MATHEMATICAL MODELING OF A MOVING SOLIDIFICATION BOUNDARY OF CONTINUOUS CAST ROUNDS

by

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YOUNGSTOWN STATE UNIVERSITY

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OF CONTINUOUS CAST ROUNDS

Chidchai Loyprasert

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ABSTRACT

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Continuous casting of rounds is an economic and efficient process and is standard industrial practice. Mathematical modeling for the continuous casting of slabs is well documented, but similar modeling has not been developed for casting of rounds. In this paper a mathematical model for the solidification of continuous cast rounds is presented. A computer program has been developed that determines the thickness of the solidified layer as a function of time for any diameter round.

ACKNOWLEDGMENTS

This thesis project could not have come to this point without the support and love of my mother and my father.

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LIST OF SYMBOLS

SYMBOL	DEFINITION	UNIT OR REFERENCE
T_l	The temperature of liquid phase	$°F$
T_s	The temperature of solid phase	$°F$
$T_{l,0}$	The temperature of liquid at time zero	$°F$
$T_{s,0}$	The temperature of cooling agent	$°F$
T_{mp}	The melting point of metal	$°F$
r	The radial distance from the center of a cylinder	ft
R	The radius of cylinder	ft
$R(t)$	The distance from center to solid-liquid interface	ft
α_l	thermal diffusivity of liquid	$\frac{ft^2}{hr}$
α_s	thermal diffusivity of solid	$\frac{ft^2}{hr}$
k_l	thermal conductivity of liquid	$\frac{Btu}{hr\ ft\ °F}$
k_s	thermal conductivity of solid	$\frac{Btu}{hr\ ft\ °F}$
ρ	density of a solid phase	$\frac{lb}{ft^3}$
ΔH	latent heat of solidification per unit mass of liquid	$\frac{Btu}{lb}$

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CHAPTER I

INTRODUCTION

Traditionally the metal working process required to produce solid round stock (referred to in the trade as "Merchant Bars") was a complex multi-step operation. The starting material in the process was a cast ingot with a square cross-section. The ingot was 28 to 54 inches on a side and 6 to 8 feet long. This ingot is hot-rolled in a blooming mill to produce a billet. The billet which still has a square cross-section is considered a semi-finished product. It is used to produce sheets, plates, and bar stock. In order to reduce the billet from 32 x 32 inches to 4 x 4 inches requires rolling the material through 27 separate blooming mills. The cross-section geometry is then changed from square (or rectangular) to circular and final dimensions (diameter) by rolling the billet in a continuous bar-mill. In order to produce a 3/4 inch diameter bar from a 4 x 4 billet requires at least 16 stands (individual rolling units) in the bar-mill, (see Fig 1.1 [(1)]). Thus, to produce one merchant bar can require at least 40 rolling operations after casting the ingot. This traditional process was made totally obsolete with the development of the continuous casting process for slabs, plated, and bar stock.

1.1 Continuous Casting

Continuous casting is an efficient and economic process developed to replace ingot casting and produced higher quality steels at reduced cost. Continuous casting of metal was first developed in the United States and Europe in the mid-1800's [2]. Commercial production started in the early 1960's [2]. A continuous casting system, shown in Fig 1.2 [3], consists of a ladle or liquid metal reservoir, a tundish or pouring system, a water-cooled mold, a cooling system, a driving system (rollers), and cutting devices. Nitrogen gas is bubbled through the molten metal for 5 to 10 minutes in order to clean the metal and stabilize the metal's temperature. Then, the metal is poured into the tundish where solid impurities are removed by filters. When the molten metal flows through the water-cooled mold, it begins to solidify. A roller system pulls the solidified stock from the mold. To start a casting process, a dummy bar or a solid starter is inserted to the bottom of the mold and the molten metal solidifies on the dummy bar. The withdrawal speed of the dummy bar is based on the pouring rate of the molten metal and solidification time of metal. At the end of the mold the stock must have a solidified shell of at least 12-18 mm [4], in order to support its own weight. This constraint limits the withdrawal rate to about 25 mm/sec [4]. The cooling system provides cooling water for the metal to solidify completely. The solidified metal is cut to the desired length by shearing or torch cutting. After cutting, the stock is ready for any needed finishing operations such as hot/cold rolling or heat treatment. Generally any finish rolling requires no more than 6-12 stands as compared to the 40-60 stands needed in the traditional process.

1.2 Continuous Casting or Rounds

The first production casting of rounds was done by Eschweiler Bergwerks in Germany in 4-stand machine in 1965 [5]. Recently, a 6-stand rounds caster was used to produce rounds with diameter ranging from 100 to 400 mm [6]. The most important applications of rounds cast is a seamless tube. The basic process of continuous casting of rounds is similar to slab continuous casting. The casting speed is usually from 1.2 to 2.9 m/min [7], with a maximum speed is 17 m/s [8].

1.3 Solidification of Rounds

The casting speed of the continuous casting depends on the thickness of solidified shell of the metal. The thickness of solidified shell can be predicted by using the Fourier's heat equation. The problem begins with unsteady state heat conduction through the liquid and solid phase. Boundary conditions are (1) the temperatures of the liquid and solid phases are equal at the solid-liquid interface, and (2) the conservation of thermal energy at the phase boundary. The solidified shell grows from the outer surface to the core of round. The temperature of the outer shell is kept at the constant using cooling water. The melting point of the metal is the temperature that molten metal solidifies. From this information, the thickness of the solidified shell can be predicted at any particular time.

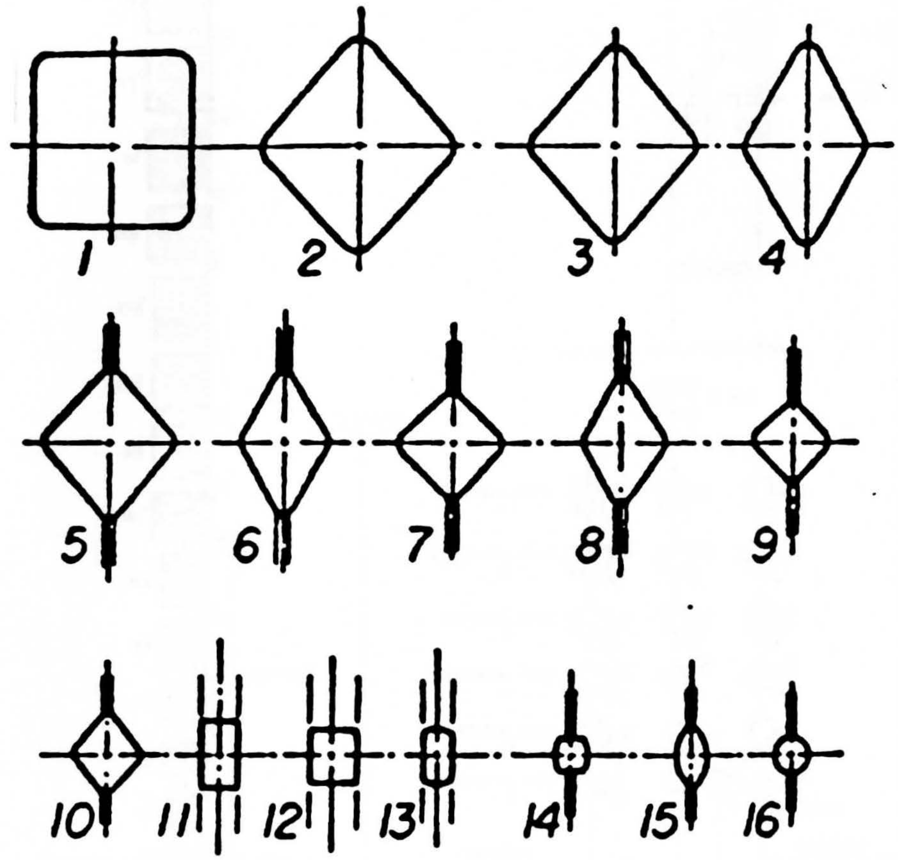

Figure 1.1 Passes and reductions of a 4 by 4-in. Billet to a 3/4-in. Round bar [1]

