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**Real-Time Simulation of Internal Flow Propulsion Systems by  
Matrix Stability Region Placement**

By

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**ABSTRACT****Real-Time Simulation of Internal Flow Propulsion Systems by****Matrix Stability Region Placement****Faramarz Mossayebi****Master of Science in Engineering****Youngstown State University, 1990**

A recently introduced real-time simulation algorithm, the Matrix Stability Region Placement (MSRP) method, is utilized to simulate a small perturbation model of the NASA Lewis Mach 2.5 40-60 mixed compression inlet. The model is representative of high speed internal flow propulsion systems which can be approximated as quasi-one-dimensional flows. The resulting system of equations, which is stiff, is also simulated by the second order Adam-Bashforth (AB-2) method. It is shown that MSRP method can be used to simulate small perturbation models of high speed internal flow propulsion systems in real-time.

Furthermore, a general closed form representation of the regression coefficients of the MSRP method is formulated by the aid of the Stirling numbers of the first kind. A heuristic argument is given for the equivalence of these methods and the Gregory-Newton  $n$ -th order hold approximation techniques.

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E	Energy	
j	$\gamma T$ or an integer	
J	Jacobian matrix	
H	The spatial discretization scheme	
m	Mass flow rate	
P	Pressure	
$S_{(n)}^{(k)}$	The Stirling numbers of the first kind	
t	Time	
T	Integration timestep	
$Y(S)$	Laplace-transform of $y(t)$	
$Y(Z)$	Z-transform of $y(kT)$	
$\alpha_{ij}$	The regression coefficients of SRP method, entries	
$\rho(Z)$	The first characteristic polynomial	
$\sigma(Z)$	The second characteristic polynomial	
$\lambda$	Eigenvalues of a given matrix	
$\Delta$	The smallest distance between	

## LIST OF SYMBOLS

SYMBOL	DEFINITION	UNITS
$A$	cross sectional area	ft <sup>2</sup>
$A$	System matrix	
$A_i, B_i$	The regression coefficients of MSRP method, matrices	
$B$	Input matrix	
$c$	Local speed of sound	
$C$	Output matrix	
$C_v$	specific heat at constant volume	ft-lb/(lb)(°R)
$C_p$	specific heat at constant pressure	ft-lb/(lb)(°R)
$e$	Specific internal energy	
$E$	Energy	ft-lbf/ft <sup>3</sup>
$j$	$\sqrt{-1}$ or an integer	
$J$	Jacobian matrix	
$H$	The spatial discretization distance	ft
$m$	Mass flow rate	
$P$	Pressure	lb/ft <sup>2</sup>
$S_{(m)}^{(n)}$	The Stirling numbers of the first kind	
$t$	Time	sec
$T$	Integration timestep	sec
$Y(S)$	Laplace-transform of $y(t)$	
$Y(Z)$	Z-Transform of $y(kT)$	
$\alpha_i, \beta_i$	The regression coefficients of SRP method, scalars	
$\rho(Z)$	The first characteristic polynomial	
$\sigma(Z)$	The second characteristic polynomial	
$\lambda$	Eigenvalues of a given matrix	
$\nabla$	The backward difference operator	

$v$  velocity ft/sec  
 $\rho$  Density slug/ft<sup>3</sup>

$\mathcal{L}$  A linear difference operator defined by Equation (3.3)

$\otimes$  The Kronecker tensor product

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dimension of physical systems. Due to these factors and other reasons, numerical simulation is now an integral part in the development and control of physical systems.

The real-time simulation of physical systems has received considerable attention in the past several decades, especially by control engineers, since this is a cost effective method for design, analysis, and testing of control systems. The major problem for real-time simulation is that of finite computing time. That is, the outputs and/or the states of the process to be controlled must be made available to the controller hardware/software in a given finite time. For very fast systems or those which are governed by a set of stiff differential equations, the available computing time is further constrained since smaller integration intervals are usually required.

Verifying controllers for propulsion systems, especially those under consideration for supersonic and hypersonic flight, requires accurate and fast algorithms for real-time simulation. The mathematical models of these complex propulsion systems are based, in part, on the Navier-Stokes equations. However, at present, the accurate simulation of these models is possible only on large scale computers and even then, these solutions cannot be utilized for real-time operations.

## CHAPTER I

### INTRODUCTION

The development of advanced technologies in the past several decades has contributed to considerable increases in the complexity and dimension of physical systems. Due to these factors and obvious economic reasons, numerical simulation is now an integral part in the research, development, and control of physical systems.

The real-time simulation of physical systems has received considerable attention in the past several decades, especially by control engineers, since this is a cost effective method for design, analysis, and testing of control systems. The major problem for real-time simulation is that of finite computing time. That is, the outputs and/or the states of the process to be controlled must be made available to the controller hardware/software in a given finite time. For very fast systems or those which are governed by a set of stiff differential equations, the simulation computing time is further constrained since smaller integration timesteps are usually required.

Verifying controllers for propulsion systems, specifically those under consideration for supersonic and hypersonic flight, requires accurate and fast algorithms for real-time simulation. The mathematical models of these complex propulsion systems are basically governed by the Navier-Stokes equations. However, at present, the accurate simulation of these models is possible only on large scale computers such as a Cray. Hence, these solutions cannot be utilized for real-time operation.

The real-time simulation of internal flow propulsion systems which are governed by quasi-one-dimensional flows is addressed in this thesis. Some of the propulsion systems which can be approximated by quasi-one-dimensional flow are ramjets, scramjets, nozzles, and inlets. Without loss of generality only the NASA Lewis Mach 2.5 40-60 mixed compression inlet [1] is considered for this study.

Traditionally these complex partial differential equations are solved by finite difference schemes, spatially as well as temporally. The stability and accuracy of these techniques are directly dependent upon the spatial and temporal discretization. The main objective of this thesis is to implement a linear multistep method to remedy the shortcomings of finite difference schemes with regards to temporal discretization. In particular, a continuous-time state space model of a typical supersonic inlet is simulated by the proposed linear multistep simulation algorithm.

Linear multistep methods are those which approximate the future values of the states of a process based upon the present and past values of the states and inputs. A large class of simulation algorithms, such as the Adams-Bashforth and Adams-Moulton methods, fall into this category. The integration timestep, i.e., temporal discretization, for accurate and stable simulation, is generally dependent upon the absolute value of the largest eigenvalue of the system. This imposes a great restriction on the timestep size for the simulation of very fast systems or stiff systems. A system is considered to be stiff if the ratio of the magnitude of its largest to smallest eigenvalues is large.

The Matrix Stability Region Placement (MSRP) [19] technique is proposed for simulation of the system which describes the dynamics of the inlet under investigation. The regression coefficients of the MSRP method are derived based upon the exact mapping of the eigenvalues of the continuous-time system. Furthermore the spurious modes of the integrator

are placed at the origin (in the Z-domain) to minimize the contribution of these modes. This particular integrator design results in the utilization of these methods for any system of equations regardless of the size of the eigenvalues of the system. It is shown here that this integration technique is indeed equivalent to the n-th order hold approximation techniques.

The aim of this thesis is to demonstrate that real-time simulation of high speed internal flows are possible by utilizing linear multistep methods. Moreover, the proposed linear multistep method, the MSRP method, for simulation of the small perturbation model of 40-60 inlet is generalized. A heuristic argument is given for the equivalence of these methods with the Gregory-Newton n-th order hold approximations.

Although the formulation of the mathematical models of the inlet is not in the scope of this thesis, the specific modeling technique used to generate the given model is briefly discussed in Chapter II. The fundamental theory of linear multistep methods are briefly reviewed in Chapter III. The MSRP method is then formulated and generalized in Chapter IV followed by the generalization of the Gregory-Newton n-th order hold approximations. The numerical results obtained by utilizing the MSRP method are compared with those obtained by using the Adams-Bashforth method in Chapter VI. Furthermore, the computational time associated with these integration routines is determined for comparison purposes. Finally, the conclusion and directions for further research are presented in Chapter VII.

## CHAPTER II

### MATHEMATICAL MODEL OF A MACH 2.5 40-60 MIXED COMPRESSION INLET

The general description of the system under investigation is presented in this chapter. The physical description and the equations that govern the 40-60 mixed compression inlet are given in the next two subsections, respectively. Finally the finite difference scheme utilized for spatial discretization of the inlet is described.

#### General Description of the Inlet

The performance of a supersonic aircraft's propulsion system is dependent upon the type of inlet system adopted. Mixed-compression inlets are recommended for aircraft that are designed to fly at Mach 2 or higher. The typical mixed compression inlet achieves compression through the external oblique shocks and normal internal shock, see Figure 2.1. The 40-60 mixed compression inlet achieves 40% of desired compression by external shocks and 60% by the internal shock.

During supersonic operation, a normal shock wave usually exists within the inlet due to the nature of these mixed-compression inlets. An isometric view of the mixed compression inlet is depicted in Figure 2.2. The basic function of the supersonic inlet is to change the kinetic energy of the air entering the inlet into a static pressure and temperature rise by converting the freestream kinetic energy into thermal energy. Optimum inlet performance can be achieved by positioning the normal shock near

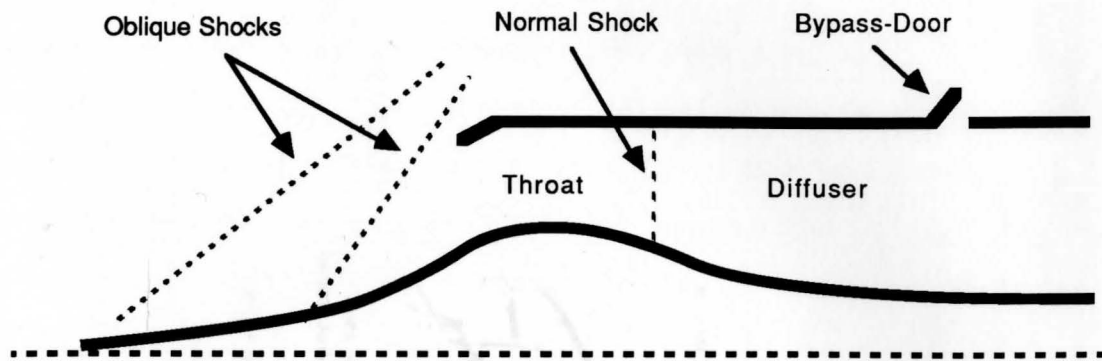


Figure 2.1 Mixed compression inlet.

the throat of the inlet [2]; this is where the greatest amount of energy conversion occurs. Positioning the normal shock further downstream degrades the inlet performance, but still allows a stable, or started, operation of the inlet. The upstream displacement of the shock on the other hand causes the shock to be expelled from the inlet. This condition is referred to as inlet unstart and has several undesirable ramifications: compressor stall, reduced thrust, and increased vehicle drag [2]. From the above discussion, it is evident that the determination and control of the shock position are of prime importance in the performance of supersonic propulsion systems.

Several inlet modeling techniques are readily available in the literature. A review of these modeling techniques can be found in [3]. In this study, only the recent modeling technique which is based on the computational fluid dynamics techniques is considered.

### Governing Equations

The inlet model is derived on the basis of quasi-one-dimensional flow. The flow inside a passage is said to be quasi-one dimensional if the cross-sectional area of the passage varies very slowly and the radius of curvature of the central axis of the passage is large compared to the passage

height. A flow model for unsteady quasi-one-dimensional turbulent flow is depicted in Figure 2.3. For this case, the area of flow is a function of the axial distance,  $x$ , as well as time,  $t$ . Furthermore it is assumed that the flow properties are uniform across all surfaces perpendicular to the axial flow direction. The governing fluid dynamic equations of motion are (4).

Continuity:

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho v A)}{\partial x} = 0$$

Momentum:

$$\frac{\partial(\rho v A)}{\partial t} + \frac{\partial(\rho v^2 A)}{\partial x} = -\frac{\partial(\rho A)}{\partial x} \frac{dP}{dx} + \rho A \frac{d^2x}{dt^2}$$

Energy:

$$\frac{\partial(\rho EA)}{\partial t} + \frac{\partial(\rho v EA)}{\partial x} = \rho A \frac{dE}{dt}$$

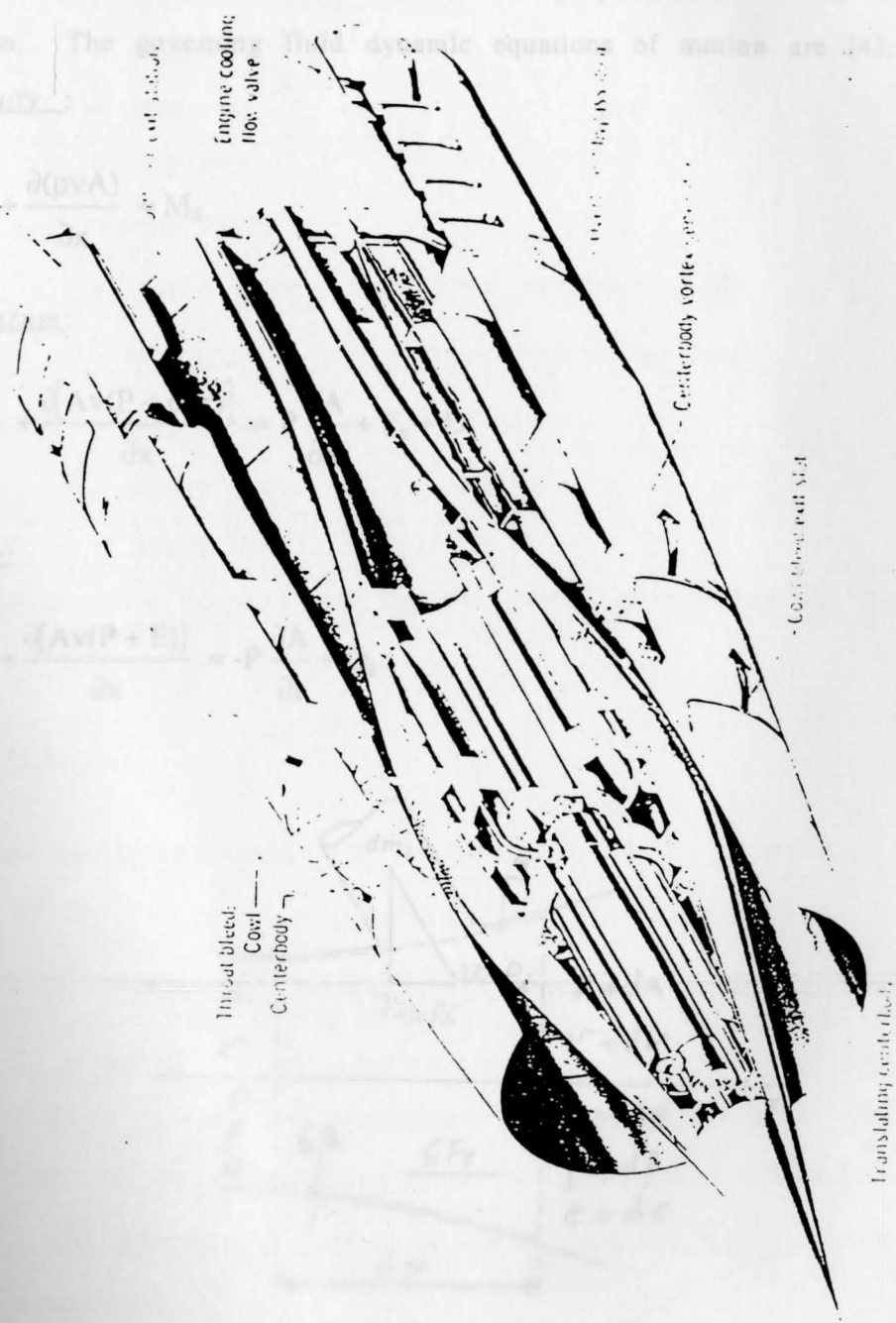


Figure 2.2 An isometric view of the mixed compression inlet [2].

Figure 2.3 Flow model for unsteady quasi-one-dimensional turbulent flow. The flow area is a function of axial distance,  $x$ , as well as time,  $t$ . The flow properties are uniform across all surfaces perpendicular to the axial flow direction.

height. A flow model for unsteady quasi-one-dimensional inviscid flow is depicted in Figure 2.3. For this case, the area of flow is a function of the axial distance,  $x$ , as well as time,  $t$ . Furthermore it is assumed that all flow properties are uniform across all surfaces perpendicular to the mean flow direction. The governing fluid dynamic equations of motion are [4]:

Continuity:

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho v A)}{\partial x} = M_s \quad (2.1)$$

Momentum:

$$\frac{\partial(\rho v A)}{\partial t} + \frac{\partial[A v (P + \rho v^2)]}{\partial x} = P \frac{\partial A}{\partial x} + F_s + D_s \quad (2.2)$$

Energy:

$$\frac{\partial(EA)}{\partial t} + \frac{\partial[A v (P + E)]}{\partial x} = -P \frac{\partial A}{\partial x} + Q_s \quad (2.3)$$

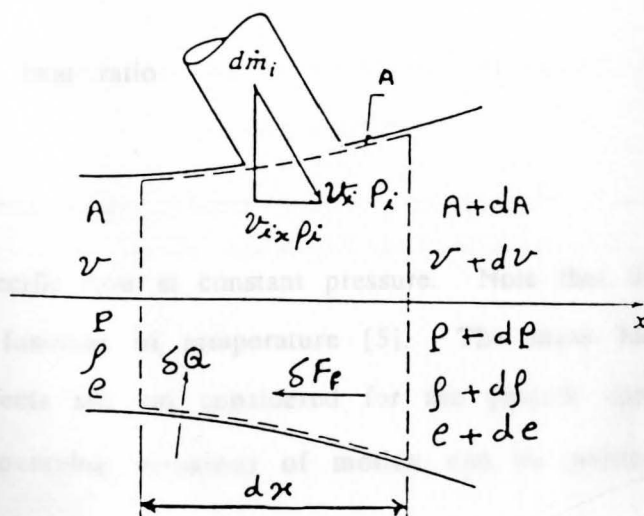


Figure 2.3 Flow model for unsteady quasi-one-dimensional inviscid flow.



where

$$E = \rho \left( e + \frac{1}{2} v^2 \right) = \rho \left( C_v T + \frac{1}{2} v^2 \right),$$

- $C_v$  = specific heat at constant volume,  
 $e$  = specific internal energy,  
 $\rho$  = density,  
 $P$  = pressure,  
 $v$  = velocity,  
 $M_s$  = mass bleed,  
 $F_s$  = friction term + mass bleed momentum term,  
 $Q_s$  = heat transfer term + mass bleed energy term,  
 $D_s$  = drag term,  
 $E$  = energy, and  
 $T$  = temperature.

Furthermore, it is assumed that the flow obeys the perfect gas equation of state:

$$P = (\gamma - 1) \left( E - \frac{\rho v^2}{2} \right) \quad (2.4)$$

with the specific heat ratio

$$\gamma = \frac{C_p}{C_v}$$

where  $C_p$  is specific heat at constant pressure. Note that these specific heats are typically a function of temperature [5]. The mass bleed, friction, and heat transfer effects are not considered for the present application.

The governing equations of motion can be written in conservative vector form,

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0 \quad (2.5)$$

where the components of  $U$  and  $F$  are

$$U = \begin{pmatrix} \rho \\ m \\ E \end{pmatrix}, \quad F = \begin{pmatrix} m \\ \frac{m^2}{\rho} + P \\ \frac{(E+P)m}{\rho} \end{pmatrix},$$

and  $m$  is mass flow rate, i.e.  $m = \rho v$  and the source terms on the right side have been ignored. The equations (2.1)-(2.3) are usually referred to as the quasi-one-dimensional inviscid equations of gasdynamics. Also, by using equation (2.4), the flux vector,  $F(U)$ , can be rewritten as

$$F = \begin{pmatrix} m \\ (\gamma-1)E + \frac{(3-\gamma)m^2}{2\rho} \\ \frac{\gamma Em}{\rho} - \frac{(\gamma-1)m^3}{2\rho^2} \end{pmatrix}. \quad (2.6)$$

### Split Flux Method for the One-Dimensional Equations of Gasdynamics

Equation (2.5) can be converted to nonconservative form, that is

$$\frac{\partial U}{\partial t} + J \frac{\partial U}{\partial x} = 0,$$

which is linearized by replacing the  $F(U)$  term by

$$\frac{\partial F}{\partial x} = J \frac{\partial U}{\partial x} \quad (2.7)$$

where  $J$  is the Jacobian of the flux vector, i.e.

$$J = \frac{\partial F}{\partial U} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{(\gamma-3)m^2}{2\rho^2} & \frac{(3-\gamma)m}{\rho} & \gamma-1 \\ \frac{(\gamma-1)m^3}{\rho^3} - \frac{\gamma Em}{\rho^2} & \frac{\gamma E}{\rho} - \frac{3(\gamma-1)m^2}{2\rho^2} & \frac{\gamma m}{\rho} \end{bmatrix}. \quad (2.8)$$

Clearly, this linearization must be carried out for each spatial lump of the system. The eigenvalues of the linearized system at each lump are:

$$\lambda_1 = v, \lambda_2 = v + c, \text{ and } \lambda_3 = v - c, \quad (2.9)$$

where  $c$  is the local speed of sound, i.e.  $c = \sqrt{\frac{\gamma P}{\rho}}$ . Note that for subsonic flow  $|v| < c$ , and the eigenvalues have mixed signs. This implies that the signals can travel in different directions with different propagation speeds. Hence, the finite difference schemes utilized to approximate  $\frac{\partial U}{\partial x}$  must take this into consideration.

The finite difference scheme utilized in this report is the so called Split Flux Method [6]. This method splits the flux vector into two subvectors which correspond to the negative and positive eigenvalues. Since the nonlinear flux vector  $F(U)$  is a homogeneous function, then applying Euler's theorem to homogeneous functions [7] gives:

$$F = J U. \quad (2.10)$$

Since the Jacobian matrix  $J$  has distinct eigenvalues then there exists a similarity transformation matrix,  $V$ , such that

$$J = V \Lambda V^{-1} \quad (2.11)$$

where  $\Lambda$  is a diagonal matrix containing the eigenvalues of  $J$  and  $V$  is the matrix which has the eigenvectors of  $J$  as its columns. Substituting equation

(2.11) into (2.10) then yields

$$F = V \Lambda V^{-1} U. \quad (2.12)$$

However the diagonal matrix  $\Lambda$  can be split into positive and negative diagonal matrices once it is realized that any eigenvalue can be expressed as

$$\lambda_i = \lambda_i^+ + \lambda_i^- \quad (2.13)$$

where

$$\lambda_i^+ = \frac{\lambda_i + |\lambda_i|}{2}, \text{ and}$$

$$\lambda_i^- = \frac{\lambda_i - |\lambda_i|}{2}.$$

That is, equation (2.13) can be used to rewrite the diagonal matrix  $\Lambda$  as

$$\Lambda = \Lambda^+ + \Lambda^- \quad (2.14)$$

where  $\Lambda^+$  and  $\Lambda^-$  are diagonal matrices with  $\lambda_i^+$  and  $\lambda_i^-$  as their coefficients, respectively. Equation (2.12) can now be written as

$$\begin{aligned} F &= V(\Lambda^+ + \Lambda^-)V^{-1}U \\ &= (J^+ + J^-)U \\ &= F^+ + F^- \end{aligned} \quad (2.15)$$

where

$$J^+ = V \Lambda^+ V^{-1},$$

$$J^- = V \Lambda^- V^{-1},$$

$$F^+ = J^+ U,$$

$$F^- = J^- U, \text{ and}$$

$$J = J^+ + J^- \quad (2.16)$$

Hence, the governing equation can be rewritten as:

$$\frac{\partial U}{\partial t} + \frac{\partial F^+}{\partial x} + \frac{\partial F^-}{\partial x} = 0. \quad (2.17)$$

Since the system has been split into positive and negative subsystems,

separate difference schemes can be utilized for each subvector. That is, the positive and negative subvectors are spatially discretized by backward and forward difference operators, respectively. Carrying out this procedure for the  $i^{\text{th}}$  lump then yields:

$$\frac{dU_i}{dt} + \frac{J_i^+}{H} (U_i - U_{i-1}) + \frac{J_i^-}{H} (U_{i+1} - U_i) = S_i \quad (2.18)$$

where  $H$  is the spatial discretization distance and  $S_i$  are the linearized source terms. It is assumed that  $H$  is constant; the spatial domain is divided into  $n$  equally spaced lumps of size  $H$ .

### Split Flux Method Applied to 40-60 Inlet

The step by step procedure for obtaining a linearized small perturbation state space model of the form

$$\begin{aligned} \frac{d\mathbf{X}}{dt} &= \dot{\mathbf{X}} = \mathbf{A} \mathbf{X} + \mathbf{B} \mathbf{U} \\ \mathbf{Y} &= \mathbf{C} \mathbf{X} + \mathbf{D} \mathbf{U} \end{aligned} \quad (2.19)$$

for the NASA Mach 2.5 40-60 mixed compression inlet, utilizing the split flux method, is [8] :

- a) Create a steady state vector,  $\mathbf{X}$ , which consists of the density, momentum, and energy for each lump respectively. The resulting vector is:

$$\mathbf{X} = \begin{bmatrix} \rho_1 \\ m_1 \\ E_1 \\ \rho_2 \\ m_2 \\ E_2 \\ \vdots \\ \rho_n \\ m_n \\ E_n \end{bmatrix} \quad (2.20)$$

where  $n$  is the total number of lumps.

- b) Calculate the Jacobian matrix for each lump by using Equation (2.8) and

determine the eigenpairs.

- c) Obtain  $J^+$  and  $J^-$  for each lump by using the similarity transformation matrix, Equation (2.15).
- d) Discretize  $J^+$  and  $J^-$  with the appropriate differencing operator, Equation (2.18).
- e) Assemble the overall state space matrix as follows:

$$A = \frac{1}{H} \begin{bmatrix} -J_1^+ + J_1^- & -J_1^- & 0 & 0 & \dots & 0 \\ J_2^+ & -J_2^+ + J_2^- & -J_2^- & 0 & \dots & 0 \\ 0 & J_3^+ & -J_3^+ + J_3^- & -J_3^- & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \vdots & \vdots \end{bmatrix}. \quad (2.21)$$

- f) Determine the B, C, and D matrices and correct the A matrix for the given boundary conditions, inputs, and outputs.

For the specific system considered, the above procedure results in a single input, single output, state space system where: the A matrix is  $(3n \times 3n)$ , the B matrix is  $(3n \times 1)$ , the C matrix is  $(1 \times 3n)$ , and the D matrix is  $(1 \times 1)$ . To carry out this procedure for the 40-60 inlet, a table of steady state values is obtained from the program LAPIN [9] and plotted as depicted in Figures 2.4-2.6. The steady state values are tabulated in TABLE 2.1 and are utilized for step one of the above procedure. The number of lumps considered is 41. The spatial discretization step, H, is .1427 for this particular number of lumps. Since it is desired to have the change in downstream pressure as the input to the system, then Equation (2.4) can be rearranged and linearized to obtain:

$$\partial E = \frac{\partial P}{(\gamma-1)} - \left(\frac{m^2}{2\rho^2}\right) \partial \rho + \left(\frac{m}{\rho}\right) \partial m. \quad (2.22)$$

The input matrix then will have one entry,  $\frac{1}{(\gamma-1)}$ , in the last row; the input

to the system is applied at the termination lump. Hence the last row of the last Jacobian matrix in the global system matrix, A, must be modified such that Equation (2.22) is satisfied. This will require the addition of the following row vector to the last row of the last diagonal block of A:

$$\left[ [J_n(1,3)] \frac{-m^2}{2\rho^2}, [J_n(2,3)] \frac{m}{\rho}, [J_n(3,3)] \right]. \quad (2.23)$$

Finally, the output matrix, C, must be determined. It is desired to determine the perturbation of shock position as a small pressure perturbation is applied to the system. However, the shock position is directly related to the pressure [10]. Hence, the required output will be the change in pressure at a lump just downstream of the shock. The output matrix is then obtained from above as

$$\partial P = \left( \frac{(\gamma-1)m^2}{\rho^2} \right) \partial \rho - \left( \frac{(\gamma-1)m}{2\rho} \right) \partial m + (\gamma-1) \partial E. \quad (2.24)$$

A MATLAB [10] program is written to compute the state space model of the inlet under consideration. The resulting system is large (123 states). The numerical values of the {A,B,C} are tabulated in the Appendix A. It must be mentioned that the system dynamics matrix, A, is scaled [8]. This does not affect the results and the conclusions of this study.

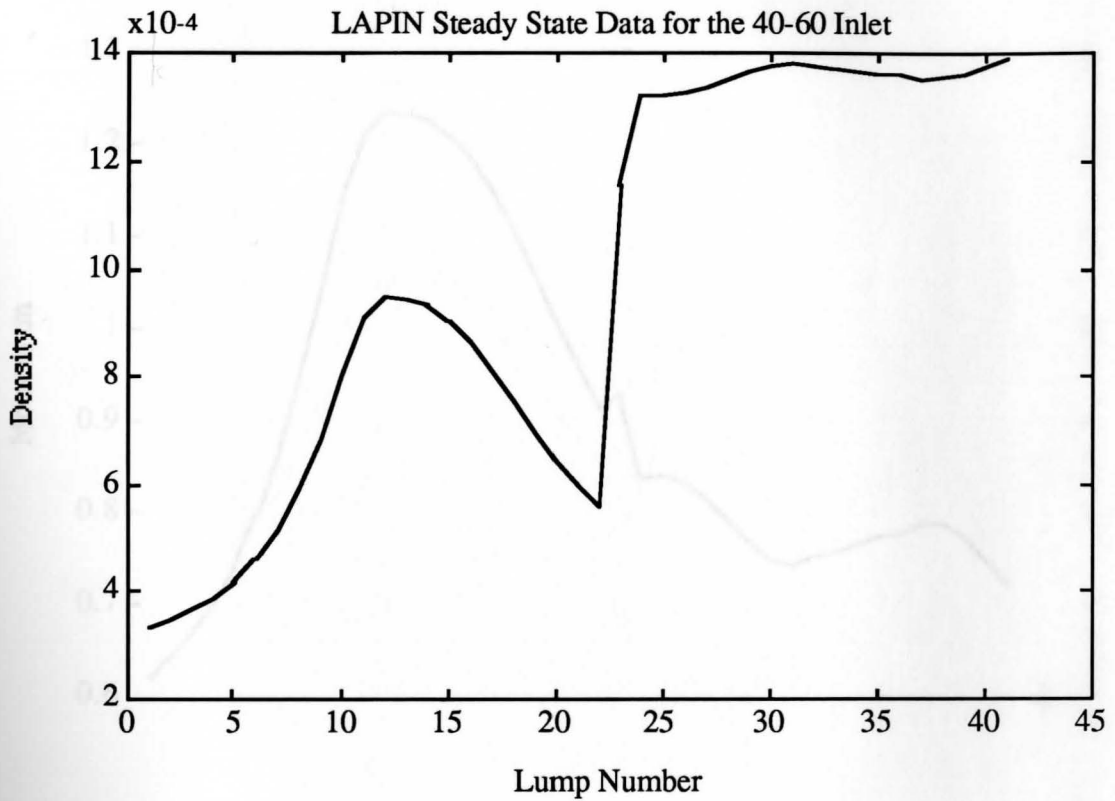


Figure 2.4 Steady state values of density at each lump.



