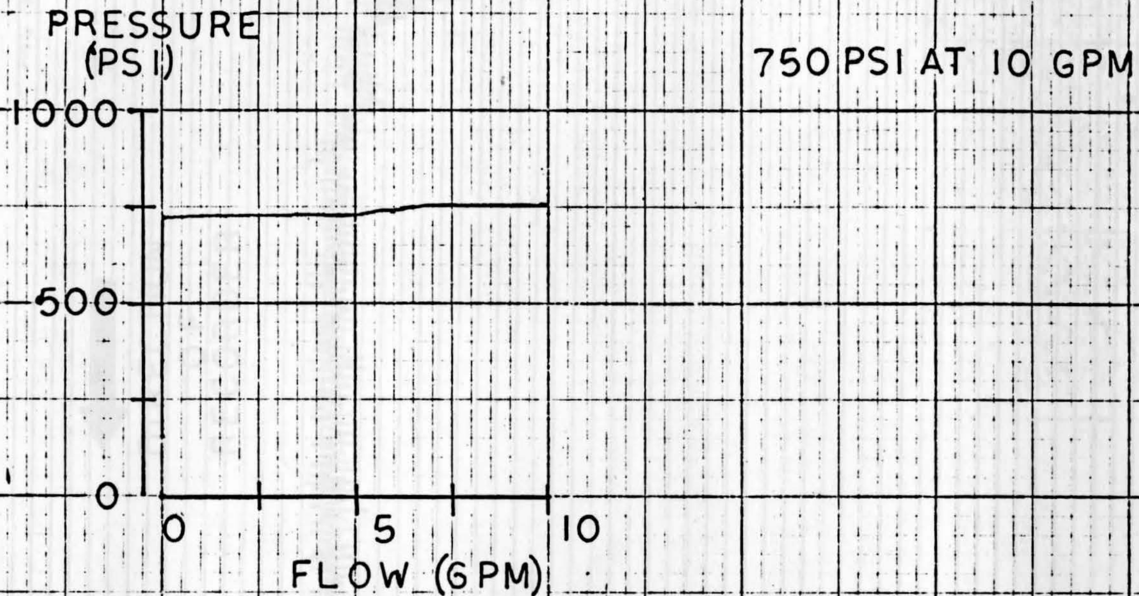
Fig. 32. Relief valve modified for direct poppet measurement

Two recording methods were used with the needle valve controlling the flow in each case.

The first method involved recording the displacement manually by means of a depth micrometer and the results are tabulated on table 8. Care must be taken with this type of measurement since too much pressure from the micrometer on the rod can change the reading.

The second method involved mounting a linear variable displacement transducer (LVDT) to the rod. The core of the LVDT was mounted on a separate platform, isolating it from the test stand in order to eliminate vibration problems. The LVDT rod was attached to the relief valve rod by wrapping wires around the two rods in an effort to minimize the weight. The addition of unnecessary weight could have resulted in inertial-type problems.

The X-Y recorder then plotted the pressure (P_1 - P_2) vs displacement as shown in Fig. 33, while the needle valve was screwed shut. The recording of pressure vs time, Fig. 34, was also made to show the rate at which the needle valve was being closed.



