IMPACT ANALYSIS OF A DROPPED OBJECT USING STATIC FINITE ELEMENT SOFTWARE

by

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ABSTRACT

The purpose of this study was to develop a method to solve impact problems using a static finite element analysis software package. This allows an engineer to approximate the stresses in a body when it is dropped and strikes the ground or supports. This is intended mainly for use when an advanced finite element package that solves dynamic problems is not available. The FEA package is utilized in two aspects. First, deflection information is found to evaluate a static load distribution equivalent to the loads experienced due to the impact. Second, this static equivalent load is placed on the FEA model, and the impact stresses and deflections are calculated. The problem of a simply supported beam was used to illustrate and verify the method.

First, the method developed in this work was used to predict stresses and strains in a test beam. Second, the test beam was dropped in the laboratory and experimental strains were recorded. The results of these two methods were then compared, which indicates the validity of this work. This method can be easily applied to more complicated structures.

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NOMENCLATURE

A - cross sectional area (in^2)

[a] - flexibility influence coefficient matrix (in)

E - Young's Modulus (psi)

F - static equivalent force (lb)

$\vec{F}$ - static equivalent force vector (lb)

h - height (in)

I - area moment of inertia (in^4)

S - section modulus (in^3)

V - potential energy (in-lb)

W_i - weight of *i*th element (lb)

$\vec{W}$ - weight vector (lb)

w - work (in-lb)

δ - deflection of centroid of element (in)

δ' - deflection component attributed to the F applied at the location where this deflection is measured (in)

δ'' - deflection component attributed to all other F's (in)

$\hat{\delta}$ - any deflection caused by a unit load (in)

$\hat{\delta}_{ij}$ - the unit load deflection at point *i* due to a force at point *j* (in)

ε - strain ($\mu\varepsilon$)

ν - Poisson's Ratio

i - index of location

j - index of location other than *i*

1. INTRODUCTION

Engineers have been plagued for years in finding fast, accurate solutions to impact problems. With today's modern computers and highly sophisticated software, which carry lofty price tags, impact solutions are becoming easier to obtain. This sophisticated software, such as ABAQUS/Explicit, is capable of advanced non-linear analyses and requires computing power greater than a personal computer (Hoffman and Ammerman, 1994). This analysis tool may not be practical for an engineer who occasionally performs somewhat detailed impact analyses. For this reason, a method for approximating impact solutions using a static finite element code has been developed.

In recent years, there have been major impact studies in the nuclear power industry. The dropping of radioactive material storage and transport packages is a major concern, leading to many finite element simulations with experimental verifications (Quercetti, et al., 1994). These studies are non-linear and deal with deformations resulting from the impact. Since radioactive leaks are such environmental disasters, very accurate analyses, provided from the type of software packages described above, are a necessity.

A method for evaluating impact results using a "Quasi-Static" method was developed for colliding bodies (Shivaswami, et al., 1994). This method was applied and experimentally verified using plates being struck by a dropped impactor. The theory was based on equations of motion for the bodies and a non-linear contact force vs. deformation curve. The somewhat complex formulation gave accurate results. In addition, there have been many other impact studies undertaken, such as aircraft fuel bladders and liquid crystal displays, using accurate, non-linear finite element analysis software (Boitnott, et al., 1994, Wong, et al., 1994).

When accuracy can be sacrificed, within a reasonable amount, the method to be described can be employed. For example, a furniture manufacturer wishes to examine the problem of an item falling from a conveyor or transport vehicle. Chances are that this company will neither have the resources available to perform a detailed, non-linear analysis, nor have the great desire to obtain exact results. The results obtained from the approximate method should give the company a good feel for the stresses induced in their product during impact loading.

The objective of the method is to obtain stresses in a body when it is dropped onto supports or the ground. This is accomplished by evaluating the static loads that are equivalent to the impact loads. The FEA package is used to help find those values and to analyze the model with the static equivalent loads.

In order to develop the method, certain assumptions were made. These are: (1) linear elastic material; (2) no yielding; (3) energy loss to the supports is neglected; and (4) stress waves resulting from higher frequency vibrations are neglected. This causes all of the external forces to be conservative, allowing the work-energy principle to be valid.

The method described herein could prove to be an effective tool for the engineer who is in need of an impact analysis, but does not perform impact analyses enough to justify the cost of more sophisticated software.

2. THEORETICAL ANALYSIS

2.1 Basic Formulation

It was desired to approximate a static equivalent loading that can be used in an FEA analysis to represent the load experienced by a body when some part of it impacts on supports or the ground. This formulation was based on the assumption that the potential energy lost by the body in falling is equal to the work done on the body by the static equivalent load, or

$$V_{LOST} = W_{STATIC\ EQUIVALENT}\ . \qquad (1)$$

This is reasonable due to the assumptions made earlier: linear elastic material, no yielding, no energy loss through ground or supports, and stress waves due to higher frequency vibrations are neglected. Figure 2.1 shows a typical body subjected to impact loading.

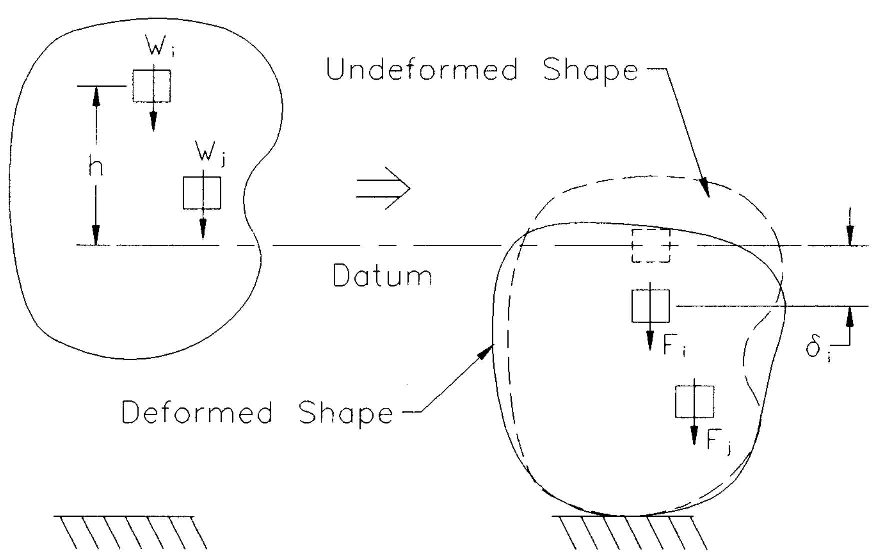

FIGURE 2.1 - FALLING BODY SHOWING IMPACT ELEMENTS

An element with weight W_i falls from a distance of *h* above a datum. The datum represents the location where the body reaches the ground or its support points. Free-fall motion occurs above the plane of the datum. This *i*th element then experiences a deflection δ_i from the impact. This deflection can also be achieved when the body is placed under the static equivalent load. In this case, the static equivalent load for the element will be denoted F_i. This leaves two unknowns, δ_i and F_i, and one equation (1).

The deflection, when related to F_i, can be broken into two parts, as shown in Figure 2.2. The first is the deflection caused by F_i, denoted as δ_i'. The second is the deflection caused by the superposition of all other static equivalent loads, or F_j's, with *j* being a location other than *i*. F_i is constant as it moves through a deflection caused by the F_j's. Let this be denoted as δ_i''. Therefore,

$$\delta_i = \delta_i' + \delta_i'' . \tag{2}$$

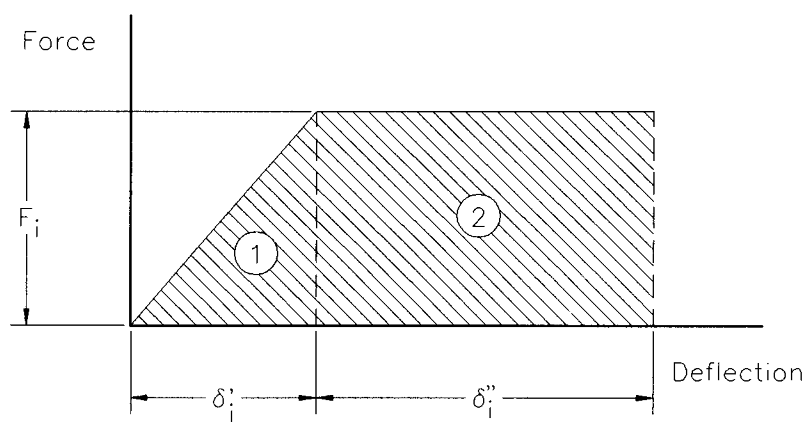

FIGURE 2.2- FORCE VS. DEFLECTION CURVE AT LOCATION *i*

At this point, the total deflection is still unknown, but it can be represented as a function of the F_i's. A deflection can be evaluated by considering it the product of a deflection due to a unit load and the actual load, or

$$\delta = F\hat{\delta} , \tag{3}$$

where $\hat{\delta}$ is the deflection due to a unit load, or the flexibility influence coefficient (Rao, 1990). Substituting (3) into (2) yields

$$\delta_i = F_i\hat{\delta}'_{ii} + \sum_j F_j\hat{\delta}''_{ij}, \tag{4}$$

where $j \neq i$. This leaves the F_i's and F_j's as the only unknowns, as the $\hat{\delta}$ values can be found using a closed form solution or an FEA package. Due to the involved calculations, it is preferred to use the latter.