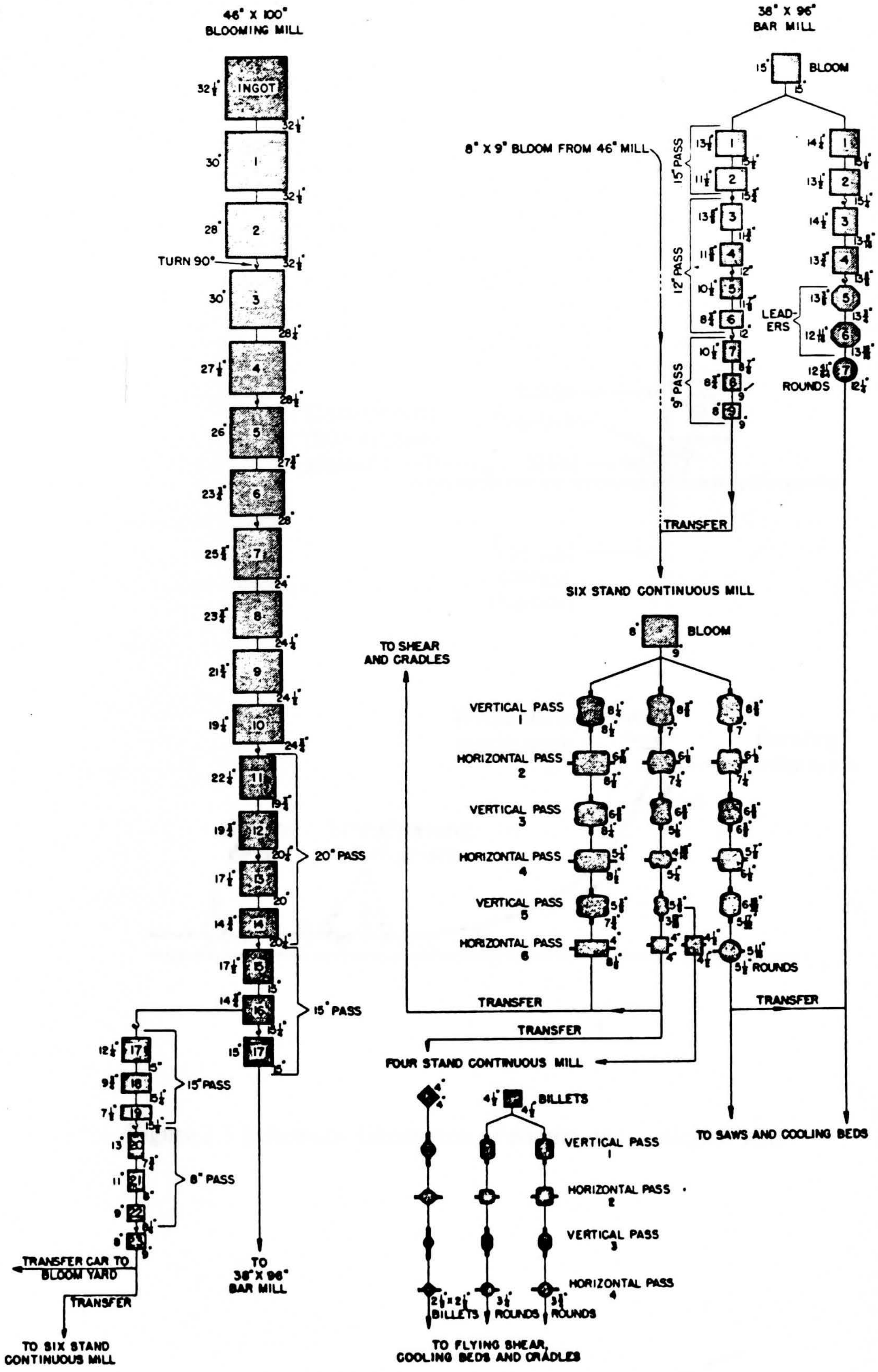

Figure 1.2 Flow chart summarizing typical series of operations possible due to the extremely flexible arrangement of the modern No. 4 Blooming, Bar and Billet Mill in the Lorain works [9].

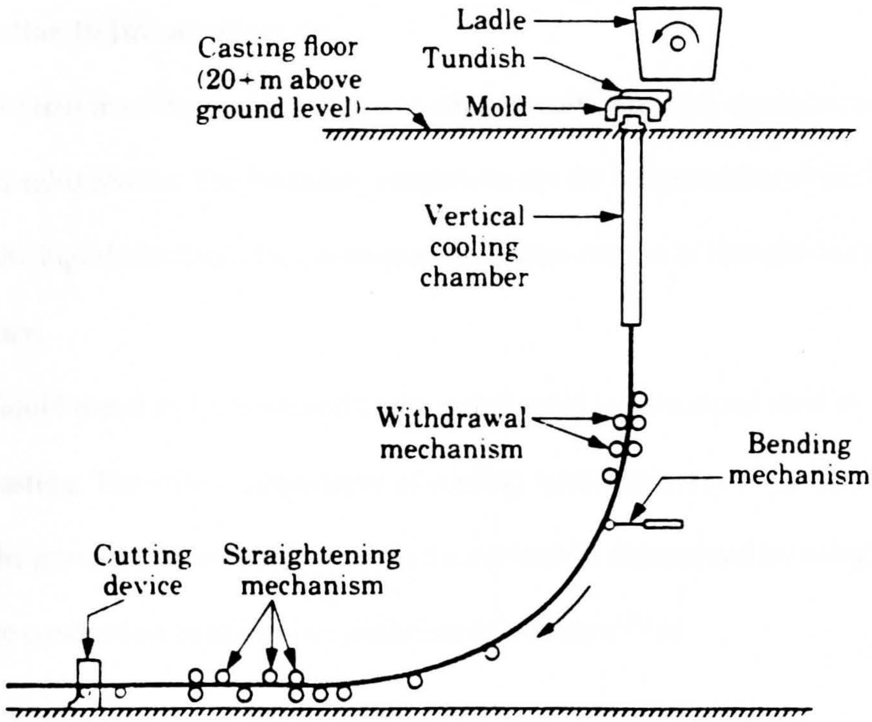

Figure 1.3 Schematic illustration of continuous casting of steel [3]

CHAPTER II

MODELING

2.1 Solidification in Infinite Rounds

The formation of the problem begins with unsteady state heat conduction through the liquid and solid phases. The boundary conditions are the temperatures of the liquid and solid at the liquid-solid interface are equal, and conservation of thermal energy at the phase boundary.

The liquid metal at $T_{l,0}$ is poured to the round mold to cast round steel in continuous casting. The initial temperature of cooling water temperature is $T_{s,0}$, as shown in Fig 2.1. The growth of a solidified layer in the bar can be determined by using the unsteady state conduction equation in cylindrical coordinates[10] as

$$\frac{\partial T_l}{\partial t} = \alpha_l \left(\frac{\partial^2 T_l}{\partial r^2} + \frac{1}{r}\frac{\partial T_l}{\partial r}\right) \tag{2.1}$$

for the liquid phase, and

$$\frac{\partial T_s}{\partial t} = \alpha_s \left(\frac{\partial^2 T_s}{\partial r^2} + \frac{1}{r}\frac{\partial T_s}{\partial r}\right) \tag{2.2}$$

for the solid phase where

$R(t)$ = distance from center to solid-liquid interface,

r = radial distance from the center of a cylinder,

R = radius of the cylinder,

α_l = thermal diffusivity of liquid,

α_s = thermal diffusivity of solid.

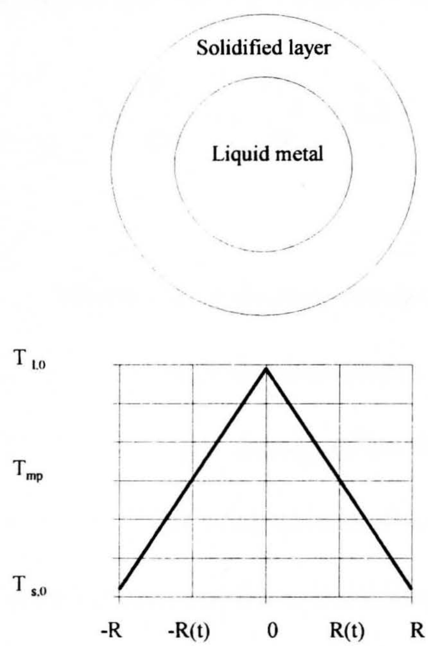

Fig 2.1 Temperature profile in the solidification of liquid metal in rod

The outer shell of a rod is cooled to $T_{s,0}$ using a water spray. The initial temperature of the liquid metal is $T_{l,0}$. At zero time, $T_{l,0}$ is constant for all r. The metal will solidify suddenly at melting point temperature. The temperature at the liquid-solid interface at $r = R(t)$ must be equal to the melting point of the metal.

The boundary conditions are

$$T_s(r,0) = T_{s,0} \quad \text{at } t = 0, R(t) < r < R \tag{2.3}$$

$$T_s(R,t) = T_{s,0} \quad \text{at } t > 0, r = R \tag{2.4}$$

$$T_l(r,0) = T_{l,0} \quad \text{at } t = 0, 0 < r < R(t) \tag{2.5}$$

$$T_l(R(t),t) = T_{mp} \quad \text{at } t > 0, r = R(t) \tag{2.6}$$

$$T_l = T_s = T_{mp} \quad \text{at } r = R(t). \tag{2.7}$$

The heat balance at the solidified interface is

$$k_l \frac{\partial T_l}{\partial r} - k_s \frac{\partial T_s}{\partial r} = \rho \Delta H \frac{d}{dt} R(t). \tag{2.8}$$

Where k_l = thermal conductivity of liquid,

k_s = thermal conductivity of solid,

ρ = density of a solid phase,

ΔH = latent heat of solidification per unit mass of liquid.

The rate of radial advance of the solidification front, $dR(t) / dt$, will be positive or negative for solidification and melting, respectively. In this case, it should be positive and ΔH will be negative.