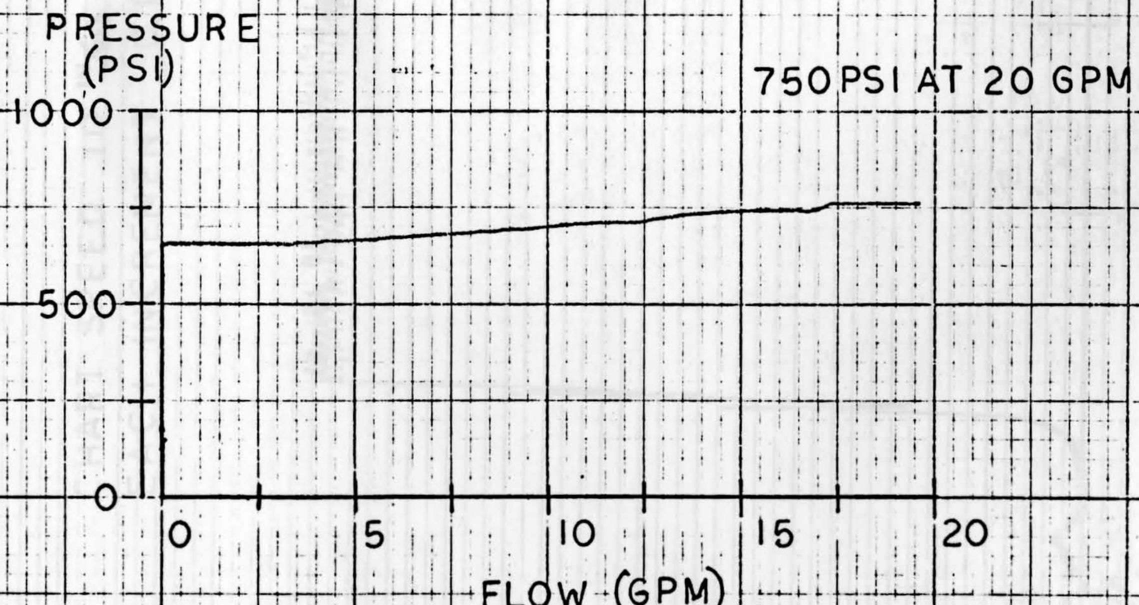


FIG. 30. PRESSURE vs FLOW.

