Current Distribution at Varying Frequencies in Hybrid Configuration of Solid Copper Bus and Braided Litz

By

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ABSTRACT

Current Distribution at Varying Frequencies in Hybrid Configuration of Solid Copper Bus and Braided Litz

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Youngstown State University, 1999

In this research, the effects of frequency on current distribution in bus bar configurations of solid copper bus and braided Litz wire is studied.

When a conductor carries an alternating current, the current tends to forsake the interior of the conductor and concentrate at the surface. The higher the frequency the more the current will concentrate at the surface. As a result of this skin-like concentration of current energy will be lost due to the cross sectional area of the conductor being underutilized by the current.

Litz wire is a special wire that is able to counter this so called "Skin Effect" and has an AC resistance very near its DC resistance over a large range of frequencies. A coupled circuit method is used to solve for the current distribution in several hybrid arrangements of normal copper conductors and Litz wire. Using Litz wire in the right application can reduce the resistance significantly and make power distribution more efficient. Also, with the presented model, electrical quantities like impedance and current distribution can be readily calculated and used to aid in a design.

ACKNOWLEDGEMENTS

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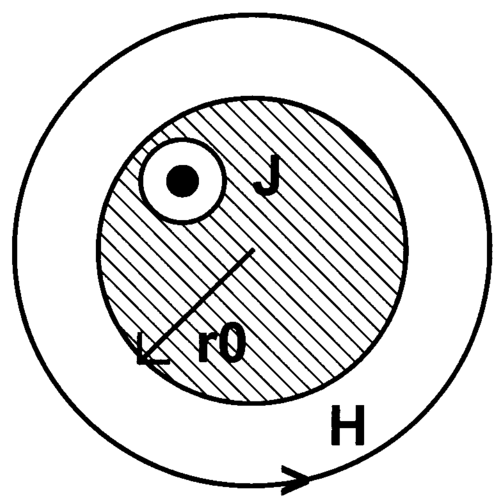
CHAPTER I

SKIN EFFECT

Skin Effect is the tendency of current density to be greatest at the surface of a conductor when it carries an alternating current. Frequency, cross-sectional area, conductivity and relative permeability all influence the extent of this concentration. To illustrate this, a long cylindrical copper conductor is analyzed under sinusoidal variation of current.



Conductor of Radius r_0 with J and H Shown with $I=I_0\cos(\omega t)$

Figure 1.1

Figure 1.1 depicts an infinitely long conductor of radius r_0 under sinusoidal excitation. The current density is directed normal to and out of the page at a particular reference instant of time.

Starting with the pertinent Maxwell equations for a conductor where there is no free charge yields

$$\nabla \bullet \overline{D} = 0 \tag{1.1}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t} \tag{1.2}$$

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t} \tag{1.3}$$

The Phasor form representing the above Maxwell's equations are defined as

$$\nabla \bullet \overline{D} = 0 \tag{1.4}$$

$$\nabla \times \overline{H} = \overline{J} + j\omega\overline{D} \tag{1.5}$$

$$\nabla \times \overline{E} = -j\omega\overline{B} \tag{1.6}$$

Where

ω is the radian frequency

Using

$$\overline{B} = \mu\overline{H} \tag{1.7}$$

$$\overline{E} = \frac{\overline{J}}{\sigma} \tag{1.8}$$

$$\overline{D} = \varepsilon\overline{E} \tag{1.9}$$

The Following Maxwell's equations are obtained.

$$\nabla \bullet \overline{D} = 0 \tag{1.10}$$

$$\nabla \times \overline{H} = (\sigma + j\omega\varepsilon)\overline{E} \tag{1.11}$$

$$\nabla \times \overline{E} = -j\omega\mu\overline{H} \tag{1.12}$$

Substitution of equation 1.8 into equation 1.12 yields

$$\nabla \times \overline{J} = -j\omega\mu\sigma\overline{H} \tag{1.13}$$

Taking equation 1.13 and solving for H yields

$$\overline{H} = \frac{\nabla \times \overline{J}}{-j\omega\mu\sigma} \tag{1.14}$$

Substitution of equation 1.14 into equation 1.11 gives

$$\nabla \times \frac{\nabla \times \overline{J}}{-j\omega\mu\sigma} = (\sigma + j\omega\varepsilon)\overline{E} \tag{1.15}$$

Simplifying and using again the relation of equation 1.8 as well as assuming that μ and σ are isotropic or put another way do not vary with direction gives

$$\nabla \times \nabla \times \overline{J} = (-j\omega\mu\sigma + \omega^2\varepsilon\mu)\overline{J} \tag{1.16}$$

From equation 1.16 it can be seen that if σ is much greater than $\omega\varepsilon$ then the term $-j\omega\mu\sigma$ dominates as the multiple of J. For the purposes of this research it is assumed that $\sigma >> \omega\varepsilon$. This is a very important assumption and only results from this derivation that adhere to this constraint are valid. This assumption removes the high frequency phenomena known as displacement current from consideration. Equation 1.16 now becomes

$$\nabla \times \nabla \times \overline{J} = -j\omega\mu\sigma\overline{J} \tag{1.17}$$

Expanding equation 1.17 gives [2]

$$\nabla(\nabla \bullet \overline{J}) - \nabla^2\overline{J} = -j\omega\mu\sigma\overline{J} \tag{1.18}$$

The first term on the left hand side of equation 1.18 is zero. This is because J can be related to E by the conductivity constant and subsequently E can be related to D by the permittivity constant. Since the Divergence of D is zero, the Divergence of J is zero provided that the conductivity and permittivity are isotropic which is a good assumption for copper. Equation 1.18 now becomes

$$\nabla^2 \overline{J} - j\omega\mu\sigma\overline{J} = 0 \tag{1.19}$$

For simplification let

$$\gamma^2 = -j\omega\mu\sigma \tag{1.20}$$

Equation 1.19 becomes

$$\nabla^2 \overline{J} + \gamma^2 \overline{J} = 0 \tag{1.21}$$

Expanding equation 1.21 into cylindrical coordinates recognizing that spatially the magnitude of J is only a function of the radial distance and that its direction is along the z-axis gives

$$\frac{\partial^2 J}{\partial r^2} + \frac{1\partial J}{r\partial r} + \gamma^2 J = 0 \tag{1.22}$$

Multiplying equation 1.22 by r^2 gives

$$r^2 \frac{\partial^2 J}{\partial r^2} + r\frac{\partial J}{\partial r} + r^2\gamma^2 J = 0 \tag{1.23}$$

Multiplying terms of equation 1.23 by appropriate unity factors gives

$$r^2 \frac{\gamma^2\partial^2 J}{\gamma^2\partial r^2} + r\frac{\gamma\partial J}{\gamma\partial r} + r^2\gamma^2 J = 0 \tag{1.24}$$

For simplicity let

$$\xi = \gamma r \tag{1.25}$$

Now equation 1.24 becomes

$$\xi^2 \frac{\partial^2 J}{\partial \xi^2} + \xi\frac{\partial J}{\partial \xi} + \xi^2 J = 0 \tag{1.26}$$

Equation 1.26 is now in the familiar form of Bessel's equation [5]. Solving for J gives

$$J(\xi) = C_1\,\mathbf{J_0}(\xi) + C_2\mathbf{Y}_0(\xi) \tag{1.27}$$

Equation 1.27 is a linear combination of a zero order Bessel function of the first kind and a zero order Bessel function of the second kind. C_1 and C_2 are constants. The bold face **J** should not be confused with the current density. Since it is common to write the Bessel function of the first kind as **J** it is kept here but made in bold face type for consistency with other literature. To solve for these constants, boundary conditions must be employed. There are two important boundary conditions and these are

$$J(R_0) = J_0 \tag{1.28}$$

$$\frac{\partial J(0)}{\partial r} = 0 \tag{1.29}$$

Equation 1.28 is the prescription of the current density on the surface while equation 1.29 is a mathematical statement of symmetry. These types of boundary conditions are known as Dirichlet and Neuman Boundary conditions respectively. The first term of 1.27 is well behaved with a finite value with an argument of zero but the second term has a logarithmic singularity at an argument of zero. Because a finite value is expected for J at an argument of zero, Y_0 should be discarded or C_2 is zero. [5]

Substituting 1.28 into 1.27 using the identity in 1.25 as well as $C_2 = 0$ gives

$$C_1 = \frac{J_0}{\mathbf{J_0}(\gamma R_0)} \tag{1.30}$$

Substituting 1.30 into 1.27 using the identity in 1.25 gives

$$J(\gamma r) = \frac{J_0}{\mathbf{J_0}(\gamma R_0)}\mathbf{J_0}(\gamma r) \tag{1.31}$$

Equation 1.31 is the solution of the current density in a cylindrical conductor as a function of γ and r. It can be seen that the Boundary condition 1.29 is also satisfied by equation 1.31 because the derivative of a zero order Bessel function of the first kind is a first order Bessel function which has a zero at an argument of zero.

To fully appreciate the relationship of the parameters including frequency, radius, conductivity and permeability to skin effect, equation 1.31 is normalized with the surface current density. This is shown in equation 1.32

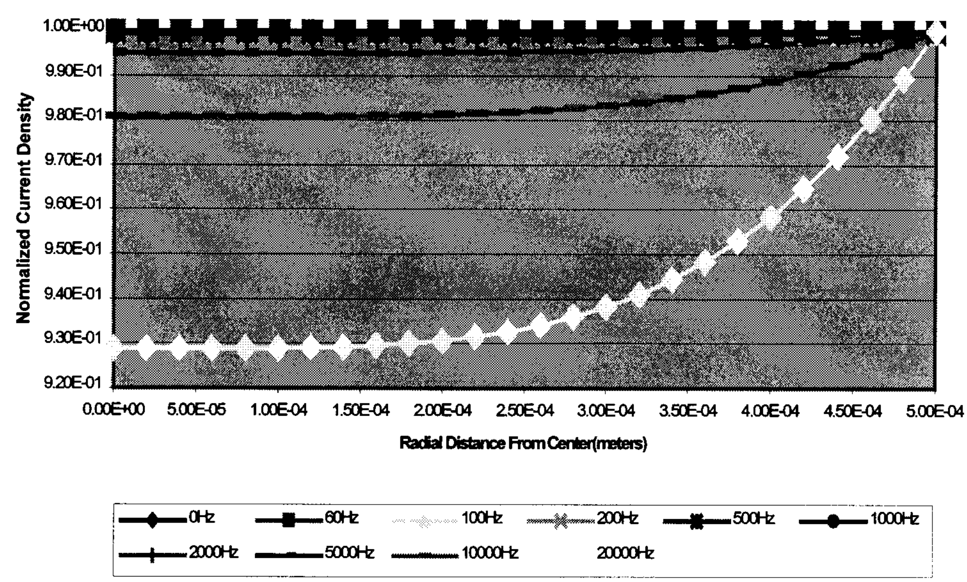
$$J_{normalized} = \frac{\mathbf{J_0}(\gamma r)}{\mathbf{J_0}(\gamma R_0)} \tag{1.32}$$

The magnitudes of equation 1.32 versus parameters are plotted in Figures 1.2-1.8. Table 1.1 represents the variables utilized in Figures 1.2-1.8.

Table 1.1

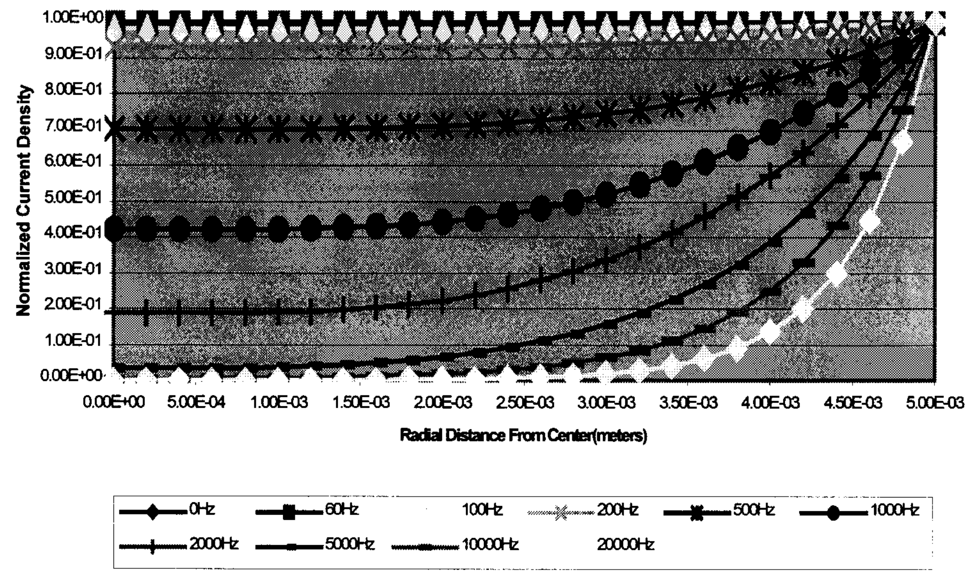
Summary of Figures 1.2 through 1.8

Figure	Radius Meters	Conductivity mhos/meter	Relative Permeability
Figure 1.2	.0005	5.7e7	1
Figure 1.3	.0050	5.7e7	1
Figure 1.4	.0500	5.7e7	1
Figure 1.5	.0050	5.7e6	1
Figure 1.6	.0050	5.7e8	1
Figure 1.7	.0050	5.7e7	300
Figure 1.8	.0050	5.7e7	3000



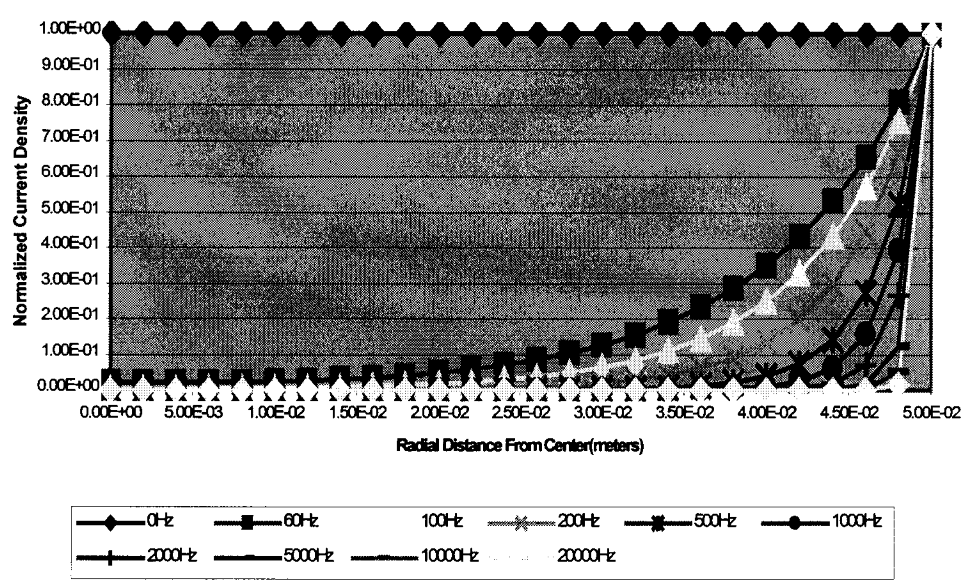
Current Density vs Radial Distance

Figure 1.2



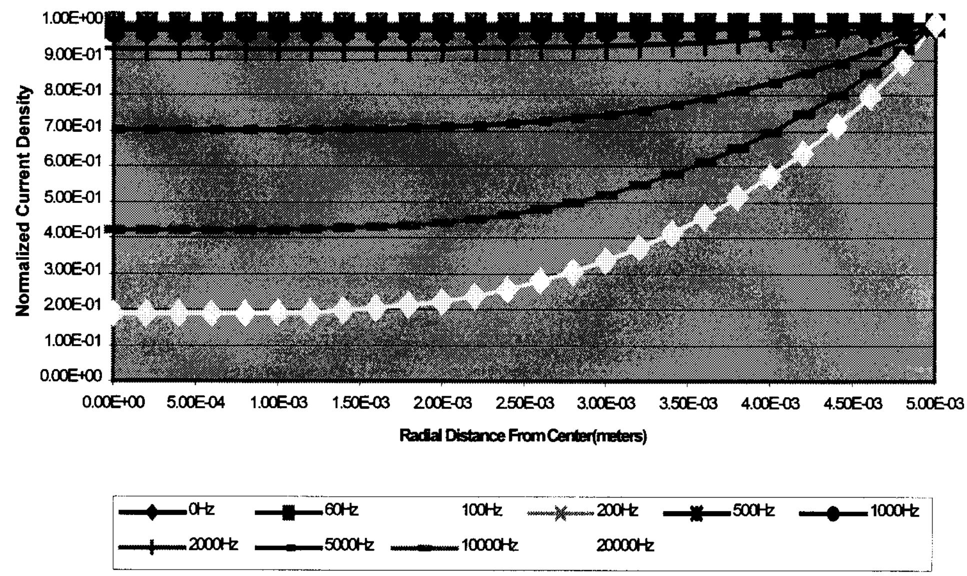
Current Density vs Radial Distance

Figure 1.3



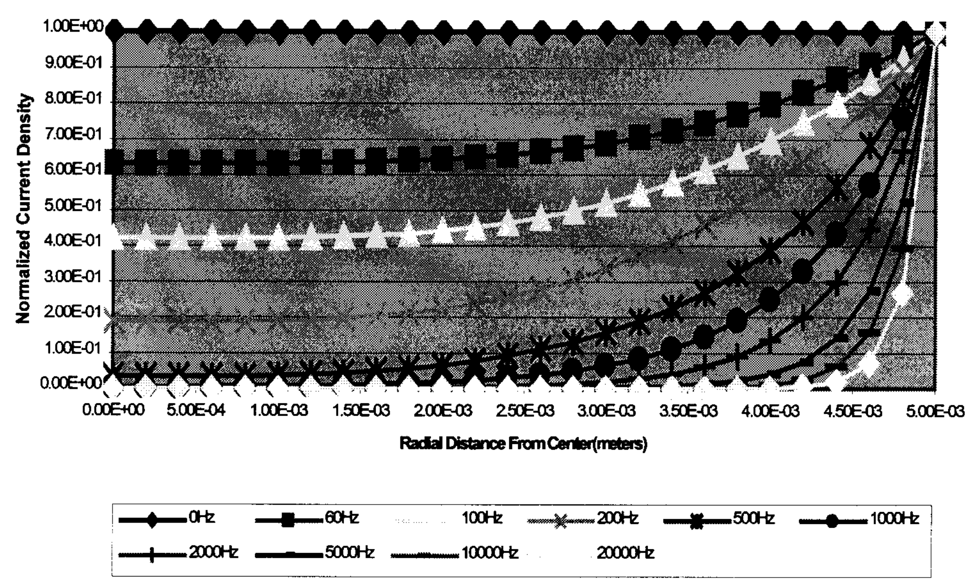
Current Density vs Radial Distance

Figure 1.4



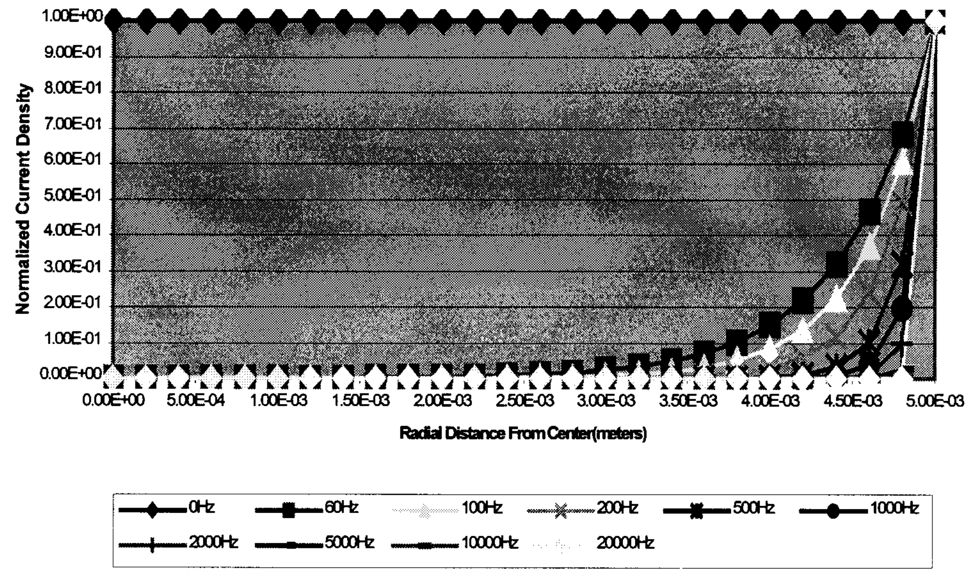
Current Density vs Radial Distance

Figure 1.5



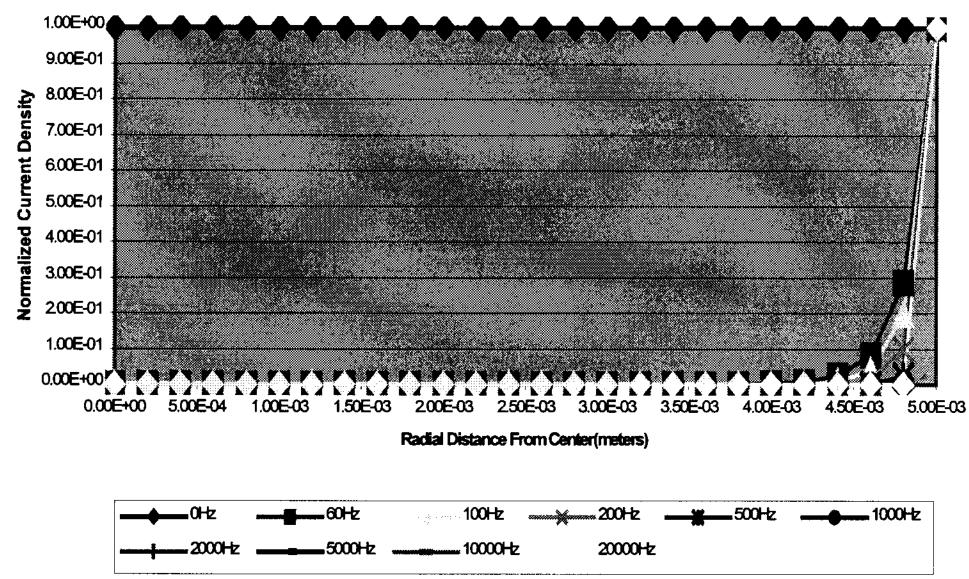
Current Density vs Radial Distance

Figure 1.6



Current Density vs Radial Distance

Figure 1.7



Current Density vs Radial Distance

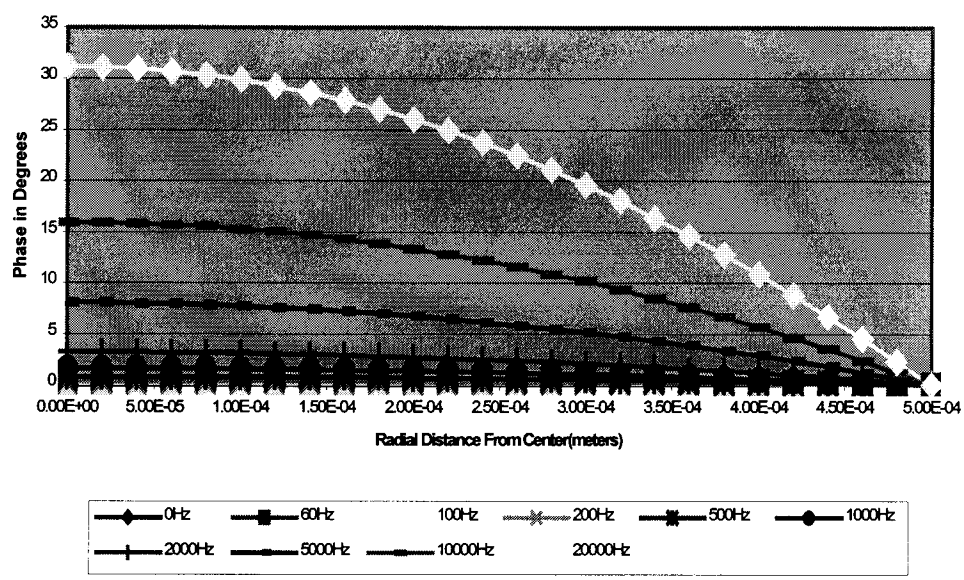
Figure 1.8

The phase angles for equation 1.32 are plotted versus parameters in Figures 1.9-1.15. Table 1.2 is a summary of the variables in Figures 1.2 through 1.8.

Table 1.2

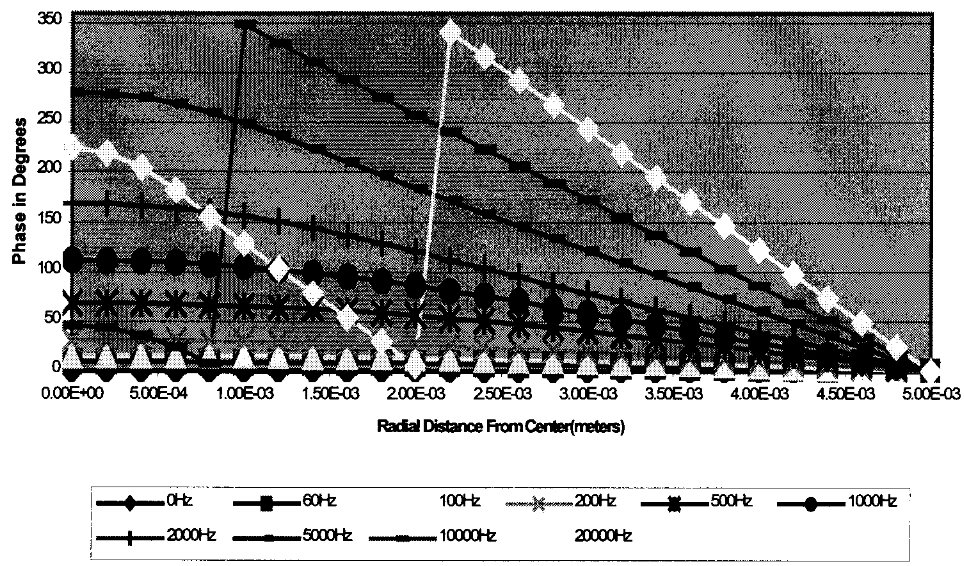
Summary of Figures 1.9 through 1.15

Figure	Radius meters	Conductivity mhos/meter	Relative Permeability
Figure 1.9	.0005	5.7e7	1
Figure 1.10	.0050	5.7e7	1
Figure 1.11	.0500	5.7e7	1
Figure 1.12	.0050	5.7e6	1
Figure 1.13	.0050	5.7e8	1
Figure 1.14	.0050	5.7e7	300
Figure 1.15	.0050	5.7e7	3000



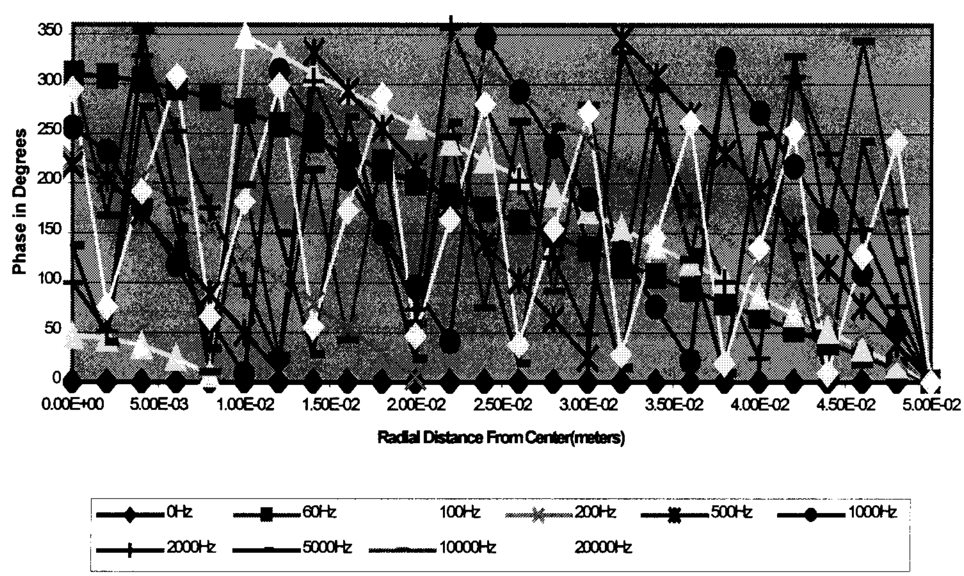
Phase of Current Density vs Radial Distance

Figure 1.9



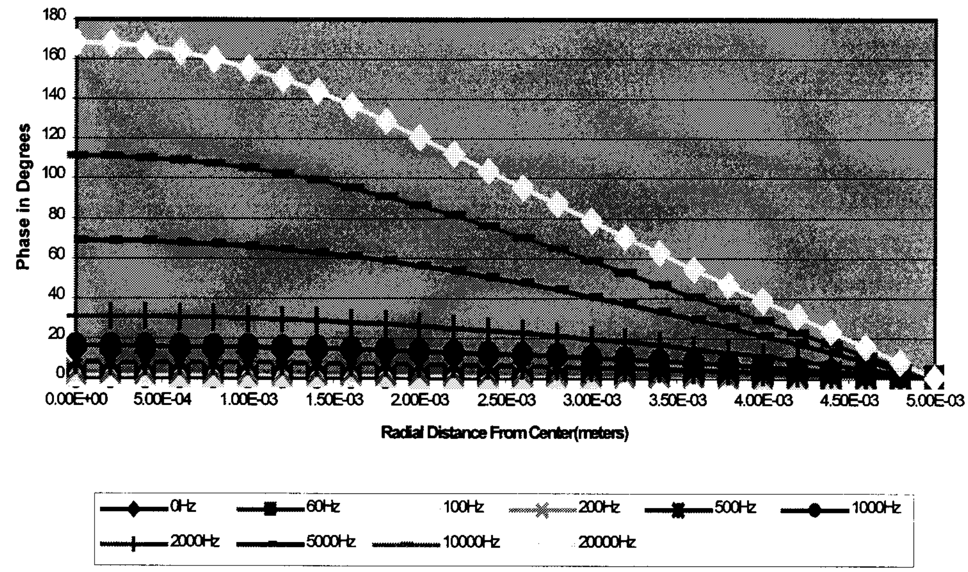
Phase of Current Density vs Radial Distance

Figure 1.10



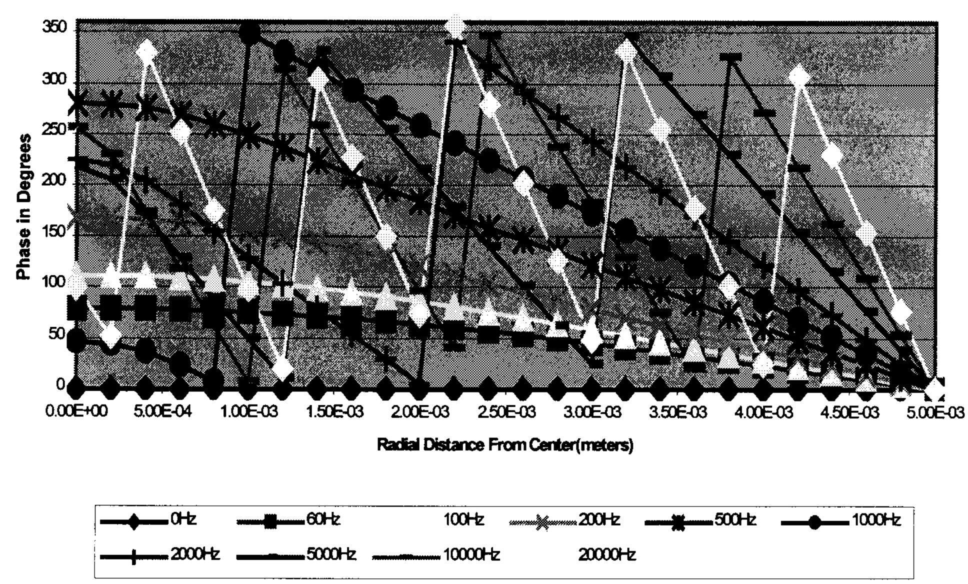
Phase of Current Density vs Radial Distance

Figure 1.11



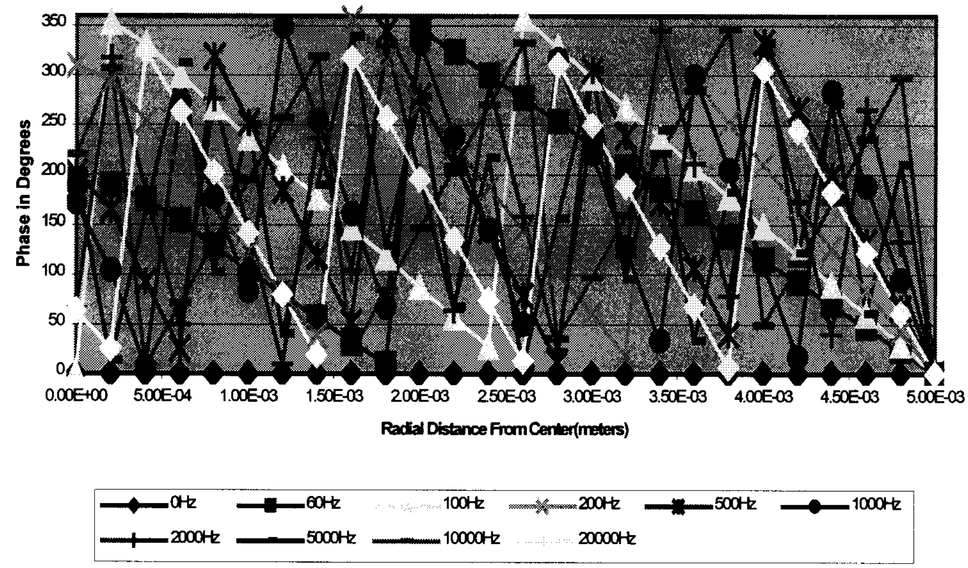
Phase of Current Density vs Radial Distance

Figure 1.12



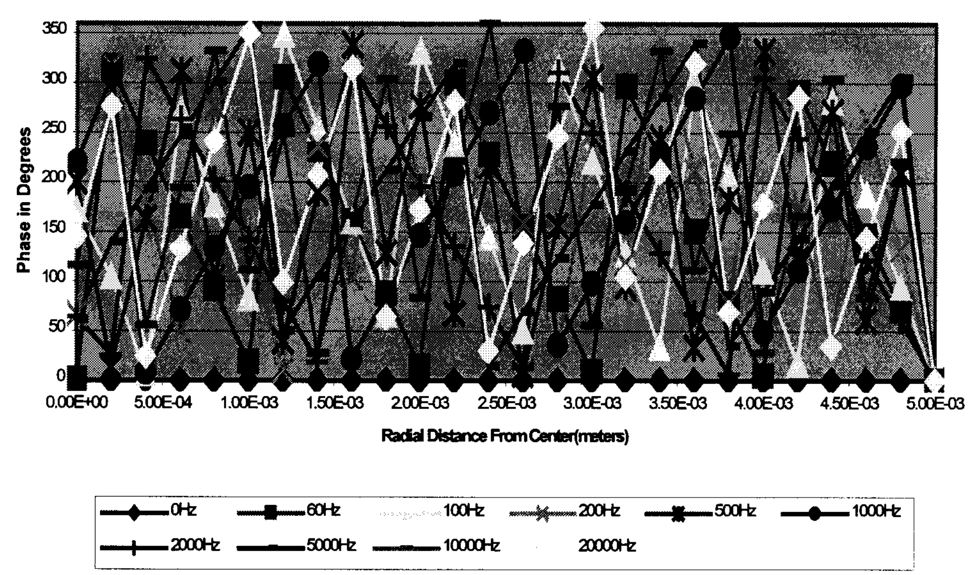
Phase of Current Density vs Radial Distance

Figure 1.13



Phase of Current Density vs Radial Distance

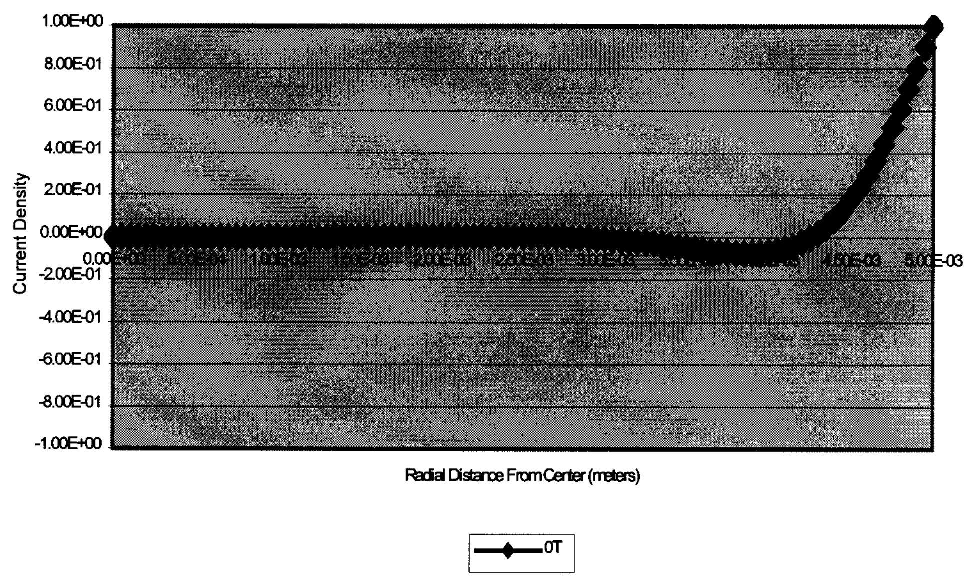
Figure 1.14



Phase of Current Density vs Radial Distance

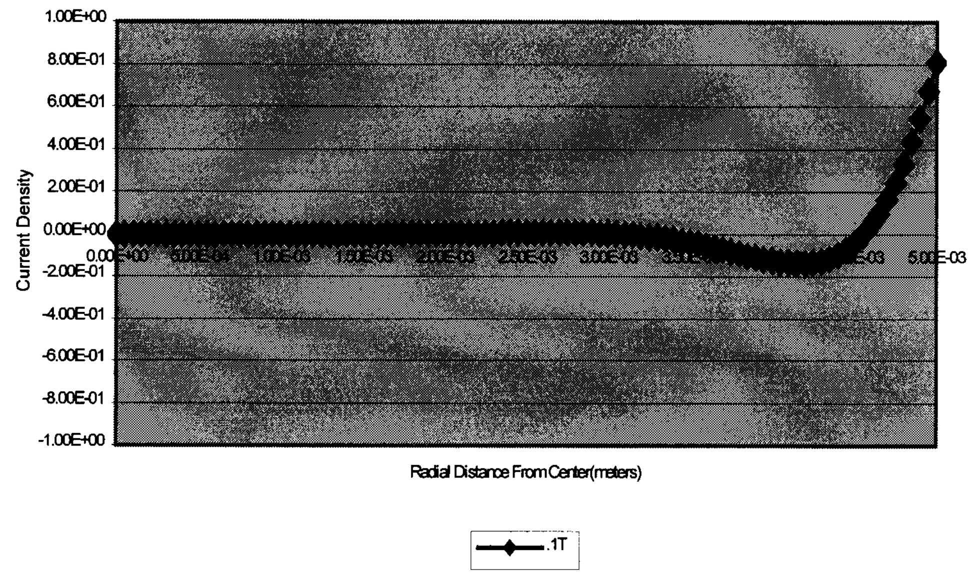
Figure 1.15

By examining Figures 1.2 through 1.15, certain relationships are evident. The magnitude of the current density has decayed exponentially. In most cases the decay is from the surface. Also, the phase angle is retarded. In most cases the phase angle retards linearly from the surface. Also at the very center of the conductor the phase flattens out. This means that the current density in the center lags the current density on the surface. In some cases it lags so much that it is 360 degrees out of phase which makes it appear to be in phase with the surface current density. These concepts are simple but sometimes hard to visualize. Figures 1.16 through 1.26 show snapshots of the 20000 Hz condition of Figure 1.3 at different points in time while Figure 1.27 shows a combined plot of Figures 1.16 to 1.26. The symbol T in Figures 1.16 to 1.26 is the period. So .1T means one tenth of a period into the cycle.