2.2 Mathematical Modeling

Consider the solid phase equation

$$\frac{\partial T_s}{\partial t} = \alpha_s \left(\frac{\partial^2 T_s}{\partial r^2} + \frac{1}{r}\frac{\partial T_s}{\partial r}\right), \tag{2.1}$$

boundary conditions

$$T_s\,(r,0) \quad = \quad T_{s,0} \qquad \text{at} \quad t = 0,\ R < r < R(t) \tag{2.3}$$

$$T_s\,(R,t) \quad = \quad T_{s,0} \qquad \text{at} \quad t > 0,\ r = R. \tag{2.4}$$

From the boundary conditions, this is a nonhomogeneous problem. To solve this problem, we have to make it to be a homogeneous problem by introducing new two function U_s and ψ_s [11], related to T_s by

$$T_s\,(r,t) \quad = \quad U_s\,(r,t) + \psi_s\,(r). \tag{2.9}$$

From the boundary condition, which we get

$$\psi_s\,(r) \quad = \quad T_{s,0}, \tag{2.10}$$

and,

$$T_s\,(r,t) \quad = \quad U_s\,(r,t) + T_{s,0}. \tag{2.11}$$

The problem for $U_s\,(r,t)$ is

$$\frac{\partial U_s}{\partial t} = \alpha_s\left(\frac{\partial^2 U_s}{\partial r^2} + \frac{1}{r}\frac{\partial U_s}{\partial r}\right). \tag{2.12}$$

The boundary conditions are

$$U_s\,(R,t) \quad = \quad 0 \tag{2.13}$$

$$U_s\,(r,0) \quad = \quad T_{s,0}. \tag{2.14}$$

To separate variables in the heat equation, let

$$U_s\,(r,t) \quad = \quad F(r)\,T(t). \tag{2.15}$$

After some algebra, the following equation is obtained (see Appendix A)

$$\frac{T'}{\alpha_s T} = \frac{F'' + \frac{1}{r} F'}{F} = -\lambda , \tag{2.16}$$

for some constant λ, then

$$F'' + \frac{1}{r} F' + \lambda F = 0, \qquad T' + \lambda T = 0. \tag{2.17}$$

Consider three cases of λ, which are $\lambda = 0$, $\lambda < 0$, and $\lambda > 0$ (see Appendix A), by using Strum-Liouville Theory [12] and Bessel Function[13], we got

$$U_s (r,t) = \sum_{n=1}^{\infty} A_n J_0(\frac{Z_n}{R} r) e^{-\frac{Z_n^2}{R^2}\alpha_s t} \tag{2.18}$$

where

$$A_n = \frac{2 T_{s,0} \int_{R(t)}^{R} \varrho J_o(\frac{Z_n}{R}\varrho) d\varrho}{R^2 [J_1(Z_n)]^2 - R^2(t) [J_1(\frac{Z_n}{R}R(t))]^2} \tag{2.19}$$

Z_n = positive zeros of Bessel Functions.

Then, the solution for temperature of a solid phase is

$$T_s (r,t) = \sum_{n=1}^{\infty} A_n J_0(\frac{Z_n}{R}r) e^{-\frac{Z_n^2}{R^2}\alpha_s t} + T_{s,o}. \tag{2.20}$$

Using the same procedure for the liquid phase give the following solution

$$T_l\ (r,t) = \sum_{n=1}^{\infty} B_n\ J_0(\frac{Z_n}{R(t)}r)\ e^{-\frac{Z_n^2}{R(t)^2}\alpha_l t} + T_{mp} \tag{2.21}$$

where

$$B_n = \frac{2\ T_{l,0} \int_0^{R(t)} \varrho\ J_o(\frac{Z_n}{R(t)}\varrho)\ d\varrho}{R(t)^2\ [J_1(Z_n)]^2}\ . \tag{2.22}$$

Consider the heat balance of the solidified front (2.8) with the temperature of solid (2.20) and liquid (2.21), we got

$$\rho\Delta H\frac{dR(t)}{dt} = k_l(\sum_{n=1}^{\infty} B_n\ J_0^{/}(\frac{Z_n}{R(t)}r)\ e^{-\frac{Z_n^2}{R^2(t)}\alpha_l t}) - k_s(\sum_{n=1}^{\infty} A_n\ J_0^{/}(\frac{Z_n}{R}r)\ e^{-\frac{Z_n^2}{R^2}\alpha_s t})\ . \tag{2.23}$$

This is equation is the solution that can be used to determine the position of the solidified front at any time.

The detail of these calculations are shown in Appendix A.

2.3 Computer Stimulation

The solidification time and the moving of solidified front can be determined by using equation (2.24) which is obtained by substituting (2.19), (2.20), (2.21), and (2.22) into (2.23).

$$\rho \Delta H(\frac{d}{dt}R(t)) =$$

$$k_l \sum_{n=1}^{\infty}[\frac{2T_{l,0}(\sum_{k=0}^{\infty}\frac{(-1)^k}{2n*2^{2k}*(k!)^2}Z_n^{2k})}{(\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{2k-1}*(k!)*(k+1)!}Z_n^{2k+1})^2}(\sum_{k=1}^{\infty}\frac{(-1)^k*(2k)}{2^{2k}*(k!)^2}(\frac{Z_n}{R(t)})^{2k})e^{-\frac{Z_n^2}{R(t)^2}\alpha_l t}]$$

$$-k_s \sum_{n=1}^{\infty}[\frac{2T_{s,0}(\sum_{k=0}^{\infty}\frac{(-1)^k}{2n*2^{2k}*(k!)^2}(\frac{Z_n}{R})^{2k}(R^{2k+2}-R(t)^{2k+2}))}{R^2(\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{2k-1}*(k!)*(k+1)!}Z_n^{2k+1})^2-R(t)^2(\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{2k-1}*(k!)*(k+1)!}(\frac{Z_n}{R}R(t))^{2k+1})}$$

$$(\sum_{k=1}^{\infty}\frac{(-1)^k*(2k)}{2^{2k}*(k!)^2}(\frac{Z_n}{R})^{2k}R(t)^{2k-1})e^{-\frac{Z_n^2}{R^2}\alpha_s t}]. \qquad (2.24)$$

The solidified front can not be determined directly by equation (2.24) because the *R(t)* terms is in both sides of equation. The right-hand side is too complicated to integrate. It can be determined by a numerical method. The concepts of solving this problem can be described as follows.

First, consider the left hand side of the equation (2.24), $\frac{dR(t)}{dt}$, and

$$dR(t) = R - R(t) \qquad (2.25)$$

The R(t) of equation (2.25) is the same *R(t)* in the right hand side of equation (2.23), or (2.24). For any *dt*, the thickness of the solidified layer can be determined by finding *R(t)* in *dR(t)* of the right hand side that matched the *R(t)* in left hand side of the equation (2.24). The thickness of the solidified layer is equaled *dR(t)* at *dt*.

Chapter III

RESULTS

3.1 Computer Results

From the computer program in Appendix B, the thickness of the solidified layer at different time and different diameter can be determined by using these data[14].

α_s = 0.44 $ft^2\ hr^{-1}$

α_l = 0.22 $ft^2\ hr^{-1}$

k_s = 20 Btu $hr^{-1}\ ft^{-1}\ °F^{-1}$

k_l = 10 Btu $hr^{-1}\ ft^{-1}\ °F^{-1}$

ΔH = 110 Btu lb^{-1}

ρ = 490 lb ft^{-3}

T_{mp} = 2700 °F

$T_{l,0}$ = 2950 °F

$T_{s,0}$ = 100 °F

The thickness is calculated for 15.15 inches (400 mm), 11.81 inches (300 mm), 7.87 inches (200 mm), and 5.91 inches (150 mm) diameter. The data are shown in Table 3.1 and Fig 3.1.

3.2 Equation Results

From the graph in Fig 3.1, the thickness of the solidified layer of the first five second can be represent by a simple equation,

$$\delta = A\ t\ B^{t} \tag{3.1}$$

where δ = the thickness of the solidification layer,

A, B = constants,

t = time.

For 15.74 inches (400 mm) diameter round,

$$\delta = 0.506\ t\ 0.973^{t}. \tag{3.2}$$

For 11.81 inches (300 mm) diameter round,

$$\delta = 3.251\ t\ 1.02^{t}. \tag{3.3}$$

For 7.87 inches (200 mm) diameter round,

$$\delta = 1.649\ t\ 1.222^{t}. \tag{3.4}$$

For 5.91 inches (150 mm) diameter round,

$$\delta = 1.523\ t\ 0.798^{t}. \tag{3.5}$$

The comparison of the thickness from the computer stimulation and from the equation (3.1) is shown in Fig 3.2, Fig 3.3, Fig 3.4, and Fig 3.5.

TABLE 3.1

SOLIDIFIED THICKNESS AS A FUNCTION OF TIME

Time (sec)	400 mm.	300 mm.	200 mm.	150 mm.
0	0.000	0.000	0.000	0.000
1	0.493	3.316	2.016	1.216
2	0.959	6.825	4.935	1.940
3	1.400	10.492	8.934	2.365
4	1.818	14.256	13.697	2.613
5	2.213	18.038	18.272	2.756
6	2.586	21.743	21.840	2.839
7	2.939	25.271	24.255	2.887
8	3.272	28.531	25.765	2.915
9	3.587	31.458	26.673	2.931
10	3.884	34.017	27.209	2.940
11	4.164	36.201	27.522	2.946
12	4.429	38.030	27.704	2.949
13	4.678	39.538	27.809	2.951
14	4.914	40.765	27.870	2.952
15	5.136	41.545	27.905	2.952
16	5.345	42.525	27.925	2.953
17	5.542	43.176	27.937	2.953
18	5.728	43.675	27.944	2.953
19	5.903	44.070	27.948	
20	6.068	44.380	27.950	
21	6.224	44.625	27.951	
22	6.370	44.817	27.952	

TABLE 3.1 (CONT.)