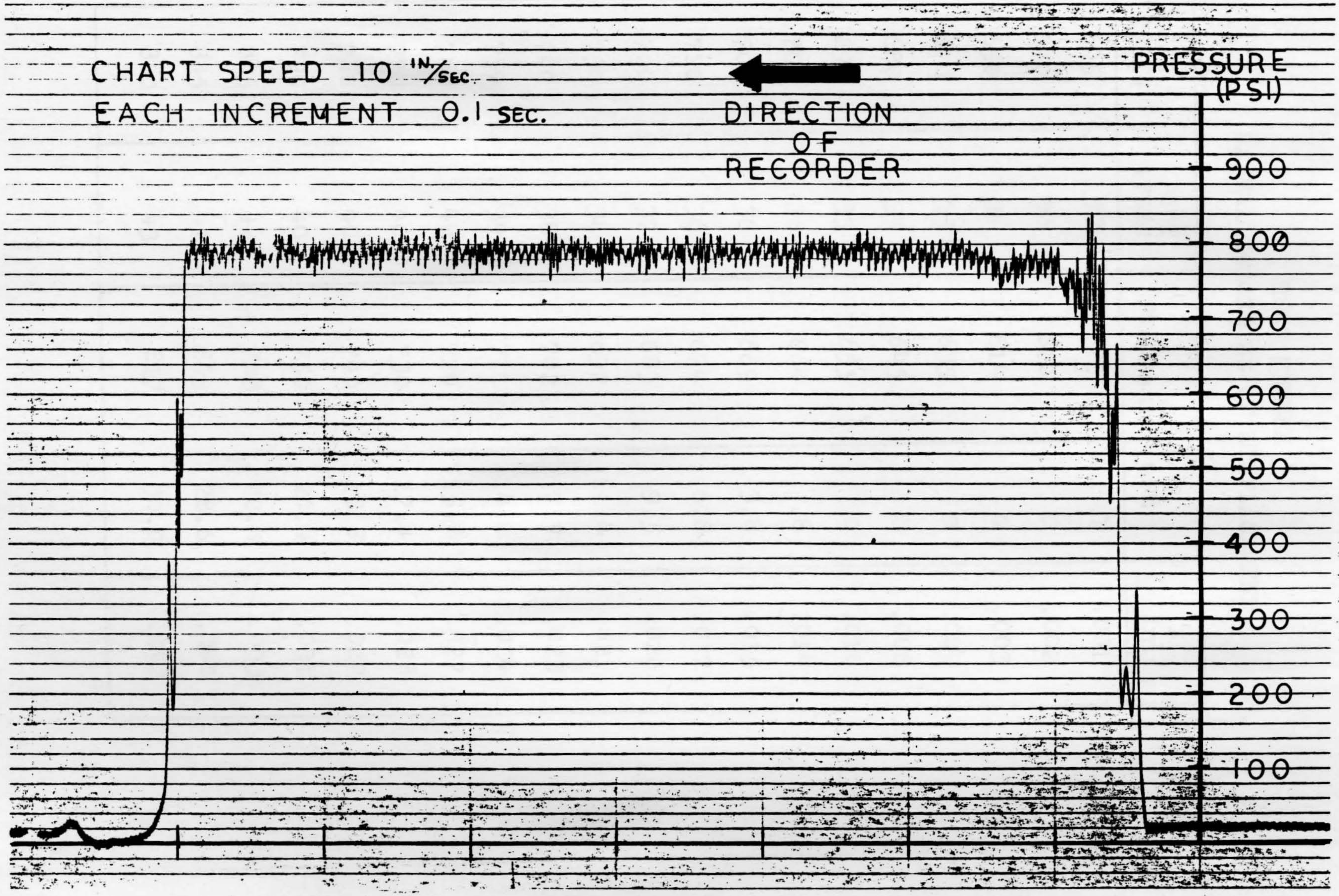


Fig. 31. Test number 2.

TABLE 8

TEST NUMBER 3
MANUAL RECORDING OF DISPLACEMENT

PIN (PSI)	POUT (PSI)	P Pin-Pout	Q_1 GPM	X in.
605	30	575	0	.001
640	30	610	0	.002
650	36	614	0	.003
670	35	635	0	.005
680	35	645	0	.008
690	35	655	0	.010
700	36	664	.8	.014
705	36	669	3.8	.015
710	36	674	6.8	.017
720	37	683	9.5	.020
730	38	692	11.4	.021
740	38	702	12.7	.023
750	39	711	14.0	.024
760	40	720	15.8	.027
770	40	730	17.4	.028
780	40	740	18.8	.029
790	40	750	20.4	.031
805	42	763	22.4	.033
845	42	803	28.6	.040
860	43	817	30.0	.042