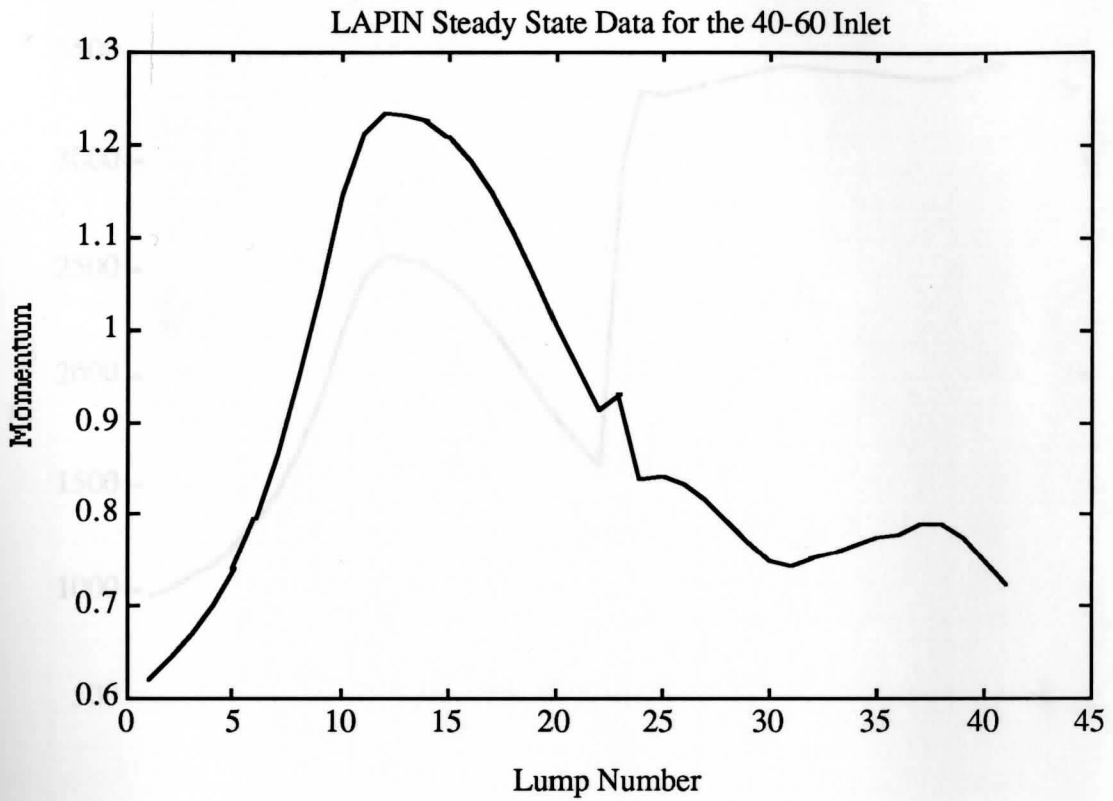


Figure 2.5 Steady state values of momentum at each lump.

TABLE 11  
LAPIN STEADY STATE DATA FOR 40-60 INLET

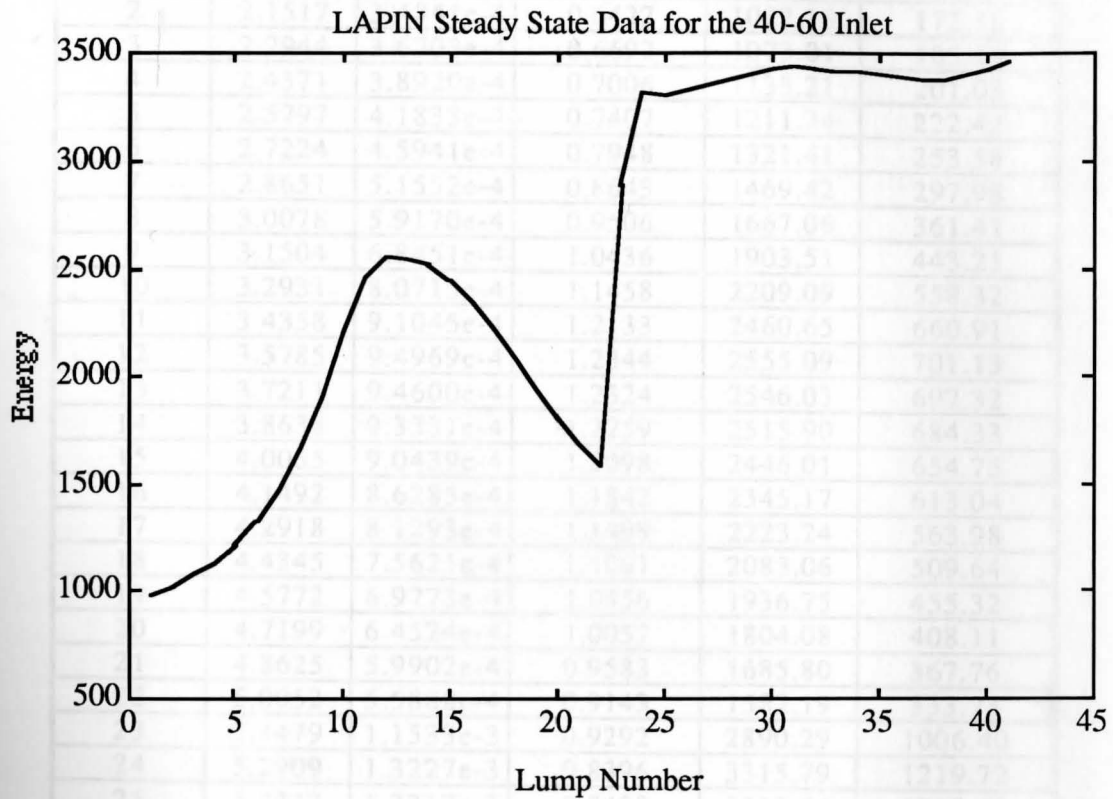


Figure 2.6 Steady state values of energy at each lump.

TABLE 2.1  
LAPIN STEADY STATE DATA FOR 40-60 INLET.

Station #	Station	Density	Momentum	Energy	Pressure
1	2.0090	3.3261e-4	0.6187	978.86	161.34
2	2.1517	3.4841e-4	0.6422	1022.18	172.16
3	2.2944	3.6703e-4	0.6692	1073.01	185.19
4	2.4371	3.8920e-4	0.7006	1133.21	201.03
5	2.5797	4.1833e-4	0.7407	1211.74	222.42
6	2.7224	4.5941e-4	0.7948	1321.41	253.58
7	2.8651	5.1552e-4	0.8643	1469.42	297.98
8	3.0078	5.9170e-4	0.9506	1667.06	361.41
9	3.1504	6.8451e-4	1.0436	1903.51	443.21
10	3.2931	8.0715e-4	1.1458	2209.09	558.32
11	3.4358	9.1045e-4	1.2133	2460.65	660.91
12	3.5785	9.4969e-4	1.2344	2555.09	701.13
13	3.7211	9.4600e-4	1.2324	2546.03	697.32
14	3.8638	9.3331e-4	1.2259	2515.90	684.33
15	4.0065	9.0439e-4	1.2098	2446.01	654.75
16	4.1492	8.6285e-4	1.1842	2345.17	613.04
17	4.2918	8.1293e-4	1.1499	2223.24	563.98
18	4.4345	7.5621e-4	1.1061	2083.06	509.64
19	4.5772	6.9773e-4	1.0556	1936.75	455.32
20	4.7199	6.4524e-4	1.0057	1804.08	408.11
21	4.8625	5.9902e-4	0.9583	1685.80	367.76
22	5.0052	5.5888e-4	0.9143	1582.19	333.73
23	5.1479	1.1533e-3	0.9292	2890.29	1006.40
24	5.2909	1.3227e-3	0.8396	3315.79	1219.72
25	5.4332	1.3217e-3	0.8408	3313.64	1218.48
26	5.5759	1.3278e-3	0.8322	3326.71	1226.35
27	5.7186	1.3401e-3	0.8141	3353.03	1242.30
28	5.8613	1.3530e-3	0.7937	3380.58	1259.11
29	6.0039	1.3678e-3	0.7685	3411.96	1278.43
30	6.1466	1.3781e-3	0.7498	3433.77	1291.92
31	6.2893	1.3807e-3	0.7450	3439.31	1295.32
32	6.4320	1.3763e-3	0.7534	3430.00	1289.52
33	6.5746	1.3733e-3	0.7587	3423.60	1285.61
34	6.7173	1.3687e-3	0.7671	3413.82	1279.54
35	6.8600	1.3633e-3	0.7763	3402.40	1272.54
36	7.0027	1.3619e-3	0.7789	3399.30	1270.61
37	7.1453	1.3552e-3	0.7900	3385.14	1261.96
38	7.2880	1.3561e-3	0.7886	3387.13	1263.13
39	7.4307	1.3646e-3	0.7744	3405.10	1274.15
40	7.5734	1.3790e-3	0.7481	3435.67	1293.09
41	7.7160	1.3920e-3	0.7229	3463.18	1310.19

## CHAPTER III

### AN INTRODUCTION TO THE THEORY OF LINEAR MULTISTEP METHODS

In this chapter the fundamental definitions and theory for numerical integration of differential equations by the so-called Linear Multistep Methods (LMM) are presented. The material, presented in this section, closely follows and occasionally draws heavily from the work of Lambert [12]. A single first-order differential equation is considered throughout this chapter unless otherwise specified.

#### The Linear Multistep Method

Consider the following initial value problem

$$\dot{x} = f(x, t), \quad x(t_0) = x_0 \quad (3.1)$$

for which a solution is sought in a finite range of  $t_0 \leq t \leq t_1$ . Assume that Equation (3.1) has a unique continuously differentiable solution,  $x(t)$ . An approximate solution can be obtained at different time intervals determined by the integration timestep; thus solution is of discrete form. Throughout this paper, it is assumed that the integration timestep,  $T$ , is constant. Let the sequence  $\{x_n\}$  be the approximate solution to Equation (3.1) and  $f_n \equiv \dot{x}_n$ . The LMM then can be defined as follows.

**Definition 3.1:** Any computational method is called a linear  $k$ -step method if the sequence is determined from a linear relationship between  $x_{n+j}$  and  $f_{n+j}$  with  $j=0, 1, \dots, k$ .

The general linear multistep method can be written as [9]

$$\sum_{j=0}^k \alpha_j x_{n+j} = T \sum_{j=0}^k \beta_j f_{n+j} \quad (3.2)$$

where  $\alpha_j$  and  $\beta_j$  are constants and  $T$  is the integration timestep. In order to remove the ambiguity in determining the constants,  $\alpha_k$  is always assumed to be unity. Furthermore, we assume that  $\beta_k = 0$ . This results in the so-called explicit methods since the present value of  $x_{n+k}$  is computed in terms of the old values of  $x$  and  $f(x,t)$ , i.e.  $x_{n+j}, f_{n+j}; j=0,1, \dots, k-1$ . Implicit methods ( $\beta_k \neq 0$ ) can be more accurate and stable than explicit ones; however, they generally require greater computational time. Moreover, the implicit methods cannot generally be used for real-time applications since the present value of  $x$  must be calculated in terms of the present and past values of  $x$  and  $f$ .

The problem at hand is to determine the constants of Equation (3.2). This is treated in [12] extensively; only the derivation through Taylor expansions is cited in this paper. A linear difference operator,  $\mathcal{L}$ , can be associated with Equation (3.2) given by

$$\mathcal{L}[x(t);T] \equiv \sum_{j=0}^k [\alpha_j x(t+jT) - T \beta_j f(t+jT)] \quad (3.3)$$

Expanding the function  $x(t+jT)$  and its derivative as Taylor series about  $t$ , and collecting terms in Equation (3.3) yields

$$\mathcal{L}[x(t);T] = C_0 x(t) + C_1 T x(t)^1 + C_2 T^2 x(t)^2 + \dots + C_q T^q x(t)^q + \dots \quad (3.4)$$

where the  $C_q$  are constants and  $x(t)^q$  is the  $q$ -th derivative of  $x(t)$ . Note that Equation (3.4) represents the local truncation error of the general multistep method. The order of accuracy associated with a LMM is defined as follows.

**Definition 3.2:** The linear multistep method, Equation (3.2), is defined to be of order  $P$  if in Equation (3.4),  $C_0 = C_1 = \dots = C_P = 0$ ,  $C_{P+1} \neq 0$ .

The constants  $C_q$  can be written in terms of the coefficients  $\alpha_j$  and  $\beta_j$  as [12] :

$$\begin{aligned} C_0 &= \alpha_0 + \alpha_1 + \dots + \alpha_k \\ C_1 &= \alpha_1 + 2\alpha_2 + \dots + k\alpha_k - (\beta_0 + \beta_1 + \beta_2 + \dots + \beta_k) \\ C_q &= \left(\frac{1}{q!}\right)(\alpha_1 + 2^q\alpha_2 + \dots + k^q\alpha_k) - \left(\frac{1}{(q-1)!}\right)(\beta_1 + 2^{(q-1)}\beta_2 + \dots + k^{(q-1)}\beta_k) \end{aligned} \quad (3.5)$$

Hence, a linear system of equations can be obtained to solve for coefficients  $\alpha_j$  and  $\beta_j$  once the number of steps and maximal order are specified. For instance, the most accurate two-step explicit method with  $\alpha_0 = 0$  is determined from Equation (3.5) as

$$\begin{aligned} C_0 &= \alpha_0 + \alpha_1 + \alpha_2 = \alpha_1 + 1 = 0 \\ C_1 &= \alpha_1 + 2\alpha_2 - (\beta_1 + \beta_2) = \alpha_1 - \beta_1 - \beta_2 + 1 = 0 \\ C_2 &= \frac{1}{2}(\alpha_1 + 4\alpha_2) - \beta_1 = \frac{\alpha_1}{2} - \beta_1 + 2 = 0 . \end{aligned}$$

Solving this system of equations yields

$$\alpha_1 = -1, \quad \beta_1 = 1.5, \quad \beta_2 = -0.5,$$

and the method is

$$x_{n+2} = x_{n+1} + T(1.5 \dot{x}_{n+1} - 0.5 \dot{x}_n) \quad (3.6)$$

which is the second order Adams-Bashforth (AB-2) method. This method is second order accurate since

$$C_3 = \frac{1}{6}(\alpha_1 + 8\alpha_2) - \frac{\beta_1}{2} = \frac{5}{12} \neq 0 .$$

### Stability of Linear K-Step Methods

Techniques from linear control theory are utilized in this section to analyze the stability of linear multistep methods. Since these methods are discrete approximations then Equation (3.2) can be Z-transformed to obtain

$$G(Z) = \frac{X(Z)}{\dot{X}(Z)} = \frac{T(\beta_k Z^k + \beta_{k-1} Z^{k-1} + \dots + \beta_1 Z + \beta_0)}{(\alpha_k Z^k + \alpha_{k-1} Z^{k-1} + \dots + \alpha_1 Z + \alpha_0)} = \frac{T\sigma(Z)}{\rho(Z)} . \quad (3.7)$$

The denominator of Equation (3.7),  $\rho(Z)$ , is called the first characteristic

polynomial and  $\sigma(Z)$  is called the second characteristic polynomial. A single first order differential equation has one eigenvalue denoted by  $\lambda$ . However an exact Z-plane representation of the system is obtained by mapping the continuous S-plane through the following function

$$Z = e^{ST} \quad (3.8)$$

Therefore the location of  $\lambda$  in Z-plane is obtained by

$$Z = e^{\lambda T} \quad (3.9)$$

Since Equation (3.7) has  $k$  poles, a  $k$ -step method will result in  $(k-1)$  extra, or spurious, roots. The root closest to the exact mapping of  $\lambda$  into the Z-plane, Equation (3.9), is called the principal root. In general, adopting a  $k$ -step method for integration of an  $n$ -th order system leads to a  $(kn)$ -th order discrete system of equations with  $n$  principal roots and  $[(k-1)n]$  spurious roots. The magnitude of the principal roots must be larger than the magnitude of the spurious roots in order to obtain a stable and accurate simulation [13]. In order to minimize the effect of the spurious roots on the integration results, they must be placed close to zero.

The following results are given in Lambert for guaranteeing the convergence of a simulation to the actual system.

**Definition 3.3 :** A linear multistep method is said to be zero-stable if all the roots of its first characteristic polynomial,  $\rho(Z)$ , are inside the unit circle and roots on the unit circle are simple.

**Definition 3.4 :** A linear multistep method is said to be consistent if it has order  $k+1$ .

Note that Definition 3.3 implies that the open-loop integration is stable. In [12] it is proved that the linear multistep method is consistent if and only if the following are satisfied

$$\sum_{j=0}^k \alpha_j = 0, \quad \sum_{j=0}^k j \alpha_j = \sum_{j=0}^k \beta_j \quad (3.10)$$

or

$$\rho(Z)|_{Z=1} = 0, \quad \left. \frac{\partial \rho(Z)}{\partial Z} \right|_{Z=1} = \sigma(Z)|_{Z=1} \quad (3.11)$$

Therefore, the first characteristic polynomial always has a root at +1 for a consistent method. This corresponds to a pole at zero in the S-plane which corresponds to an integrator. It was shown in [13] that these requirements can be obtained by applying the final value theorem to an integrator being forced by an impulse.

A fundamental theorem for convergence of the linear multistep method which was originally proposed by Dahlquist [14] and is proved in [15] is stated as follows.

**Theorem 3.1 :** The necessary and sufficient conditions for a linear multistep method to be convergent are that it be consistent and zero-stable.

Qualitatively, the magnitude of the local truncation error at each step of the calculation is controlled by consistency whereas the fashion by which this error is propagated as the calculation proceeds is controlled by zero-stability.

Thus far, the stability of the integration routine by itself, open-loop stability, has been considered. The performance of the integration on an actual dynamic system is determined by closed loop stability which is defined as weak stability theory by mathematicians. This analysis is obtained by utilizing a linear test equation, i.e.,

$$\dot{x} = \lambda x + u \quad (3.12)$$

where  $\lambda$  is an eigenvalue. A block diagram representation of integrating the test equation is depicted in Figure (3.1). In order to investigate the accuracy and stability of the overall system, the closed-loop poles of the system must be determined. The closed-loop transfer function of Figure (3.1) can be determined as



$$H(Z) = \frac{X(Z)}{U(Z)} = \frac{T \sigma(Z)}{\rho(Z) - \lambda T \sigma(Z)} \quad (3.13)$$

Hence, the stability of the integration process is determined by the roots of the closed-loop characteristic polynomial, i.e.,

$$\rho(Z) - \lambda T \sigma(Z) = 0. \quad (3.14)$$

**Definition 3.5:** A linear multistep method is said to be absolutely stable for a given  $\lambda T$  product if all the roots of the Equation (3.14) are inside the unit circle.

**Definition 3.6:** A linear multistep method is said to be relatively stable for a given  $\lambda T$  product if the principal root is larger or equal to all the spurious roots.

The concept of absolute stability introduced by numerical analysts is related to the control engineering stability analysis of constructing the root locus. That is, from a control system point of view, considering the closed-loop characteristic equation, the  $\lambda T$  product resembles the gain of a system with unity feedback. To clarify, consider the AB-2 method which was previously derived. The open-loop transfer function of this method can be written as

$$G(Z) = \frac{X(Z)}{\dot{X}(Z)} = \frac{T(1.5z - 0.5)}{z^2 - z} = \frac{T \sigma(Z)}{\rho(Z)}. \quad (3.15)$$

The root locus of the AB-2 method applied to the linear test equation, depicted in Figure (3.2), is obtained by substituting the values of  $\rho(Z)$  and  $\sigma(Z)$  into Equation (3.14). It must be noted that the system to be integrated is assumed to be stable. Hence, the product  $\lambda T$  is always negative. Furthermore, it is evident that all explicit methods have a zero at infinity. Therefore, these methods will always become unstable for large values of the  $\lambda T$  product.

The major drawback of analyzing the stability of linear multistep methods by constructing the root locus is its lack of generality; the stability results obtained by this analysis are limited only to a particular system

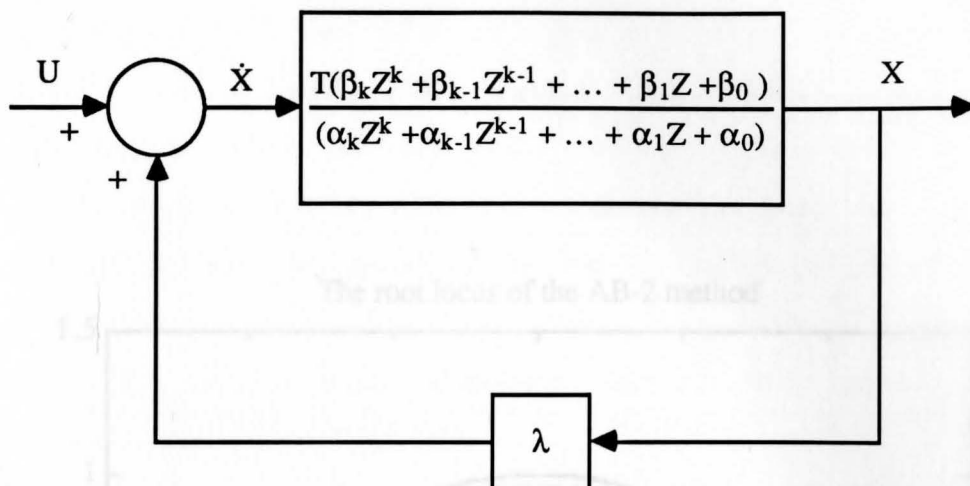


Figure 3.1 Block diagram representation for integrating the test equation.

whose eigenvalues are real. The stability of a specific linear multistep method for systems with complex eigenvalues can be generalized by means of conformal mappings. This is discussed in the next section.

### Stability Region of Linear Multistep Methods

To obtain a generalized stability region for any linear multistep method, Equation (3.14) can be solved for the product  $\lambda T$  to obtain

$$\lambda T = \frac{\rho(Z)}{\sigma(Z)}. \quad (3.16)$$

The boundary of the stability region in the Z-Plane, the unit circle, then can be conformally mapped into a new complex plane called the  $\lambda T$ -plane by substituting  $Z = e^{j\theta}$ , viz.

$$\lambda T = \frac{\rho(e^{j\theta})}{\sigma(e^{j\theta})}. \quad (3.17)$$

Once the boundary of the stability region in the  $\lambda T$ -plane is determined, one can investigate the stability and convergence of the method by observing

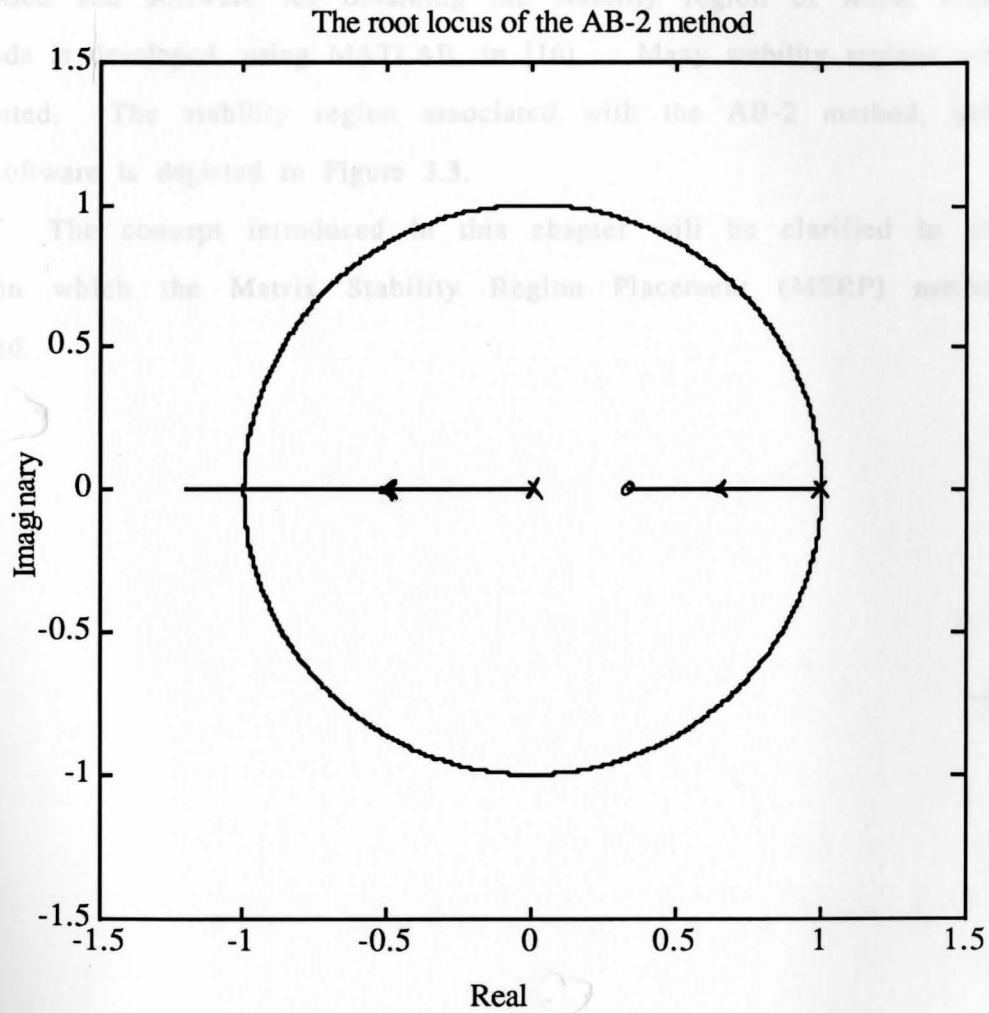


Figure 3.2 Root locus of AB-2 method.

whether the product  $\lambda T$  lies inside the region enclosed in this complex plane. The stability region for a given method is equivalent to the inverse frequency response of that method [17].

The rules associated with this conformal mapping are thoroughly discussed and software for obtaining the stability region of linear multistep methods is developed, using MATLAB, in [16]. Many stability regions are also presented. The stability region associated with the AB-2 method, utilizing this software is depicted in Figure 3.3.

The concept introduced in this chapter will be clarified in chapter IX, in which the Matrix Stability Region Placement (MSRP) method is derived.

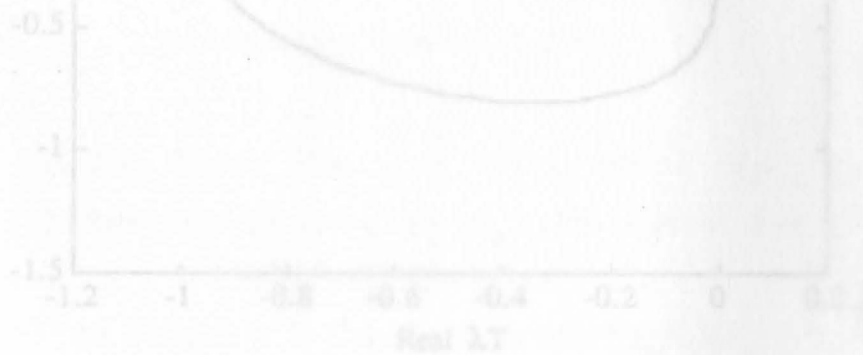


Figure 3.3 The stability region of AB-2 method.

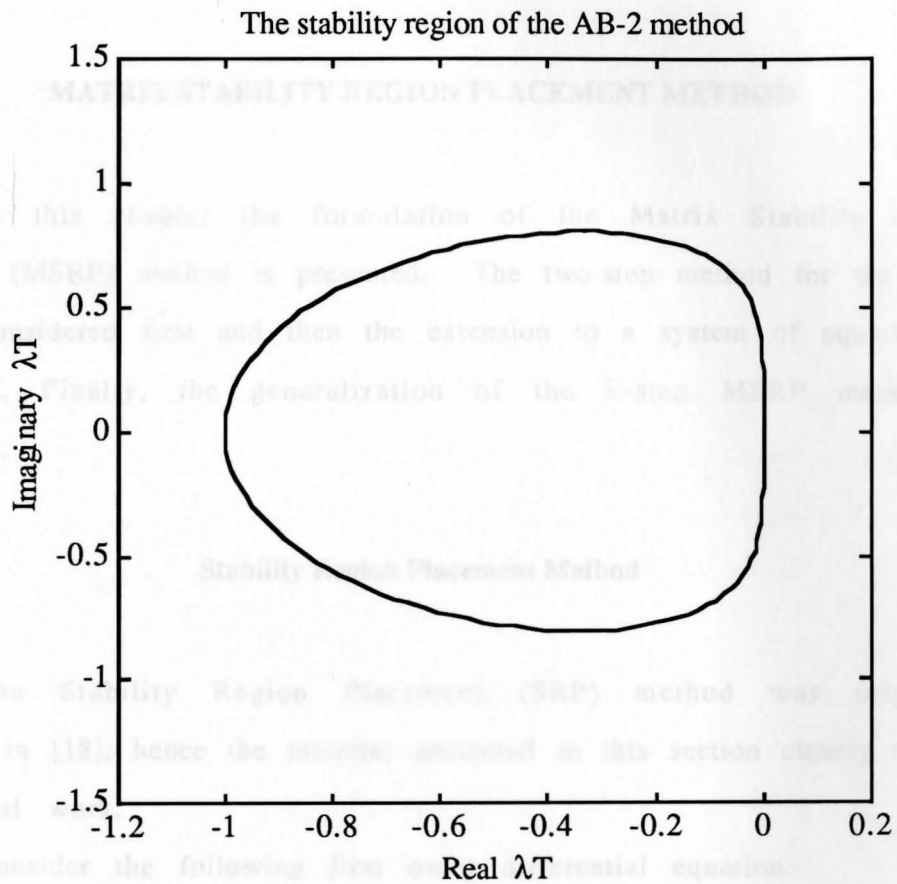


Figure 3.3 The stability region of AB-2 method.

## CHAPTER IV

### MATRIX STABILITY REGION PLACEMENT METHOD

In this chapter the formulation of the Matrix Stability Region Placement (MSRP) method is presented. The two-step method for the scalar case is considered first and then the extension to a system of equations is considered. Finally, the generalization of the k-step MSRP method is presented.

#### Stability Region Placement Method

The Stability Region Placement (SRP) method was originally developed in [18], hence the material presented in this section closely follows the original work.