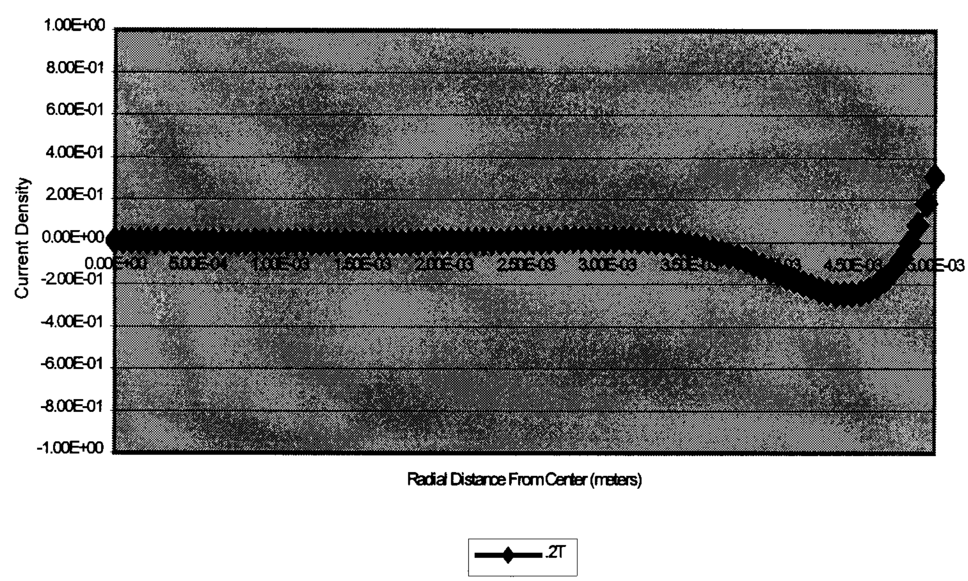
Instantaneous Current Density vs Radial Distance at 0T

Figure 1.16



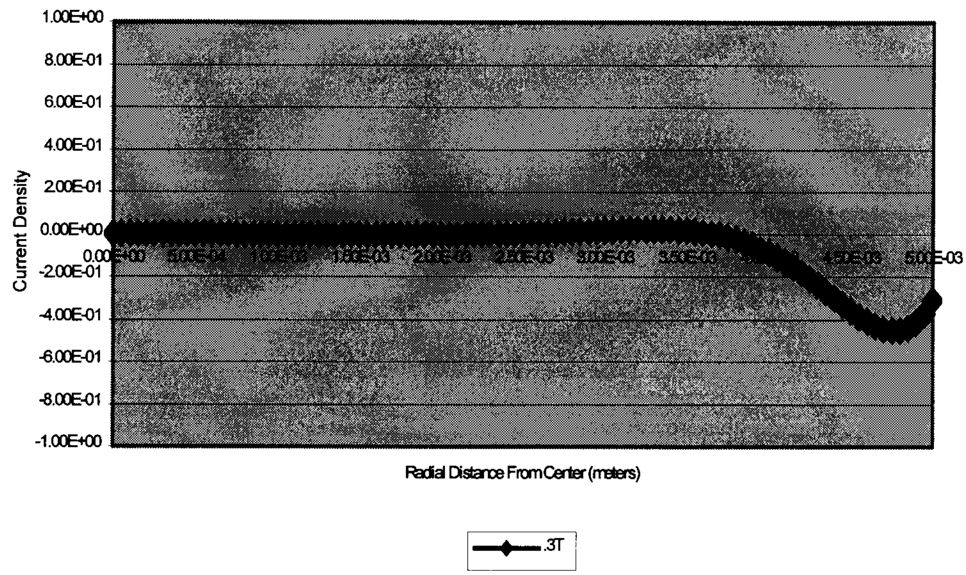
Instantaneous Current Density vs Radial Distance at 0T

Figure 1.17



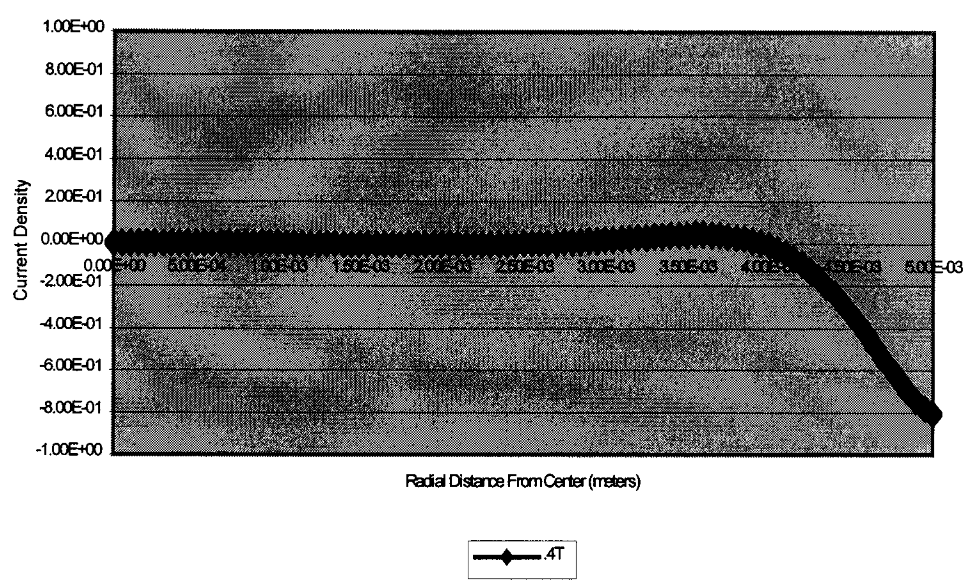
Instantaneous Current Density vs Radial Distance at 0T

Figure 1.18



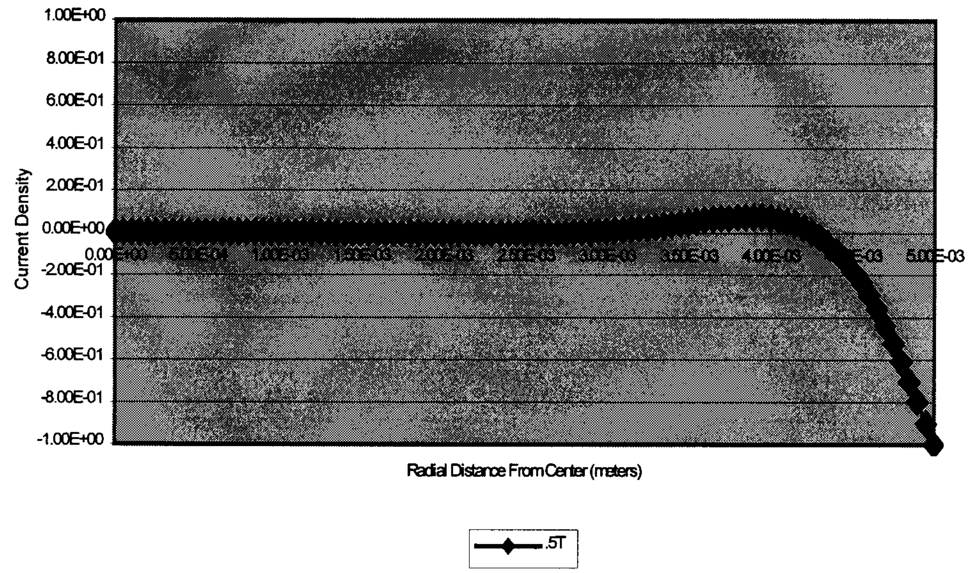
Instantaneous Current Density vs Radial Distance at 0T

Figure 1.19



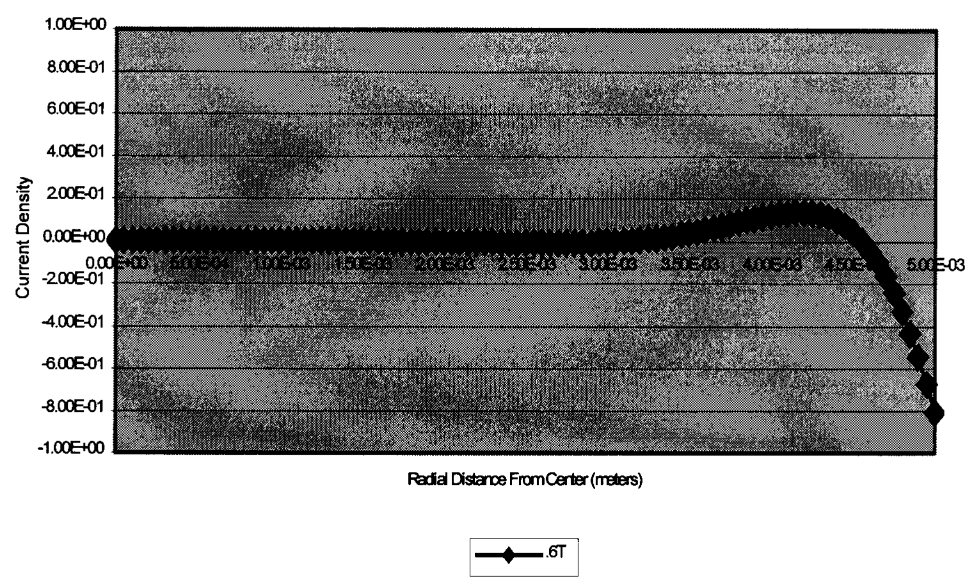
Instantaneous Current Density vs Radial Distance at 0T

Figure 1.20



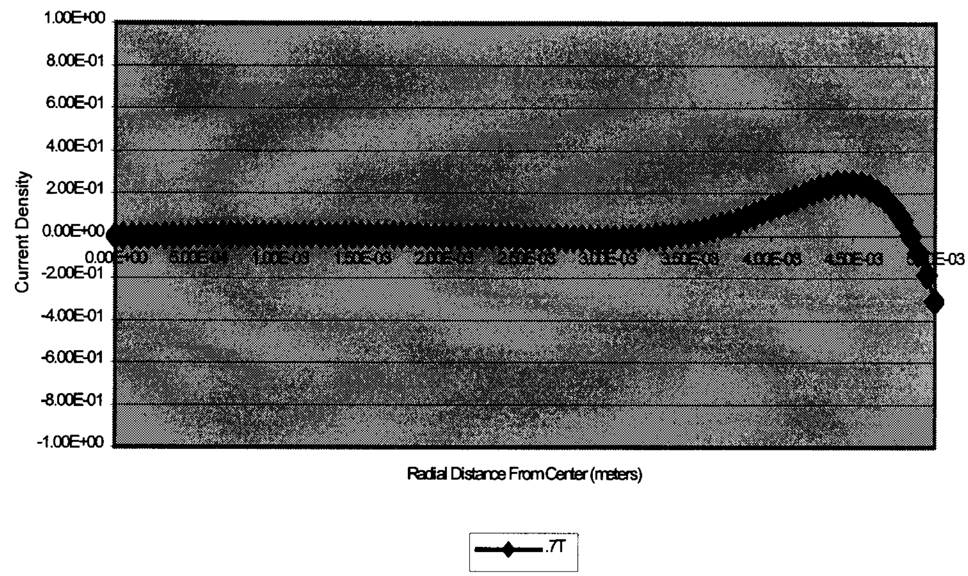
Instantaneous Current Density vs Radial Distance at 0T

Figure 1.21



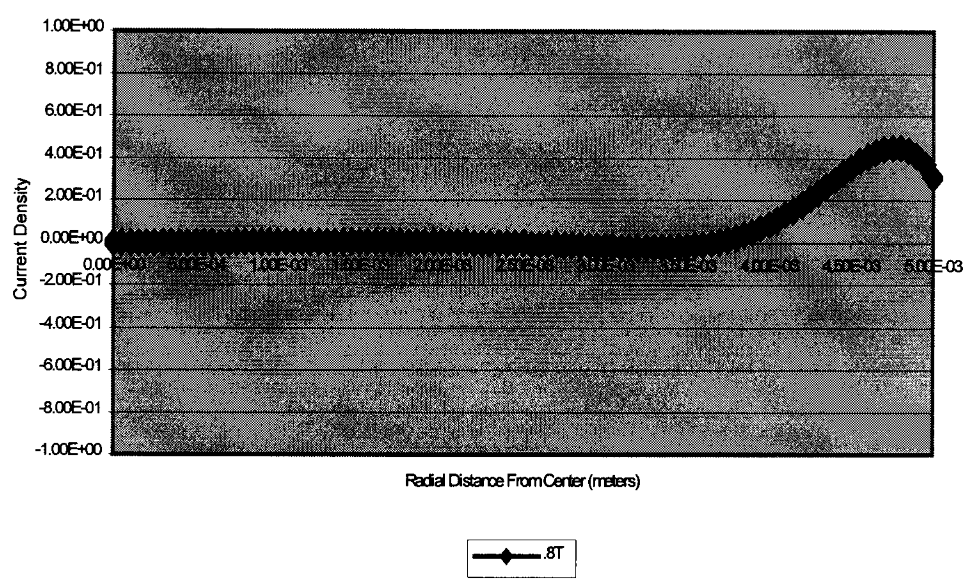
Instantaneous Current Density vs Radial Distance at 0T

Figure 1.22



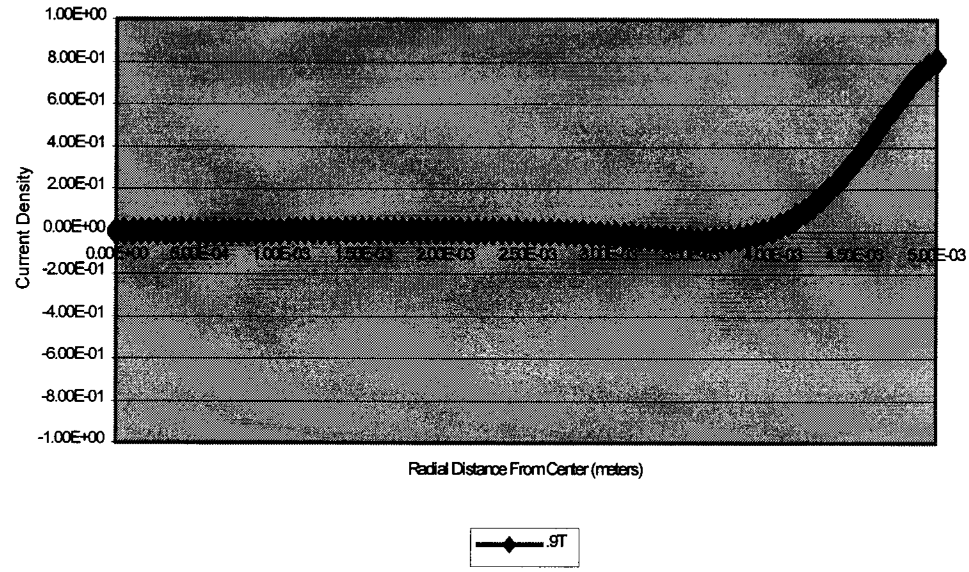
Instantaneous Current Density vs Radial Distance at 0T

Figure 1.23



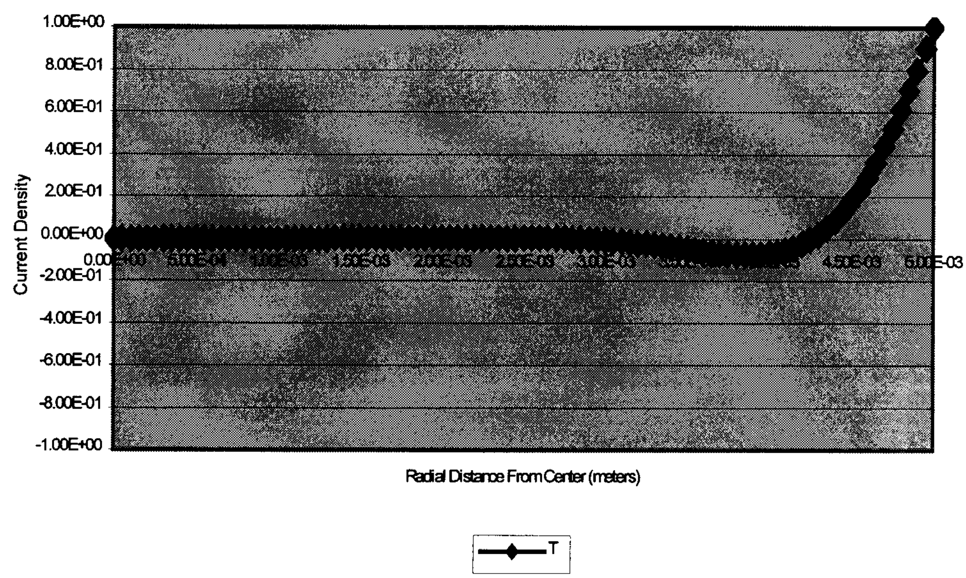
Instantaneous Current Density vs Radial Distance at 0T

Figure 1.24



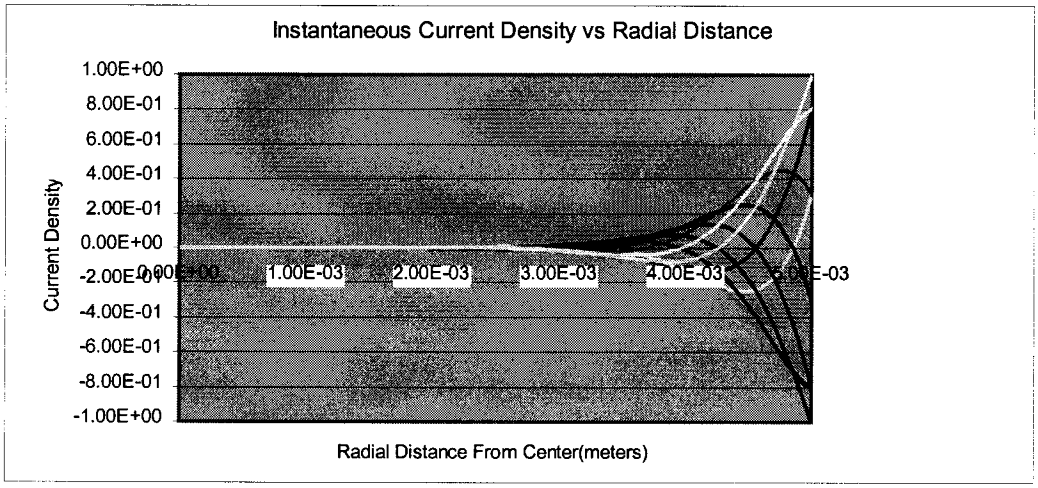
Instantaneous Current Density vs Radial Distance at 0T

Figure 1.25



Instantaneous Current Density vs Radial Distance at 0T

Figure 1.26



Instantaneous Current Density vs Radial Distance for 1 Period

Figure 1.27

It is apparent that under the right circumstances the phase angle of the current density can be the same at several radial points in the conductor. This means that at a given instant in time, the surface current is moving in one direction while the current somewhere deeper is moving in the completely opposite direction. Also, somewhere deeper still, the current is in the same direction as the current on the surface. These statements are general and there is a more standard way to describe the degree of skin effect as well as the effect of various physical parameters on it. The degree of skin effect is sometimes referred to as the Skin Depth and is defined as the distance into the conductor at which the current density falls to 1/e of its value on the surface. A good approximation for the Skin Depth is shown in equation 1.33 [1].

$$\delta = \frac{1}{\sqrt{f\pi\mu\sigma}} \tag{1.33}$$

Where

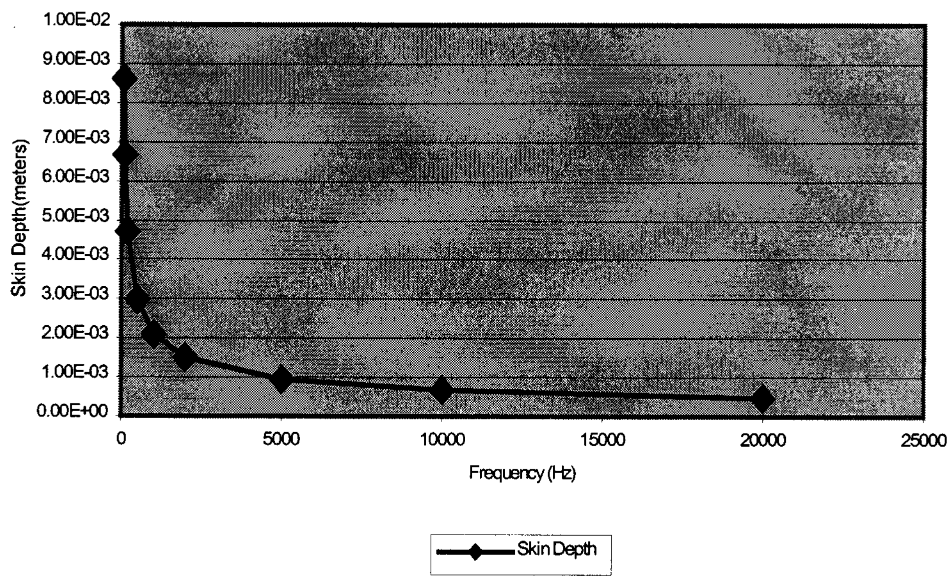
μ is the permeability of the medium in henrys per meter

σ is the conductivity of the conductor in mhos per meter

f is the frequency in Hz

δ is the Skin Depth in meters

Equation 1.33 is nothing more than the reciprocal of the real part of γ. Figure 1.28 shows the Skin Depth of the conductor from Figure 1.3 as a function of frequency.



Skin Depth vs Frequency

Figure 1.28

In Figure 1.28 it can be seen that the current has concentrated on the surface as a function of frequency. Because of this concentration, the resistance is higher than the DC case. A good approximation of the resistance of a conductor in the DC case is shown in equation 1.34 [6].

$$R = \frac{l}{A\sigma} \tag{1.34}$$

Where

R is the resistance

L is the length of the conductor

σ is the conductivity of the conductor

A is the cross sectional area of the conductor

In the AC case, however, the approximation for the resistance is shown in equation 1.35.

$$R = \frac{l}{\pi\sigma(r-\delta)^2} \tag{1.35}$$

Where

R is the resistance

δ is the Skin Depth

σ is the conductivity

r is the radius of the conductor

In this chapter the phenomena of Skin Effect was developed from Maxwell's equations. Several important results were revealed. Namely, the frequency, conductivity, permeability, and radius all play a major role in the current distribution throughout a conductor. In turn, the current distribution determines the AC resistance of the conductor and subsequently the real power loss in the conductor. In the next chapter, a special type of conductor known as Litz that counteracts Skin Effect is introduced.

CHAPTER II

LITZ WIRE

Litz wire is a type of conductor that is made up of small diameter wires that are insulated from one another. These wires are twisted, stranded or braided in a consistent pattern along the length of the wire. The word, Litz, is derived from the German word, "Litzendraht", meaning many stranded [1]. Because of this construction, any component strand of the Litz wire is forced to travel throughout the cross section of the whole Litz wire over a certain distance. Stated another way, there is no opportunity for the current to concentrate on the surface conductors of the Litz wire because the wires on the surface at one location might be in the center at another location along the length.

Because of this construction the Litz wire has an AC resistance equal to its DC resistance. It may be unclear as to why the wires are braided. If they are insulated from one another, then it may seem reasonable to conclude that they would have the same current. After all, they are all in parallel and physically identical. The reason that they need to be braided is to overcome the phenomena known as proximity effect.

The proximity effect is the tendency of the current to concentrate on the outer wires when parallel wires are in close proximity to one another. If the conductors in a Litz wire were straight and not braided, then the current would concentrate on the outer conductors. Being braided, there are no wires that can be considered either exclusively outer or inner. They zigzag throughout the wire and occupy the entire cross-section over a certain length. In this way no conductor is favored by position and as a consequence, each conductor in a Litz wire carries essentially the same current.

It was stated in closing in Chapter I that Litz wire could overcome skin effect but nothing was said about proximity effect. This is because they are one in the same. If the conductor analyzed in Chapter I was broken up conceptually into many conductors in parallel with each other then the skin effect could be explained precisely by proximity effect. Proximity effect and skin effect can be effectively explained by the mutual coupling of current carrying conductors whether real like in proximity effect or conceptual like in skin effect. For now, skin effect is used when referring to a single conductor and proximity effect is used when referring to multiple conductors.

Even if the proximity effect of all the insulated conductors can be overcome, this does not mean that the current distribution in a single insulated wire is constant. This may not be the case. Certain considerations must be kept in mind when designing a power distribution system with Litz wire. The individual strand size, for example, can cause the AC resistance to be greater than the DC resistance. This is because of what was found in Chapter I. Each component conductor of a Litz wire is just a normal conductor and subject to skin effect with the right combination of frequency, conductivity, and size.

Another design consideration is the insulation between each strand. For Litz wire to be effective, each component strand must be insulated from every other component strand. This insulation is usually a nylon shellac or film, which is good to a temperature of about 300 °F. If the application is in high temperature due to sufficient I squared R heating or due to the environment, then the result can be catastrophic if the insulation breaks down. That is why in high current applications, it is sometimes necessary to provide cooling for the Litz wire.

The insulation, component size, and construction of the Litz wire counteracts the skin/proximity effect but it also contributes to the resistance in a more fundamental way. Because of the insulation, there is less copper in the cross section than there would be if it were a normal conductor. That is, the whole cross section of the Litz wire is not all copper. Typical percent copper numbers or "packing" range from 55-70% copper. In addition to this, because of the twisting, braiding and stranding, the component strands are longer than the Litz wire. This adds to the resistance as well. These factors and their influence on the strand and total resistance can be seen in equation 2.1.

$$R_{strand} = \frac{\rho L L_f}{A A_f} \qquad 2.1$$

Where

R_{strand} is the Resistance of a strand of the Litz wire

L is the length of the Litz wire

L_f is the Length factor

A is the cross sectional area of a strand of Litz Wire including insulation

A_f is the packing factor

Taken by themselves the current in a single normal conductor or a Litz conductor can be easily calculated. For the case of multiple conductors and especially if some are Litz and some are not, then the problem becomes complex. Even the simple case of two cylindrical conductors carrying an alternating current has no closed form solution and the only practical method is to utilize a computer model. In the next chapter, a model is developed to calculate the current distribution in bus bar configurations where both normal and Litz wires are present.

CHAPTER III

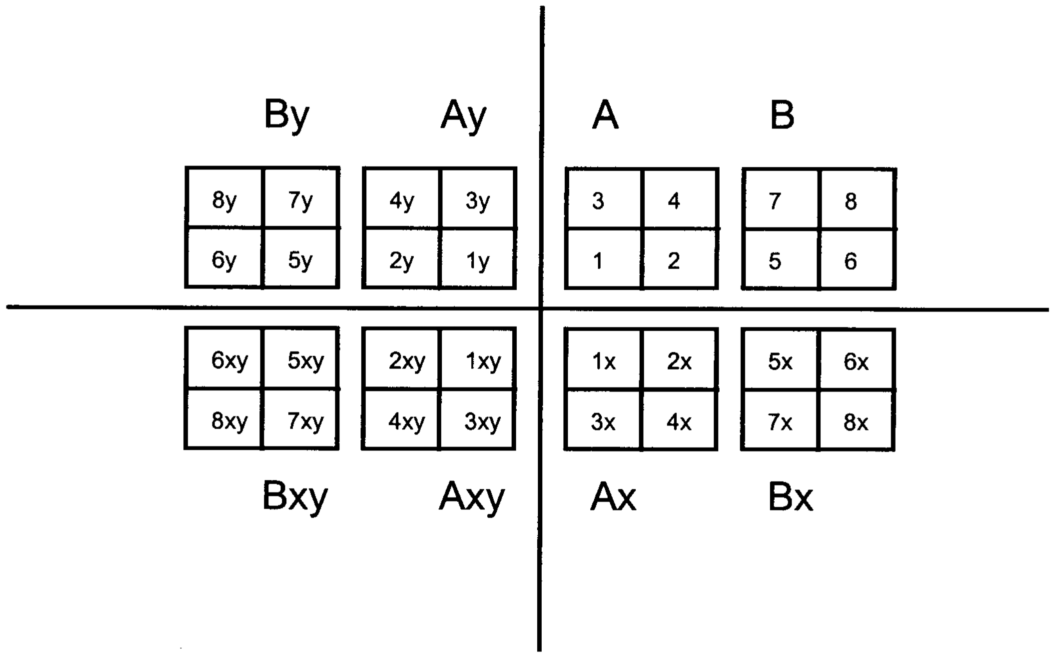
MODEL

The purpose of the model is to solve for the current distribution in conductor configurations that are made up of both Litz and regular conductors at different frequencies. The model presented in this research is a modification of the coupled circuit method that uses resistance and self and mutual inductances. The model equations that are derived below are for the specific configuration that has both Litz and regular conductors as well as symmetry. In this way, the core components of the general model can be derived from this specific case. The general model is then shown utilizing pseudo code.

Figure 3.1 shows eight conductors labeled A, B, Ax, Bx, Ay, By, Axy, Bxy. Each conductor is arbitrarily broken up conceptually into four smaller segments or elements. These are square segments and all are of identical dimensions. The x, y and xy notation denotes the type of symmetry. Ax is the same type of conductor as A and is symmetrical with A about the x-axis. Bxy is the same type of conductor as B and is symmetrical with B about the origin. In this specific case, A denotes a Litz type wire and B denotes a normal conductor.

The numbers in Figure 3.1 are the segment numbers and utilize the same nomenclature for symmetry as the conductors did. The configuration shown in Figure 3.1 has both Litz wire and regular conductors with all type of symmetry. The reason for this is to derive not only the model for the coupling of Litz and regular conductors but to derive the utilization of symmetry as well. The symmetry utilization drastically reduces the computation time and in addition allows more efficient memory usage.

The method that is used here is a coupled circuit method. Familiar circuit theory quantities such as voltage, current, resistance, self and mutual inductance are used [4].



General Arrangement of Conductors with Symmetry for Model Derivation

Figure 3.1

For this specific example, the conductors of Figure 3.1 are constrained to have a sinusoidal voltage of magnitude V_0 across them with the passive sign convention such that the front of the conductors are the positive polarity. Now circuit equations can be written for any segment of any conductor. Writing Ohm's Law for segment 1 gives equation 3.1 [3].

$$\begin{aligned}
V_1 &= Z_{11}I_1 + Z_{12}I_2 + Z_{13}I_3 + Z_{14}I_4 \\
&+ Z_{15}I_5 + Z_{16}I_6 + Z_{17}I_7 + Z_{18}I_8 \\
&+ Z_{11y}I_{1y} + Z_{12y}I_{2y} + Z_{13y}I_{3y} + Z_{14y}I_{4y} \\
&+ Z_{15y}I_{5y} + Z_{16y}I_{6y} + Z_{17y}I_{7y} + Z_{18y}I_{8y} \\
&+ Z_{11xy}I_{1xy} + Z_{12xy}I_{2xy} + Z_{13xy}I_{3xy} + Z_{14xy}I_{4xy} \\
&+ Z_{15xy}I_{5xy} + Z_{16xy}I_{6xy} + Z_{17xy}I_{7xy} + Z_{18xy}I_{8xy} \\
&+ Z_{11x}I_{1x} + Z_{12x}I_{2x} + Z_{13x}I_{3x} + Z_{14x}I_{4x} \\
&+ Z_{15x}I_{5x} + Z_{16x}I_{6x} + Z_{17x}I_{7x} + Z_{18x}I_{8x}
\end{aligned} \tag{3.1}$$

Where

V is the voltage across the segment

I is the Current through the segment

Z is the impedance either self or mutual

The two digit subscripts on the impedance should be read as the impedance at conductor "1st digit" due to the conductor at "2nd digit". Conductor A segment 1 has 3 symmetrical counterparts. In this case the x and y symmetry are both positive because all the conductors have the same voltage across them. This means that the sign of the current in A is the same as all its symmetrical counterparts. Symmetry about an axis is either positive (+1) or negative (–1) and is determined by the user of the model for each specific case. Table 3.1 shows the relationships of the different symmetries on current and voltage. In the 2nd quadrant, for example, the value of the current solved for in quadrant 1 must be multiplied by the sign of the Y symmetry to get the solution in quadrant 2.

Table 3.1

Symmetry Relationship

Quadrant	Current	Voltage
1st	I	V
2nd	(I)(Ysym)	(V)(Ysym)
3rd	(I)(Xsym)(Ysym)	(V)(Xsym)(Ysym)
4th	(I)(Xsym)	(V)(Ysym)

Utilizing symmetry equation 3.1 can be rewritten as

$$\begin{aligned} V_1 = {} & (Z_{11} + Z_{11y} + Z_{11xy} + Z_{11x})I_1 \\ & + (Z_{12} + Z_{12y} + Z_{12xy} + Z_{12x})I_2 \\ & + (Z_{13} + Z_{13y} + Z_{13xy} + Z_{13x})I_3 \\ & + (Z_{14} + Z_{14y} + Z_{14xy} + Z_{14x})I_4 \\ & + (Z_{15} + Z_{15y} + Z_{15xy} + Z_{15x})I_5 \\ & + (Z_{16} + Z_{16y} + Z_{16xy} + Z_{16x})I_6 \\ & + (Z_{17} + Z_{17y} + Z_{17xy} + Z_{17x})I_7 \\ & + (Z_{18} + Z_{18y} + Z_{18xy} + Z_{18x})I_8 \end{aligned} \qquad (3.2)$$

Similar equations can also be written at Segments 2-8.

$$\begin{aligned} V_2 = &(Z_{21} + Z_{21y} + Z_{21xy} + Z_{21x})I_1 \\ &+ (Z_{22} + Z_{22y} + Z_{22xy} + Z_{22x})I_2 \\ &+ (Z_{23} + Z_{23y} + Z_{23xy} + Z_{23x})I_3 \\ &+ (Z_{24} + Z_{24y} + Z_{24xy} + Z_{24x})I_4 \\ &+ (Z_{25} + Z_{25y} + Z_{25xy} + Z_{25x})I_5 \\ &+ (Z_{26} + Z_{26y} + Z_{26xy} + Z_{26x})I_6 \\ &+ (Z_{27} + Z_{27y} + Z_{27xy} + Z_{27x})I_7 \\ &+ (Z_{28} + Z_{28y} + Z_{28xy} + Z_{28x})I_8 \end{aligned} \tag{3.3}$$

$$\begin{aligned} V_3 = &(Z_{31} + Z_{31y} + Z_{31xy} + Z_{31x})I_1 \\ &+ (Z_{32} + Z_{32y} + Z_{32xy} + Z_{32x})I_2 \\ &+ (Z_{33} + Z_{33y} + Z_{33xy} + Z_{33x})I_3 \\ &+ (Z_{34} + Z_{34y} + Z_{34xy} + Z_{34x})I_4 \\ &+ (Z_{35} + Z_{35y} + Z_{35xy} + Z_{35x})I_5 \\ &+ (Z_{36} + Z_{36y} + Z_{36xy} + Z_{36x})I_6 \\ &+ (Z_{37} + Z_{37y} + Z_{37xy} + Z_{37x})I_7 \\ &+ (Z_{38} + Z_{38y} + Z_{38xy} + Z_{38x})I_8 \end{aligned} \tag{3.4}$$

$$\begin{aligned} V_4 = &(Z_{41} + Z_{41y} + Z_{41xy} + Z_{41x})I_1 \\ &+ (Z_{42} + Z_{42y} + Z_{42xy} + Z_{42x})I_2 \\ &+ (Z_{43} + Z_{43y} + Z_{43xy} + Z_{43x})I_3 \\ &+ (Z_{44} + Z_{44y} + Z_{44xy} + Z_{44x})I_4 \\ &+ (Z_{45} + Z_{45y} + Z_{45xy} + Z_{45x})I_5 \\ &+ (Z_{46} + Z_{46y} + Z_{46xy} + Z_{46x})I_6 \\ &+ (Z_{47} + Z_{47y} + Z_{47xy} + Z_{47x})I_7 \\ &+ (Z_{48} + Z_{48y} + Z_{48xy} + Z_{48x})I_8 \end{aligned} \tag{3.5}$$

$$
\begin{aligned}
V_5 = &(Z_{51} + Z_{51y} + Z_{51xy} + Z_{51x})I_1 \\
&+(Z_{52} + Z_{52y} + Z_{52xy} + Z_{52x})I_2 \\
&+(Z_{53} + Z_{53y} + Z_{53xy} + Z_{53x})I_3 \\
&+(Z_{54} + Z_{54y} + Z_{54xy} + Z_{54x})I_4 \\
&+(Z_{55} + Z_{55y} + Z_{55xy} + Z_{55x})I_5 \\
&+(Z_{56} + Z_{56y} + Z_{56xy} + Z_{56x})I_6 \\
&+(Z_{57} + Z_{57y} + Z_{57xy} + Z_{57x})I_7 \\
&+(Z_{58} + Z_{58y} + Z_{58xy} + Z_{58x})I_8
\end{aligned}
\tag{3.6}
$$

$$
\begin{aligned}
V_6 = &(Z_{61} + Z_{61y} + Z_{61xy} + Z_{61x})I_1 \\
&+(Z_{62} + Z_{62y} + Z_{62xy} + Z_{62x})I_2 \\
&+(Z_{63} + Z_{63y} + Z_{63xy} + Z_{63x})I_3 \\
&+(Z_{64} + Z_{64y} + Z_{64xy} + Z_{64x})I_4 \\
&+(Z_{65} + Z_{65y} + Z_{65xy} + Z_{65x})I_5 \\
&+(Z_{66} + Z_{66y} + Z_{66xy} + Z_{66x})I_6 \\
&+(Z_{67} + Z_{67y} + Z_{67xy} + Z_{67x})I_7 \\
&+(Z_{68} + Z_{68y} + Z_{68xy} + Z_{68x})I_8
\end{aligned}
\tag{3.7}
$$

$$
\begin{aligned}
V_7 = &(Z_{71} + Z_{71y} + Z_{71xy} + Z_{71x})I_1 \\
&+(Z_{72} + Z_{72y} + Z_{72xy} + Z_{72x})I_2 \\
&+(Z_{73} + Z_{73y} + Z_{73xy} + Z_{73x})I_3 \\
&+(Z_{74} + Z_{74y} + Z_{74xy} + Z_{74x})I_4 \\
&+(Z_{75} + Z_{75y} + Z_{75xy} + Z_{75x})I_5 \\
&+(Z_{76} + Z_{76y} + Z_{76xy} + Z_{76x})I_6 \\
&+(Z_{77} + Z_{77y} + Z_{77xy} + Z_{77x})I_7 \\
&+(Z_{78} + Z_{78y} + Z_{78xy} + Z_{78x})I_8
\end{aligned}
\tag{3.8}
$$

$$
\begin{aligned}
V_8 = &(Z_{81} + Z_{81y} + Z_{81xy} + Z_{81x})I_1 \\
&+ (Z_{82} + Z_{82y} + Z_{82xy} + Z_{82x})I_2 \\
&+ (Z_{83} + Z_{83y} + Z_{83xy} + Z_{83x})I_3 \\
&+ (Z_{84} + Z_{84y} + Z_{84xy} + Z_{84x})I_4 \\
&+ (Z_{85} + Z_{85y} + Z_{85xy} + Z_{85x})I_5 \\
&+ (Z_{86} + Z_{86y} + Z_{86xy} + Z_{86x})I_6 \\
&+ (Z_{87} + Z_{87y} + Z_{87xy} + Z_{87x})I_7 \\
&+ (Z_{88} + Z_{88y} + Z_{88xy} + Z_{88x})I_8
\end{aligned}
\tag{3.9}
$$

Now there are 8 equations and 8 unknowns. The unknowns are the currents while the voltages are known and replaced with *Vo*. *Vo* is the constrained voltage of all segments because they are in parallel. Now the impedance can be calculated and here lies the actual modeling. It can be seen from equations 3.2 through 3.9 that each multiple of each current consists of four terms. The first term is the 1st quadrant impedance. The remaining three terms are the 2nd , 3rd, and 4th quadrant components due to symmetry. It can also be seen that the diagonal impedances consist of a self impedance of the segment in the 1st quadrant and in addition three mutual impedances with its symmetrical counterparts. The off diagonal terms are all mutual impedances.