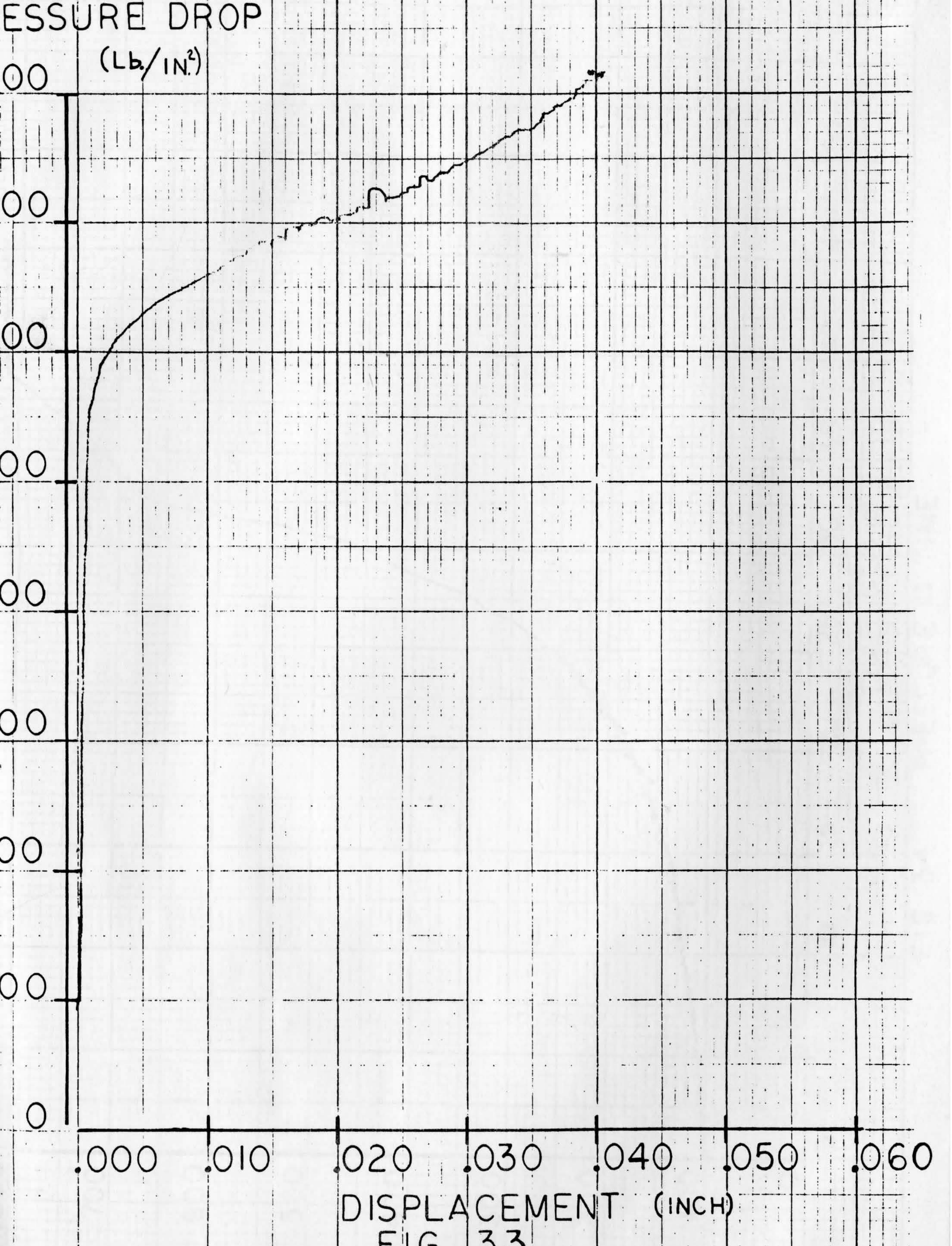


FIG. 33
PRESSURE vs DISPLACEMENT

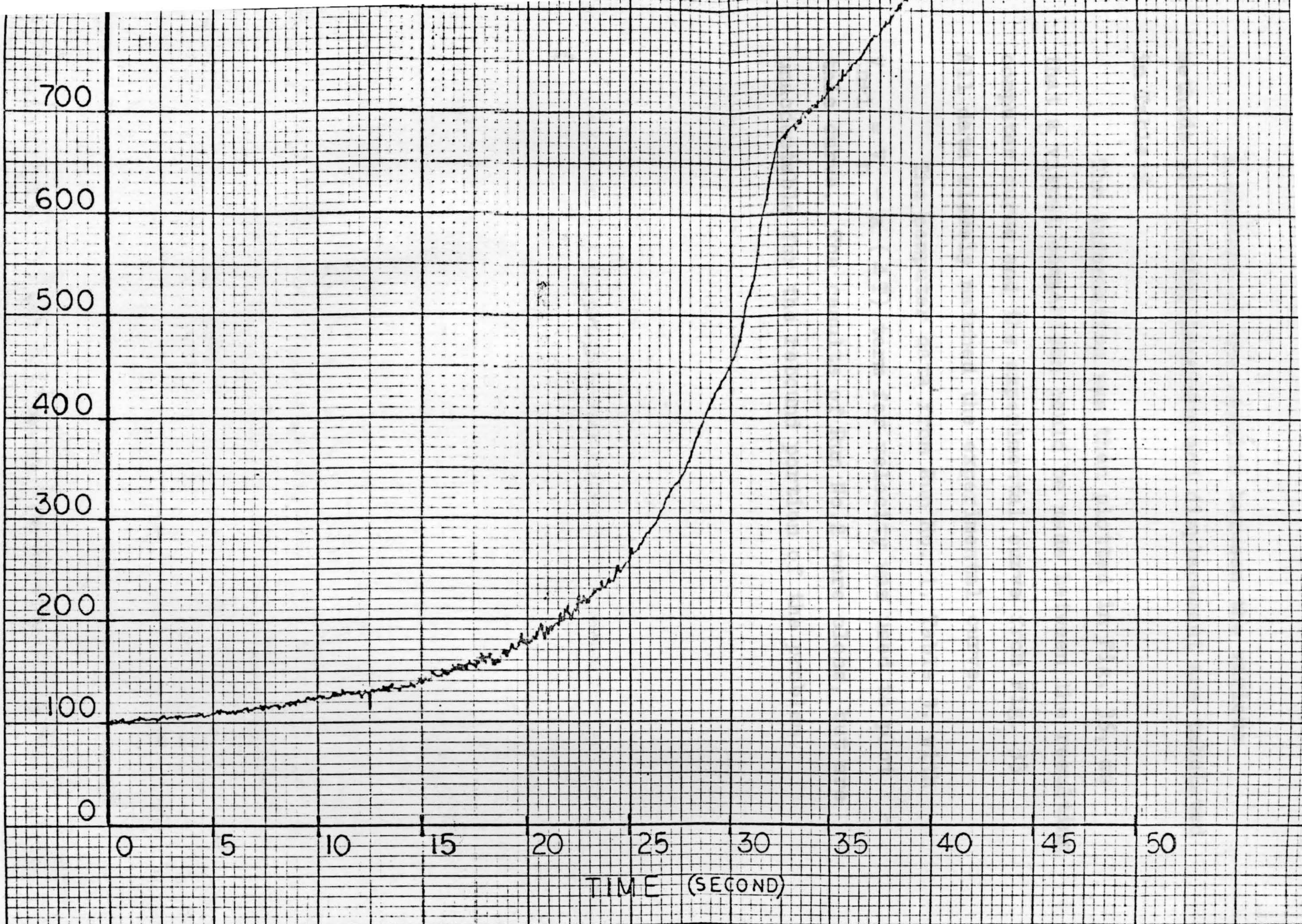


FIG 34 PRESSURE vs TIME

A summary of the results between the various methods of relating pressure and displacement are tabulated in Table 9.

The information was then plotted in Fig. 35, so that a visual comparison could be made between the resulting computer curve and the experimental curve from Fig. 33. All data closely matched the experimental curve.

The computer data from Run No. 1, Table 3, with $V_{n+1} = V_n + a\ (\Delta T)$ term for velocity was used to plot this curve. The results of Run No.2 were close, but are unacceptable for the rebound portion of the problem.