Consider the following first order differential equation

$$\dot{x} = \lambda x + bu \quad (4.1)$$

Suppose that an explicit two-step linear multistep method, Equation (3.2), is considered for simulation of Equation (4.1), i.e.

$$x_{n+2} + \alpha_1 x_{n+1} + \alpha_0 x_n = T(\beta_1 \dot{x}_{n+1} + \beta_0 \dot{x}_n) \quad (4.2)$$

Substituting Equation (4.1) into Equation (4.2) and rearranging terms gives

$$x_{n+2} + (\alpha_1 - \lambda T\beta_1) x_{n+1} + (\alpha_0 - \lambda T\beta_1) x_n = T b (\beta_1 u_{n+1} + \beta_0 u_n) \quad (4.3)$$

In order to determine the coefficients of the integrator, four independent equations are necessary. Since the integrator must be consistent and zero-stable (Theorem 3.1), the following conditions must be satisfied

$$\alpha_1 + \alpha_0 = -1$$

$$\alpha_1 - \beta_0 - \beta_1 = -2 \quad (4.4)$$

Clearly, two more equations are needed to determine the coefficients uniquely. The SRP method determines the additional equations by placing the principal roots at the exact mapping of the poles of the system at their exact mapping in the Z-plane and placing the spurious roots at the Z-plane origin for obtaining the maximum region of stability. Taking the Z-transform of Equation (4.3) then gives

$$\frac{X(Z)}{U(Z)} = \frac{T b (\beta_1 Z + \beta_0)}{Z^2 + (\alpha_1 - \lambda T \beta_1) Z + (\alpha_0 - \lambda T \beta_0)} \quad (4.5)$$

Clearly, the pole of Equation (4.1) is located at  $\lambda$  in the S-plane, hence its exact mapping in the Z-plane is  $e^{\lambda T}$ . However the discrete-time approximation has resulted in a system with two poles. The SRP method then requires that the spurious roots be placed at the origin. These conditions result in the following two equations

$$\begin{aligned} (\alpha_1 - \lambda T \beta_1) &= -e^{\lambda T} \\ (\alpha_0 - \lambda T \beta_0) &= 0 \end{aligned} \quad (4.6)$$

The above equation along with Equation (4.4) can be solved simultaneously to obtain the coefficients  $\alpha_0$ ,  $\alpha_1$ ,  $\beta_0$  and  $\beta_1$ .

For illustrative purposes, suppose that  $\lambda = -10$ , and the integration timestep is restricted by the hardware to be  $T=1.0$ . Interestingly, this timestep is ten times that of the largest timestep allowed by most of the explicit linear multistep methods available in the literature. Utilizing the SRP method then gives the following system of equations to be solved for the integrator coefficients:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 1 & 0 & -\lambda T & 0 \\ 0 & 1 & 0 & -\lambda T \end{bmatrix} \begin{Bmatrix} \alpha_0 \\ \alpha_1 \\ \beta_0 \\ \beta_1 \end{Bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 0 \\ -e^{\lambda T} \end{bmatrix}.$$

Which upon solving gives :

$$\begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} 0.9 \\ -1.9 \\ -0.09 \\ 0.19 \end{bmatrix}.$$

The stability region of this integrator can be determined by the techniques presented in the previous chapter, i.e.,

$$\lambda T = \frac{\rho(Z)}{\sigma(Z)} = \frac{Z^2 + \alpha_1 Z + \alpha_0}{\beta_1 Z + \beta_0} \Big|_{Z=e^{j\theta}; \theta=0, \dots, 2\pi}$$

Using MATLAB software [8], the absolute stability region of the above integrator is determined as depicted in Figure 4.1 .

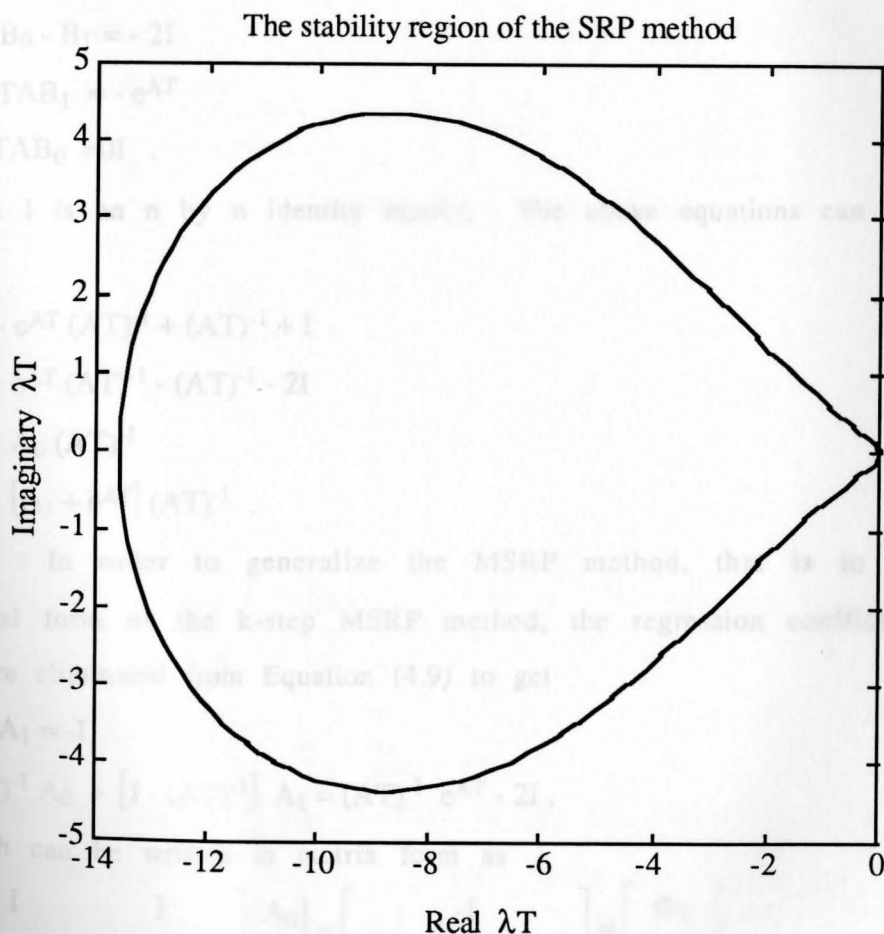


Figure 4.1 Stability region of SRP method for  $|\lambda T| = 10$ .



Extending the above argument to a system of equations

$$\dot{x} = A x + B u \quad (4.7)$$

where  $x$  is an  $n$ -dimensional state vector,  $u$  is an  $m$ -vector of input functions,  $A$  is an  $n$  by  $n$  matrix and  $B$  is an  $n$  by  $m$  matrix, results in the Matrix Stability Region Placement (MSRP) method [19]. The two-step MSRP (MSRP2) method then is written as :

$$x_{n+2} = -A_1 x_{n+1} - A_0 x_n + T (B_1 \dot{x}_{n+1} + B_0 \dot{x}_n) \quad (4.8)$$

where  $A_i$  and  $B_i$  ;  $i=0,1$  are  $n$  by  $n$  regression coefficient matrices. The regression coefficients are determined from the matrix version of Equation (4.4) and Equation (4.6), that is

$$\begin{aligned} A_0 + A_1 &= -I \\ A_1 - B_0 - B_1 &= -2I \\ A_1 - TAB_1 &= -e^{AT} \\ A_0 - TAB_0 &= 0I \end{aligned} \quad (4.9)$$

where  $I$  is an  $n$  by  $n$  identity matrix. The above equations can be solved to get

$$\begin{aligned} A_0 &= -e^{AT} (AT)^{-1} + (AT)^{-1} + I \\ A_1 &= e^{AT} (AT)^{-1} - (AT)^{-1} - 2I \\ B_0 &= A_0 (AT)^{-1} \\ B_1 &= [A_1 + e^{AT}] (AT)^{-1} \end{aligned} \quad (4.10)$$

In order to generalize the MSRP method, that is to develop the general form of the  $k$ -step MSRP method, the regression coefficients  $B_0$  and  $B_1$  are eliminated from Equation (4.9) to get

$$\begin{aligned} A_0 + A_1 &= -I \\ -(AT)^{-1} A_0 + [I - (AT)^{-1}] A_1 &= (AT)^{-1} e^{AT} - 2I \end{aligned}$$

Which can be written in matrix form as

$$\begin{bmatrix} I & I \\ -(AT)^{-1} & I - (AT)^{-1} \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \end{bmatrix} = \begin{bmatrix} -I \\ (AT)^{-1} e^{AT} - 2I \end{bmatrix} \equiv \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} \quad (4.11)$$

Post-multiplying the first row by  $(AT)^{-1}$  and adding to the second row gives

$$\begin{bmatrix} I & I \\ 0 & I \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \end{bmatrix} = \begin{bmatrix} \Phi_1 \\ \Phi_2 + \Phi_1 (AT)^{-1} \end{bmatrix} \equiv \begin{bmatrix} \hat{\Phi}_1 \\ \hat{\Phi}_2 \end{bmatrix}. \quad (4.12)$$

From the above block upper triangular system of equations, the coefficients  $A_0$  and  $A_1$  are directly obtained as :

$$\begin{aligned} A_1 &= \hat{\Phi}_2 = \Phi_2 + \Phi_1 (AT)^{-1} = (AT^{-1})e^{AT} - (AT)^{-1} - 2I \\ A_0 &= \hat{\Phi}_1 - A_1 = \Phi_1 - A_1 = - (AT)^{-1} e^{AT} + (AT)^{-1} + I. \end{aligned} \quad (4.13)$$

Note that the  $B_j; j=0,1$  coefficients are computed directly from knowledge of the  $A_j; j=0,1$  coefficients, that is

$$\begin{aligned} B_1 &= (A_1 + e^{AT}) (AT)^{-1} \\ B_0 &= A_0 (AT)^{-1}. \end{aligned} \quad (4.14)$$

It must be noted that the introduction of the  $\Phi_j$  notation is to facilitate the transition from the two-step method to the k-step method presented in the following subsection.

### K-Step Matrix Integrators

The two-step method of the previous subsection is generalized to the k-step case in [20]. The explicit k-step MSRP integrator is described by a difference equation similar to that of Equation (4.8), viz.,

$$x_{n+k} = \sum_{j=0}^{k-1} (-A_j x_{n+j} + T B_j \dot{x}_{n+j}) \quad (4.15)$$

where  $x$  is given by Equation (4.7). Substituting Equation (4.7) into Equation (4.15) and rearranging gives :

$$x_{n+k} = \sum_{j=0}^{k-1} (T B_j A - A_j) x_{n+j} + T \sum_{j=0}^{k-1} B_j B u_{n+j} \quad (4.16)$$

In order to investigate the stability of the resulting closed-loop system, the Z-transform can be utilized. Hence, taking the Z-transform of Equation (4.16) and grouping similar terms yields :

$$\left\{ Z^k I + \sum_{j=0}^{k-1} (A_j - T B_j A) Z^j \right\} X(Z) = \left\{ T \sum_{j=0}^{k-1} B_j B Z^j \right\} U(Z). \quad (4.17)$$

This equation can be written in the input-output form as

$$X(Z) = H(Z) U(Z) \quad (4.18)$$

where the transfer function matrix,  $H(z)$ , is given by :

$$H(Z) = \left[ Z^k I + \sum_{j=0}^{k-1} (A_j - T B_j A) Z^j \right]^{-1} \left[ T \sum_{j=0}^{k-1} B_j B Z^j \right].$$

A block diagram representation of Equation (4.17) is depicted in Figure 4.2.

The state variables shown in this figure are related to the actual states by [19]  $x_j(i) = x_{i+k-j-1}$  ;  $j=1,2,\dots, k-1$  .

Note that the block diagram representation of the well known observer canonical form for multivariable linear system is precisely that of Figure 4.2. Therefore, the resulting closed-loop system, Equation (4.16), is always observable, which in turn guarantees that the integrated states can be completely recovered from output observations. The closed-loop system can be represented in state space form as:

$$\begin{aligned} x(i+1) &= \tilde{A} x(i) + \tilde{B} u(i) \\ y(i) &= \tilde{C} x(i) = x_{k-1}(i) = x_i \end{aligned} \quad (4.19)$$

where

$$\tilde{A} = \begin{bmatrix} 0 & 0 & \dots & \dots & 0 & -(A_0 - T B_0 A) \\ I & 0 & \dots & \dots & 0 & -(A_1 - T B_1 A) \\ 0 & I & \dots & \dots & 0 & -(A_2 - T B_2 A) \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & 0 & -(A_{k-2} - T B_{k-2} A) \\ 0 & 0 & \dots & \dots & I & -(A_{k-1} - T B_{k-1} A) \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} T B_0 B \\ T B_1 B \\ T B_2 B \\ \dots \\ \dots \\ T B_{k-2} B \\ T B_{k-1} B \end{bmatrix}, \quad \text{and}$$

$$\tilde{C} = [0 \ 0 \ \dots \ 0 \ I].$$

Therefore, the closed-loop stability of the integrator is determined by the system described by Equation (4.19). The philosophy used in the two-step method can then be applied to place the poles of the closed-loop integrator at the desired locations. Since the system to be integrated, Equation (4.7), is  $n$ -th order then utilizing a  $k$ -step method results in a  $kn$ -th order discrete-time

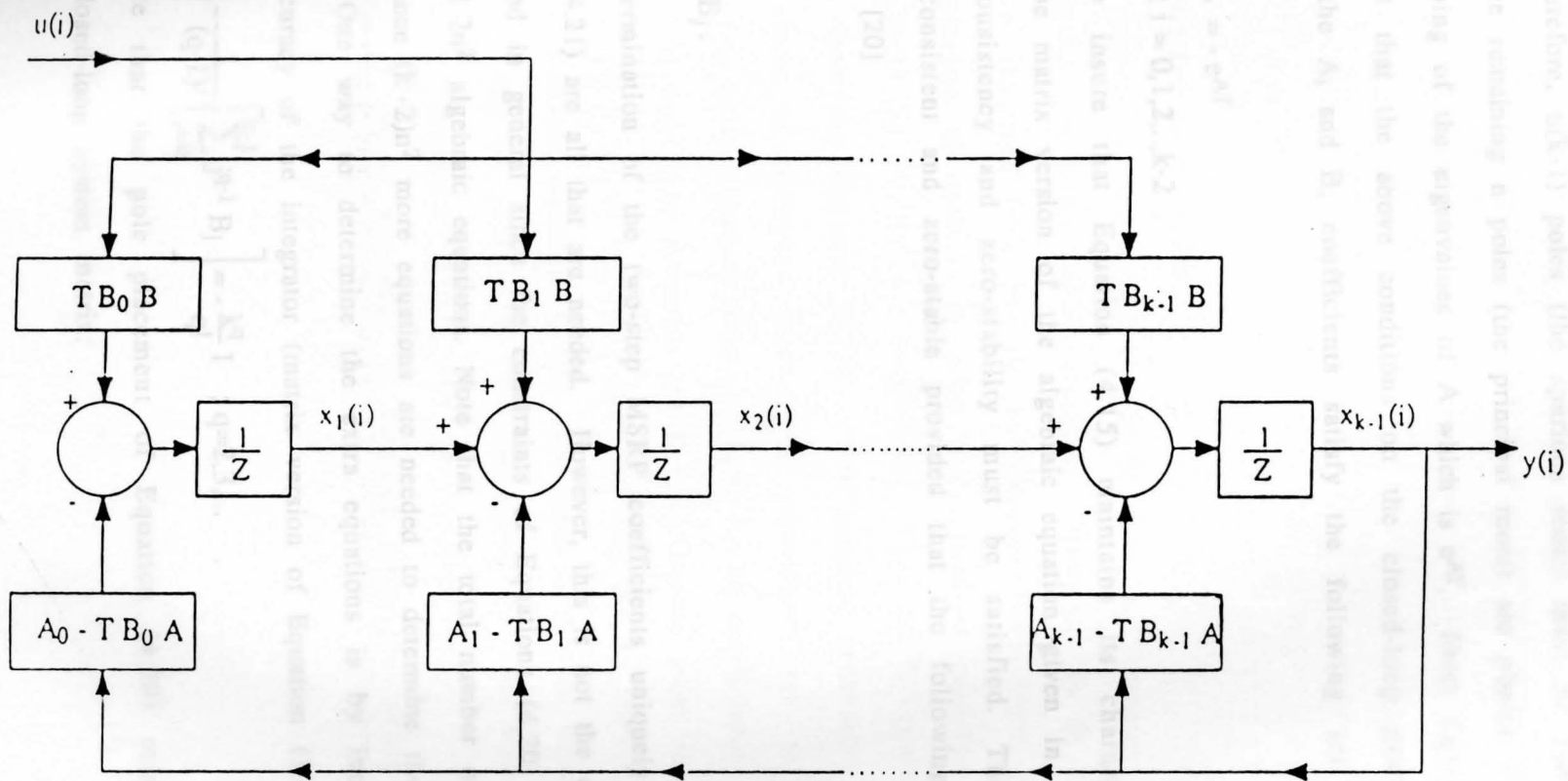


Figure 4.2 Block diagram implementation of k-step MSRP [20].

system. Therefore,  $n(k-1)$  poles (the spurious roots) must be placed at the origin and the remaining  $n$  poles (the principal roots) are placed at the exact Z-plane mapping of the eigenvalues of  $A$  which is  $e^{AT}$ . From Equation (4.17), it is evident that the above conditions on the closed-loop poles will be satisfied if the  $A_i$  and  $B_i$  coefficients satisfy the following set of matrix equations :

$$\begin{aligned} A_{k-1} - TB_{k-1} A &= -e^{AT} \\ A_i &= TB_i A \quad ; i = 0, 1, 2, \dots, k-2 \end{aligned} \quad (4.20)$$

In order to insure that Equation (4.15) maintains its character of an integrator, the matrix version of the algebraic equation given in chapter III regarding consistency and zero-stability must be satisfied. That is, the method is consistent and zero-stable provided that the following equations are satisfied [20]

$$\begin{aligned} \sum_{j=0}^k A_j &= 0 \\ \sum_{j=0}^k j A_j &= \sum_{j=0}^k B_j \end{aligned} \quad (4.21)$$

For the determination of the two-step MSRP coefficients uniquely, Equations (4.20) and (4.21) are all that are needed. However, this is not the case for the  $k$ -step method in general since the constraints of Equations (4.20) and (4.21) give  $kn^2$  and  $2n^2$  algebraic equations. Note that the total number of unknowns are  $2kn^2$ , hence  $(k-2)n^2$  more equations are needed to determine the  $A_i$  and  $B_i$  uniquely. One way to determine the extra equations is by increasing the order of accuracy of the integrator (matrix version of Equation (3.5)), i.e.

$$\frac{1}{q!} \left[ \sum_{j=1}^{k-1} j^q A_j \right] - \frac{1}{(q-1)!} \left[ \sum_{j=0}^{k-1} j^{q-1} B_j \right] = -\frac{k^q}{q!} I \quad ; q=2, 3, \dots \quad (4.22)$$

Note that the pole placement of Equation (4.20) results in the following closed-loop system matrix:

$$\tilde{A} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ I & 0 & 0 & \dots & 0 & 0 \\ 0 & I & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & I & e^{AT} \end{bmatrix} \quad (4.23)$$

which is guaranteed to be stable.

The regression coefficients  $A_i$  and  $B_i$  must be computed off-line in order to implement the  $k$ -step MSRP integrator. However, the  $B_i$  coefficients can be easily computed once the  $A_i$  coefficients have been determined. Hence, the problem at hand is to solve the system of equations given by Equations (4.20)-(4.22) for the  $A_i$  coefficients. Then the  $B_i$  coefficients can be computed from  $A_i$ , Equation (4.20). The standard technique utilized to solve this system of equations imitates the procedure used earlier in the MSRP2 method to solve Equations (4.4) and (4.6) for  $A_i$  coefficients.

The first step is to eliminate the  $B_i$  coefficients from the Equations (4.20)-(4.22) to get a set of equations involving only the  $A_i$  coefficients. This step results in a matrix equation of the same form as that of Equation (4.11). Performing a series of elementary row operations on this matrix equation yields the upper triangular form given in Equation (4.12). Therefore, a linear system of matrix equations is obtained from performing these two steps [20], i.e.,

$$M \phi = R \Phi \quad (4.24)$$

where the parameters are :

1)  $M$  is a  $kn$  by  $kn$  matrix, that is,  $M = [m_{i,j} \ I]$ , with  $I$  being the identity

matrix of size  $n$  and the  $m_{i,j}$ 's given by

$$m_{i,j} = \begin{cases} 1, & i=1 & j=1,2,\dots,k \\ 0, & i=2,3,\dots,k & j=i-1,i-2,\dots,1 \\ (j-i+1)m_{(i-1),j}, & i=2,3,\dots,k & j=i,i+1,\dots,k \end{cases} \quad (4.25)$$

II)  $\phi$  is a  $kn$  by  $n$  vector whose entries are the  $A_i$  coefficients, that is,

$$\phi^T = [A_0^T, A_1^T, \dots, A_{k-1}^T]. \quad (4.26)$$

III)  $R$  is a  $kn$  by  $kn$  matrix, i.e.  $R=[r_{i,j}]$ , with the  $r_{i,j}$ 's given as:

$$r_{i,j} = \begin{cases} 0, & i=2,3,\dots,k \quad j=1,i+1,\dots,k-1 \\ 1, & i=1,2,\dots,k \quad j=i \\ (-1)^{j+i} \{ (i-2)|r_{i-1,j}| + |r_{i-1,j-1}| \}, & i=3,4,\dots,k \quad j=i-1,i-2,\dots,2 \end{cases}. \quad (4.27)$$

IV)  $\Phi$  is a  $kn$  by  $n$  vector whose entries are specified as follows:

$$\Phi^T = [\hat{\Phi}_1^T, \hat{\Phi}_2^T, \dots, \hat{\Phi}_k^T]$$

$$\hat{\Phi}_m = \Phi_m + (m-1)(AT)^{-1} \hat{\Phi}_{m-1}; \quad m=1,2, \dots, k$$

$$\Phi_m = (m-1)(k-1)^{m-2} e^{AT}(AT)^{-1} - k^{m-1} I. \quad (4.28)$$

Note that Equation (4.24) can not be solved for the coefficients  $A_i$  directly since the vector containing the  $A_i$ 's is being pre-multiplied by the matrix  $M$ . Therefore, the inverse of  $M$  must be computed in order to determine the regression coefficients  $A_i$ . However, the upper triangular nature of  $M$  not only guarantees the existence of this inverse, but also can be exploited to obtain a closed-form inverse of  $M$ . An algorithm to determine the inverse of  $M$ , element by element, is given in [20]. This algorithm does not require the use of any special software packages to compute the inverse of  $M$ . Rather, it utilizes only the usual arithmetic operations supported by most hardware.

The entries  $w_{i,j}$  of  $W$ , the inverse of the matrix  $M$  in Equation (4.24) above, are given by:

$$w_{i,j} = \begin{cases} 0, & i = 2,3,\dots,k \quad j = i-1,i-2,\dots,1 \\ (i!)^{-1}, & i = 0,1,\dots,k-1 \quad j = i \\ (-1)^{i+j} (j-i) w_{i-1,j}, & i = 1,2,\dots,k \quad j = i+1,i+2,\dots,k \end{cases}. \quad (4.29)$$

Therefore, Equation (4.24) can be rewritten as:

$$\phi = M^{-1} R \Phi = W R \Phi. \quad (4.30)$$

To illustrate the procedure just presented, consider the case when a 3-step MSRP method is to be utilized for the simulation. Thus, letting  $k=3$  in Equation (4.15) yields the integrator equation as:

$$x_{n+3} = -A_2 x_{n+2} - A_1 x_{n+1} - A_0 x_n + T (B_2 \dot{x}_{n+2} + B_1 \dot{x}_{n+1} + B_0 \dot{x}_n) .$$

Clearly, knowledge of  $x_{n+2}$  and  $x_{n+1}$  is essential for determining  $x_{n+3}$ . To start the algorithm, Euler's method can be used with  $x_0$  as initial conditions to obtain  $x_1$ . Once this computation has been completed,  $x_2$  can be easily determined using two-step MSRP or Euler's method. Three-step MSRP can then be used to find  $x_{n+3}$ ;  $n=0,1,2,\dots$ . To implement the three-step integrator it is necessary to compute the integrator coefficients, namely  $A_i$  and  $B_i$ ;  $i=0,1,2$ . Following the process outlined in Equations (4.24)-(4.28) the matrices  $M$  and  $R$ , and vectors  $\varphi$  and  $\Phi$  can be determined as:

$$\begin{bmatrix} I & I & I \\ 0 & I & 2I \\ 0 & 0 & 2I \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & -I & I \end{bmatrix} \begin{bmatrix} \hat{\Phi}_1 \\ \hat{\Phi}_2 \\ \hat{\Phi}_3 \end{bmatrix} .$$

The inverse of  $M$ , that is  $W$ , can be found from Equation (4.29). For this numerical example  $W$  is:

$$W = \begin{bmatrix} I & -I & \frac{I}{2} \\ 0 & I & -I \\ 0 & 0 & \frac{I}{2} \end{bmatrix} .$$

Hence, the coefficients  $A_i$  are determined from

$$\begin{bmatrix} A_0 \\ A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} I & -I & \frac{I}{2} \\ 0 & I & -I \\ 0 & 0 & \frac{I}{2} \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & -I & I \end{bmatrix} \begin{bmatrix} \hat{\Phi}_1 \\ \hat{\Phi}_2 \\ \hat{\Phi}_3 \end{bmatrix} .$$

To obtain the  $A_i$  coefficients the matrix product indicated above can be carried out to get:

$$A_0 = \hat{\Phi}_1 - \frac{3}{2} \hat{\Phi}_2 + \frac{1}{2} \hat{\Phi}_3$$

$$A_1 = 2 \hat{\Phi}_2 - \hat{\Phi}_3$$

$$A_2 = -\frac{1}{2} \hat{\Phi}_2 + \frac{1}{2} \hat{\Phi}_3$$

From Equation (4.28) the  $\hat{\Phi}_i$ 's can be found as:



$$\hat{\Phi}_1 = \Phi_1 = -I$$

$$\hat{\Phi}_2 = \Phi_2 + \hat{\Phi}_1 (TA)^{-1} = e^{AT} (TA)^{-1} - (TA)^{-1} - 3I$$

$$\hat{\Phi}_3 = \Phi_3 + 2\hat{\Phi}_2 (TA)^{-1} = 2e^{AT} (TA)^{-2} + 4e^{AT} (TA)^{-1} - 2(TA)^{-2} - 6(TA)^{-1} - 9I$$

Substituting these expressions for the  $\hat{\Phi}_i$ 's into those for the  $A_i$ 's yields:

$$A_0 = e^{AT} (TA)^{-2} + \frac{3}{2} e^{AT} (TA)^{-1} - (TA)^{-2} - \frac{3}{2} (TA)^{-1} - I$$

$$A_1 = -2e^{AT} (TA)^{-2} - 2e^{AT} (TA)^{-1} + 2(TA)^{-2} + 4(TA)^{-1} - I \quad (4.31)$$

$$A_2 = e^{AT} (TA)^{-2} + \frac{3}{2} e^{AT} (TA)^{-1} - (TA)^{-2} - \frac{5}{2} (TA)^{-1} - 3I$$

The  $B_i$  coefficients can then be determined from Equation (4.20) as:

$$B_0 = (TA)^{-1} A_0$$

$$B_1 = (TA)^{-1} A_1$$

$$B_2 = (TA)^{-1} (A_2 + e^{AT})$$

The Equation (4.24) for determination of the  $A_i$  regression coefficients can be further simplified to obtain a closed-form solution for  $A_i$  coefficients. The procedure leading to this simplification is illustrated in the following section.

### A Closed-Form Representation of the K-step MSRP Method

The Equation (4.24) describing the regression coefficients of the MSRP method can be further simplified. In order to illustrate this procedure, the Equation (4.24) for a 7-step method is determined to be:

$$\phi = M^{-1} R \Phi = W R \Phi$$

or

$$\begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix} = \begin{bmatrix} 1I & -1I & \frac{1}{2} & \frac{-1}{6} & \frac{1}{24} & \frac{-1}{120} & \frac{1}{720} \\ 0I & 1I & -1I & \frac{1}{2} & \frac{-1}{6} & \frac{1}{24} & \frac{-1}{120} \\ 0I & 0I & \frac{1}{2} & \frac{-1}{2} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{48} \\ 0I & 0I & 0I & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-1}{36} \\ 0I & 0I & 0I & 0I & \frac{1}{24} & \frac{-1}{24} & \frac{1}{48} \\ 0I & 0I & 0I & 0I & 0I & \frac{1}{120} & \frac{-1}{120} \\ 0I & 0I & 0I & 0I & 0I & 0I & \frac{1}{720} \end{bmatrix} \begin{bmatrix} 1I & 0I & 0I & 0I & 0I & 0I & 0I \\ 0I & 1I & 0I & 0I & 0I & 0I & 0I \\ 0I & -1I & 1I & 0I & 0I & 0I & 0I \\ 0I & 2I & -3I & 1I & 0I & 0I & 0I \\ 0I & -6I & 11I & -6I & 1I & 0I & 0I \\ 0I & 24I & -50I & 35I & -10I & 1I & 0I \\ 0I & -120I & 274I & -225I & 35I & -15I & 1I \end{bmatrix} \begin{bmatrix} \hat{\Phi}_1 \\ \hat{\Phi}_2 \\ \hat{\Phi}_3 \\ \hat{\Phi}_4 \\ \hat{\Phi}_5 \\ \hat{\Phi}_6 \\ \hat{\Phi}_7 \end{bmatrix}$$

The following facts can be observed

- a) The coefficients of the matrix  $\Phi$ , given by Equation (4.28), can alternatively be determined from the following equation given the desired step number,  $k$ :

$$\hat{\Phi}_i^{(k)} = \sum_{m=1}^{i-1} \left[ \frac{(k-1)^{m-1} (i-1)!}{(m-1)!} e^{AT} (AT)^{(m-1)} \right] - \sum_{m=1}^i \left[ \frac{(k)^{m-1} (i-1)!}{(m-1)!} (AT)^{(m-1)} \right]$$

for  $i = 1, 2, \dots, k-1$ . (4.32)

- b) The coefficients of the matrix  $R$  are indeed the Stirling numbers of the first kind. There are numerous notations for describing these numbers; the notation adopted in this paper is that of Tuma's [21]. A short table of the Stirling numbers of the first kind is depicted in TABLE 4.1. The matrix  $R$  then can be rewritten as

$$R = \left[ S_1^{(i)} \quad S_2^{(i)} \quad S_3^{(i)} \quad \dots \quad S_7^{(i)} \quad \dots \right] \otimes I$$

where the column vectors  $S_{1,2,\dots}^{(i)}$  are

$$S_j^{(i)} = \left[ S_j^{(1)} \quad S_j^{(2)} \quad S_j^{(3)} \quad \dots \quad \dots \right]^T,$$

and  $\otimes$  denotes the Kronecker tensor product.