For simplification let

$$
\mathbf{Z}_{ij} = Z_{ij} + Z_{ijy} + Z_{ijxy} + Z_{ijx} \tag{3.10}
$$

In equation 3.10 the boldface **Z** is the sum of all the impedance terms that multiply the segment current denoted by the j subscript. The subscript i is the segment at which the equation is being written. The x, y and xy subscripts denote the different symmetry terms.

Substituting *Vo* into equations 3.2 through 3.9 and using equation 3.10 yields

$$V_o = \mathbf{Z}_{11}I_1 + \mathbf{Z}_{12}I_2 + \mathbf{Z}_{13}I_3 + \mathbf{Z}_{14}I_4 + \mathbf{Z}_{15}I_5 + \mathbf{Z}_{16}I_6 + \mathbf{Z}_{17}I_7 + \mathbf{Z}_{18}I_8 \quad (3.11)$$

$$V_o = \mathbf{Z}_{21}I_1 + \mathbf{Z}_{22}I_2 + \mathbf{Z}_{23}I_3 + \mathbf{Z}_{24}I_4 + \mathbf{Z}_{25}I_5 + \mathbf{Z}_{26}I_6 + \mathbf{Z}_{27}I_7 + \mathbf{Z}_{28}I_8 \quad (3.12)$$

$$V_o = \mathbf{Z}_{31}I_1 + \mathbf{Z}_{32}I_2 + \mathbf{Z}_{33}I_3 + \mathbf{Z}_{34}I_4 + \mathbf{Z}_{35}I_5 + \mathbf{Z}_{36}I_6 + \mathbf{Z}_{37}I_7 + \mathbf{Z}_{38}I_8 \quad (3.13)$$

$$V_o = \mathbf{Z}_{41}I_1 + \mathbf{Z}_{42}I_2 + \mathbf{Z}_{43}I_3 + \mathbf{Z}_{44}I_4 + \mathbf{Z}_{45}I_5 + \mathbf{Z}_{46}I_6 + \mathbf{Z}_{47}I_7 + \mathbf{Z}_{48}I_8 \quad (3.14)$$

$$V_o = \mathbf{Z}_{51}I_1 + \mathbf{Z}_{52}I_2 + \mathbf{Z}_{53}I_3 + \mathbf{Z}_{54}I_4 + \mathbf{Z}_{55}I_5 + \mathbf{Z}_{56}I_6 + \mathbf{Z}_{57}I_7 + \mathbf{Z}_{58}I_8 \quad (3.15)$$

$$V_o = \mathbf{Z}_{61}I_1 + \mathbf{Z}_{62}I_2 + \mathbf{Z}_{63}I_3 + \mathbf{Z}_{64}I_4 + \mathbf{Z}_{65}I_5 + \mathbf{Z}_{66}I_6 + \mathbf{Z}_{67}I_7 + \mathbf{Z}_{68}I_8 \quad (3.16)$$

$$V_o = \mathbf{Z}_{71}I_1 + \mathbf{Z}_{72}I_2 + \mathbf{Z}_{73}I_3 + \mathbf{Z}_{74}I_4 + \mathbf{Z}_{75}I_5 + \mathbf{Z}_{76}I_6 + \mathbf{Z}_{77}I_7 + \mathbf{Z}_{78}I_8 \quad (3.17)$$

$$V_o = \mathbf{Z}_{81}I_1 + \mathbf{Z}_{82}I_2 + \mathbf{Z}_{83}I_3 + \mathbf{Z}_{84}I_4 + \mathbf{Z}_{85}I_5 + \mathbf{Z}_{86}I_6 + \mathbf{Z}_{87}I_7 + \mathbf{Z}_{88}I_8 \quad (3.18)$$

These are simultaneous equations relating the current in each 1st quadrant segment to the voltage across them. The 2nd, 3rd and 4th quadrant currents are calculated using symmetry after the 1st quadrant currents are found. Rewriting these in matrix form gives

$$\begin{bmatrix} V_o \\ V_o \\ V_o \\ V_o \\ V_o \\ V_o \\ V_o \\ V_o \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \mathbf{Z}_{13} & \mathbf{Z}_{14} & \mathbf{Z}_{14} & \mathbf{Z}_{16} & \mathbf{Z}_{17} & \mathbf{Z}_{18} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \mathbf{Z}_{23} & \mathbf{Z}_{24} & \mathbf{Z}_{25} & \mathbf{Z}_{26} & \mathbf{Z}_{27} & \mathbf{Z}_{28} \\ \mathbf{Z}_{31} & \mathbf{Z}_{32} & \mathbf{Z}_{33} & \mathbf{Z}_{34} & \mathbf{Z}_{35} & \mathbf{Z}_{36} & \mathbf{Z}_{37} & \mathbf{Z}_{38} \\ \mathbf{Z}_{41} & \mathbf{Z}_{42} & \mathbf{Z}_{43} & \mathbf{Z}_{44} & \mathbf{Z}_{45} & \mathbf{Z}_{46} & \mathbf{Z}_{47} & \mathbf{Z}_{48} \\ \mathbf{Z}_{51} & \mathbf{Z}_{52} & \mathbf{Z}_{53} & \mathbf{Z}_{54} & \mathbf{Z}_{55} & \mathbf{Z}_{56} & \mathbf{Z}_{57} & \mathbf{Z}_{58} \\ \mathbf{Z}_{61} & \mathbf{Z}_{62} & \mathbf{Z}_{63} & \mathbf{Z}_{64} & \mathbf{Z}_{65} & \mathbf{Z}_{66} & \mathbf{Z}_{67} & \mathbf{Z}_{68} \\ \mathbf{Z}_{71} & \mathbf{Z}_{72} & \mathbf{Z}_{73} & \mathbf{Z}_{74} & \mathbf{Z}_{75} & \mathbf{Z}_{76} & \mathbf{Z}_{77} & \mathbf{Z}_{78} \\ \mathbf{Z}_{81} & \mathbf{Z}_{82} & \mathbf{Z}_{83} & \mathbf{Z}_{84} & \mathbf{Z}_{85} & \mathbf{Z}_{86} & \mathbf{Z}_{87} & \mathbf{Z}_{88} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} \quad (3.19)$$

Conductor A is a Litz wire. For this specific example the conductor has been broken up into 4 segments. In the computer model it can be broken up into the exact number of wires composing the Litz. The model further constrains the segment currents of the Litz wire to be identical. After all it was stated in Chapter II that the conductors comprising the Litz wire have identical currents provided that the Litz wire is designed and utilized properly. Let

$$I_1 = I_2 = I_3 = I_4 \tag{3.20}$$

Substitution of equation 3.20 into equation 3.19 yields

$$\begin{bmatrix} V_o \\ V_o \\ V_o \\ V_o \\ V_o \\ V_o \\ V_o \\ V_o \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{14} & Z_{16} & Z_{17} & Z_{18} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} & Z_{26} & Z_{27} & Z_{28} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} & Z_{35} & Z_{36} & Z_{37} & Z_{38} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} & Z_{45} & Z_{46} & Z_{47} & Z_{48} \\ Z_{51} & Z_{52} & Z_{53} & Z_{54} & Z_{55} & Z_{56} & Z_{57} & Z_{58} \\ Z_{61} & Z_{62} & Z_{63} & Z_{64} & Z_{65} & Z_{66} & Z_{67} & Z_{68} \\ Z_{71} & Z_{72} & Z_{73} & Z_{74} & Z_{75} & Z_{76} & Z_{77} & Z_{78} \\ Z_{81} & Z_{82} & Z_{83} & Z_{84} & Z_{85} & Z_{86} & Z_{87} & Z_{88} \end{bmatrix} \begin{bmatrix} I_A \\ I_A \\ I_A \\ I_A \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} \tag{3.21}$$

Where

I_A is the segment current in any of the Litz segments

Equation 3.21 is no longer a set of independent equations. The problem is that there are 8 equations and 5 unknowns. Selecting one of the equations in I_A for use is not very wise due to the fact that whichever one is selected, a particular segment of the Litz wire is favored and would give a different result if used. The solution to this problem is to use the average of all equations in I_A. This is accomplished by summing the equations in I_A and generating the following single equation.

$$\begin{aligned} V_o &= \frac{1}{4}(\mathbf{Z}_{11} + \mathbf{Z}_{12} + \mathbf{Z}_{13} + \mathbf{Z}_{14} + \mathbf{Z}_{21} + \mathbf{Z}_{22} + \mathbf{Z}_{23} + \mathbf{Z}_{24} \\ &+ \mathbf{Z}_{31} + \mathbf{Z}_{32} + \mathbf{Z}_{33} + \mathbf{Z}_{34} + \mathbf{Z}_{41} + \mathbf{Z}_{42} + \mathbf{Z}_{43} + \mathbf{Z}_{44})I_A \\ &+ \frac{1}{4}(\mathbf{Z}_{15} + \mathbf{Z}_{25} + \mathbf{Z}_{35} + \mathbf{Z}_{45})I_5 \\ &+ \frac{1}{4}(\mathbf{Z}_{16} + \mathbf{Z}_{26} + \mathbf{Z}_{36} + \mathbf{Z}_{46})I_6 \\ &+ \frac{1}{4}(\mathbf{Z}_{17} + \mathbf{Z}_{27} + \mathbf{Z}_{37} + \mathbf{Z}_{47})I_7 \\ &+ \frac{1}{4}(\mathbf{Z}_{18} + \mathbf{Z}_{28} + \mathbf{Z}_{38} + \mathbf{Z}_{48})I_8 \end{aligned} \tag{3.22}$$

It is intended that the averaging of the Litz equations reflects that each Litz segment eventually occupies each part of the cross section. It should be noted that the 4 in equation 3.22 is nothing more than the number of segments the Litz wire is broken into in this specific case.

Again for Simplicity let

$$\begin{aligned} S_A &= \frac{1}{4}(\mathbf{Z}_{11} + \mathbf{Z}_{12} + \mathbf{Z}_{13} + \mathbf{Z}_{14} + \mathbf{Z}_{21} + \mathbf{Z}_{22} + \mathbf{Z}_{23} + \mathbf{Z}_{24} \\ &+ \mathbf{Z}_{31} + \mathbf{Z}_{32} + \mathbf{Z}_{33} + \mathbf{Z}_{34} + \mathbf{Z}_{41} + \mathbf{Z}_{42} + \mathbf{Z}_{43} + \mathbf{Z}_{44})I_A \end{aligned} \tag{3.23}$$

$$M_{Aj} = \frac{1}{4}(\mathbf{Z}_{1j} + \mathbf{Z}_{2j} + \mathbf{Z}_{3j} + \mathbf{Z}_{4j}) \tag{3.24}$$

$$M_{jA} = \mathbf{Z}_{j1} + \mathbf{Z}_{j2} + \mathbf{Z}_{j3} + \mathbf{Z}_{j4} \tag{3.25}$$

Substitution of equations 3.23 and 3.24 and 3.25 into 3.21 yield

$$\begin{bmatrix} V_o \\ V_o \\ V_o \\ V_o \\ V_o \end{bmatrix} = \begin{bmatrix} S_A & M_{A5} & M_{A6} & M_{A7} & M_{A8} \\ M_{5A} & \mathbf{Z}_{55} & \mathbf{Z}_{56} & \mathbf{Z}_{57} & \mathbf{Z}_{58} \\ M_{6A} & \mathbf{Z}_{65} & \mathbf{Z}_{66} & \mathbf{Z}_{67} & \mathbf{Z}_{68} \\ M_{7A} & \mathbf{Z}_{75} & \mathbf{Z}_{76} & \mathbf{Z}_{77} & \mathbf{Z}_{78} \\ M_{8A} & \mathbf{Z}_{85} & \mathbf{Z}_{87} & \mathbf{Z}_{87} & \mathbf{Z}_{88} \end{bmatrix} \begin{bmatrix} I_A \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} \qquad (3.26)$$

Solving equation 3.26 for the current vector yields

$$\tilde{\mathbf{I}} = \mathbf{Z}^{-1}\tilde{\mathbf{V}} \qquad (3.27)$$

Where

$\tilde{\mathbf{I}}$ is the column matrix representing the current

$\mathbf{Z}^{-1}$ is the inverse of the impedance matrix

$\tilde{\mathbf{V}}$ is the column matrix representing the voltage

Before the above solution will work, however, all the impedances must be calculated. There are only three types of calculations that need to be performed. These are resistance, self inductance and mutual inductance. Capacitance is omitted in this model. Because displacement current gives rise to capacitance in wires, the capacitance can be ignored provided that the inequality $\sigma >> \omega\varepsilon$ of Chapter I holds true. Z_{11} in equation 3.1 is made up of the resistance and self reactance of segment 1. To calculate the resistance of segment 1, equation 1.34 can be used. It should be noted that the use of equation 1.34 assumes constant current density within the segment. If however the conductors are broken up into many segments, then the current density can be assumed to be constant over each segment. This should be a good assumption provided that the size of each segment is smaller than the skin depth defined in Chapter I. To calculate the self reactance, the self inductance must be calculated. The self inductance of a segment is calculated using the following equation with input units in cm and output units of micro henries [7].

$$L = .002l\left[\ln\left(\frac{2l}{B+C}\right) + \frac{1}{2} - K\right] \qquad (3.29)$$

Where

l is the depth of the conductor

B and C are the length and width of the conductor cross section, respectively

K is given by the following expression

$$K = \left[\ln\left(\frac{r}{B + C} \right) + 1.5 \right] \tag{3.30}$$

Where

r is the Self Geometric Mean Distance of the segment and is given by

$$r = \exp(T1 + T2 + T3 + T4 + T5 + T6) \tag{3.31}$$

Where

$$T1 = 0.5 \ln(B^2 + C^2) \tag{3.32}$$

$$T2 = \frac{-C^2 \ln\left(1 + \frac{B^2}{C^2} \right)}{12 B^2} \tag{3.33}$$

$$T3 = \frac{-B^2 \ln\left(1 + \frac{C^2}{B^2} \right)}{12 C^2} \tag{3.34}$$

$$T4 = \frac{2C \arctan\left(\frac{B}{C} \right)}{3B} \tag{3.35}$$

$$T5 = \frac{2B \arctan\left(\frac{C}{B} \right)}{3C} \tag{3.36}$$

$$T6 = -\frac{25}{12} \tag{3.37}$$

From the self inductance the self reactance can be calculated by simply multiplying by the self inductance by the radian frequency and making the result imaginary.

In equation 3.1, the 2nd impedance term is Z_{12}. Another way of writing this term is

$$Z_{12} = j\omega M_{12} \tag{3.38}$$

Where

M_{12} is the mutual inductance between segment 1 with 2.

The mutual inductance is calculated from the following equation with input units in cm and output units of micro henries [7].

$$M = .002l\left[\ln\left(\frac{l}{d}+\sqrt{1+\frac{l^2}{d^2}}\right)-\sqrt{1+\frac{d^2}{l^2}}+\frac{d}{l}\right] \tag{3.39}$$

Where

l is the depth of the segment

d is the distance between two segments

It should be noted that the preceding inductance equations are based on Neuman's inductance formula which is shown below and integrating it over the cross section of the conductor/conductors [7].

$$M = \frac{\mu}{4\pi}\frac{\int\int ds \bullet ds'}{d} \tag{3.40}$$

Where

M is the mutual inductance of element ds from element ds',

ds and ds' are differential current carrying circuit elements, and

d is the distance between the elements.

The program shown in Appendix B is developed to model and solve equation 3.27. It reads in the input data with the subroutine LITZIO. Next the voltage vector is made with VMAKE. The construction of the voltage vector is trivial because it is simply input into the model by the user. The Z matrix, however, is very complicated and it is at the heart of the model. What follows is a detailed description of how the subroutine ZMAKE constructs the impedance matrix.

The impedance matrix in equation 3.26 is made up of rows and columns. The row number can be thought of as the segment at which the equation is being written. Even though the Litz wire can be broken up into many segments, there is always only one current to solve for in a Litz wire because all the Litz wire segment currents are equal. Consequently, Litz wires have only one corresponding segment. ZMAKE starts at row 1 and loops though all columns and then proceeds to row 2 and loops through all columns and so on until it traverses all rows making the impedance matrix. The following page contains the pseudo code for ZMAKE.

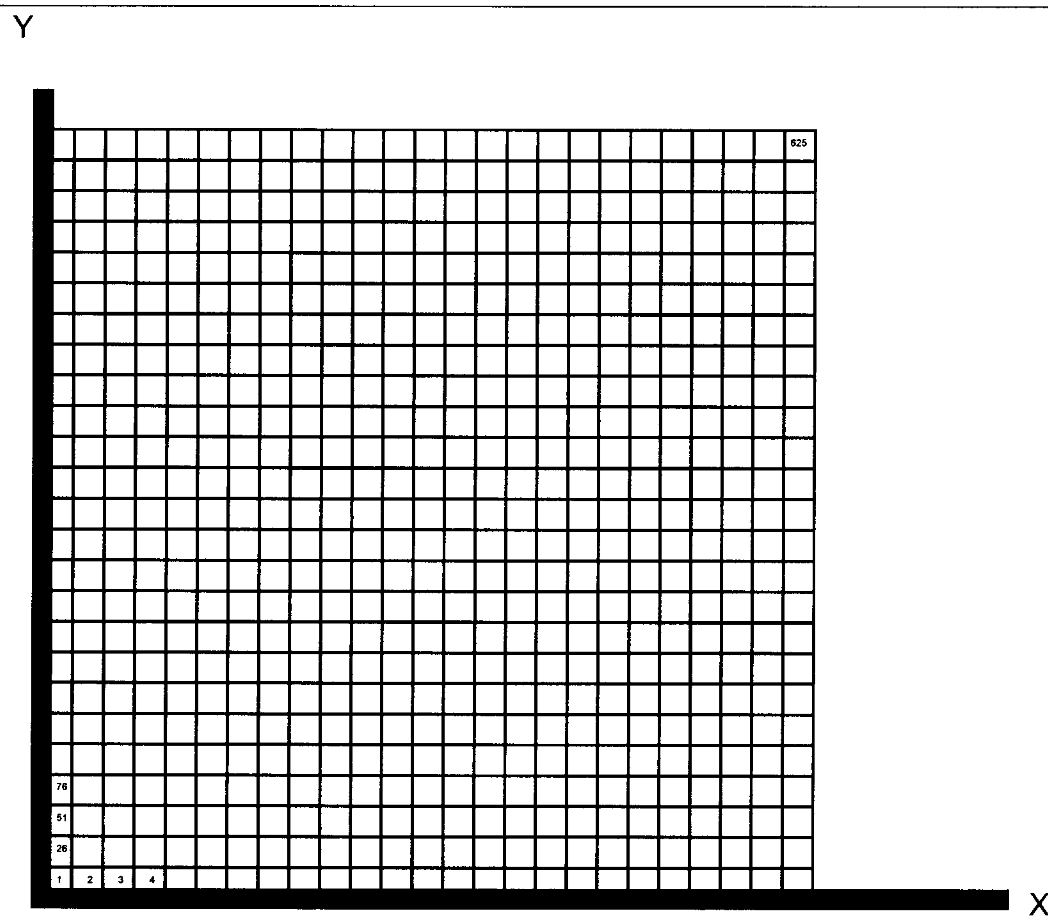
```
LOOP THROUGH ALL ROWS
 LOOP THROUGH ALL COLUMNS
  IF ROW = COLUMN THEN DIAGONAL OF IMPEDANCE MATRIX
     IF ROW = LITZ THEN
        CALCULATE SA  FROM EQUATION 3.26 BY CALLING
        LITZRESI TO CALCULATE RESISTANCE
        LITZSELF TO CALCULATE SELF IMPEDANCE
        LITZSYMM TO CALCULATE SYMMETRICAL MUTUAL TERMS
     ELSE
       CALCULATE ZNN  FROM EQUATION 3.26 BY CALLING
       RESIST TO CALCULATE RESISTANCE
       SELFL TO CALCULATE SELF IMPEDANCE
       MUTSYMSL TO CALCULATE SYMMETRICAL MUTUAL TERMS
    ENDIF
  ELSE NON DIAGONAL OF IMPEDANCE MATRIX
     IF ROW=LITZ AND COLUMN=LITZ THEN
        MUTUAL IMPEDANCE BETWEEN TWO LITZWIRES CALL
        LSYMMZ TO CALCULATE MUTAL IMPEDANCE BETWEEN TWO
        LITZ
       CONDUCTORS AS WELL AS SYMMETRICAL TERMS
     ENDIF
     IF ROW=LITZ AND COLUMN=NON LITZ THEN
        CALCULATE MAN FROM EQUATION 3.26 BY CALLING
        LITZREG TO CALCULATE MUTUAL IMPEDANCE BETWEEN A
        LITZ WIRE AND A REGULAR SEGMENT
     ENDIF
     IF ROW=NON LITZ AND COLUMN=LITZ THEN
       CALCULATE MNA FROM EQUATION 3.26 BY CALLING
       REGLITZ TO CALCULATE THE MUTUAL INDUCTANCE
       BETWEEN A REGULAR SEGMENT AND A LITZWIRE AND
       SYMMETRICAL TERMS
     ENDIF
     IF ROW=NON LITZ AND COLUMN=NON LITZ THEN
       CALCULATE ZIJ FROM EQUATION 3.26 BY CALLING
       CALL MUTSYM
     ENDIF
 END LOOP THROUGH ALL COLUMNS
END LOOP THROUGH ALL ROWS
```

In the next chapter the program in Appendix B will be utilized to solve for the current distribution in a number of interesting conductor configurations.

CHAPTER IV

RESULTS

In Chapter I, some cylindrical conductors were analyzed with differing properties to find the current distribution. Like in Chapter I this chapter starts out with a single conductor situation and then proceeds to more complex configurations. Figure 4.1 shows a configuration consisting of a regular copper conductor with the properties shown in Table 4.1.



1st Quadrant of Single Rectangular Normal Conductor

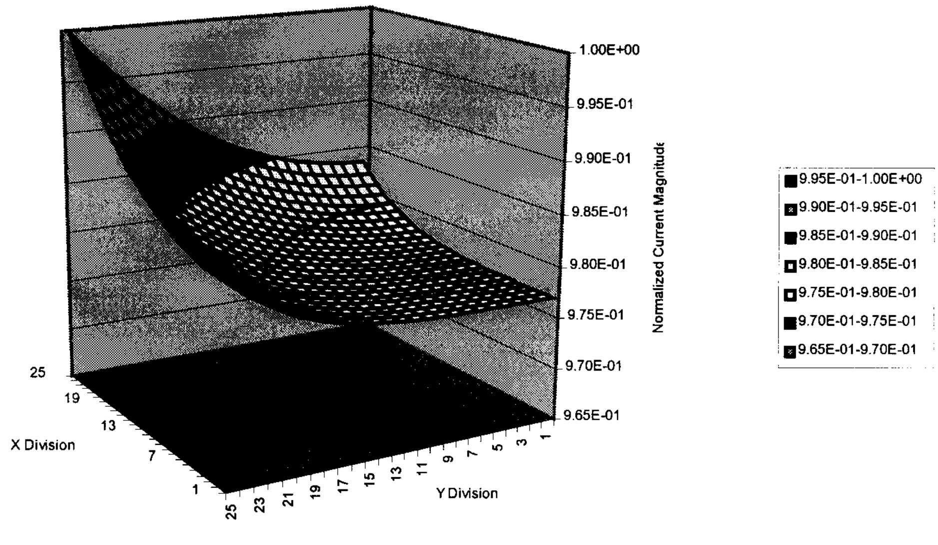
Figure 4.1

Table 4.1

Properties of Partial Regular Conductor in Figure 4.1

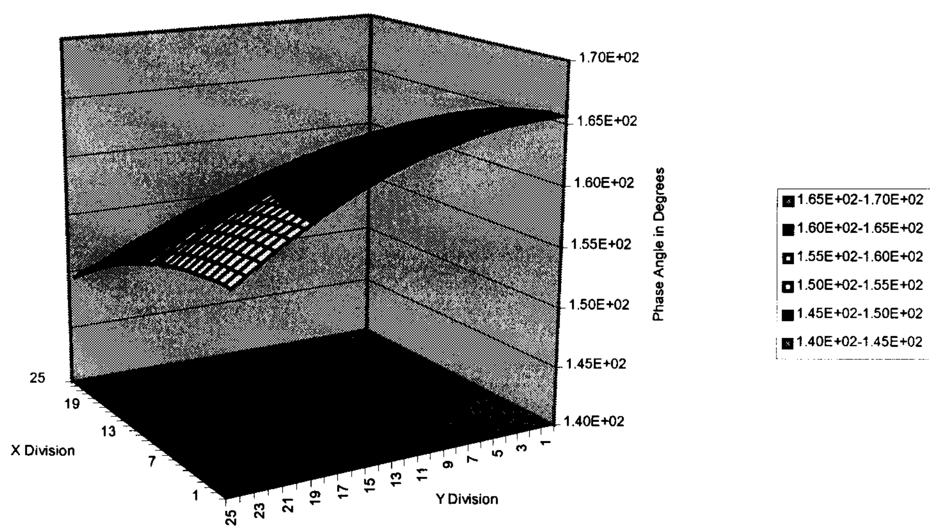
Voltage	Cos(wt) volts
X Symmetry	1
Y Symmetry	1
X Divisions	25
Y divisions	25
Total segments	625
Height	0.005 meters
Width	0.005 meters
Length	3.048 meters
Conductivity	5.7e7 mhos/meter
Relative Permeability	1

The partial conductor in Figure 4.1 is broken up into 625 segments by dividing it up in 25 divisions in the x direction and 25 divisions in the y direction. Some of the segments are numbered for clarity. In addition to this, there is positive x and y symmetry. This means that the entire conductor is 4 times the size of the portion shown in Figure 4.1. The whole conductor is comprised of 2500 segments but because of symmetry only 625 of them need to be solved for. After these are solved, then symmetry can be used to find the remaining ones. Figures 4.2, 4.4 and 4.6 shows the normalized current distribution for the partial conductor in Figure 4.1. Figures 4.3, 4.5 and 4.7 show the phase of the current in the partial conductor in Figure 4.1. It should be noted that in Chapter I, the phase of the current density was always referenced to the surface current density. That is, the surface current density had a phase angle of zero. This in no way implied that it was in phase with the voltage. The current density had a phase of zero on the surface because it was prescribed that way. The voltage magnitude or voltage phase was in fact never calculated in Chapter I. Now, in this model the voltage magnitude and phase are prescribed and the resulting currents are calculated and have a phase angle referenced from the voltage. Tables 4.2, 4.3, and 4.4 shows the circuit parameters used and calculated by the model for three frequencies.



Normalized Current Distribution

Figure 4.2



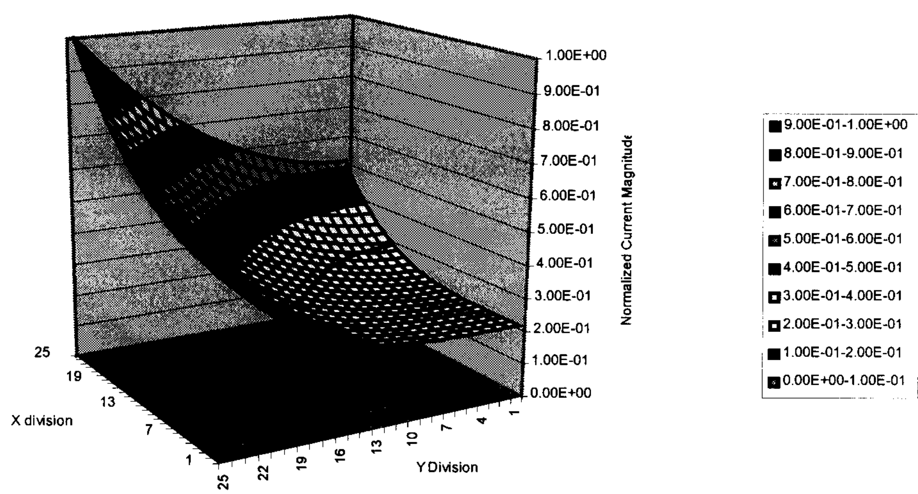
Phase Angle

Figure 4.3

Table 4.2

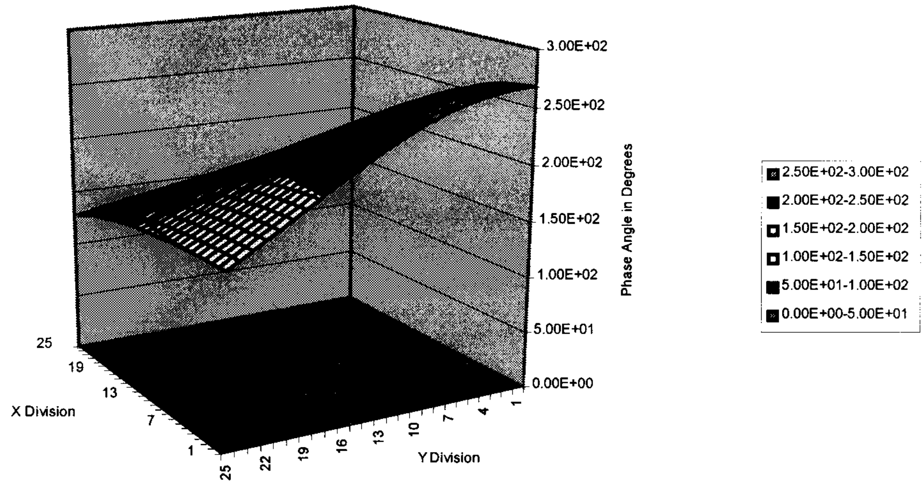
Model Output of Configuration Shown in Figure 4.1

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.002149352894009	ohms
Reactance	0.005716875085973	ohms
Current Magnitude	163.731308	amps
Current Phase	159.3954163	degrees
Real Power	28.809863811933050	watts



Normalized Current Distribution

Figure 4.4



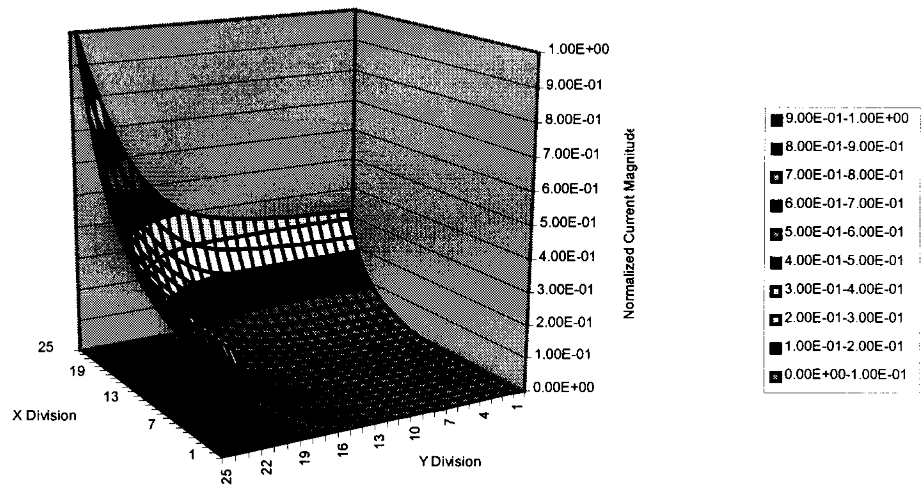
Phase Angle

Figure 4.5

Table 4.3

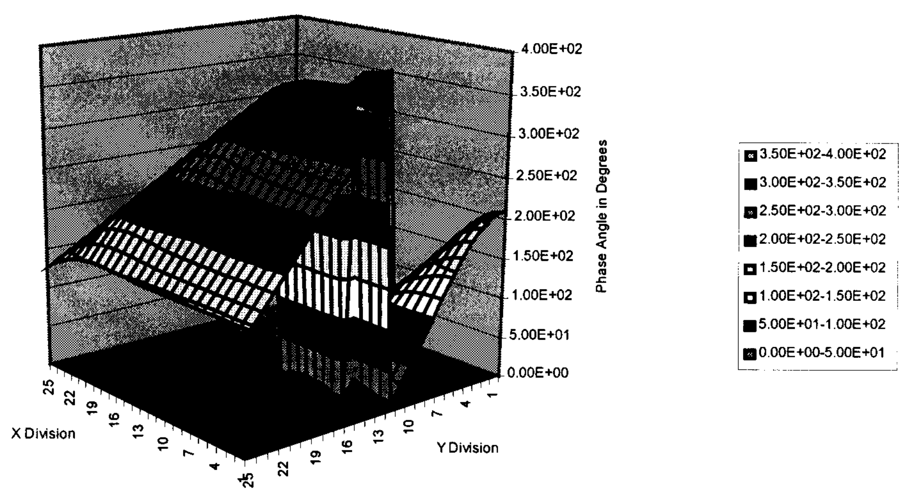
Model Output of Configuration Shown in Figure 4.1

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	0.003562962234577	ohms
Reactance	0.094045896606623	ohms
Current Magnitude	10.6254835	amps
Current Phase	177.8303680	degrees
Real Power	0.201130820453723	watts



Normalized Current Distribution

Figure 4.6



Phase Angle

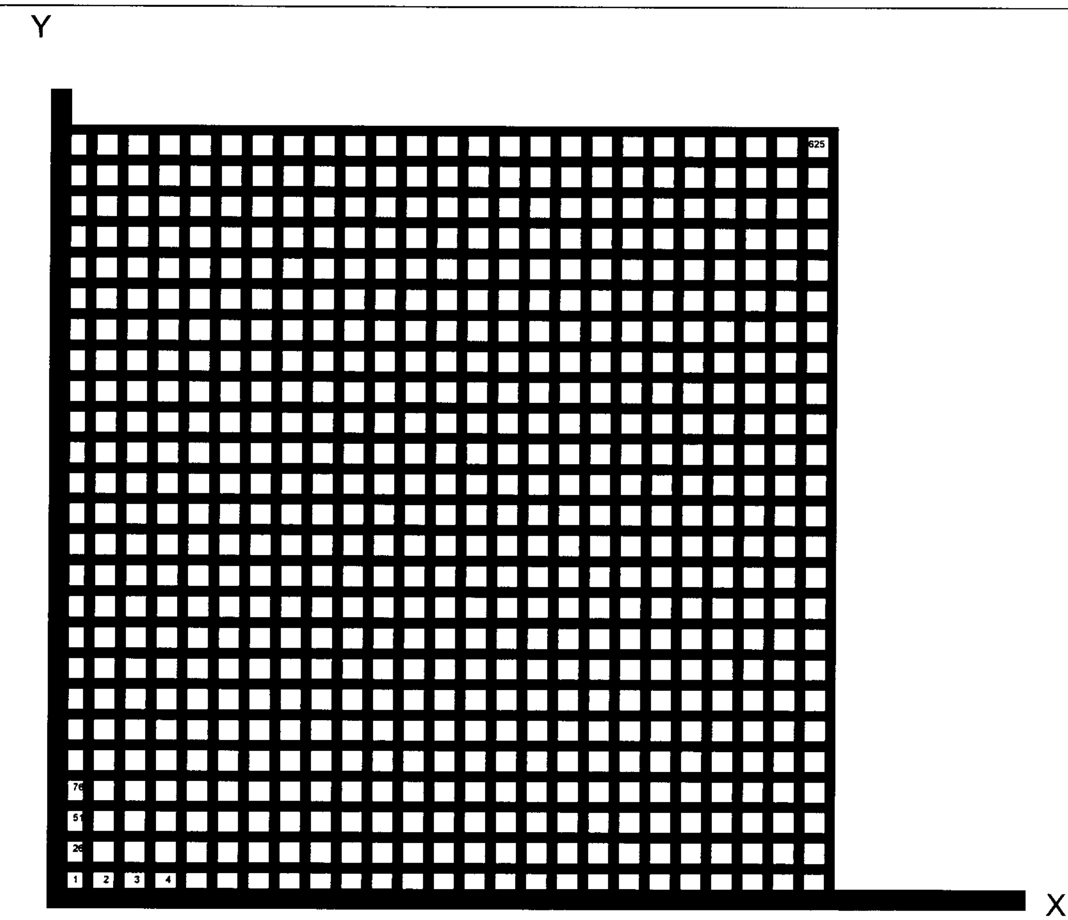
Figure 4.7

Table 4.4

Model Output of Configuration Shown in Figure 4.1

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.010185283828687	ohms
Reactance	0.920374836929470	ohms
Current Magnitude	1.0864474	amps
Current Phase	179.3659668	degrees
Real Power	0.006011190399594	watts

Figure 4.8 shows a configuration consisting of one copper Litz conductor with the properties shown in Table 4.5.



1st Quadrant of Single Rectangular Litz Conductor

Figure 4.8

Table 4.5

Properties of Partial Litz Conductor in Figure 4.8

Voltage	Cos(wt) volts
X Symmetry	+1
Y Symmetry	+1
X Divisions	25
Y divisions	25
Total segments	625
Height	.005 meters
Width	.005 meters
Length	3.048 meters
Space Factor	.65
Length Factor	1.1
Conductivity	5.7e7 mhos/meter
Relative Permeability	1

The partial conductor in Figure 4.8 is broken up into 625 segments like in Figure 4.1 with the same numbering and symmetry as well. The lines are drawn thicker in an attempt to portray the insulation. Tables 4.6, 4.7 and 4.8 shows the circuit parameters used and calculated by the model for three frequencies. Because the model calculates a constant current distribution and constant phase for the Litz wire, these are not plotted.