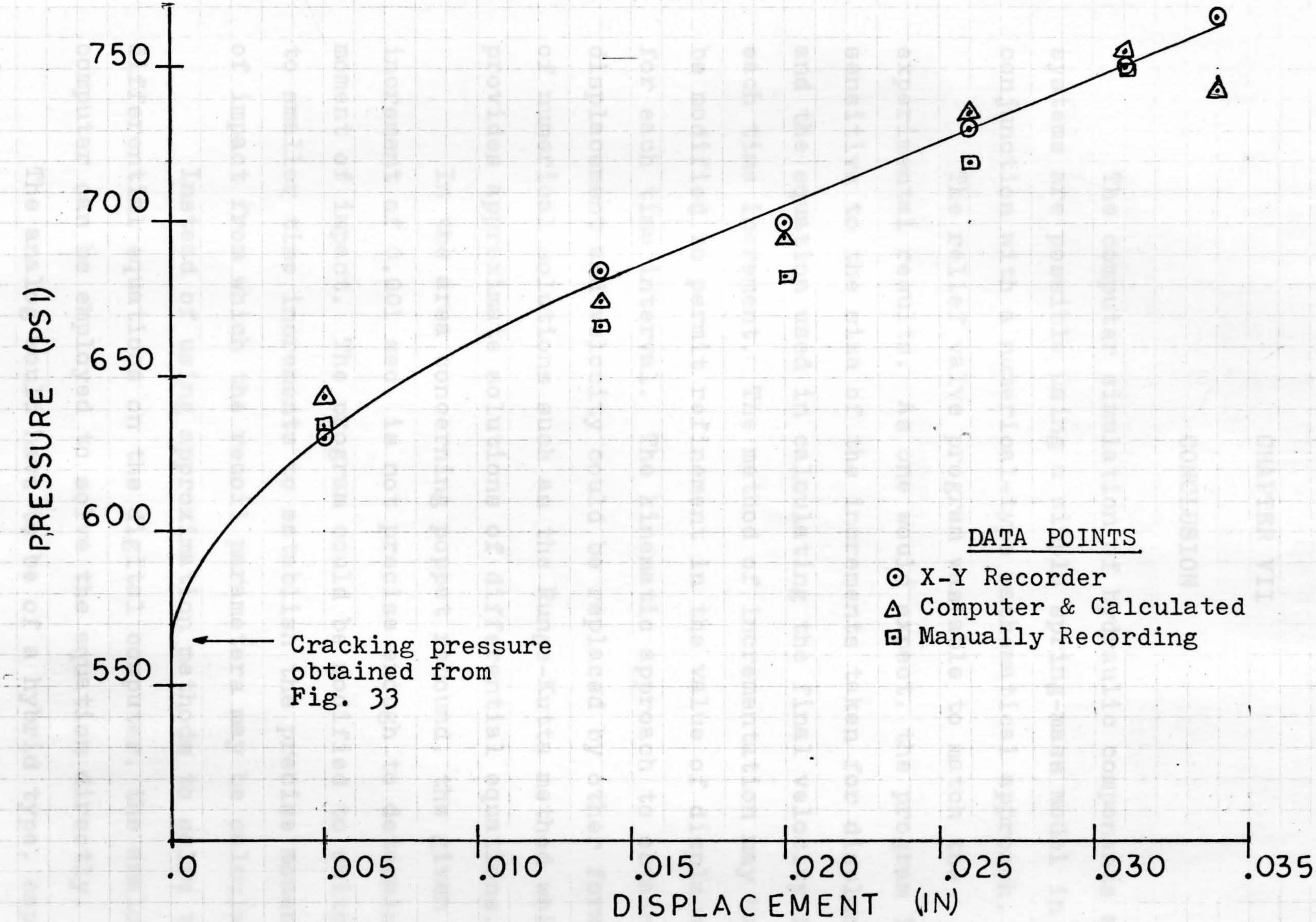


Fig. 35. Pressure vs Displacement Comparison Graph

CHAPTER VII

CONCLUSION

The computer simulation of hydraulic components and systems are possible using a simple spring-mass model in conjunction with a numerical-type mathematical approach.

The relief valve program was able to match the experimental results. As one would expect, the program is sensitive to the size of the increments taken for displacement and the equation used in calculating the final velocity for each time increment. The method of incrementation may have to be modified to permit refinement in the value of displacement for each time interval. The kinematic approach to obtain the displacement and velocity could be replaced by other forms of numerical solutions such as the Runge-Kutta method which provides approximate solutions of differential equations.

In the area concerning poppet rebound, the given time increment of 0.001 sec. is not precise enough to determine the moment of impact. The program could be modified to switch to smaller time increments to establish the precise moment of impact from which the recoil parameters may be calculated.

Instead of using approximation methods to solve the differential equations on the digital computer, the analog computer can be employed to solve the equation directly.

The analog would have to be of a hybrid type, capable of performing "IF" type decisions. This feature will permit switching from the spring equation to the impact equations.

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