The potential energy lost by the element can be expressed as

$$V_{LOST} = W_i(h + \delta_i). \tag{5}$$

The work done by the static equivalent force is not as evident as the potential energy lost by the element. The work is made of two parts, which is related to the total deflection of the element (Budynas, 1977). The area under the curve in Figure 2.2 is the work done by the static equivalent load. This can be expressed by

$$w_{\text{STATIC EQUIVALENT}} = \frac{1}{2}F_i\delta'_i + F_i\delta''_i. \tag{6}$$

Substituting (2) and (4) into (6) yields

$$w_{\text{STATIC EQUIVALENT}} = \frac{1}{2}F_i^2\hat{\delta}'_{ii} + F_i\sum_j F_i\hat{\delta}''_{ij}. \tag{7}$$

Setting (5) equal to (7) will yield the general equation

$$W_i\left(h + F_i\hat{\delta}'_{ii} + \sum_j F_j\hat{\delta}''_{ij}\right) = \frac{1}{2}F_i^2\hat{\delta}'_{ii} + F_i\sum_j F_i\hat{\delta}''_{ij} \tag{8}$$

where $j \neq i$. This is to be written at each location i where there is to be a static equivalent load solved for. This results in a system of n nonlinear equations in n unknowns. The use of modern software such as Mathcad® or Matlab® is strongly encouraged.

2.2 Example

It will be the objective of this example to use the above method to approximate the stresses and deflections in the falling beam given in Figure 2.3. All length units are in inches.

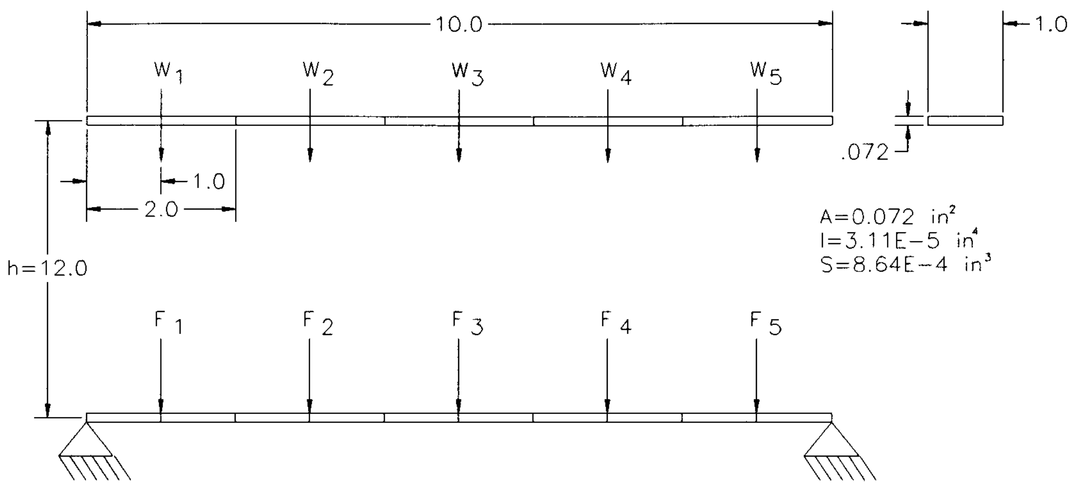

FIGURE 2.3 - EXAMPLE PROBLEM

The following is a step by step illustration of the complete method using a simply supported beam divided into ten finite elements. Using ten elements will allow a static equivalent load to be placed at a node, since a node is required for both the application of a concentrated load, and to obtain a deflection. This will yield a solution for five forces, since there are two shared finite elements for each location *i*. The beam's characteristics can be given as follows:

$E = 30E6$ psi (mild steel)
$\nu = 0.3$
$W_i = 0.0408$ lb

The first step is to model the beam with a suitable FEA package using beam elements and suitable boundary conditions, as shown in Figure 2.4. Place a one pound load on the beam at each location where a static equivalent force is desired. Be sure that each force is in it's own load case, and solve the model.

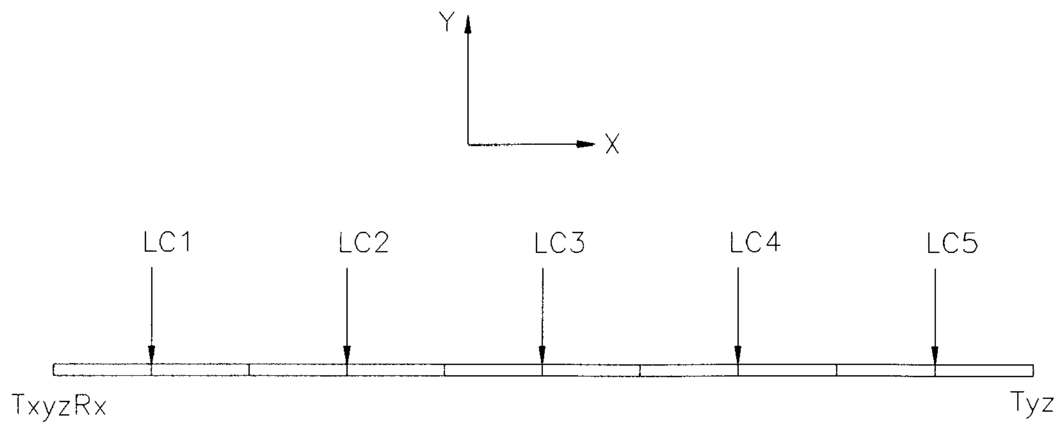

FIGURE 2.4 - PRELIMINARY LOADING OF BEAM

The second step is to obtain the deflections of all load points in each load case, forming a table of values. Table 2.1 shows the values obtained for the given beam. For example, i=2, j=3 gives a value of 0.017685 which is the deflection at i=2 due to a unit force at j=3.

$i\Rightarrow$ $j\Downarrow$	1	2	3	4	5
1	0.002894	0.006252	0.00661	0.004823	0.001751
2	0.006252	0.015756	0.017685	0.013183	0.004823
3	0.00661	0.017685	0.022329	0.017685	0.00661
4	0.004823	0.013183	0.017685	0.015756	0.006252
5	0.001751	0.004823	0.00661	0.006252	0.002894

TABLE 2.1 - UNIT LOAD DEFLECTIONS FOR EXAMPLE PROBLEM (in.)

The third step is to use the above data as coefficients in equation (8). The equations were solved using Mathcad® by setting up a solve block with the five nonlinear equations.

Given

$$W\cdot(h+.002894F1+.006252F2+.00661F3+.004823F4+.001751F5)=.5\cdot F1^2\cdot.002894+F1\cdot(.006252F2+.00661F3+.004823F4+.001751F5)$$

$$W\cdot(h+.006252F1+.015756F2+.017685F3+.013183F4+.004823F5)=.5\cdot F2^2\cdot.015756+F2\cdot(.006252F1+.017685F3+.013183F4+.004823F5)$$

$$W\cdot(h+.00661F1+.017685F2+.022329F3+.017685F4+.00661F5)=.5\cdot F3^2\cdot.022329+F3\cdot(.00661F1+.017685F2+.017685F4+.00661F5)$$

$$W\cdot(h+.004823F1+.013183F2+.017685F3+.015756F4+.006252F5)=.5\cdot F4^2\cdot.015756+F4\cdot(.004823F1+.013183F2+.017685F3+.006252F5)$$

$$W\cdot(h+.001751F1+.004823F2+.00661F3+.006252F4+.002894F5)=.5\cdot F5^2\cdot.002894+F5\cdot(.001751F1+.004823F2+.00661F3+.006252F4)$$

$$\mathrm{Find}(F1,F2,F3,F4,F5)=\begin{bmatrix}7.151\\2.789\\2.282\\2.789\\7.151\end{bmatrix}$$

Once the static equivalent loads are obtained, the last step is to place them on the FEA model and run it. Figure 2.5 shows the loads placed on the model, and Figure 2.6 shows the deformed geometry with the bending stresses shown.