Table 4.6

Model Output of Configuration Shown in Figure 4.8

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0 .003621327590942	ohms
Reactance	0.005717332458496	ohms
Current Magnitude	147.7604523	amps
Current Phase	147.6500397	degrees
Real Power	39.532497621998580	watts

Table 4.7

Model Output of Configuration Shown in Figure 4.8

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	0.003621327590942	ohms
Reactance	0.095288873291016	ohms
Current Magnitude	10.4868345	amps
Current Phase	177.8235931	degrees
Real Power	0.199125377220933	watts

Table 4.8

Model Output of Configuration Shown in Figure 4.8

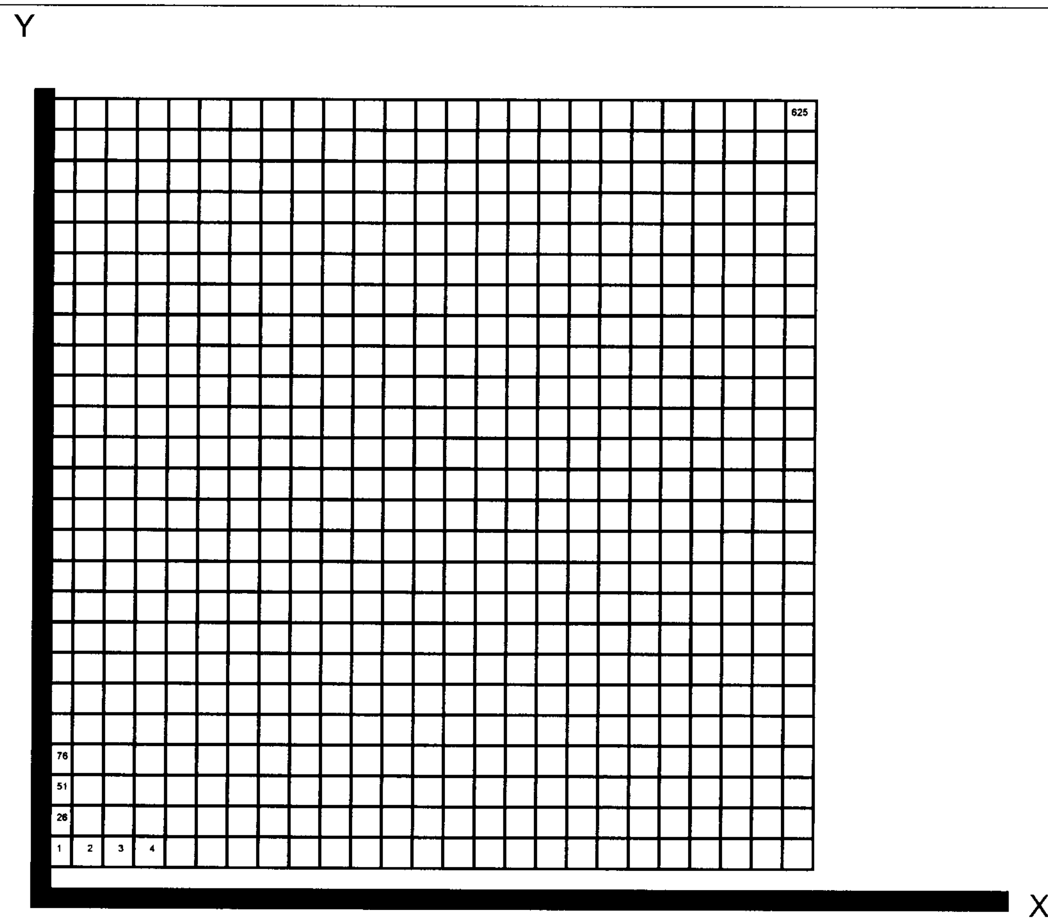
Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.003621327590942	ohms
Reactance	0.952888769531250	ohms
Current Magnitude	1.0494329	amps
Current Phase	179.7822571	degrees
Real Power	0.001994100745528	watts

Table 4.9

Summary of Results for Single Conductor

Frequency	Regular Resistance (ohms)	Litz Resistance (ohms)	Regular Reactance (ohms)	Litz Reactance (ohms)
60	0.0021	0.0036	0.0057	0.0057
1000	0.0036	0.0036	0.0940	0.0953
10000	0.0102	0.0036	0.9204	0.9528

Figure 4.9 shows the 1st quadrant of a transmission line. There are two rectangular conductors implied in Figure 4.9 because of the Y Offset. The properties of this conductor configuration are shown in Table 4.10.



1st Quadrant of a Configuration of 2 Regular Conductors

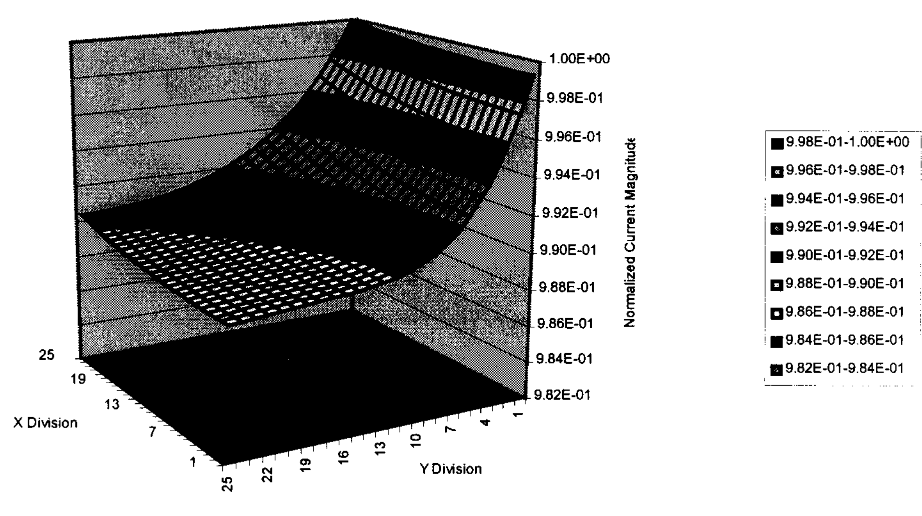
Figure 4.9

Table 4.10

Properties of Configuration Shown in Figure 4.8

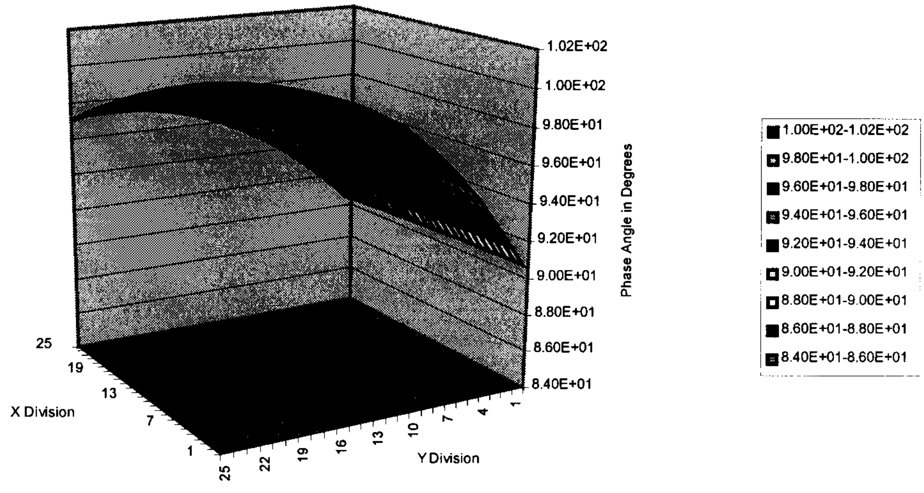
Voltage	Cos(wt) volts
X Symmetry	+1
Y Symmetry	-1
X Divisions	25
Y divisions	25
Total segments	625
Height	.005 meters
Width	.005 meters
Length	3.048 meters
X Offset	0
Y Offset	.00005 meters
Conductivity	5.7e7 mhos/meter
Relative Permeability	1

Figures 4.10, 4.12 and 4.14 show the current distribution in the part of the conductor shown in Figure 4.9 for three frequencies. Figures 4.11, 4.13 and 4.15 show the phase of the part of the conductor shown in Figure 4.9 for three frequencies. Tables 4.10, 4.11 and 4.12 show the circuit parameters used and calculated by the model for three frequencies.



Normalized Current Distribution

Figure 4.10



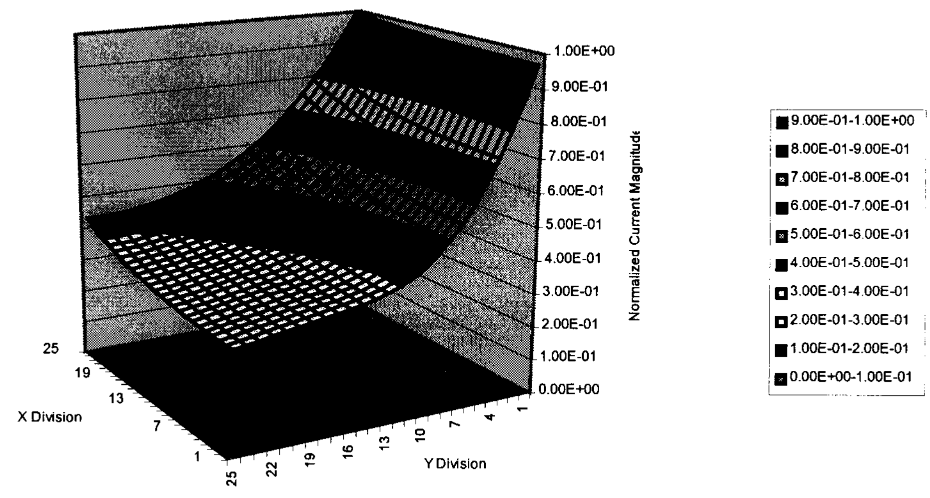
Phase Angle

Figure 4.11

Table 4.11

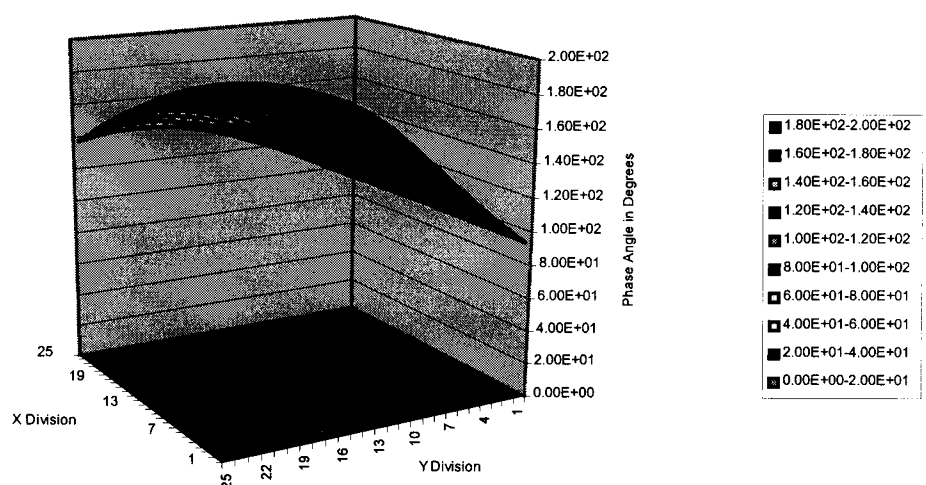
Model Output of Configuration Shown in Figure 4.9

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.002145486613672	ohms
Reactance	0.000270454969892	ohms
Current Magnitude	462.4350281	amps
Current Phase	97.1846695	degrees
Real Power	229.402044726375600	watts



Normalized Current Distribution

Figure 4.12



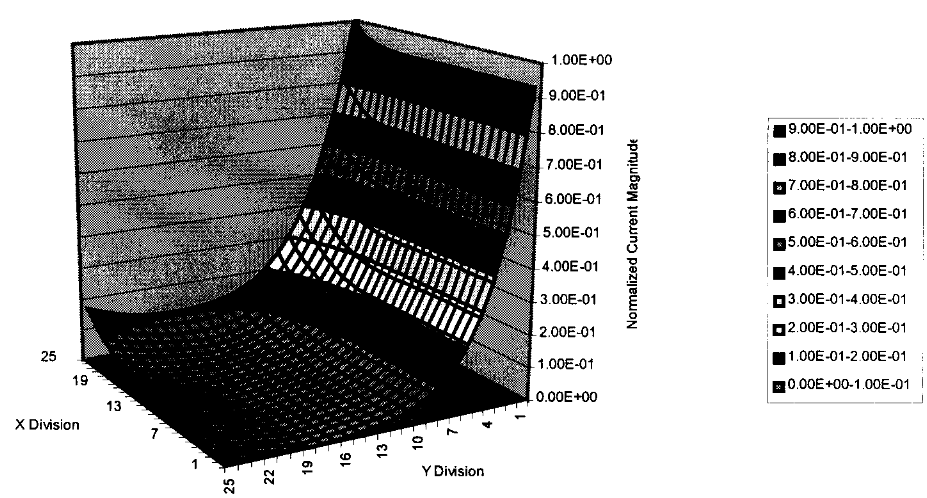
Phase Angle

Figure 4.13

Table 4.12

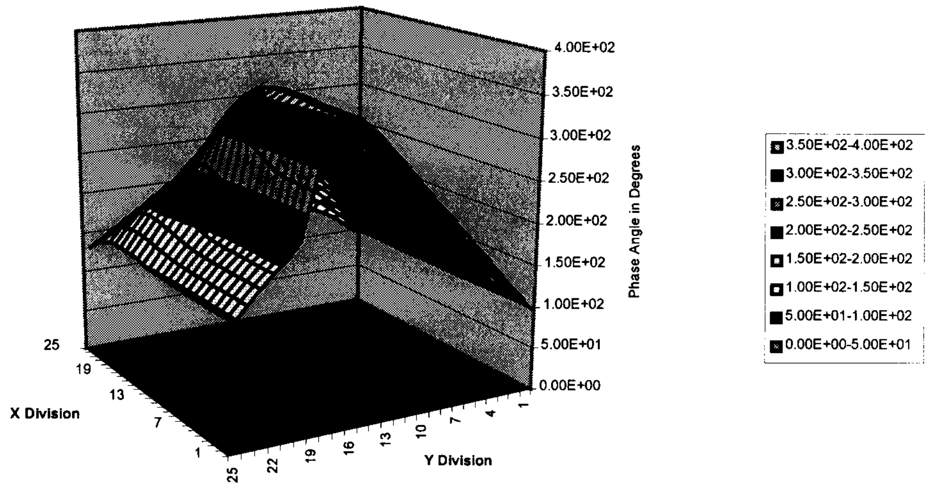
Model Output of Configuration Shown in Figure 4.9

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	0.003264728493435	ohms
Reactance	0.003857746004026	ohms
Current Magnitude	197.8718262	amps
Current Phase	139.7594604	degrees
Real Power	63.912384405171280	watts



Normalized Current Distribution

Figure 4.14



Phase Angle

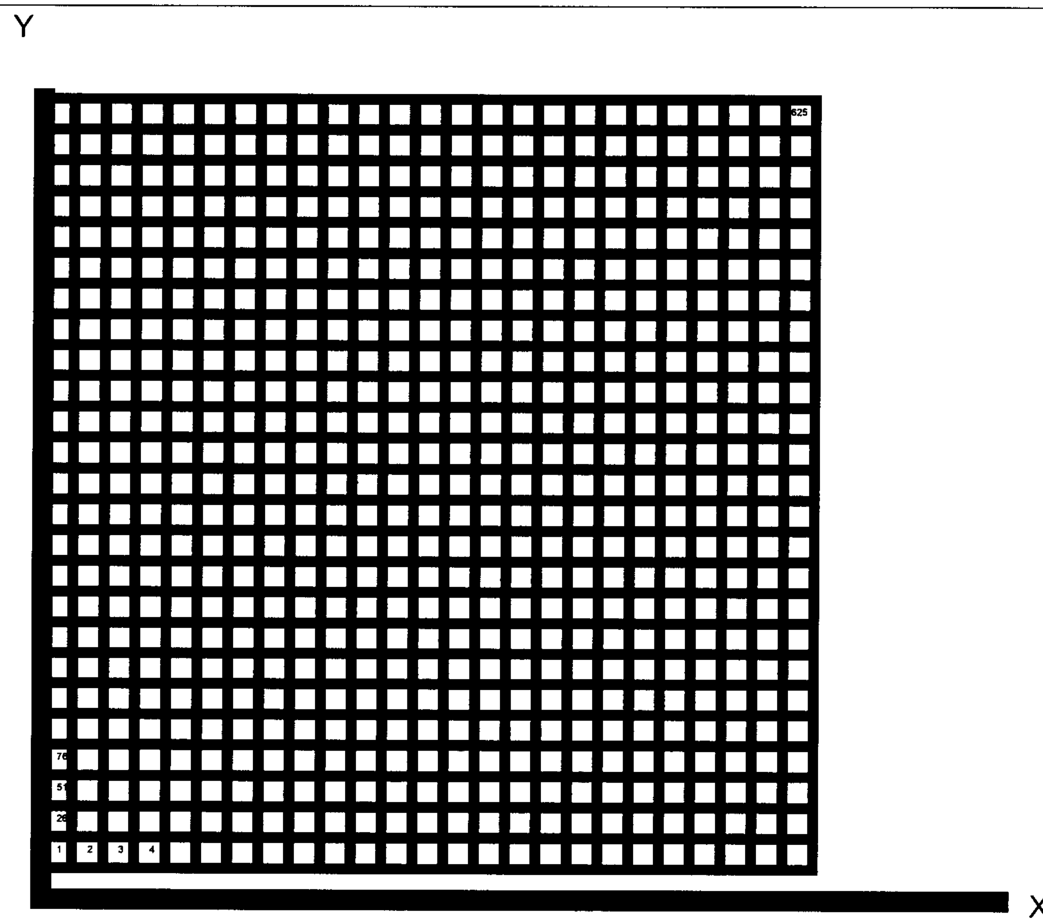
Figure 4.15

Table 4.13

Model Output of Configuration Shown in Figure 4.9

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.012162784630579	ohms
Reactance	0.016570878842499	ohms
Current Magnitude	48.6488495	amps
Current Phase	143.7218628	degrees
Real Power	14.392896077985050	watts

Figure 4.16 shows the 1st quadrant of a transmission line. There are two rectangular Litz conductors implied in Figure 4.16 because of the Y Offset. The properties of this conductor configuration are shown in Table 4.14.



1st Quadrant of a Configuration of 2 Litz Conductors

Figure 4.16

Table 4.14

Properties of Configuration Shown in Figure 4.16

Voltage	Cos(wt) volts
X Symmetry	+1
Y Symmetry	-1
X Divisions	25
Y divisions	25
Total segments	625
Height	.005 meters
Width	.005 meters
Length	3.048 meters
X Offset	0
Y Offset	.00005 meters
Space Factor	.65
Length Factor	1.1
Conductivity	5.7e7 mhos/meter
Relative Permeability	1

Tables 4.15, 4.16 and 4.17 show the circuit parameters used and calculated by the model for three frequencies.

Table 4.15

Model Output of Configuration Shown in Figure 4.16

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.003621327590942	ohms
Reactance	0.000270637273788	ohms
Current Magnitude	275.3738708	amps
Current Phase	94.2740097	degrees
Real Power	137.304028776925600	watts

Table 4.16

Model Output of Configuration Shown in Figure 4.16

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	0.003621327590942	ohms
Reactance	0.004510621261597	ohms
Current Magnitude	172.8776550	amps
Current Phase	141.2409515	degrees
Real Power	54.114732121237560	watts

Table 4.17

Model Output of Configuration Shown in Figure 4.16

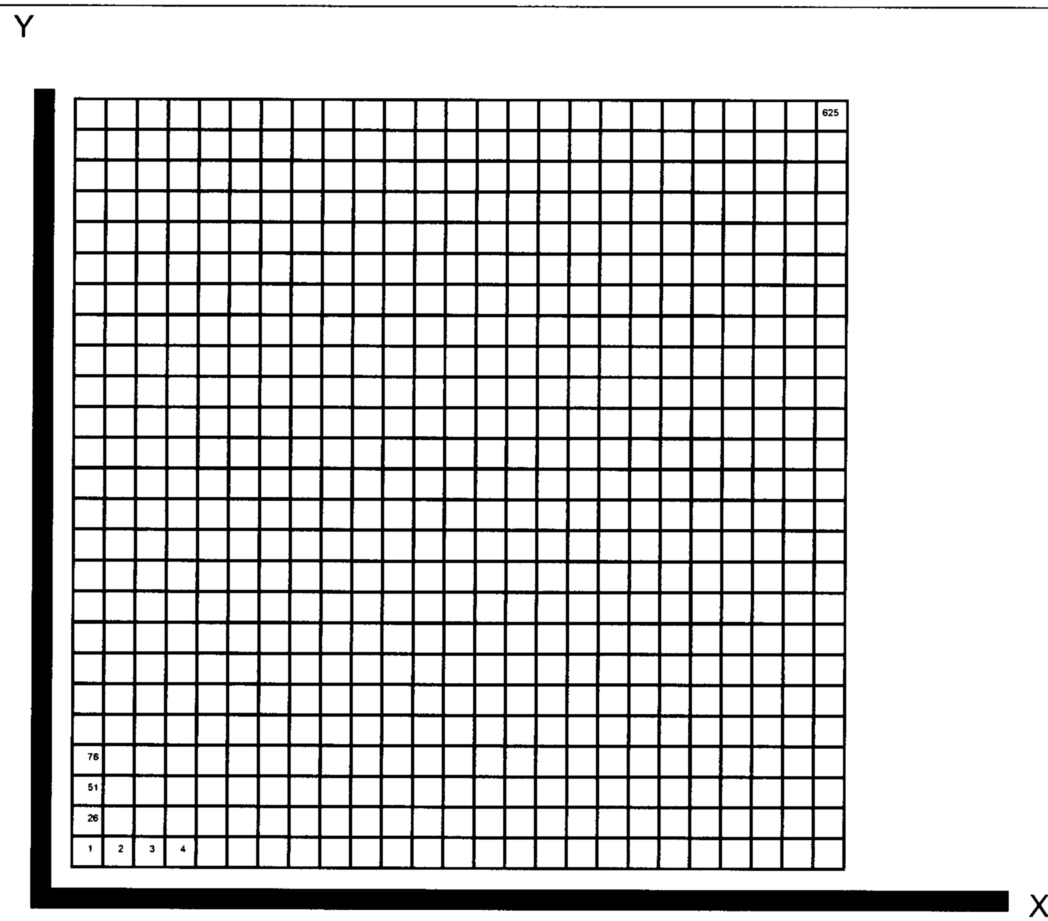
Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.003621327590942	ohms
Reactance	0.045106213378906	ohms
Current Magnitude	22.0987892	amps
Current Phase	175.4098816	degrees
Real Power	0.884249336470242	watts

Table 4.18

Summary of Results for 2 Conductor Configuration

Frequency	Regular Resistance (ohms)	Litz Resistance (ohms)	Regular Reactance (ohms)	Litz Reactance (ohms)
60	0.0021	0.0036	0.0003	0.0003
1000	0.0033	0.0036	0.0039	0.0045
10000	0.0122	0.0036	0.0166	0.0451

Figure 4.17 shows the 1st quadrant of a transmission line. There are four rectangular conductors implied in Figure 4.17 because of the Y Offset and X Offset. The properties of this conductor configuration are shown in Table 4.19.



1st Quadrant of a Configuration of 4 Regular Conductors

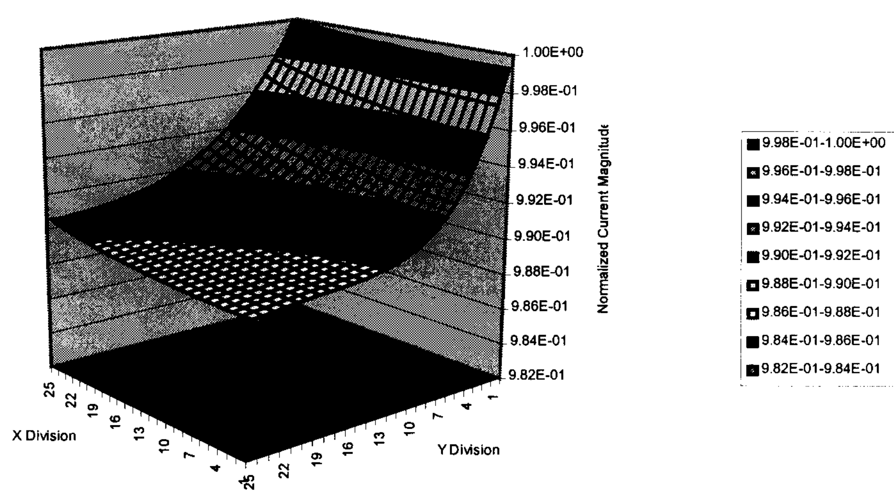
Figure 4.17

Table 4.19

Properties of Configuration Shown in Figure 4.17

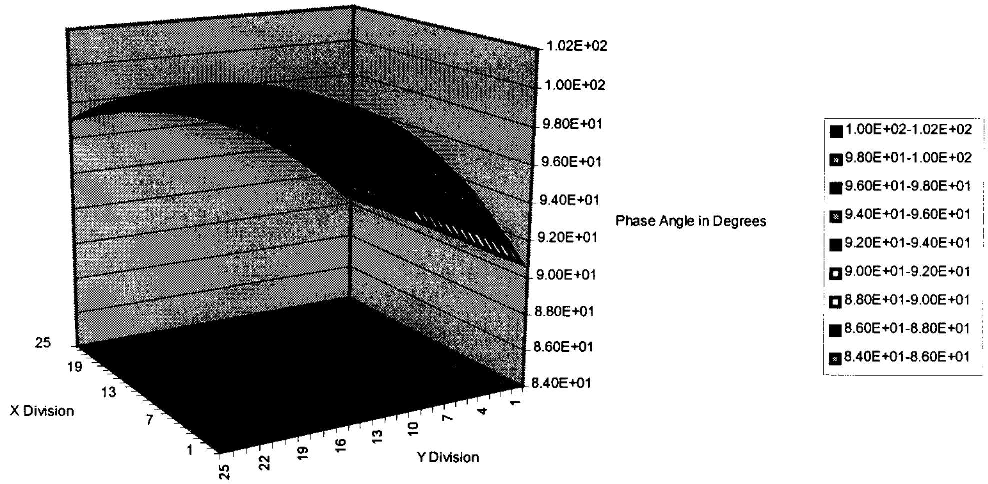
Voltage	Cos(wt) volts
X Symmetry	+1
Y Symmetry	-1
X Divisions	25
Y divisions	25
Total segments	625
Height	.005 meters
Width	.005 meters
Length	3.048 meters
X Offset	.00005 meters
Y Offset	.00005 meters
Conductivity	5.7e7 mhos/meter
Relative Permeability	1

Figures 4.18, 4.20 and 4.22 show the current distribution in the part of the conductor shown in Figure 4.17 for three frequencies. Figures 4.19, 4.21 and 4.23 show the phase of the part of the conductor shown in Figure 4.17 for three frequencies. Tables 4.20, 4.21 and 4.22 show the circuit parameters used and calculated by the model for three frequencies.



Normalized Current Distribution

Figure 4.18



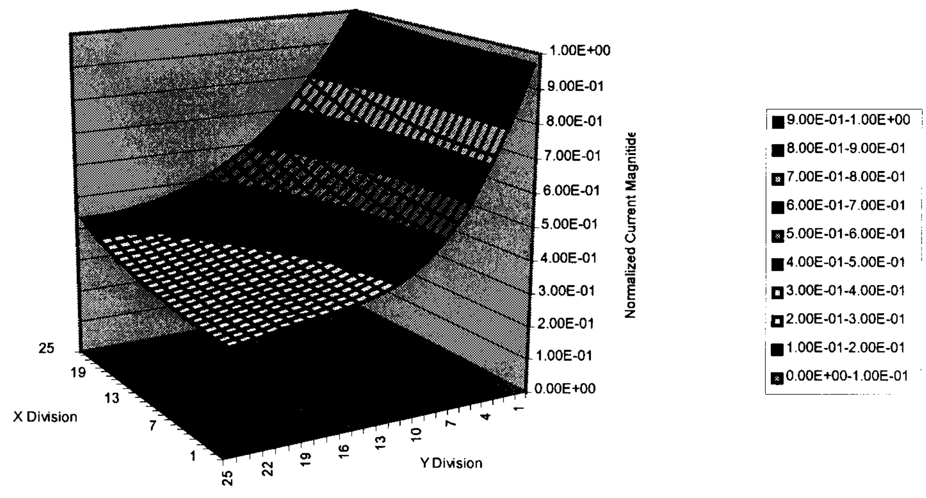
Phase Angle

Figure 4.19

Table 4.20

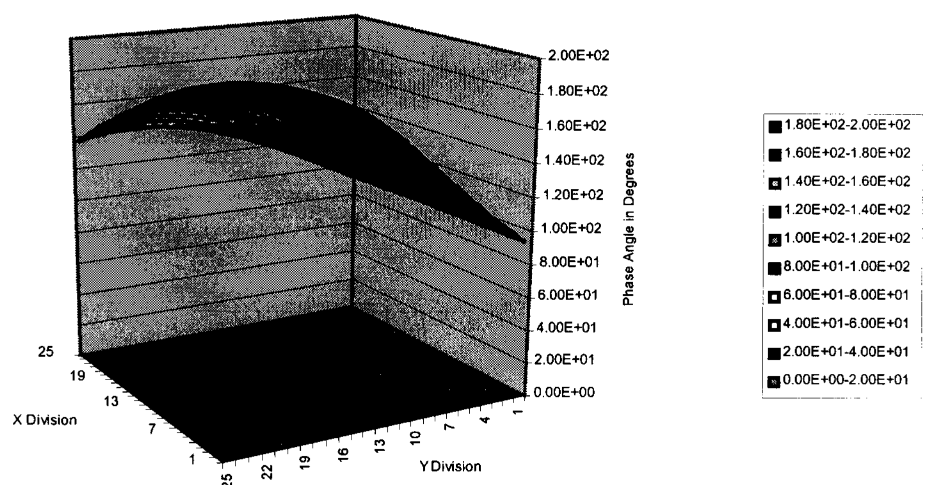
Model Output of Configuration Shown in Figure 4.17

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.002145380051700	ohms
Reactance	0.000268244087508	ohms
Current Magnitude	462.5165710	amps
Current Phase	97.1268997	degrees
Real Power	229.471532748759300	watts



Normalized Current Distribution

Figure 4.20



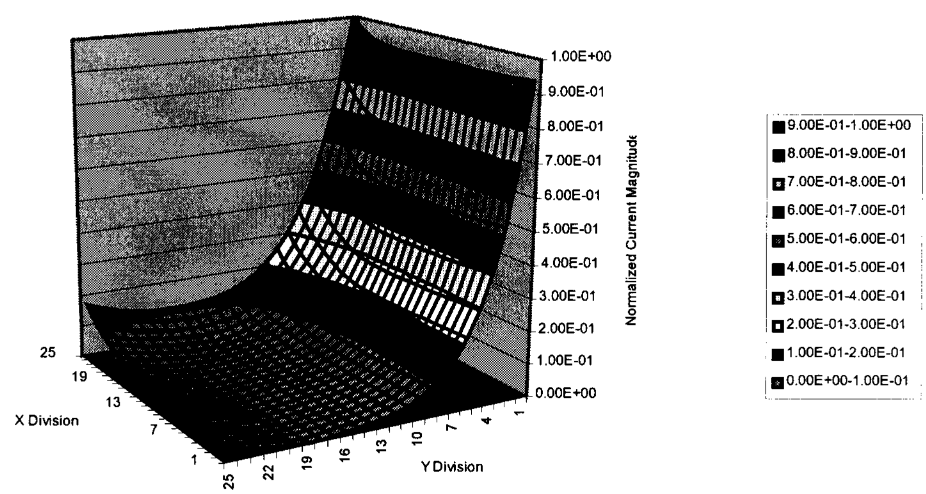
Phase Angle

Figure 4.21

Table 4.21

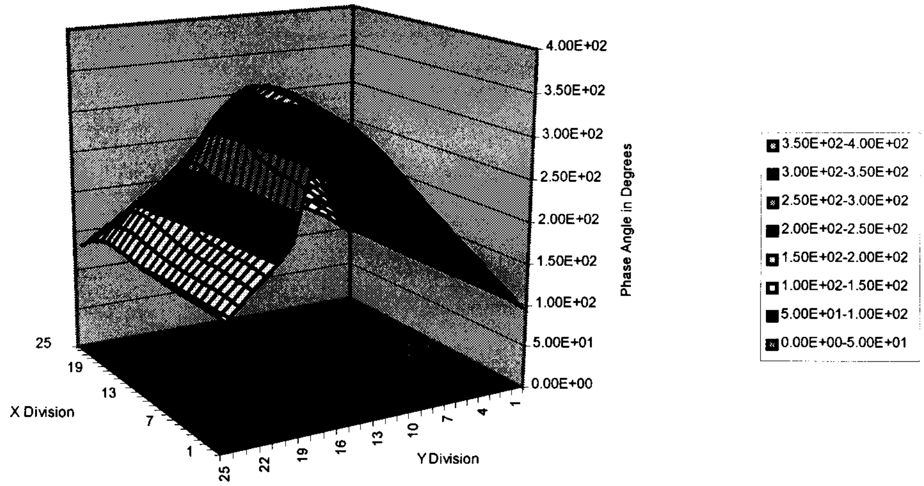
Model Output of Configuration Shown in Figure 4.17

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	0.003250211815405	ohms
Reactance	0.003836134021354	ohms
Current Magnitude	198.8901062	amps
Current Phase	139.726654	degrees
Real Power	64.284758566264670	watts



Normalized Current Distribution

Figure 4.22



Phase Angle

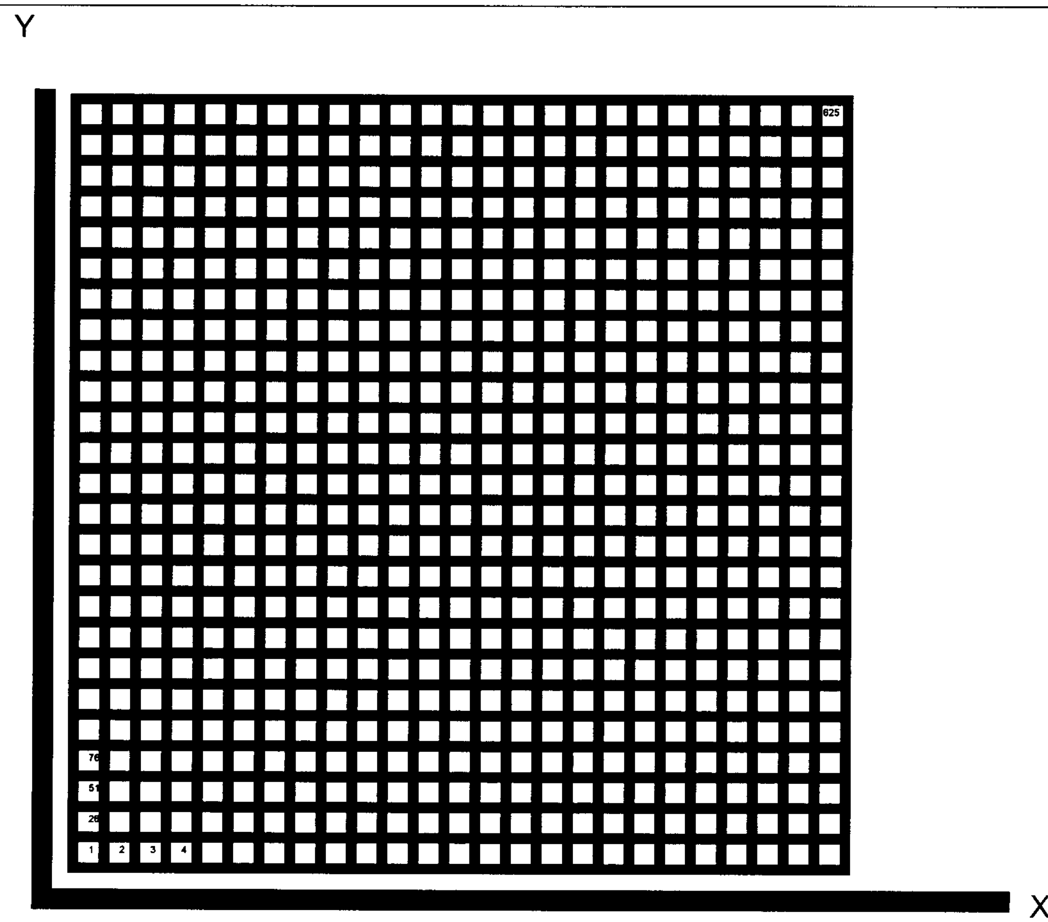
Figure 4.23

Table 4.22

Model Output of Configuration Shown in Figure 4.17

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.012114961282335	ohms
Reactance	0.016490557980533	ohms
Current Magnitude	48.8700676	amps
Current Phase	143.6967468	degrees
Real Power	14.466980069498500	watts

Figure 4.24 shows the 1st quadrant of a transmission line. There are four rectangular Litz conductors implied in Figure 4.24 because of the Y Offset and X Offset. The properties of this conductor configuration are shown in Table 4.23.



1st Quadrant of a Configuration of 4 Litz Conductors

Figure 4.24

Table 4.23

Properties of Configuration Shown in Figure 4.24

Voltage	Cos(wt) volts
X Symmetry	+1
Y Symmetry	-1
X Divisions	25
Y divisions	25
Total segments	625
Height	.005 meters
Width	.005 meters
Length	3.048 meters
X Offset	.00005 meters
Y Offset	.00005 meters
Space Factor	.65
Length Factor	1.1
Conductivity	5.7e7 mhos/meter
Relative Permeability	1

Tables 4.24, 4.25 and 4.26 show the circuit parameters used and calculated by the model for three frequencies.

Table 4.24

Model Output of Configuration Shown in Figure 4.24

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.003621327590942	ohms
Reactance	0.000268607068061	ohms
Current Magnitude	275.3853149	amps
Current Phase	94.2420654	degrees
Real Power	137.315428399605600	watts

Table 4.25

Model Output of Configuration Shown in Figure 4.24

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	0.003621327590942	ohms
Reactance	0.004476784515381	ohms
Current Magnitude	173.6686707	amps
Current Phase	141.0301666	degrees
Real Power	54.611074541387450	watts

Table 4.26

Model Output of Configuration Shown in Figure 4.24

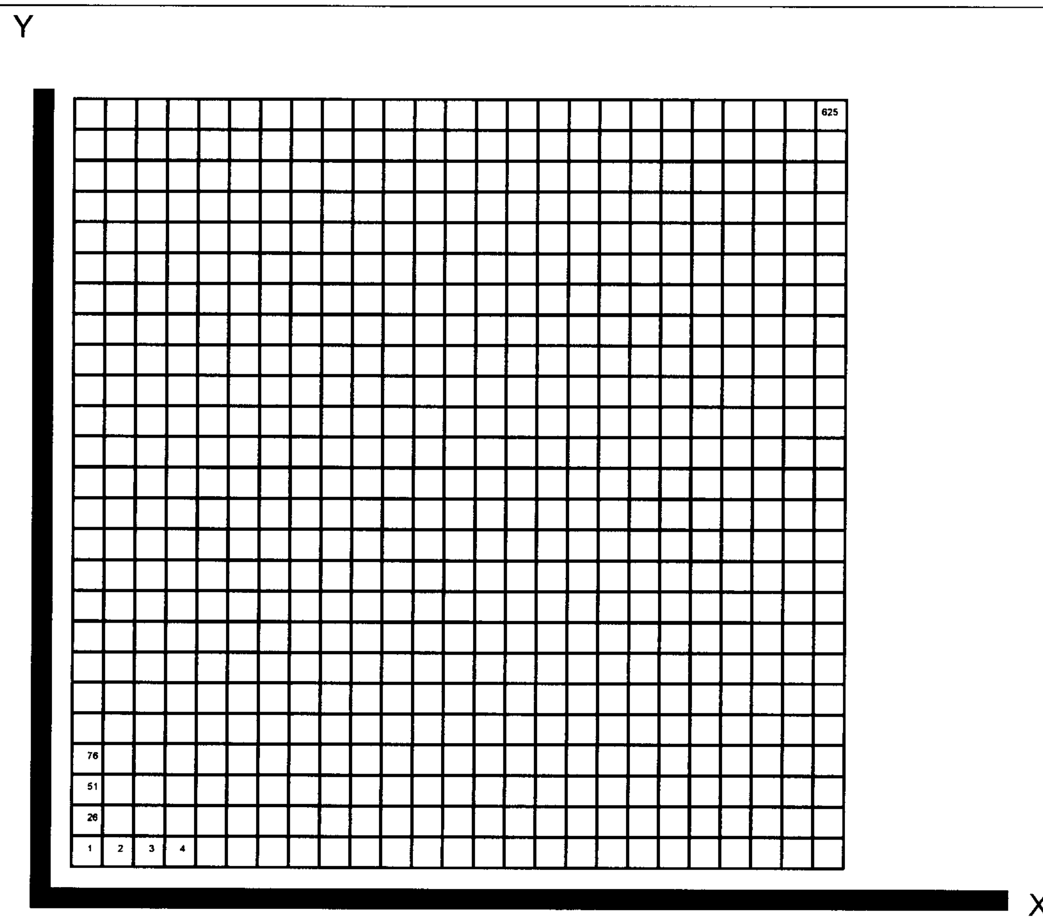
Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.003621327590942	ohms
Reactance	0.044767846679688	ohms
Current Magnitude	22.2647362	amps
Current Phase	175.3753357	degrees
Real Power	0.897579387257878	watts

Table 4.27

Summary of Results for 4 Conductor Configuration

Frequency	Regular Resistance (ohms)	Litz Resistance (ohms)	Regular Reactance (ohms)	Litz Reactance (ohms)
60	0.0021	0.0036	0.0003	0.0003
1000	0.0033	0.0036	0.0038	0.0045
10000	0.0121	0.0036	0.0165	0.0448

Figure 4.25 shows the 1st quadrant of a transmission line. There are four rectangular conductors implied in Figure 4.25 because of the Y Offset and X Offset. The properties of this conductor configuration are shown in Table 4.28. Notice the symmetry. This has the effect of transposing the conductors so that polarity alternates from quadrant to quadrant.