SOLIDIFIED THICKNESS AS A FUNCTION OF TIME

Time (sec)	400 mm.	300 mm.	200 mm.
23	6.508	44.968	27.952
24	6.637	45.086	27.953
25	6.760	45.175	27.953
26	6.874	45.251	27.953
27	6.983	45.308	
28	7.085	45.353	
29	7.180	45.387	
30	7.271	45.415	
31	7.355	45.436	
32	7.435	45.452	
33	7.510	45.465	
34	7.581	45.476	
35	7.647	45.484	
36	7.710	45.490	
37	7.769	45.495	
38	7.824	45.498	
39	7.876	45.501	
40	7.925	45.504	
41	7.971	45.506	
42	8.014	45.507	
43	8.054	45.508	
44	8.093	45.509	
45	8.128	45.510	

TABLE 3.1 (CONT.)

SOLIDIFIED THICKNESS AS A FUNCTION OF TIME

Time (sec)	400 mm.	300 mm.
46	8.162	45.510
47	8.194	45.511
48	8.224	45.511
49	8.252	45.511
50	8.278	45.511
51	8.303	45.512
52	8.326	45.512
53	8.348	
54	8.369	
55	8.388	
56	8.406	
57	8.423	
58	8.439	
59	8.454	
60	8.469	

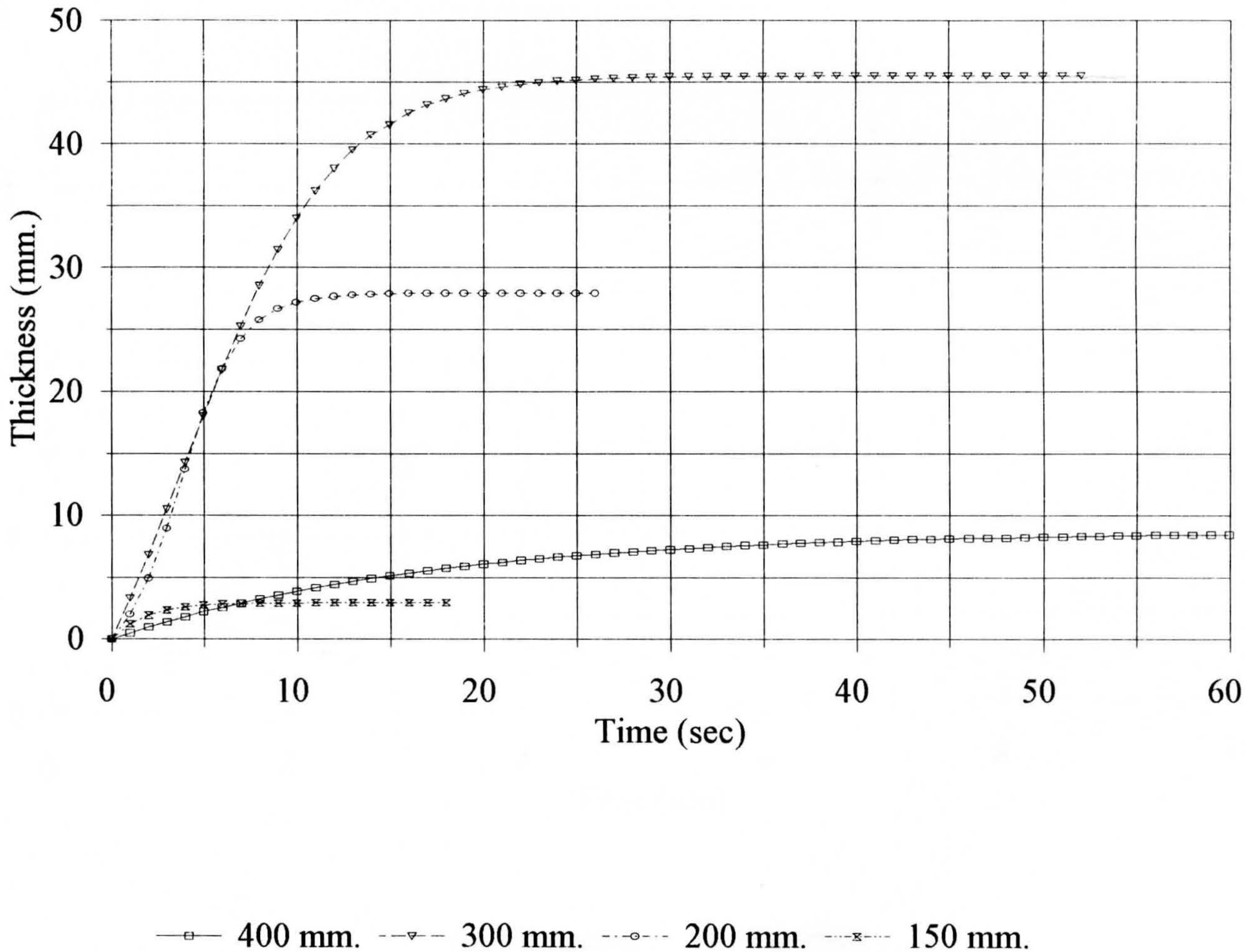

Fig 3.1 Solidified thickness as a function of time

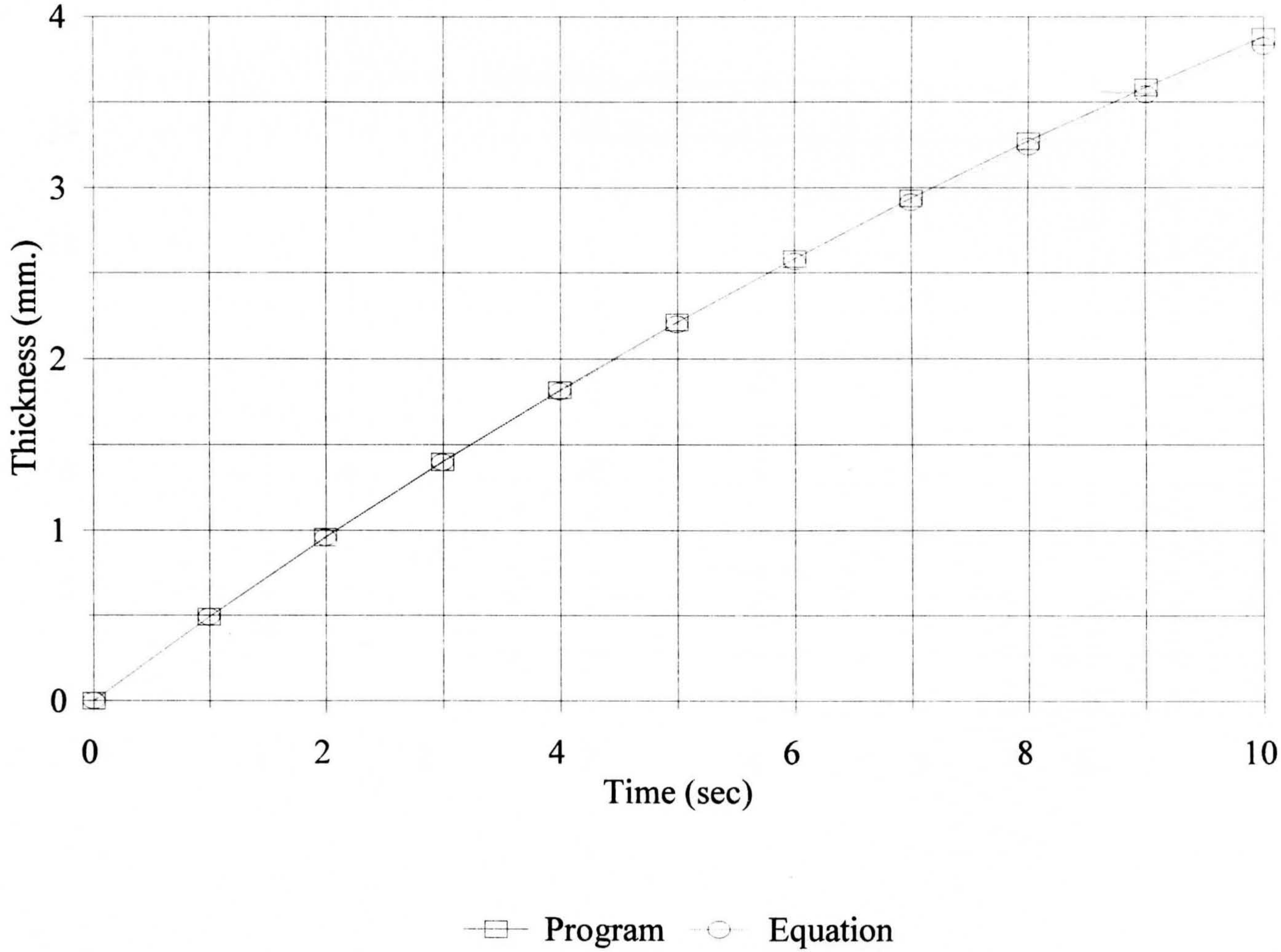

Fig 3.2 Comparison of solidified thickness of 400 mm diameter round

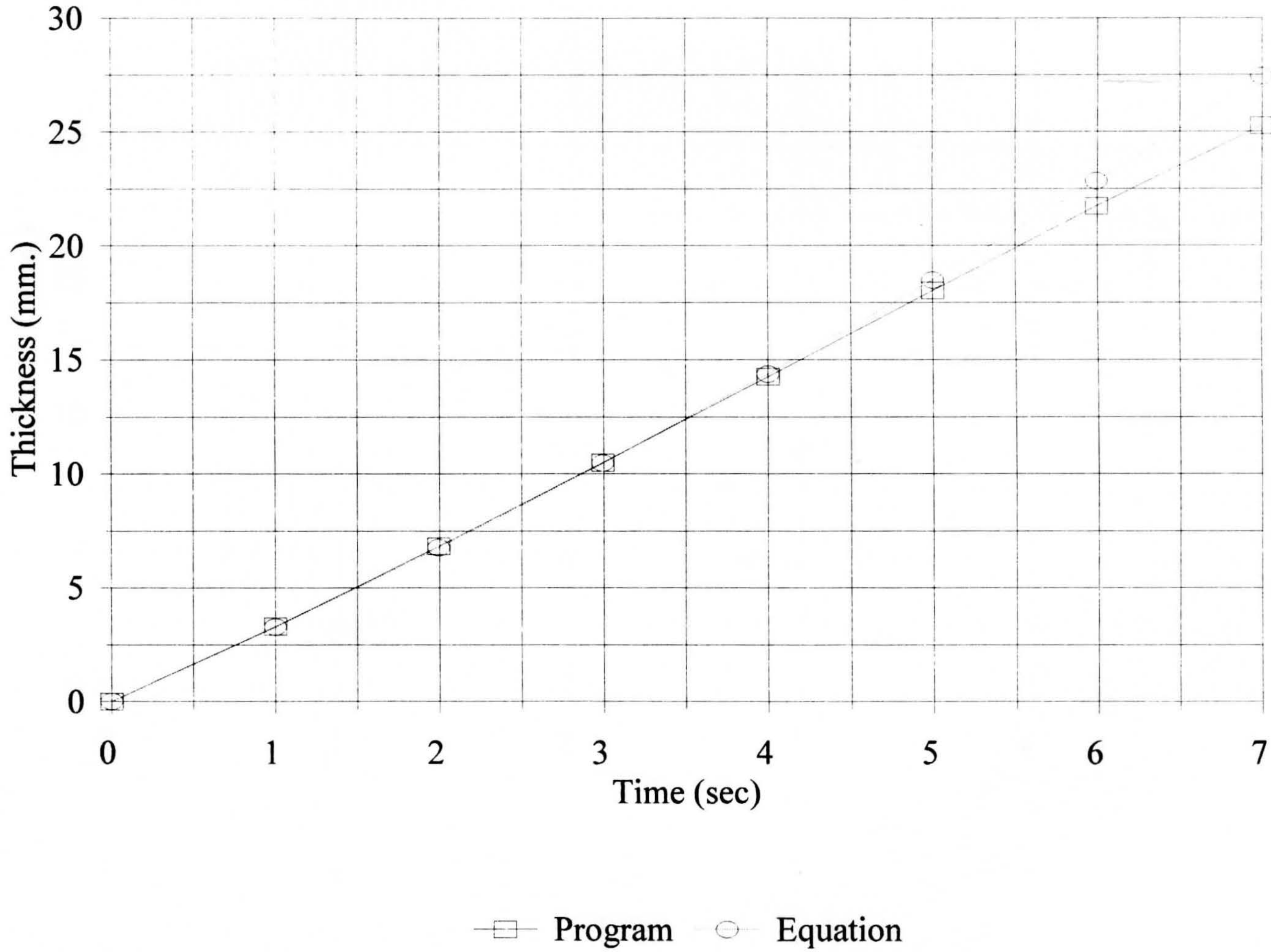

Fig 3.3 Comparison of solidified thickness of 300 mm diameter round

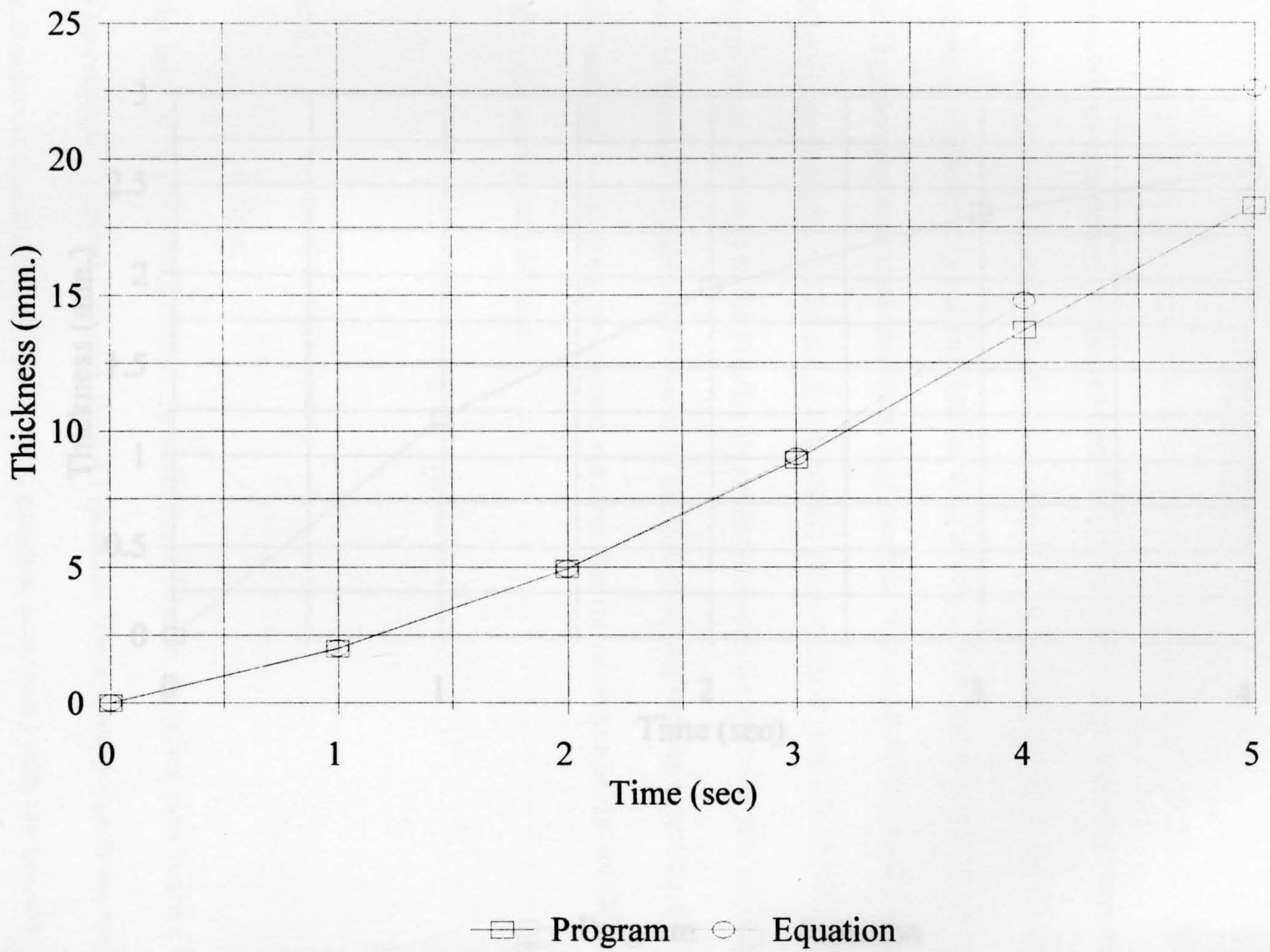

Fig 3.4 Comparison of solidified thickness of 200 mm diameter round

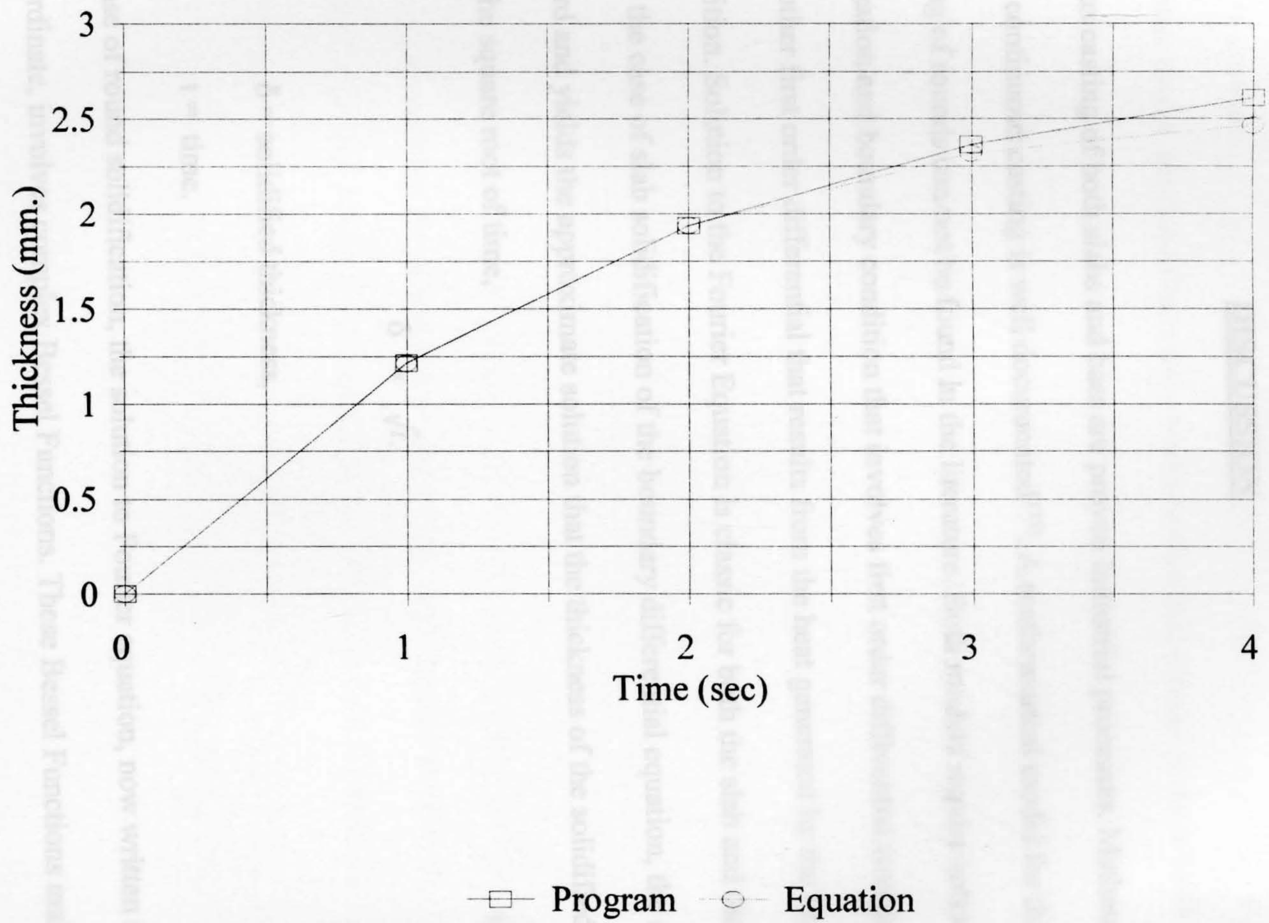

Fig 3.5 Comparison of solidified thickness of 150 mm diameter round

CHAPTER IV

DISCUSSION

Continuous casting of both slabs and bars are proven industrial processes. Mathematical modeling of slab continuous casting is well documented [15]. A mathematical model for the continuous casting of rounds can not be found in the literature. Both models require solution to Fourier Heat Equation and boundary condition that involves first order differential (temperature gradient) and another first order differential that results from the heat generated by the liquid to solid phase transition. Solution to the Fourier Equation is classic for both the slab and the round solidification. In the case of slab solidification of the boundary differential equation, the solution is straight forward and yields the approximate solution that the thickness of the solidified crust is proportional to the square root of time,

$$\delta \propto \sqrt{t}. \tag{4.1}$$

where δ = solidified thickness,

t = time.

In the case of round solidification, the solution to Fourier equation, now written in cylindrical co-ordinate, involves complex Bessel Functions. These Bessel Functions make it impossible to solve in closed form the differential equation associated with the boundary condition. A computer model was developed to solve the boundary condition. This approach leads to the approximate relationship for the thickness of the solidified crust

$$\delta \propto tB^{t}, \tag{4.2}$$

where B is a constant.