- c) The inverse of the matrix  $M$ , the matrix  $W$ , can be written as:

$$\begin{bmatrix} \frac{1}{0!} & \frac{-1}{0!x1!} & \frac{1}{0!x2!} & \frac{-1}{0!x3!} & \frac{1}{0!x4!} & \frac{-1}{0!x5!} & \frac{1}{0!x6!} \\ 0I & \frac{1}{1!} & \frac{-1}{1!x1!} & \frac{1}{1!x2!} & \frac{-1}{1!x3!} & \frac{1}{1!x4!} & \frac{-1}{1!x5!} \\ 0I & 0I & \frac{1}{2!} & \frac{-1}{2!x1!} & \frac{1}{2!x2!} & \frac{-1}{2!x3!} & \frac{1}{2!x4!} \\ 0I & 0I & 0I & \frac{1}{3!} & \frac{-1}{3!x1!} & \frac{1}{3!x2!} & \frac{-1}{3!x3!} \\ 0I & 0I & 0I & 0I & \frac{1}{4!} & \frac{-1}{4!x1!} & \frac{1}{4!x2!} \\ 0I & 0I & 0I & 0I & 0I & \frac{1}{5!} & \frac{-1}{5!x1!} \\ 0I & 0I & 0I & 0I & 0I & 0I & \frac{1}{6!} \end{bmatrix}$$

Utilizing the above simplifications results in the following closed-form expression for the regression coefficients :

$$A_h^{(k)} = \sum_{j=h+1}^k \frac{(-1)^{j-h+1}}{h!(j-h-1)!} \left\{ \sum_{i=1}^j (i-1)! S_{(i-1)}^{(j-1)} \left[ \sum_{m=1}^{i-1} \frac{(k-1)^{m-1}}{(m-1)!} e^{AT} (AT)^{(m-i)} \sum_{m=1}^i \frac{(k)^{m-1}}{(m-1)!} (AT)^{(m-i)} \right] \right\}$$

for  $h=0,1,\dots,k-1$  . (4.33)

Since the  $B_i$  coefficients are directly related to the  $A_i$  coefficients, i.e.

$$B_h^{(k)} = (TA)^{-1} A_h^{(k)} ; h=0,1,2,\dots,k-2$$

$$B_{k-1}^{(k)} = (TA)^{-1} (A_{k-1}^{(k)} + e^{AT})$$

then the following closed form expression for a  $k$ -step MSRP method can be formulated.

$$x_{n+k} = e^{(AT)} x_{n+k-1} + \left\{ e^{(AT)} u_{n+k-1} + \sum_{g=1}^k \frac{u_{n+k-g}}{(k-g)!} \left[ \sum_{j=k-g+1}^k \frac{(-1)^{j-k+g+1}}{(j-k+g-1)!} \sum_{i=1}^j S_{(i-1)}^{(j-1)} \right] \right\} \left\{ \left[ \sum_{m=1}^i \frac{(i-1)!}{(m-1)!} [(k-1)^{(m-1)} e^{AT} (AT)^{(m-i)} - (k)^{(m-1)} (AT)^{(m-i)}] - (k-1)^{i-1} e^{AT} \right] \right\} A^{-1} B$$

(4.34)

TABLE 4.1  
THE STIRLING NUMBERS OF THE FIRST KIND

$n$	$S_n^0$	$S_n^1$	$S_n^2$	$S_n^3$	$S_n^4$	$S_n^5$	$S_n^6$
0	1	0	0	0	0	0	0
1	0	1	-1	2	-6	24	-120
2	0	0	1	-3	11	-50	274
3	0	0	0	1	-6	35	-225
4	0	0	0	0	1	-10	85
5	0	0	0	0	0	1	-15
6	0	0	0	0	0	0	1

calculus with the MSRP method.

#### Gregory-Newton Interpolation Formula

The derivation of Gregory-Newton extrapolation formula using frequency domain analysis is derived in [22] and will be presented here for sake of completeness. The derivation of the formula is based upon an expression for prediction in the frequency domain. That is, a system which has an output equal to its input  $\tau$  seconds in the future can be described by the following transfer function.

$$H(s) = e^{s\tau}$$

Equation (5.1) can be rearranged as

$$H(s) = [1 - (-e^{-s\tau})]^{-1}$$

where  $\tau$  is the sampling time. Expanding the function in (5.2) as a power series in binomial expansion yields

## CHAPTER V

### A CLASS OF N-TH ORDER HOLD TECHNIQUES AND ITS RELATION TO MSRP METHOD

In this chapter the relationship between the n-th order hold technique based on the Gregory-Newton interpolation formula and the MSRP method is exploited. The derivation of the Gregory-Newton interpolation formula from the sampling point of view is presented in the first subsection followed by its utilization for an n-th order hold technique. Finally, the general n-th order hold approximation technique is formulated and a heuristic argument is given for the equivalence of these n-th order hold techniques with the MSRP method.

#### Gregory-Newton Interpolation Formula

The derivation of Gregory-Newton extrapolation formula using frequency domain analysis is derived in [22] and will be presented here for sake of completeness. The derivation of the formula is based upon an expression for prediction in the frequency domain. That is, a system which has an output equal to its input  $\tau$  seconds in the future can be described by the following transfer function

$$H(S) = e^{\tau s} \quad (5.1)$$

Equation (5.1) can be rearranged as

$$H(S) = [1 - (1 - e^{-Ts})] \left( \frac{-\tau}{T} \right) \quad (5.2)$$

where  $T$  is the sampling time. Expanding the Equation (5.2) by means of binomial expansion yields

$$H(S) = 1 + \frac{(1 - e^{-Ts})\tau}{T} + \frac{(1 - e^{-Ts})^2}{2T^2} (T + \tau)\tau + \dots \quad (5.3)$$

The above transfer function represents the transfer function of a device whose output will be the future value of its input if an infinite number of terms are considered. Then the Laplace transform of the output,  $Y(S)$ , is obtained by

$$Y(S) = H(S) U(S) \quad (5.4)$$

where  $U(S)$  is the Laplace transform of the input signal. Substituting Equation (5.3) into (5.4) gives

$$Y(S) = \left[ 1 + \frac{(1 - e^{-Ts})\tau}{T} + \frac{(1 - e^{-Ts})^2}{2T^2} (T + \tau)\tau + \dots \right] U(S) \quad (5.5)$$

Since  $u(t + \tau) = y(t)$ ,

inverting Equation (5.5) gives the following expression for extrapolation of the input  $\tau$  seconds into the future

$$u(t + \tau) = u(t) + \frac{(u(t) - u(t - T))\tau}{T} + \frac{(u(t) - 2u(t - T) + u(t - 2T))}{2T^2} (T + \tau)\tau + \dots \quad (5.7)$$

Since the input signal is available at particular sampling times, substituting  $t = kT$  in Equation (5.7) results in

$$u(kT + \tau) = u(kT) + \frac{\tau}{T} (u(kT) - u[(k-1)T]) + \frac{(T + \tau)\tau}{2T^2} (u(kT) - 2u[(k-1)T] + u[(k-2)T]) + \dots \quad (5.8)$$

From the foregoing it can easily be seen that the input between the two sampling intervals can be approximated by Equation (5.7). To clarify, let the prediction time,  $\tau$ , be restricted by

$$0 \leq \tau \leq kT \quad (5.9)$$

Furthermore, let the time between two consecutive sampling intervals be denoted as  $t$ , i.e.

$$t = \tau + kT \quad (5.10)$$

Then upon the substitution of Equation (5.10) into Equation (5.8) the following expression for extrapolation of the input signal between the

sampling intervals is obtained;

$$u(t) = u(kT) + \frac{(t - kT)}{T} (u(kT) - u[(k-1)T]) + \frac{(T+t-kT)(t-kT)}{2T^2} (u(kT) - 2u[(k-1)T] + u[(k-2)T]) + \dots ; kT \leq t \leq (k+1)T. \quad (5.11)$$

The Gregory-Newton extrapolation formula, Equation (5.11), can be written in a compact form by utilizing the backward difference operator, viz.

$$u(t) = u(kT) + \frac{(t - kT)}{T} \nabla u(kT) + \frac{(T+t-kT)(t-kT)}{2T^2} \nabla^2 u(kT) + \dots + \frac{((n-1)T + \tau - kT) \dots (T + \tau - kT)(\tau - kT)}{n! T^n} \nabla^n u(kT) ; kT \leq t \leq (k+1)T. \quad (5.12)$$

In the next section this equation will be utilized to derive a class of  $n$ -th order hold approximations.

### A Class of Generalized $n$ -th Order Hold Approximations

Consider the  $n$ -th order linear system, Equation (4.7), i.e.

$$\dot{x} = Ax + Bu \quad (5.13)$$

where  $A$  and  $B$  are matrices of appropriate size. Then the exact solution of Equation (5.13) is given by [23]

$$x(t) = e^{A(t-t_0)} x(t_0) + \int_{t_0}^t e^{A(t-\tau)} B u(\tau) d\tau \quad (5.14)$$

where  $t_0$  is the initial time and can be considered to be zero without loss of generality. Evaluation of Equation (5.14) at the sampling time  $t=kT$  gives

$$x(kT) = e^{AkT} x(0) + \int_0^{kT} e^{A(kT-\tau)} B u(\tau) d\tau. \quad (5.15)$$

At the next sampling time,  $t=(k+1)T$ , the exact solution is obtained by

$$x[(k+1)T] = e^{A(k+1)T} x(0) + \int_0^{(k+1)T} e^{A((k+1)T-\tau)} B u(\tau) d\tau. \quad (5.16)$$

Multiplying Equation (5.15) by  $e^{AT}$  and subtracting from Equation (5.16)

yields

$$x[(k+1)T] = e^{AT} x(kT) + \int_{kT}^{(k+1)T} e^{A((k+1)T-\tau)} B u(\tau) d\tau. \quad (5.17)$$

Assuming that the input signal is sampled by a sample and hold device, then the integral in Equation (5.17) can be evaluated by some approximation method.

Employing the Gregory-Newton extrapolation formula to approximate the integral in Equation (5.17) then gives the following difference equation

$$x[(k+1)T] = e^{AT} x(kT) + \int_{kT}^{(k+1)T} e^{A((k+1)T-\tau)} B \left[ u(kT) + \frac{(\tau - kT)}{T} \nabla u(kT) + \frac{(T + \tau - kT)(\tau - kT)}{2T^2} \nabla^2 u(kT) + \dots + \frac{(nT + \tau - kT) \dots (T + \tau - kT)(\tau - kT)}{n! T^n} \nabla^n u(kT) \right] d\tau \quad (5.18)$$

Evaluating the integral by one or two terms results in the standard Zero Order or First Order Hold approximations, respectively. However, the standard higher order hold ( which are derived based on the Taylor series expansion ) will not be obtained by Equation (5.18). Rather, this substitution gives the higher order closed-loop MSRP methods. To demonstrate this, consider the case for which three terms in the brackets are utilized for the approximation. Introducing the following change of variables,

$$\xi \equiv (k+1)T - \tau, \quad (5.19)$$

in Equation (5.18) and considering only three terms gives

$$x[(k+1)T] = e^{AT} x(kT) + \int_0^T e^{A\xi} B \left[ u(kT) + \frac{(T - \xi)}{T} \nabla u(kT) + \frac{(2T - \xi)(T - \xi)}{2T^2} \nabla^2 u(kT) \right] d\xi. \quad (5.20)$$

Performing the integration and simplifying yields



$$\begin{aligned}
 x[(k+1)T] = & \phi x(kT) + \\
 & \left[ \{ (AT)^{-2} + 1.5(AT)^{-1} + I \} \phi - (AT)^{-2} - 2.5(AT)^{-1} - 3I \right] (AT)^{-1} TB u(kT) + \\
 & \left[ \{ -2(AT)^{-2} - 2(AT)^{-1} \} \phi + 2(AT)^{-2} + 4(AT)^{-1} + 3I \right] (AT)^{-1} TB u[(k-1)T] + \\
 & \left[ \{ (AT)^{-2} + 0.5(AT)^{-1} \} \phi - (AT)^{-2} - 1.5(AT)^{-1} - I \right] (AT)^{-1} TB u[(k-2)T]
 \end{aligned}$$

where

$$\phi \equiv e^{AT} \quad (5.21)$$

and subject to the existence of the inverse of A. Equation (5.21) is identical to the 3-step MSRP method developed in Chapter III. This can be easily verified if the above equation is rearranged as

$$\begin{aligned}
 x[(k+1)T] = & \phi x(kT) + (\phi + A_2) (AT)^{-1} TB u(kT) + A_1 (AT)^{-1} TB u[(k-1)T] + \\
 & A_0 (AT)^{-1} TB u[(k-2)T]
 \end{aligned}$$

where

$$\begin{aligned}
 A_2 = & (AT)^{-2} \phi + 1.5(AT)^{-1} \phi - (AT)^{-2} - 2.5(AT)^{-1} - 3I \\
 A_1 = & -2(AT)^{-2} \phi - 2(AT)^{-1} \phi + 2(AT)^{-2} + 4(AT)^{-1} + 3I \\
 A_0 = & (AT)^{-2} \phi + 0.5(AT)^{-1} \phi - (AT)^{-2} - 1.5(AT)^{-1} - I .
 \end{aligned} \quad (5.22)$$

Furthermore, if p denotes the number of terms for evaluating the integrand of Equation (5.18), then one can adopt the following change of variables

$$f[(k+1)T] \equiv f_{n+p} \quad (5.23)$$

in Equation (5.22) to get:

$x_{n+p} = e^{AT} x_{n+p-1} + (e^{AT} + A_2)(AT)^{-1} TB u_{n+p-1} + A_1(AT)^{-1} TB u_{n+p-2} + A_0(AT)^{-1} TB u_{n+p-3}$  with p=3 for this particular case. The above equation is precisely the MSRP-3 which is derived in the previous chapter, see Equation (4.25). Therefore the second order Gregory-Newton hold approximation is equivalent to the 3-step MSRP method. Conversely, the standard Taylor series second order hold is

$$x[(k+1)T] = e^{AT} x(kT) + \int_0^T e^{A\xi} B \left[ u(kT) + \frac{(T-\xi)}{T} \nabla u(kT) + \frac{(T-\xi)^2}{2T^2} \nabla^2 u(kT) \right] d\xi.$$

Integrating and simplifying yields:

$$\begin{aligned}
 x[(k+1)T] = & \phi x(kT) + \\
 & \left[ \{ (AT)^{-2} + (AT)^{-1} + I \} \phi - (AT)^{-2} - 2(AT)^{-1} - 2.5I \right] (AT)^{-1} TB u(kT) + \\
 & \left[ \{ -2(AT)^{-2} - (AT)^{-1} \} \phi + 2(AT)^{-2} + 3(AT)^{-1} + 2I \right] (AT)^{-1} TB u[(k-1)T] + \\
 & \left[ \{ (AT)^{-2} \} \phi - (AT)^{-2} - (AT)^{-1} - 0.5I \right] (AT)^{-1} TB u[(k-2)T]
 \end{aligned}$$

with  $\phi \equiv e^{AT}$ . It is evident that the above approximation is not equivalent to the previous results, Equation (5.21). Hence, by contradiction the (n+1)-step MSRP method cannot be equivalent to the standard Taylor series n-th order hold approximations.

A closed-form representation of the n-th order hold technique just presented can be obtained as follows. The Gregory-Newton interpolation formula can be alternatively written as :

$$u(\theta) = u(kT) + \frac{\theta}{T} \nabla^1 u(kT) + \frac{\theta(\theta+T)}{2T^2} \nabla^2 u(kT) + \dots + \frac{((n-1)T+\theta) \dots (T+\theta) \theta}{n! T^n} \nabla^n u(kT). \quad (5.24)$$

by the following change of variables:

$$\tau - kT \equiv \theta. \quad (5.25)$$

Interestingly, the numerators of the above equation are indeed factorial polynomials which can be expressed in terms of the Stirling numbers of the first kind [21], i.e.

$$((n-1)T+\theta) \dots (T+\theta) \theta = (-T)^n \sum_{m=1}^n \left( \frac{\theta}{-T} \right)^m S_{(m)}^{(n)} \quad (5.26)$$

where  $S_{(m)}^{(n)}$  are the Stirling numbers. Hence, the Gregory-Newton formula can be expressed as:

$$u(\theta) = \sum_{j=1}^{n+1} \frac{\nabla^{(j-1)} u(kT)}{(j-1)! T^{j-1}} \left[ \sum_{m=1}^n (-T)^n \left( \frac{\theta}{-T} \right)^m S_{(m)}^{(n)} \right]. \quad (5.27)$$

Equation (5.18) can be simplified by utilizing the above equation along with the notation given by Equation (5.23) as:

$$x_{k+p} = e^{(AT)} x_{k+p-1} + e^{(AT)} \left[ \sum_{j=0}^{p-1} \frac{(\nabla^{(j)} u_{k+p-1})}{(j)!} \left[ \sum_{m=0}^j \frac{(-1)^{j-m}}{T^m} S_{(m)}^{(j)} \int_0^T \theta^m e^{-A\theta} d\theta \right] \right] B. \quad (5.28)$$

But the integral can be evaluated as :

$$\frac{e^{(AT)}}{T^m} \int_0^T \theta^m e^{-A\theta} d\theta = A^{-1} \left[ (m)! (AJ)^{-m} e^{AT} - \sum_{i=0}^m \frac{(m)!}{(i)!} (AT)^{(i-m)} \right] \quad (5.29)$$

subject to the existence of  $A^{-1}$ . Substituting Equation (5.29) into (5.28) yields

$$x_{k+p} = e^{(AT)} x_{k+p-1} + \left[ \sum_{j=0}^{p-1} \frac{(\nabla^j) u_{k+p-1}}{(j)!} \left[ \sum_{m=0}^j (-1)^{j-m} S_{(m)}^{(j)} \left[ (m)! (AT)^{-m} e^{AT} - \sum_{i=0}^m \frac{(m)!}{(i)!} (AT)^{(i-m)} \right] \right] \right] A^{-1} B. \quad (5.30)$$

Furthermore, the backward difference of any discrete function can be expressed as:

$$\nabla^j y_n = \sum_{i=0}^j (-1)^i \binom{j}{i} y_{n+i-j} = \sum_{i=0}^j \frac{(-1)^i j!}{(j-i)! (i)!} y_{n+i-j}. \quad (5.31)$$

Hence, the  $(p-1)$ -th order Gregory-Newton hold technique can be written as

$$x_{k+p} = e^{(AT)} x_{k+p-1} + \left\{ \sum_{j=0}^{p-1} \sum_{n=0}^j \frac{(-1)^n}{(j-n)! n!} u_{k+p-n-1} \left[ \sum_{m=0}^j (-1)^{j-m} S_{(m)}^{(j)} \left[ (m)! (AT)^{-m} e^{AT} - \sum_{i=0}^m \frac{(m)!}{(i)!} (AT)^{(i-m)} \right] \right] \right\} A^{-1} B. \quad (5.32)$$

Note that, Equation (5.32) is very similar to the generalized MSRP method, Equation (4.34). In fact, these equations should be equivalent to each other. The mathematical proof of this statement is involved and presently not complete. However, the equivalence of these two methods, up to  $p=6$  ( and  $k=6$  for MSRP) has been established by symbolic evaluation of the two equations.

### Simulation Algorithms

The  $\nabla^j$  integrator was derived in Chapter III as

$$x_{k+2} = x_{k+1} + T \cdot \nabla^2 x_k - 0.5 T^2 \nabla^3 x_k. \quad (6.2)$$

## CHAPTER VI

### SIMULATION OF THE SMALL PERTURBATION MODEL OF MACH 2.5 40-60 INLET BY MSRP-2 AND AB-2 METHODS

In this chapter, the simulation results of the 40-60 mixed compression inlet by the second order Adam-Bashforth (AB-2) and the second order Matrix Stability Region Placement (MSRP-2) methods are presented. The small perturbation model of the inlet contains 41 spatial lumps and is described by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{6.1}$$

where  $A \in \mathbf{R}^{123 \times 123}$ ,  $B \in \mathbf{R}^{123 \times 1}$ , and  $C \in \mathbf{R}^{1 \times 123}$  as formulated in chapter II. The output of the system corresponds to the change in pressure of the lump just before the shock position, the 23-rd lump. The input of the system is assumed to be a negative unit step function applied at the last lump at time  $t=0.002$  seconds. The numerical values of the  $\{A,B,C\}$  triple are included in Appendix A. The system under investigation is indeed stiff since the ratio of the magnitudes of the largest to smallest eigenvalues is about 50. The eigenvalues,  $\lambda_i$ ;  $i=1,2,\dots,123$ , of the system matrix,  $A$ , are determined by utilizing MATLAB built in function and are plotted in the S-plane, as shown in Figure 6.1.

#### Simulation Algorithms

The AB-2 integrator was derived in Chapter III as:

$$x_{n+2} = x_{n+1} + T(1.5 \dot{x}_{n+1} - 0.5 \dot{x}_n). \tag{6.2}$$

Since the system of equations to be simulated is stiff, an extremely small time-step is required for this method. This is evident from the stability region of the AB-2 method, Figure 3.3. The largest eigenvalue of the system matrix is:  $\max|\lambda(A)| = 23 \times 10^4$ , hence to place the  $\lambda T$  products within the stability region of AB-2, the time-step  $T$  must be less than  $10^{-5}$ .

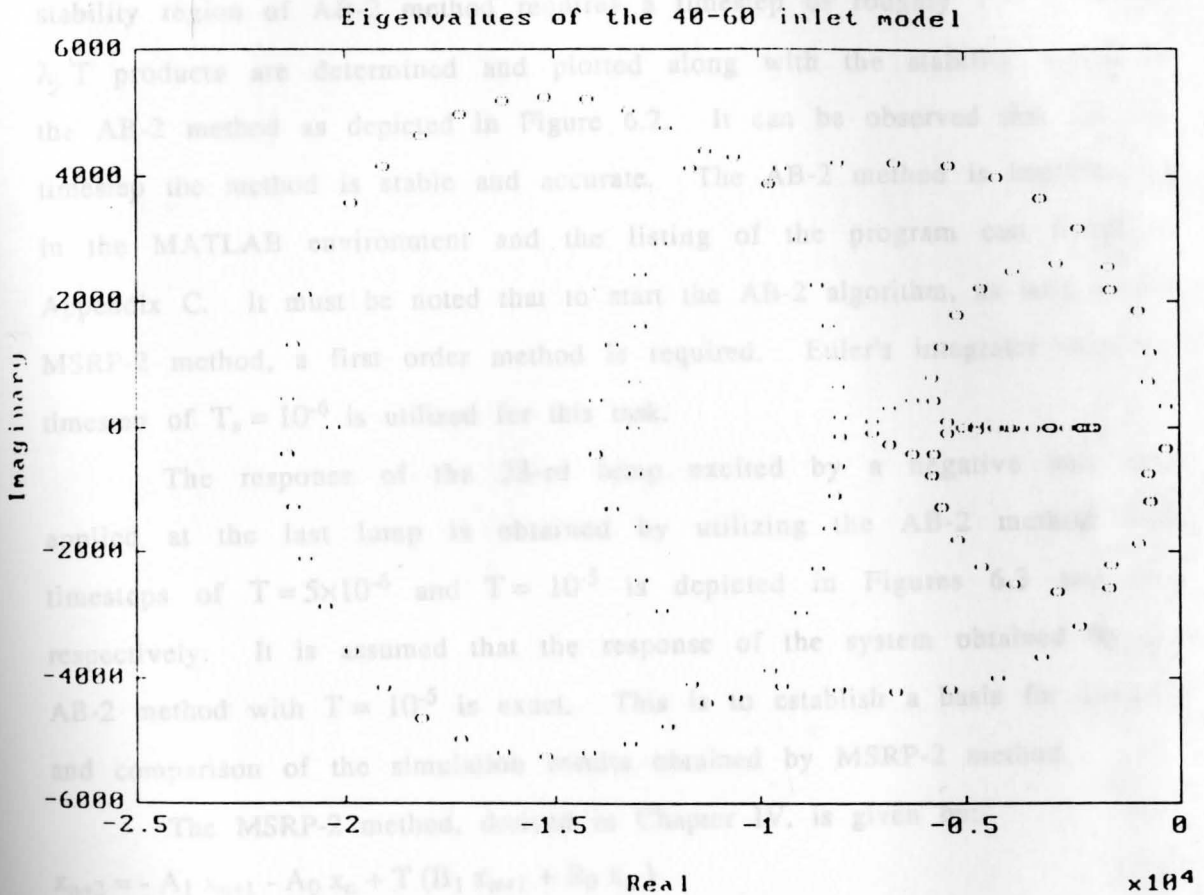


Figure 6.1 Eigenvalues of the inlet model

where the regression coefficients of the integrator are calculated

$$[A_i, B_i; i=0,1] \in \mathbb{R}^{123 \times 123}$$

only the numerical value of  $A_0$  is included in this thesis, see Appendix C. The MATLAB programs for calculating the regression coefficients and performing the simulation are included in Appendix C. A copy is attached

Since the system of equations to be simulated is stiff, an extremely small timestep is required for this method. This is evident from the stability region of the AB-2 method, Figure 3.3. The largest eigenvalue of the system matrix is:  $\max|\lambda(A)| \approx 2.1 \times 10^4$ , hence to place the  $\lambda_i T$  products well inside the stability region of AB-2 method requires a timestep of roughly  $T = 10^{-5}$ . The  $\lambda_i T$  products are determined and plotted along with the stability region of the AB-2 method as depicted in Figure 6.2. It can be observed that for this timestep the method is stable and accurate. The AB-2 method is implemented in the MATLAB environment and the listing of the program can be found in Appendix C. It must be noted that to start the AB-2 algorithm, as well as the MSRP-2 method, a first order method is required. Euler's integrator with the timestep of  $T_s = 10^{-6}$  is utilized for this task.

The response of the 23-rd lump excited by a negative unit step applied at the last lump is obtained by utilizing the AB-2 method with timesteps of  $T = 5 \times 10^{-6}$  and  $T = 10^{-5}$  is depicted in Figures 6.3 and 6.4, respectively. It is assumed that the response of the system obtained by the AB-2 method with  $T = 10^{-5}$  is exact. This is to establish a basis for analysis and comparison of the simulation results obtained by MSRP-2 method.

The MSRP-2 method, derived in Chapter IV, is given by:

$$x_{n+2} = -A_1 x_{n+1} - A_0 x_n + T(B_1 \dot{x}_{n+1} + B_0 \dot{x}_n) \quad (6.3)$$

where the regression coefficients of the integrator are calculated by utilizing Equation (4.10). Since all of these regression coefficients have the same size as the system matrix, i.e.,

$$\{A_i, B_i ; i=0,1\} \in \mathbf{R}^{123 \times 123} \quad (6.4)$$

only the numerical value of  $A_0$  is included in this thesis, see Appendix B. The MATLAB programs for calculating the regression coefficients and performing the simulation are included in Appendix C. In order to establish

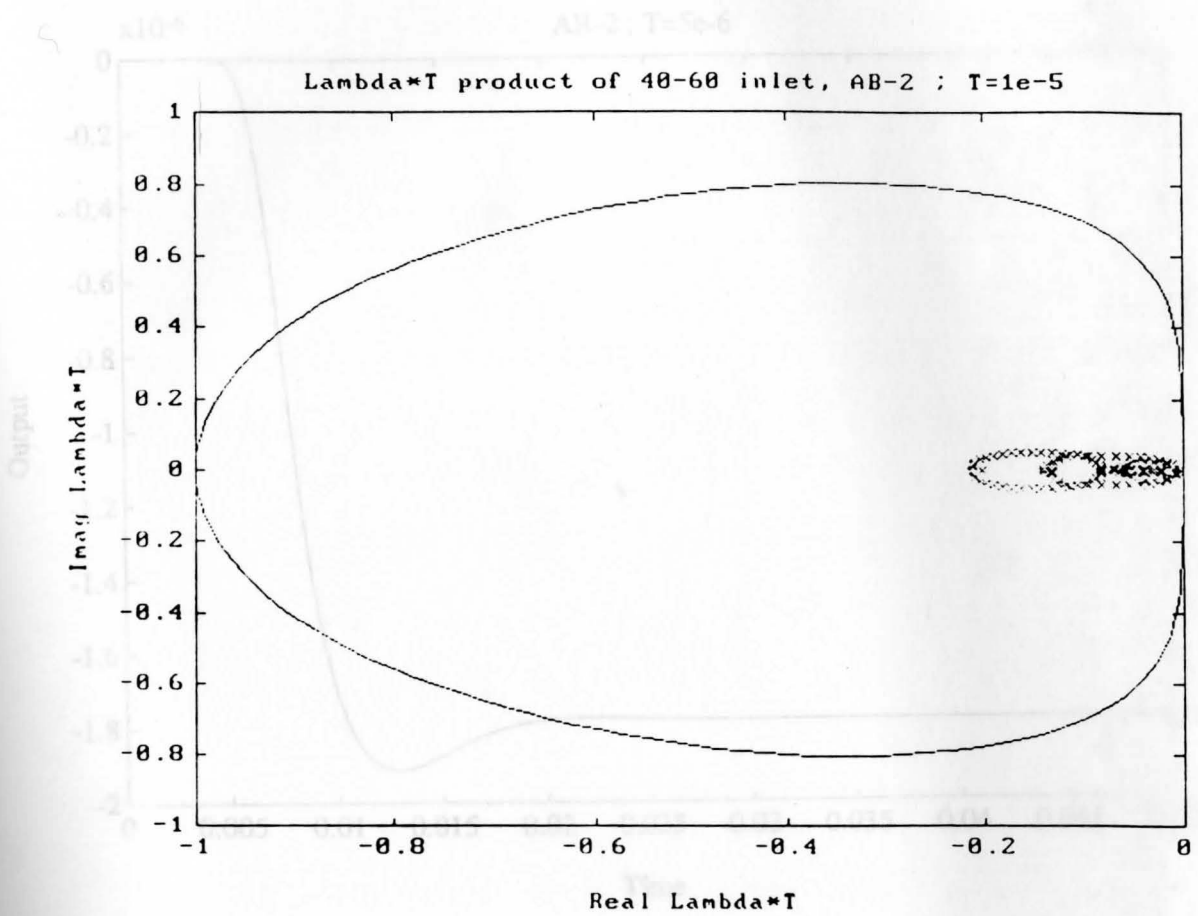


Figure 6.2  $\lambda T$  product and AB-2's stability region.