1st Quadrant of a Configuration of 4 Transposed Regular Conductors

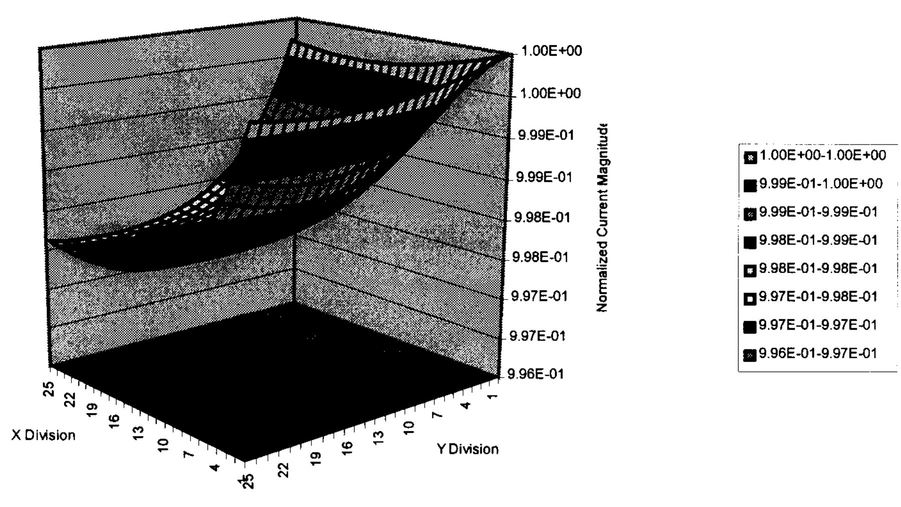
Figure 4.25

Table 4.28

Properties of Configuration Shown in Figure 4.25

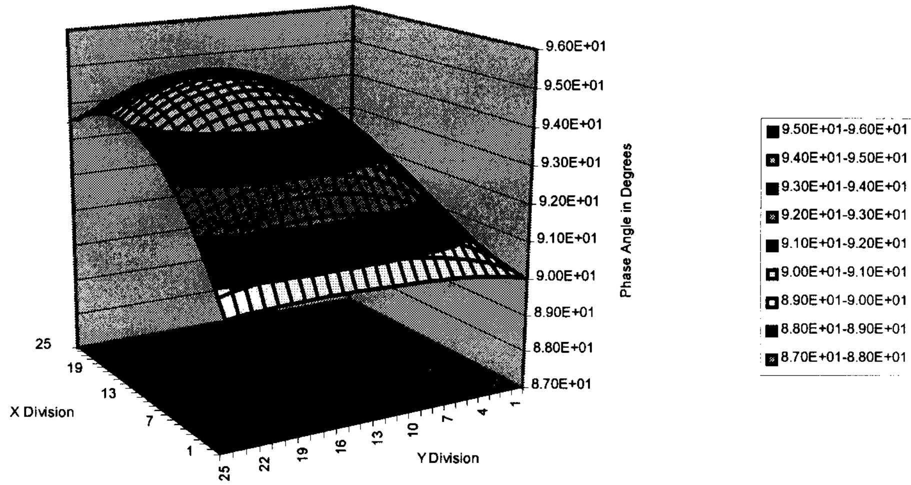
Voltage	Cos(wt) volts
X Symmetry	-1
Y Symmetry	-1
X Divisions	25
Y divisions	25
Total segments	625
Height	.005 meters
Width	.005 meters
Length	3.048 meters
X Offset	.00005 meters
Y Offset	.00005 meters
Conductivity	5.7e7 mhos/meter
Relative Permeability	1

Figures 4.26, 4.28 and 4.30 show the current distribution in the part of the conductor shown in Figure 4.25 for three frequencies. Figures 4.27, 4.29 and 4.31 show the phase of the part of the conductor shown in Figure 4.17 for three frequencies. Tables 4.29, 4.30 and 4.31 show the circuit parameters used and calculated by the model for three frequencies.



Normalized Current Distribution

Figure 4.26



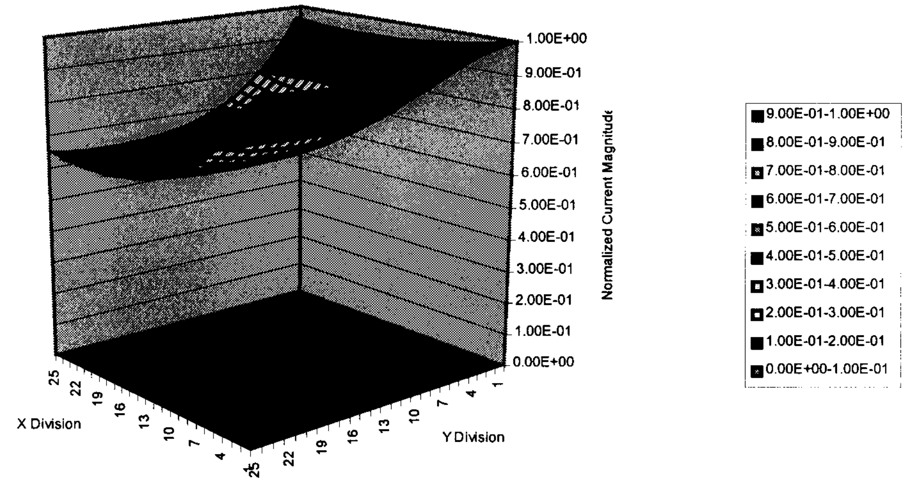
Phase Angle

Figure 4.27

Table 4.29

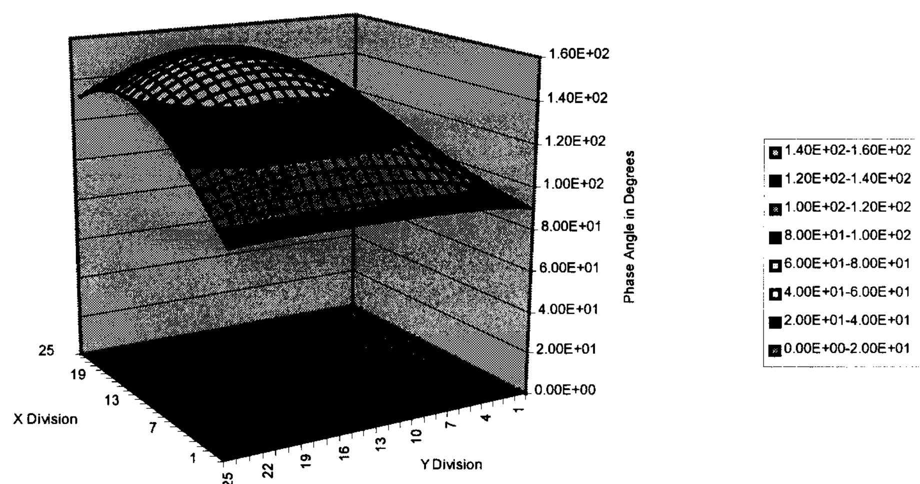
Model Output of Configuration Shown in Figure 4.25

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.002141247256829	ohms
Reactance	0.000113196258062	ohms
Current Magnitude	466.3663025	amps
Current Phase	93.0261002	degrees
Real Power	232.858002746250000	watts



Normalized Current Distribution

Figure 4.28



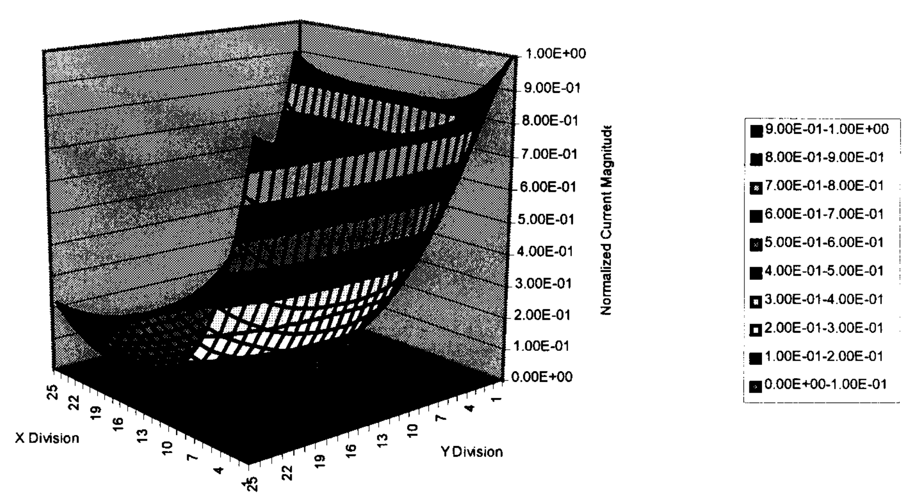
Phase Angle

Figure 4.29

Table 4.30

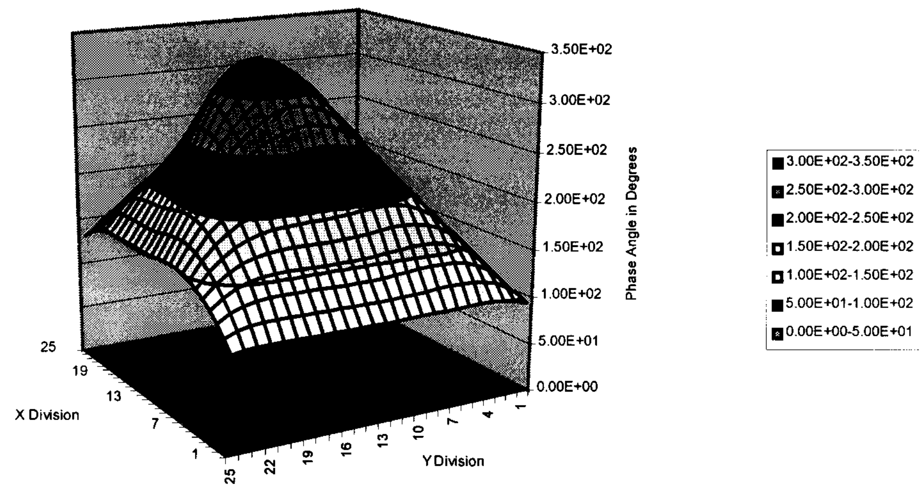
Model Output of Configuration Shown in Figure 4.25

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	0.002475271704481	ohms
Reactance	0.001770857633251	ohms
Current Magnitude	328.5688171	amps
Current Phase	125.5806656	degrees
Real Power	133.612038203796300	watts



Normalized Current Distribution

Figure 4.30



Phase Angle

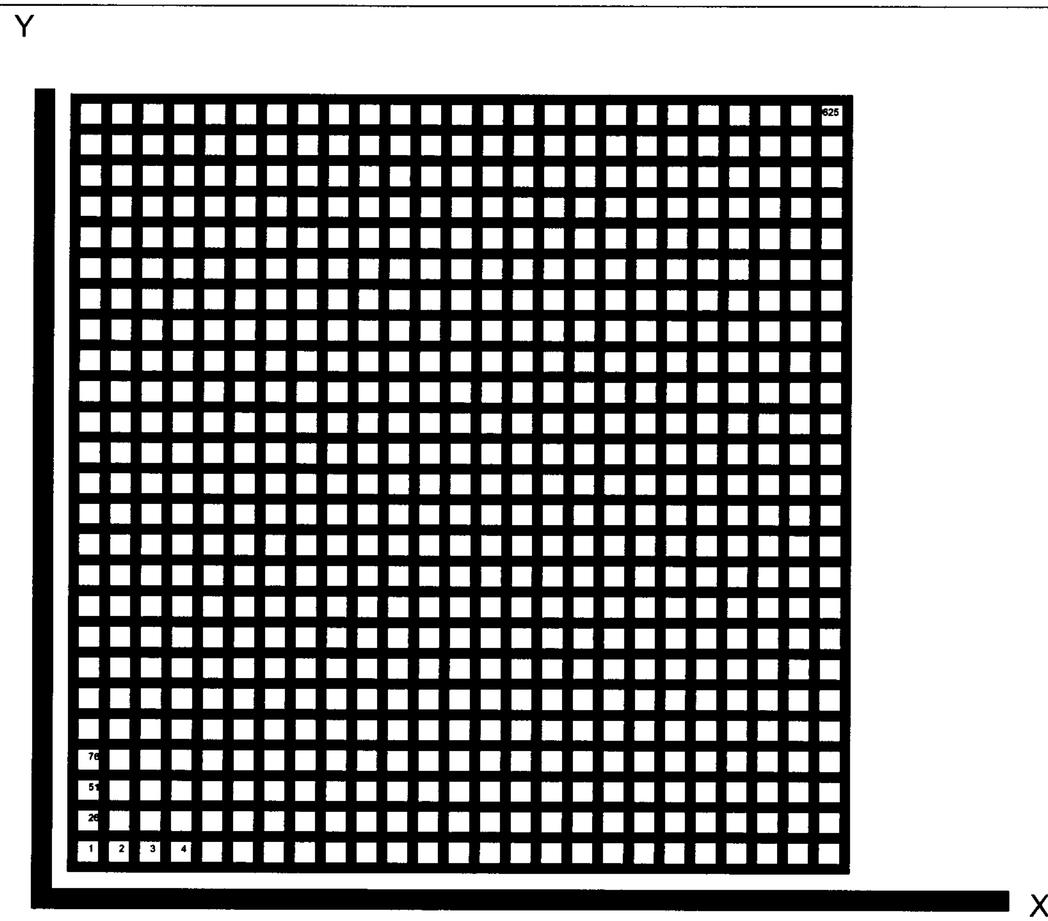
Figure 4.31

Table 4.31

Model Output of Configuration Shown in Figure 4.25

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.007010928019247	ohms
Reactance	0.008446258467791	ohms
Current Magnitude	91.1003799	amps
Current Phase	140.3051758	degrees
Real Power	29.092824047526710	watts

Figure 4.32 shows the 1st quadrant of a transmission line. There are four rectangular Litz conductors implied in Figure 4.24 because of the Y Offset and X Offset. The properties of this conductor configuration are shown in Table 4.23. Notice the symmetry. This has the effect of transposing the conductors so that polarity alternates from quadrant to quadrant.



1st Quadrant of a Configuration of 4 Litz Conductors

Figure 4.32

Table 4.32

Properties of Configuration Shown in Figure 4.32

Voltage	Cos(wt) volts
X Symmetry	-1
Y Symmetry	-1
X Divisions	25
Y divisions	25
Total segments	625
Height	.005 meters
Width	.005 meters
Length	3.048 meters
X Offset	.00005 meters
Y Offset	.00005 meters
Space Factor	.65
Length Factor	1.1
Conductivity	5.7e7 mhos/meter
Relative Permeability	1

Tables 4.33, 4.34 and 4.35 show the circuit parameters used and calculated by the model for three frequencies.

Table 4.33

Model Output of Configuration Shown in Figure 4.32

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.003621327590942	ohms
Reactance	0.000113061892986	ohms
Current Magnitude	276.0073242	amps
Current Phase	91.7882538	degrees
Real Power	137.936446087453700	watts

Table 4.34

Model Output of Configuration Shown in Figure 4.32

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	0.003621327590942	ohms
Reactance	0.001884364891052	ohms
Current Magnitude	244.9623718	amps
Current Phase	117.4903030	degrees
Real Power	108.651698043246100	watts

Table 4.35

Model Output of Configuration Shown in Figure 4.32

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.003621327590942	ohms
Reactance	0.018843649291992	ohms
Current Magnitude	52.1146469	amps
Current Phase	169.1216583	degrees
Real Power	4.917647372297274	watts

Table 4.36

Summary of Results for 4 Conductor Configuration

Frequency	Regular Resistance (ohms)	Litz Resistance (ohms)	Regular Reactance (ohms)	Litz Reactance (ohms)
60	0.0021	0.0036	0.0001	0.0001
1000	0.0028	0.0036	0.0018	0.0019
10000	0.0070	0.0036	0.0084	0.0188

Figure 4.33 shows the 1st quadrant of a transmission line. There are four rectangular conductors implied in Figure 4.33 because of the Y Offset. The properties of this conductor configuration are shown in Table 4.37.

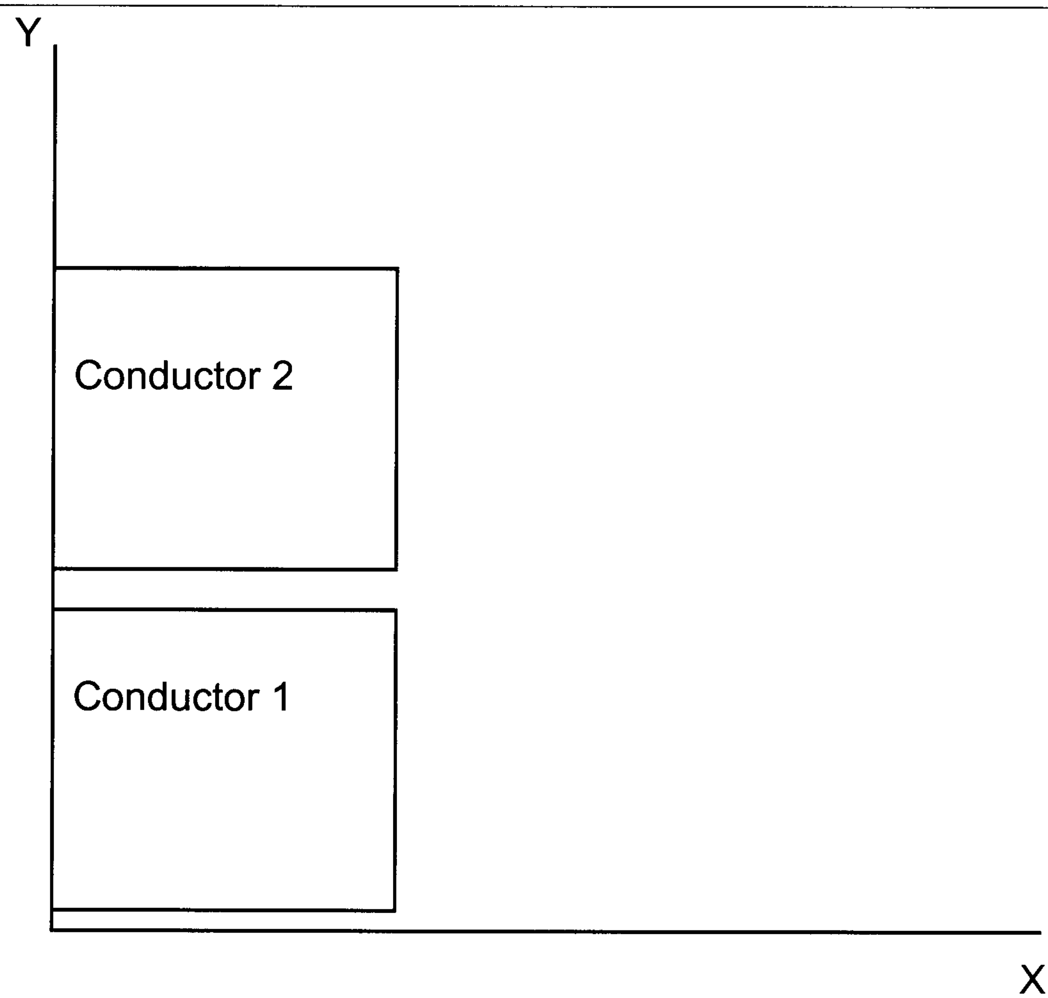


Figure 4.33

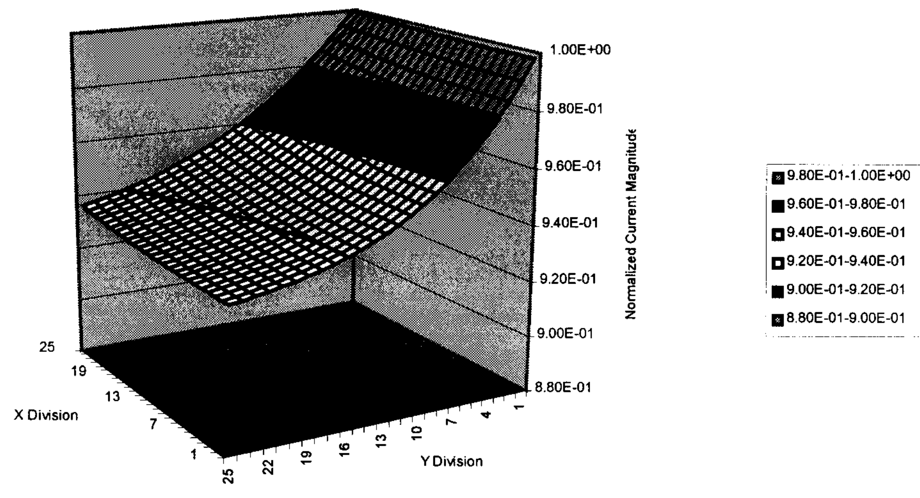
1st Quadrant of a Configuration of 4 Regular Conductors

Table 4.37

Properties of Configuration Shown in Figure 4.33

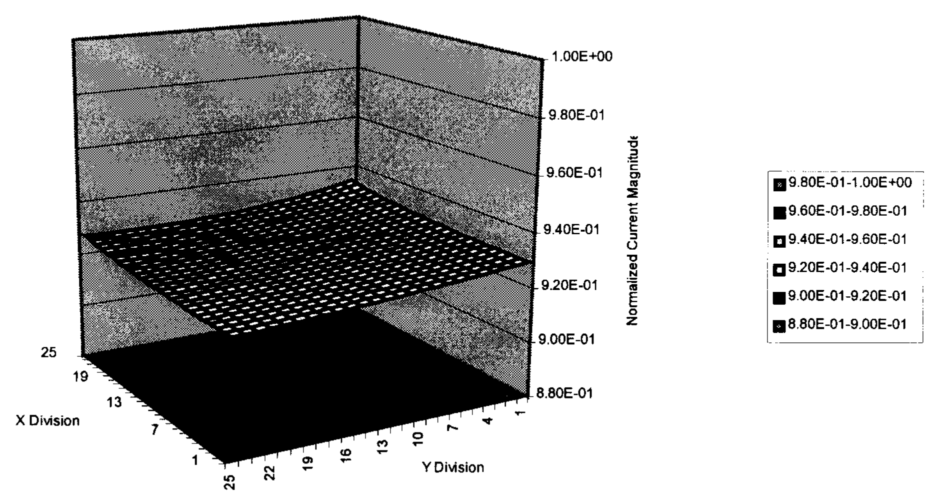
Conductor 1 and 2 Voltage	Cos(wt) volts
X Symmetry	+1
Y Symmetry	-1
Conductor 1 and 2 X Divisions	25
Conductor 1 and 2 Y Divisions	25
Total segments	1250
Conductor 1 Height	.005 meters
Conductor 2 Width	.005 meters
Conductor 1 and 2 Length	3.048 meters
Conductor 1 X Offset	0 meters
Conductor 1 Y Offset	.00005 meters
Conductor 2 X Offset	0 meters
Conductor 2 Y Offset	.00515 Meters
Conductor 1 and 2 Conductivity	5.7e7 mhos/meter
Conductor 1 and 2 Relative Permeability	1

Figures 4.34, 4.35, 4.38, 4.39, 4.42 and 4.43 show the current distribution in the part of the conductors shown in Figure 4.33 for three frequencies. Figures 4.36, 4.37, 4.40, 4.41, 4.44 and 4.45 show the phase of the part of the conductors shown in Figure 4.33 for three frequencies. Tables 4.38, 4.39, 4.40, 4.41, 4.42, 4.43, 4.44, 4,45 and 4.46 show the circuit parameters used and calculated by the model for three frequencies.



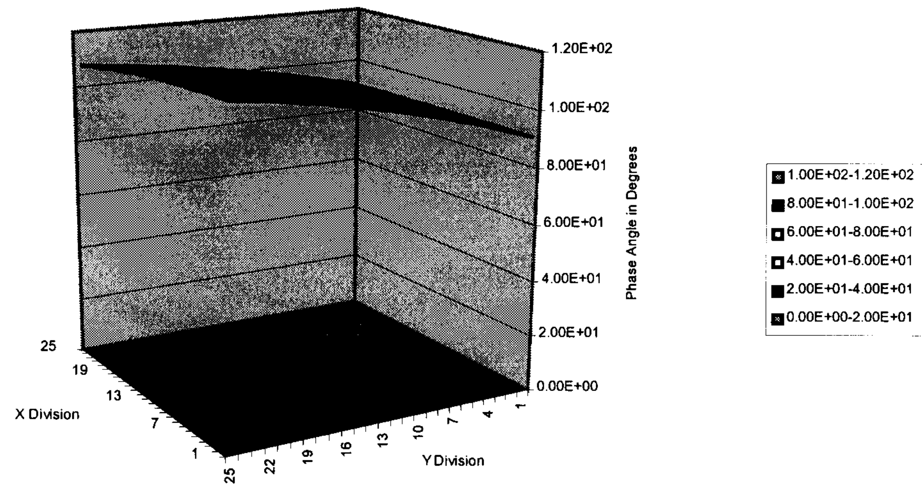
Normalized Current Distribution in Conductor 1

Figure 4.34



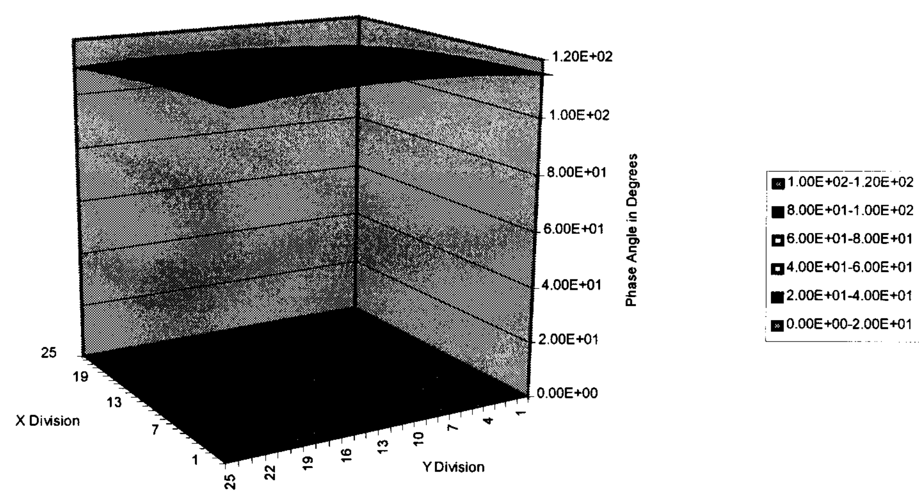
Normalized Current Distribution in Conductor 2

Figure 4.35



Phase Angle for Conductor 1

Figure 4.36



Phase Angle for Conductor 2

Figure 4.37

Table 4.38

Model Output for Conductor 1 of Configuration Shown in Figure 4.33

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.002204362308852	ohms
Reactance	0.000521602185662	ohms
Current Magnitude	441.4556274	amps
Current Phase	103.3126221	degrees
Real Power	211.451331098216000	watts

Table 4.39

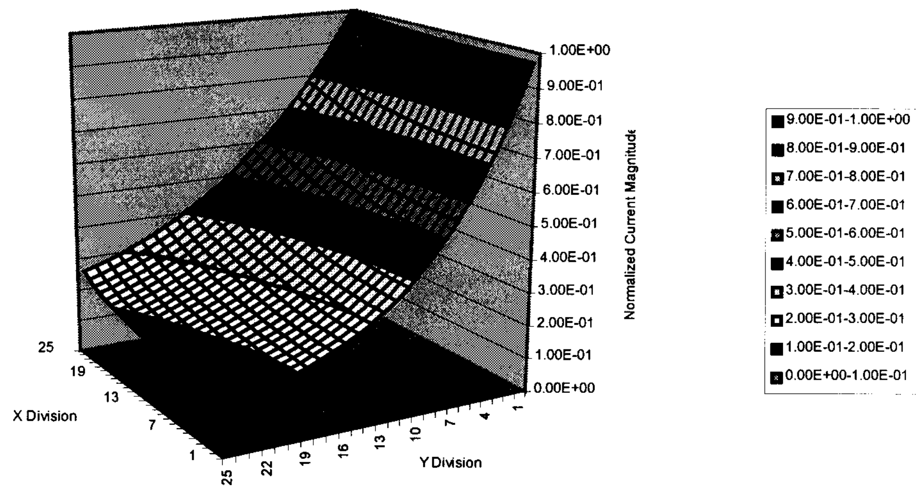
Model Output for Conductor 2 of Configuration Shown in Figure 4.33

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.002107301083709	ohms
Reactance	0.000973769931087	ohms
Current Magnitude	430.7726135	amps
Current Phase	114.8013458	degrees
Real Power	198.865835760934100	watts

Table 4.40

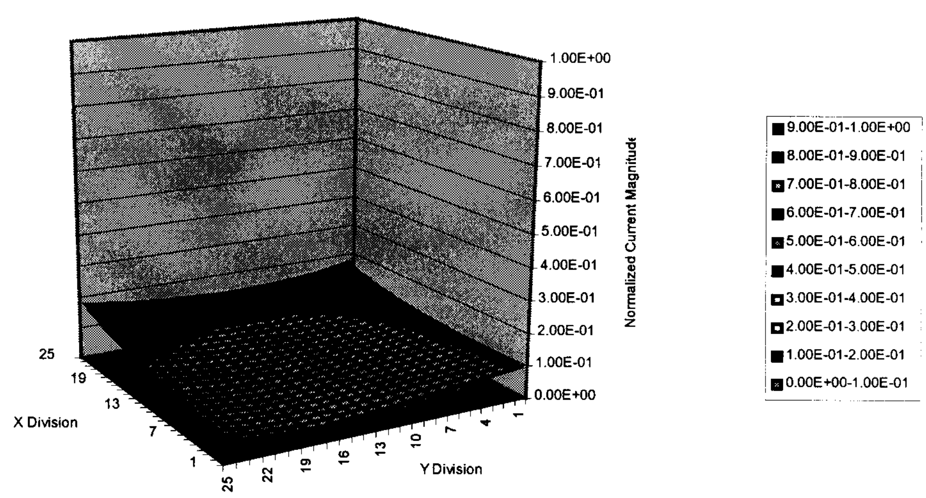
Model Output for Both Conductors Shown in Figure 4.33

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.001089585815740	ohms
Reactance	0.000374885094188	ohms
Current Magnitude	867.848907149822800	amps
Current Phase	108.986391321046300	degrees
Real Power	410.317166859150100	watts



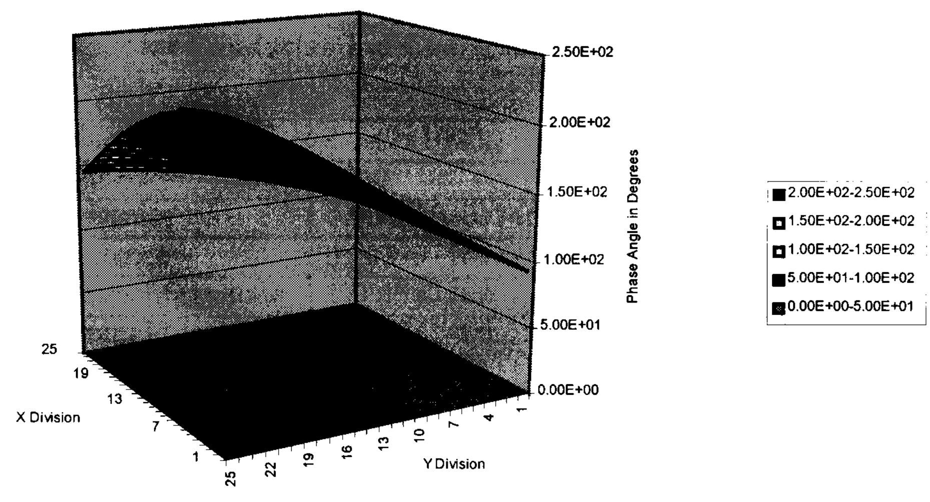
Normalized Current Distribution for Conductor 1

Figure 4.38



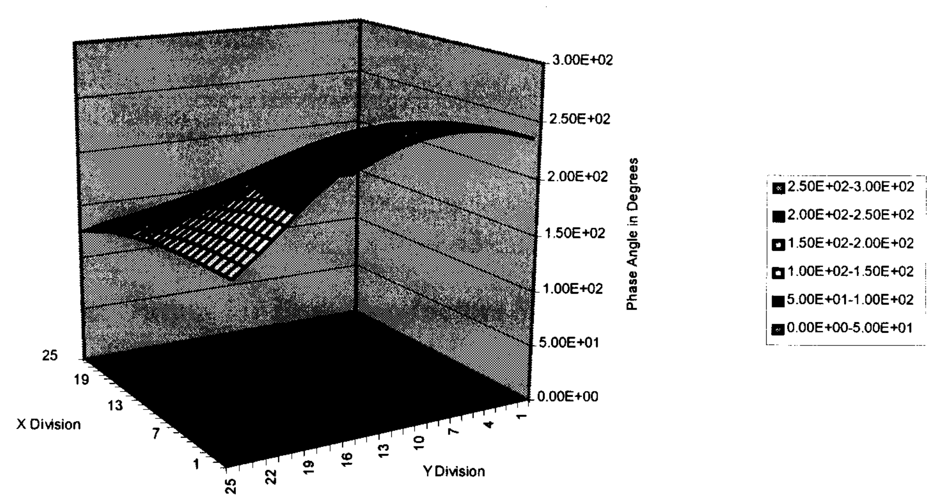
Normalized Current Distribution for Conductor 2

Figure 4.39



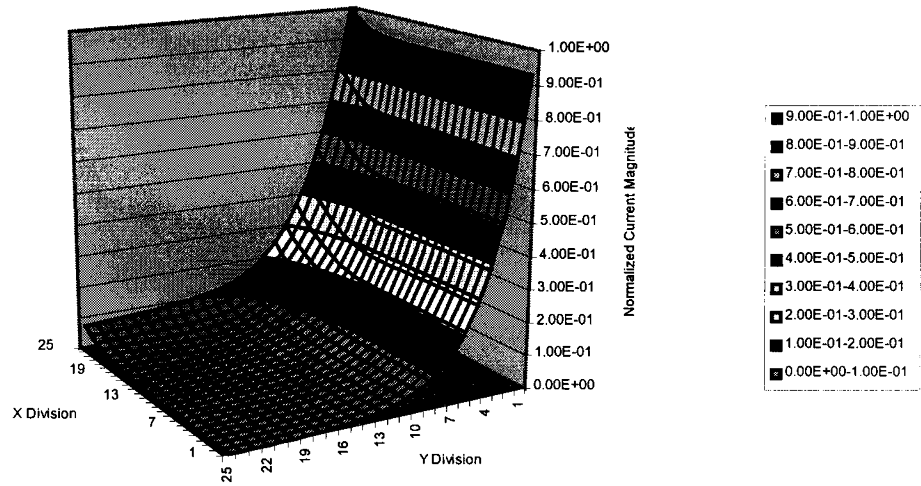
Phase Angle for Conductor 1

Figure 4.40



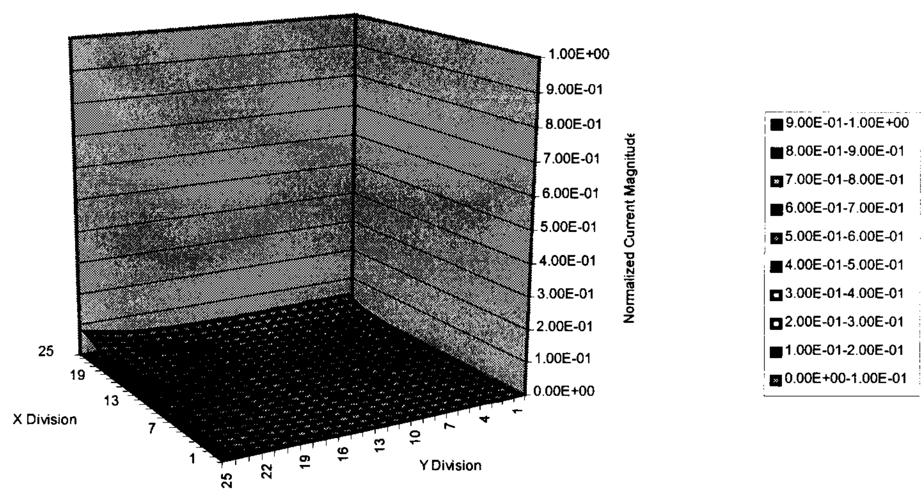
Phase Angle for Conductor 2

Figure 4.41



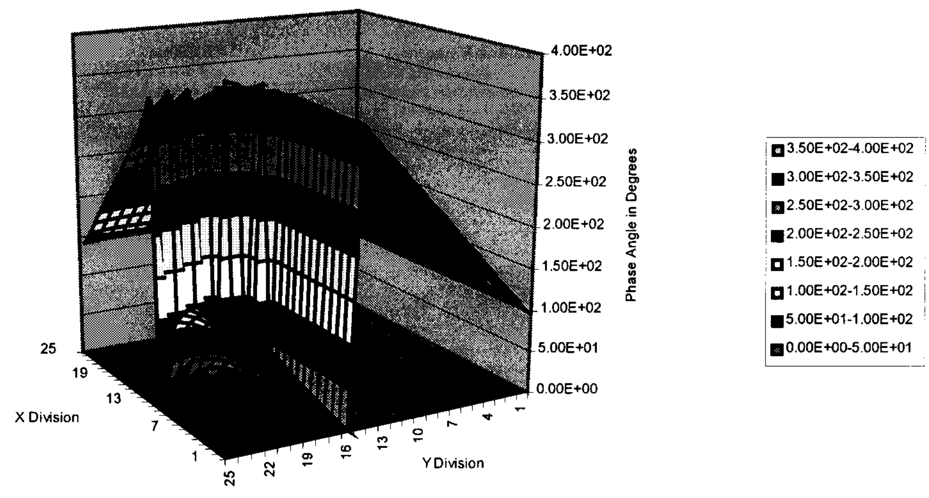
Normalized Current Distribution for Conductor 1

Figure 4.42



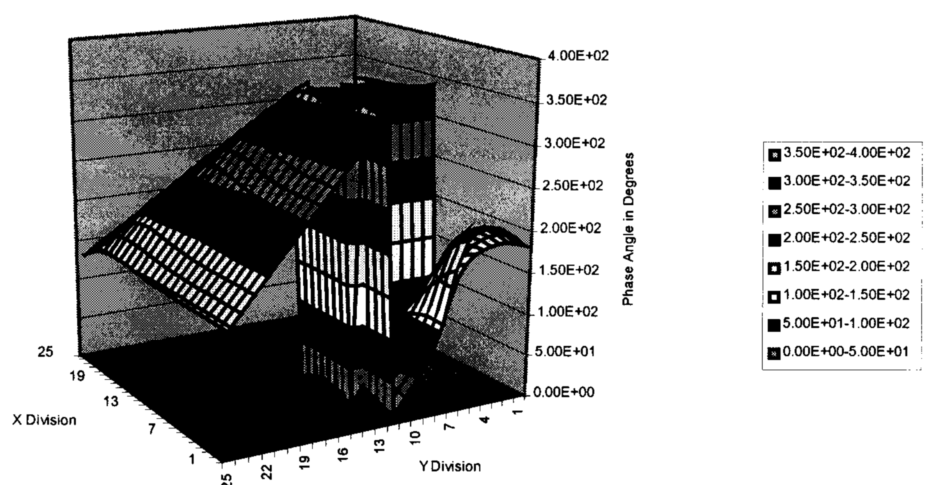
Normalized Current Distribution for Conductor 2

Figure 4.43



Phase Angle for Conductor 1

Figure 4.44



Phase Angle for Conductor 2

Figure 4.45

Table 4.44

Model Output for Conductor 1 of Configuration Shown in Figure 4.33

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.013379163767824	ohms
Reactance	0.016980186513714	ohms
Current Magnitude	46.2582016	amps
Current Phase	141.7644348	degrees
Real Power	14.261737156585920	watts

Table 4.45

Model Output for Conductor 2 of Configuration Shown in Figure 4.33

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.004399226397529	ohms
Reactance	0.229065424883950	ohms
Current Magnitude	4.3647599	amps
Current Phase	178.8997650	degrees
Real Power	0.094677363940418	watts

Table 4.46

Model Output for Both Conductors Shown in Figure 4.33

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.011574038693127	ohms
Reactance	0.016405427516851	ohms
Current Magnitude	49.807589895617720	amps
Current Phase	144.797004968626400	degrees
Real Power	14.356414520526340	watts

Figure 4.46 shows the 1st quadrant of a transmission line. There are four rectangular Litz conductors implied in Figure 4.46 because of the Y Offset. The properties of this conductor configuration are shown in Table 4.47.