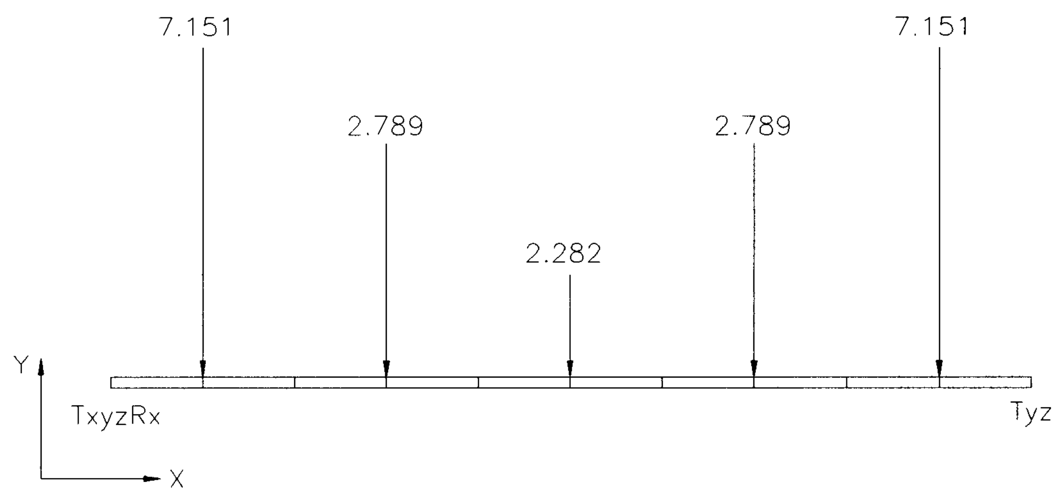

FIGURE 2.5 - STATIC EQUIVALENT LOADING (lb.)

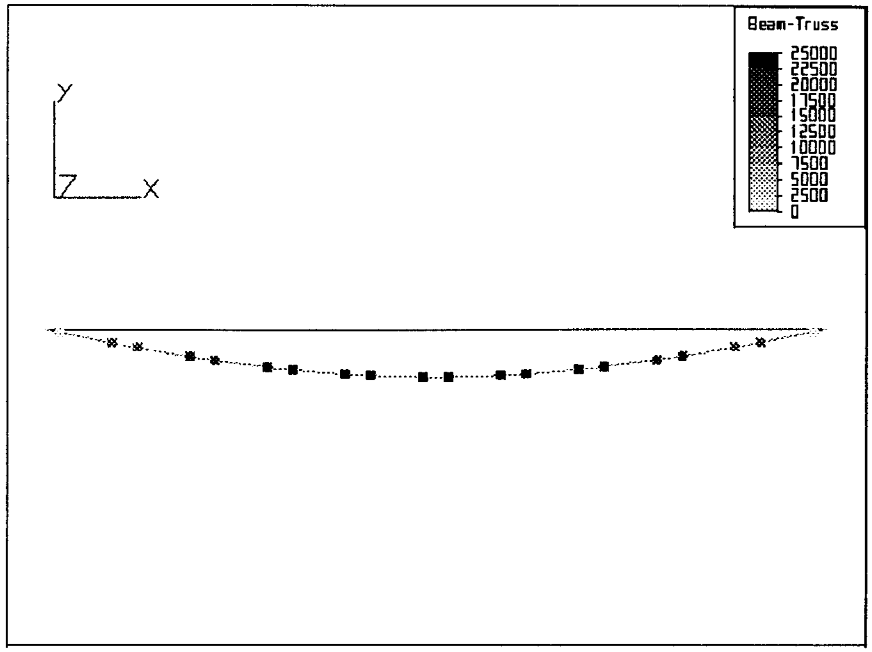

FIGURE 2.6 - BENDING STRESSES (psi) FOR EXAMPLE PROBLEM

It should be noted that the solution process can become very tedious as the number of desired forces increases. Equation writing becomes cumbersome and very time consuming, but it could be the only available option an engineer has to effectively study an impact problem.

2.3 Results

A study similar to the previous example was performed in order to supply data for experimental verification. The same beam was used and dropped from heights of 8", 10", and 12". The major difference between the previous example and this investigation was the number of forces required. Ten forces were desired to have a better distribution of static equivalent forces. The following graphs and tables show the load and strain distributions across the beam. Strain information was used in order to more easily correlate the data with the experimental results. Discussion of results will follow the experimental analysis.

FEA Load and Strain Results

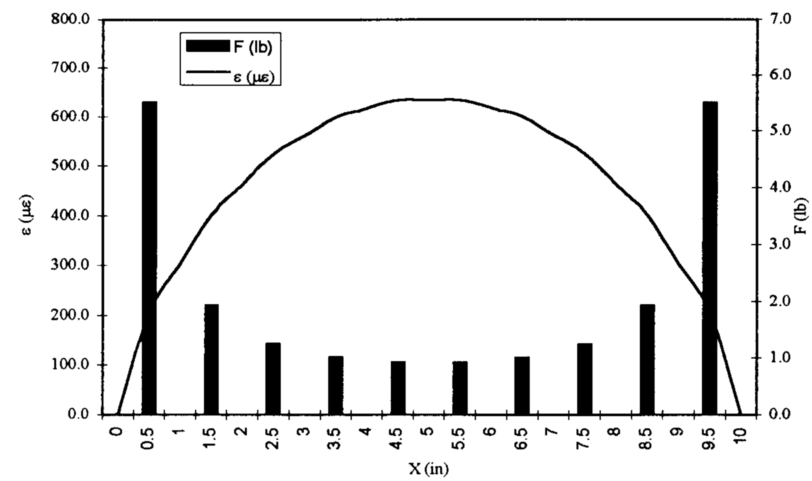

FIGURE 2.7 - 8" HEIGHT

FEA Load and Strain Results

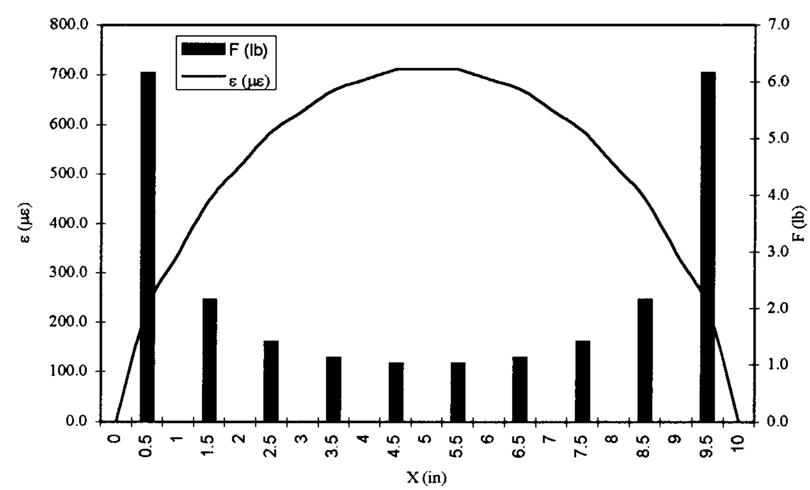

FIGURE 2.8 - 10" HEIGHT

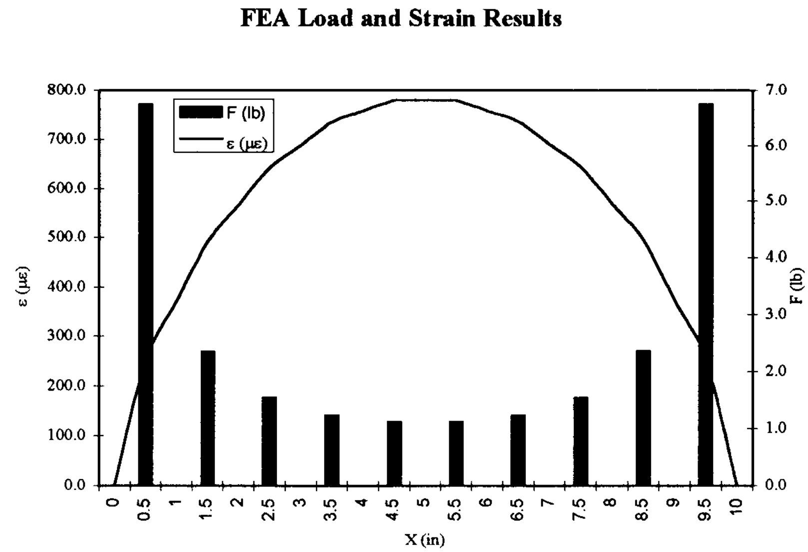

FIGURE 2.9 - 12" HEIGHT

X (in)	8" Height	10" Height	12" Height
0.5	5.504	6.156	6.746
1.5	1.926	2.153	2.358
2.5	1.26	1.408	1.541
3.5	1.015	1.133	1.239
4.5	0.923	1.03	1.127
5.5	0.923	1.03	1.127
6.5	1.015	1.133	1.239
7.5	1.26	1.408	1.541
8.5	1.926	2.153	2.358
9.5	5.504	6.156	6.746

TABLE 2.2 - STATIC FORCES (lb.)