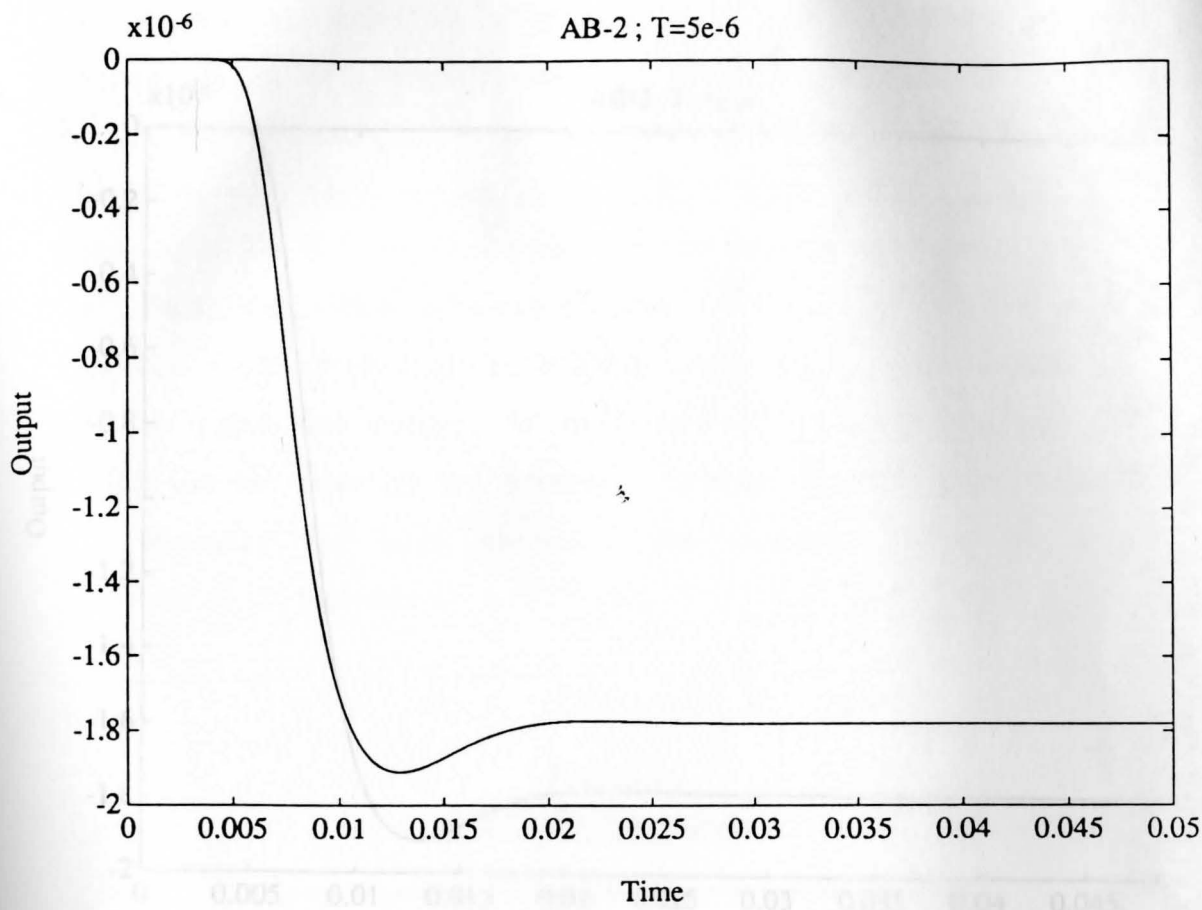


Figure 6.3 Step response utilizing AB-2 method with  $T=0.000005$ .

Figure 6.4 Step response utilizing AB-2 method with  $T=0.00001$ .



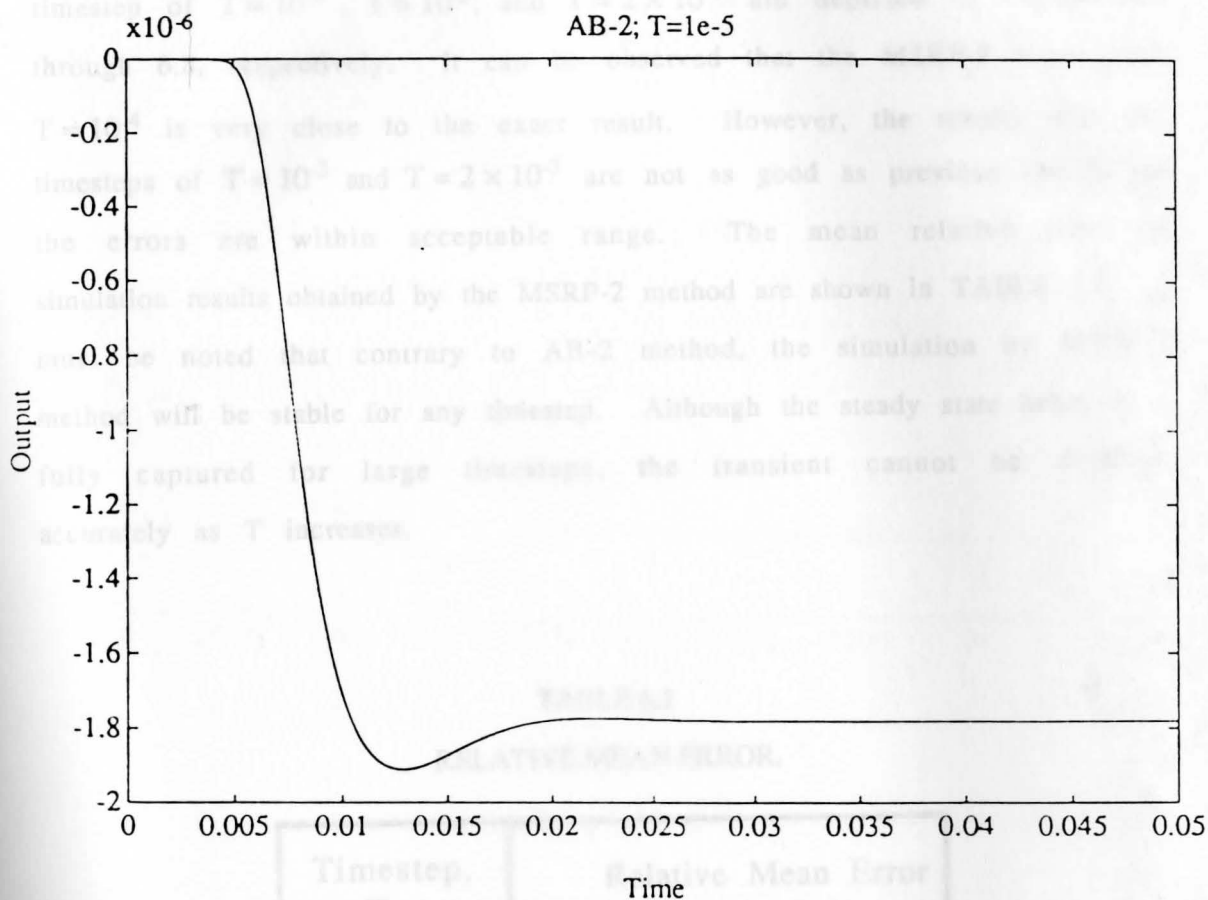


Figure 6.4 Step response utilizing AB-2 method with T=0.00001.

that MSRP-2 results can be as accurate as the exact results, the model is simulated with timestep of  $T = 10^{-5}$  and the output,  $y$ , is plotted as depicted in Figure 6.5. It is evident that the MSRP-2 and the exact results are the same, compare Figures 6.3 and 6.5. The exact results and the MSRP-2 results with timestep of  $T = 10^{-4}$ ,  $T = 10^{-3}$ , and  $T = 2 \times 10^{-3}$  are depicted in Figures 6.6 through 6.8, respectively. It can be observed that the MSRP-2 result with  $T = 10^{-4}$  is very close to the exact result. However, the results with the timesteps of  $T = 10^{-3}$  and  $T = 2 \times 10^{-3}$  are not as good as previous results but the errors are within acceptable range. The mean relative error of simulation results obtained by the MSRP-2 method are shown in TABLE 6.1. It must be noted that contrary to AB-2 method, the simulation by MSRP-2 method will be stable for any timestep. Although the steady state behavior is fully captured for large timesteps, the transient cannot be obtained accurately as  $T$  increases.

TABLE 6.1  
RELATIVE MEAN ERROR.

Timestep, T	Relative Mean Error
T=0.00001	0.00376
T=0.0001	0.01853
T=0.001	0.035297
T=0.002	0.041239

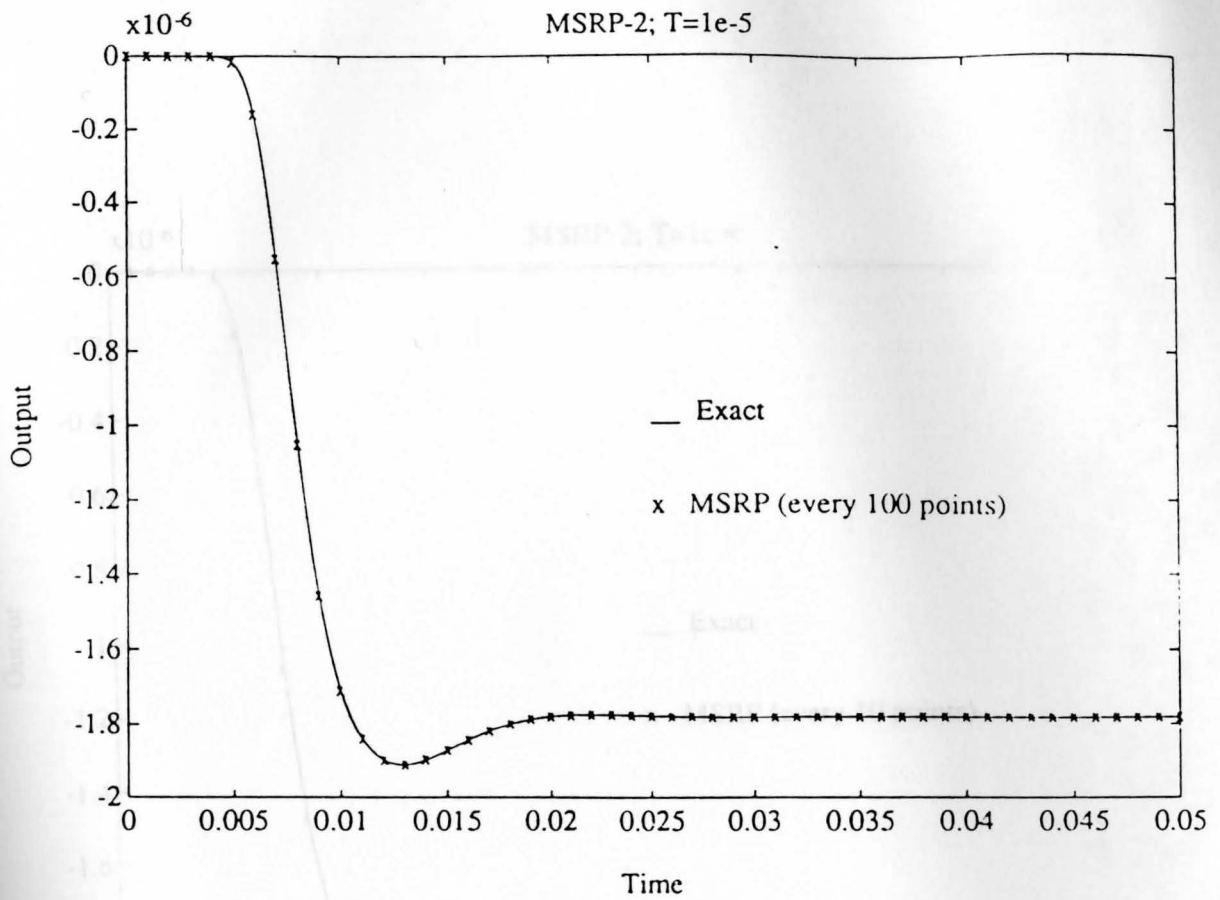


Figure 6.5 Step response utilizing MSRP-2 method with  $T=0.00001$ .

Figure 6.6 Step response utilizing MSRP-2 method with  $T=0.0001$ .

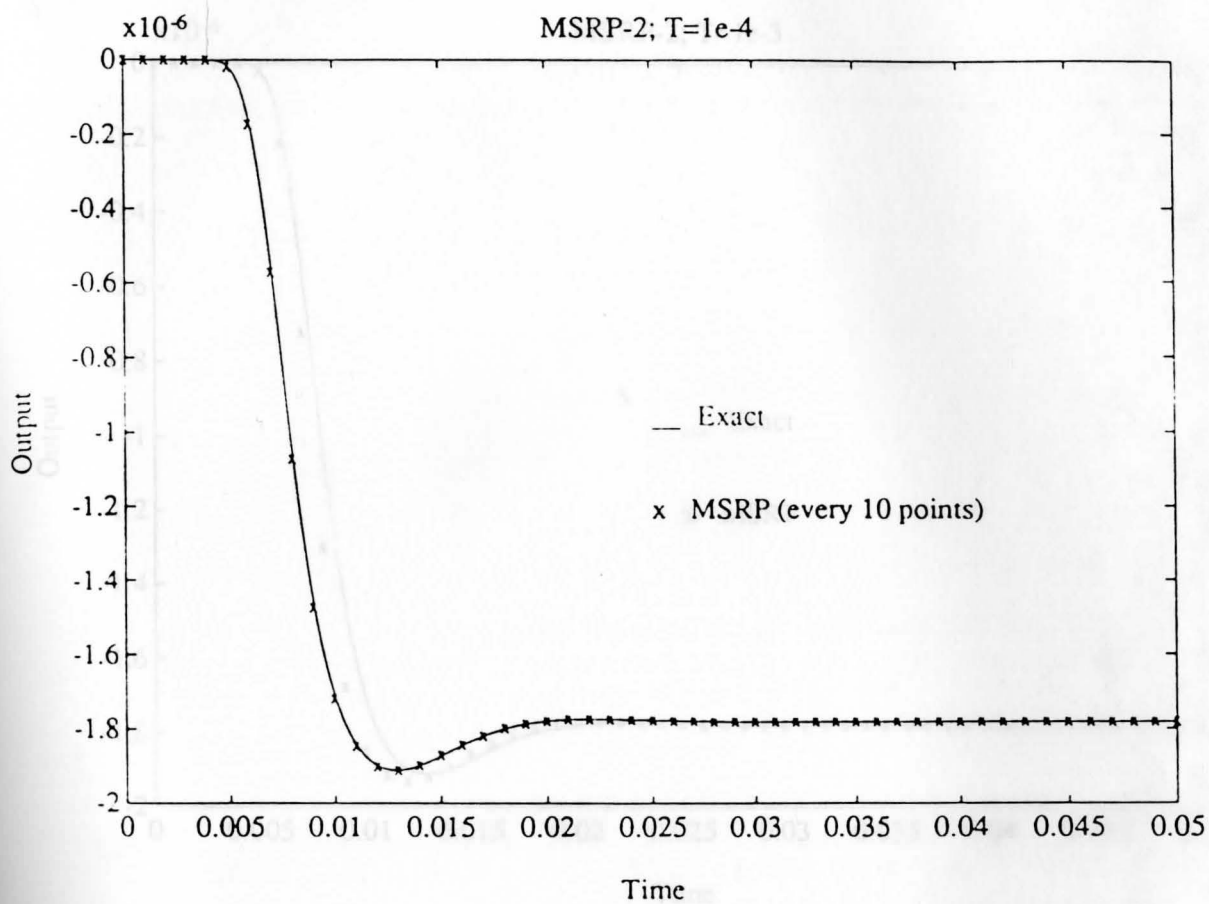


Figure 6.6 Step response utilizing MSRP-2 method with  $T=0.0001$ .

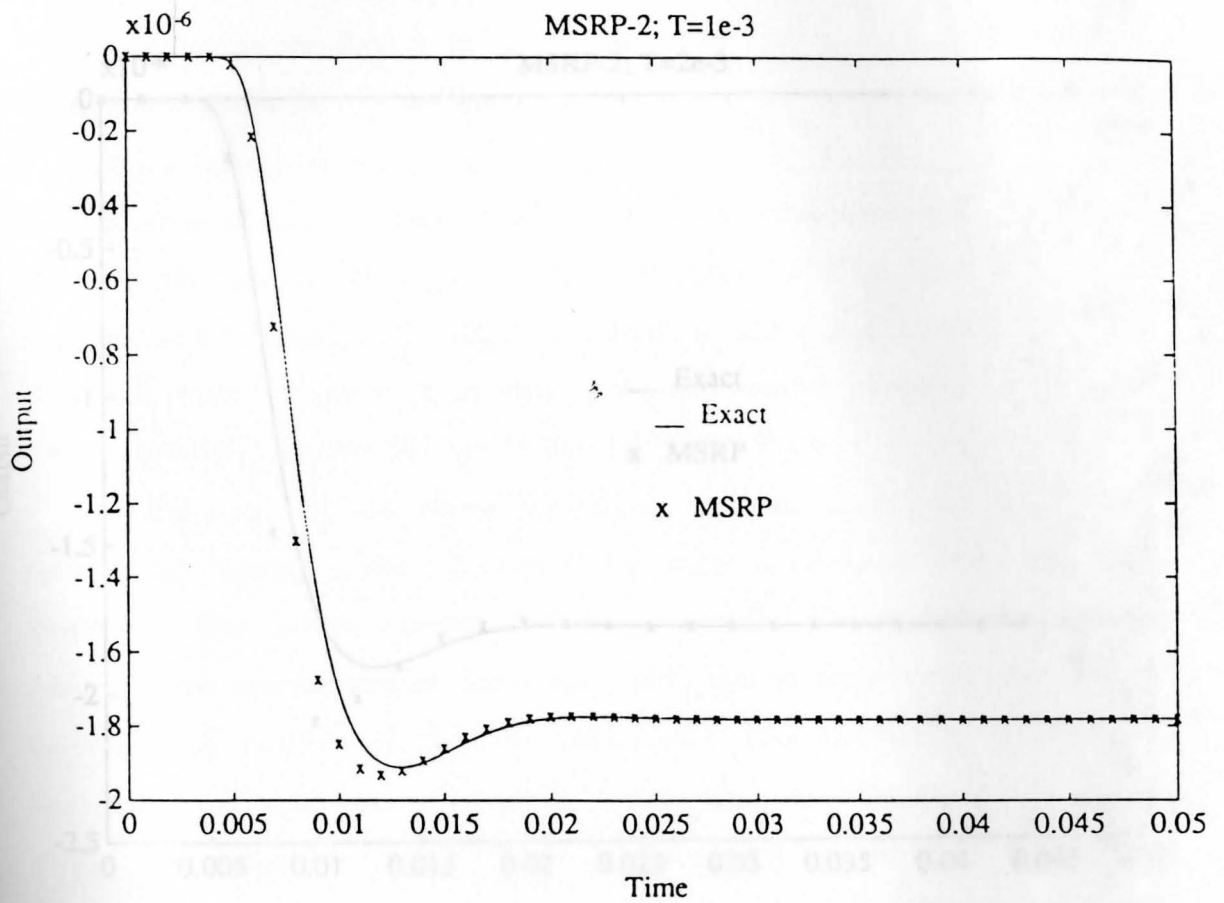


Figure 6.7 Step response utilizing MSRP-2 method with  $T=0.001$ .

## Computational Cost

The computational cost associated with the simulation of the 40-60 plant is approximately obtained from the amount of work for the floating point operations, i.e. flops. Consider the matrices  $A$  and  $B$ . Then, to calculate the product of  $A$  and  $B$  requires

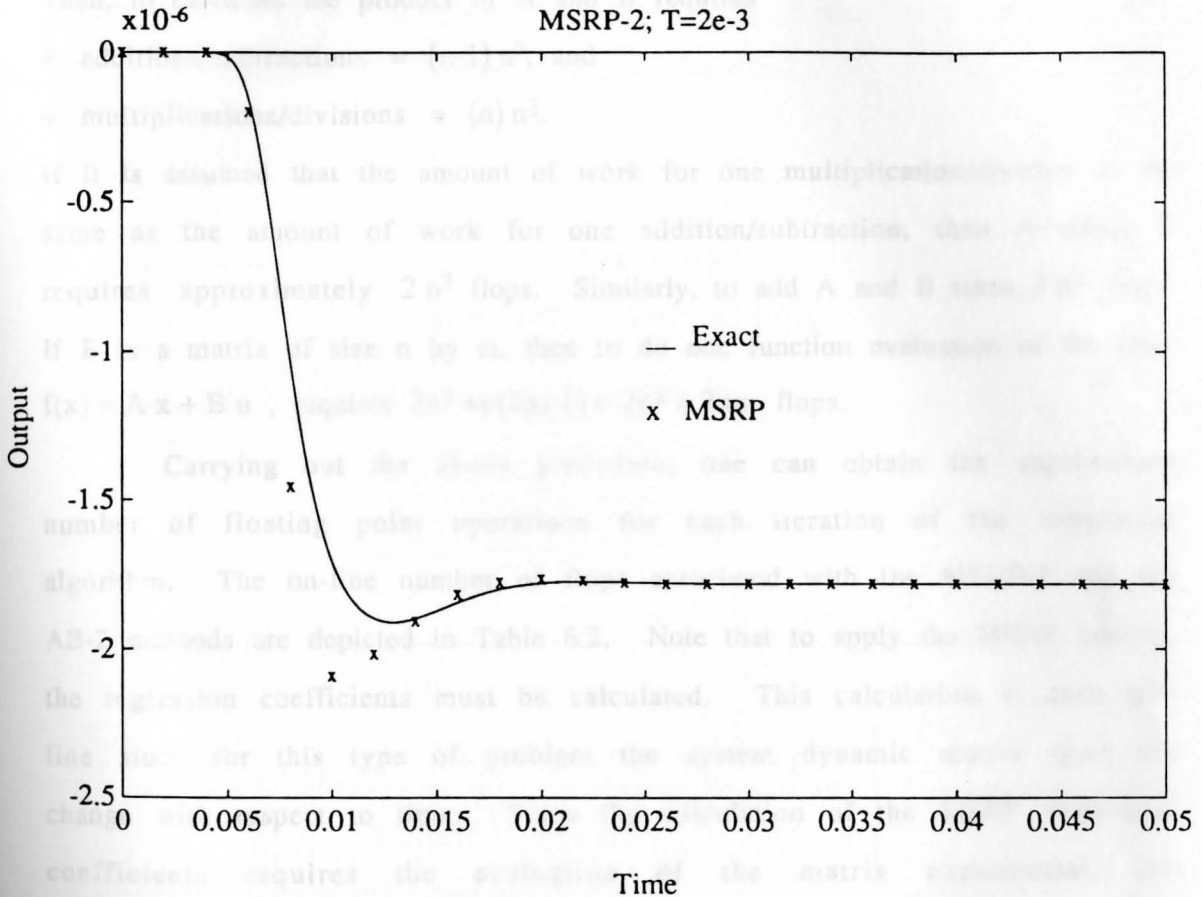


Figure 6.8 Step response utilizing MSRP-2 method with  $T=0.002$ .

### Computational Cost

The computational cost associated with the simulation of the 40-60 inlet is approximately obtained from the amount of work to perform one floating point operation, i.e. flops. Consider the matrices A and B of size n. Then, to calculate the product of A and B requires

# additions/subtractions =  $(n-1)n^2$ , and

# multiplications/divisions =  $(n)n^2$ .

If it is assumed that the amount of work for one multiplication/division is the same as the amount of work for one addition/subtraction, then A times B requires approximately  $2n^3$  flops. Similarly, to add A and B takes  $2n^2$  flops. If B is a matrix of size n by m, then to do one function evaluation of the form  $f(x) = Ax + Bu$ , requires  $2n^2 + n(2m-1) \approx 2n^2 + 2nm$  flops.

Carrying out the above procedure, one can obtain the approximate number of floating point operations for each iteration of the simulation algorithm. The on-line number of flops associated with the MSRP-2 and the AB-2 methods are depicted in Table 6.2. Note that to apply the MSRP method, the regression coefficients must be calculated. This calculation is done off-line since for this type of problem the system dynamic matrix does not change with respect to time. Since the calculation of the MSRP regression coefficients requires the evaluation of the matrix exponential, the determination of the required number of flops is difficult. This is due to the fact that matrix exponential can only be evaluated by some approximation method such as Pade approximation. The number of flops will then be dependent upon the nature of the approximation as well as the termination constraint.

It turns out that a MATLAB built in function, called FLOPS, can be utilized to calculate the number of flops. Note that, the mechanism by which

MATLAB calculates the number of floating point operations is involved. Hence, to be consistent, MATLAB's built in function is utilized to calculate the flop counts for simulation of the system by both AB-2 and MSRP-2 methods with a fixed timestep and fixed final time. The MATLAB flop counts are tabulated in Table 6.3. It is clear that the off-line calculations are not considered; then to break even the timestep associated with MSRP-2 method must be approximately five times that of the AB-2.

T=0.001	7,492,424	88,850,552	76,292,976
T=0.002	3,685,574	72,522,040	76,207,614

TABLE 6.2  
APPROXIMATE COMPUTATIONAL COST.

Required Operations	Flops
Function evaluation $x = A x + B u$	$2n^2 + 2nm$
AB-2 method $x_{k+2} = x_{k+1} + T(1.5 x_{k+1} - 0.5 x_k)$	$4n$
MSRP-2 method $x_{k+2} = -A_1 x_{k+1} - A_0 x_k + T(B_1 x_{k+1} + B_0 x_k)$	$8n^2 + 4n$
Total # flops for AB-2	$2n^2 + 2nm + 4n$
Total # flops for MSRP-2	$10n^2 + 4n + 2nm$



TABLE 6.3  
COMPUTATIONAL COST FOR SIMULATING THE 40-60 INLET BY MATLAB.

Timestep T	MSRP-2 Method			AB-2 Method
	# on-line flops	# off-line flops	Total # flops	Total # flops
T=0.00001	761,248,722	46,471,256	807,719,978	156,214,754
T=0.0001	76,015,722	53,914,478	129,930,200	—
T=0.001	7,492,424	68,800,552	76,292,976	—
T=0.002	3,685,574	72,522,040	76,207,614	—

## CHAPTER VII

### CONCLUSION AND DIRECTION FOR FUTURE RESEARCH

This study provides an alternative procedure in the field of computational fluid dynamics. That is, the temporal restriction imposed by the customary explicit finite difference methods can be eliminated for the class of problems which are linear or linearizable. Although the MSRP method can only be directly applied to small perturbation models, the methodology of departing from the simulation algorithms traditionally used by computational fluid dynamicists, provides further motivation for future developments for the field of real-time computational fluid dynamics.

Several linear multistep algorithms were utilized to simulate an 123-rd order small perturbation model of the NASA Lewis Mach 2.5 40-60 mixed compression inlet. It is shown that the Matrix Stability Region Placement method adopted in this thesis provides a stable and accurate simulation with extremely large integration timestep.

The MSRP-2 method can be utilized to simulate the inlet model with timestep of more than 100 times that of the AB-2 method and still preserve the accuracy of simulation. Hence, this method, where applicable, can be adopted for real-time simulation. This is of considerable importance for real-time simulation of stiff systems such as the inlet model considered in this investigation.

The algorithms considered in this study, the AB-2 and MSRP-2 methods, are programmed in the MATLAB environment and are included in the Appendix C. The computational costs associated with these methods are determined by utilizing the MATLAB built in function FLOPS [10]. It is

observed that MSRP-2 method can provide speed up time of about 40. The timestep of the AB-2 method is restricted to  $T=0.000005$ , whereas the MSRP-2's has no restriction in general for stability.

As a part of this thesis, a closed form representation of the MSRP method was also derived. Furthermore, the equivalence of the MSRP method and the  $n$ -th order hold technique based on Gregory-Newton interpolation formula was investigated. Although the equivalence of the methods in general form is not complete at present, this equivalence was demonstrated for several cases, and thought to be generally true.

#### Directions For Future Research

There are several areas which can be further investigated:

- I) The general proof of the equivalence of the MSRP and higher order hold technique, presented in this paper, needs to be completed. This is not an easy task since the general form of these methods is very complicated.
- II) It is possible to determine recursive relationships between the coefficients of the MSRP methods. This can be utilized to change the order of the simulation on-line. That is, the calculation of the  $k$ -step MSRP regression coefficients can be performed with less computational effort if the regression coefficients of  $(k-1)$ -step MSRP method are known. Note that this is important since the off-line computational cost associated with the MSRP method can be significantly decreased
- III) The MSRP method can be used on nonlinear systems close to a steady state operating point. An adaptive version of the MSRP method has been developed in [25]-[26]. This algorithm approximates the spatially and time varying Jacobian of the system and recalculates the regression

coefficients of the integrator. The applicability of this method is somewhat restrictive since for large systems the computational burden of estimating the Jacobian and recalculating the regression coefficients might not be feasible. The method can be made more attractive if efficient algorithms are developed for estimation and recalculation of the Jacobian and the regression coefficients. Furthermore, bounds on the acceptable nonlinear perturbation as a function of  $T$  are presently being developed.

in the vicinity of the numerical solution of the (A,B,C) triple  
 perturbation matrix of the 40-60 model system. The matrices are given  
 system dynamic matrix, A, is in the block diagonal form,  
 matrices are included, that is, the original A matrix is used in the  
 new matrix.

**APPENDIX A**

**NUMERICAL VALUES OF {A,B,C} TRIPLE**

$$A = \begin{bmatrix} -J_1 + I_1 & & & \\ & -J_1 & & \\ & & & \\ 0 & & & \end{bmatrix}$$

Compare the  $\tilde{A}$  to A given by Equation (2.21). The numerical values of  
 matrices are followed by that of the  $\tilde{A}$  matrix.

In this appendix the numerical values of the {A,B,C} triple for the small perturbation model of the 40-60 mixed compression inlet are presented. Since the system dynamic matrix, A, is in the block-tri-diagonal form, only the diagonal matrices are included. that is, the original A matrix is written in banded form. The new matrix has dimension of 9 by 123 with is assembled as follows.

$$\hat{A} = \frac{1}{H} \begin{bmatrix} -J_1^+ + J_1^- & J_2^+ & J_3^+ & \dots & \dots & \dots \\ -J_1^- & -J_2^+ + J_2^- & -J_3^+ + J_3^- & \dots & \dots & \dots \\ 0 & -J_2^- & -J_3^- & \dots & \dots & \dots \end{bmatrix}$$

Compare the  $\hat{A}$  to A given by Equation (2.21). The numerical values of the B and C matrices are followed by that of the  $\hat{A}$  matrix.