The computer model developed here only works for the initial stage of the solidification process. The problem with the method results from trying to sum the infinite series associated with the Bessel Functions. The work presented here indicates the solidification of rounds is initially faster but then slower than solidification of slabs. No experimental data could be found to compare with the theoretical work presented there.

CHAPTER V

CONCLUSION

The mathematical model presented here for the solidification of continuously cast rounds predicts the kinetics of the solidified layer which is considerably different from the kinetics of the solidification of slab material. The model only works for the initial stages of solidification. As the solidified layer approaches the center of the round the model fails to properly converge. The problem is probably associated with the boundary condition (equation (2.8)) near the end of the solidification process. Unfortunately, no experimental data could be found to determine the correctness of the equations developed in this work.

It is recommended that a numerical methods are used to solve equation (2.1), (2.2), and (2.8), and that the results compared to the results presented here.

Having a valid model for the solidification process should facilitate the industrial process and allow maximum yield of the casting process.

APPENDIX A

MATHEMATICAL MODELING

A.1 Solidification condition

The unsteady state conduction equation in cylindrical coordination[10]

$$\frac{\partial T_l}{\partial t} = \alpha_l \left(\frac{\partial^2 T_l}{\partial r^2} + \frac{1}{r}\frac{\partial T_l}{\partial r}\right) \tag{A.1}$$

for the liquid phase, and

$$\frac{\partial T_s}{\partial t} = \alpha_s \left(\frac{\partial^2 T_s}{\partial r^2} + \frac{1}{r}\frac{\partial T_s}{\partial r}\right) \tag{A.2}$$

for the solid phase where T_l = temperature of liquid phase which is the function of (r,t),

T_s = temperature of solid phase which is the function of (r,t),

$R(t)$ = distance from center to solid-liquid interface,

r = radial distance from the center of a cylinder,

R = radius of the cylinder,

α_l = thermal diffusivity of liquid ,

α_s = thermal diffusivity of solid.

The boundary conditions are

$$T_s(r,0) = T_{s,0} \quad \text{at} \quad t = 0,\ R(t) < r < R \tag{A.3}$$

$$T_s(R,t) = T_{s,0} \quad \text{at } t > 0, r = R \tag{A.4}$$

$$T_l(r,0) = T_{l,0} \quad \text{at } t = 0, 0 < r < R(t) \tag{A.5}$$

$$T_l(R(t),t) = T_{mp} \quad \text{at } t > 0, r = R(t) \tag{A.6}$$

$$T_l = T_s = T_{mp} \quad \text{at } r = R(t). \tag{A.7}$$

The heat balance of the solidified front

$$k_l \frac{\partial T_l}{\partial r} - k_s \frac{\partial T_s}{\partial r} = \rho \Delta H \frac{d}{dt} R(t) \tag{A.8}$$