X (in)	8" Height	10" Height	12" Height
0.0	0.0	0.0	0.0
0.5	205.0	229.2	251.0
1.0	303.9	339.7	372.0
1.5	402.7	450.0	492.7
2.0	464.3	519.0	568.0
2.5	526.0	587.7	643.3
3.0	563.3	629.3	689.0
3.5	601.0	671.3	734.7
4.0	618.7	691.0	756.3
4.5	636.3	711.0	778.3
5.0	636.3	711.0	778.3
5.5	636.3	711.0	778.3
6.0	618.7	691.0	756.3
6.5	601.0	671.3	734.7
7.0	563.3	629.3	689.0
7.5	526.0	587.7	643.3
8.0	464.3	519.0	568.0
8.5	402.7	450.0	492.7
9.0	303.9	339.7	372.0
9.5	205.0	229.2	251.0
10.0	0.0	0.0	0.0

TABLE 2.3 - NODAL STRAINS ($\mu\varepsilon$)

3. EXPERIMENTAL ANALYSIS

3.1 Apparatus

The apparatus used for experimental verification of the above results is shown in schematic form in Figure 3.1. The basic setup consisted of a computer/data acquisition system taking strain readings from the beam as it impacted on simple supports on the test fixture. Each component will be shown, giving details necessary to repeat the experiment.

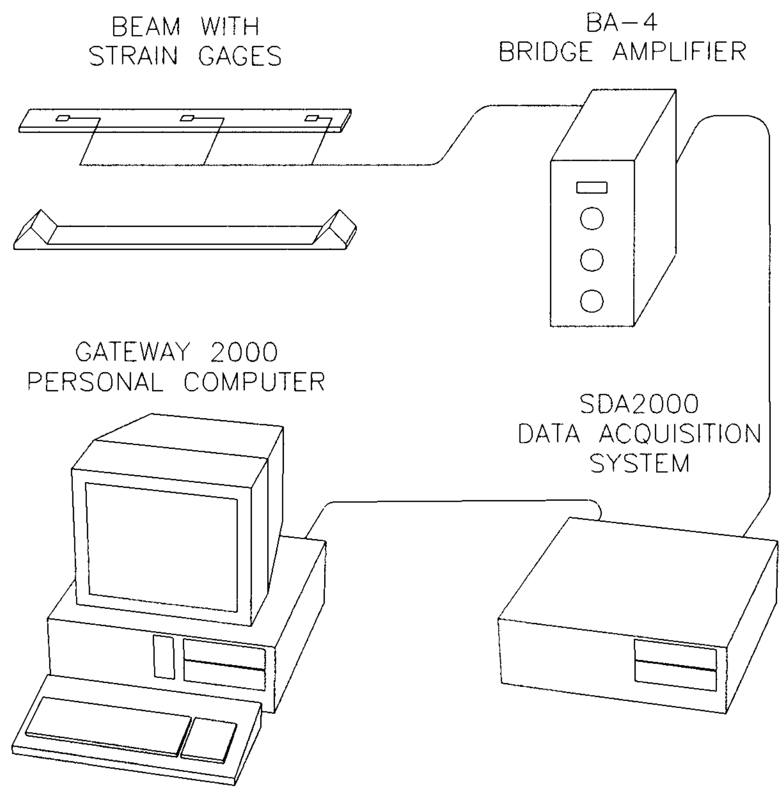

FIGURE 3.1 - EXPERIMENTAL APPARATUS

The beam being dropped was made of mild steel and its dimensions, strain gage locations, and channels are given in Figure 3.2. Length dimensions are in inches.

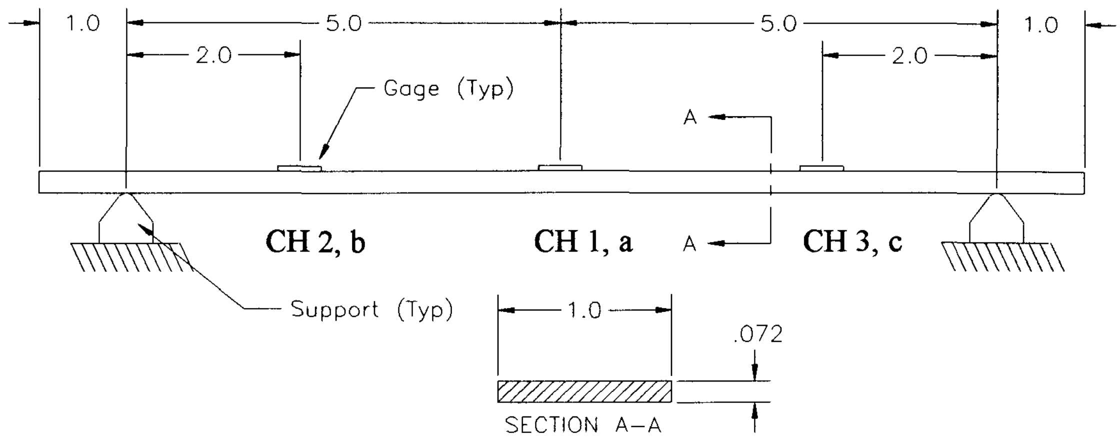

FIGURE 3.2 - DETAILS OF BEAM

The fixture assembly is shown in Figure 3.3. Detailed drawings of the assembly are given in the appendix. The fixture operates in a manner such that the beam will rest on a length of monofilament fishing line stretched over the vertical poles. When the line is cut, the beam will fall onto the supports below. Notches were turned on the poles starting at 8" from the supports and increasing by 1" thereafter up to 12". The fishing line was tied into a loop to a length that would allow it to be stretched enough to keep the beam from sagging when it was placed on the line. The line used was 8 lb. test Berkley® Trilene XL®. The plate that the vertical poles are attached to is not physically connected to the plate on which the supports lie. This prevents vibrations being transmitted to the supports from the vertical poles as the line is cut.

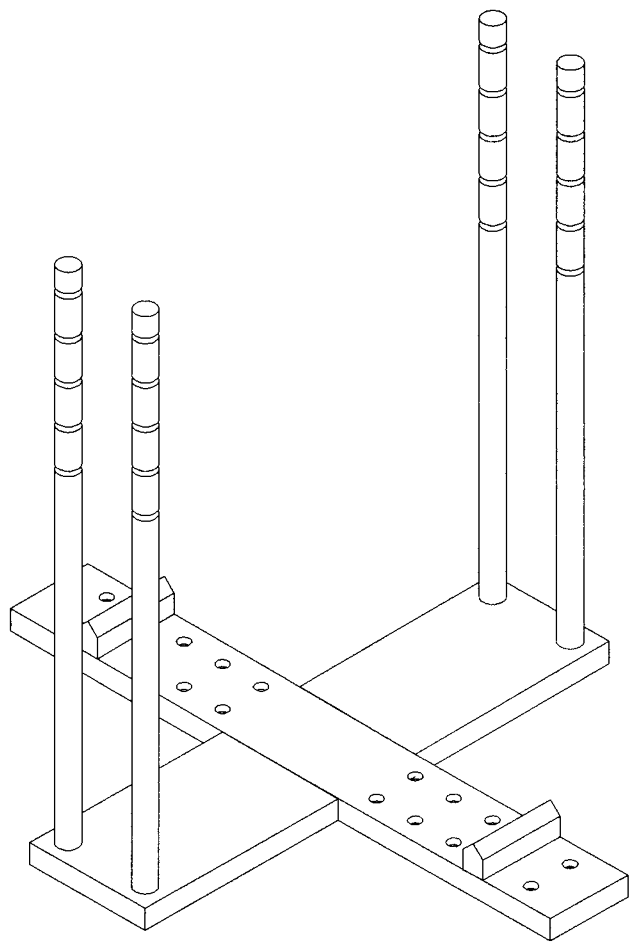

FIGURE 3.3 - TEST FIXTURE

The strain gages used were Measurements Group, Inc. type CEA-06-240UZ-120. Table 3.1 gives all of the specifications. The gages were bonded to the beam as shown in Figure 3.2 using M-Bond 200 cement according to the procedure outlined in the Measurements Group, Inc. Bulletin 309C. The leadwires were soldered in place and the gage was coated with M-Coat A to provide a rubberized, protective seal. The wires were 30 gage, single strand, nickel plated copper conductors in a twisted pair arrangement. The wires were six feet long to allow enough slack for the drop. The small diameter and light weight of the wires also helped to minimize their effect on the beam.