Columns 13 through 16

5.5003e-12	-9.0082e-03	-1.1817e+03	1.1817e-12		
1.3872e+04	1.3872e+04	9.9251e-12	1.3872e-04		
-1.3872e-12	1.3872e+03	1.7372e+03	4.4678e-12		
5.5003e-12	9.0082e+03	1.1817e+03	-1.1817e-12		
1.3872e+04	-1.3872e+04	-9.9251e+03	-1.3872e+04		
1.3872e-12	-4.4678e+03	-1.7372e+03	-4.4678e-12		
0	0	0	0		
0	0	0	0		
0	0	0	0		

Columns 19 through 24

-7.2425e-12	-6.8772e+03	-1.1817e+03	1.1817e-12		
1.3872e+04	1.3872e+04	1.3872e-04	1.3872e-04		
-2.3477e-12	4.2987e+03	1.1817e+03	1.1817e-12		
7.2425e-12	6.8772e+03	1.1817e+03	-1.1817e-12		
-1.3872e+04	-1.3872e+04	-1.3872e+04	-1.3872e+04		
2.3477e-12	-4.2987e+03	-1.1817e+03	-1.1817e-12		
0	0	0	0		
0	0	0	0		
0	0	0	0		

Columns 25 through 30

7.1817e-12	-6.8772e+03	-1.1817e+03	1.1817e-12		
1.3872e+04	1.3872e+04	1.3872e-04	1.3872e-04		
-2.1157e-12	4.2987e+03	1.1817e+03	1.1817e-12		
-7.1817e-12	6.8772e+03	1.1817e+03	-1.1817e-12		
-1.3872e+04	-1.3872e+04	-1.3872e+04	-1.3872e+04		
2.1157e-12	-4.2987e+03	-1.1817e+03	-1.1817e-12		
0	0	0	0		
0	0	0	0		
0	0	0	0		

Columns 31 through 36

-9.3014e-12	-9.1032e+03	-9.5930e+03	1.3872e-12		
1.3872e+04	1.4942e+04	1.2421e+04	1.3872e+04		
9.6011e-12	4.2987e+03	1.3874e+03	-2.3477e-12		
-9.3014e-12	1.1812e+03	-9.5930e+03	-1.0000e-12		
-1.3872e+04	-1.4942e+04	-1.2421e+04	-1.3872e+04		
-9.6011e-12	1.2987e+03	-1.3874e+03	2.3477e-12		
0	0	0	0		
0	0	0	0		
0	0	0	0		

, at '=

Columns 1 through 6

-2.8684e-12	9.9429e+03	1.1921e+04	5.5306e-12	-9.7629e+03	-1.1865e+04
-1.3672e+04	-2.0856e+04	-9.3405e+03	1.3672e+04	2.0667e+04	9.4538e+03
-1.0987e-10	-4.2987e+03	-1.8249e+04	-2.7551e-11	4.2987e+03	1.8084e+04
0	0	0	-5.5306e-12	9.7629e+03	1.1865e+04
0	0	0	-1.3672e+04	-2.0667e+04	-9.4538e+03
0	0	0	2.7551e-11	-4.2987e+03	-1.8084e+04
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Columns 7 through 12

2.0485e-12	-9.5528e+03	-1.1800e+04	-4.9560e-12	-9.3114e+03	-1.1722e+04
1.3672e+04	2.0443e+04	9.5881e+03	1.3672e+04	2.0183e+04	9.7426e+03
-6.6375e-13	4.2987e+03	1.7888e+04	-3.7627e-12	4.2987e+03	1.7660e+04
-2.0485e-12	9.5528e+03	1.1800e+04	4.9560e-12	9.3114e+03	1.1722e+04
-1.3672e+04	-2.0443e+04	-9.5881e+03	-1.3672e+04	-2.0183e+04	-9.7426e+03
6.6375e-13	-4.2987e+03	-1.7888e+04	3.7627e-12	-4.2987e+03	-1.7660e+04
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Columns 13 through 18

5.8009e-12	-9.0088e+03	-1.1617e+04	5.9491e-12	-8.6008e+03	-1.1466e+04
1.3672e+04	1.9853e+04	9.9351e+03	1.3672e+04	1.9398e+04	1.0195e+04
-3.3321e-12	4.2987e+03	1.7371e+04	4.4676e-12	4.2987e+03	1.6973e+04
-5.8009e-12	9.0088e+03	1.1617e+04	-5.9491e-12	8.6008e+03	1.1466e+04
-1.3672e+04	-1.9853e+04	-9.9351e+03	-1.3672e+04	-1.9398e+04	-1.0195e+04
3.3321e-12	-4.2987e+03	-1.7371e+04	-4.4676e-12	-4.2987e+03	-1.6973e+04
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Columns 19 through 24

-7.2425e-12	-8.0772e+03	-1.1255e+04	-5.3823e-12	-7.4168e+03	-1.0957e+04
1.3672e+04	1.8798e+04	1.0528e+04	1.3672e+04	1.8013e+04	1.0947e+04
-2.3487e-12	4.2987e+03	1.6448e+04	-2.8859e-12	4.2987e+03	1.5762e+04
7.2425e-12	8.0772e+03	1.1255e+04	5.3823e-12	7.4168e+03	1.0957e+04
-1.3672e+04	-1.8798e+04	-1.0528e+04	-1.3672e+04	-1.8013e+04	-1.0947e+04
2.3487e-12	-4.2987e+03	-1.6448e+04	2.8859e-12	-4.2987e+03	-1.5762e+04
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Columns 25 through 30

7.1632e-12	-6.6793e+03	-1.0582e+04	-2.1465e-12	-5.7907e+03	-1.0060e+04
1.3672e+04	1.7094e+04	1.1417e+04	1.3672e+04	1.5917e+04	1.1984e+04
-4.1157e-12	4.2987e+03	1.4957e+04	-1.3941e-12	4.2987e+03	1.3927e+04
-7.1632e-12	6.6793e+03	1.0582e+04	2.1465e-12	5.7907e+03	1.0060e+04
-1.3672e+04	-1.7094e+04	-1.1417e+04	-1.3672e+04	-1.5917e+04	-1.1984e+04
4.1157e-12	-4.2987e+03	-1.4957e+04	1.3941e-12	-4.2987e+03	-1.3927e+04
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Columns 31 through 36

-8.3014e-13	-5.1032e+03	-9.5930e+03	1.0585e-12	-4.8548e+03	-9.4097e+03
1.3672e+04	1.4942e+04	1.2421e+04	1.3672e+04	1.4574e+04	1.2580e+04
9.6011e-13	4.2987e+03	1.3074e+04	-2.9947e-12	4.2987e+03	1.2752e+04
8.3014e-13	5.1032e+03	9.5930e+03	-1.0585e-12	4.8548e+03	9.4097e+03
-1.3672e+04	-1.4942e+04	-1.2421e+04	-1.3672e+04	-1.4574e+04	-1.2580e+04
-9.6011e-13	-4.2987e+03	-1.3074e+04	2.9947e-12	-4.2987e+03	-1.2752e+04
0	0	0	0	0	0

## Columns 37 through 42

1.5035e-12	-4.8769e+03	-9.4257e+03	-5.1349e-14	-4.9577e+03	-9.4874e+03
1.3672e+04	1.4607e+04	1.2564e+04	1.3672e+04	1.4727e+04	1.2515e+04
5.0281e-13	4.2987e+03	1.2781e+04	2.2632e-12	4.2987e+03	1.2886e+04
-1.5035e-12	4.8769e+03	9.4257e+03	5.1349e-14	4.9577e+03	9.4874e+03
-1.3672e+04	-1.4607e+04	-1.2564e+04	-1.3672e+04	-1.4727e+04	-1.2515e+04
-5.0281e-13	-4.2987e+03	-1.2781e+04	-2.2632e-12	-4.2987e+03	-1.2886e+04
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

## Columns 43 through 48

-8.8227e-13	-5.1421e+03	-9.6209e+03	-3.1786e-12	-5.4125e+03	-9.8097e+03
1.3672e+04	1.4999e+04	1.2396e+04	1.3672e+04	1.5388e+04	1.2223e+04
1.3031e-12	4.2987e+03	1.3124e+04	3.4176e-12	4.2987e+03	1.3465e+04
8.8227e-13	5.1421e+03	9.6209e+03	3.1786e-12	5.4125e+03	9.8097e+03
-1.3672e+04	-1.4999e+04	-1.2396e+04	-1.3672e+04	-1.5388e+04	-1.2223e+04
-1.3031e-12	-4.2987e+03	-1.3124e+04	-3.4176e-12	-4.2987e+03	-1.3465e+04
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

## Columns 49 through 54

-3.3400e-12	-5.7496e+03	-1.0034e+04	3.5687e-12	-6.1479e+03	-1.0280e+04
1.3672e+04	1.5860e+04	1.2010e+04	1.3672e+04	1.6400e+04	1.1756e+04
2.0490e-12	4.2987e+03	1.3877e+04	-3.3632e-13	4.2987e+03	1.4350e+04
3.3400e-12	5.7496e+03	1.0034e+04	-3.5687e-12	6.1479e+03	1.0280e+04
-1.3672e+04	-1.5860e+04	-1.2010e+04	-1.3672e+04	-1.6400e+04	-1.1756e+04
-2.0490e-12	-4.2987e+03	-1.3877e+04	3.3632e-13	-4.2987e+03	-1.4350e+04
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

## Columns 55 through 60

6.0183e-12	-6.5773e+03	-1.0526e+04	-4.3277e-12	-6.9810e+03	-1.0743e+04
1.3672e+04	1.6963e+04	1.1482e+04	1.3672e+04	1.7476e+04	1.1226e+04
-3.9392e-12	4.2987e+03	1.4843e+04	2.2343e-12	4.2987e+03	1.5292e+04
-6.0183e-12	6.5773e+03	1.0526e+04	4.3277e-12	6.9810e+03	1.0743e+04
-1.3672e+04	-1.6963e+04	-1.1482e+04	-1.3672e+04	-1.7476e+04	-1.1226e+04
3.9392e-12	-4.2987e+03	-1.4843e+04	-2.2343e-12	-4.2987e+03	-1.5292e+04
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

## Columns 61 through 66

-5.9817e-12	-7.3528e+03	-1.0927e+04	-7.1499e-13	-7.6906e+03	-1.1085e+04
1.3672e+04	1.7935e+04	1.0988e+04	1.3672e+04	1.8343e+04	1.0773e+04
3.2943e-12	4.2987e+03	1.5693e+04	6.6231e-12	4.2987e+03	1.6050e+04
5.9817e-12	7.3528e+03	1.0927e+04	7.1499e-13	7.6906e+03	1.1085e+04
-1.3672e+04	-1.7935e+04	-1.0988e+04	-1.3672e+04	-1.8343e+04	-1.0773e+04
-3.2943e-12	-4.2987e+03	-1.5693e+04	-6.6231e-12	-4.2987e+03	-1.6050e+04
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

## Columns 67 through 72

8.7677e+02	-2.0000e+03	-5.4018e+03	1.0913e+03	-1.4383e+03	-3.9906e+03
1.1278e+04	9.4012e+03	1.2262e+04	9.9805e+03	8.0659e+03	1.1592e+04
1.0284e+03	4.1408e+03	8.7597e+03	1.6288e+03	3.8801e+03	7.7016e+03
-1.7535e+03	2.1346e+03	4.6727e+03	-2.1825e+03	1.7188e+03	3.0030e+03
-8.8853e+03	-9.7687e+03	-1.0272e+04	-6.2894e+03	-9.0147e+03	-8.2513e+03
-2.0567e+03	-3.9829e+03	-9.6149e+03	-3.2576e+03	-3.4614e+03	-9.1757e+03
8.7677e+02	-1.3464e+02	7.2915e+02	1.0913e+03	-2.8050e+02	9.8762e+02
-2.3932e+03	3.6751e+02	-1.9903e+03	-3.6911e+03	9.4875e+02	-3.3405e+03
1.0284e+03	-1.5792e+02	8.5523e+02	1.6288e+03	-4.1866e+02	1.4741e+03



## Columns 73 through 78

1.0907e+03	-1.4424e+03	-4.0021e+03	1.0943e+03	-1.4151e+03	-3.9249e+03
9.9898e+03	8.0762e+03	1.1599e+04	9.9271e+03	8.0069e+03	1.1551e+04
1.6242e+03	3.8825e+03	7.7103e+03	1.6551e+03	3.8658e+03	7.6516e+03
-2.1814e+03	1.7219e+03	3.0155e+03	-2.1886e+03	1.7013e+03	2.9314e+03
-6.3080e+03	-9.0196e+03	-8.2686e+03	-6.1826e+03	-8.9865e+03	-8.1518e+03
-3.2484e+03	-3.4663e+03	-9.1795e+03	-3.3102e+03	-3.4328e+03	-9.1543e+03
-1.0907e+03	-2.7949e+02	9.8658e+02	1.0943e+03	-2.8627e+02	9.9351e+02
-3.6818e+03	9.4346e+02	-3.3303e+03	-3.7445e+03	9.7958e+02	-3.3996e+03
1.6242e+03	-4.1620e+02	1.4692e+03	1.6551e+03	-4.3298e+02	1.5027e+03

## Columns 79 through 84

1.0995e+03	-1.3601e+03	-3.7684e+03	1.1018e+03	-1.3022e+03	-3.6009e+03
9.8005e+03	7.8665e+03	1.1453e+04	9.6660e+03	7.7165e+03	1.1346e+04
1.7180e+03	3.8305e+03	7.5316e+03	1.7857e+03	3.7909e+03	7.4022e+03
-2.1989e+03	1.6598e+03	2.7626e+03	-2.2035e+03	1.6155e+03	2.5847e+03
-5.9293e+03	-8.9216e+03	-7.9120e+03	-5.6603e+03	-8.8556e+03	-7.6515e+03
-3.4361e+03	-3.3622e+03	-9.1033e+03	-3.5713e+03	-3.2831e+03	-9.0493e+03
1.0995e+03	-2.9966e+02	1.0058e+03	1.1018e+03	-3.1332e+02	1.0162e+03
-3.8712e+03	1.0551e+03	-3.5414e+03	-4.0057e+03	1.1391e+03	-3.6946e+03
1.7180e+03	-4.6825e+02	1.5717e+03	1.7857e+03	-5.0780e+02	1.6470e+03

## Columns 85 through 90

1.1001e+03	-1.2353e+03	-3.4050e+03	1.0961e+03	-1.1886e+03	-3.2666e+03
9.5100e+03	7.5413e+03	1.1217e+04	9.4004e+03	7.4174e+03	1.1123e+04
1.8653e+03	3.7422e+03	7.2494e+03	1.9219e+03	3.7061e+03	7.1403e+03
-2.2002e+03	1.5635e+03	2.3805e+03	-2.1922e+03	1.5266e+03	2.2387e+03
-5.3483e+03	-8.7829e+03	-7.3409e+03	-5.1293e+03	-8.7343e+03	-7.1174e+03
-3.7305e+03	-3.1858e+03	-8.9865e+03	-3.8439e+03	-3.1136e+03	-8.9426e+03
1.1001e+03	-3.2821e+02	1.0246e+03	1.0961e+03	-3.3796e+02	1.0279e+03
-4.1616e+03	1.2416e+03	-3.8758e+03	-4.2712e+03	1.3169e+03	-4.0055e+03
1.8653e+03	-5.5649e+02	1.7372e+03	1.9219e+03	-5.9259e+02	1.8024e+03

## Columns 91 through 96

1.0947e+03	-1.1769e+03	-3.2318e+03	1.0970e+03	-1.1973e+03	-3.2924e+03
9.3730e+03	7.3862e+03	1.1099e+04	9.4208e+03	7.4405e+03	1.1141e+04
1.9362e+03	3.6968e+03	7.1127e+03	1.9113e+03	3.7130e+03	7.1607e+03
-2.1894e+03	1.5172e+03	2.2034e+03	-2.1940e+03	1.5335e+03	2.2649e+03
-5.0743e+03	-8.7225e+03	-7.0607e+03	-5.1700e+03	-8.7432e+03	-7.1593e+03
-3.8725e+03	-3.0949e+03	-8.9317e+03	-3.8227e+03	-3.1272e+03	-8.9508e+03
1.0947e+03	-3.4029e+02	1.0284e+03	1.0970e+03	-3.3620e+02	1.0274e+03
-4.2986e+03	1.3363e+03	-4.0384e+03	-4.2508e+03	1.3027e+03	-3.9813e+03
1.9362e+03	-6.0190e+02	1.8190e+03	1.9113e+03	-5.8576e+02	1.7901e+03

## Columns 97 through 102

1.0983e+03	-1.2105e+03	-3.3317e+03	1.0999e+03	-1.2316e+03	-3.3941e+03
9.4519e+03	7.4756e+03	1.1167e+04	9.5014e+03	7.5315e+03	1.1209e+04
1.8952e+03	3.7233e+03	7.1917e+03	1.8697e+03	3.7395e+03	7.2408e+03
-2.1965e+03	1.5440e+03	2.3051e+03	-2.1997e+03	1.5606e+03	2.3692e+03
-5.2321e+03	-8.7568e+03	-7.2229e+03	-5.3311e+03	-8.7789e+03	-7.3233e+03
-3.7905e+03	-3.1479e+03	-8.9632e+03	-3.7394e+03	-3.1802e+03	-8.9830e+03
1.0983e+03	-3.3346e+02	1.0266e+03	1.0999e+03	-3.2900e+02	1.0249e+03
-4.2197e+03	1.2812e+03	-3.9443e+03	-4.1703e+03	1.2474e+03	-3.8859e+03
1.8952e+03	-5.7544e+02	1.7715e+03	1.8697e+03	-5.5927e+02	1.7422e+03

## Columns 103 through 108

1.1011e+03	-1.2556e+03	-3.4646e+03	1.1013e+03	-1.2622e+03	-3.4841e+03
9.5573e+03	7.5945e+03	1.1256e+04	9.5730e+03	7.6121e+03	1.1269e+04
1.8410e+03	3.7573e+03	7.2960e+03	1.8330e+03	3.7623e+03	7.3113e+03
-2.2022e+03	1.5794e+03	2.4421e+03	-2.2026e+03	1.5845e+03	2.4625e+03
-5.4429e+03	-8.8045e+03	-7.4360e+03	-5.4744e+03	-8.8115e+03	-7.4671e+03
-3.6820e+03	-3.2159e+03	-9.0055e+03	-3.6660e+03	-3.2259e+03	-9.0116e+03
1.1011e+03	-3.2381e+02	1.0225e+03	1.1013e+03	-3.2231e+02	1.0216e+03
-4.1143e+03	1.2099e+03	-3.8205e+03	-4.0986e+03	1.1995e+03	-3.8019e+03
1.8410e+03	-5.4139e+02	1.7095e+03	1.8330e+03	-5.3644e+02	1.7003e+03

## Columns 109 through 114

1.1018e+03	-1.2921e+03	-3.5715e+03	1.1018e+03	-1.2882e+03	-3.5603e+03
9.6426e+03	7.6902e+03	1.1327e+04	9.6335e+03	7.6802e+03	1.1320e+04
1.7975e+03	3.7838e+03	7.3794e+03	1.8021e+03	3.7811e+03	7.3707e+03
-2.2036e+03	1.6077e+03	2.5538e+03	-2.2036e+03	1.6047e+03	2.5420e+03
-5.6135e+03	-8.8443e+03	-7.6053e+03	-5.5954e+03	-8.8401e+03	-7.5876e+03
-3.5951e+03	-3.2689e+03	-9.0397e+03	-3.6042e+03	-3.2634e+03	-9.0361e+03
1.1018e+03	-3.1561e+02	1.0177e+03	1.1018e+03	-3.1650e+02	1.0183e+03
-4.0291e+03	1.1541e+03	-3.7214e+03	-4.0381e+03	1.1600e+03	-3.7320e+03
1.7975e+03	-5.1490e+02	1.6603e+03	1.8021e+03	-5.1767e+02	1.6655e+03

## Columns 115 through 120

1.1009e+03	-1.2504e+03	-3.4492e+03	1.0956e+03	-1.1845e+03	-3.2543e+03
9.5452e+03	7.5809e+03	1.1246e+04	9.3907e+03	7.4064e+03	1.1114e+04
1.8472e+03	3.7535e+03	7.2840e+03	1.9270e+03	3.7029e+03	7.1305e+03
-2.2018e+03	1.5753e+03	2.4263e+03	-2.1912e+03	1.5233e+03	2.2262e+03
-5.4188e+03	-8.7988e+03	-7.4116e+03	-5.1099e+03	-8.7301e+03	-7.0974e+03
-3.6944e+03	-3.2083e+03	-9.0005e+03	-3.8539e+03	-3.1070e+03	-8.9388e+03
1.1009e+03	-3.2493e+02	1.0230e+03	1.0956e+03	-3.3879e+02	1.0281e+03
-4.1264e+03	1.2179e+03	-3.8344e+03	-4.2809e+03	1.3237e+03	-4.0171e+03
1.8472e+03	-5.4521e+02	1.7165e+03	1.9270e+03	-5.9587e+02	1.8082e+03

## Columns 121 through 123

0	0	0
0	0	0
0	0	0
1.0866e+03	-1.1251e+03	-3.0766e+03
9.2510e+03	7.2472e+03	1.0991e+04
2.0003e+03	3.6542e+03	6.9891e+03
-1.0866e+03	1.1251e+03	3.0766e+03
-9.2510e+03	-7.2472e+03	-1.0991e+04
-2.0003e+03	-3.6542e+03	-8.8832e+03

## Columns 49 through 54

## Columns 55 through 60

## Columns 61 through 66

## Columns 67 through 72

## Columns 73 through 78

## Columns 79 through 84

## Columns 85 through 90

## Columns 91 through 96

. bn' =

Columns 1 through 6

0 0 0 0 0 0

Columns 7 through 12

0 0 0 0 0 0

Columns 13 through 18

0 0 0 0 0 0

Columns 19 through 24

0 0 0 0 0 0

Columns 25 through 30

0 0 0 0 0 0

Columns 31 through 36

0 0 0 0 0 0

Columns 37 through 42

0 0 0 0 0 0

Columns 43 through 48

0 0 0 0 0 0

Columns 49 through 54

0 0 0 0 0 0

Columns 55 through 60

0 0 0 0 0 0

Columns 61 through 66

0 0 0 0 0 0

Columns 67 through 72

0 0 0 0 0 0

Columns 73 through 78

0 0 0 0 0 0

Columns 79 through 84

0 0 0 0 0 0

Columns 85 through 90

0 0 0 0 0 0

Columns 91 through 96

0 0 0 0 0 0

Columns 97 through 102  
 0 0 0 0 0 0  
 Columns 103 through 108  
 0 0 0 0 0 0  
 Columns 109 through 114  
 0 0 0 0 0 0  
 Columns 115 through 120  
 0 0 0 0 0 0  
 Columns 121 through 123  
 0 0 9.3696e-04

Columns 17 through 22  
 0 0 0 0 0 0  
 Columns 23 through 28  
 0 0 0 0 0 0  
 Columns 29 through 34  
 0 0 0 0 0 0  
 Columns 35 through 40  
 0 0 0 0 0 0  
 Columns 41 through 46  
 0 0 0 0 0 0  
 Columns 47 through 52  
 0 0 0 0 0 0  
 Columns 53 through 58  
 0 0 0 0 0 0  
 Columns 59 through 64  
 0 0 0 0 0 0  
 Columns 65 through 70  
 0 0 0 0 0 0  
 Columns 71 through 76  
 0 0 0 0 0 0  
 Columns 77 through 82  
 0 0 0 0 0 0  
 Columns 83 through 88  
 0 0 0 0 0 0  
 Columns 89 through 94  
 0 0 0 0 0 0

> CE =

Columns 1 through 6

0 0 0 0 0 0

Columns 7 through 12

0 0 0 0 0 0

Columns 13 through 18

0 0 0 0 0 0

Columns 19 through 24

0 0 0 0 0 0

Columns 25 through 30

0 0 0 0 0 0

Columns 31 through 36

0 0 0 0 0 0

Columns 37 through 42

0 0 0 0 0 0

Columns 43 through 48

0 0 0 0 0 0

Columns 49 through 54

0 0 0 0 0 0

Columns 55 through 60

0 0 0 0 0 0

Columns 61 through 66

0 0 0 0 0 0

Columns 67 through 72

5.9657e-01 -7.2229e-01 2.7496e+00 0 0 0

Columns 73 through 78

0 0 0 0 0 0

Columns 79 through 84

0 0 0 0 0 0

Columns 85 through 90

0 0 0 0 0 0

Columns 91 through 96

0 0 0 0 0 0

Columns 97 through 102

0 0 0 0 0 0

Columns 103 through 108

0 0 0 0 0 0

Columns 109 through 114

0 0 0 0 0 0

Columns 115 through 120

0 0 0 0 0 0

Columns 121 through 123

0 0 0 0 0 0







-8.6886e-03	9.0646e-03	-7.5990e-03	-8.8582e-03	9.3742e-03	-7.7450e-03
-1.9000e-02	1.3828e-02	-3.2244e-03	-1.9300e-02	1.4244e-02	-3.3583e-03
-7.2777e-03	-5.4809e-05	2.2510e-05	-7.2831e-03	-4.7036e-05	1.9288e-05
-8.6657e-03	9.0303e-03	-7.5849e-03	-8.8385e-03	9.3445e-03	-7.7328e-03
-1.6129e-02	9.2799e-03	4.0460e-03	-1.6009e-02	9.2797e-03	4.0814e-03
-7.7078e-03	6.2742e-04	-1.0873e-03	-7.7790e-03	7.0111e-04	-1.1174e-03
-6.2865e-03	5.2609e-03	-1.5438e-03	-6.1094e-03	5.2275e-03	-1.5503e-03
-1.5485e-02	9.4276e-03	4.0756e-03	-1.5380e-02	9.4210e-03	4.1112e-03
-7.8813e-03	5.5031e-04	-1.0848e-03	-7.9507e-03	6.2956e-04	-1.1163e-03
-5.6937e-03	5.4350e-03	-1.5268e-03	-5.5280e-03	5.3920e-03	-1.5317e-03
-1.5060e-02	9.3456e-03	4.1017e-03	-1.4975e-02	9.3463e-03	4.1355e-03
-7.9916e-03	5.2174e-04	-1.0789e-03	-8.0585e-03	6.0382e-04	-1.1117e-03
-5.3075e-03	5.4146e-03	-1.5161e-03	-5.1572e-03	5.3733e-03	-1.5208e-03
-1.4628e-02	9.1840e-03	4.1366e-03	-1.4567e-02	9.1957e-03	4.1683e-03
-8.0989e-03	5.0150e-04	-1.0724e-03	-8.1632e-03	5.8630e-04	-1.1066e-03
-4.9232e-03	5.3385e-03	-1.5011e-03	-4.7905e-03	5.3012e-03	-1.5057e-03
-1.4210e-02	8.9499e-03	4.1848e-03	-1.4174e-02	8.9748e-03	4.2139e-03
-8.1994e-03	4.8663e-04	-1.0657e-03	-8.2614e-03	5.7445e-04	-1.1017e-03
-4.5599e-03	5.2167e-03	-1.4793e-03	-4.4445e-03	5.1841e-03	-1.4835e-03
-1.3814e-02	8.6518e-03	4.2490e-03	-1.3800e-02	8.6905e-03	4.2754e-03
-8.2886e-03	4.7339e-04	-1.0588e-03	-8.3492e-03	5.6491e-04	-1.0968e-03
-4.2276e-03	5.0605e-03	-1.4495e-03	-4.1274e-03	5.0320e-03	-1.4531e-03
-1.3433e-02	8.2939e-03	4.3316e-03	-1.3440e-02	8.3460e-03	4.3552e-03
-8.3658e-03	4.5862e-04	-1.0514e-03	-8.4261e-03	5.5472e-04	-1.0919e-03
-3.9253e-03	4.8790e-03	-1.4116e-03	-3.8373e-03	4.8533e-03	-1.4143e-03
-1.3058e-02	7.8786e-03	4.4342e-03	-1.3083e-02	7.9428e-03	4.4549e-03
-8.4294e-03	4.3958e-04	-1.0429e-03	-8.4905e-03	5.4106e-04	-1.0860e-03
-3.6488e-03	4.6775e-03	-1.3660e-03	-3.5702e-03	4.6527e-03	-1.3673e-03
-1.2675e-02	7.4076e-03	4.5575e-03	-1.2716e-02	7.4820e-03	4.5757e-03
-8.4794e-03	4.1490e-04	-1.0328e-03	-8.5422e-03	5.2226e-04	-1.0788e-03
-3.3878e-03	4.4560e-03	-1.3124e-03	-3.3162e-03	4.4308e-03	-1.3119e-03
-1.2274e-02	6.8846e-03	4.7010e-03	-1.2327e-02	6.9661e-03	4.7175e-03
-8.5176e-03	3.8456e-04	-1.0210e-03	-8.5825e-03	4.9794e-04	-1.0698e-03
-3.1321e-03	4.2128e-03	-1.2502e-03	-3.0656e-03	4.1857e-03	-1.2477e-03
-1.1851e-02	6.3120e-03	4.8645e-03	-1.1914e-02	6.3973e-03	4.8803e-03
-8.5467e-03	3.4971e-04	-1.0076e-03	-8.6140e-03	4.6897e-04	-1.0592e-03
-2.8762e-03	3.9475e-03	-1.1794e-03	-2.8133e-03	3.9174e-03	-1.1746e-03
-1.1411e-02	5.6966e-03	5.0462e-03	-1.1481e-02	5.7824e-03	5.0624e-03
-8.5674e-03	3.1106e-04	-9.9270e-04	-8.6370e-03	4.3562e-04	-1.0468e-03
-2.6195e-03	3.6607e-03	-1.1001e-03	-2.5593e-03	3.6272e-03	-1.0929e-03
-1.0955e-02	5.0452e-03	5.2439e-03	-1.1029e-02	5.1286e-03	5.2614e-03
-8.5811e-03	2.6961e-04	-9.7668e-04	-8.6525e-03	3.9861e-04	-1.0330e-03
-2.3609e-03	3.3535e-03	-1.0126e-03	-2.3029e-03	3.3165e-03	-1.0032e-03
-1.0488e-02	4.3652e-03	5.4547e-03	-1.0565e-02	4.4439e-03	5.4743e-03
-8.5891e-03	2.2663e-04	-9.5994e-04	-8.6619e-03	3.5896e-04	-1.0179e-03
-2.1018e-03	3.0295e-03	-9.1814e-04	-2.0461e-03	2.9899e-03	-9.0696e-04
-1.0016e-02	3.6683e-03	5.6739e-03	-1.0095e-02	3.7413e-03	5.6962e-03
-8.5906e-03	1.8169e-04	-9.4258e-04	-8.6642e-03	3.1610e-04	-1.0017e-03
-1.8440e-03	2.6917e-03	-8.1784e-04	-1.7914e-03	2.6508e-03	-8.0554e-04
-9.5420e-03	2.9611e-03	5.8982e-03	-9.6226e-03	3.0281e-03	5.9233e-03
-8.5873e-03	1.3631e-04	-9.2522e-04	-8.6610e-03	2.7163e-04	-9.8482e-04
-1.5897e-03	2.3456e-03	-7.1387e-04	-1.5411e-03	2.3054e-03	-7.0125e-04
-9.0718e-03	2.2544e-03	6.1223e-03	-9.1553e-03	2.3163e-03	6.1504e-03
-8.5782e-03	9.0273e-05	-9.0819e-04	-8.6519e-03	2.2549e-04	-9.6777e-04
-1.3466e-03	2.0020e-03	-6.0979e-04	-1.3033e-03	1.9646e-03	-5.9779e-04
-8.6068e-03	1.5581e-03	6.3405e-03	-8.6953e-03	1.6164e-03	6.3715e-03
-8.5617e-03	4.2746e-05	-8.9192e-04	-8.6352e-03	1.7719e-04	-9.5100e-04
-1.1208e-03	1.6705e-03	-5.0897e-04	-1.0839e-03	1.6381e-03	-4.9847e-04
-8.5139e-03	1.4061e-03	6.3852e-03	-8.6015e-03	1.4573e-03	6.4210e-03
-8.5984e-03	8.0517e-05	-9.0388e-04	-8.6710e-03	2.1336e-04	-9.6194e-04
-8.4326e-04	1.2516e-03	-3.8144e-04	-8.1579e-04	1.2277e-03	-3.7372e-04