where k_l = thermal conductivity of liquid

k_s = thermal conductivity of solid

ρ = density of a solid phase

ΔH = latent heat of solidification per unit mass of liquid.

A.2 Temperature of solid phase

Consider the solid phase of Fourier heat equation for cylindrical coordination[10],

$$\frac{\partial T_s}{\partial t} = \alpha_s \left(\frac{\partial^2 T_s}{\partial r^2} + \frac{1}{r}\frac{\partial T_s}{\partial r}\right), \tag{A.2}$$

and boundary conditions are

$$T_s(r,0) = T_{s,0} \quad \text{at } t = 0, R(t) < r < R \tag{A.3}$$

$$T_s(R,t) = T_{s,0} \quad \text{at } t > 0, r = R. \tag{A.4}$$

Because of the boundary conditions equation (A.3) and (A.4), this is a nonhomogeneous problem (equation (A.2)). To solve this problem, it has to be transformed to a homogeneous problem. Consider the functions U_s and ψ_s[11], related to T_s by

$$T_s(r,t) = U_s(r,t) + \psi_s(r). \tag{A.9}$$

Partial differentiation of equation (A.9) is shown as

$$\frac{\partial T_s}{\partial t} = U_t \tag{A.10}$$

$$\frac{\partial T_s}{\partial r} = U_r + \psi_s^{/} \tag{A.11}$$

$$\frac{\partial^2 T_s}{\partial r^2} = U_{rr} + \psi_s^{//}. \tag{A.12}$$

Then, substitute equation (A.10), (A.11), and (A.12) into (A.2), yields

$$U_t = \alpha_s \left(U_{rr} + \frac{1}{r} U_r \right) + \alpha_s \left(\psi_s^{//} + \frac{1}{r} \psi_s^{/} \right) \tag{A.13}$$

assume

$$\psi_s^{//} + \frac{1}{r} \psi_s^{/} = 0 \tag{A.14}$$

Consider the boundary condition (A.4)

$$T_s(R,t) = U_s(R,t) + \psi_s(R) = T_{s,0} \tag{A.15}$$

Let $U_s(R,t) = 0$, *and* $\psi_s(R) = T_{s,0}$

Let $\phi = \psi_s'$

$$r\phi' + \phi = 0$$

$$[\ r\phi\ ]' = 0$$

$$\frac{d}{dr}(r\phi) = 0$$

$$r\,\phi = C_1$$

$$r\psi_s' = C_1$$

$$\psi_s = C_1 \ln(r) + C_2$$

as $r = R(t)$ which goes to 0, $\ln(r)$ go to $-\infty$

so C_1 must be 0

$$\psi_s = C_2$$

$$\psi_s(R) = T_{s,0} = C_2$$

$$\psi_s(r) = T_{s,0} \tag{A.16}$$

and equation is now

$$U_t = \alpha_s\,(U_{rr} + \frac{1}{r}U_r) \tag{A.17}$$

and, the boundary conditions are

$$U_s(R,t) = 0 \text{ for } t > 0$$

$$U_s(r,0) = T_{s,0} \qquad \text{for } R(t) < r < R$$

To separate variables in the heat equation, let

$$U_s\,(r,t) \;=\; F_s\,(r)\,T_s\,(t)$$

$$U_t \;=\; \frac{\partial U_s}{\partial t} \;=\; T'(t).$$

$$U_r \;=\; \frac{\partial U_s}{\partial t} \;=\; F'(r).$$

$$U_{rr} \;=\; \frac{\partial^2 U_s}{\partial t^2} \;=\; F''(r).$$

These three equations are substituted to equation (A.17) and after some algebra, the following equation is obtained

$$\frac{T'}{\alpha_s T} \;=\; \frac{F'' + \frac{1}{r}F'}{F} \;=\; -\lambda$$

for some constant λ. Then

$$F'' + \frac{1}{r}F' + \lambda F = 0, \qquad T' + \lambda\alpha_s T = 0$$

Now consider cases on λ.