Gage Type	CEA-06-240UZ-120
Resistance (Ω)	120.0±0.3%
Gage Factor	2.05±0.5%
K_t	(+0.8±0.2)%
Lot Number	R-A38AD639
Code	991117

TABLE 3.1 - STRAIN GAGE INFORMATION

Three Ellis Associates Model BA-4 Bridge Amplifiers were used to connect the strain gages to the data acquisition system. Typical connections are shown in Figure 3.4, and all pertinent information regarding the amplifiers is given in Table 3.2. The amplifiers have a frequency range from 0 to 10,000 Hz.

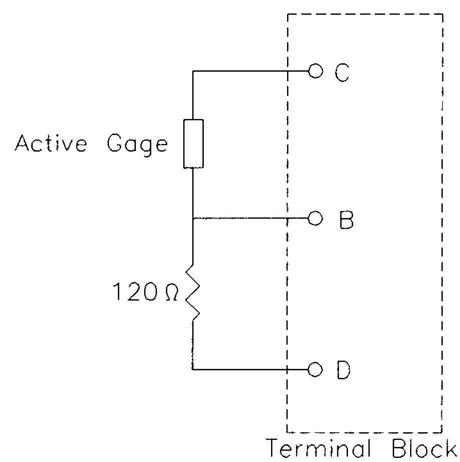

FIGURE 3.4 - GAGE HOOKUP

Calibration Switch Setting	5
Gain	16
Bridge Current	15 ma
Serial Numbers	3465, 3467, 3470

TABLE 3.2 - BA-4 SETTINGS

A Soltec Industries SDA 2000 Transient Waveform Recorder was used to capture the voltage readings from the amplifiers. The SDA 2000 software was used on a Gateway 2000 486DX-33 Personal Computer. All set up information for the SDA is provided in

Table 3.3. The SDA is capable of sampling at most four channels at a rate of 800 ns per reading or a single channel at a rate of 100 ns per reading.

Recorder Memory Segment Controls Setup			
Multiplexer Mode		4 Channels	
Multiplexer Speed		Low X-Talk	
Channel Segment Size		131,072 Samples	
Recording Mode		Single Segment	
Timebase Setup/Segment			
Sampling Interval		1 µs	
Pre-trigger Time		6.552 ms	
Timebase #1		124.6 ms	
Segment Total		131.1 ms	
Channel Pre-amplifier Setup			
	Range (V)	Lower Level (V)	Upper Level(V)
Channel 1	5.0	-2.50	2.50
Channel 2	5.0	-2.50	2.50
Channel 3	5.0	-2.50	2.50
Recorder Trigger Setup			
Source		Channel 1	
Coupling		DC	
Trigger Condition		-Slope/Low Limit	
Upper Trigger Limit		200.0 mV	
Lower Trigger Limit		-200 mV	

TABLE 3.3 - SDA 2000 SETTINGS

3.2 Procedure

The apparatus was connected and configured according to the above figures, tables, and manufacturer's instructions. First, it was necessary to obtain calibration constants from the bridge amplifiers. The calibration strain value can be found from the following equation:

$$\frac{400}{\mathrm{GF}} \times \frac{\text{Calibration Switch Setting}}{\text{\# Active Arms}} = \text{Calibration } \varepsilon \,. \qquad (9)$$

For this setup, the number of active arms was 1, the switch was set to 5, and the gage factor was 2.05. This yielded a calibration strain of 975.6 $\mu\varepsilon$. Each channel had to be calibrated independently, and constants were found to convert voltage to strain. Table 3.4 lists average calibration constants for the three channels used.

Channel 1	503.7
Channel 2	579.9
Channel 3	507.5

TABLE 3.4 - AVERAGE CALIBRATION CONSTANTS IN $\mu\varepsilon$/V

In order to drop the beam successfully, the tension in the fishing line had to be great enough so that the beam would not cause the line to sag, and not so large to cause preliminary vibrations in the beam. For a drop to be qualified as successful, the traces of the gages in channels 2 and 3 (refer to Figure 3.2) had to be very similar and in phase. This indicated that both ends of the beam struck their respective support at the same time. It should be noted that there was difficulty in obtaining successful drops. The beam was then dropped from heights of 8", 10", and 12".

The captured data was converted to strain using the SDA software and manipulated further. The mean and RMS values of the trace could be found between two markers. Other options available were finding the maximum and minimum values, cycle frequency, cycle period, and number of cycles. Screen dump output of the traces captured are presented in the next section.

3.3 Results

The following figures and tables present the experimental data taken from dropping the beam. The graphs include captured data that has been converted to strain.

The curves show the real time data taken, which show the high frequency vibrations that occur during the impact. The RMS value was found in an interval that contained the maximum absolute values of the strains.

The graphs that show three traces have the reading from the middle strain gage first, followed by the left and right end, respectively, as "a", "b", and "c" data traces. The graphs that contain only one trace are the strain readings for the center gage, or "a" data showing the RMS value. The RMS values were found over a small interval containing the peak of the curve. At least three cycles of high frequency vibrations were contained in this interval. The average RMS values for the middle and end gages are given in Table 3.5.

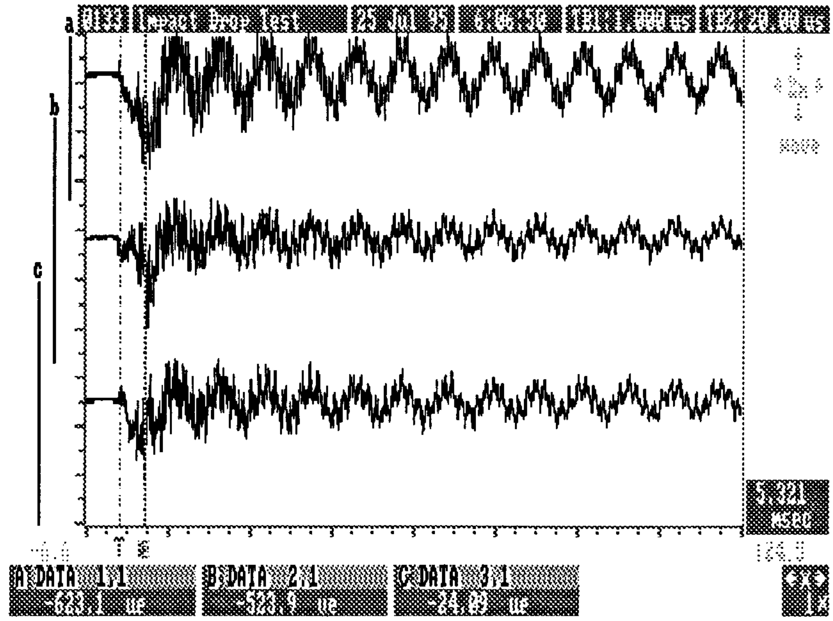

FIGURE 3.5 - STRAINS FROM 8" HEIGHT (με)

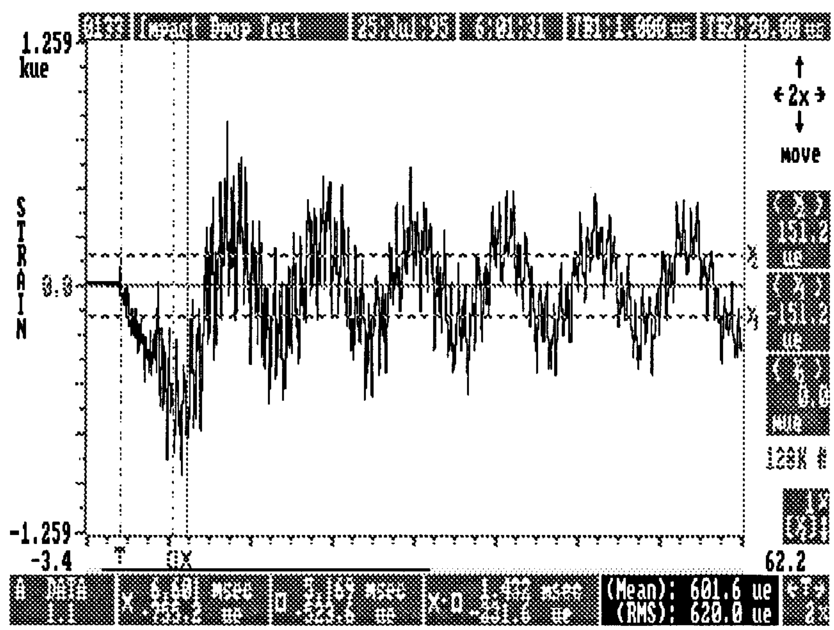

FIGURE 3.6 - "A" DATA STRAINS FROM 8" HEIGHT (με)