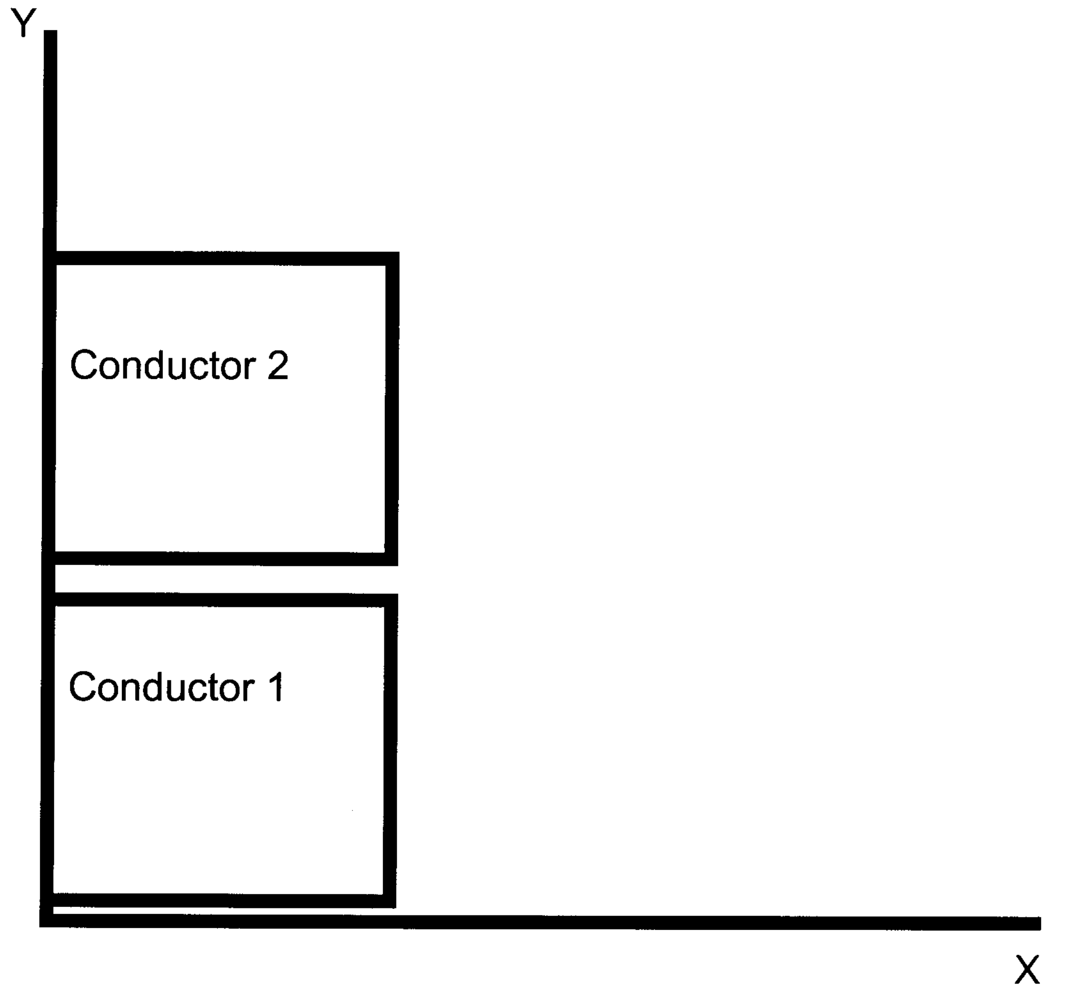


Figure 4.46

1st Quadrant of a Configuration of 4 Litz Conductors

Table 4.47

Properties of Configuration Shown in Figure 4.46

Conductor 1 and 2 Voltage	Cos(wt) volts
X Symmetry	-1
Y Symmetry	-1
Conductor 1 and 2 X Divisions	25
Conductor 1 and 2 Y divisions	25
Total segments	1250
Conductor 1 and 2 Height	.005 meters
Conductor 1 and 2 Width	.005 meters
Conductor 1 and 2 Length	3.048 meters
Conductor 1 X Offset	0.0 meters
Conductor 1 Y Offset	0.00005 meters
Conductor 2 X Offset	0.0 meters
Conductor 2 Y Offset	0.00515 meters
Conductor 1 and 2 Space Factor	.65
Conductor 1 and 2 Length Factor	1.1
Conductor 1 and 2 Conductivity	5.7e7 mhos/meter
Conductor 1 and 2 Relative Permeability	1

Tables 4.48, 4.49, 4.50, 5.51, 4.52, 4.53, 4.54, 4.55 and 4.56 show the circuit parameters used and calculated by the model for three frequencies.

Table 4.48

Model Output of Conductor 1 Shown in Figure 4.46

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.003652801596339	ohms
Reactance	0.000531194283465	ohms
Current Magnitude	270.9129333	amps
Current Phase	98.2740097	degrees
Real Power	132.891508230706800	watts

Table 4.49

Model Output of Conductor 2 Shown in Figure 4.46

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.003589378218865	ohms
Reactance	0.000972635892567	ohms
Current Magnitude	268.9021606	amps
Current Phase	105.1616898	degrees
Real Power	130.926147681288300	watts

Table 4.50

Model Output for Both Conductors Shown in Figure 4.33

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.001817245494517	ohms
Reactance	0.000376499146630	ohms
Current Magnitude	268.9021606	amps
Current Phase	105.1616898	degrees
Real Power	263.817655911995200	watts

Table 4.51

Model Output of Conductor 1 Shown in Figure 4.46

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	0.005337575325308	ohms
Reactance	0.005412581820775	ohms
Current Magnitude	131.5496216	amps
Current Phase	135.3997498	degrees
Real Power	31.334084731343260	watts

Table 4.52

Model Output of Conductor 2 Shown in Figure 4.46

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	-0.005246304041381	ohms
Reactance	0.016462706952048	ohms
Current Magnitude	57.8756027	amps
Current Phase	-162.3240509	degrees
Real Power	6.064972245491898	watts

Table 4.53

Model Output for Both Conductors Shown in Figure 4.33

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	0.002696432761985	ohms
Reactance	0.005364701671767	ohms
Current Magnitude	166.549259207410300	amps
Current Phase	153.314753345633200	degrees
Real Power	37.399056976835160	watts

Table 4.54

Model Output of Conductor1 Shown in Figure 4.46

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.005743995782858	ohms
Reactance	0.045825870791066	ohms
Current Magnitude	21.6523075	amps
Current Phase	172.8555756	degrees
Real Power	0.848879667479485	watts

Table 4.55

Model Output of Conductor 2 Shown in Figure 4.46

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	-0.817049926566540	ohms
Reactance	0.396259663825388	ohms
Current Magnitude	1.1012359	amps
Current Phase	-115.8728409	degrees
Real Power	0.002195828958604	watts

Table 4.56

Model Output for Both Conductors Shown in Figure 4.33

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.003506893578731	ohms
Reactance	0.045255748856524	ohms
Current Magnitude	22.030594688032610	amps
Current Phase	175.568967718291000	degrees
Real Power	0.851075496438089	watts

Table 4.57

Summary of Results for 4 Conductor Configuration

Frequency	Regular Resistance (ohms)	Litz Resistance (ohms)	Regular Reactance (ohms)	Litz Reactance (ohms)
60	0.0010	0.0018	0.0004	0.0004
1000	0.0029	0.0027	0.0040	0.0053
10000	0.0116	0.0035	0.0164	0.0453

Figure 4.47 shows the 1st quadrant of a transmission line. There are two Litz conductors and two regular conductors implied in Figure 4.47. The properties of this conductor configuration are shown in Table 4.58.

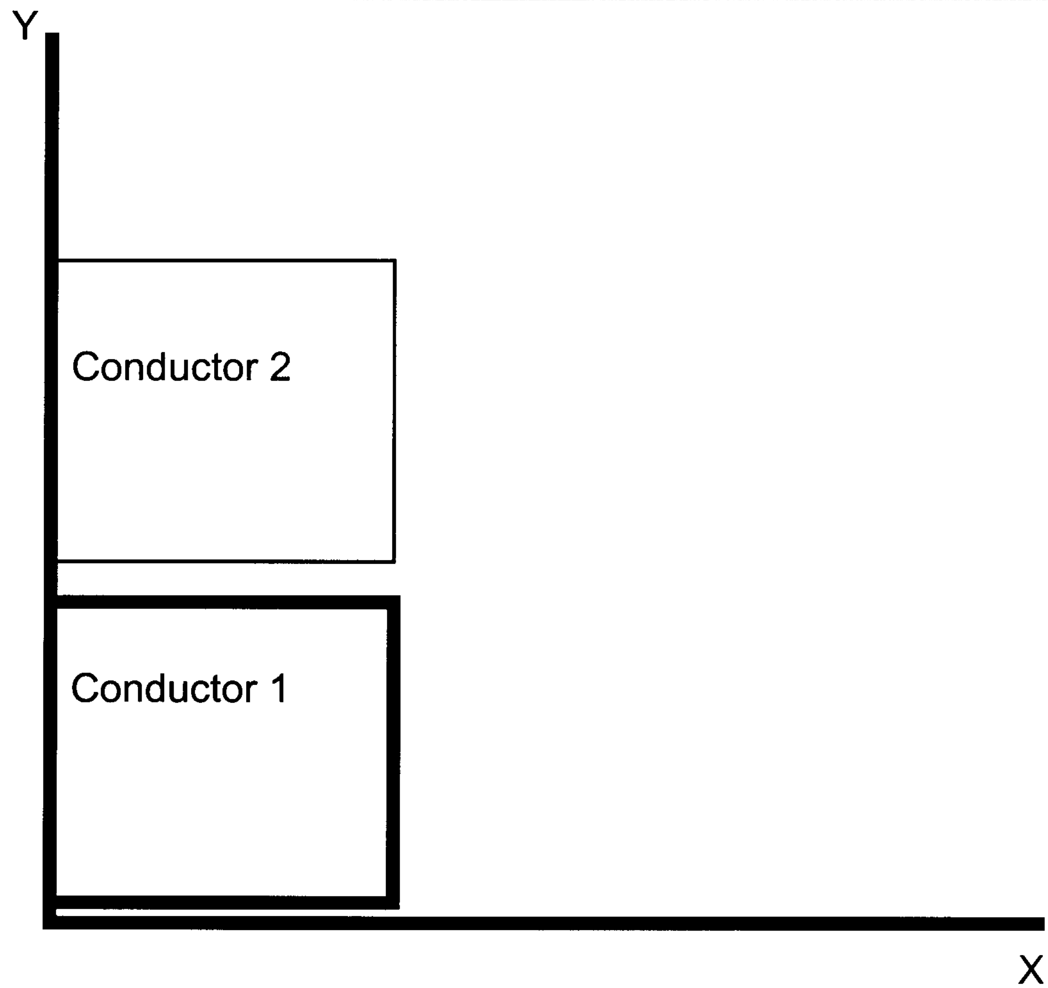


Figure 4.47

1st Quadrant of a Configuration of 2 Litz and 2 Regular Conductors

Table 4.58

Properties of Configuration Shown in Figure 4.58

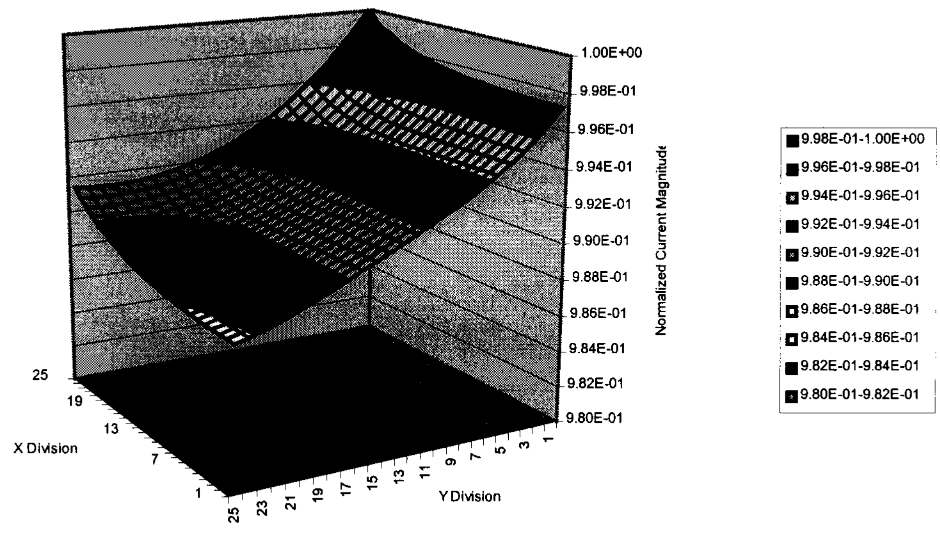
Conductor 1 and 2 Voltage	Cos(wt) volts
X Symmetry	+1
Y Symmetry	-1
Conductor 1 and 2 X Divisions	25
Conductor 1 and 2 Y divisions	25
Total segments	1250
Conductor 1 and 2 Height	.005 meters
Conductor 1 and 2 Width	.005 meters
Conductor 1 and 2 Length	3.048 meters
Conductor 1 X Offset	0.0 meters
Conductor 1 Y Offset	0.00005 meters
Conductor 2 X Offset	0.0 meters
Conductor 2 Y Offset	0.00515 meters
Conductor 1 Space Factor	.65
Conductor 1 Length Factor	1.1
Conductor 1 and 2 Conductivity	5.7e7 mhos/meter
Conductor 1 and 2 Relative Permeability	1

Figures 4.48, 4.50 and 4.52 show the current distribution in the regular conductor shown in Figure 4.47 for three frequencies. Figures 4.49, 4.51 and 4.53 show the phase of the regular conductor shown in Figure 4.47 for three frequencies. Tables 4.59, 4.60, 4.61, 4.62, 4.63, 4.64, 4.65, 4.66 and 4.67 show the circuit parameters used and calculated by the model for three frequencies.

Table 4.59

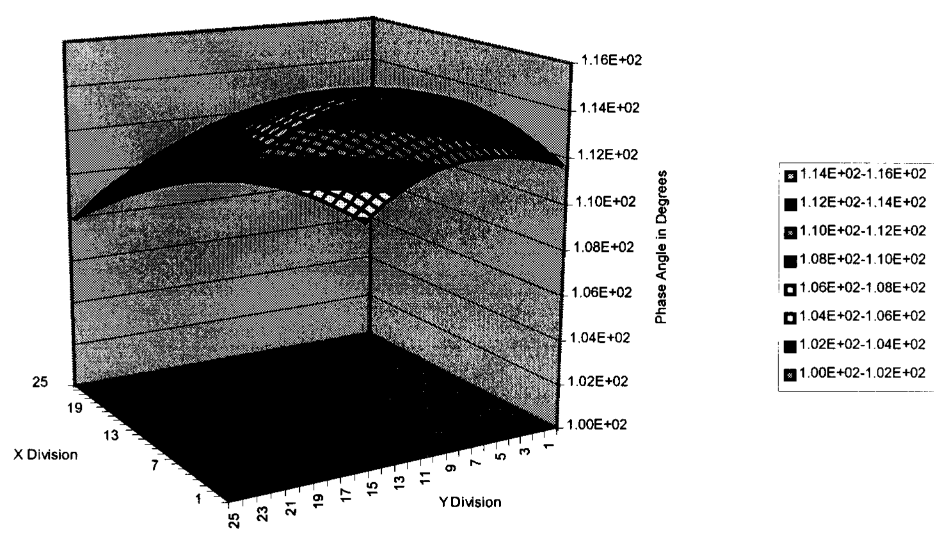
Model Output of Conductor 1 Shown in Figure 4.47

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.003709770992471	ohms
Reactance	0.000699558314532	ohms
Current Magnitude	264.8898926	amps
Current Phase	100.6789703	degrees
Real Power	127.048222925375200	watts



Normalized Current Distribution for Conductor 2

Figure 4.48



Phase Angle for Conductor 2

Figure 4.49

Table 4.60

Model Output of Conductor 2 Shown in Figure 4.47

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.002110662297273	ohms
Reactance	0.000864014471908	ohms
Current Magnitude	438.4692993	amps
Current Phase	112.2620850	degrees
Real Power	205.995924398428200	watts

Table 4.61

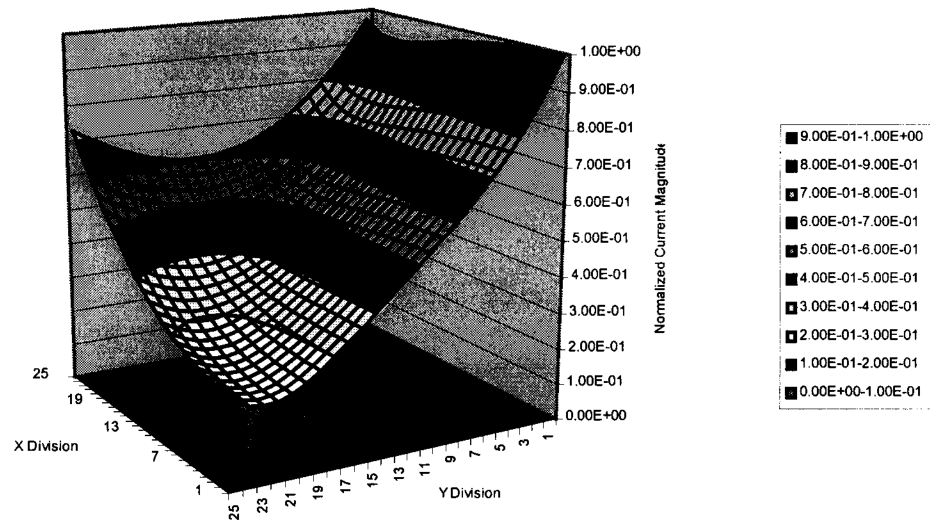
Model Output for Both Conductors Shown in Figure 4.47

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.001359409919075	ohms
Reactance	0.000439192531182	ohms
Current Magnitude	699.988152910071400	amps
Current Phase	107.904389445341600	degrees
Real Power	333.044147323803400	watts

Table 4.62

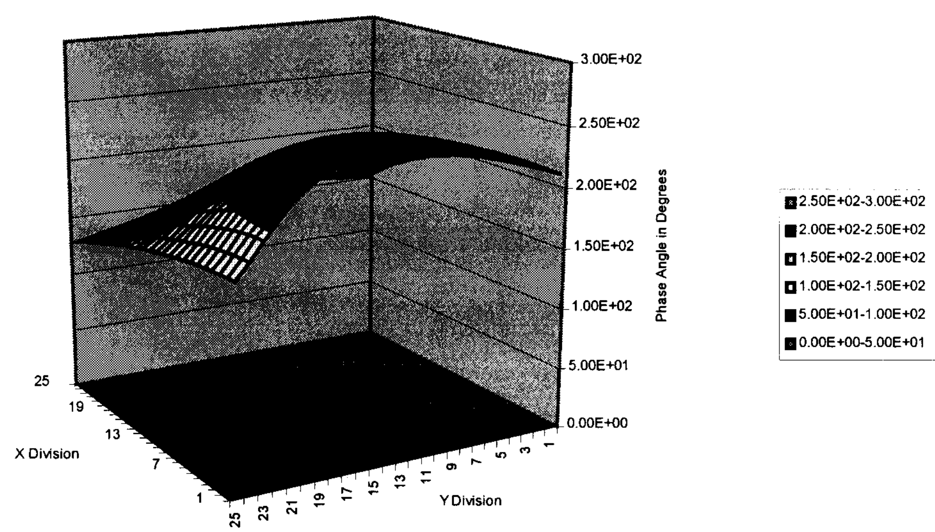
Model Output of Conductor 1 Shown in Figure 4.47

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	0.005883058630894	ohms
Reactance	0.005220896111572	ohms
Current Magnitude	127.1353989	amps
Current Phase	131.5873108	degrees
Real Power	29.266498135611580	watts



Normalized Current Distribution for Conductor 2

Figure 4.50



Phase Angle for Conductor 2

Figure 4.51

Table 4.63

Model Output of Conductor 2 Shown in Figure 4.47

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	-0.005453235985206	ohms
Reactance	0.014504489489429	ohms
Current Magnitude	64.5338593	amps
Current Phase	-159.3952942	degrees
Real Power	6.923317145636013	watts

Table 4.64

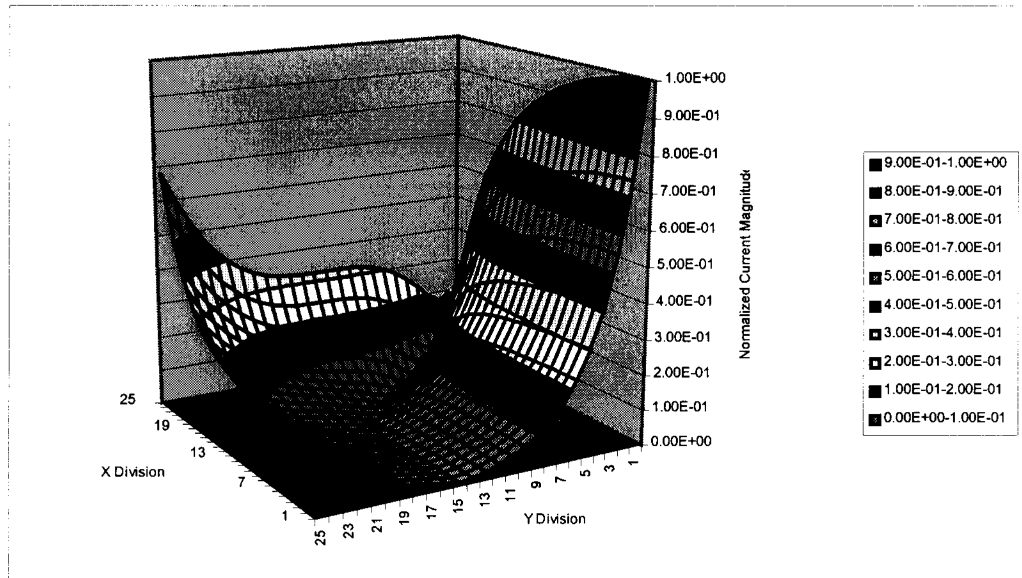
Model Output for Both Conductors Shown in Figure 4.47

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	0.002762173542221	ohms
Reactance	0.005525640245989	ohms
Current Magnitude	161.876092841472400	amps
Current Phase	153.440308359849300	degrees
Real Power	36.189815281247600	watts

Table 4.65

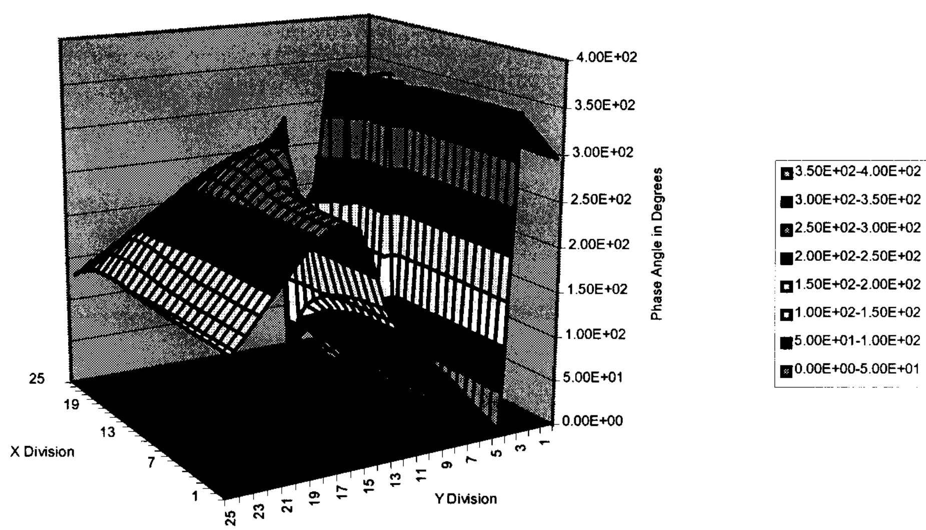
Model Output of Conductor1 Shown in Figure 4.47

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.008800589123809	ohms
Reactance	0.039038157471005	ohms
Current Magnitude	24.9888496	amps
Current Phase	167.2958679	degrees
Real Power	1.130655600959582	watts



Normalized Current Distribution for Conductor 2

Figure 4.52



Phase Angle for Conductor 2

Figure 4.53

Table 4.66

Model Output of Conductor 2 Shown in Figure 4.47

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	-0.365231772661355	ohms
Reactance	-0.107694118736626	ohms
Current Magnitude	2.6261988	amps
Current Phase	-73.5710449	degrees
Real Power	0.357588829162316	watts

Table 4.67

Model Output for Both Conductors Shown in Figure 4.47

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.005245458458735	ohms
Reactance	0.041650719234258	ohms
Current Magnitude	23.821023556738570	amps
Current Phase	172.822000395356000	degrees
Real Power	1.488244430121898	watts

Figure 4.54 shows the 1st quadrant of a transmission line. There are two Litz conductors and two regular conductors implied in Figure 4.54. The properties of this conductor configuration are shown in Table 4.68.

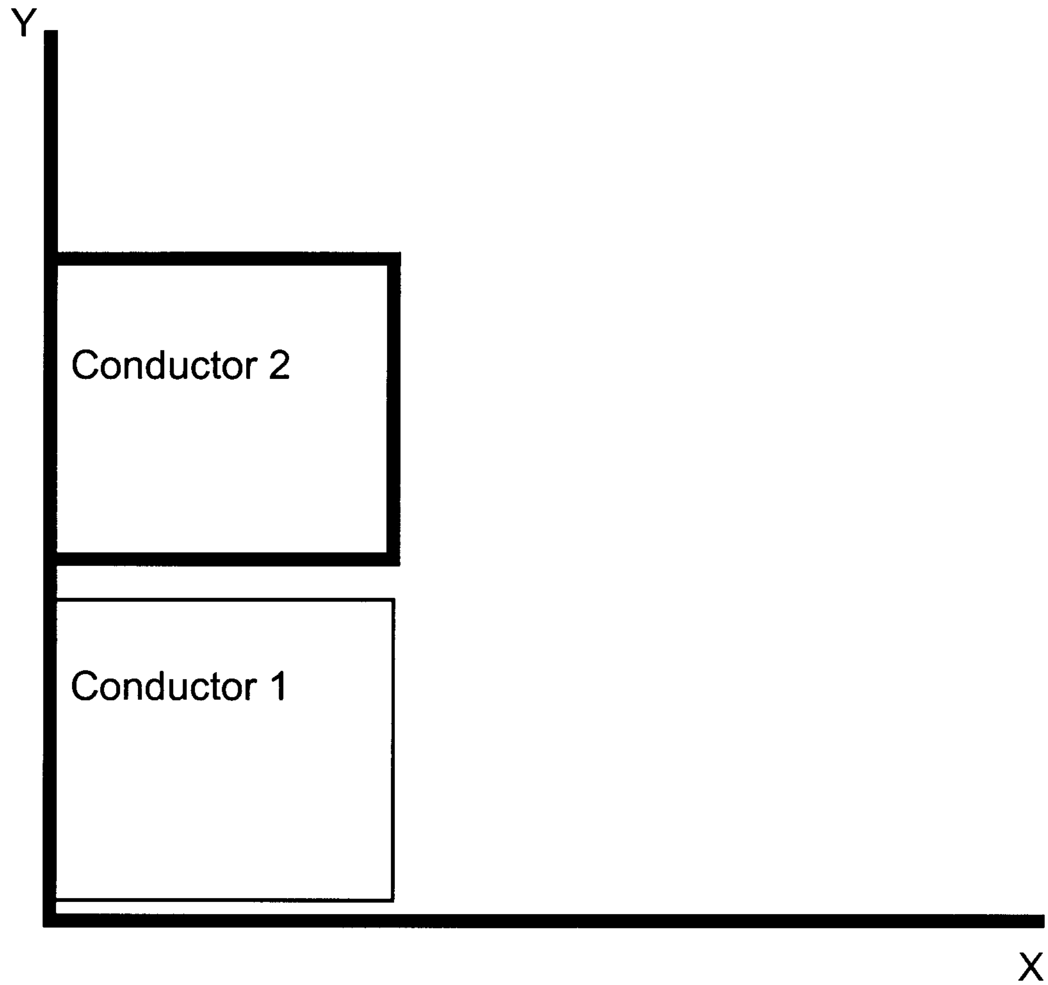


Figure 4.54

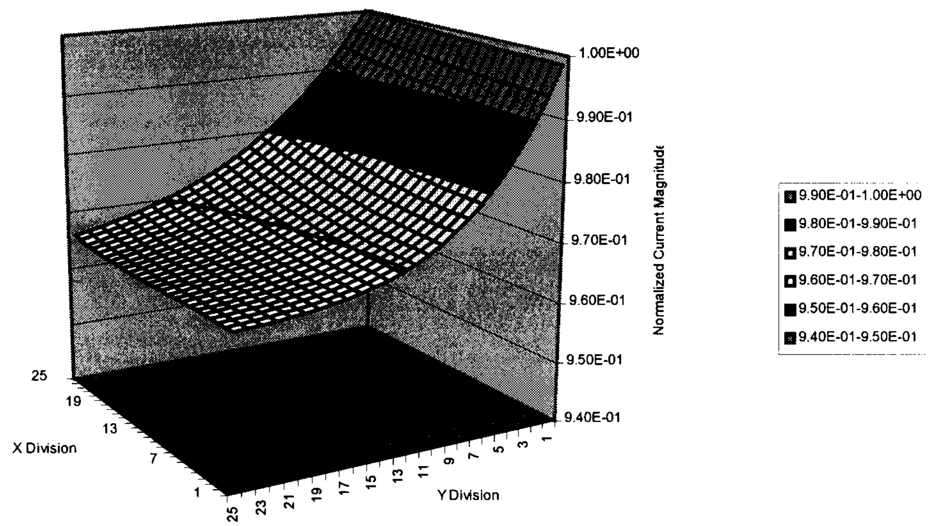
1st Quadrant of a Configuration of 2 Litz and 2 Regular Conductors

Table 4.68

Properties of Configuration Shown in Figure 4.54

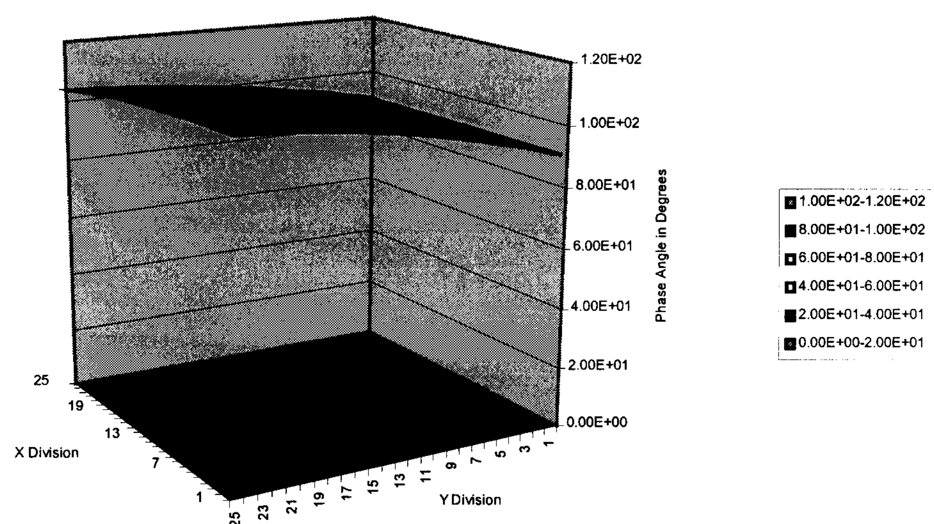
Conductor 1 and 2 Voltage	Cos(wt) volts
X Symmetry	+1
Y Symmetry	-1
Conductor 1 and 2 X Divisions	25
Conductor 1 and 2 Y Divisions	25
Total segments	1250
Conductor 1 and 2 Height	.005 meters
Conductor 1 and 2 Width	.005 meters
Conductor 1 and 2 Length	3.048 meters
Conductor 1 X Offset	0.0 meters
Conductor 1 Y Offset	0.00005 meters
Conductor 2 X Offset	0.0 meters
Conductor 2 Y Offset	0.00515 meters
Conductor 2 Space Factor	.65
Conductor 2 Length Factor	1.1
Conductor 1 and 2 Conductivity	5.7e7 mhos/meter
Conductor 1 and 2 Relative Permeability	1

Figures 4.55, 4.57 and 4.59 show the current distribution in the regular conductor shown in Figure 4.54 for three frequencies. Figures 4.56, 4.58 and 4.60 show the phase of the regular conductor shown in Figure 4.47 for three frequencies. Tables 4.69, 4.70, 4.71, 4.72, 4.73, 4.74, 4.75, 4.76 and 4.77 show the circuit parameters used and calculated by the model for three frequencies.



Normalized Current Distribution for Conductor 2

Figure 4.55



Phase Angle for Conductor 2

Figure 4.56

Table 4.69

Model Output of Conductor 1 Shown in Figure 4.54

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.002167987283849	ohms
Reactance	0.0004.23529965831	ohms
Current Magnitude	452.6997681	amps
Current Phase	101.0538712	degrees
Real Power	221.098872888959500	watts

Table 4.70

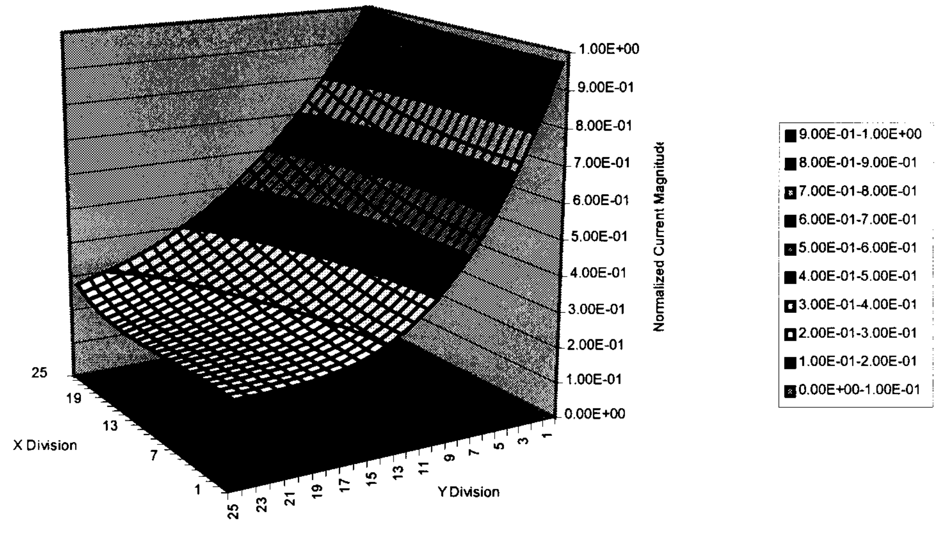
Model Output of Conductor 2 Shown in Figure 4.54

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.003591383395245	ohms
Reactance	0.001157231825761	ohms
Current Magnitude	265.0253296	amps
Current Phase	107.8602142	degrees
Real Power	127.178173496533700	watts

Table 4.71

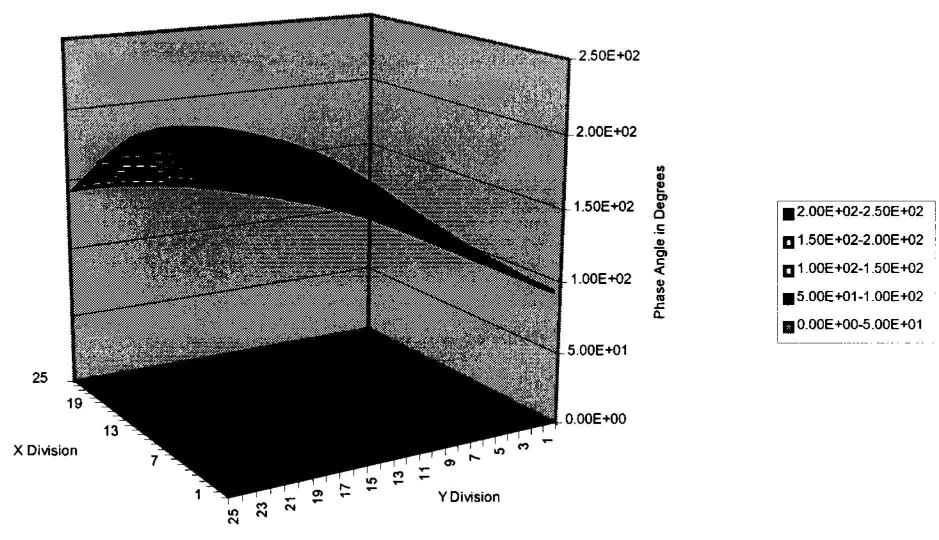
Model Output for Both Conductors Shown in Figure 4.54

Voltage	Cos(2π60t)	volts
Frequency	60	Hz
Resistance	0.001356646376749	ohms
Reactance	0.000327360004655	ohms
Current Magnitude	716.546040343060900	amps
Current Phase	103.566186707212600	degrees
Real Power	348.277046385493200	watts



Normalized Current Distribution for Conductor 2

Figure 4.57



Phase Angle for Conductor 2

Figure 4.58

Table 4.72

Model Output of Conductor 1 Shown in Figure 4.54

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	0.004085578460831	ohms
Reactance	0.004189352065560	ohms
Current Magnitude	170.8899384	amps
Current Phase	135.7184906	degrees
Real Power	55.903201315113960	watts

Table 4.73

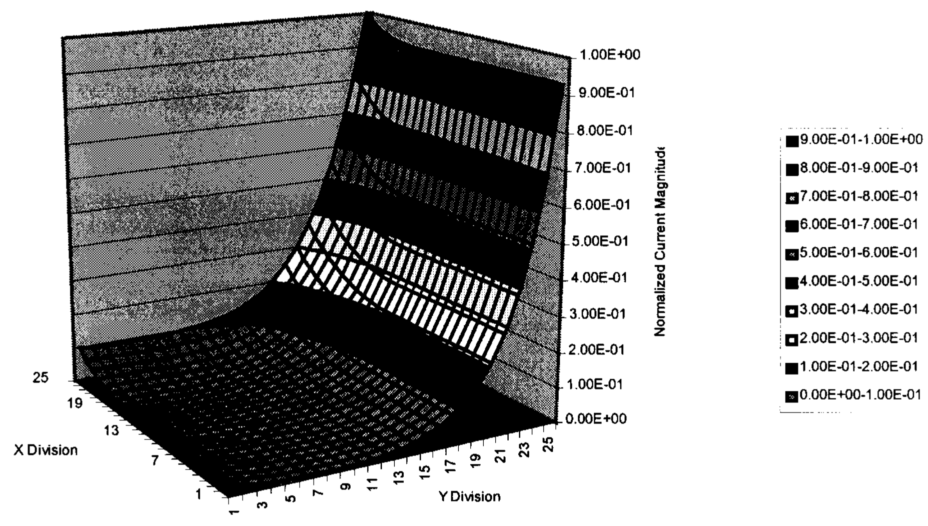
Model Output of Conductor 2 Shown in Figure 4.54

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	-0.001031965503429	ohms
Reactance	0.024876828224260	ohms
Current Magnitude	40.1635094	amps
Current Phase	-177.6245575	degrees
Real Power	2.920794901215245	watts

Table 4.74

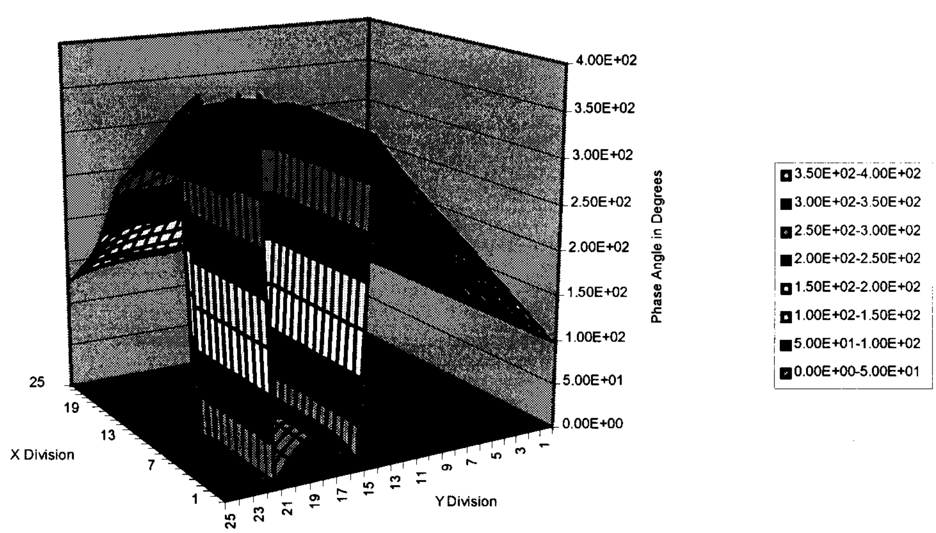
Model Output for Both Conductors Shown in Figure 4.54

Voltage	Cos(2π1000t)	volts
Frequency	1000	Hz
Resistance	0.002923783909597	ohms
Reactance	0.004037753533480	ohms
Current Magnitude	200.594778944805800	amps
Current Phase	144.091265050364700	degrees
Real Power	58.823996216329210	watts



Normalized Current Distribution for Conductor 2

Figure 4.59



Phase Angle for Conductor 2

Figure 4.60

Table 4.75

Model Output of Conductor1 Shown in Figure 4.54

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.013511657985639	ohms
Reactance	0.017068886080606	ohms
Current Magnitude	45.9357948	amps
Current Phase	141.6350098	degrees
Real Power	14.290966954367900	watts

Table 4.76

Model Output of Conductor 2 Shown in Figure 4.54

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.007520830960684	ohms
Reactance	0.234202681522120	ohms
Current Magnitude	4.2676063	amps
Current Phase	178.1607208	degrees
Real Power	0.032976642786342	watts

Table 4.77

Model Output for Both Conductors Shown in Figure 4.54

Voltage	Cos(2π10000t)	volts
Frequency	10000	Hz
Resistance	0.011724720928086	ohms
Reactance	0.016486386015760	ohms
Current Magnitude	49.430504359016470	amps
Current Phase	144.580474166522600	degrees
Real Power	14.323943597154240	watts

Table 4.78

Summary of Results for 4 Conductor Hybrid Configuration

	Conductor 2 is Regular and Conductor 1 is Litz	Conductor 1 is Regular and Conductor 2 is Litz	Conductor 2 is Regular and Conductor 1 is Litz	Conductor 1 is Regular and Conductor 2 is Litz
Frequency	Resistance (ohms)	Resistance (ohms)	Reactance (ohms)	Reactance (ohms)
60	0.0014	0.0013	0.0004	0.0003
1000	0.0028	0.0029	0.0055	0.0040
10000	0.0052	0.0117	0.0416	0.0165

CHAPTER V

ANALYSIS OF RESULTS

In Chapter IV, six cases were studied with the aid of the model. Three frequencies were considered on the regular conductor configuration, the Litz configuration and the hybrid configuration as well. The highest frequency used was 10 kHz. Taking into account the size, the conductivity, and the frequency, 25 divisions in the x direction and 25 divisions in the y direction forced each segment size smaller than the skin depth (equation 1.33). This was necessary as each segment in the model is assumed to have a constant current density.