Case 1: $\lambda = 0$ The differential equation for F is $F'' + \frac{1}{r}F' = 0$, with general solution

$$F(r) = C\,ln\,(r) + K.$$

But $ln\,(r) \to -\infty$ as $r \to 0$ (as $R(t)$ is at the center of the cylinder), so C must be 0 to make bounded solution. Then $F(r) = K$.

If $\lambda = 0$, the differential equation for T is

$$T' = 0,$$

so $T' =$ constant also. Hence, when $\lambda = 0$, $U_s =$ constant. The function $U_s =$ constant will satisfy $U_s(R,t) = 0$ for $t > 0$ only if the constant is zero. Thus, the solution $U_s(r,t) = 0$ is trivial in this case. So, this case is eliminated.

Case 2: λ < 0 Write $\lambda = -k^2$, with k positive. Then $T' - \alpha_s k^2 T = 0$ has general solution $T = ce^{\alpha_s k^2 t}$, which is unbounded if $c \neq 0$. Thus, there is no bounded solution for $\lambda < 0$.

Case 3: λ > 0 Write $\lambda = k^2$, with k positive. Then $T' - \alpha_s k^2 T = 0$, so $T = ce^{-\alpha_s k^2 t}$.

The equation for F is

$$r^2 F'' + rF' + k^2 r^2 F = 0.$$

The general solution of this equation is

$$F(r) = AJ_0(kr) + BY_0(kr),$$

in which J_0 and Y_0 are Bessel Function[13] of order zero of the first and second kind, respectively.

As $r \rightarrow 0$, $Y_0(kr) \rightarrow -\infty$, which gives an unbounded solution. Choose B equal to 0.

The new solution is

$$F(r) = AJ_0(kr).$$

For every $k > 0$, the function is

$$U_s(r,t) = A_n J_0(kr) e^{-k^2 \alpha_s t},$$

which satisfies the heat equation (A.17).

Now, consider the boundary condition, $U_s(R,t) = 0$.

$$U_s(R,t) = A_n J_0(kR) e^{-k^2 \alpha_s t} = 0.$$

To satisfy this condition with $A_n \neq 0$, $J_0(kR)$ has to be equaled 0. Let $Z_n = k_n R$, for n = 1, 2, 3,...., and Z_n is the positive zero number of $J_0(Z_n)$. So, $k = (Z_n / R)$. Corresponding to each positive integer n,

$$U_{sn}(r,t) = A_n J_0\left(\frac{Z_n r}{R}\right) e^{-\frac{Zn^2}{R^2}\alpha_s t}.$$

So, the solution is

$$U_s(r,t) = \sum_{n=1}^{\infty} U_{sn}(r,t) = \sum_{n=1}^{\infty} A_n J_0\left(\frac{Z_n}{R} r\right) e^{-\frac{Z_n^2}{R^2}\alpha_s t}. \qquad \text{(A.18)}$$

Choose the A_n's to satisfy $U_s(r,0) = T_{s,0}$. Require that

$$U_s(r,0) = T_{s,0} = \sum_{n=1}^{\infty} A_n J_0\left(\frac{Z_n r}{R}\right). \qquad \text{(A.19)}$$

The orthogonality relationship[14] for the function $J_0\,(Z_n r/R)$. This relationship is

$$\int_0^R rJ_0\left(\frac{Z_{nr}}{R}\right)J_0\left(\frac{Z_{mr}}{R}\right)dr = 0 \qquad \text{If } n \neq m. \tag{A.20}$$

Rearrange the equation (A.19) by multiplying both sides of equation by $rJ_0(Z_k r/R)$, with k any positive integer.

$$rT_{s,0}J_0\left(\frac{Z_k r}{R}\right) = \sum_{n=1}^{\infty} A_n rJ_0\left(\frac{Z_n r}{R}\right)J_0\left(\frac{Z_k r}{R}\right).$$

Integrate both sides from $R(t)$ to R, interchanging the summation and the integral,

$$\int_{R(t)}^{R} rT_{s,0}J_0\left(\frac{Z_n r}{R}\right)dr = \sum_{n=1}^{\infty} A_n \int_{R(t)}^{R} rJ_0\left(\frac{Z_n r}{R}\right)J_0\left(\frac{Z_k r}{R}\right)dr.$$

By the orthogonality relationship[16] (A.20), all of the integrals on the right are zero except the one in which $n = k$. The last equation therefore reduces to

$$\int_{R(t)}^{R} rT_{s,0}J_0\left(\frac{Z_k r}{R}\right)dr = A_k \int_{R(t)}^{R} r\left[J_0\left(\frac{Z_k r}{R}\right)\right]^2 dr.$$

Solve this equation for A_k to obtain

$$A_k = \frac{\int_{R(t)}^{R} r T_{s,0} J_0(\frac{Z_k r}{R}) dr}{\int_{R(t)}^{R} r [J_0(\frac{Z_k r}{R})]^2 dr}$$

for $k = 1, 2, 3, \ldots$. These numbers are the Fourier-Bessel coefficients[17], which can be changed to A_n's. From the Bessel Function and Strum-Liouville Theory[18],

$$\int_{0}^{R} r [J_0(\frac{Z_n r}{R})]^2 dr = \frac{1}{2} R^2 [J_1(Z_n)]^2.$$

Then, the equation for A_n is

$$A_n = \frac{2 T_{s,0} \int_{R(t)}^{R} \varrho J_0(\frac{Z_n}{R}\varrho) d\varrho}{R^2 [J_1(Z_n)]^2 - R(t)^2 [J_1(\frac{Z_n}{R} R(t))])^2}. \tag{A.21}$$

Then, substitute equation (A.18) with (A.21)

$$U_s(r,t) = 2\ T_{s,0} \sum_{n=1}^{\infty} \frac{\int_{R(t)}^{R} \varrho J_0(\frac{Z_n}{R}\varrho) d\varrho}{R^2 [J_1(Z_n)]^2 + R(t)^2 [J_1(\frac{Z_n}{R} R(t))]^2} J_0(\frac{Z_n}{R} r) e^{-\frac{Z_n^2}{R^2}\alpha_s t}. \tag{A.22}$$

From equation (A.9), the solution for the temperature of a solid phase is

$$T_s(r,t) = \sum_{n=1}^{\infty} A_n J_0(\frac{Z_n}{R} r) e^{-\frac{Z_n^2}{R^2}\alpha_s t} + T_{s,0}, \tag{A.23}$$

where (A.21)

$$A_n = \frac{2T_{s,0}\int_{R(t)}^{R} \varrho J_0(\frac{Z_n}{R}\varrho)d\varrho}{R^2[J_1(Z_n)]^2 - R(t)^2[J_1(\frac{Z_n}{R}R(t))]^2}.$$

A.3 Temperature of liquid phase

From the same procedure as the solid temperature, the solution of the liquid temperature is

$$T_l(r,t) = \sum_{n=1}^{\infty} B_n J_0(\frac{Z_n}{R(t)}r)e^{-\frac{Z_n^2}{R(t)^2}\alpha_l t} + T_{mp} \tag{A.24}$$

where

$$B_n = \frac{2\ T_{l,0}\int_{0}^{R(t)} \varrho J_0(\frac{Z_n}{R(t)}\varrho)d\varrho}{R(t)^2[J_1(Z_n)]^2}. \tag{A.25}$$

A.4 Heat balance at solidified front

Consider the heat balance of the solidified front (A.8) with the temperature of solid (A.23) and the temperature of liquid (A.24),

$$\rho\Delta H\frac{dR(t)}{dt} = k_l(\sum_{n=1}^{\infty} B_n J_0^{/}(\frac{Z_n}{R(t)}r)e^{-\frac{Z_n^2}{R(t)^2}\alpha_l t}) - k_s(\sum_{n=1}^{\infty} A_n J_0^{/}(\frac{Z_n}{R}r)e^{-\frac{Z_n^2}{R^2}\alpha_s t}). \tag{A.26}$$

At $r = R(t)$, this equation (A.26) can be expanded by given more detail of the Bessel Functions. The first term in the right side of (A.26) can be expanded as

$$k_l \sum_{n=1}^{\infty} \left[\frac{2\, T_{l,0} \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{2n * 2^{2k} * (k!)^2} Z_n^{2k} \right)}{\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{2k-1} (k!)(k+1)!} Z_n^{2k+1} \right)^2} \left(\sum_{k=1}^{\infty} \frac{(-1)^k (2k)}{2^{2k} (k!)^2} \left(\frac{Z_n}{R(t)} \right)^{2k} \right) e^{-\frac{Z_n^2}{R(t)^2} \alpha_l t} \right]. \qquad (A.27)$$

The second term can be expressed as

$$k_s \sum_{n=1}^{\infty} \left[\frac{2\, T_{s,0} \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{2n * 2^{2k} * (k!)^2} \left(\frac{Z_n}{R} \right)^{2k} \left(R^{2k+2} - R(t)^{2k+2} \right) \right)}{R^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{2k-1} (k!)(k+1)!} Z_n^{2k+1} \right)^2 - R(t)^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{2k-1} (k!)(k+1)!} \left(\frac{Z_n}{R} R(t) \right)^{2k+1} \right)^2} \right. \qquad (A.28)$$

$$\left. \left(\sum_{k=1}^{\infty} \frac{(-1)^k (2k)}{2^{2k} (k!)^2} \left(\frac{Z_n}{R} \right)^{2k} R(t)^{2k-1} \right) e^{-\frac{Z_n^2}{R^2} \alpha_s t} \right].$$

APPENDIX B

COMPUTER PROGRAM

The following is the computer program for predicting the thickness of solidified layer of the infinite round. This program is written by using the Turbo Pascal for Windows. The flow chart shows in Fig B.1.