Columns 7 through 12

0	0	0	0	0	0
0	0	0	0	0	0

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
9.8025e-01	1.4887e-02	-3.5777e-03	-4.1166e-19	6.9474e-34	1.6852e-34	
-7.3144e-03	1.0000e+00	1.5008e-19	-9.2548e-19	1.4662e-33	-7.5156e-34	
-9.1094e-03	9.8208e-03	9.9205e-01	1.6856e-19	5.3046e-34	-4.9465e-34	
-1.9752e-02	1.4887e-02	-3.5777e-03	9.7983e-01	1.5484e-02	-3.7690e-03	
-7.3144e-03	-1.7539e-18	1.5008e-19	-7.3144e-03	1.0000e+00	-3.6132e-18	
-9.1094e-03	9.8208e-03	-7.9505e-03	-9.3545e-03	1.0277e-02	9.9184e-01	
-1.9752e-02	1.4887e-02	-3.5777e-03	-2.0173e-02	1.5484e-02	-3.7690e-03	
-7.3144e-03	-1.7539e-18	1.5008e-19	-7.3144e-03	8.4416e-18	-3.6132e-18	
-9.1094e-03	9.8208e-03	-7.9505e-03	-9.3545e-03	1.0277e-02	-8.1640e-03	
-1.9752e-02	1.4887e-02	-3.5777e-03	-2.0173e-02	1.5484e-02	-3.7690e-03	
-7.3144e-03	-1.7539e-18	1.5008e-19	-7.3144e-03	8.4416e-18	-3.6132e-18	
-9.1094e-03	9.8208e-03	-7.9505e-03	-9.3545e-03	1.0277e-02	-8.1640e-03	
-1.9752e-02	1.4887e-02	-3.5777e-03	-2.0173e-02	1.5484e-02	-3.7690e-03	
-7.3144e-03	-1.7539e-18	1.5009e-19	-7.3144e-03	8.4416e-18	-3.6132e-18	
-9.1094e-03	9.8208e-03	-7.9505e-03	-9.3545e-03	1.0277e-02	-8.1640e-03	
-1.9752e-02	1.4887e-02	-3.5777e-03	-2.0173e-02	1.5484e-02	-3.7690e-03	
-7.3144e-03	-1.7808e-18	1.6064e-19	-7.3144e-03	8.4329e-18	-3.6098e-18	
-9.1094e-03	9.8208e-03	-7.9505e-03	-9.3545e-03	1.0277e-02	-8.1640e-03	
-1.9752e-02	1.4887e-02	-3.5777e-03	-2.0173e-02	1.5484e-02	-3.7690e-03	
-7.3144e-03	-2.8674e-17	1.0750e-17	-7.3144e-03	-3.3831e-18	1.0423e-18	
-9.1094e-03	9.8208e-03	-7.9505e-03	-9.3545e-03	1.0277e-02	-8.1640e-03	
-1.9752e-02	1.4887e-02	-3.5777e-03	-2.0173e-02	1.5484e-02	-3.7690e-03	
-7.3144e-03	-7.7923e-14	3.0911e-14	-7.3144e-03	-4.5671e-14	1.8117e-14	
-9.1094e-03	9.8208e-03	-7.9505e-03	-9.3545e-03	1.0277e-02	-8.1640e-03	
-1.9752e-02	1.4887e-02	-3.5777e-03	-2.0173e-02	1.5484e-02	-3.7690e-03	
-7.3144e-03	-6.1941e-11	2.4835e-11	-7.3144e-03	-4.3718e-11	1.7523e-11	
-9.1094e-03	9.8208e-03	-7.9505e-03	-9.3545e-03	1.0277e-02	-8.1640e-03	
-1.9752e-02	1.4887e-02	-3.5777e-03	-2.0173e-02	1.5484e-02	-3.7690e-03	
-7.3144e-03	-2.5846e-09	1.0426e-09	-7.3144e-03	-1.9540e-09	7.8769e-10	
-9.1093e-03	9.8208e-03	-7.9505e-03	-9.3545e-03	1.0277e-02	-8.1640e-03	
-1.9752e-02	1.4887e-02	-3.5776e-03	-2.0173e-02	1.5484e-02	-3.7690e-03	
-7.3144e-03	-2.4964e-08	1.0101e-08	-7.3144e-03	-1.9429e-08	7.8534e-09	
-9.1093e-03	9.8207e-03	-7.9504e-03	-9.3544e-03	1.0277e-02	-8.1639e-03	
-1.9752e-02	1.4887e-02	-3.5773e-03	-2.0173e-02	1.5483e-02	-3.7689e-03	
-7.3143e-03	-1.3347e-07	5.4129e-08	-7.3143e-03	-1.0609e-07	4.2975e-08	
-9.1089e-03	9.8202e-03	-7.9502e-03	-9.3542e-03	1.0277e-02	-8.1638e-03	
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-5.2065e-04	9.7768e-04	-1.3316e-05	-2.3465e-04	5.5397e-04	1.3196e-04
-1.3421e-02	6.7021e-03	6.6664e-03	-1.2485e-02	5.6779e-03	6.7783e-03
-1.1247e-02	5.5466e-03	-3.2304e-03	-1.0825e-02	4.7070e-03	-2.8681e-03
-4.1130e-04	8.3987e-04	1.3650e-05	-1.5851e-04	4.6615e-04	1.4105e-04
-1.3358e-02	6.5626e-03	6.6575e-03	-1.2463e-02	5.5986e-03	6.7491e-03
-1.1305e-02	5.5981e-03	-3.2554e-03	-1.0874e-02	4.7449e-03	-2.8878e-03
-3.1770e-04	7.1585e-04	3.4079e-05	-9.5439e-05	3.8771e-04	1.4538e-04
-1.3302e-02	6.4264e-03	6.6461e-03	-1.2447e-02	5.5207e-03	6.7181e-03
-1.1355e-02	5.6417e-03	-3.2776e-03	-1.0918e-02	4.7773e-03	-2.9056e-03
-2.3812e-04	6.0371e-04	4.8990e-05	-4.4210e-05	3.1763e-04	1.4557e-04
-1.3252e-02	6.2949e-03	6.6325e-03	-1.2437e-02	5.4454e-03	6.6856e-03
-1.1399e-02	5.6785e-03	-3.2972e-03	-1.0955e-02	4.8046e-03	-2.9214e-03
-1.7205e-04	5.0317e-04	5.8877e-05	-4.4566e-06	2.5598e-04	1.4198e-04
-1.3210e-02	6.1689e-03	6.6172e-03	-1.2433e-02	5.3740e-03	6.6521e-03
-1.1435e-02	5.7079e-03	-3.3140e-03	-1.0987e-02	4.8265e-03	-2.9353e-03





-2.1210e-17	1.2504e-17	-5.7382e-18	-7.4881e-18	6.3180e-18	1.3959e-18
-1.6845e-17	9.6382e-18	-2.7723e-18	-1.5949e-17	1.3375e-17	-5.9851e-18
-2.8231e-18	-4.9510e-23	-4.1520e-24	-7.8973e-19	-3.1768e-23	-2.7960e-24
-1.7740e-17	9.0345e-18	-3.5704e-18	-7.4880e-18	6.3179e-18	-2.0735e-18
9.7173e-01	2.8339e-02	-8.2076e-03	-1.3787e-17	7.2162e-19	3.5609e-18
-7.3144e-03	1.0000e+00	4.7787e-20	-7.8967e-19	-6.4449e-23	-5.2815e-24
-1.4390e-02	2.0098e-02	9.8744e-01	-1.0737e-17	5.9227e-18	-1.6433e-18
-2.8269e-02	2.8339e-02	-8.2076e-03	9.7356e-01	2.5246e-02	-7.0971e-03
-7.3144e-03	-8.2468e-18	6.0861e-19	-7.3144e-03	1.0000e+00	-3.6219e-19
-1.4390e-02	2.0098e-02	-1.2558e-02	-1.3207e-02	1.7736e-02	9.8847e-01
-2.8269e-02	2.8339e-02	-8.2076e-03	-2.6444e-02	2.5246e-02	-7.0971e-03
-7.3144e-03	-1.6810e-17	3.9736e-18	-7.3144e-03	5.0655e-18	-3.5229e-19
-1.4390e-02	2.0098e-02	-1.2558e-02	-1.3207e-02	1.7736e-02	-1.1525e-02
-2.8269e-02	2.8339e-02	-8.2076e-03	-2.6444e-02	2.5246e-02	-7.0971e-03
-7.3144e-03	-5.5293e-17	1.9096e-17	-7.3144e-03	4.8868e-18	-2.8230e-19
-1.4390e-02	2.0098e-02	-1.2558e-02	-1.3207e-02	1.7736e-02	-1.1525e-02
-1.3272e-02	6.3396e-03	6.4147e-03	-1.2290e-02	5.3145e-03	6.5615e-03
-9.6175e-03	3.3784e-03	-2.2455e-03	-9.4879e-03	3.0608e-03	-2.0975e-03
-1.9175e-03	1.8020e-03	-3.9748e-04	-1.4370e-03	1.1599e-03	-1.6637e-04
-1.2642e-02	5.8615e-03	6.6899e-03	-1.1764e-02	4.9825e-03	6.7767e-03
-9.8098e-03	3.5240e-03	-2.3302e-03	-9.6489e-03	3.1621e-03	-2.1640e-03
-1.3141e-03	1.3443e-03	-1.3284e-04	-9.3228e-04	8.4189e-04	4.0885e-05
-1.2368e-02	5.6412e-03	6.7900e-03	-1.1542e-02	4.8353e-03	6.8480e-03
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-1.0299e-03	1.1194e-03	-2.5010e-05	-7.0011e-04	6.9022e-04	1.1963e-04
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-1.0007e-02	3.6771e-03	-2.4070e-03	-9.8115e-03	3.2666e-03	-2.2213e-03
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-1.0094e-02	3.7439e-03	-2.4382e-03	-9.8833e-03	3.3125e-03	-2.2443e-03
-5.8180e-04	7.6020e-04	1.1920e-04	-3.3761e-04	4.5003e-04	2.1797e-04
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-1.0173e-02	3.8037e-03	-2.4654e-03	-9.9489e-03	3.3540e-03	-2.2644e-03
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-1.0245e-02	3.8575e-03	-2.4898e-03	-1.0009e-02	3.3918e-03	-2.2826e-03
-2.9045e-04	5.2045e-04	1.8701e-04	-1.0545e-04	2.9026e-04	2.5651e-04
-1.1730e-02	5.0624e-03	6.8917e-03	-1.1061e-02	4.4687e-03	6.8738e-03
-1.0311e-02	3.9054e-03	-2.5118e-03	-1.0065e-02	3.4258e-03	-2.2993e-03
-1.8983e-04	4.3360e-04	2.0259e-04	-2.7131e-05	2.3274e-04	2.6186e-04
-1.1711e-02	5.0152e-03	6.8622e-03	-1.1059e-02	4.4446e-03	6.8377e-03
-1.0369e-02	3.9474e-03	-2.5319e-03	-1.0115e-02	3.4561e-03	-2.3149e-03
-1.1027e-04	3.6122e-04	2.1036e-04	3.3159e-05	1.8529e-04	2.6121e-04
-1.1703e-02	4.9718e-03	6.8273e-03	-1.1070e-02	4.4239e-03	6.7964e-03
-1.0422e-02	3.9841e-03	-2.5504e-03	-1.0160e-02	3.4829e-03	-2.3296e-03
-4.5654e-05	2.9834e-04	2.1267e-04	8.0338e-05	1.4465e-04	2.5622e-04
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-1.0470e-02	4.0166e-03	-2.5677e-03	-1.0202e-02	3.5068e-03	-2.3434e-03
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-1.1710e-02	4.8874e-03	6.7480e-03	-1.1116e-02	4.3857e-03	6.7043e-03
-1.0513e-02	4.0447e-03	-2.5835e-03	-1.0240e-02	3.5276e-03	-2.3564e-03
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-1.1722e-02	4.8463e-03	6.7057e-03	-1.1147e-02	4.3678e-03	6.6556e-03
-1.0551e-02	4.0687e-03	-2.5980e-03	-1.0273e-02	3.5455e-03	-2.3685e-03
8.1037e-05	1.4957e-04	1.9698e-04	1.6179e-04	5.2449e-05	2.2273e-04
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-1.0584e-02	4.0890e-03	-2.6110e-03	-1.0302e-02	3.5606e-03	-2.3796e-03
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-1.1762e-02	4.7699e-03	6.6180e-03	-1.1225e-02	4.3363e-03	6.5554e-03
-1.0612e-02	4.1052e-03	-2.6226e-03	-1.0327e-02	3.5728e-03	-2.3895e-03
1.1387e-04	8.2412e-05	1.7071e-04	1.7032e-04	1.5135e-05	1.8760e-04
-1.1788e-02	4.7359e-03	6.5734e-03	-1.1270e-02	4.3233e-03	6.5049e-03
-1.0635e-02	4.1182e-03	-2.6328e-03	-1.0349e-02	3.5825e-03	-2.3984e-03
1.1634e-04	5.8454e-05	1.5354e-04	1.6257e-04	3.6089e-06	1.6690e-04
-1.1820e-02	4.7058e-03	6.5286e-03	-1.1318e-02	4.3130e-03	6.4546e-03
-1.0655e-02	4.1278e-03	-2.6415e-03	-1.0367e-02	3.5899e-03	-2.4062e-03



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-1.8271e-17	6.3399e-18	-8.7023e-19	-1.5351e-17	9.5781e-18	-3.1617e-18
-8.4139e-19	-2.5178e-22	1.3782e-22	-4.9741e-19	-2.0465e-22	1.0309e-22
-1.4610e-17	8.7628e-18	-6.3964e-18	-8.6883e-18	6.7358e-18	-6.0095e-18
9.7496e-01	2.2937e-02	-6.2830e-03	-1.5350e-17	5.4085e-18	-4.1687e-18
-7.3144e-03	1.0000e+00	-1.5918e-19	-4.9725e-19	-3.5188e-22	1.7743e-22
-1.2314e-02	1.5971e-02	9.8925e-01	-8.6880e-18	9.5070e-18	-8.3625e-18
-2.5041e-02	2.2937e-02	-6.2830e-03	9.7606e-01	2.1179e-02	-5.6724e-03
-7.3144e-03	6.1861e-18	-1.5878e-19	-7.3144e-03	1.0000e+00	4.6198e-18
-1.2314e-02	1.5971e-02	-1.0747e-02	-1.1628e-02	1.4627e-02	9.8985e-01
-1.1534e-02	4.6022e-03	6.6243e-03	-1.0933e-02	4.0874e-03	6.6428e-03
-9.3887e-03	2.8157e-03	-1.9821e-03	-9.3126e-03	2.6246e-03	-1.8912e-03
-1.0808e-03	7.2233e-04	-1.2522e-05	-8.0653e-04	4.1300e-04	9.3686e-05
-1.1089e-02	4.3780e-03	6.7949e-03	-1.0555e-02	3.9439e-03	6.7797e-03
-9.5248e-03	2.8842e-03	-2.0350e-03	-9.4286e-03	2.6685e-03	-1.9338e-03
-6.5437e-04	5.0745e-04	1.5202e-04	-4.4320e-04	2.7543e-04	2.2589e-04
-1.0905e-02	4.2810e-03	6.8464e-03	-1.0400e-02	3.8827e-03	6.8170e-03
-9.5956e-03	2.9206e-03	-2.0586e-03	-9.4886e-03	2.6917e-03	-1.9520e-03
-4.6135e-04	4.0690e-04	2.1041e-04	-2.8066e-04	2.1169e-04	2.6955e-04
-1.0757e-02	4.2012e-03	6.8761e-03	-1.0277e-02	3.8325e-03	6.8355e-03
-9.6609e-03	2.9542e-03	-2.0785e-03	-9.5438e-03	2.7132e-03	-1.9671e-03
-2.9675e-04	3.2011e-04	2.5159e-04	-1.4267e-04	1.5660e-04	2.9844e-04
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-1.0579e-02	4.0966e-03	6.8782e-03	-1.0134e-02	3.7681e-03	6.8239e-03
-9.7764e-03	3.0134e-03	-2.1106e-03	-9.6416e-03	2.7515e-03	-1.9910e-03
-5.5999e-05	1.8955e-04	2.9157e-04	5.6880e-05	7.3868e-05	3.2090e-04
-1.0543e-02	4.0679e-03	6.8571e-03	-1.0110e-02	3.7521e-03	6.7993e-03
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-9.8746e-03	3.0632e-03	-2.1370e-03	-9.7251e-03	2.7841e-03	-2.0109e-03
8.7772e-05	1.0508e-04	2.9533e-04	1.7175e-04	2.1610e-05	3.1393e-04
-1.0546e-02	4.0402e-03	6.7857e-03	-1.0131e-02	3.7425e-03	6.7242e-03
-9.9176e-03	3.0845e-03	-2.1492e-03	-9.7619e-03	2.7982e-03	-2.0203e-03
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-1.0569e-02	4.0331e-03	6.7412e-03	-1.0163e-02	3.7437e-03	6.6783e-03
-9.9569e-03	3.1035e-03	-2.1609e-03	-9.7959e-03	2.8111e-03	-2.0297e-03
1.6762e-04	4.9375e-05	2.7941e-04	2.2976e-04	-1.0610e-05	2.9093e-04
-1.0601e-02	4.0277e-03	6.6931e-03	-1.0204e-02	3.7470e-03	6.6287e-03
-9.9932e-03	3.1207e-03	-2.1722e-03	-9.8275e-03	2.8228e-03	-2.0389e-03
1.9242e-04	2.7768e-05	2.6692e-04	2.4494e-04	-2.2092e-05	2.7557e-04
-1.0641e-02	4.0237e-03	6.6424e-03	-1.0254e-02	3.7522e-03	6.5763e-03
-1.0026e-02	3.1358e-03	-2.1830e-03	-9.8564e-03	2.8332e-03	-2.0479e-03
2.0801e-04	9.9955e-06	2.5188e-04	2.5174e-04	-3.0727e-05	2.5804e-04
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-1.0056e-02	3.1489e-03	-2.1932e-03	-9.8827e-03	2.8424e-03	-2.0566e-03
2.1520e-04	-4.2125e-06	2.3458e-04	2.5094e-04	-3.6739e-05	2.3861e-04
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-1.0082e-02	3.1600e-03	-2.2027e-03	-9.9064e-03	2.8504e-03	-2.0649e-03
2.1454e-04	-1.4894e-05	2.1534e-04	2.4319e-04	-4.0240e-05	2.1760e-04
-1.0792e-02	4.0183e-03	6.4827e-03	-1.0435e-02	3.7753e-03	6.4109e-03
-1.0105e-02	3.1692e-03	-2.2115e-03	-9.9273e-03	2.8572e-03	-2.0727e-03
2.0609e-04	-2.1913e-05	1.9417e-04	2.2863e-04	-4.1205e-05	1.9506e-04
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-1.0911e-02	4.0216e-03	6.3755e-03	-1.0573e-02	3.7960e-03	6.3002e-03
-1.0141e-02	3.1824e-03	-2.2265e-03	-9.9617e-03	2.8674e-03	-2.0866e-03
1.7022e-04	-2.5890e-05	1.4792e-04	1.8374e-04	-3.6618e-05	1.4731e-04
-1.0974e-02	4.0254e-03	6.3231e-03	-1.0644e-02	3.8076e-03	6.2462e-03
-1.0155e-02	3.1865e-03	-2.2328e-03	-9.9752e-03	2.8710e-03	-2.0926e-03
1.4423e-04	-2.2892e-05	1.2384e-04	1.5506e-04	-3.1350e-05	1.2316e-04
-1.1001e-02	4.0287e-03	6.3085e-03	-1.0675e-02	3.8131e-03	6.2309e-03
-1.0161e-02	3.1893e-03	-2.2325e-03	-9.9804e-03	2.8729e-03	-2.0921e-03

1.0594e-04 -1.6058e-05 9.2121e-05 1.1438e-04 -2.2795e-05 9.1767e-05

Columns 67 through 72

4.6497e-05	0	0	0	0	0	7.3374e-03	0
-9.4813e-05	0	0	0	0	0	8.764e-01	0
-1.6287e-05	0	0	0	0	0	9.017e-03	0
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-5.4899e-05	0	0	0	0	0	1.236e-03	0
-4.2377e-05	0	0	0	0	0	1.258e-03	0
-1.7401e-05	0	0	0	0	0	1.280e-03	0
-5.5133e-05	0	0	0	0	0	1.302e-03	0
-1.2387e-05	0	0	0	0	0	1.324e-03	0
-1.7401e-05	0	0	0	0	0	1.346e-03	0
-5.5389e-05	0	0	0	0	0	1.368e-03	0
-1.0432e-05	0	0	0	0	0	1.390e-03	0
-1.7387e-05	0	0	0	0	0	1.412e-03	0
-5.5541e-05	0	0	0	0	0	1.434e-03	0
-2.2397e-05	0	0	0	0	0	1.456e-03	0
-1.7331e-05	0	0	0	0	0	1.478e-03	0
-5.5741e-05	0	0	0	0	0	1.500e-03	0
-2.2643e-05	0	0	0	0	0	1.522e-03	0
-1.7279e-05	0	0	0	0	0	1.544e-03	0
-5.5899e-05	0	0	0	0	0	1.566e-03	0
-1.8073e-05	0	0	0	0	0	1.588e-03	0
-1.7270e-05	0	0	0	0	0	1.610e-03	0
-5.6038e-05	0	0	0	0	0	1.632e-03	0
-1.4911e-05	0	0	0	0	0	1.654e-03	0
-1.7244e-05	0	0	0	0	0	1.676e-03	0
-5.6159e-05	0	0	0	0	0	1.698e-03	0
-1.3154e-05	0	0	0	0	0	1.720e-03	0
-1.7228e-05	0	0	0	0	0	1.742e-03	0
-5.6262e-05	0	0	0	0	0	1.764e-03	0
-5.7742e-05	0	0	0	0	0	1.786e-03	0
-1.7203e-05	0	0	0	0	0	1.808e-03	0
-5.6358e-05	0	0	0	0	0	1.830e-03	0
-7.7588e-05	0	0	0	0	0	1.852e-03	0
-1.7183e-05	0	0	0	0	0	1.874e-03	0
-5.6471e-05	0	0	0	0	0	1.896e-03	0
-6.1015e-05	0	0	0	0	0	1.918e-03	0
-1.7158e-05	0	0	0	0	0	1.940e-03	0
-5.6579e-05	0	0	0	0	0	1.962e-03	0
-4.7497e-05	0	0	0	0	0	1.984e-03	0
-1.7133e-05	0	0	0	0	0	2.006e-03	0
-5.6675e-05	0	0	0	0	0	2.028e-03	0
-1.7108e-05	0	0	0	0	0	2.050e-03	0
-1.7083e-05	0	0	0	0	0	2.072e-03	0
-5.6771e-05	0	0	0	0	0	2.094e-03	0
-1.7058e-05	0	0	0	0	0	2.116e-03	0
-5.6867e-05	0	0	0	0	0	2.138e-03	0
-1.7033e-05	0	0	0	0	0	2.160e-03	0
-5.6963e-05	0	0	0	0	0	2.182e-03	0
-1.7008e-05	0	0	0	0	0	2.204e-03	0
-5.7059e-05	0	0	0	0	0	2.226e-03	0
-1.6983e-05	0	0	0	0	0	2.248e-03	0
-5.7155e-05	0	0	0	0	0	2.270e-03	0
-1.6958e-05	0	0	0	0	0	2.292e-03	0
-5.7251e-05	0	0	0	0	0	2.314e-03	0
-1.6933e-05	0	0	0	0	0	2.336e-03	0
-5.7347e-05	0	0	0	0	0	2.358e-03	0
-1.6908e-05	0	0	0	0	0	2.380e-03	0
-5.7443e-05	0	0	0	0	0	2.402e-03	0
-1.6883e-05	0	0	0	0	0	2.424e-03	0
-5.7539e-05	0	0	0	0	0	2.446e-03	0
-1.6858e-05	0	0	0	0	0	2.468e-03	0
-5.7635e-05	0	0	0	0	0	2.490e-03	0
-1.6833e-05	0	0	0	0	0	2.512e-03	0
-5.7731e-05	0	0	0	0	0	2.534e-03	0
-1.6808e-05	0	0	0	0	0	2.556e-03	0
-5.7827e-05	0	0	0	0	0	2.578e-03	0
-1.6783e-05	0	0	0	0	0	2.600e-03	0
-5.7923e-05	0	0	0	0	0	2.622e-03	0
-1.6758e-05	0	0	0	0	0	2.644e-03	0
-5.8019e-05	0	0	0	0	0	2.666e-03	0
-1.6733e-05	0	0	0	0	0	2.688e-03	0
-5.8115e-05	0	0	0	0	0	2.710e-03	0
-1.6708e-05	0	0	0	0	0	2.732e-03	0
-5.8211e-05	0	0	0	0	0	2.754e-03	0
-1.6683e-05	0	0	0	0	0	2.776e-03	0
-5.8307e-05	0	0	0	0	0	2.798e-03	0
-1.6658e-05	0	0	0	0	0	2.820e-03	0
-5.8403e-05	0	0	0	0	0	2.842e-03	0
-1.6633e-05	0	0	0	0	0	2.864e-03	0
-5.8499e-05	0	0	0	0	0	2.886e-03	0
-1.6608e-05	0	0	0	0	0	2.908e-03	0
-5.8595e-05	0	0	0	0	0	2.930e-03	0
-1.6583e-05	0	0	0	0	0	2.952e-03	0
-5.8691e-05	0	0	0	0	0	2.974e-03	0
-1.6558e-05	0	0	0	0	0	2.996e-03	0
-5.8787e-05	0	0	0	0	0	3.018e-03	0
-1.6533e-05	0	0	0	0	0	3.040e-03	0
-5.8883e-05	0	0	0	0	0	3.062e-03	0
-1.6508e-05	0	0	0	0	0	3.084e-03	0
-5.8979e-05	0	0	0	0	0	3.106e-03	0
-1.6483e-05	0	0	0	0	0	3.128e-03	0
-5.9075e-05	0	0	0	0	0	3.150e-03	0
-1.6458e-05	0	0	0	0	0	3.172e-03	0
-5.9171e-05	0	0	0	0	0	3.194e-03	0
-1.6433e-05	0	0	0	0	0	3.216e-03	0
-5.9267e-05	0	0	0	0	0	3.238e-03	0
-1.6408e-05	0	0	0	0	0	3.260e-03	0
-5.9363e-05	0	0	0	0	0	3.282e-03	0
-1.6383e-05	0	0	0	0	0	3.304e-03	0
-5.9459e-05	0	0	0	0	0	3.326e-03	0
-1.6358e-05	0	0	0	0	0	3.348e-03	0
-5.9555e-05	0	0	0	0	0	3.370e-03	0
-1.6333e-05	0	0	0	0	0	3.392e-03	0
-5.9651e-05	0	0	0	0	0	3.414e-03	0
-1.6308e-05	0	0	0	0	0	3.436e-03	0
-5.9747e-05	0	0	0	0	0	3.458e-03	0
-1.6283e-05	0	0	0	0	0	3.480e-03	0
-5.9843e-05	0	0	0	0	0	3.502e-03	0
-1.6258e-05	0	0	0	0	0	3.524e-03	0
-5.9939e-05	0	0	0	0	0	3.546e-03	0
-1.6233e-05	0	0	0	0	0	3.568e-03	0
-6.0035e-05	0	0	0	0	0	3.590e-03	0
-1.6208e-05	0	0	0	0	0	3.612e-03	0
-6.0131e-05	0	0	0	0	0	3.634e-03	0
-1.6183e-05	0	0	0	0	0	3.656e-03	0
-6.0227e-05	0	0	0	0	0	3.678e-03	0
-1.6158e-05	0	0	0	0	0	3.700e-03	0
-6.0323e-05	0	0	0	0	0	3.722e-03	0
-1.6133e-05	0	0	0	0	0	3.744e-03	0
-6.0419e-05	0	0	0	0	0	3.766e-03	0
-1.6108e-05	0	0	0	0	0	3.788e-03	0
-6.0515e-05	0	0	0	0	0	3.810e-03	0
-1.6083e-05	0	0	0	0	0	3.832e-03	0
-6.0611e-05	0	0	0	0	0	3.854e-03	0
-1.6058e-05	0	0	0	0	0	3.876e-03	0
-6.0707e-05	0	0	0	0	0	3.898e-03	0
-1.6033e-05	0	0	0	0	0	3.920e-03	0
-6.0803e-05	0	0	0	0	0	3.942e-03	0
-1.6008e-05	0	0	0	0	0	3.964e-03	0
-6.0899e-05	0	0	0	0	0	3.986e-03	0
-1.5983e-05	0	0	0	0	0	4.008e-03	0
-6.0995e-05	0	0	0	0	0	4.030e-03	0
-1.5958e-05	0	0	0	0	0	4.052e-03	0
-6.1091e-05	0	0	0	0	0	4.074e-03	0
-1.5933e-05	0	0	0	0	0	4.096e-03	0
-6.1187e-05	0	0	0	0	0	4.118e-03	0
-1.5908e-05	0	0	0	0	0	4.140e-03	0
-6.1283e-05	0	0	0	0	0	4.162e-03	0
-1.5883e-05	0	0	0	0	0	4.184e-03	0
-6.1379e-05	0	0	0	0	0	4.206e-03	0
-1.5858e-05	0	0	0	0	0	4.228e-03	0
-6.1475e-05	0	0	0	0	0	4.250e-03	0
-1.5833e-05	0	0	0	0	0	4.272e-03	0