Case 1 is the single conductor in Figures 4.1 and 4.8. These are summarized again in Table 5.1. At 60 Hz, due to the space factor and the length factor, the Litz wire has a larger resistance than the regular conductor. At 10 kHz, the Litz conductor has an AC resistance that is 35% of the regular conductor resistance. The reactance of the Litz conductor, however, is 104% of the reactance of the regular conductor.

Table 5.1

Summary of Results for Single Conductor

Frequency (Hz)	Regular Resistance (ohms)	Litz Resistance (ohms)	Regular Reactance (ohms)	Litz Reactance (ohms)
60	0.0021	0.0036	0.0057	0.0057
1000	0.0036	0.0036	0.0940	0.0953
10000	0.0102	0.0036	0.9204	0.9528

Case 2 is the two-conductor configuration shown in Figures 4.9 and 4.16. These are summarized again in Table 5.2. It can be seen that in this case, the Litz conductor has an AC resistance that is 30% of the regular conductor resistance at 10 kHz. The reactance of the Litz conductor, however, is 272% of the reactance of the regular conductor at 10 kHz.

Table 5.2

Summary of Results for 2 Conductor Configuration

Frequency (Hz)	Regular Resistance (ohms)	Litz Resistance (ohms)	Regular Reactance (ohms)	Litz Reactance (ohms)
60	0.0021	0.0036	0.0003	0.0003
1000	0.0033	0.0036	0.0039	0.0045
10000	0.0122	0.0036	0.0166	0.0451

Case 3 is the four-conductor configuration shown in Figures 4.17 and 4.24. These are summarized again in Table 5.3. It can be seen that in this case, the Litz conductor has an AC resistance that is 30% of the regular conductor resistance at 10 kHz. The reactance of the Litz conductor, however, is 272% of the reactance of the regular conductor at 10 kHz. Notice that Case 2 and Case 3 are the same except that the top and bottom conductors are split into 2. This illustrates the equivalence of proximity effect and skin effect.

Table 5.3

Summary of Results for 4 Conductor Configuration

Frequency (Hz)	Regular Resistance (ohms)	Litz Resistance (ohms)	Regular Reactance (ohms)	Litz Reactance (ohms)
60	0.0021	0.0036	0.0003	0.0003
1000	0.0033	0.0036	0.0038	0.0045
10000	0.0121	0.0036	0.0165	0.0448

Case 4 is the four-conductor configuration shown in Figures 4.25 and 4.32. These are summarized again in Table 5.4. It can be seen that in this case, the Litz conductor has an AC resistance that is 51% of the regular conductor resistance at 10 kHz. The reactance of the Litz conductor, however, is 223% of the reactance of the regular conductor at 10 kHz. It is interesting to compare Case 3 to Case 4. Really they are the same except the conductors are transposed. This is commonly used and has the net effect of lowering the reactance which the model predicts very well.

Table 5.4

Summary of Results for 4 Conductor Configuration

Frequency (Hz)	Regular Resistance (ohms)	Litz Resistance (ohms)	Regular Reactance (ohms)	Litz Reactance (ohms)
60	0.0021	0.0036	0.0001	0.0001
1000	0.0028	0.0036	0.0018	0.0019
10000	0.0070	0.0036	0.0084	0.0188

Case 5 is the four-conductor configuration shown in Figures 4.33 and 4.46. These are summarized again in Table 5.5. It can be seen that in this case, the all Litz configuration has an AC resistance that is 30% of the regular conductor resistance at 10 kHz. The reactance of the Litz conductor, however, is 276% of the reactance of the regular conductor at 10 kHz.

Table 5.5

Summary of Results for 4 Conductor Configuration

Frequency (Hz)	Regular Resistance (ohms)	Litz Resistance (ohms)	Regular Reactance (ohms)	Litz Reactance (ohms)
60	0.0010	0.0018	0.0004	0.0004
1000	0.0029	0.0027	0.0040	0.0053
10000	0.0116	0.0035	0.0164	0.0453

Case 6 is the four-conductor configuration shown in Figures 4.47 and 4.54. These are summarized again in Table 5.6. At 10 kHz, it can be seen that when the Litz is on the inside, the configuration has an AC resistance that is 44% of when it's on the outside. At 10 kHz, the reactance of the configuration where the Litz is on the inside, however, is 252% of when its on outside.

Table 5.6

Summary of Results for 4 Conductor Hybrid Configuration

	Conductor 2 is Regular and Conductor 1 is Litz	Conductor 1 is Regular and Conductor 2 is Litz	Conductor 2 is Regular and Conductor 1 is Litz	Conductor 1 is Regular and Conductor 2 is Litz
Frequency (Hz)	Resistance (ohms)	Resistance (ohms)	Reactance (ohms)	Reactance (ohms)
60	0.0014	0.0013	0.0004	0.0003
1000	0.0028	0.0029	0.0055	0.0040
10000	0.0052	0.0117	0.0416	0.0165

In this research, skin effect was introduced and a model specific to regular and Litz conductors was developed to solve different wire configurations. In general, the Litz wire saved power by having a much lower resistance. However, there are some practical problems in utilizing Litz wire. Namely, it has a high reactance. In order to take advantage of the Litz wire, you must be prepared to take a large voltage drop across the bus. Therefore, a larger power source will be required. Also, the capacitance necessary for power factor correction would be more expensive for the Litz configuration. Therefore, when the primary area of concern is real power loss, a good choice may be the Litz wire.

For further research on this topic, the model can be extended to the axi-symmetric case. Therefore, with right modeling of the segments, the current distribution and impedance of multi-layered coils composed of regular and Litz conductors could be solved. The segments would be current carrying rings. If a good model for the self inductance of a ring and the mutual inductance between two rings was developed, the program could be modified with some effort to handle this case. Another direction of further research might be to modify the model to solve for cylindrical conductors. All that would be needed is the right inductance formula for the segments. This would no doubt apply to the latest developments in using hybrid conductors in speaker wire. Finally, the model can be adapted to solve for the current distribution and impedance in both rectangular and cylindrical hollow conductors or coax conductors.

APPENDIX A

MATLAB PROGRAMS FOR FIGURES 1.2-1.15

Figure 1.2 and Figure 1.9

```
Clear
radius=.0005
r(1:26)=0:radius/25:radius
sigma=5.700e+7
mu=4*pi*1e-7
freq(1)=0
freq(2)=60
freq(3)=100
freq(4)=200
freq(5)=500
freq(6)=1000
freq(7)=2000
freq(8)=5000
freq(9)=10000
freq(10)=20000
for n=1:10,
gg(n)=-sqrt(-1)*2*pi*freq(n)*mu*sigma,
g(n)=sqrt(gg(n)),
gr(n,:)=r*g(n),
j(n,:)=bessela(0,sqrt(-1)*gr(n,:))/bessela(0,sqrt(-1)*g(n)*radius),
end
jmag=abs(j)'
jphase=angle(j)'
x=r'
results=[x,jmag,jphase]
save c:\thesis\thesis1.2 results -ascii -double –tabs
```

Figure 1.3 and Figure 1.10

```
clear
radius=.005
r(1:26)=0:radius/25:radius
sigma=5.700e+7
mu=4*pi*1e-7
freq(1)=0
freq(2)=60
freq(3)=100
freq(4)=200
freq(5)=500
freq(6)=1000
freq(7)=2000
freq(8)=5000
freq(9)=10000
freq(10)=20000
for n=1:10,
gg(n)=-sqrt(-1)*2*pi*freq(n)*mu*sigma,
g(n)=sqrt(gg(n)),
gr(n,:)=r*g(n),
j(n,:)=bessela(0,sqrt(-1)*gr(n,:))/bessela(0,sqrt(-1)*g(n)*radius),
end
jmag=abs(j)'
jphase=angle(j)'
x=r'
results=[x,jmag,jphase]
save c:\thesis\thesis1.3 results -ascii -double –tabs
```

Figure 1.4 and Figure 1.11

```
clear
radius=.05
r(1:26)=0:radius/25:radius
sigma=5.700e+7
mu=4*pi*1e-7
freq(1)=0
freq(2)=60
freq(3)=100
freq(4)=200
freq(5)=500
freq(6)=1000
freq(7)=2000
freq(8)=5000
freq(9)=10000
freq(10)=20000
for n=1:10,
gg(n)=-sqrt(-1)*2*pi*freq(n)*mu*sigma,
g(n)=sqrt(gg(n)),
gr(n,:)=r*g(n),
j(n,:)=bessela(0,sqrt(-1)*gr(n,:))/bessela(0,sqrt(-1)*g(n)*radius),
end
jmag=abs(j)'
jphase=angle(j)'
x=r'
results=[x,jmag,jphase]
save c:\thesis\thesis1.4 results -ascii -double –tabs
```

Figure 1.5 and Figure 1.12

```
clear
radius=.005
r(1:26)=0:radius/25:radius
sigma=5.700e+6
mu=4*pi*1e-7
freq(1)=0
freq(2)=60
freq(3)=100
freq(4)=200
freq(5)=500
freq(6)=1000
freq(7)=2000
freq(8)=5000
freq(9)=10000
freq(10)=20000
for n=1:10,
gg(n)=-sqrt(-1)*2*pi*freq(n)*mu*sigma,
g(n)=sqrt(gg(n)),
gr(n,:)=r*g(n),
j(n,:)=bessela(0,sqrt(-1)*gr(n,:))/bessela(0,sqrt(-1)*g(n)*radius),
end
jmag=abs(j)'
jphase=angle(j)'
x=r'
results=[x,jmag,jphase]
save c:\thesis\thesis1.5 results -ascii -double –tabs
```

Figure 1.6 and Figure 1.13

```
clear
radius=.005
r(1:26)=0:radius/25:radius
sigma=5.700e+8
mu=4*pi*1e-7
freq(1)=0
freq(2)=60
freq(3)=100
freq(4)=200
freq(5)=500
freq(6)=1000
freq(7)=2000
freq(8)=5000
freq(9)=10000
freq(10)=20000
for n=1:10,
gg(n)=-sqrt(-1)*2*pi*freq(n)*mu*sigma,
g(n)=sqrt(gg(n)),
gr(n,:)=r*g(n),
j(n,:)=bessela(0,sqrt(-1)*gr(n,:))/bessela(0,sqrt(-1)*g(n)*radius),
end
jmag=abs(j)'
jphase=angle(j)'
x=r'
results=[x,jmag,jphase]
save c:\thesis\thesis1.6 results -ascii -double –tabs
```

Figure 1.7 and Figure 1.14

```
clear
radius=.005
r(1:26)=0:radius/25:radius
sigma=5.700e+7
mu=300*4*pi*1e-7
freq(1)=0
freq(2)=60
freq(3)=100
freq(4)=200
freq(5)=500
freq(6)=1000
freq(7)=2000
freq(8)=5000
freq(9)=10000
freq(10)=20000
for n=1:10,
gg(n)=-sqrt(-1)*2*pi*freq(n)*mu*sigma,
g(n)=sqrt(gg(n)),
gr(n,:)=r*g(n),
j(n,:)=bessela(0,sqrt(-1)*gr(n,:))/bessela(0,sqrt(-1)*g(n)*radius),
end
jmag=abs(j)'
jphase=angle(j)'
x=r'
results=[x,jmag,jphase]
save c:\thesis\thesis1.7 results -ascii -double –tabs
```

Figure 1.8 and Figure 1.15

```
clear
radius=.005
r(1:26)=0:radius/25:radius
sigma=5.700e+7
mu=3000*4*pi*1e-7
freq(1)=0
freq(2)=60
freq(3)=100
freq(4)=200
freq(5)=500
freq(6)=1000
freq(7)=2000
freq(8)=5000
freq(9)=10000
freq(10)=20000
for n=1:10,
gg(n)=-sqrt(-1)*2*pi*freq(n)*mu*sigma,
g(n)=sqrt(gg(n)),
gr(n,:)=r*g(n),
j(n,:)=bessela(0,sqrt(-1)*gr(n,:))/bessela(0,sqrt(-1)*g(n)*radius),
end
jmag=abs(j)'
jphase=angle(j)'
x=r'
results=[x,jmag,jphase]
save c:\thesis\thesis1.8 results -ascii -double -tabs
```

APPENDIX B

COMPUTER MODEL

FILE LITZ.FOR

```
C*******************************************************************
C*******************************************************************
C*******************************************************************
C*******************************************************************
C**Litz Program for calculating current distribution
C**in a conductor configuration involving both litz wire and regular
C**non-magnetic conductors. At present, conductors must be
C**solid rectangular.
C*******************************************************************
C*******************************************************************
C*******************************************************************
C parms has all the parameter statements
       include 'PARMS'
C positions for defining conductor boundaries for all conductors
      REAL*8 Xdim(MPOS)
C     X coordinates of all physical conductors
C     There are four x coordinates per conductor
C     starting from the lower left and proceeding to each corner
C     counter clockwise
C
      REAL*8 Ydim(MPOS)
C     Y coordinates of all physical conductors
C     There are four y coordinates per conductor
C     starting from the lower left and proceeding to each corner
C     counter clockwise
C
      Complex*16 Volt(MAXCOND)
C     The voltages across each conductor
C
C     DIMENSION depth(MAXCOND)  Not currently implemented.
C     Depth of penetration for conductors. In future a sanity check for
determining
C     if the coductors element are fine enough.
C     The elements of the conductors should have dimensions of about
C     1/2 or less than the depth of penetration if they are located
C     within the first three depths of the conductor.
C
      REAL*8 Rho(MAXCOND)
C     Resistivity of conductors
C
      real len
C     Length of all conductors (could be changed to an array)
C
C
      real freq
C     frequency in hz
C
      real phase,cmag,delx,dely,area
C
```

```
      integer  ixdiv(MAXCOND)
C     Divisions in x direction of conductors
C
      integer  iydiv(MAXCOND)
C     Divisions in y direction of conductors
C
      integer  ttype(MAXCOND)
C     ttype is type of conductor
C     0=solid-rectangular-nonlitz
C     1=hollow-rectangular-nonlitz
C     2=solid-rectangular-litz
C
      Complex*16 V(MAXELE),cur(MAXCOND),pow(MAXCOND),amp,watt
C     V is voltage vector the modified to the current vector after
return of slnpd
C
      Complex*16 ZZ(MAXELE,MAXELE),DD
C     ZZ is the impedance matrix
C
      integer cnum, num,idim,icond,ibeg,iend,i,iflag,imult,ic
      integer iltzx,iltzy,ix
C
      real pi,sf,lf,xsym,ysym
C DEFINE CONSTANTS
      num=0
      pi=3.14159265358979323
C
C      READ in the input data
C
      CALL LITZIO(cnum,xsym,ysym,ttype,len,ixdiv,iydiv,Xdim,Ydim
     +,Rho,freq,Volt,sf,lf)

C     cnum is number of conductors
      write(20,*) 'Number of conductors=',cnum
      write(20,*) 'X SYMMETRY FACTOR=',xsym
      write(20,*) 'Y SYMMETRY FACTOR=',ysym
      write(20,*) 'Length of conductors=',len
      write(20,*) 'Frequency=',freq
      write(40,*) 'Number of conductors=',cnum
      write(40,*) 'X SYMMETRY FACTOR=',xsym
      write(40,*) 'Y SYMMETRY FACTOR=',ysym
      write(40,*) 'Length of conductors=',len
      write(40,*) 'Frequency=',freq

      if(cnum.gt.MAXCOND) then
       write(6,*) 'Too Many conductors'
       goto 8000
      endif
C*******CONVERT INPUT DATA TO PROPER DIMENSIONS********
Clength in inductance formulas must be in cm to give micro H
      len=len*2.54
C DEFAULT SPACE FACTOR AND LENGTH FACTOR FOR ALL LITZ
      if (sf.eq.0.0) sf=.65
      if (lf.eq.0.0) lf=1.1
C Convert coordinates from inches to centimeters
      do 500 idim=1,cnum*4
       Xdim(idim)=2.54*Xdim(idim)
```

```
       Ydim(idim)=2.54*Ydim(idim)
500    CONTINUE
C
C compute the number of simultaneous equations
       do 99 icond=1,cnum
          RHO(icond)=2.54*RHO(icond)/1000000
        if(ttype(icond).ne.2) then
         num=num+ixdiv(icond)*iydiv(icond)
        else
         num=num+1
        endif
99     continue
C
C      MAKE THE VOLTAGE VECTOR
C
       write(6,*) 'Making the Voltage Vector'
       CALL VMAKE(cnum,num,V,Volt,ixdiv,iydiv,ttype)
C
C        Make the Impedance Matrix
C
       write(6,*) 'Making the Impedance Matrix'
C
       CALL ZMAKE(cnum,ttype,num,freq,len,xsym,ysym
     +,Xdim,Ydim,Rho,ixdiv,iydiv,ZZ,sf,lf)
C
C solve for current vector. Vector V changes to current
C
      write(6,*) 'Solve for the current vector'
C
      write(6,*) 'num,maxele',num,MAXELE
      CALL SLNPD(ZZ,V,DD,num,MAXELE)

C
C
C OUTPUT RESULTS (BE VERY CAREFUL INTERPRETING WHEN THERE IS SYMMETRY)
C
C
C
7100  FORMAT(3A24)
7120  FORMAT(I12,4F15.9)
3333  FORMAT(E15.6,A1,$)
      watt=0.0
      amp=0.0
      ibeg=0
      iend=0
      DO 7777 i=1,cnum
       cur(i)=dcmplx(0.00,0.00)
       if(i.eq.1) then
C IF FIRST CONDUCTOR THEN FIRST ELEMENT OF FIRST CONDUCTOR IS 1
        ibeg=1
       else
C Check previous conductor type if litz then previous conductor only
has one element
        if(ttype(i-1).eq.2) then
         ibeg=ibeg+1
        else
```

```
C If previous conductor is regular then calculate the begin element of
next conductor
          ibeg=ibeg+ixdiv(i-1)*iydiv(i-1)
         endif
        endif
C if conductor is litz get the end element
        if(ttype(i).eq.2) then
         iend=iend+1
        else
C conductor is regular get end element
         iend=iend+ixdiv(i)*iydiv(i)
        endif
        write(20,*) 'Conductor # ',i
        write(40,*) 'Conductor # ',i
        iflag=1
        imult=1
        write(20,3333) imult*1.0,','
        write(40,3333) imult*1.0,','

        do 7766 ic=iend,ibeg,-1
         if(ttype(i).eq.2) then
          cur(i)=v(ic)*ixdiv(i)*iydiv(i)
          delx=(Xdim((i-1)*4+2)-Xdim((i-1)*4+1))/ixdiv(i)
          dely=(Ydim((i-1)*4+3)-Ydim((i-1)*4+2))/iydiv(i)
          area=delx*dely*sf
          pow(i)=ixdiv(i)*iydiv(i)*Rho(i)*len*lf
     +*(abs(v(ic))/2**.5)**2/area
          write(20,3333) abs(v(ic)),','
          write(20,*)
          write(40,3333) atan2(dreal(v(ic)),dimag(v(ic)))*180/pi,','
          write(40,*)
         else
          ix=iend-imult*ixdiv(i)+iflag
          write(20,3333) abs(v(ix)),','
          write(40,3333) atan2(dreal(v(ix)),dimag(v(ix)))*180/pi,','

          if(iflag.eq.ixdiv(i)) then
           iflag=0
           imult=imult+1
           write(20,*)
           write(20,3333) imult*1.0,','
           write(40,*)
           write(40,3333) imult*1.0,','

          endif
          iflag=iflag+1
          cur(i)=cur(i)+v(ic)
          delx=(Xdim((i-1)*4+2)-Xdim((i-1)*4+1))/ixdiv(i)
          dely=(Ydim((i-1)*4+3)-Ydim((i-1)*4+2))/iydiv(i)
          area=delx*dely
          pow(i)=pow(i)+Rho(i)*len*(abs(v(ic))/sqrt(2.0))**2/area
         endif
 7766   CONTINUE
        watt=watt+pow(i)
        amp=amp+cur(i)
        phase=atan2(dreal(cur(i)),dimag(cur(i)))*180.0/pi
        cmag = abs(cur(i))
```

```
        write(6,*) 'Conductor Current (real,imag) =',cur(i)
        write(6,*) 'Conductor Current (mag,phase) =',cmag,phase
        write(6,*) 'Z=R + JWL =',Volt(i)/cur(i)
        write(6,*) 'Power Loss = ',pow(i)
        write(6,*) 'Effective Resistance= ',2.0*pow(i)/cmag**2.0
        write(20,*) 'Conductor Current (real,imag) =',cur(i)
        write(20,*) 'Conductor Current (mag,phase) =',cmag,phase
        write(20,*) 'Z=R + JWL =',Volt(i)/cur(i)
        write(20,*) 'Power Loss = ',pow(i)
        write(20,*) 'Effective Resistance= ',2.0*pow(i)/cmag**2.0
        write(40,*) 'Conductor Current (real,imag) =',cur(i)
        write(40,*) 'Conductor Current (mag,phase) =',cmag,phase
        write(40,*) 'Z=R + JWL =',Volt(i)/cur(i)
        write(40,*) 'Power Loss = ',pow(i)
        write(40,*) 'Effective Resistance= ' ,2.0*pow(i)/cmag**2.0

7777   CONTINUE

8000   CONTINUE
        write(6,*) 'TOTAL Current(real,imag) =',amp
        write(6,*) 'TOTAL
I',abs(amp),atan2(dreal(amp),dimag(amp))*180/pi
        write(6,*) 'ZTotal = R + JWL =',dcmplx(1,0)/amp
        write(6,*) 'Power Loss = ',watt
        write(6,*) 'Effective Resistance= ' ,2.0*watt/abs(amp)**2.0
        write(20,*) 'TOTAL Current(real,imag) =',amp
        write(20,*)
'I(M,P)',abs(amp),atan2(dreal(amp),dimag(amp))*180/pi
        write(20,*) 'ZTotal = R + JWL =',dcmplx(1,0)/amp
        write(20,*) 'Power Loss = ',watt
        write(20,*) 'Effective Resistance= ' ,2.0*watt/abs(amp)**2.0
        write(40,*) 'TOTAL Current(real,imag) =',amp
        write(40,*)
'I(M,P)',abs(amp),atan2(dreal(amp),dimag(amp))*180/pi
        write(40,*) 'ZTotal = R + JWL =',dcmplx(1,0)/amp
        write(40,*) 'Power Loss = ',watt
        write(40,*) 'Effective Resistance= ' ,2.0*watt/abs(amp)**2.0

       STOP
       END
```

FILE PARMS.FOR

```
C positions for defining conductor boundaries for all conductors
C      IMPLICIT NONE
C number of conductors*4
      PARAMETER (MPOS=32)
C maximum number of conductors
      PARAMETER (MAXCOND=4)
C maximum number of elements number of conductors*numelemx*numelemy
      PARAMETER (MAXELE=4000)
```

FILE LITZIO.FOR

```
C*******************************************************************
C*******************************************************************
C*******************************************************************
C*******************************************************************
      SUBROUTINE LITZIO(cnum,xsym,ysym,ttype,len,ixdiv,iydiv,Xdim
     +,Ydim,Rho,freq,Volt,sf,lf)
C parms has all the parameter statements
      include 'PARMS'
C positions for defining conductor boundaries for all conductors
      REAL*8 Xdim(MPOS)
C     X coordinates of all physical conductors
C
      REAL*8 Ydim(MPOS)
C     Y coordinates of all physical conductors
C
      COMPLEX*16 Volt(MAXCOND)
C     The voltages across each conductor
C
      REAL*8 Rho(MAXCOND)
C     Resistivity of conductors
C
      real len
C     Length of conductors
C
      real freq
C     frequency in hz
C
      integer  ixdiv(MAXCOND)
C     Divisions in x direction of conductors
C
      integer  iydiv(MAXCOND)
C     Divisions in y direction of conductors
C
      integer  ttype(MAXCOND)
C     ttype is type of conductor
C     0=solid-rectangular-nonlitz
C     1=hollow-rectangular-nonlitz
C     2=solid-rectangular-litz
C
      integer cnum,lur,INDEXX,INDEXY,II,JJ
      character filename*17
      character*48 atitle
      real rl,img,xsym,ysym,sf,lf
C******************** Input Data ****************************
      filename='litz.dat'
        lur=21
      write(6,*) 'Reading DATA from ',filename
      OPEN(lur,FILE=FILENAME)
9100  FORMAT(A8)
9101  FORMAT(I4)
9102  FORMAT(F11.5)
9103  FORMAT(2F11.5)
      indexx=1
```

```
      indexy=1
C READ IN TITLE
      read(lur,9100) atitle
C READ IN NUMBER OF CONDUCTORS
      read(lur,9101) cnum
C READ IN X SYMMETRY INFORMATION
      read(lur,9102) xsym
C READ IN Y SYMMETRY INFORMATION
      read(lur,9102) ysym
C READ IN CONDUCTOR TYPES
      DO 200 ii=1,cnum
       read(lur,9101) ttype(ii)
200   CONTINUE
C READ IN CONDUCTOR LENGTH
      read(lur,9102) len
C READ IN X divisions of conductors
      DO 201 ii=1,cnum
       read(lur,9101) ixdiv(ii)
201   CONTINUE
C READ IN Y divisions of conductors
      DO 202 ii=1,cnum
       read(lur,9101) iydiv(ii)
202   CONTINUE
C READ IN X COORDINATES
C every 4 places is one conductor
      DO 203 ii=1,cnum
       DO 303 jj=1,4
        read(lur,9102) Xdim(indexx)
        indexx=indexx+1
303    CONTINUE
203   CONTINUE
C READ IN Y COORDINATES
C every 4 places is one conductor
      DO 204 ii=1,cnum
       DO 304 jj=1,4
        read(lur,9102) Ydim(indexy)
        indexy=indexy+1
304    CONTINUE
204   CONTINUE
C READ IN RESISTIVITIES
      DO 205 ii=1,cnum
       read(lur,9102) Rho(ii)
205   CONTINUE
C READ IN THE FREQUENCY
       read(lur,9102) freq
C READ IN THE LITZ SPACE FACTOR
       read(lur,9102) sf
C READ IN THE LITZ LENGTH FACTOR
       read(lur,9102) lf
C READ IN CONDUCTOR VOLTAGES
      DO 206 ii=1,cnum
       read(lur,9103) rl,img
       Volt(ii)=dcmplx(rl,img)
206   CONTINUE
      RETURN
      end
```

FILE VMAKE.FOR

```
      SUBROUTINE  VMAKE(cnum,num,V,Volt,ixdiv,iydiv,ttype)
C     parms has all the parameter statements in it
      include'PARMS'
C positions for defining conductor boundaries for all conductors
C number of conductors*4

      complex*16 Volt(MAXCOND)
      complex*16 V(MAXELE)
      logical litz
      integer ixdiv(MAXCOND)
      integer iydiv(MAXCOND)
      integer ttype(MAXCOND)
      integer range,cnum,num,inum
      integer iicond,ielem
      litz=.false.
      iicond=0
      range=0
      inum=0
      do 100 inum=1,num
       call ltest(inum,litz,cnum,iicond,ttype,ixdiv,iydiv)
       if(litz) then
        V(inum)=dreal(Volt(iicond))
       else
        call ctest(ielem,inum,cnum,iicond,ixdiv,iydiv,ttype)
        V(inum)=dreal(Volt(iicond))
       endif
100   continue
      RETURN
      END
```

FILE ZMAKE.FOR

```
      SUBROUTINE ZMAKE(cnum,ttype,num,freq,len,xsym,ysym
     + ,Xdim,Ydim,Rho,ixdiv,iydiv,ZZ,sf,lf)
C     parms has all the parameter statements in it
     include 'PARMS'
C positions for defining conductor boundaries for all conductors
C number of conductors*4
      Complex*16 ZZ(MAXELE,MAXELE)
      REAL*8 Xdim(MPOS)
      REAL*8 Ydim(MPOS)
      REAL*8 Rho(MAXCOND)
      integer ixdiv(MAXCOND)
      integer iydiv(MAXCOND)
      integer ttype(MAXCOND)
      logical litz,litzrow,litzcol
      integer cnum,num,row,col,iicond2,iicond1,iicond,ielem
      integer ielem1,ielem2,ircond1,ircond2
      real freq
      real resis,react,l,len,M,pi,xsym,ysym,sf,lf
      litz=.false.
      ielem=0
      ielem1=0
      ielem2=0
      ircond1=0
      ircond2=0
      iicond1=0
      iicond2=0
      iicond=0
      M=0.000
      pi=3.1415926536
C row can be thought of as the element number at which the equation is
C being written
      do 200 row=1,num
       do 300 col=1,num
        if(row.eq.col) then
C*******************************************************************
C*******************************************************************
C**************DIAGONAL OF SYSTEM MATRIX*************************
C*******************************************************************
C*******************************************************************
C find out if the element is a litz conductor
C if it is, then iicond is the conductor number
         call ltest(row,litz,cnum,iicond,ttype,ixdiv,iydiv)
         if(litz) then
C         ***************************************************
C         *FOR LITZ WIRE DIAGONAL TERMS OF THE SYSTEM MATRIX:*
C         ***************************************************
C The impedance entry for the litz wire should be the sum of all the
C litz elemental self and mutual impedances divided by the total number
C of elements comprising the litzwire. This is done due to all litz
C elemental currents assumed as being equal and when writing equations
C for each litz element, introducing many extra equations. This takes
C all the extra equations and averages them hopefully in the same
C manner in which each individual wire in a litz conductor is
```

```
C averaged or transposed throughout the cross section.
C For the case of symmetry, one or three additional terms must be
C added.
C If there is x symmetry( symmetry about the y-axis)
C then the mutual impedance of the litzwire with its counterpart
C on the other side of the x-axis must be added not forgeting the
C sign of the symmetry.(this is also divided by the number or elems)
C If there is y symmetry(symmentry about the x-axis)
C then the mutual impedance of the litzwire with its counterpart
C on the other side of the x-axis must be added not forgetting the
C sign of the symmetry.(this is also divided by the number or elems)
C If there is both x and y symmetry, then the previous two terms
C plus the mutual impedance of the litzwire with its counterpart
C in the 3rd quadrant must be added. The sign of the 3rd quadrant
C is the product of the x symmetry sign and the y symmetry sign.
C (this is also divided by the number or elems)
C          calculate the resistance of a litz element
           CALL LITZRESI(iicond,Xdim,Ydim,Rho,len,sf,lf,resis,
     +     ixdiv,iydiv)
C          Calculate the mean self L of a litzwire element
           CALL LITZSELF(iicond,ixdiv,iydiv,Xdim,Ydim,len,sf,l)
C          Calculate the symmetrical mutual terms if any
             if(xsym.ne.0.0.or.ysym.ne.0.0) then
            CALL LITZSYMM(row,iicond,ixdiv,iydiv,Xdim,Ydim,
     +      len,M,xsym,ysym)
           endif
          else
C         ****************************************************
C         *FOR NON-LITZ WIRE DIAGONAL TERMS OF THE SYSTEM MATRIX:*
C         ****************************************************
C          find out which conductor it is and which element
             call ctest(ielem,row,cnum,iicond,ixdiv,iydiv,ttype)
C          calculate the elemental resistance
           CALL RESIST(iicond,Xdim,Ydim,Rho,ixdiv,iydiv,len,resis)
C          Calculate the elemental self inductance using Grover's Self
C          inductance formula for a rectangle
             CALL SELFL(iicond,ixdiv,iydiv,Xdim,Ydim,len,l)
C          Calculate the symmetrical mutual terms
           CALL MUTSYMSL(ielem,iicond,ixdiv,iydiv,Xdim,Ydim,
     +     len,M,xsym,ysym)
          endif
C         diagonal of system matrix is resistance + reactance
C         system matrix entry
          react=freq*2.0*pi*(l+M)

          M=0.00
          l=0.00
          ZZ(row,col)=dcmplx(resis,react)
         else
C******************************************************************
C******************************************************************
C**************NON-DIAGONAL OF SYSTEM MATRIX***********************
C******************************************************************
C******************************************************************
          call ltest(row,litzrow,cnum,iicond1,ttype,ixdiv,iydiv)
          call ltest(col,litzcol,cnum,iicond2,ttype,ixdiv,iydiv)
           if(litzrow.and.litzcol) then
```

```
C           Calculate the mutual inductance between two litzwires
C           and the symmetrical terms of the second litzwire
            CALL LSYMMZ(iicond1,iicond2,ixdiv,iydiv,Xdim,Ydim,len
     +      ,xsym,ysym,M)
            endif
          if(litzrow.and.(.not.litzcol)) then
C          find out which normal conductor it is and which element
             call ctest(ielem,col,cnum,iicond,ixdiv,iydiv,ttype)
C          Calculate the mutual inductance between a litzwire and
C          a normal conductor element (M is divided by elems)
C           and the symmetrical terms of the conductor element
           CALL LITZREG(iicond1,iicond,ielem,ixdiv,iydiv,Xdim,
     +     Ydim,len,M,xsym,ysym)
          endif
          if((.not.litzrow).and.litzcol) then
C          find out which normal conductor it is and which element
           call ctest(ielem,row,cnum,iicond,ixdiv,iydiv,ttype)
C           Calculate the mutual inductance between a normal conductor
C           element and a litzwire (M is not divided by elems)
C           and the symmetrical terms of the litzwire
            CALL REGLITZ(iicond,iicond2,ielem,ixdiv,iydiv,Xdim,
     +      Ydim,len,M,xsym,ysym)
          endif
          if((.not.litzrow).and.(.not.litzcol)) then
C          find out which conductors they are and which elements
           call ctest(ielem1,row,cnum,ircond1,ixdiv,iydiv,ttype)
           call ctest(ielem2,col,cnum,ircond2,ixdiv,iydiv,ttype)
C          calculate the mutual reactance between elements
C          and the symmetrical terms
           CALL MUTSYM(ielem1,ielem2,ircond1,ircond2,ixdiv,iydiv
     +     ,Xdim,Ydim,len,M,xsym,ysym)
          endif
          react=M*freq*2.0*pi
          M=0.00
          resis=0.00
          ZZ(row,col)=dcmplx(resis, react)
           endif

300    CONTINUE


200   CONTINUE
      RETURN
      END
```

FILE LITZRESI.FOR

```
      SUBROUTINE LITZRESI(iicond,Xdim,Ydim,Rho,len,sf,lf,resis,
     +ixdiv,iydiv)
C The purpose of this subroutine is to calculate the resistance of
C a litz conductor by assuming is is just one element with a
C constant current distribution. The area is multiplied by the
C space factor of the litz wire while the length is multiplied
C by the length factor. Then its R is broken up into an elemental R
C by dividing the area by the total elements
C to portray only one element
C     parms has all the parameter statements in it
      include 'PARMS'
C positions for defining conductor boundaries for all conductors

      REAL*8 Xdim(MPOS)
      REAL*8 Ydim(MPOS)
      REAL*8 Rho(MAXCOND)
      integer ixdiv(MAXCOND)
      integer iydiv(MAXCOND)
      integer iicond
      real delx, dely,resis,area
      real sf,lf,len
C     physical width of litz
      delx=Xdim((iicond-1)*4+2)-Xdim((iicond-1)*4+1)
C     physical height of litz
      dely=Ydim((iicond-1)*4+3)-Ydim((iicond-1)*4+2)
C     calculate the actual litz copper area
      area=delx*dely*sf/(ixdiv(iicond)*iydiv(iicond))
C     calculate the resistance
      resis=(Rho(iicond)*len*lf)/area
      RETURN
      end
```