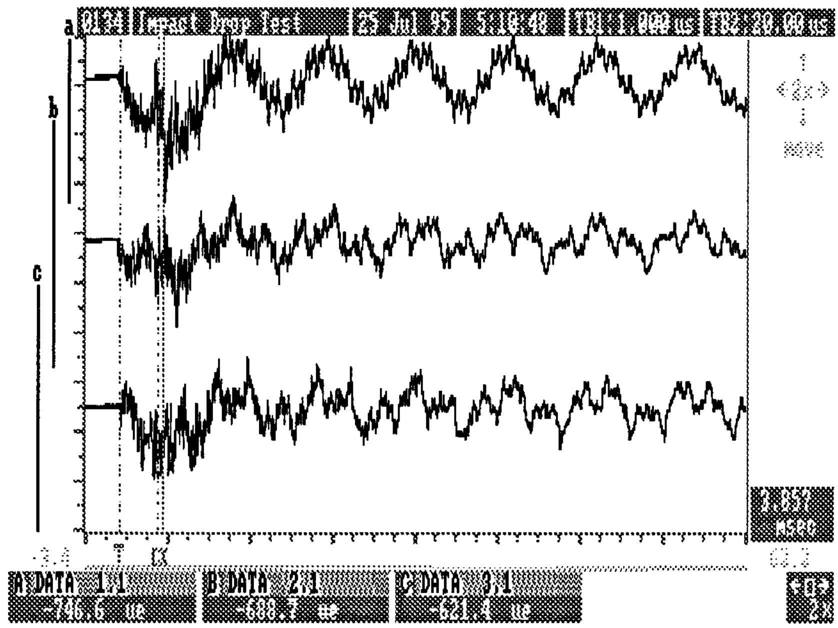

FIGURE 3.7 - STRAINS FROM 10” HEIGHT ($\mu\varepsilon$)

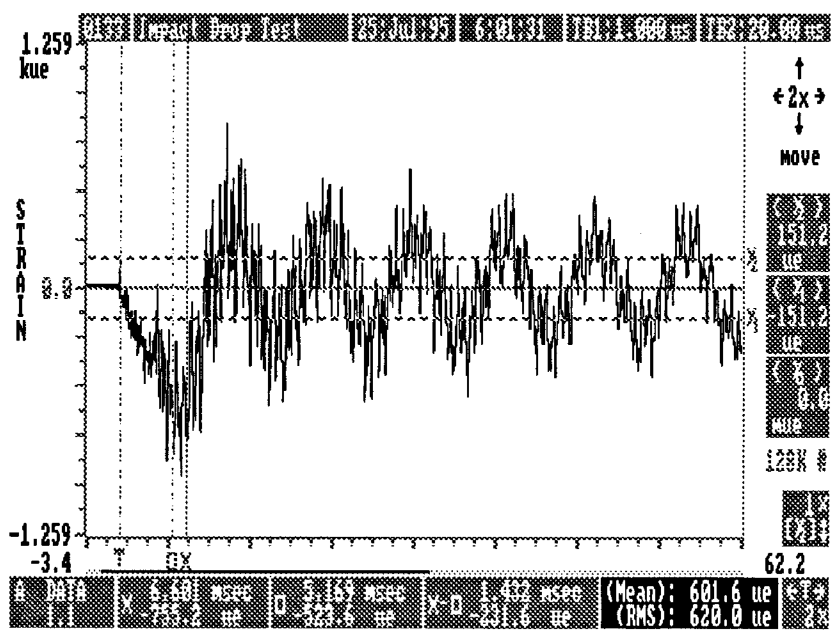

FIGURE 3.8 - “A” DATA STRAINS FROM 10” HEIGHT ($\mu\varepsilon$)

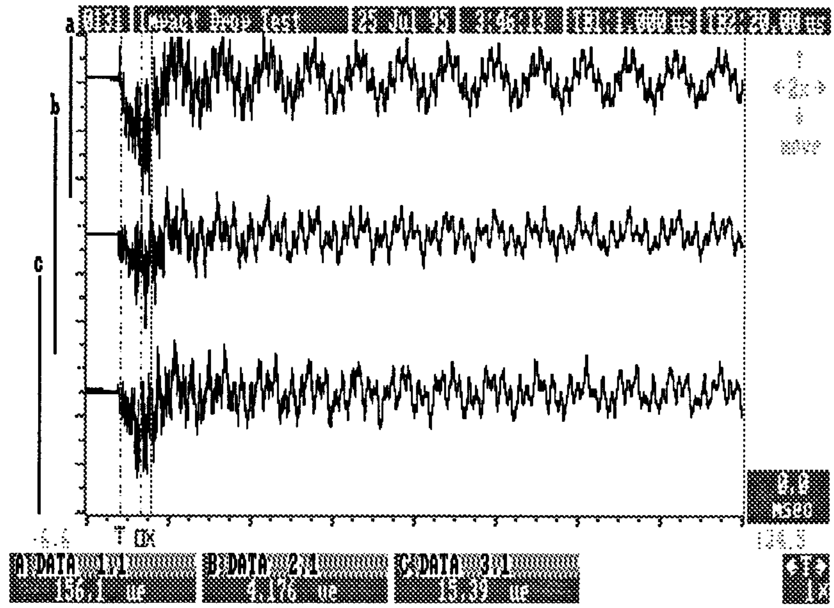

FIGURE 3.9 - STRAINS FROM 12" HEIGHT ($\mu\varepsilon$)

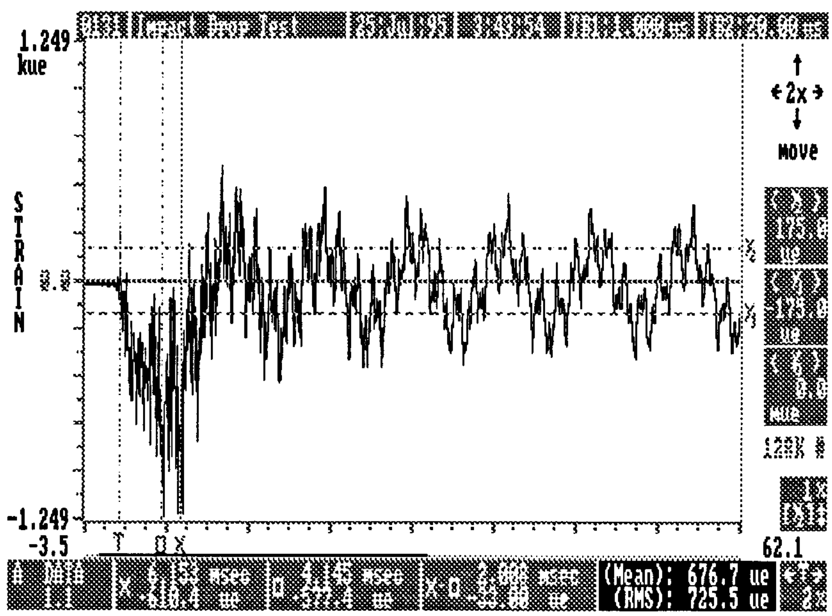

FIGURE 3.10 - "A" DATA STRAINS FROM 12" HEIGHT ($\mu\varepsilon$)

Height	Middle Gage	End Gage
8"	610	449
10"	700	505
12"	730	540

TABLE 3.5 - AVERAGE RMS STRAIN VALUES ($\mu\varepsilon$)

4. THEORETICAL AND EXPERIMENTAL COMPARISON

Table 4.1 shows the comparison of the results obtained in both the experimental and theoretical analyses. It can be seen that the experimental results agree very well with the theoretical results. It should be noted that the experimental values are smaller than the theoretical values. This can be attributed to the fact that not all of the energy from the drop was dissipated through the beam, i.e. there was energy loss through the supports. This violation of an assumption can be justified by the idea that the theory will predict values that are somewhat greater than the actual values, being a conservative estimate, which contributes to the method.