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0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
9.6487e-01	4.6593e-02	-3.6833e-03	-1.3764e-02	4.1251e-02	-7.7337e-03
-9.3953e-04	9.8239e-01	6.2511e-03	2.1137e-03	-6.3348e-03	1.1876e-03
-1.6381e-02	4.2561e-02	9.8221e-01	-1.1447e-02	3.4306e-02	-6.4317e-03
-1.7752e-02	-2.6840e-04	1.6596e-02	9.6862e-01	2.8343e-02	8.2018e-03
-5.4113e-03	-5.5744e-03	1.0346e-03	-3.3095e-03	9.8569e-01	4.8738e-03
-6.5054e-04	1.6192e-04	5.6986e-04	-8.7651e-03	2.7272e-02	9.8874e-01
-1.7672e-02	-2.0490e-04	1.6589e-02	-2.2456e-02	-1.5107e-03	2.1396e-02
-5.4375e-03	-5.6020e-03	1.0309e-03	-5.6003e-03	-6.6705e-03	1.4867e-03
-5.7227e-04	2.3229e-04	5.7052e-04	-6.8869e-04	2.8183e-04	6.7967e-04
-1.7594e-02	-1.6280e-04	1.6566e-02	-2.2376e-02	-1.4658e-03	2.1365e-02
-5.4637e-03	-5.6264e-03	1.0302e-03	-5.6249e-03	-6.6965e-03	1.4864e-03
-4.9538e-04	2.8610e-04	5.5747e-04	-6.1198e-04	3.3924e-04	6.6236e-04
-1.7523e-02	-1.4214e-04	1.6529e-02	-2.2298e-02	-1.4442e-03	2.1320e-02
-5.4893e-03	-5.6481e-03	1.0314e-03	-5.6493e-03	-6.7197e-03	1.4882e-03
-4.2372e-04	3.2425e-04	5.3482e-04	-5.3814e-04	3.7974e-04	6.3428e-04
-1.7461e-02	-1.4121e-04	1.6483e-02	-2.2227e-02	-1.4440e-03	2.1265e-02
-5.5134e-03	-5.6677e-03	1.0335e-03	-5.6723e-03	-6.7404e-03	1.4907e-03
-3.5988e-04	3.4871e-04	5.0669e-04	-4.7073e-04	4.0530e-04	6.0034e-04
-1.7409e-02	-1.5780e-04	1.6431e-02	-2.2165e-02	-1.4631e-03	2.1203e-02
-5.5359e-03	-5.6858e-03	1.0355e-03	-5.6936e-03	-6.7596e-03	1.4931e-03
-3.0435e-04	3.6215e-04	4.7608e-04	-4.1088e-04	4.1875e-04	5.6397e-04
-1.7367e-02	-1.8958e-04	1.6375e-02	-2.2110e-02	-1.4990e-03	2.1136e-02
-5.5562e-03	-5.7028e-03	1.0369e-03	-5.7124e-03	-6.7776e-03	1.4944e-03
-2.5707e-04	3.6655e-04	4.4478e-04	-3.5882e-04	4.2223e-04	5.2712e-04
-1.7331e-02	-2.3335e-04	1.6315e-02	-2.2059e-02	-1.5484e-03	2.1065e-02
-5.5741e-03	-5.7190e-03	1.0372e-03	-5.7287e-03	-6.7948e-03	1.4943e-03
-2.1645e-04	3.6384e-04	4.1327e-04	-3.1309e-04	4.1783e-04	4.9022e-04
-1.7299e-02	-2.8602e-04	1.6253e-02	-2.2012e-02	-1.6080e-03	2.0990e-02
-5.5899e-03	-5.7347e-03	1.0365e-03	-5.7425e-03	-6.8114e-03	1.4928e-03
-1.8079e-04	3.5546e-04	3.8149e-04	-2.7197e-04	4.0709e-04	4.5308e-04
-1.7270e-02	-3.4597e-04	1.6189e-02	-2.1967e-02	-1.6761e-03	2.0913e-02
-5.6038e-03	-5.7499e-03	1.0348e-03	-5.7544e-03	-6.8276e-03	1.4903e-03
-1.4911e-04	3.4247e-04	3.4954e-04	-2.3451e-04	3.9119e-04	4.1577e-04
-1.7244e-02	-4.1189e-04	1.6123e-02	-2.1924e-02	-1.7512e-03	2.0834e-02
-5.6159e-03	-5.7645e-03	1.0323e-03	-5.7643e-03	-6.8433e-03	1.4866e-03
-1.2154e-04	3.2512e-04	3.1768e-04	-2.0082e-04	3.7044e-04	3.7850e-04
-1.7220e-02	-4.8232e-04	1.6057e-02	-2.1884e-02	-1.8316e-03	2.0755e-02
-5.6262e-03	-5.7786e-03	1.0291e-03	-5.7724e-03	-6.8584e-03	1.4822e-03
-9.7742e-05	3.0393e-04	2.8597e-04	-1.7057e-04	3.4544e-04	3.4130e-04
-1.7200e-02	-5.5629e-04	1.5992e-02	-2.1846e-02	-1.9162e-03	2.0676e-02
-5.6350e-03	-5.7919e-03	1.0255e-03	-5.7789e-03	-6.8729e-03	1.4772e-03
-7.7588e-05	2.7949e-04	2.5464e-04	-1.4365e-04	3.1692e-04	3.0442e-04
-1.7183e-02	-6.3218e-04	1.5927e-02	-2.1811e-02	-2.0031e-03	2.0598e-02
-5.6421e-03	-5.8044e-03	1.0214e-03	-5.7839e-03	-6.8866e-03	1.4718e-03
-6.1015e-05	2.5203e-04	2.2373e-04	-1.1999e-04	2.8517e-04	2.6785e-04
-1.7168e-02	-7.0918e-04	1.5864e-02	-2.1779e-02	-2.0913e-03	2.0521e-02
-5.6479e-03	-5.8161e-03	1.0172e-03	-5.7877e-03	-6.8996e-03	1.4662e-03
-4.7497e-05	2.2247e-04	1.9340e-04	-9.9166e-05	2.5126e-04	2.3183e-04
-1.7155e-02	-7.8656e-04	1.5802e-02	-2.1749e-02	-2.1800e-03	2.0446e-02
-5.6525e-03	-5.8270e-03	1.0129e-03	-5.7904e-03	-6.9117e-03	1.4606e-03
-3.7205e-05	1.9162e-04	1.6424e-04	-8.1502e-05	2.1611e-04	1.9706e-04
-1.7145e-02	-8.6336e-04	1.5744e-02	-2.1723e-02	-2.2682e-03	2.0375e-02
-5.6560e-03	-5.8369e-03	1.0087e-03	-5.7921e-03	-6.9230e-03	1.4551e-03
-2.9957e-05	1.6035e-04	1.3670e-04	-6.6982e-05	1.8074e-04	1.6409e-04
-1.7151e-02	-8.9090e-04	1.5731e-02	-2.1723e-02	-2.3005e-03	2.0358e-02
-5.6610e-03	-5.8384e-03	1.0118e-03	-5.7974e-03	-6.9248e-03	1.4584e-03
-2.3267e-05	1.1933e-04	1.0235e-04	-5.0922e-05	1.3456e-04	1.2285e-04

Columns 73 through 78

-1.2154e-04	0	4.2561e-02	0	-9.2071e-05	0	-1.1919e-04	0	1.3456e-04	0	-7.7337e-03	0
1.8958e-04	0	-3.4121e-04	0	1.4121e-04	0	1.8222e-04	0	0	0	1.1876e-03	0













-8.0014e-03	3.2084e-02	-5.8348e-03	-7.5599e-03	3.1621e-02	-5.6355e-03
-9.6239e-03	3.8500e-02	-7.0893e-03	-9.0949e-03	3.7942e-02	-6.8437e-03
1.4113e-03	-5.9255e-03	1.0333e-03	1.3332e-03	-5.8260e-03	9.9183e-04
-7.9493e-03	3.2045e-02	-5.8339e-03	-7.5116e-03	3.1570e-02	-5.6278e-03
9.6835e-01	2.4327e-02	1.2067e-02	-9.0540e-03	3.7868e-02	-6.8639e-03
-4.0202e-03	9.8658e-01	4.4355e-03	1.3174e-03	-5.8167e-03	9.8185e-04
-6.4313e-03	2.3879e-02	9.8998e-01	-7.4708e-03	3.1519e-02	-5.6260e-03
-2.5090e-02	-4.3396e-04	2.3096e-02	9.6794e-01	2.3575e-02	1.3008e-02
-6.0469e-03	-5.8064e-03	1.0150e-03	-4.1650e-03	9.8675e-01	4.3571e-03
-2.7385e-04	6.8354e-04	3.4520e-04	-6.0455e-03	2.3250e-02	9.9019e-01
-2.5055e-02	-5.1253e-04	2.3051e-02	-2.5883e-02	-4.6730e-04	2.3748e-02
-6.0588e-03	-5.8039e-03	1.0061e-03	-6.0873e-03	-5.7970e-03	9.9595e-04
-2.4204e-04	6.4056e-04	3.3356e-04	-2.3996e-04	6.9282e-04	3.0545e-04
-2.5020e-02	-5.9858e-04	2.3000e-02	-2.5850e-02	-5.5985e-04	2.3702e-02
-6.0685e-03	-5.8031e-03	9.9666e-04	-6.0964e-03	-5.7934e-03	9.8558e-04
-2.1351e-04	5.9584e-04	3.1902e-04	-2.1288e-04	6.4142e-04	2.9457e-04
-2.4985e-02	-6.9180e-04	2.2943e-02	-2.5816e-02	-6.5930e-04	2.3648e-02
-6.0766e-03	-5.8039e-03	9.8675e-04	-6.1039e-03	-5.7918e-03	9.7485e-04
-1.8729e-04	5.4933e-04	3.0203e-04	-1.8773e-04	5.8887e-04	2.8073e-04
-2.4950e-02	-7.9102e-04	2.2881e-02	-2.5781e-02	-7.6454e-04	2.3590e-02
-6.0829e-03	-5.8062e-03	9.7652e-04	-6.1097e-03	-5.7920e-03	9.6391e-04
-1.6323e-04	5.0142e-04	2.8263e-04	-1.6442e-04	5.3553e-04	2.6407e-04
-2.4915e-02	-8.9508e-04	2.2815e-02	-2.5745e-02	-8.7461e-04	2.3527e-02
-6.0878e-03	-5.8099e-03	9.6611e-04	-6.1141e-03	-5.7942e-03	9.5287e-04
-1.4102e-04	4.5239e-04	2.6106e-04	-1.4266e-04	4.8160e-04	2.4487e-04
-2.4881e-02	-1.0029e-03	2.2747e-02	-2.5710e-02	-9.8851e-04	2.3461e-02
-6.0915e-03	-5.8151e-03	9.5571e-04	-6.1172e-03	-5.7981e-03	9.4188e-04
-1.2063e-04	4.0294e-04	2.3768e-04	-1.2249e-04	4.2777e-04	2.2358e-04
-2.4847e-02	-1.1127e-03	2.2678e-02	-2.5674e-02	-1.1046e-03	2.3393e-02
-6.0938e-03	-5.8216e-03	9.4553e-04	-6.1191e-03	-5.8038e-03	9.3118e-04
-1.0206e-04	3.5350e-04	2.1257e-04	-1.0395e-04	3.7442e-04	2.0035e-04
-2.4814e-02	-1.2240e-03	2.2608e-02	-2.5639e-02	-1.2227e-03	2.3325e-02
-6.0952e-03	-5.8295e-03	9.3574e-04	-6.1198e-03	-5.8113e-03	9.2092e-04
-8.5231e-05	3.0483e-04	1.8641e-04	-8.7023e-05	3.2226e-04	1.7592e-04
-2.4782e-02	-1.3360e-03	2.2539e-02	-2.5605e-02	-1.3422e-03	2.3258e-02
-6.0956e-03	-5.8387e-03	9.2656e-04	-6.1195e-03	-5.8207e-03	9.1135e-04
-7.0573e-05	2.5827e-04	1.5986e-04	-7.2182e-05	2.7269e-04	1.5097e-04
-2.4751e-02	-1.4493e-03	2.2472e-02	-2.5571e-02	-1.4641e-03	2.3193e-02
-6.0949e-03	-5.8496e-03	9.1825e-04	-6.1182e-03	-5.8323e-03	9.0280e-04
-5.8264e-05	2.1478e-04	1.3357e-04	-5.9632e-05	2.2669e-04	1.2616e-04
-2.4743e-02	-1.4944e-03	2.2450e-02	-2.5560e-02	-1.5151e-03	2.3171e-02
-6.0972e-03	-5.8504e-03	9.1778e-04	-6.1196e-03	-5.8338e-03	9.0190e-04
-4.4344e-05	1.6089e-04	9.9686e-05	-4.5337e-05	1.6989e-04	9.4104e-05

Columns 91 through 96

-8.1750e-03	0	0	0	0	0
-9.6239e-03	0	0	0	0	0
1.4113e-03	0	0	0	0	0
-7.9493e-03	0	0	0	0	0
9.6835e-01	0	0	0	0	0
-4.0202e-03	0	0	0	0	0
-6.4313e-03	0	0	0	0	0
-2.5090e-02	0	0	0	0	0
-6.0469e-03	0	0	0	0	0
-2.7385e-04	0	0	0	0	0
-2.5055e-02	0	0	0	0	0
-6.0588e-03	0	0	0	0	0
-2.4204e-04	0	0	0	0	0
-2.5020e-02	0	0	0	0	0
-6.0685e-03	0	0	0	0	0
-2.1351e-04	0	0	0	0	0
-2.4985e-02	0	0	0	0	0
-6.0766e-03	0	0	0	0	0
-1.8729e-04	0	0	0	0	0
-2.4950e-02	0	0	0	0	0
-6.0829e-03	0	0	0	0	0
-1.6323e-04	0	0	0	0	0
-2.4915e-02	0	0	0	0	0
-6.0878e-03	0	0	0	0	0
-1.4102e-04	0	0	0	0	0
-2.4881e-02	0	0	0	0	0
-6.0915e-03	0	0	0	0	0
-1.2063e-04	0	0	0	0	0
-2.4847e-02	0	0	0	0	0
-6.0938e-03	0	0	0	0	0
-1.0206e-04	0	0	0	0	0
-2.4814e-02	0	0	0	0	0
-6.0952e-03	0	0	0	0	0
-8.5231e-05	0	0	0	0	0
-2.4782e-02	0	0	0	0	0
-6.0956e-03	0	0	0	0	0
-7.0573e-05	0	0	0	0	0
-2.4751e-02	0	0	0	0	0
-6.0949e-03	0	0	0	0	0
-5.8264e-05	0	0	0	0	0
-2.4743e-02	0	0	0	0	0
-6.0972e-03	0	0	0	0	0
-4.4344e-05	0	0	0	0	0



-7.2608e-03	3.1597e-02	-5.7521e-03	-7.2978e-03	3.2028e-02	-6.0668e-03
-8.7664e-03	3.7888e-02	-7.0383e-03	-8.8129e-03	3.8411e-02	-7.4073e-03
1.2684e-03	-5.8064e-03	9.9091e-04	1.2766e-03	-5.8677e-03	1.0451e-03
-7.2284e-03	3.1537e-02	-5.7498e-03	-7.2663e-03	3.1957e-02	-6.0585e-03
9.6784e-01	2.3491e-02	1.3128e-02	-8.7835e-03	3.8307e-02	-7.4310e-03
-4.1875e-03	9.8680e-01	4.3253e-03	1.2656e-03	-5.8551e-03	1.0332e-03
-5.9433e-03	2.3177e-02	9.9016e-01	-7.2386e-03	3.1889e-02	-6.0556e-03
-2.6029e-02	-5.4665e-04	2.3870e-02	9.6811e-01	2.3857e-02	1.2548e-02
-6.0645e-03	-5.8611e-03	1.0100e-03	-4.1181e-03	9.8675e-01	4.3361e-03
-2.0822e-04	6.9858e-04	2.5477e-04	-6.0648e-03	2.3482e-02	9.8999e-01
-2.5996e-02	-6.5406e-04	2.3823e-02	-2.5587e-02	-5.6443e-04	2.3438e-02
-6.0718e-03	-5.8570e-03	9.9879e-04	-6.0302e-03	-5.8668e-03	1.0064e-03
-1.8400e-04	6.3902e-04	2.4497e-04	-1.7803e-04	6.9633e-04	1.9829e-04
-2.5963e-02	-7.6715e-04	2.3771e-02	-2.5555e-02	-6.8648e-04	2.3394e-02
-6.0776e-03	-5.8553e-03	9.8749e-04	-6.0362e-03	-5.8628e-03	9.9494e-04
-1.6140e-04	5.7929e-04	2.3199e-04	-1.5587e-04	6.2933e-04	1.8979e-04
-2.5928e-02	-8.8513e-04	2.3714e-02	-2.5521e-02	-8.1356e-04	2.3345e-02
-6.0818e-03	-5.8558e-03	9.7620e-04	-6.0408e-03	-5.8617e-03	9.8363e-04
-1.4019e-04	5.1954e-04	2.1622e-04	-1.3509e-04	5.6293e-04	1.7833e-04
-2.5892e-02	-1.0072e-03	2.3653e-02	-2.5487e-02	-9.4502e-04	2.3292e-02
-6.0848e-03	-5.8586e-03	9.6503e-04	-6.0439e-03	-5.8632e-03	9.7256e-04
-1.2044e-04	4.6039e-04	1.9817e-04	-1.1579e-04	4.9773e-04	1.6444e-04
-2.5875e-02	-1.1318e-03	2.3591e-02	-2.5453e-02	-1.0796e-03	2.3238e-02
-6.0863e-03	-5.8637e-03	9.5423e-04	-6.0456e-03	-5.8678e-03	9.6199e-04
-1.0222e-04	4.0222e-04	1.7802e-04	-9.8052e-05	4.3409e-04	1.4834e-04
-2.5821e-02	-1.2590e-03	2.3528e-02	-2.5417e-02	-1.2178e-03	2.3183e-02
-6.0867e-03	-5.8711e-03	9.4393e-04	-6.0459e-03	-5.8752e-03	9.5205e-04
-8.5577e-05	3.4567e-04	1.5660e-04	-8.1912e-05	3.7256e-04	1.3088e-04
-2.5786e-02	-1.3887e-03	2.3466e-02	-2.5382e-02	-1.3596e-03	2.3129e-02
-6.0860e-03	-5.8810e-03	9.3443e-04	-6.0449e-03	-5.8859e-03	9.4307e-04
-7.0977e-05	2.9224e-04	1.3452e-04	-6.7835e-05	3.1470e-04	1.1262e-04
-2.5750e-02	-1.5222e-03	2.3406e-02	-2.5345e-02	-1.5072e-03	2.3079e-02
-6.0839e-03	-5.8939e-03	9.2612e-04	-6.0424e-03	-5.9005e-03	9.3550e-04
-5.8632e-05	2.4287e-04	1.1242e-04	-5.6001e-05	2.6147e-04	9.4153e-05
-2.5736e-02	-1.5811e-03	2.3385e-02	-2.5328e-02	-1.5762e-03	2.3062e-02
-6.0846e-03	-5.8964e-03	9.2496e-04	-6.0421e-03	-5.9044e-03	9.3437e-04
-4.4552e-05	1.8207e-04	8.3792e-05	-4.2547e-05	1.9607e-04	7.0088e-05

Columns 97 through 102

-8.1645e-03	3.3420e-02	0	0	4.3028e-02	0
-8.7811e-03	-1.0550e-03	0	0	-6.1574e-03	0
-7.7879e-03	-1.2811e-03	0	0	2.3345e-02	0
-2.2582e-02	1.3389e-03	0	0	4.8013e-03	0
1.8080e-03	3.0427e-03	0	0	-8.1757e-03	0
-7.8846e-03	1.2734e-03	0	0	3.2274e-02	0
-2.2000e-02	3.9314e-03	0	0	9.8228e-03	0
1.3889e-03	-4.0220e-03	0	0	1.1722e-02	0
-7.3200e-02	3.2688e-03	0	0	-1.1318e-03	0
-1.4874e-03	3.9314e-03	0	0	1.9832e-03	0
-1.3677e-03	-1.9880e-03	0	0	1.0533e-03	0
-2.3748e-02	1.2802e-03	0	0	1.3099e-03	0
-9.0977e-03	3.9110e-03	0	0	9.9990e-03	0
1.3500e-03	-5.9744e-03	0	0	-0.0547e-03	0
-2.3761e-03	1.2510e-03	0	0	1.2901e-03	0
-9.0567e-03	2.9001e-03	0	0	3.8972e-03	0
-2.4147e-02	-3.2220e-03	0	0	-0.2888e-03	0
-7.4834e-03	3.2430e-03	0	0	1.8888e-03	0
-9.0217e-03	3.8891e-03	0	0	3.2449e-03	0
1.2188e-03	-8.9282e-03	0	0	-8.0020e-03	0
-7.4489e-03	1.2334e-03	0	0	3.2790e-03	0
1.3911e-03	3.8780e-03	0	0	3.3326e-03	0
1.3087e-03	-8.9087e-03	0	0	-1.7885e-03	0
-7.4153e-03	3.2783e-03	0	0	1.2696e-03	0
8.9822e-03	3.8660e-03	0	0	3.2618e-03	0
1.2845e-03	-4.8924e-03	0	0	-8.9588e-03	0



-7.3873e-03	3.2174e-02	-6.1941e-03	-7.5528e-03	3.2608e-02	-6.4410e-03
-8.9330e-03	3.8545e-02	-7.6168e-03	-9.1301e-03	3.9069e-02	-7.9010e-03
1.2850e-03	-5.8800e-03	1.0447e-03	1.3170e-03	-5.9431e-03	1.0880e-03
-7.3617e-03	3.2097e-02	-6.1910e-03	-7.5262e-03	3.2521e-02	-6.4330e-03
9.6825e-01	2.4113e-02	1.2147e-02	-9.0996e-03	3.8931e-02	-7.9227e-03
-4.0866e-03	9.8670e-01	4.3584e-03	1.3086e-03	-5.9309e-03	1.0758e-03
-6.1538e-03	2.3714e-02	9.8984e-01	-7.5013e-03	3.2436e-02	-6.4288e-03
-2.5305e-02	-6.5356e-04	2.3175e-02	9.6846e-01	2.4495e-02	1.1576e-02
-6.0109e-03	-5.9227e-03	1.0372e-03	-4.0214e-03	9.8663e-01	4.3867e-03
-1.5687e-04	6.8447e-04	1.4917e-04	-6.3089e-03	2.4057e-02	9.8965e-01
-2.5272e-02	-7.9101e-04	2.3134e-02	-2.4884e-02	-7.5643e-04	2.2799e-02
-6.0157e-03	-5.9200e-03	1.0260e-03	-5.9734e-03	-5.9823e-03	1.0659e-03
-1.3581e-04	6.1098e-04	1.4181e-04	-1.3965e-04	6.6629e-04	1.0159e-04
-2.5238e-02	-9.3338e-04	2.3090e-02	-2.4850e-02	-9.1197e-04	2.2764e-02
-6.0190e-03	-5.9206e-03	1.0152e-03	-5.9769e-03	-5.9821e-03	1.0554e-03
-1.1629e-04	5.3929e-04	1.3190e-04	-1.1955e-04	5.8720e-04	9.6168e-05
-2.5203e-02	-1.0797e-03	2.3044e-02	-2.4814e-02	-1.0725e-03	2.2728e-02
-6.0207e-03	-5.9249e-03	1.0050e-03	-5.9786e-03	-5.9866e-03	1.0458e-03
-9.8401e-05	4.6971e-04	1.1971e-04	-1.0119e-04	5.1083e-04	8.8354e-05
-2.5167e-02	-1.2307e-03	2.2997e-02	-2.4777e-02	-1.2392e-03	2.2692e-02
-6.0209e-03	-5.9329e-03	9.9562e-04	-5.9785e-03	-5.9955e-03	1.0373e-03
-8.2165e-05	4.0275e-04	1.0608e-04	-8.4543e-05	4.3763e-04	7.8991e-05
-2.5130e-02	-1.3870e-03	2.2953e-02	-2.4738e-02	-1.4132e-03	2.2658e-02
-6.0195e-03	-5.9450e-03	9.8740e-04	-5.9766e-03	-6.0095e-03	1.0302e-03
-6.8033e-05	3.4002e-04	9.1502e-05	-7.0054e-05	3.6929e-04	6.8496e-05
-2.5092e-02	-1.5515e-03	2.2911e-02	-2.4696e-02	-1.5981e-03	2.2629e-02
-6.0163e-03	-5.9618e-03	9.8085e-04	-5.9727e-03	-6.0294e-03	1.0251e-03
-5.6155e-05	2.8247e-04	7.6531e-05	-5.7833e-05	3.0673e-04	5.7361e-05
-2.5071e-02	-1.6330e-03	2.2899e-02	-2.4671e-02	-1.6948e-03	2.2623e-02
-6.0151e-03	-5.9678e-03	9.8008e-04	-5.9705e-03	-6.0381e-03	1.0250e-03
-4.2617e-05	2.1180e-04	5.6885e-05	-4.3818e-05	2.2995e-04	4.2535e-05

Columns 103 through 108

-8.7872e-03	0	0	0	0	0
-1.4030e-03	0	0	0	0	0
-4.1394e-03	0	0	0	0	0
-7.9081e-03	0	0	0	0	0
-1.4790e-03	0	0	0	0	0
-4.8505e-03	0	0	0	0	0
-9.6410e-03	0	0	0	0	0
-1.4875e-03	0	0	0	0	0
-7.9505e-03	0	0	0	0	0
-9.3894e-03	0	0	0	0	0
1.4490e-03	0	0	0	0	0
-7.9475e-03	0	0	0	0	0
-9.7471e-03	0	0	0	0	0
-1.4220e-03	0	0	0	0	0
-7.9000e-03	0	0	0	0	0
-1.4802e-03	0	0	0	0	0
-1.4069e-03	0	0	0	0	0
-7.9571e-03	0	0	0	0	0
-1.4535e-03	0	0	0	0	0
-1.3913e-03	0	0	0	0	0
-7.8100e-03	0	0	0	0	0
-9.4307e-03	0	0	0	0	0
-1.3282e-03	0	0	0	0	0
-7.7853e-03	0	0	0	0	0
-1.4007e-03	0	0	0	0	0
-1.3618e-03	0	0	0	0	0
-9.7551e-03	0	0	0	0	0
-9.3695e-03	0	0	0	0	0
-1.3833e-03	0	0	0	0	0
-7.7271e-03	0	0	0	0	0
-9.2778e-03	0	0	0	0	0
-1.3470e-03	0	0	0	0	0





















```
function [lam]=ysu41
% The Stability region of the SRP-2 method
%
% Faramarz Mossayebi
% 1/12/1990
%
j=sqrt(-1);
theta=0:0.05:2.05*pi;
n=length(theta);
for i=1:n
    z=exp(j*theta(i));
    lam(i)=(z*z-1.9*z+.9)/(0.19*z-.09);
end;
plot(real(lam),imag(lam))

% plot the unit circle
% plot the stability region
% plot the real and imaginary parts
% plot the real and imaginary parts
% plot the real and imaginary parts
% plot the real and imaginary parts
% plot the real and imaginary parts
% plot the real and imaginary parts
```

```
function [r1,r2]=ysu32(maxgain)
% The root locus of the AB-2 method
% maxgain = Maximum gain desirable
%
% Faramarz Mossayebi
% 1/12/1990
%
r1=[];
for k=0:0.01:maxgain,
    rr=[1 -1 0]+k*[0 1.5 -.5];
    r1=[r1 roots(rr)];
end;
axis('square');
axis([-1.5 1.5 -1.5 1.5]);
r2=0:0.01:2.05*pi;
si=sin(r2);
ci=cos(r2);
% Plot the unit circle
plot(si,ci);
hold;
% plot(real(r1),imag(r1))
x1=[1.0 1/3];y1=[0 0];
x2=[0.0 -1.2];
plot(x1,y1,x2,y1)
```

```

function [lam]=ysu41
% The Stability region of the SRP-2 method
%
% Faramarz Mossayebi
% 1/12/1990
%
j=sqrt(-1);
theta=0:0.05:2.05*pi;
n=length(theta);
for i=1:n
    z=exp(j*theta(i));
    lam(i)=(z*z-1.9*z+.9)/(0.19*z-.09);
end;
plot(real(lam),imag(lam))

```

```

function [y,t]=ab2inlet(T);
% SIMULATION OF 40-60 INLET BY AB-2 METHOD
% T =Time Step of Integration
%
% [y,t]=ab2ysu(T)
%
ts=1e-6;
flops(0);
load inlabc
n=size(an);
t=0:T:.002;
u=zeros(t);
t2=.002+T:T:.05;
u2=-ones(t2);
u=[u u2];
t=[t t2];
nn=length(u);
xk=zeros(n,1);
y=[];
yk=cn*xk;
xdk=an*xk;
xkpl=xk+ts*xdk;
ykpl=cn*xkpl;
y=[y yk ykpl];
disp('----- Simulating the 40-60 Inlet by AB-2 -----')
%k=0;
%***** Main loop
for nn=3:1:nn
    xdkpl=an*xkpl+bn*u(nn);
    xkp2=xkpl+T*.5*(3*xdkpl-xdk);
    ykp2=cn*xkp2;
    xdk=xdkpl;
    xkpl=xkp2;
    y=[y ykp2];
end;
flops

```

```

function [y,t,numflop]=milmsrp(T)
% This program performs the simulation of the 40-60 inlet
% using the MSRP2 method.
% T = Integration timestep
% y = Outputs
%
% [y,t,numflop]=milmsrp(T)
%
% Faramarz Mossayebi
% 11/20/89
%
flops(0);
load c:\f-m\inlabc
n=size(an);
t=0:T:.002;
u=zeros(t);
t2=.002+T:T:.05;
u2=-ones(t2);
u=[u u2];
t=[t t2];
nn=length(u);
xk=zeros(n,1);
y=[];
yk=cn*xk;
% compute the Regression Coefficients
[a0,a1,b0,b1]=msrp2(an,T);
% Start the algorithm by euler's
xdk=an*xk+bn*u(1);
xkp1=xk+.000001*xdk;
ykp1=cn*xkp1;
y=[y yk ykp1];
disp('—— Simulating the 40-60 Inlet ——')
%***** Main loop
for nn=3:1:nn
    xdkp1=an*xkp1+bn*u(nn);
    xkp2=-a1*xkp1-a0*xk+T*(b1*xdkp1+b0*xdk);
    ykp2=cn*xkp2;
    xk=xkp1;
    xdk=xdkp1;
    xkp1=xkp2;
    y=[y ykp2];
end;
numflop=flops;

```

```

function [a0,a1,b0,b1,numflops]=msrp2(A,T)
% This routine calculates the coefficients of the 2-step matrix
% stability region placement method.
% A = System's dynamic matrix
% T = Simulation timestep
% numflops = Number of flops
%
% [a0,a1,b0,b1,numflops]=msrp2(A,T)
%
% Faramarz Mossayebi
% 10/10/89
%
if nargin ==1,
    T=1;
end;
flops(0);
AT=A*T;
ATi=inv(AT);
I=eye(A);
exAT=expm(AT);
a0=I+ATi-exAT*ATi;
a1=-I-a0;
b0=a0*ATi;
b1=(a1+exAT)*ATi;
numflops=flops;

```

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