FILE LITZSELF.FOR

```
      SUBROUTINE LITZSELF(iicond,ixdiv,iydiv,Xdim,Ydim,len,sf,l)
C The purpose of this subroutine is to calculate the
C average self inductance of a litz conductor element by breaking the
C Litz wire up and looping over the litz cross-section
C and summing all the inductances in much the same way as
C you would loop over a conductor cross-section to get
C its self geometric mean distance. The result of this will
C be entered into the diagonal of the impedance matrix
C along with the resistance from LITZRESIS and any
C symmetrical terms.
C     parms has all the parameter statements in it
      include 'PARMS'
C positions for defining conductor boundaries for all conductors

      REAL*8 Xdim(MPOS)
      REAL*8 Ydim(MPOS)
      integer ixdiv(MAXCOND),iydiv(MAXCOND)
      integer row1,col1,row2,col2,iicond,isweep1,isweep2
      real xc1,xc2,yc1,yc2
      real divx, divy
      real llx1,llx2,urx1,urx2
      real lly1,lly2,ury1,ury2
      real delx, dely,sf
      real len,ltot,l,d
      ltot=0.0
      divx = ixdiv(iicond)
      divy = iydiv(iicond)
      do 500 isweep1=1,ixdiv(iicond)*iydiv(iicond)
       do 400 isweep2=1,ixdiv(iicond)*iydiv(iicond)
        if(isweep1.eq.isweep2) then
C        calculate x dimension of element including sf
         delx=sf**.5*(Xdim((iicond-1)*4+2)-Xdim((iicond-1)*4+1))/divx
C        calculate y dimension of element including sf
         dely=sf**.5*(Ydim((iicond-1)*4+3)-Ydim((iicond-1)*4+2))/divy
          call self( delx, dely, len, l)
        else
C        calculate x dimension of conductor
         delx=Xdim((iicond-1)*4+2)-Xdim((iicond-1)*4+1)
C        calculate y dimension of conductor
         dely=Ydim((iicond-1)*4+3)-Ydim((iicond-1)*4+2)
C        Center coordinates for litz elements
         CALL GETRC(row2,col2,isweep2,divx)
         llx2=(col2-1)*delx/divx+Xdim((iicond-1)*4+1)
         lly2=(row2-1)*dely/divy+Ydim((iicond-1)*4+1)
         urx2=col2*delx/divx+Xdim((iicond-1)*4+1)
         ury2=row2*dely/divy+Ydim((iicond-1)*4+1)
         xc2=(urx2+llx2)/2.00
         yc2=(ury2+lly2)/2.00
         CALL GETRC(row1,col1,isweep1,divx)
         llx1=(col1-1)*delx/divx+Xdim((iicond-1)*4+1)
         lly1=(row1-1)*dely/divy+Ydim((iicond-1)*4+1)
         urx1=col1*delx/divx+Xdim((iicond-1)*4+1)
         ury1=row1*dely/divy+Ydim((iicond-1)*4+1)
```

```
          xc1=(urx1+llx1)/2.00
          yc1=(ury1+lly1)/2.00
C         calculate distance between elements
          d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
          CALL MUT(d,len,l)
         endif
         ltot=ltot+l
400     CONTINUE
500    CONTINUE
       l=ltot/(divx*divy)
       RETURN
       end
```

FILE LITZSYMM.FOR

```
      SUBROUTINE LITZSYMM(row,iicond,ixdiv,iydiv,Xdim,Ydim,
     +len,MTOT,xsym,ysym)
C The purpose of this subroutine is to calculate the mutual impedance
C between the base litz wire with its symmetrical counterparts
C     parms has all the parameter statements in it
      include 'PARMS'
C positions for defining conductor boundaries for all conductors
C number of conductors*4

      REAL*8 Xdim(MPOS)
      REAL*8 Ydim(MPOS)
      integer ixdiv(MAXCOND),iydiv(MAXCOND)
      integer row1,col1,row2,col2,iicond,isweep1,isweep2
      real xsym,ysym
      real xc1,xc2,yc1,yc2
      real divx,divy
      real llx1,llx2,urx1,urx2
      real lly1,lly2,ury1,ury2
      real delx, dely
      real len,MTOT,M,MX,MY,MXY,d
C     calculate x dimension of conductor iicond
      delx=Xdim((iicond-1)*4+2)-Xdim((iicond-1)*4+1)
C     calculate y dimension of conductor iicond
      dely=Ydim((iicond-1)*4+3)-Ydim((iicond-1)*4+2)
      MTOT=0.0
      M=0.0
      MX=0.0
      MY=0.0
      MXY=0.0
      divx = ixdiv(iicond)
      divy = iydiv(iicond)
       do 500 isweep1=1,ixdiv(iicond)*iydiv(iicond)
C       calculate center coordinates of 1st quadrant element
        CALL GETRC(row1,col1,isweep1,divx)
        llx1=(col1-1)*delx/divx+Xdim((iicond-1)*4+1)
        lly1=(row1-1)*dely/divy+Ydim((iicond-1)*4+1)
        urx1=col1*delx/divx+Xdim((iicond-1)*4+1)
        ury1=row1*dely/divy+Ydim((iicond-1)*4+1)
        xc1=(urx1+llx1)/2.00
        yc1=(ury1+lly1)/2.00
C*************************X SYMMETRY*****************************
        do 400 isweep2=1,ixdiv(iicond)*iydiv(iicond)
C        calculate center coordinates of 2nd quadrant element
        CALL GETRC(row2,col2,isweep2,divx)
         llx2=(col2-1)*delx/divx+Xdim((iicond-1)*4+1)
         lly2=(row2-1)*dely/divy+Ydim((iicond-1)*4+1)
         urx2=col2*delx/divx+Xdim((iicond-1)*4+1)
         ury2=row2*dely/divy+Ydim((iicond-1)*4+1)
C*************************X SYMMETRY*****************************
      if(xsym.ne.0.0) then
         xc2=-(urx2+llx2)/2.00
         yc2=(ury2+lly2)/2.00
C        calculate distance between elements
```

```
         d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
         CALL MUT(d,len,M)
         MX=MX+M
      endif
C************************Y SYMMETRY******************************
      if(ysym.ne.0.0) then
         xc2=(urx2+llx2)/2.00
         yc2=-(ury2+lly2)/2.00
C        calculate distance between elements
         d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
         CALL MUT(d,len,M)
         MY=MY+M
      endif
C************************XY SYMMETRY*****************************
      if(xsym*ysym.ne.0.0) then
         xc2=-(urx2+llx2)/2.00
         yc2=-(ury2+lly2)/2.00
C        calculate distance between elements
         d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
         CALL MUT(d,len,M)
         MXY=MXY+M
      endif
400     CONTINUE
500    CONTINUE
       MTOT=(MX*xsym+MY*ysym+MXY*xsym*ysym)/(divx*divy)
      RETURN
      end
```

FILE RESIST.FOR

```
.     SUBROUTINE RESIST(iicond,Xdim,Ydim,Rho,ixdiv,iydiv
     +,len,resis)
C The purpose of this subroutine is to calculate the elemental
C resistance of a conductor.
C     parms has all the parameter statements in it
C positions for defining conductor boundaries for all conductors
      include 'PARMS'
      REAL*8 Xdim(MPOS)
      REAL*8 Ydim(MPOS)
      REAL*8 Rho(MAXCOND)
      integer iicond
      real delx, dely,resis
      real len
      real divx, divy,area
      integer iydiv(MAXCOND),ixdiv(MAXCOND)
      divx = ixdiv(iicond)
      divy = iydiv(iicond)
      delx=(Xdim((iicond-1)*4+2)-Xdim((iicond-1)*4+1))/divx
      dely=(Ydim((iicond-1)*4+3)-Ydim((iicond-1)*4+2))/divy
      area=delx*dely
      resis=Rho(iicond)*len/area
      RETURN
      end
```

FILE SELFL.FOR

```
      SUBROUTINE SELFL(iicond,ixdiv,iydiv,Xdim,Ydim,len,l)
C The purpose of this subroutine is to calculate the
C self inductance of an element of a particular conductor
C     parms has all the parameter statements in it
      include 'PARMS'
C positions for defining conductor boundaries for all conductors

      REAL*8 Xdim(MPOS)
      REAL*8 Ydim(MPOS)
      integer ixdiv(MAXCOND),iydiv(MAXCOND)
      integer iicond
      real divx, divy
      real delx, dely
      real len,l
      divx = ixdiv(iicond)
      divy = iydiv(iicond)
C     calculate x dimension of element
      delx=(Xdim((iicond-1)*4+2)-Xdim((iicond-1)*4+1))/divx
C     calculate y dimension of element
      dely=(Ydim((iicond-1)*4+3)-Ydim((iicond-1)*4+2))/divy
      call self( delx, dely, len, l)
      RETURN
      end
```

FILE MUTSYMSL.FOR

```
      SUBROUTINE MUTSYMSL(ielem,iicond,ixdiv,iydiv,Xdim,Ydim,
     +len,MTOT,xsym,ysym)
C The purpose of this subroutine is to calculate the mutual impedance
C between the base element its symmetrical counterparts
C     parms has all the parameter statements in it
      include 'PARMS'
C positions for defining conductor boundaries for all conductors

      REAL*8 Xdim(MPOS)
      REAL*8 Ydim(MPOS)
      integer ixdiv(MAXCOND),iydiv(MAXCOND)
      integer ielem,row1,col1,iicond
      real xsym,ysym
      real xc1,xc2,yc1,yc2
      real divx,divy
      real llx1,urx1
      real lly1,ury1
      real delx, dely
      real len,Mtot,M,MX,MY,MXY,d
      Mtot=0.0
      M=0.0
      MX=0.0
      MY=0.0
      MXY=0.0
      divx = ixdiv(iicond)
      divy = iydiv(iicond)
      delx=(Xdim((iicond-1)*4+2)-Xdim((iicond-1)*4+1))
      dely=(Ydim((iicond-1)*4+3)-Ydim((iicond-1)*4+2))
C     calculate center coordinates of base element
      CALL GETRC(row1,col1,ielem,divx)
      llx1=(col1-1)*delx/divx+Xdim((iicond-1)*4+1)
      lly1=(row1-1)*dely/divy+Ydim((iicond-1)*4+1)
      urx1=col1*delx/divx+Xdim((iicond-1)*4+1)
      ury1=row1*dely/divy+Ydim((iicond-1)*4+1)
      xc1=(urx1+llx1)/2.00
      yc1=(ury1+lly1)/2.00
C*************************X SYMMETRY******************************
      if(xsym.ne.0.0) then
       xc2=-xc1
       yc2=yc1
C      calculate distance between elements
       d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
       CALL MUT(d,len,M)
       MX=M
      endif
C*************************Y SYMMETRY******************************
      if(ysym.ne.0.0) then
       xc2=xc1
       yc2=-yc1
C      calculate distance between elements
       d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
       CALL MUT(d,len,M)
       MY=M
```

```
      endif
C************************XY SYMMETRY*****************************
      if(xsym*ysym.ne.0.0) then
       xc2=-xc1
       yc2=-yc1
C      calculate distance between elements
       d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
       CALL MUT(d,len,M)
       MXY=M
      endif
       MTOT=MX*xsym+MY*ysym+MXY*xsym*ysym
      RETURN
      end
```

FILE LSYMMZ.FOR

```
      SUBROUTINE LSYMMZ(iicond1,iicond2,ixdiv,iydiv,Xdim,Ydim,len,
     +xsym,ysym,M)
C The purpose of this subroutine is to calculate the
C the symmetrical mutual inductances between two litzwires remembering
C to divide the result by the number of elements in iicond1
C     parms has all the parameter statements in it
      include 'PARMS'
C positions for defining conductor boundaries for all conductors

      REAL*8 Xdim(MPOS)
      REAL*8 Ydim(MPOS)
      integer ixdiv(MAXCOND),iydiv(MAXCOND)
      integer row1,col1,row2,col2,iicond1,iicond2,isweep1,isweep2
      real xsym,ysym
      real xc1,xc2,yc1,yc2,M,MX,MY,MXY,MM
      real MXTOT,MYTOT,MXYTOT,MMTOT
      real divx1,divx2,divy1,divy2
      real llx1,llx2,urx1,urx2
      real lly1,lly2,ury1,ury2
      real delx1,delx2,dely1, dely2
      real len,d
      MMTOT=0.0
      MXTOT=0.0
      MYTOT=0.0
      MXYTOT=0.0
      MM=0.0
      MX=0.0
      MY=0.0
      MXY=0.0
      M=0.0
      divx1 = ixdiv(iicond1)
      divx2 = ixdiv(iicond2)
      divy1 = iydiv(iicond1)
      divy2 = iydiv(iicond2)
C     calculate x dimension of base conductor iicond1
      delx1=Xdim((iicond1-1)*4+2)-Xdim((iicond1-1)*4+1)
C     calculate y dimension of base conductor iicond1
      dely1=Ydim((iicond1-1)*4+3)-Ydim((iicond1-1)*4+2)
C     calculate x dimension of conductor iicond2
      delx2=Xdim((iicond2-1)*4+2)-Xdim((iicond2-1)*4+1)
C     calculate y dimension of conductor iicond2
      dely2=Ydim((iicond2-1)*4+3)-Ydim((iicond2-1)*4+2)
C Mutual inductance between iicond1 and iicond2 in
C first quadrant
      do 500 isweep1=1,ixdiv(iicond1)*iydiv(iicond1)
C        calculate center coordinates of base element
         CALL GETRC(row1,col1,isweep1,divx1)
         llx1=(col1-1)*delx1/divx1+Xdim((iicond1-1)*4+1)
         lly1=(row1-1)*dely1/divy1+Ydim((iicond1-1)*4+1)
         urx1=col1*delx1/divx1+Xdim((iicond1-1)*4+1)
         ury1=row1*dely1/divy1+Ydim((iicond1-1)*4+1)
         xc1=(urx1+llx1)/2.00
         yc1=(ury1+lly1)/2.00
```

```
       do 400 isweep2=1,ixdiv(iicond2)*iydiv(iicond2)
C         calculate center coordinates of element
          CALL GETRC(row2,col2,isweep2,divx2)
          llx2=(col2-1)*delx2/divx2+Xdim((iicond2-1)*4+1)
          lly2=(row2-1)*dely2/divy2+Ydim((iicond2-1)*4+1)
          urx2=col2*delx2/divx2+Xdim((iicond2-1)*4+1)
          ury2=row2*dely2/divy2+Ydim((iicond2-1)*4+1)
          xc2=(urx2+llx2)/2.00
          yc2=(ury2+lly2)/2.00
C         calculate distance between elements
          d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
          CALL MUT(d,len,MM)
          MMTOT=MMTOT+MM
C Mutual inductance between iicond1 and iicond2 in
C second quadrant
      if(xsym.ne.0.0) then
          xc2=-(urx2+llx2)/2.00
          yc2=(ury2+lly2)/2.00
C         calculate distance between elements
          d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
          CALL MUT(d,len,MX)
          MXTOT=MXTOT+MX
      endif
C Mutual inductance between iicond1 and iicond2 in
C third quadrant
      if(ysym.ne.0.0.and.xsym.ne.0.0) then
          xc2=-(urx2+llx2)/2.00
          yc2=-(ury2+lly2)/2.00
C         calculate distance between elements
          d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
          CALL MUT(d,len,MXY)
          MXYTOT=MXYTOT+MXY
      endif
C Mutual inductance between iicond1 and iicond2 in
C fourth quadrant
      if(ysym.ne.0.0) then
          xc2=(urx2+llx2)/2.00
          yc2=-(ury2+lly2)/2.00
C         calculate distance between elements
          d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
          CALL MUT(d,len,MY)
          MYTOT=MYTOT+MY
      endif
400    CONTINUE
500   CONTINUE
      M=(MMTOT+MXTOT*xsym+MXYTOT*xsym*ysym+MYTOT*ysym)/(divx1*divy1)
      RETURN
      end
```

FILE LITZREG.FOR

```
      SUBROUTINE LITZREG(iicond1,iicond2,ielem,ixdiv,iydiv,Xdim,
     +Ydim,len,M,xsym,ysym)
C The purpose of this subroutine is to calculate the
C the mutual inductance between a litzwire and a regular
C conductor remembering to divide the result by the number of
C elements in iicond1
C     parms has all the parameter statements in it
      include 'PARMS'
C positions for defining conductor boundaries for all conductors

      REAL*8 Xdim(MPOS)
      REAL*8 Ydim(MPOS)
      integer ixdiv(MAXCOND),iydiv(MAXCOND)
      integer row1,col1,row2,col2
      real xsym,ysym
      integer ielem,iicond1,iicond2,isweep1
      real xc1,xc2,yc1,yc2,M,MX,MY,MXY,MM
      real divx1,divx2,divy1,divy2
      real llx1,llx2,urx1,urx2
      real lly1,lly2,ury1,ury2
      real delx1,delx2,dely1, dely2
      real len,d
      MM=0.0
      MX=0.0
      MY=0.0
      MXY=0.0
      M=0.0
      divx1 = ixdiv(iicond1)
      divx2 = ixdiv(iicond2)
      divy1 = iydiv(iicond1)
      divy2 = iydiv(iicond2)
C     calculate x dimension of base conductor iicond1
      delx1=Xdim((iicond1-1)*4+2)-Xdim((iicond1-1)*4+1)
C     calculate y dimension of base conductor iicond1
      dely1=Ydim((iicond1-1)*4+3)-Ydim((iicond1-1)*4+2)
C     calculate x dimension of conductor iicond2
      delx2=Xdim((iicond2-1)*4+2)-Xdim((iicond2-1)*4+1)
C     calculate y dimension of conductor iicond2
      dely2=Ydim((iicond2-1)*4+3)-Ydim((iicond2-1)*4+2)
C Mutual inductance between iicond1 and iicond2 in
C first quadrant
C         calculate center coordinates of regular element
          CALL GETRC(row2,col2,ielem,divx2)
          llx2=(col2-1)*delx2/divx2+Xdim((iicond2-1)*4+1)
          lly2=(row2-1)*dely2/divy2+Ydim((iicond2-1)*4+1)
          urx2=col2*delx2/divx2+Xdim((iicond2-1)*4+1)
          ury2=row2*dely2/divy2+Ydim((iicond2-1)*4+1)
          xc2=(urx2+llx2)/2.00
          yc2=(ury2+lly2)/2.00
      do 500 isweep1=1,ixdiv(iicond1)*iydiv(iicond1)
C         calculate center coordinates of base element
          CALL GETRC(row1,col1,isweep1,divx1)
          llx1=(col1-1)*delx1/divx1+Xdim((iicond1-1)*4+1)
```

```
          lly1=(row1-1)*dely1/divy1+Ydim((iicond1-1)*4+1)
          urx1=col1*delx1/divx1+Xdim((iicond1-1)*4+1)
          ury1=row1*dely1/divy1+Ydim((iicond1-1)*4+1)
          xc1=(urx1+llx1)/2.00
          yc1=(ury1+lly1)/2.00
C         calculate distance between elements
          d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
          CALL MUT(d,len,MM)
          M=M+MM
500   CONTINUE
C Mutual inductance between iicond1 and iicond2 in
C second quadrant
      if(xsym.ne.0.0) then
       xc2=-(urx2+llx2)/2.00
       yc2=(ury2+lly2)/2.00
       do 501 isweep1=1,ixdiv(iicond1)*iydiv(iicond1)
C         calculate center coordinates of base element
          CALL GETRC(row1,col1,isweep1,divx1)
          llx1=(col1-1)*delx1/divx1+Xdim((iicond1-1)*4+1)
          lly1=(row1-1)*dely1/divy1+Ydim((iicond1-1)*4+1)
          urx1=col1*delx1/divx1+Xdim((iicond1-1)*4+1)
          ury1=row1*dely1/divy1+Ydim((iicond1-1)*4+1)
          xc1=(urx1+llx1)/2.00
          yc1=(ury1+lly1)/2.00
C         calculate distance between elements
          d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
          CALL MUT(d,len,MX)
          M=M+MX*xsym
501    CONTINUE
      endif
C Mutual inductance between iicond1 and iicond2 in
C third quadrant
      if(ysym.ne.0.0.and.xsym.ne.0.0) then
          xc2=-(urx2+llx2)/2.00
          yc2=-(ury2+lly2)/2.00
       do 502 isweep1=1,ixdiv(iicond1)*iydiv(iicond1)
C         calculate center coordinates of base element
          CALL GETRC(row1,col1,isweep1,divx1)
          llx1=(col1-1)*delx1/divx1+Xdim((iicond1-1)*4+1)
          lly1=(row1-1)*dely1/divy1+Ydim((iicond1-1)*4+1)
          urx1=col1*delx1/divx1+Xdim((iicond1-1)*4+1)
          ury1=row1*dely1/divy1+Ydim((iicond1-1)*4+1)
          yc1=(ury1+lly1)/2.00
         !DAS
          xc1=(urx1+llx1)/2.00
C         calculate distance between elements
          d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
          CALL MUT(d,len,MXY)
          M=M+MXY*xsym*ysym
502    CONTINUE
      endif
C Mutual inductance between iicond1 and iicond2 in
C fourth quadrant
      if(ysym.ne.0.0) then
          xc2=(urx2+llx2)/2.00
          yc2=-(ury2+lly2)/2.00
       do 503 isweep1=1,ixdiv(iicond1)*iydiv(iicond1)
```

```
C         calculate center coordinates of base element
          CALL GETRC(row1,col1,isweep1,divx1)
          llx1=(col1-1)*delx1/divx1+Xdim((iicond1-1)*4+1)
          lly1=(row1-1)*dely1/divy1+Ydim((iicond1-1)*4+1)
          urx1=col1*delx1/divx1+Xdim((iicond1-1)*4+1)
          ury1=row1*dely1/divy1+Ydim((iicond1-1)*4+1)
          xc1=(urx1+llx1)/2.00
          yc1=(ury1+lly1)/2.00
C         calculate distance between elements
          d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
          CALL MUT(d,len,MY)
          M=M+MY*ysym
503     CONTINUE
      endif
C THIS ASSUMES THAT ALL ELEMENTAL VOLTAGES IN THE LITZ IRE ARE
C THE SAME WHICH SHOULD BE THE CASE PROVIDED ALL STRANDS ARE CONNECTED
C (I really should be dividing by the sum
C of all elemental voltages ) but I just divide by the number of
C elements so I don't have to modify the voltage vector
      M=M/(divx1*divy1)
      RETURN
      END
```

FILE REGLITZ.FOR

```
      SUBROUTINE REGLITZ(iicond1,iicond2,ielem,ixdiv,iydiv,Xdim,
     +Ydim,len,M,xsym,ysym)
C The purpose of this subroutine is to calculate the
C the mutual inductance between a regular conductor element and a
C whole litzwire conductor remembering not to divide by the number of
C elements in iicond2
C     parms has all the parameter statements in it
      include 'PARMS'
C positions for defining conductor boundaries for all conductors

      REAL*8 Xdim(MPOS)
      REAL*8 Ydim(MPOS)
      integer ixdiv(MAXCOND),iydiv(MAXCOND)
      integer row1,col1,row2,col2
      real xsym,ysym
      integer ielem,iicond1,iicond2,isweep2
      real xc1,xc2,yc1,yc2,M,MX,MY,MXY,MM
      real divx1,divx2,divy1,divy2
      real llx1,llx2,urx1,urx2
      real lly1,lly2,ury1,ury2
      real delx1,delx2,dely1,dely2
      real len,d
      MM=0.0
      MX=0.0
      MY=0.0
      MXY=0.0
      M=0.0
      divx1 = ixdiv(iicond1)
      divx2 = ixdiv(iicond2)
      divy1 = iydiv(iicond1)
      divy2 = iydiv(iicond2)
C     calculate x dimension of base conductor iicond1
      delx1=Xdim((iicond1-1)*4+2)-Xdim((iicond1-1)*4+1)
C     calculate y dimension of base conductor iicond1
      dely1=Ydim((iicond1-1)*4+3)-Ydim((iicond1-1)*4+2)
C     calculate x dimension of conductor iicond2
      delx2=Xdim((iicond2-1)*4+2)-Xdim((iicond2-1)*4+1)
C     calculate y dimension of conductor iicond2
      dely2=Ydim((iicond2-1)*4+3)-Ydim((iicond2-1)*4+2)
C Mutual inductance between iicond1 and iicond2 in
C first quadrant
C        calculate center coordinates of base element
         CALL GETRC(row1,col1,ielem,divx1)
         llx1=(col1-1)*delx1/divx1+Xdim((iicond1-1)*4+1)
         lly1=(row1-1)*dely1/divy1+Ydim((iicond1-1)*4+1)
         urx1=col1*delx1/divx1+Xdim((iicond1-1)*4+1)
         ury1=row1*dely1/divy1+Ydim((iicond1-1)*4+1)
         xc1=(urx1+llx1)/2.00
         yc1=(ury1+lly1)/2.00
      do 500 isweep2=1,ixdiv(iicond2)*iydiv(iicond2)
C        calculate center coordinates of litz element
         CALL GETRC(row2,col2,isweep2,divx2)
         llx2=(col2-1)*delx2/divx2+Xdim((iicond2-1)*4+1)
```

```
          lly2=(row2-1)*dely2/divy2+Ydim((iicond2-1)*4+1)
          urx2=col2*delx2/divx2+Xdim((iicond2-1)*4+1)
          ury2=row2*dely2/divy2+Ydim((iicond2-1)*4+1)
          xc2=(urx2+llx2)/2.00
          yc2=(ury2+lly2)/2.00
C         calculate distance between elements
          d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
          CALL MUT(d,len,MM)
          M=M+MM
500    CONTINUE
C Mutual inductance between iicond1 and iicond2 in
C second quadrant
       if(xsym.ne.0.0) then
        do 501 isweep2=1,ixdiv(iicond2)*iydiv(iicond2)
C         calculate center coordinates of base element
          CALL GETRC(row2,col2,isweep2,divx2)
          llx2=(col2-1)*delx2/divx2+Xdim((iicond2-1)*4+1)
          lly2=(row2-1)*dely2/divy2+Ydim((iicond2-1)*4+1)
          urx2=col2*delx2/divx2+Xdim((iicond2-1)*4+1)
          ury2=row2*dely2/divy2+Ydim((iicond2-1)*4+1)
          xc2=-(urx2+llx2)/2.00
          yc2=(ury2+lly2)/2.00
C         calculate distance between elements
          d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
          CALL MUT(d,len,MX)
          M=M+MX*xsym
501     CONTINUE
       endif
C Mutual inductance between iicond1 and iicond2 in
C third quadrant
       if(ysym.ne.0.0.and.xsym.ne.0.0) then
        do 502 isweep2=1,ixdiv(iicond2)*iydiv(iicond2)
C         calculate center coordinates of base element
          CALL GETRC(row2,col2,isweep2,divx2)
          llx2=(col2-1)*delx2/divx2+Xdim((iicond2-1)*4+1)
          lly2=(row2-1)*dely2/divy2+Ydim((iicond2-1)*4+1)
          urx2=col2*delx2/divx2+Xdim((iicond2-1)*4+1)
          ury2=row2*dely2/divy2+Ydim((iicond2-1)*4+1)
          xc2=-(urx2+llx2)/2.00
          yc2=-(ury2+lly2)/2.00
C         calculate distance between elements
          d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
          CALL MUT(d,len,MXY)
          M=M+MXY*xsym*ysym
502     CONTINUE
       endif
C Mutual inductance between iicond1 and iicond2 in
C fourth quadrant
       if(ysym.ne.0.0) then
        do 503 isweep2=1,ixdiv(iicond2)*iydiv(iicond2)
C         calculate center coordinates of base element
          CALL GETRC(row2,col2,isweep2,divx2)
          llx2=(col2-1)*delx2/divx2+Xdim((iicond2-1)*4+1)
          lly2=(row2-1)*dely2/divy2+Ydim((iicond2-1)*4+1)
          urx2=col2*delx2/divx2+Xdim((iicond2-1)*4+1)
          ury2=row2*dely2/divy2+Ydim((iicond2-1)*4+1)
          xc2=(urx2+llx2)/2.00
```

```
          yc2=-(ury2+lly2)/2.00
C         calculate distance between elements
          d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
          CALL MUT(d,len,MY)
          M=M+MY*ysym
503     CONTINUE
        endif
        RETURN
        END
```

FILE MUTSYM.FOR

```
      SUBROUTINE MUTSYM(ielem1,ielem2,ircond1,ircond2,ixdiv,iydiv
     +,Xdim,Ydim,len,MTOT,xsym,ysym)
C The purpose of this subroutine is to calculate the mutual impedance
C between the base element and another element plus
C its symmetrical counterparts
C     parms has all the parameter statements in it
      include 'PARMS'
C positions for defining conductor boundaries for all conductors

      REAL*8 Xdim(MPOS)
      REAL*8 Ydim(MPOS)
      integer ixdiv(MAXCOND),iydiv(MAXCOND)
      integer row1,col1,row2,col2,ircond1,ircond2
      integer ielem1,ielem2
      real xsym,ysym
      real xc1,xc2,yc1,yc2
      real divx1,divx2,divy1,divy2
      real llx1,llx2,urx1,urx2
      real lly1,lly2,ury1,ury2
      real delx1,delx2,dely1,dely2
      real len,MTOT,M,MM,MX,MY,MXY
      real d,dtmpx,dtmpy
      MTOT=0.0
      M=0.0
      MM=0.0
      MX=0.0
      MY=0.0
      MXY=0.0
      divx1= ixdiv(ircond1)
      divy1= iydiv(ircond1)
      divx2= ixdiv(ircond2)
      divy2= iydiv(ircond2)
      delx1=(Xdim((ircond1-1)*4+2)-Xdim((ircond1-1)*4+1))
      dely1=(Ydim((ircond1-1)*4+3)-Ydim((ircond1-1)*4+2))
      delx2=(Xdim((ircond2-1)*4+2)-Xdim((ircond2-1)*4+1))
      dely2=(Ydim((ircond2-1)*4+3)-Ydim((ircond2-1)*4+2))
C     calculate center coordinates of both elements
      CALL GETRC(row1,col1,ielem1,divx1)
      llx1=(col1-1)*delx1/divx1+Xdim((ircond1-1)*4+1)
      lly1=(row1-1)*dely1/divy1+Ydim((ircond1-1)*4+1)
      urx1=col1*delx1/divx1+Xdim((ircond1-1)*4+1)
      ury1=row1*dely1/divy1+Ydim((ircond1-1)*4+1)
      xc1=(urx1+llx1)/2.00
      yc1=(ury1+lly1)/2.00
      CALL GETRC(row2,col2,ielem2,divx2)
      llx2=(col2-1)*delx2/divx2+Xdim((ircond2-1)*4+1)
      lly2=(row2-1)*dely2/divy2+Ydim((ircond2-1)*4+1)
      urx2=col2*delx2/divx2+Xdim((ircond2-1)*4+1)
      ury2=row2*dely2/divy2+Ydim((ircond2-1)*4+1)
      xc2=(urx2+llx2)/2.00
      yc2=(ury2+lly2)/2.00
C     calculate distance between elements
```

```
      dtmpx=(xc1-xc2)

      dtmpy=(yc1-yc2)
      d=(dtmpx**2+dtmpy**2)**.5
C     write(20,*) 'In Mutsym dtmpx,dtmpy,d=',dtmpx,dtmpy,d
      CALL MUT(d,len,M)
      MM=M
C     calculate
C*************************X SYMMETRY******************************
      if(xsym.ne.0.0) then
C       calculate center coordinates of 2nd quadrant element
        xc2=-(urx2+llx2)/2.00
        yc2=(ury2+lly2)/2.00
C       calculate distance between elements
        d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
        CALL MUT(d,len,M)
        MX=M
      endif
C*************************Y SYMMETRY******************************
      if(ysym.ne.0.0) then
        xc2=(urx2+llx2)/2.00
        yc2=-(ury2+lly2)/2.00
C       calculate distance between elements
        d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
        CALL MUT(d,len,M)
        MY=M
      endif
C*************************XY SYMMETRY*****************************
      if(xsym*ysym.ne.0.0) then
        xc2=-(urx2+llx2)/2.00
        yc2=-(ury2+lly2)/2.00
C       calculate distance between elements
        d=((xc1-xc2)**2.0+(yc1-yc2)**2.0)**.5
        CALL MUT(d,len,M)
        MXY=M
      endif
       MTOT=MM+MX*xsym+MY*ysym+MXY*xsym*ysym
      RETURN
      end
```

FILE MUT.FOR

```
      SUBROUTINE MUT(d,len,M)
C This subroutine calculates the mutual inductance of two
C areas. M is calculated by putting current filaments
C at the center of their areas and using the current
C filament mutual inductance formula from page 31 in Grover's
C Inductance Calculations
C
C parms has all the parameter statements
      include 'PARMS'
C positions for defining conductor boundaries for all conductors

      real M,len,d
C calculate the mutual inductance for two parallel filaments spaced
C d apart with a length of len Grover page 31
C         write(20,*) 'In Mut d=',d,len

        M=(.002*len*(log(len/d +(1.00+len*len/d/d)**.5)
     +  -(1.00+d*d/len/len)**.5 + d/len))/1000000.0

      RETURN
      END
```

FILE SELF.FOR

```
      SUBROUTINE self( a, b, len, l)
C GMD Taken from The Theory and Practice of Absolute Measurements
C in Electricity and Magnetism by Andrew Gray MA Volume II
C Self Inductance Formula taken from Grover's Inductance Calculations
C Page 35
      real t1,t2,t3,t4,t5,t6
      real a,b
      real len,l
C a and b are the x and y dimension of the element
      real r
C Geometric mean distance between two rectangles
      t1=0.5*log(a*a+b*b)
      t2=-b*b*log(1.0+a*a/b/b)/12.0/a/a
      t3=-a*a*log(1.0+b*b/a/a)/12.0/b/b
      t4=2.0*b*atan(a/b)/3.0/a
      t5=2.0*a*atan(b/a)/3.0/b
      t6=-25.0/12.0
      r=exp(t1+t2+t3+t4+t5+t6)
      l=.002*len*(log(2.0*len/(a+b))+.5-(log(r/(a+b))+1.5))/(1.0e+6)

      RETURN
      end
```

FILE CTEST.FOR

```
      subroutine  ctest(ielem,row,cnum,iicond,ixdiv,iydiv,ttype)
C     The purpose of this subroutine is to return the conductor
C     number in iicond and the number of element it is in ielem
C     parms has all the parameter statements in it
      INCLUDE 'PARMS'
C positions for defining conductor boundaries for all conductors
      integer ixdiv(MAXCOND)
      integer iydiv(MAXCOND)
      integer ttype(MAXCOND)
      integer range,cnum
      integer iicond,ielem,row,icond,inum,irange,irangeprev
      ielem=0
      inum=0
      range=0
      irange=0
      iicond=0
      irangeprev=0
      do 99 icond=1,cnum
        irangeprev=range
       if(ttype(icond).eq.2) then
        range=range+1
      else
         range=range+ixdiv(icond)*iydiv(icond)
       endif
       if(row.le.range.and.row.gt.irangeprev) then
        iicond=icond
        irange=range
       endif
 99   continue
     ielem=row - (irange-ixdiv(iicond)*iydiv(iicond))
       return
      end
```

FILE LTEST.FOR

```
      SUBROUTINE LTEST(element,litz,cnum,iicond,ttype,ixdiv,iydiv)
C     parms has all the parameter statements in it
      include 'PARMS'
C positions for defining conductor boundaries for all conductors
C

      integer ttype(MAXCOND)
      integer ixdiv(MAXCOND)
      integer iydiv(MAXCOND)
      integer element,icond,cnum,iicond,inum
      logical litz
      litz=.false.
      inum=0
      do 99 icond=1,cnum
      if(ttype(icond).ne.2) then
       inum=inum+ixdiv(icond)*iydiv(icond)
      else
       inum=inum+1
       if(element.eq.inum) then
        litz=.true.
        iicond=icond
       endif
      endif
99     continue
       RETURN
       END
```

FILE GETRC.FOR

```
SUBROUTINE GETRC(row,col,ielem,divx)
integer row,col,ielem
real divx
integer itest
itest=ielem/divx
if(ielem.eq.(itest*divx)) then
 row=itest
 col=divx
else
 row=ielem/divx+1
 col=ielem-divx*(row-1)
endif
RETURN
END
```

FILE SLNPD.FOR

```
      SUBROUTINE SLNPD(A,B,D,N,NX)
      implicit real*8(a-h,o-z)
C
C SOLUTION OF LINEAR SYSTEMS OF EQUATIONS
C BY THE GAUSS ELIMINATION METHOD PROVIDING
C FOR INTERCHANGING OF ROWS WHEN ENCOUNTERING A
C ZERO DIAGONAL COEFFICIENT
C
C A:SYSTEM MATRIX
C B:ORIGINALLY IT CONTAINS THE INDEPENDANT COEFFICIENTS.
C   AFTER SOLUTION IT CONTAINS THE VALUES OF THE SYSTEM UNKNOWNS.
C
C N:ACTUAL NUMBER OF UNKNOWNS
C NX:ROW AND COLUMN DIMENSION OF A
C
      COMPLEX*16 A(NX,NX),B(NX),C,D
C     DO 9999 I99=1,N
C      WRITE(6,*)' A(',I99,',J)=',(A(I99,J),J=1,N)
C9999 CONTINUE
      TOL = 1.E-35
      TOL1 = 1.E-15
      N1 = N-1
       DO 100 K=1,N1
        K1 = K + 1
        C = A(K,K)
         IF(abs(C)-TOL1) 1,1,3
 1      DO 7 J=K1,N
C
C TRY TO INTERCHANGE ROWS TO GET NONZERO DIAGONAL COEFFICIENT
C
         IF(abs(A(J,K))-TOL1) 7,7,5
 5        DO 6 L=K,N
           C=A(K,L)
           A(K,L) = A(J,L)
 6        A(J,L) = C
         C=B(K)
         B(K) = B(J)
         B(J) = C
         C = A(K,K)
         GO TO 3
 7      CONTINUE
        GO TO 8
C        NO DIAGONAL FOUND -> ERROR RETURN
C
C DIVIDE ROW BY DIAGONAL COEFFICIENT
C
 3     C=A(K,K)
        DO 4 J=K1,N
         A(K,J) = A(K,J)/C
 4      CONTINUE
       B(K) = B(K)/C
C
C ELIMINATE UNKNOWN X(K) FROM ROW I
```

```
C
       DO 10 I=K1,N
         IF(abs(A(I,K)).LT.TOL) GO TO 10
        C = A(I,K)
         DO 9 J=K1,N
c           IF(abs(A(K,J)).LT.TOL) GO TO 9
          A(I,J) = A(I,J) - C*A(K,J)
 9       CONTINUE
        B(I) = B(I) -C*B(K)
 10    CONTINUE
 100  CONTINUE
C
C COMPUTE LAST UNKNOWN
C
       IF(abs(A(N,N))-TOL1) 8,8,101
 101  B(N) = B(N)/A(N,N)
C
C APPLY BACKSUBSTITUTION PROCESS TO COMPUTE REMAINING UNKNOWNS
C
      DO 200 L=1,N1
       K=N-L
       K1=K+1
        DO 200 J=K1,N
         B(K) = B(K) - A(K,J)*B(J)
 200  CONTINUE
C
C COMPUTE VALUE OF DETERMINANT
C
      D = 1.
C      DO 250 I=1,N
C       D = D*A(I,I)
C250   CONTINUE
      RETURN
 8     WRITE(3,2) K
 2    FORMAT('****** SINGULARITY IN ROW',I5)
       D=0.
      RETURN
      END
```

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