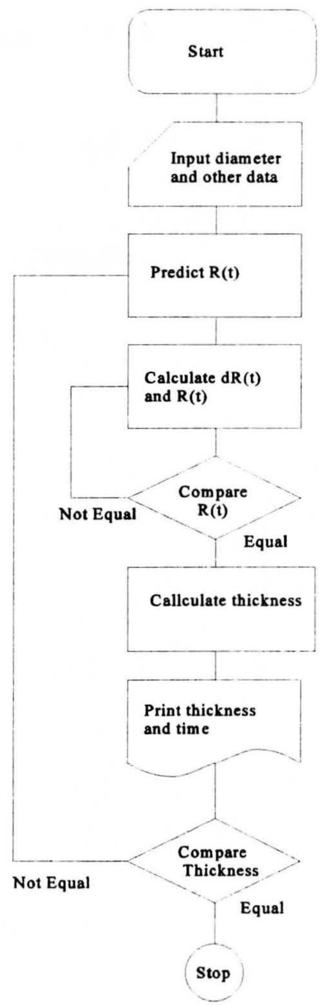

Fig B.1 Flow chart for computer program.

```
Program Thesis (Input, Output);

Uses Wincrt;

Function Power (Base:Real; Time:Integer):Real;

Var
  Count  : Integer;
  V1, V2 : Real;
Begin {Function Power}
   V1 := 1;
   V2 := 1;
   If Time = 0 Then Power := 1
     Else
     Begin
        If Time = -1 Then Power := 1/base
        Else
        Begin
           If Base = 0 Then Power := 0
           Else
           Begin
              For Count := 1 to Time Do
              Begin
                 V2 := Base * V1;
                 V1 := V2;
              End;
              Power := V2;
           End;
        End;
     End;
End; {Function Power}

Function Fac (k:Integer):Real;

Var
  Count : Integer;
  F     : Real;
Begin
   Count := 1;
   F     := 1;
   Fac   := 1;
   If k = 0 Then k := 1
     Else
     Begin
```

```
        While Count <= k Do
        Begin
            F := Count * F;
            Count := Count + 1;
        End;
    End;
    Fac := F;
End;

Function Sum1 (Zn:Real):Real;

Var
  Count  : Integer;
  S1, S2 : Real;
Begin
   S1 := 0;
   S2 := 0;
   For Count := 1 To 15 Do
   Begin
      S1 := Power(-1,Count)*Power(Zn,2*Count)/(2 * Count
          * Power(2,2*Count) * Fac(Count)*Fac(Count));
      S2 := S1 + S2;
   End;
   Sum1 := S2;
End;

Function Sum2 (Zn:Real):Real;

Var
  Count  : Integer;
  S1, S2 : Real;
Begin
   S1 := 0;
   S2 := 0;
   For Count := 0 To 15 Do
   Begin
      S1 := Power(-1,Count) * Power(Zn,(2*Count+1))/(Power(2,(2*Count+1))
            * Fac(Count) * Fac(Count+1));
      S2 := S1 + S2;
   End;
   Sum2 := S2 * S2;
End;
```

```
Function Sum3 (Zn,Rt:Real):Real;

Var
  Count  : Integer;
  S1, S2 : Real;
Begin
   S1 := 0;
   S2 := 0;
   For Count := 1 To 15 Do
   Begin
      S1 := Power(-1,Count)*2*Count*Power((Zn/Rt),(2*Count))
         /(Power(2,(2*Count))*Fac(Count)*Fac(Count));
      S2 := S1 + S2;
   End;
   Sum3 := S2;
End;

Function Sum4 (Zn,R,Rt:Real):Real;

Var
  Count : Integer;
  S1,S2 : Real;
Begin
   S1 := 0;
   S2 := 0;
   For Count := 1 To 15 Do
   Begin
      S1 := Power(-1,Count)*Power((Zn/R),(2*Count))
         *(Power(R,(2*Count+2))-Power(Rt,(2*Count+2)))
         /( 2 * Count * power (2,2*Count) *Fac(Count)*Fac(Count));
      S2 := S1 + S2;
   End;
   Sum4 := S2;
End;

Function Sum5 (Zn,R,Rt:Real):Real;

Var
  Count : Integer;
  S1,S2 : Real;
Begin
   S1 := 0;
   S2 := 0;
   For Count := 0 To 15 Do
```

```
    Begin
      S1 := Power(-1,Count)*Power((Zn*Rt/R),(2*Count+1))
          /(Power(2,(2*Count+1))*Fac(Count)*Fac(Count+1));
      S2 := S1 + S2;
    End;
    Sum5 := S2 * S2;
End;

Function Sum6 (Zn,R,Rt:Real):Real;

Var
  Count : Integer;
  S1,S2 : Real;
Begin
    S1 := 0;
    S2 := 0;
    For Count :=1 To 15 Do
    Begin
      S1 := Power(-1,Count)*2*Count*Power((Zn/R),(2*Count))
          *Power(Rt,(2*Count-1))
          /(Power(2,(2*Count))*Fac(Count)*Fac(Count));
      S2 := S1 + S2;
    End;
    Sum6 := S2;
End;

Type
   Table = Array [1..9] of Real;
Var
  Zn : Table;
  n, I, J, K : Integer;
  R, Rt, Ra, Rb, Rc, Rd, Tlo, Tso, dRt, t : Real;
  An, Bn, d, H, kl, kKs, Tmp, Al, As : Real;
Begin { Main Program }
  Zn[1]:= 2.405;
  Zn[2]:= 5.520;
  Zn[3]:= 8.654;
  t     := 1/360;
  Bn    := 0;
  An    := 0;
  Tc    := 0;
  K     := 0;
  I     := 0;
  Rt    := 0;
```

```
Write ('Enter the density of solid phase (lb/sq(ft)) = ');
Readln (d);
Write ('Enter the latent heat of solidification per unit mass of liquid (Btu/lb) =');
Readln (H);
Write ('Enter the thermal conductivity of liquid (Btu/(hr ft F)) = ');
Readln (kl);
Write ('Enter the thermal conductivity of solid  (Btu/(hr ft F)) = ');
Readln (ks);
Write ('Enter the thermal diffusivity of liquid (sq(ft)/hr) = ');
Readln (Al);
Write ('Enter the thermal diffusivity of solid  (sq(ft)/hr) = ');
Readln (As);
Write ('Enter the melting point temperature (F) = ');
Readln (Tmp);
Write ('Enter the liquid steel temperature (F) = ');
Readln (Tlo);
Write ('Enter the cooling Temperature (F) = ');
Readln (Tso);
Write ('Enter the radius of steel (mm.) = ');
Readln (Rd);
Writeln (I:3,'    ',Rt:8:3);
Rc := Rd / (12 * 25.4);
While K = 0 Do
Begin
   J  := 0;
   R  := Rc;
   Rt := R - 1e-9;
   While J = 0 Do
   Begin
      For n := 1 To 1 Do
      Begin
         Bn := 2 * Tlo * Sum1(Zn[n])/(Sum2(Zn[n]))
             * Sum3(Zn[n],Rt)
             * Exp ( -Zn[n] * Zn[n] * Al * t / ( Rt*RT));
         An := 2 * Tso * Sum4(Zn[n],R,Rt)* Sum6(Zn[n],R,Rt)
             /( ( R*R * Sum2(Zn[n]) )
               - ( Rt * Rt * Sum5(Zn[n],R,Rt)))
             * Exp (- Zn[n] * Zn[n] * As * t /( R*R));
      End;
      dRt := Abs ( ((kl * Bn) - (ks * An)) / ( d*H));
      Ra := ( R - Abs(dRt * t));
      Rb := Abs ( Rt - Ra);

      If Rb < 5e-8 Then J := 1
```

```
        Else Rt := (Rt + Ra) /2;
    End;
    I := I + 1;
    Writeln (I:3,'    ',(Rd -(Ra * 12 * 25.4)) : 8:3);
    Rc := Abs( R - Ra );
    If Rc < 5e-7 Then K := 1
    Else K:= 0;
    Rc := Rt;
  End;
End.
```

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