RMS Strains: Middle Gage ($\mu\varepsilon$)			
Drop Height	8"	10"	12"
Experimental	610	700	730
Theoretical	636	711	778
% Difference	4.14%	1.55%	6.21%
RMS Strains: Average of End Gages ($\mu\varepsilon$)			
Experimental	449	505	540
Theoretical	464	519	568
% Difference	3.30%	2.70%	4.93%

TABLE 4.1 - STRAIN RESULTS COMPARISON

The difficulty in dropping the beam can lead to problems interpreting the results. The experimental setup used had problems with the strain gage leadwires affecting the fall of the beam. Minimization of those effects were assumed with the use of the small diameter wire. If further studies in this area are conducted, it is recommended to use a different means of dropping the beam and obtaining strain readings. A wireless setup would undoubtedly be better. Nevertheless, as more good drops are obtained, the reliability of results increases. In this case, though, there is confidence that the results are accurate.

5. DISCUSSION

The use of this method could be practical for many simplified impact analyses. There are a few drawbacks to the method, but for the engineer without sophisticated software, the practicality is evident. However, the method could be improved to make it easier to study more complex structures.

5.1 Limitations

One concern is how to handle the instance of a statically unstable impact. For example, if a chair were dropped and one leg contacted the ground first, it is likely that a rotation will occur. Static equivalent forces can not be found the same way, because a static system no longer exists. This is an area that may deserve further study.

This method is also limited by the assumptions made, especially the assumption of linearity. For example, a design engineer wishes to study the effect of adding a viscoelastic damping material to protect a dropped object from impact loads. Since most of these types of materials have nonlinear force-deflection curves, the work done by the static equivalent force will be only approximate. The force-deflection curve may be linearized to obtain approximate results. If the material deflects outside of the linearized range, the process can be iterated to closer approximate a spring constant for the material. However, if linear springs are used in place of the viscoelastic material, the springs can be easily added to the model and the analysis should be accurate.

Another concern is the magnitude of the static equivalent force near the supports. The results obtained above will not be correct near the supports, but are valid away from the supports. True contact forces and stresses can not be found. In models that have members with large spans, the worst stresses will occur away from the supports in those spans. Providing that the contact stresses do not cause yielding, the method should prove reasonably accurate.

This formulation was intended for the study of the objects that are dropped, and not impacted by a dropped object or projectile. Those cases are simpler due to the fact that the number of static equivalent forces can be reduced to one concentrated load, as studied in engineering texts, such as Spotts' Design of Machine Elements (1985).

5.2 Future Applications

Beam elements have been the main focus of this study. If an FEA modeler had to use plate elements instead of beam elements, the user would have to modify the method very little. Groups of four plate elements can be used in the same manner that two beam elements were used. This places a node at the center of the large "impact element". The problem can be solved normally, and a beam's distributed loading becomes a plate's pressure loading. This is not necessary if the FEA package of choice allows the user to apply concentrated loads at points other than nodes.

An adaptation to perform analyses on horizontal impacts could be developed. The case of a body colliding with a stationary object could be investigated using the same energy technique. The major difference would be substituting a change in kinetic energy for a change in potential energy for the body. The development could proceed along the same lines as the previous one. The method would be straightforward for pure translation. If rotations were involved, the same problem with statically unstable impacts would be present.

The structure of the equations used to solve for the F_i's, (8) would lead one to believe that there would be a way to write them in matrix format. This was investigated using the flexibility matrix, [a]. Placing equation (8) in matrix format will eventually yield

$$\vec{W}\left\{h[I]+[a]\vec{F}\right\}=\vec{F}\left\{\left([a]-\frac{1}{2}\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & a_{nn}\end{bmatrix}\right)\right\}, \tag{10}$$

where $\vec{W}$ and $\vec{F}$ are the weight and static equivalent force vectors, respectively, and [I] is the identity matrix. The large *n* by *n* matrix on the right hand side is part of the flexibility matrix, containing only the main diagonal. At the present time, an algorithm for this particular solution has not been found. Once a solution is available, the method will become more efficient, as the complete equations will not have to be entered into a package such as Mathcad®. It is not intended to mislead the reader that Mathcad® is an unfavorable choice to solve the equations. On the contrary, it is well suited to solve the nonlinear equations.

It may be possible for a solution algorithm, as discussed above, to be incorporated into a commercially available FEA code. This being the case, the process could be automated. Once the model were created and the static load points defined, the static equivalent loading could be applied by the software, and the model could then be solved. This yields one processing stage, thus eliminating chances for human errors. This may be redundant with impact analysis software available, but could possibly be developed at a lower cost than other advanced, non-linear software if there were a demand.

5.3 Conclusion

The analysis technique developed in this study has the potential to open doors to answers that were locked either by financial or technical restrictions. Today's engineers have recently started to simulate real life with great accuracy. This causes greater economic pressures to optimize designs for cost effectiveness, and vice versa. Even though the method is not exact, an engineer can expose himself or herself to a wider range of analyses. Although this is not the most glamorous use of software for impact analysis, it can get the job done to predict the worst when objects fall.

6. REFERENCES

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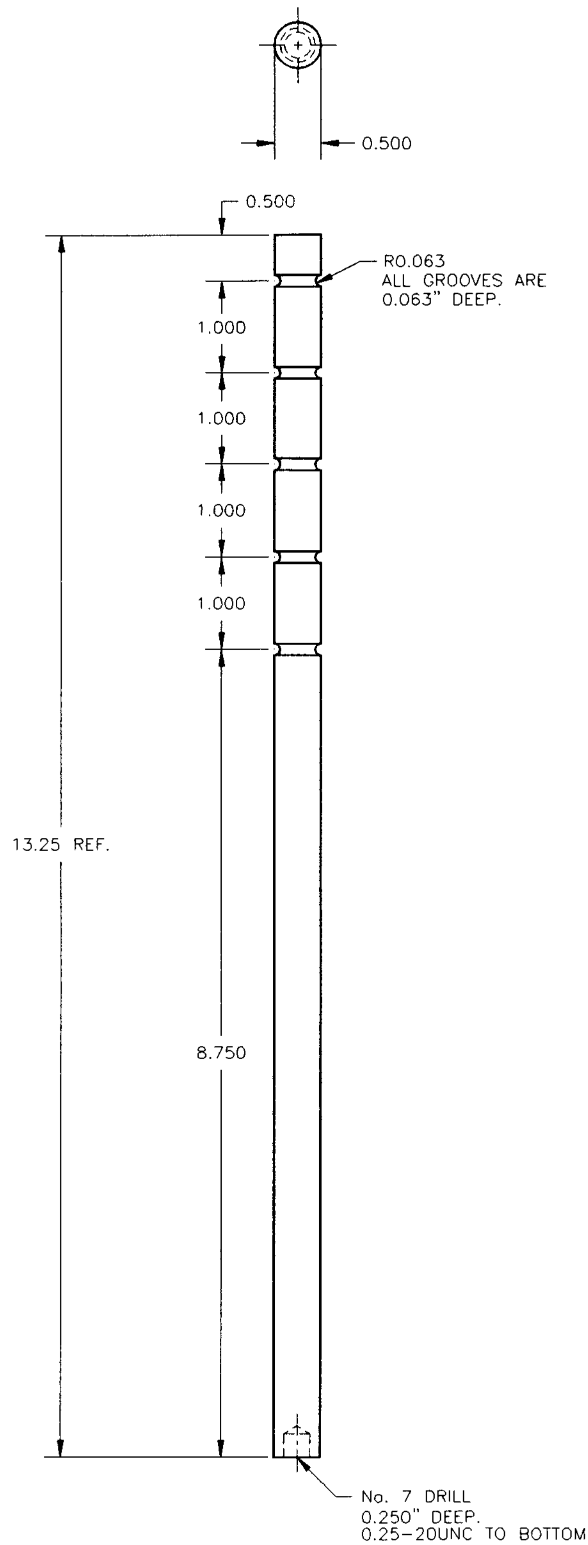
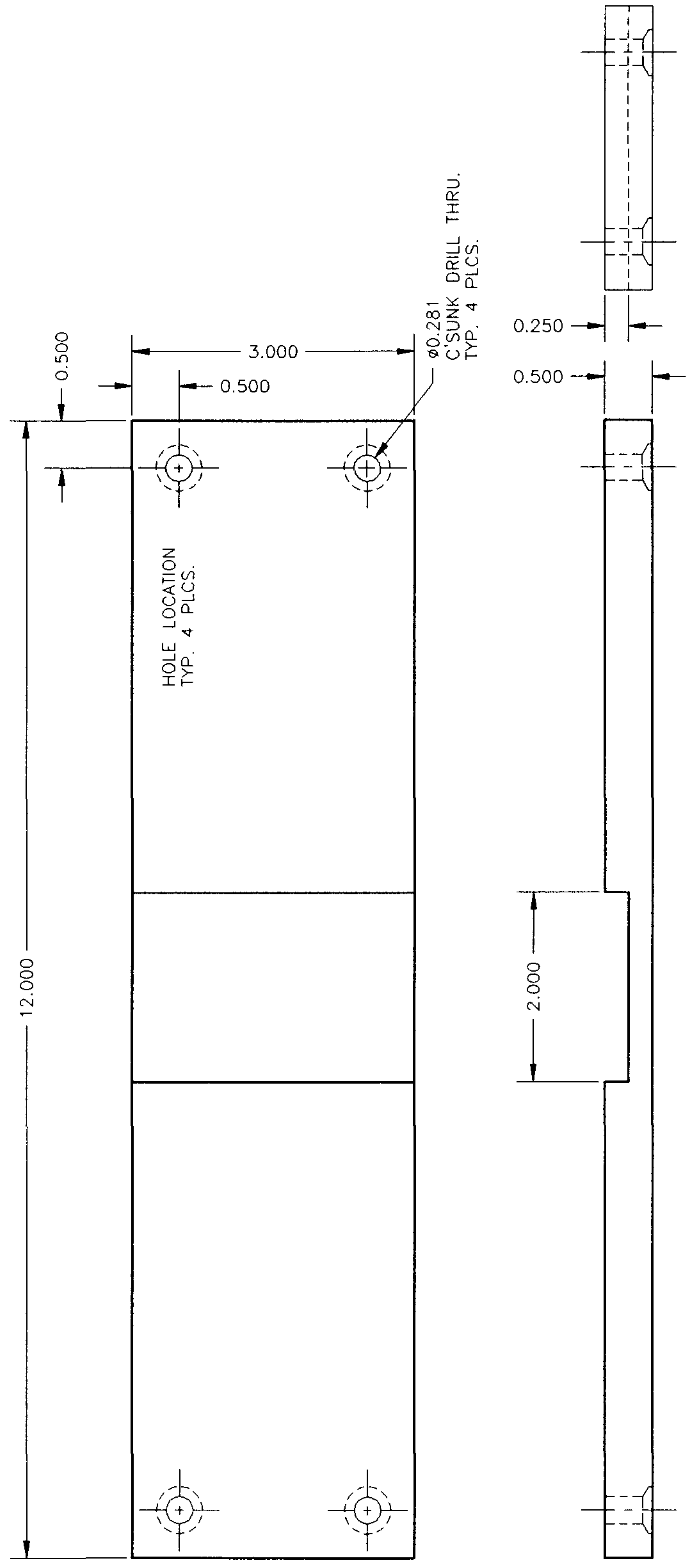
7. APPENDIX

Part #	Description	Mat'l	# Req'd
1	Vertical Post	Aluminum	4
2	Base Plate 1	Aluminum	1
3	Base Plate 2	Aluminum	1
4	Support	Steel	2

NOTE:
1. DRAWINGS ARE NOT TO SCALE.
2. LENGTH DIMENSIONS ARE IN INCHES.
3. GENERAL TOLERANCE IS ±0.010" FOR LINEAR DIMENSIONS AND ±0.5° FOR ANGLES.

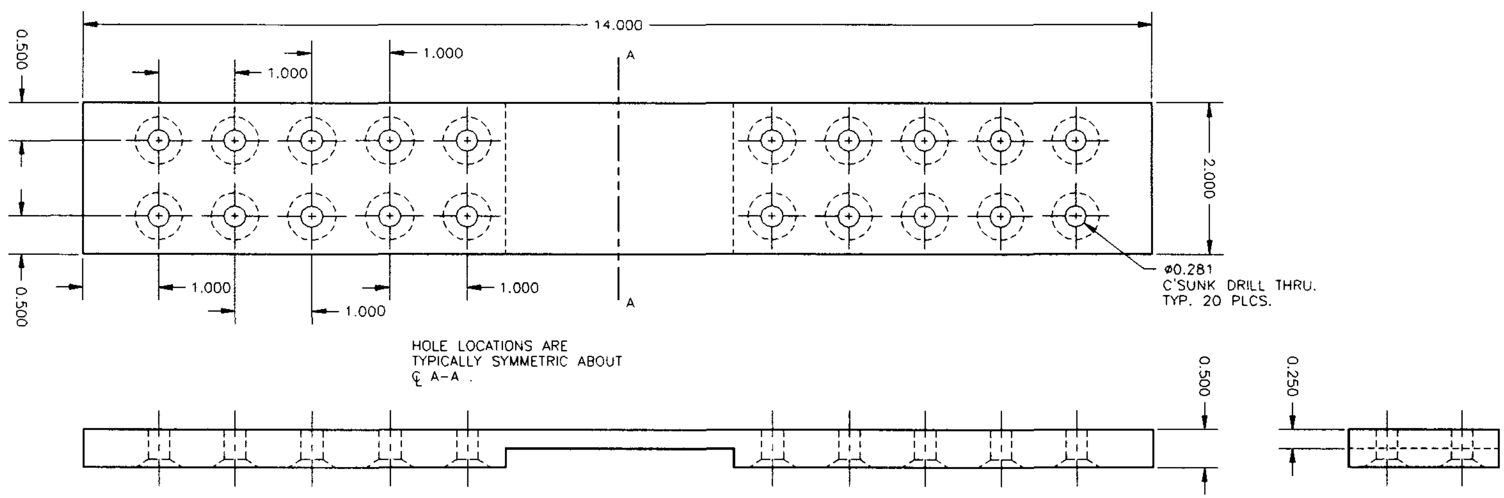


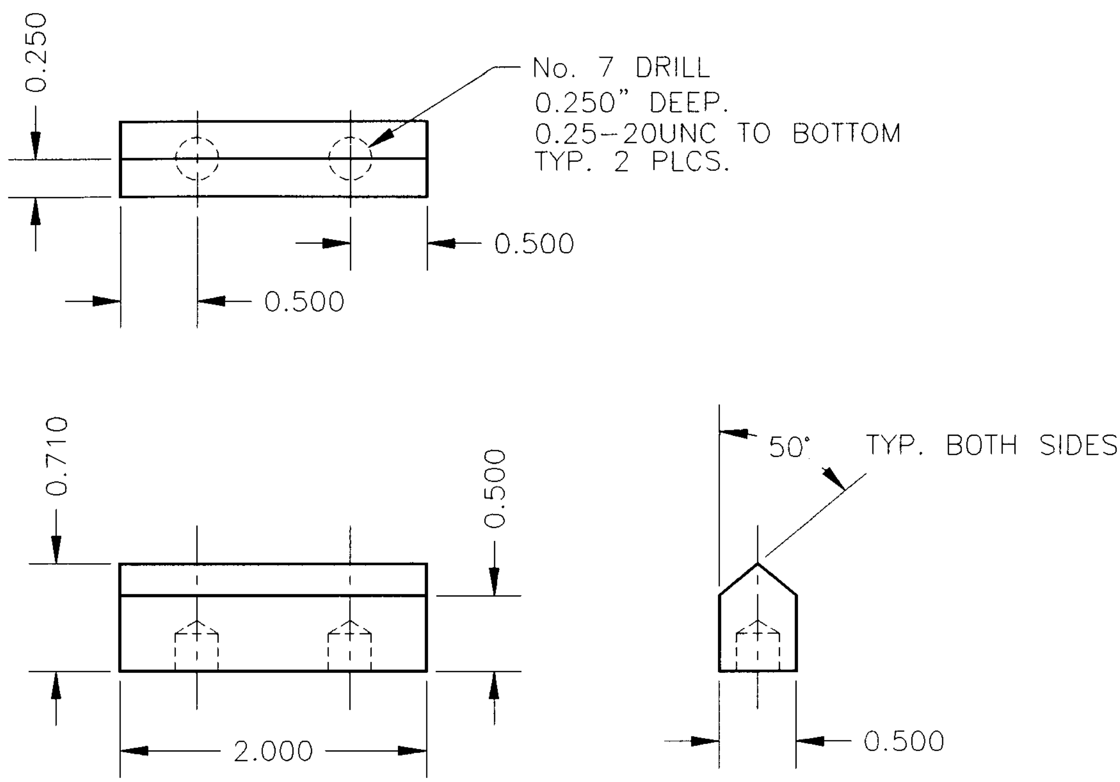
PART 1 - VERTICAL POST



PART 2 - BASE PLATE 1

PART 3 - BASE PLATE 2





PART 4 - SUPPORT